

## FREQUENCY RESPONSE

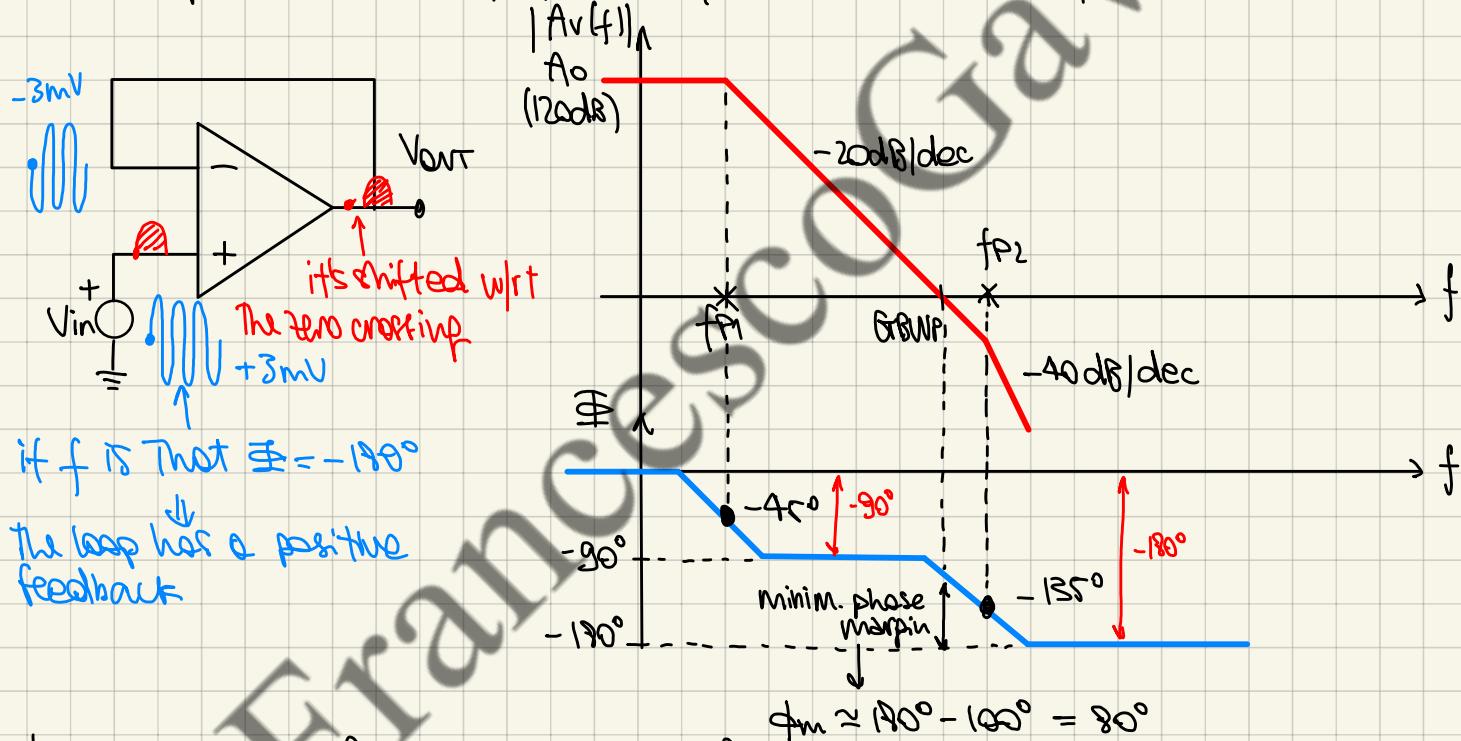
The frequency response study is a small-signal study. That is a study that analyzes components when they deviate a bit from their operating point and do not go out of their linear operation range, reaching the saturation or even the w/o off.

Opamps cannot operate w/o an open loop, but they have to be used in a negative FB configuration w/ the output signal coming back to the inverting OpAmp pin. The "loop closure" leads to a reduction of the OpAmp gain, but at the same time it makes the OpAmp gain more precise, widens the BW, reduces the output impedance raises the input impedance (seen from the positive terminal).

A noticeable disadvantage of the FB is that it may cause the instability of the loop.

### • OPEN-LOOP FREQUENCY RESPONSE

The OpAmp introduces amplification, but also phase shift.

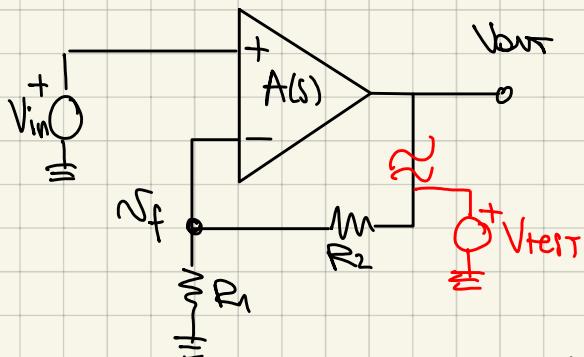


if  $f$  is that  $\angle = -180^\circ$   
↓  
the loop has a positive feedback

$\phi_m$  is the residual phase before the loop becomes positive, and so unstable

Commonly we say that an opamp is compensated if the 2nd pole of its open-loop gain  $A(f)$  is at a frequency higher than the intersection of  $G_{FB}(s)$  and the 0dB axis

## NON-INVERTING STAGE



### 1. FEEDBACK GAIN

$$\beta = \frac{N_f}{V_{test}} = \frac{R_1}{R_1 + R_2}$$

### 2. LOOP GAIN

$$Gloop = -A(s) \cdot \beta = -A(s) \frac{R_1}{R_1 + R_2}$$

\* IDEAL GAIN

3. REAL GAIN  $\rightarrow$  BEWARE (IMP!!): This is valid only for the NI config.

$$G_{NI} = \frac{A}{1 - A\beta} = \frac{A}{1 + Gloop} = \frac{A}{\beta \left( \frac{1}{\beta} - A \right)} = \begin{cases} \text{if } \frac{1}{\beta} \ll A \quad (\text{Gloop} \gg 1) \rightarrow G_{NI} \approx \frac{1}{\beta} \\ \text{if } \frac{1}{\beta} \gg A \quad (\text{Gloop} \ll 1) \rightarrow G_{NI} \approx A \end{cases}$$

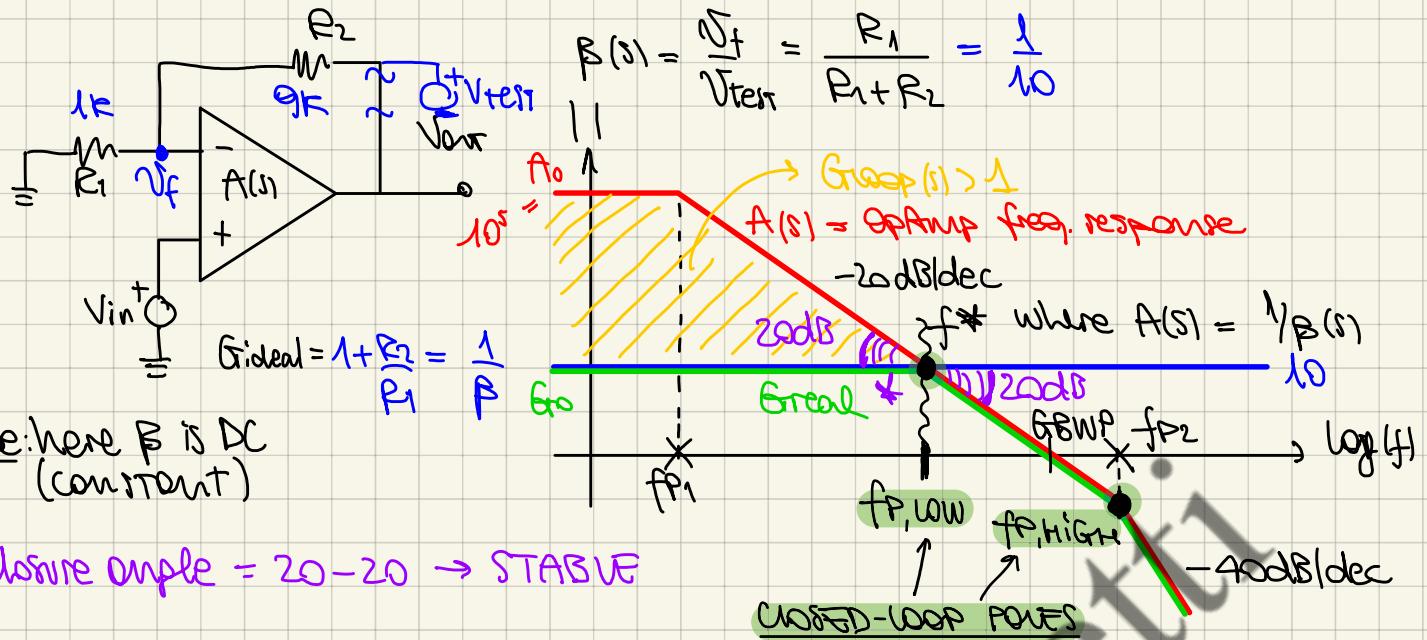
\* We love this gain b/c it's independent of the Opamp itself

## GLOOP ASSESSMENT AND REAL GAIN EXTRACTION

In order to assess  $Gloop$ , to check FB quality and stage stability, and to be able to draw the real closed-loop frequency response and not just the ideal one, do follow these hints:

- ① DRAW  $A(s)$  (the freq. response of the Opamp)
- ② COMPUTE  $\beta(s)$  (No Laplace, just asymptotic analysis @ DC and @ HF)  
and draw  $1/\beta(s)$  (remember: poles of  $\beta(s)$  lift up, zeros of  $\beta(s)$  put down  $1/\beta(s)$ )
- ③ THE SPLIT BETWEEN  $A(s)$  AND  $1/\beta(s)$  IS  $Gloop(s)$ :
  - THE LARGER THE SPLIT (i.e.  $Gloop$ ), THE BETTER THE FEEDBACK QUALITY  
(remember: if  $Gloop \gg 1 \rightarrow G \approx 1/\beta$ : ideal gain)
  - @  $f=f^*$  WHERE  $A(f^*) = 1/\beta(f^*) \rightarrow Gloop(f^*) = 1$
- ④ DRAW THE EXPECTED IDEAL GAIN (don't care about  $A(s)$  nor  $1/\beta(s)$ )
- ⑤ THE REAL CLOSED-LOOP FREQUENCY RESPONSE:
  - follows the ideal one when "there is  $Gloop$ " (i.e.  $Gloop(s) > 1$ )
  - beyond  $f^*$  there is no more feedback, hence the real gain rolls off from the ideal trend, experiencing all following poles and zeros of  $A(s)$
- ⑥ STAGE STABILITY DEPENDS ON THE "CLOSURE ANGLE" BETWEEN  $A(s)$  AND  $1/\beta(s)$  AROUND  $f^*$ 
  - STABLE  $\rightarrow$  20dB/dec before and after  $f^*$
  - MARGINALLY STABLE  $\rightarrow$  20dB/dec before and 40dB/dec after  $f^*$ , or viceversa
  - UNSTABLE  $\rightarrow$  40dB/dec or higher before and after  $f^*$

Let's apply the procedure to the NI stage seen before:

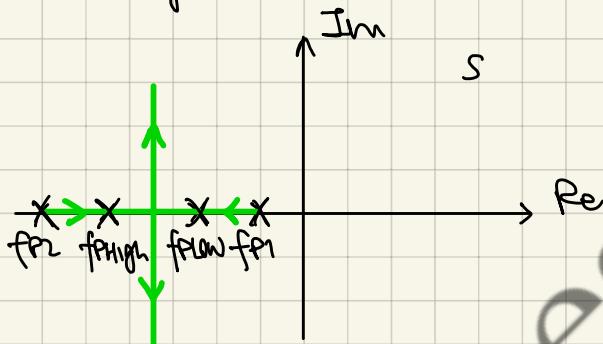


Notice: here  $\beta$  is DC (constant)

\* closure angle = 20 - 20 → STABILE

$$f_{p1} \cdot A_0 = f^* \cdot 10 \rightarrow f^* = f_{p1} \frac{A_0}{10} = 10 \text{ Hz} \cdot \frac{100 \text{ K}}{10} = 100 \text{ KHz}$$

We can study this behavior also using the root locus:



Why does the "closure angle" matter on the stability?

a. Given that the split b/w  $A(s)$  and  $1/\beta(s)$  is  $G_{\text{group}}(s)$

b. The "closure angle" @  $f^*$  measures the slope of  $G_{\text{group}}(s)$  versus freq. @  $f^*$

c. If before  $f^*$  the group experienced:

- 1 pole  $\rightarrow -20 \text{ dB/dec slope @ } f^*$
- 2 poles  $\rightarrow -40 \text{ dB/dec slope @ } f^*$
- $n$  poles  $\rightarrow n \times (-20 \text{ dB/dec}) \text{ slope @ } f^*$
- 2 poles, 1 zero  $\rightarrow -20 \text{ dB/dec slope @ } f^*$
- $p$  poles,  $z$  zeros  $\rightarrow (p-z) \times (-20 \text{ dB/dec}) \text{ slope @ } f^*$
- { 1 pole before  $f^*$  }  $\rightarrow -20 \text{ dB/dec slope before } f^*, -40 \text{ dB/dec slope after } f^*$   
{ 1 pole @  $f^*$  }

d. Each pole (zero) adds a phase shift of  $-90^\circ (+90^\circ)$  to the feedback signal after one decade from it; instead the pole (zero) adds just  $-45^\circ (+45^\circ)$  if the  $f^*$  is coincident w/ the pole (zero) itself

e. Therefore, by measuring the "closure angle" it's possible to infer the difference ( $\phi - \phi_0$ ) and eventually the overall phase shift accumulated along the feedback path.

f. The stage is STABLE if the feedback stays negative and does not accumulate  $-180^\circ$  phase shift, in which case it turns to be positive and the stage is UNSTABLE

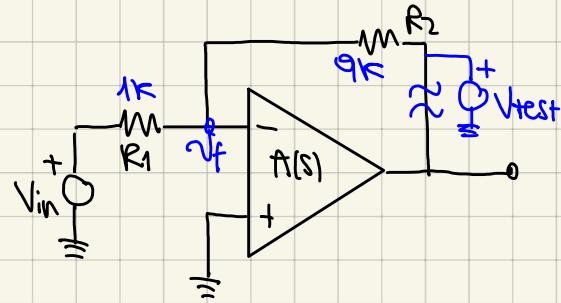
**When loop is poor?**, when the attenuation introduced by the loop is really strong.

Ex: in the N1 stage analyzed so far, loop is poor if  $R_1 = 1\text{k}\Omega$  and  $R_2 = 1\text{M}\Omega$

Let's look at other three examples:

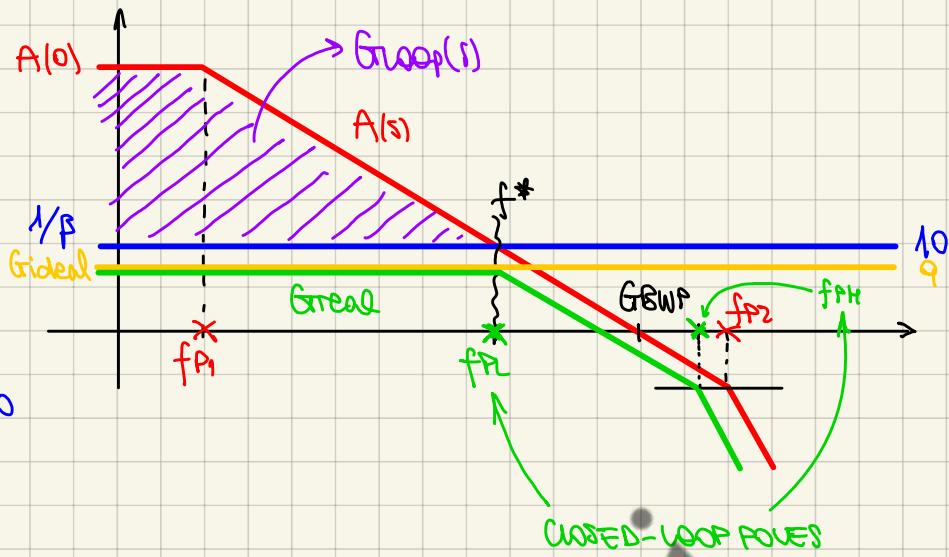
	ROOT LOCUS	CLOSED-LOOP FREQ. RESPONSE	TIME RESPONSE
Loop POOR ( $\frac{1}{\beta}$ STRONG) $\beta$ POOR)	<p>Greal has 2 real separated poles</p>		
Loop MODERATE ( $\frac{1}{\beta}$ DECR.) $\beta$ INCR.)	<p>Greal has 2 real coincident poles</p>		<p>faster than the previous case</p>
Loop STRONG ( $\frac{1}{\beta}$ POOR) $\beta$ STRONG)	<p>Greal has 2 complex conjugated poles</p>		<p>overshootings and undershootings in time</p>

## • INVERTING STAGE



$$\frac{V_T}{V_{\text{test}}} = \beta = \frac{R_1}{R_1 + R_2} = \frac{1}{10} \rightarrow \frac{1}{\beta} = 10$$

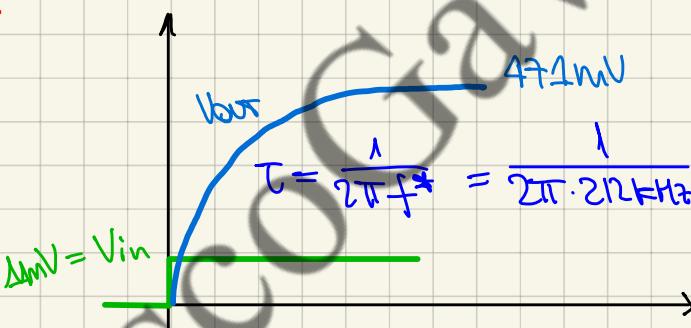
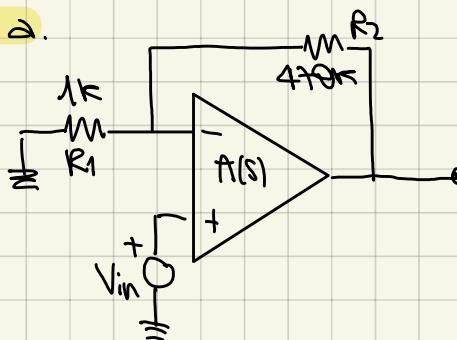
$$|G_{ideal}| = \left| -\frac{R_2}{R_1} \right| = |-q|$$



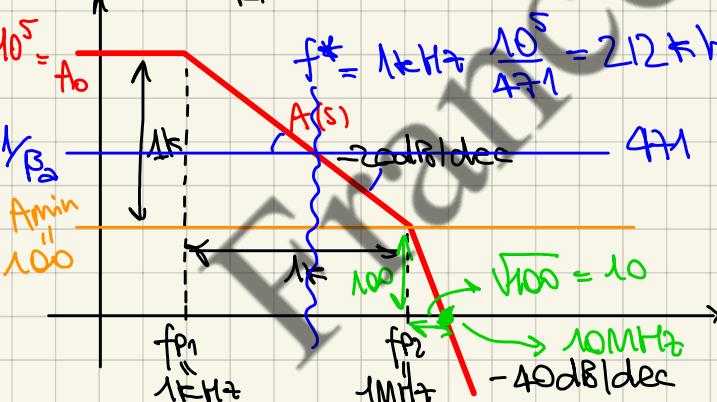
- if  $f_{P2} > f_{GSWP}$  → The OpAmp is "compensated"
  - if  $f_{P2} < f_{GSWP}$  → The OpAmp is "uncompensated"

## "UNCOMPENSATED" OPAMP

## CASE 2.

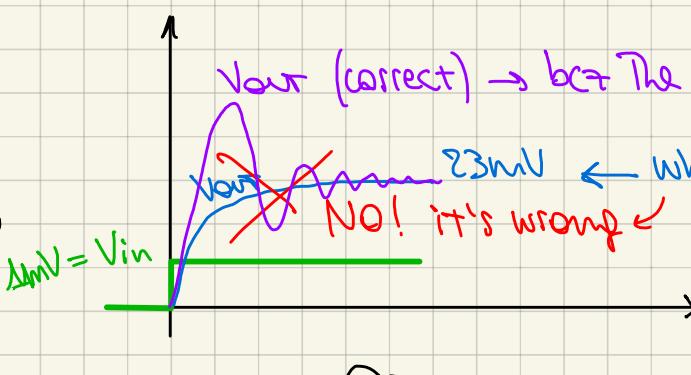
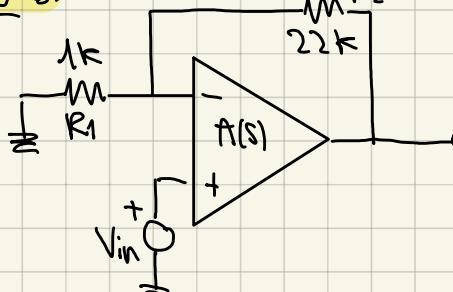


$$\text{Fideal} = 1 + R_2 = 471 \rightarrow \text{Amin} \Rightarrow \text{STAB VE}$$

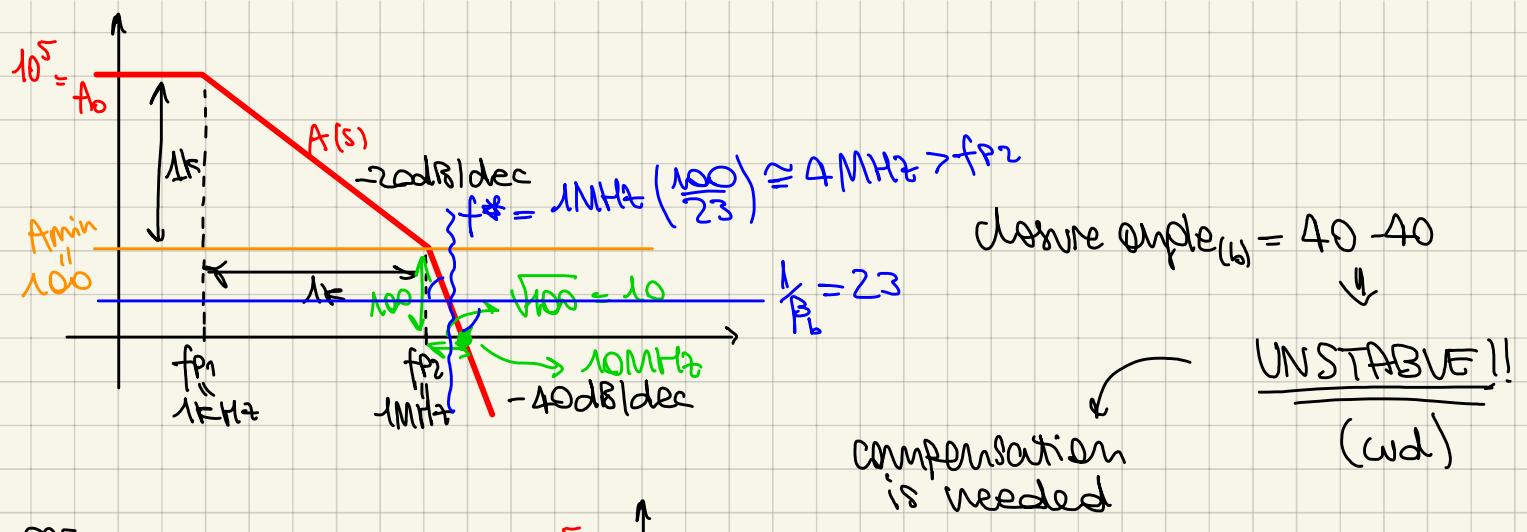


$$\text{Closure Angle} = 2\alpha - \omega \Rightarrow \delta T A B V \bar{v} \quad (\underline{\text{cud}})$$

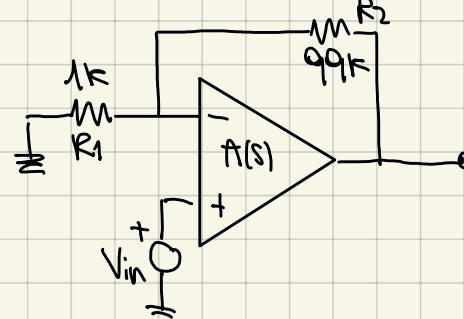
## CASE b.



$$f_{ideal} = 1 + \frac{P_2}{P_1} = 23 < f_{imin} \Rightarrow \text{UNSTABLE}$$

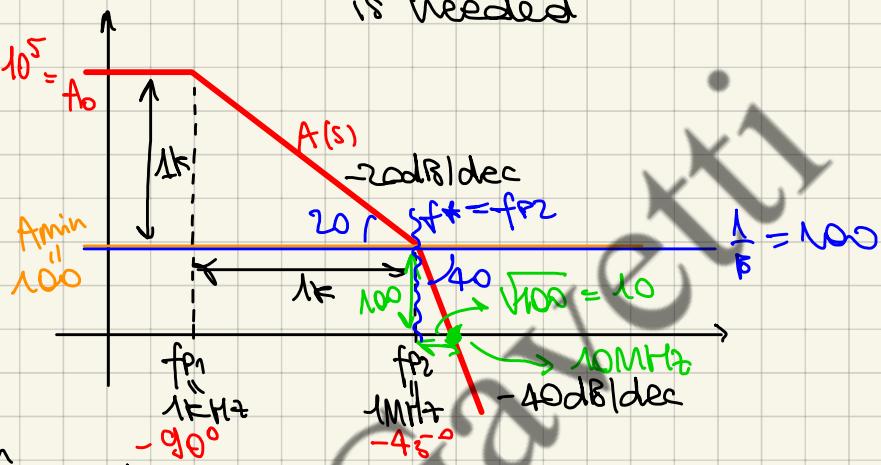


CASE C.



$$G_{ideal} = 1 + \frac{R_2}{R_1} = 100 = A_{min}$$

$$\frac{1}{P_o} = 1 + \frac{R_2}{R_1} = 100$$

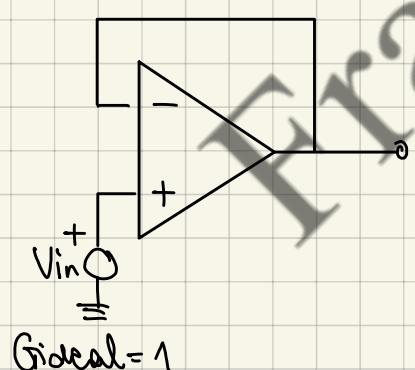


STILL STABLE !!

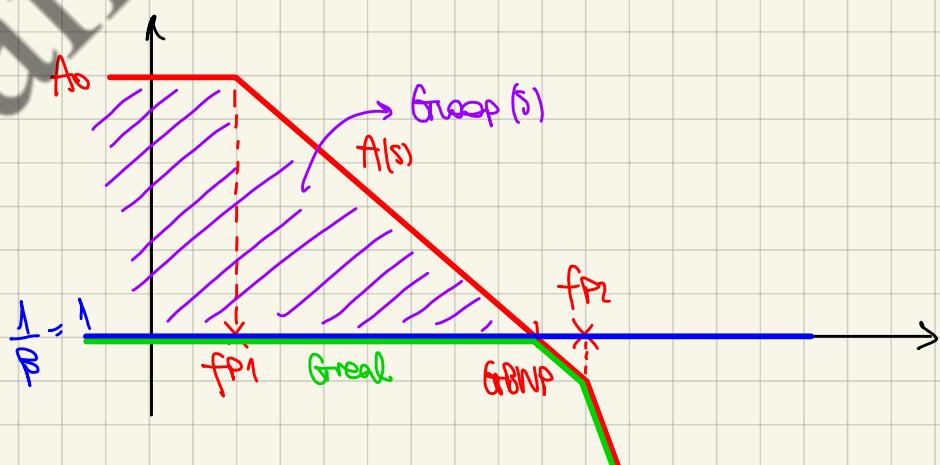
BEST CONDITION → The time response will be very fast even though there is a minor peaking

$$PM = 180^\circ - 90^\circ - 45^\circ = 45^\circ$$

NON-INVERTING STAGE : THE WORST CASE



$$G_{ideal} = 1$$



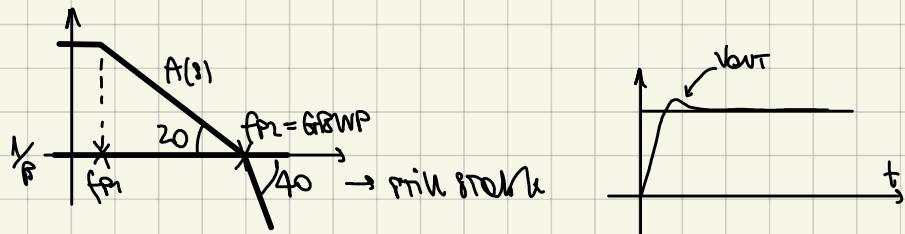
**UNCOMPENSATED OPAMP** = is an OpAmp that has the two poles before the frequency at which the gain ( $A(s)$ ) becomes 1.

in this case it can be stable or unstable depending on where  $1/\beta$  cuts  $A(s)$

**COMPENSATED OPAMP** = is an OpAmp that has the 2nd pole  $f_{p2}$  at a frequency higher than the frequency at which the gain ( $A(s)$ ) becomes 1

The MAIN ADVANTAGE of a compensated OpAmp is that even if you connect the OpAmp in the worst possible way for stability that is  $1/\beta = 1$  (so the buffer) the OpAmp, since it has  $f_{p2} > f_{p1}$ , is stable for sure

The manufacturer could design the OpAmp also to have  $f_{p2} = f_{GWP}$



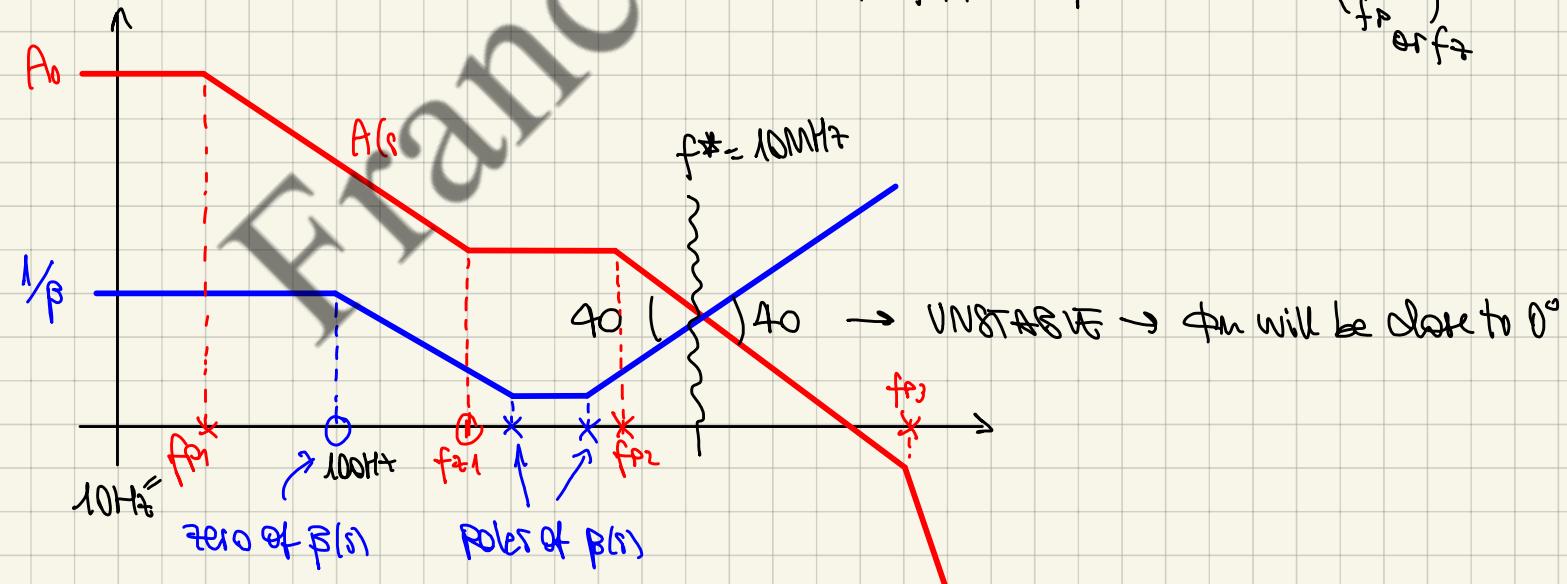
### HOW TO COMPUTE THE PHASE MARGIN

1. Plot  $A(s)$
2. Plot  $1/\beta(s)$
3. Find  $f^*$  (where  $A(1) = 1/\beta(s)$ )
- 4.

$$\Rightarrow \phi_m = 180^\circ - \sum_i \text{atan}\left(\frac{f^*}{f_{p,i}}\right) + \sum_j \text{atan}\left(\frac{f^*}{f_{z,j}}\right)$$

NOTICE:  
where  $f_{p,i}$  and  $f_{z,j}$  are all the poles and zeros of  $A(s)$  and  $\beta(s)$

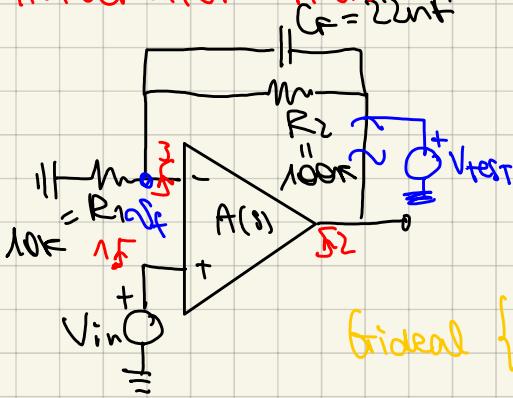
Example:



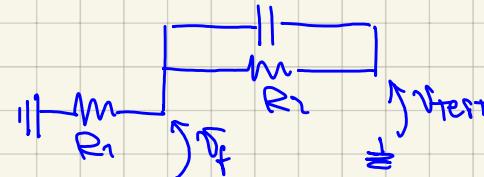
$$PM = 180^\circ - \text{arctan}\left(\frac{10\text{MHz}}{10\text{MHz}}\right) + \text{arctan}\left(\frac{10\text{MHz}}{100\text{Hz}}\right)$$

ROUTE OF C

INTEGRATOR STAGE



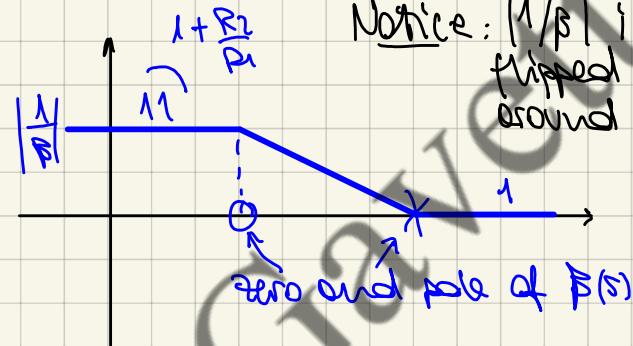
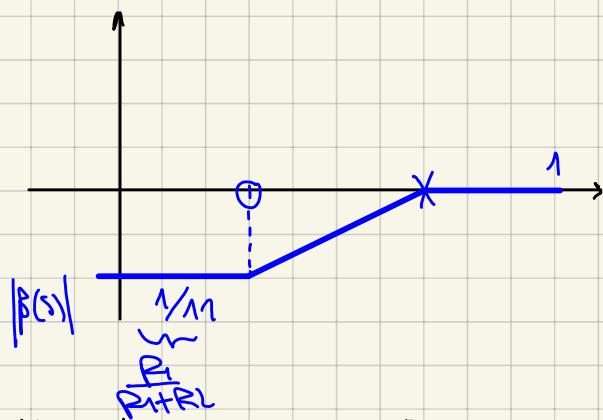
NEGATIVE FB



$$\text{fideal} \left\{ \begin{array}{l} \text{fideal}(0) = 1 + R_2/R_1 = 11 \\ \text{fideal}(\infty) = 1 \end{array} \right.$$

$$@DC \rightarrow C \text{ open} \rightarrow \beta(0) = \frac{V_f}{V_{test}}(0) = \frac{R_1}{R_1 + R_2} = \frac{1}{11}$$

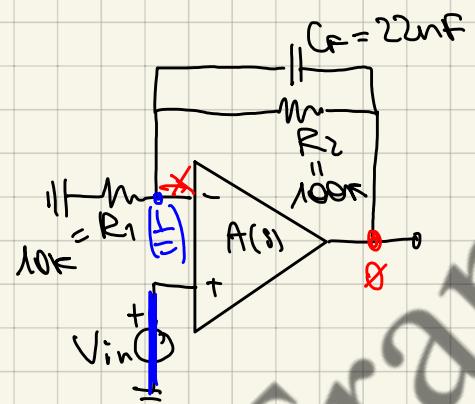
$$@HF \rightarrow C \text{ close} \rightarrow \beta(\infty) = 1$$



Notice:  $|1/\beta|$  is simply  $|\beta|$  flipped vertically around the  $\text{dB}$  axis

zero and pole of  $\beta(s)$

Now, let's compute the zero and the pole of the circuit; (not of  $\beta$ )

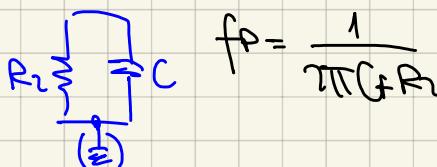


zero:

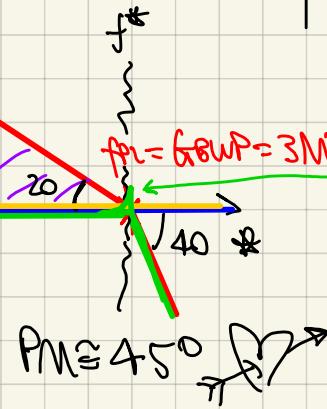
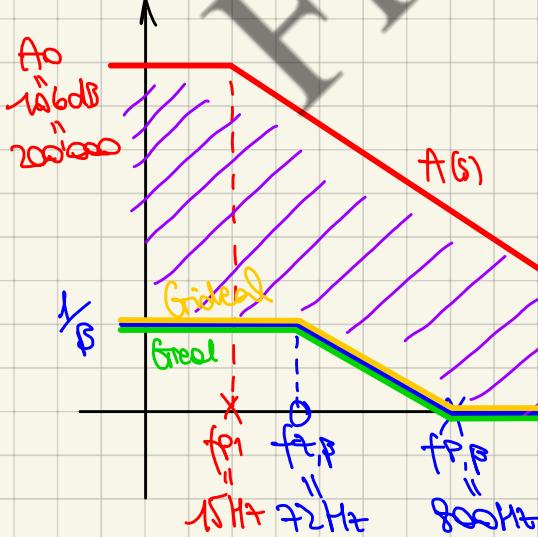


$$f_z = \frac{1}{2\pi C_F R_1 R_2}$$

pole:



$$f_p = \frac{1}{\pi C_F R_2}$$

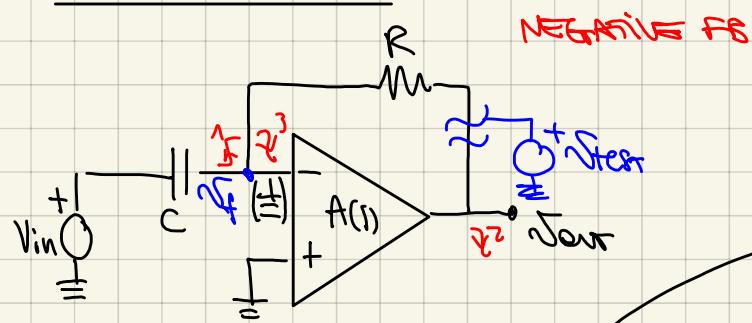


$$\text{Notice: } \frac{1}{\beta}(s) = \text{fideal}(s)$$

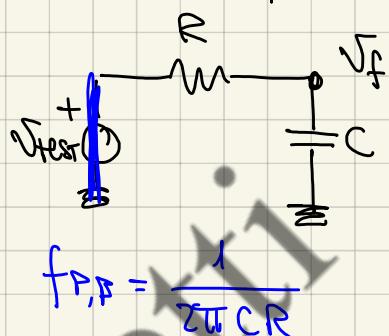
indeed it is a  
NON-INVERTING CONFIG

## DERIVATOR STAGE

### IDEAL DERIVATOR



$\beta$  should have a pole



$$@DC \rightarrow C \text{ open} \rightarrow \beta(0) = 1 \rightarrow \frac{1}{\beta}(0) = 1$$

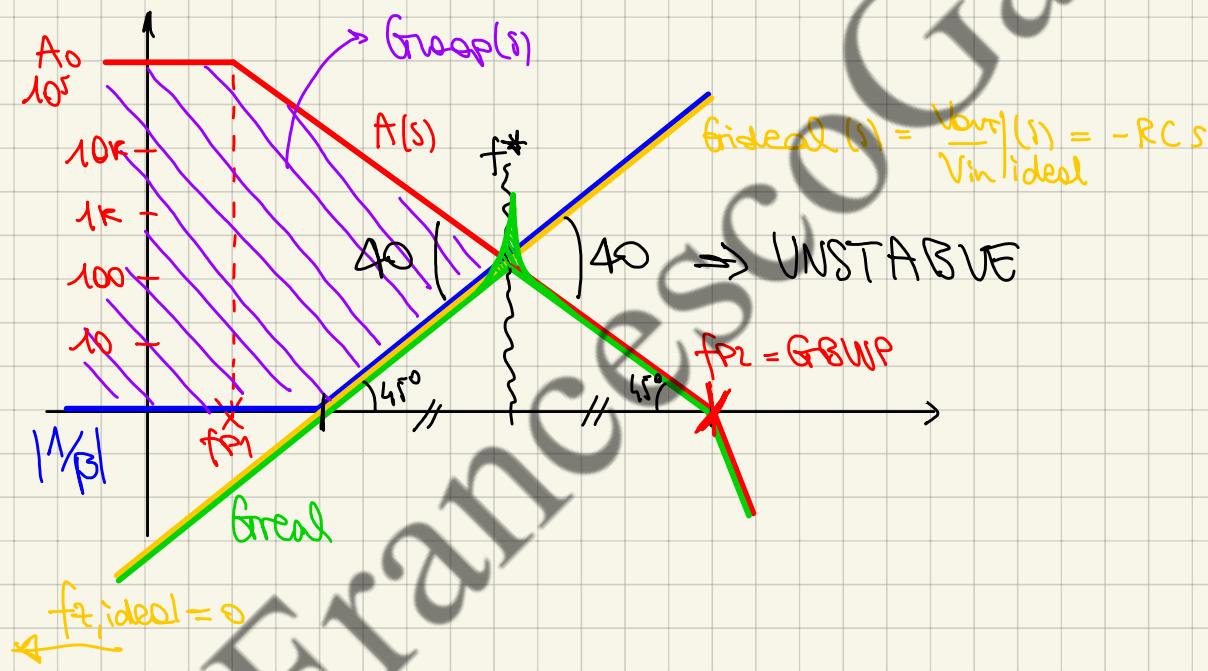
$$@HF \rightarrow C \text{ short} \rightarrow \beta(\infty) = \infty \rightarrow \frac{1}{\beta}(\infty) = 0$$

$G_{ideal} = ?$

$$@DC \rightarrow G_{ideal}(0) > 0$$

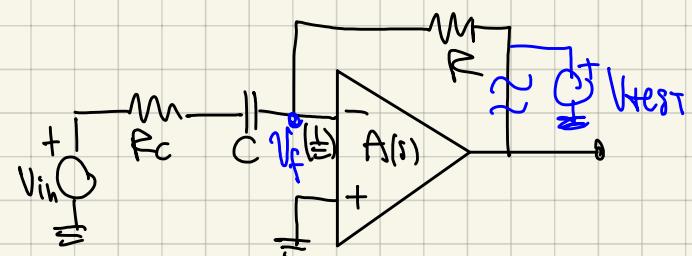
$$@HF \rightarrow G_{ideal}(\infty) = \infty$$

} in the ideal gain there is no pole, there is only a zero at the origin  
 $f_{z,ideal} = \infty$



### REAL DERIVATOR

NEG FB



$$@DC \rightarrow \beta(0) = 1$$

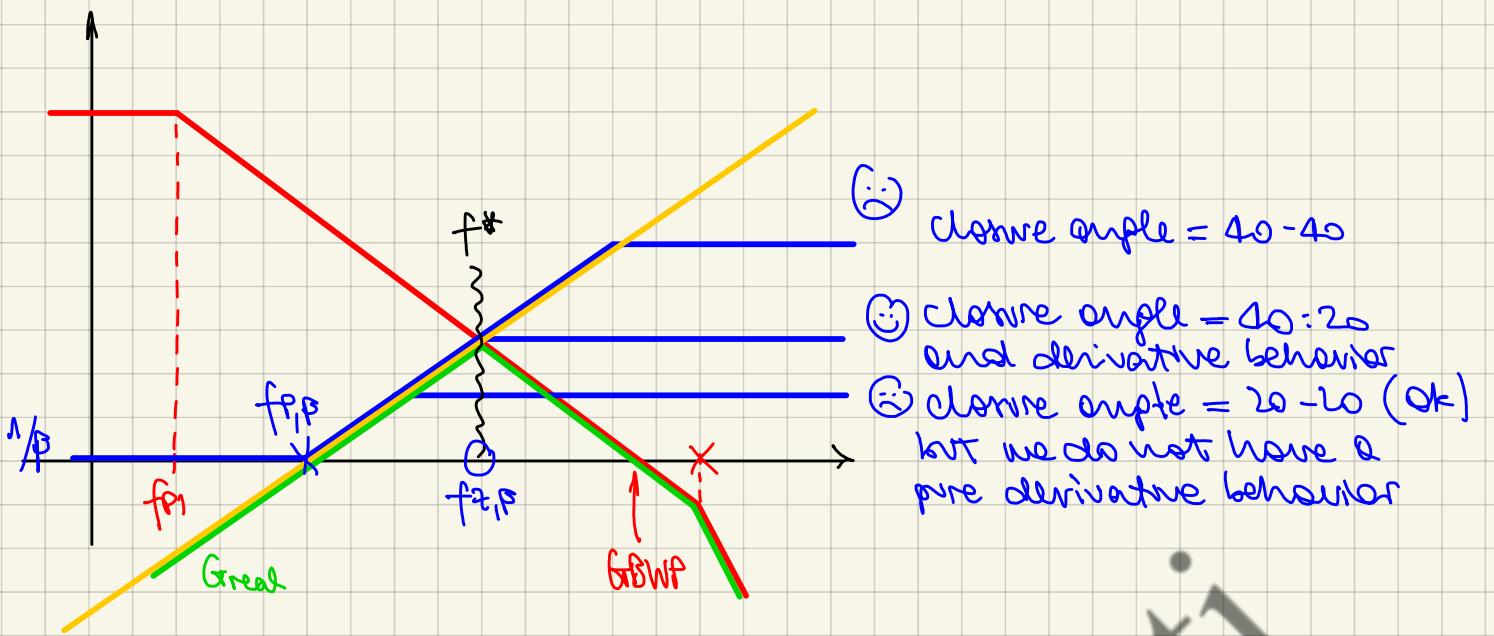
$$@HF \rightarrow \beta(\infty) = \frac{R_C}{R_C + R}$$

$$f_{P,B} = \frac{1}{2\pi C(R_C + R)}$$

$G_{ideal}$

$$@DC \rightarrow G_{ideal}(0) = 0$$

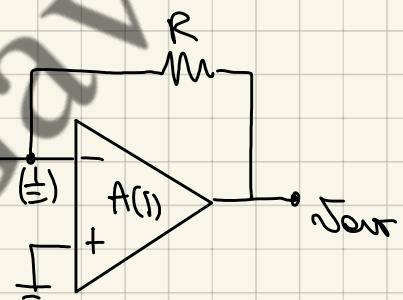
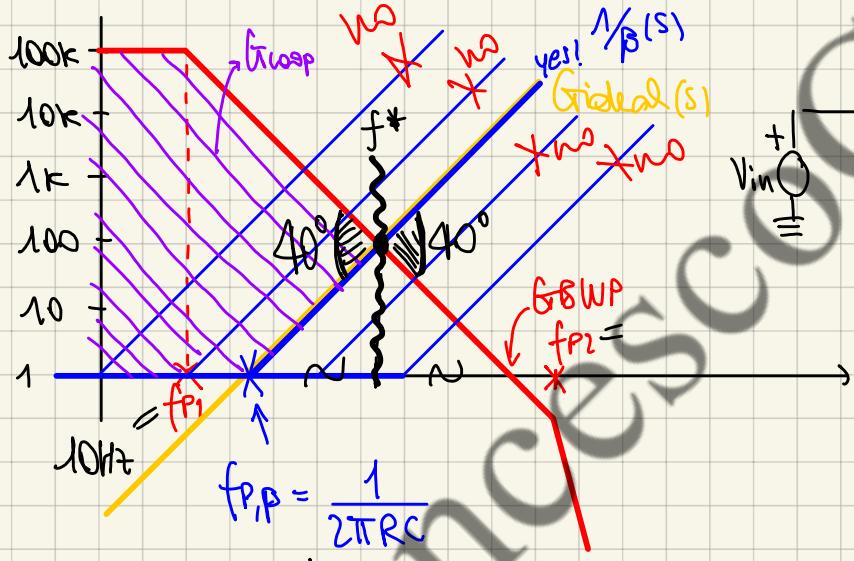
$$@HF \rightarrow G_{ideal}(\infty) = -\frac{R}{R_C}$$



## ES04 - FREQUENCY COMPENSATION (2)

30/09/2021

Let's look again at the ideal derivator



The pole of  $\beta$  should be where  $G_{ideal}(s)$  cuts the 0dB axis

$f^*$  is in the middle b/w  $f_{p,p}$  and  $G_{BW}$   $\Rightarrow$

$$f^* = \sqrt{f_{p,p} \cdot G_{BW}}$$

closure angle = 40 - 40  $\rightarrow$  the circuit is UNSTABLE

$$PM = 180^\circ - \text{atan}\left(\frac{f^*}{f_m}\right) - \text{atan}\left(\frac{f^*}{f_{p,p}}\right) - \text{atan}\left(\frac{f^*}{f_{p2}}\right)$$

$$f^* \gg f_m, f_{p,p} \Rightarrow PM \approx 180^\circ - 90^\circ - 90^\circ - \text{atan}\left(\frac{f^*}{f_{p2}}\right) \approx 0^\circ$$

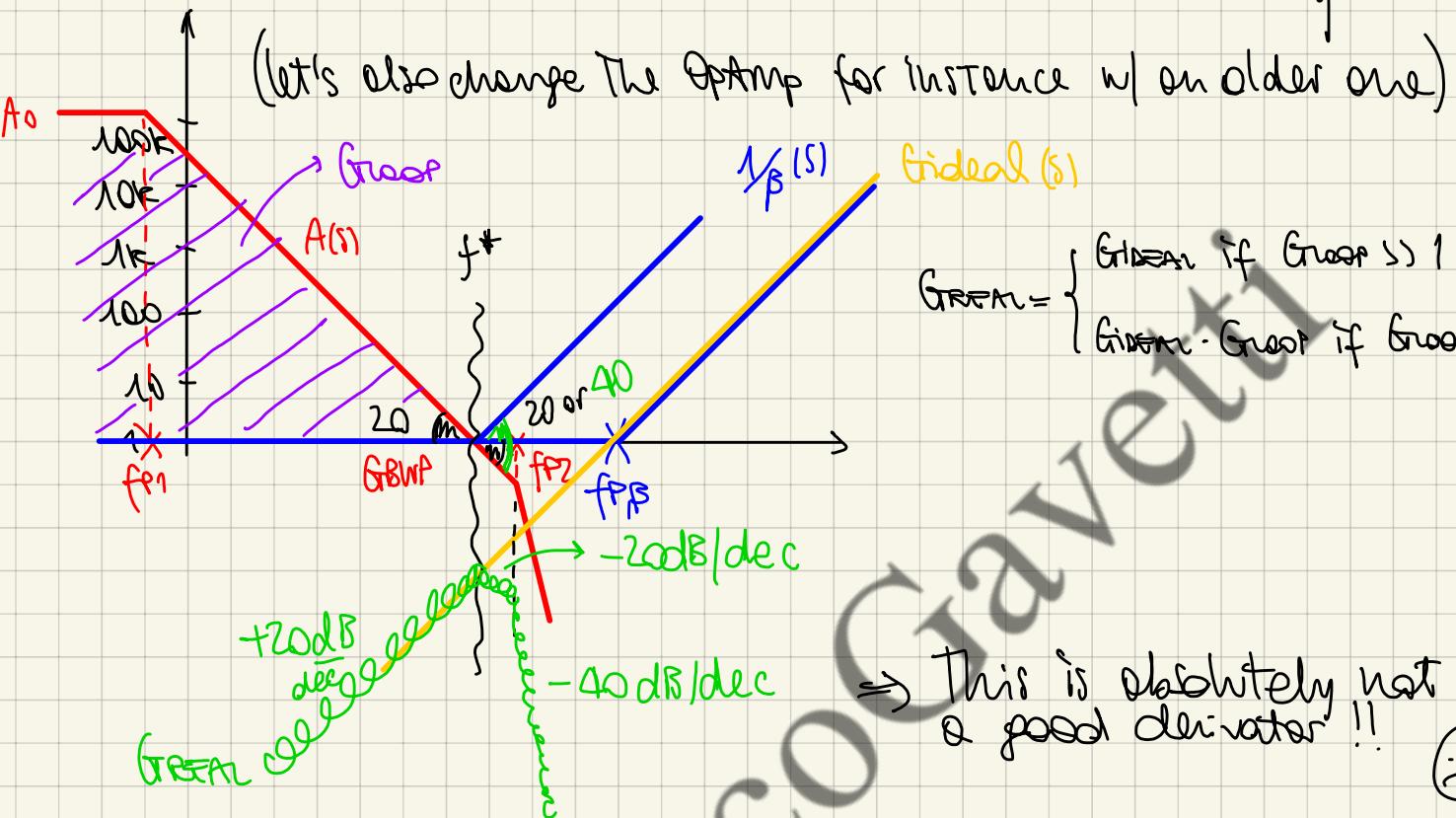
$$\Rightarrow \text{atan}\left(\frac{f^*}{f_{p2}}\right) \approx 0^\circ \rightarrow f_{p2} \gg f^*$$

We should try to avoid this bad closure angle

How can we do that?

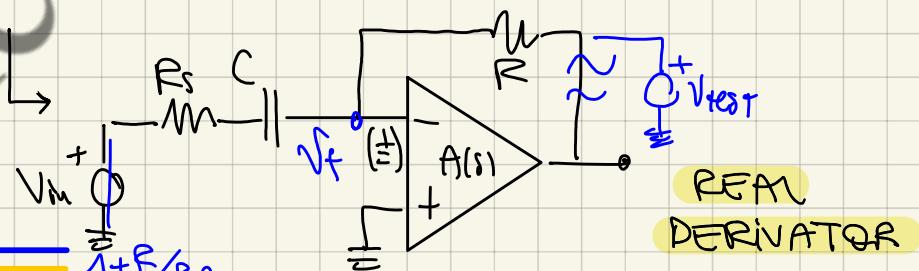
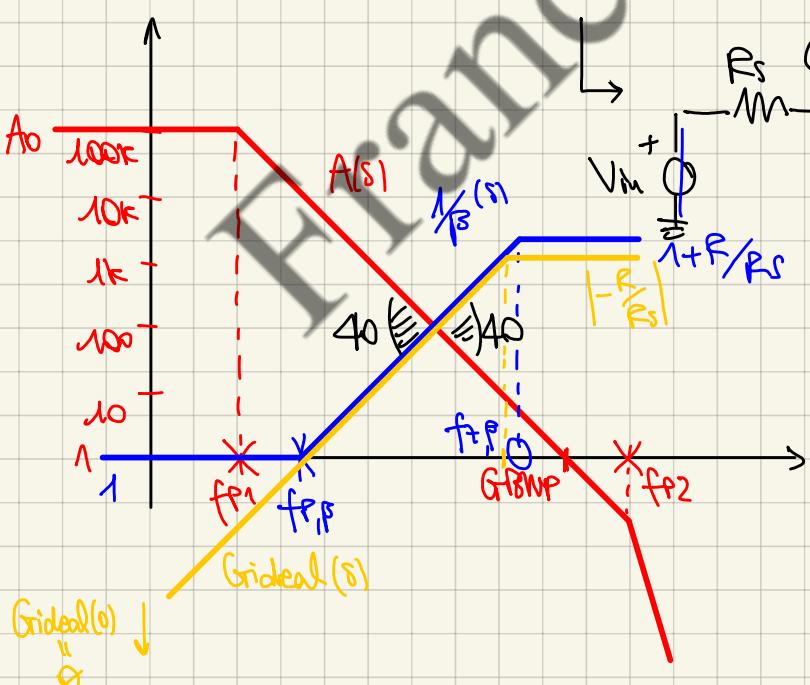
SOLUTION ①: we move  $f_{P,P}$  at very high frequency

The GBWP stays  
at a lower freq.  
↓



→ Move  $f_{P,P}$  at very high frequency is not a good way to compensate!!

SOLUTION ②: we add a COMPENSATION RESISTOR  $R_c$  in series to the capacitor



$$\text{Grdeal}(0) = 0$$

$$\text{Grdeal}(\infty) = -R/R_s$$

$$\beta(0) = 1 \rightarrow \frac{1}{\beta(0)} = 1$$

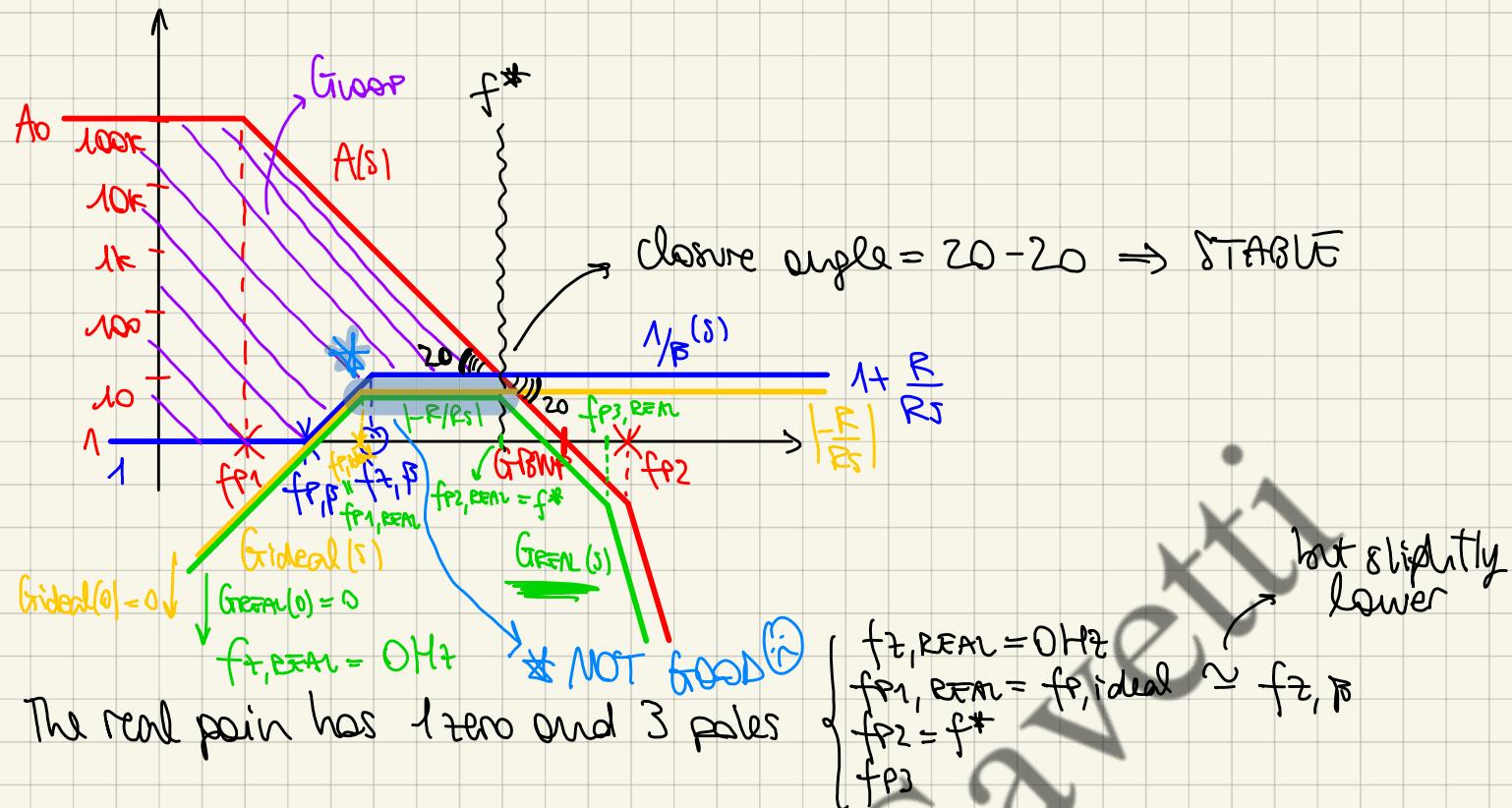
$$\beta(\infty) = \frac{R_s}{R+R_s} \rightarrow \frac{1}{\beta(\infty)} = 1 + \frac{R}{R_s}$$

$$f_{z,\beta} = \frac{1}{2\pi R_s C}$$

$$f_{P,\beta} = \frac{1}{2\pi C(R+R_s)}$$

The closure angle is still bad!!

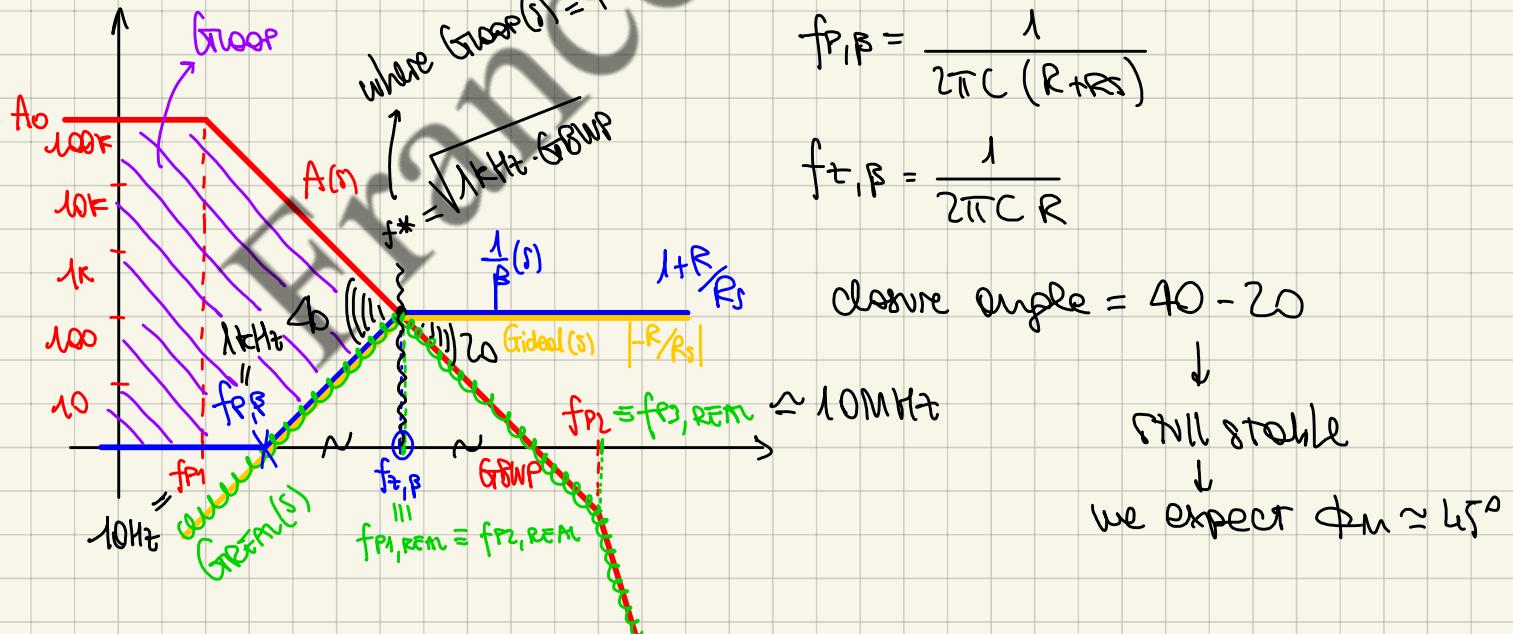
- we can choose  $R_S$  in such a way that  $1/\beta(s)$  cuts  $A(s)$  when it is constant



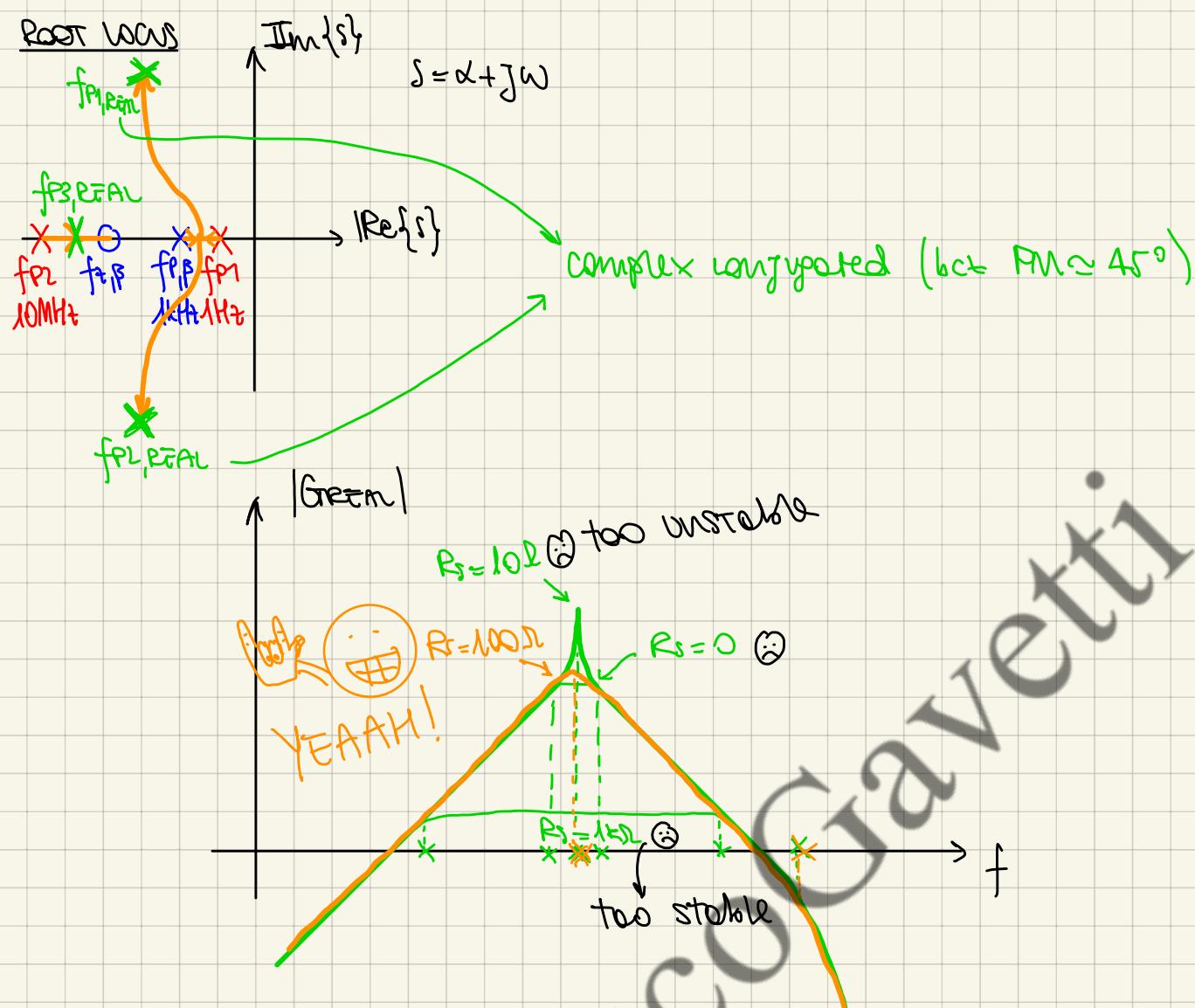
Now the closure angle is passed, the circuit is stable, but the WSTimer is still not satisfied, bcz he asked vs a deviator and we are talking to him a deviator till  $f_{p1, \text{REAL}}$  but then it is an amplifier till  $f_{p2, \text{REAL}}$

### $\Downarrow$ (FINAL SOLUTION)

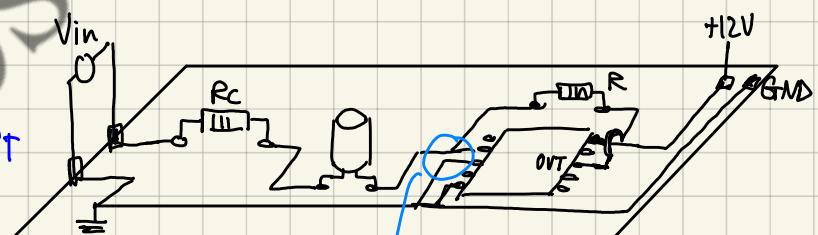
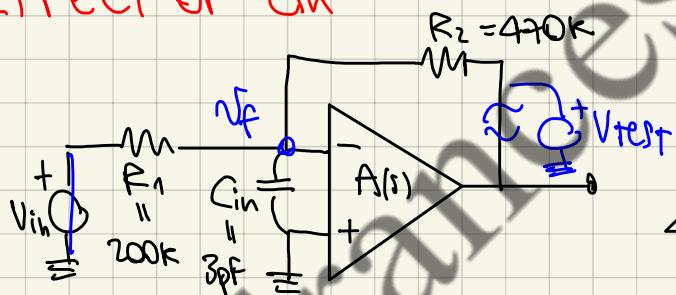
- we can choose  $R_S$  in such a way that  $G_{\text{ideal}}(s)$  cuts  $A(s)$  @  $f^*$



$$\begin{aligned} PM &= 180^\circ - \tan^{-1}\left(\frac{f^*}{f_{p1}}\right) - \tan^{-1}\left(\frac{f^*}{f_{p1, \beta}}\right) + \tan^{-1}\left(\frac{f^*}{f_{z, \beta}}\right) - \tan^{-1}\left(\frac{f^*}{f_{p2}}\right) \\ &= 180^\circ - 90^\circ - 90^\circ + 45^\circ - 0^\circ \approx 45^\circ \end{aligned}$$



## EFFECT OF $C_{in}$



very close: There might be a  
parasitic effect in

$$G_{\text{DEAL}}(0) = -R_2/R_1 \approx -2.3$$

$$F_{\text{ideal}}(\alpha) = -P_2/P_1$$

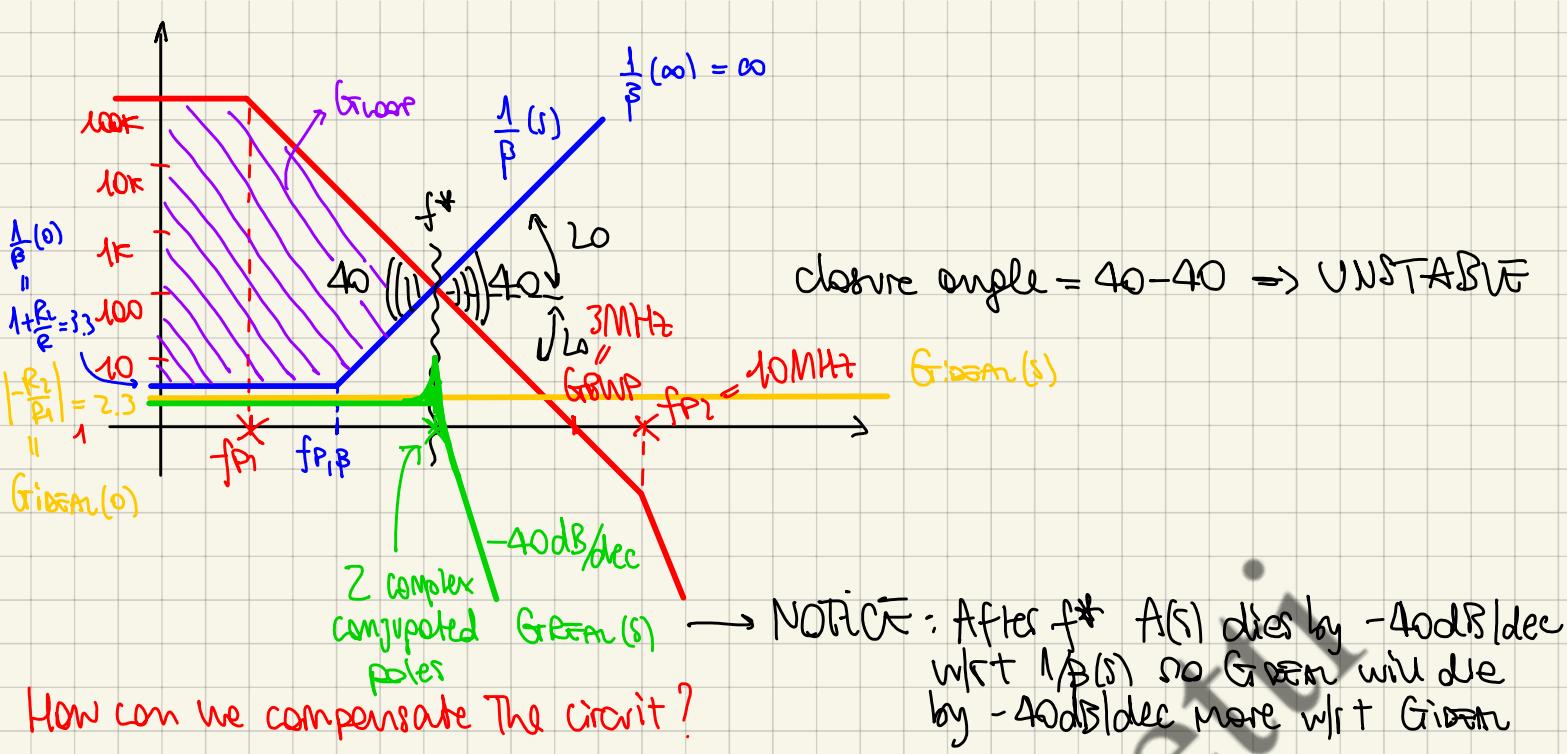
The capacitor  $C_{in}$  has no effect on  $G_{ideal}$  bcz @ DC it's open and  $V^-$  is @ v.g. and @ HF it's short and  $V^-$  is @ ground, so nothing changes & we can say that, since  $C_{in}$  stays b/w v. f. and ground, no current can flow through it so it doesn't influence  $G_{ideal}$  which remains constant.

$$\beta(0) = \frac{R_1}{R+R_2} \approx 0.3 \rightarrow \frac{1}{\beta}(0) = 1 + \frac{R_2}{R_1} \approx 3.3$$

$$\beta(\infty) = 0 \quad \rightarrow \quad \frac{1}{\beta}(n) = \infty$$

$\beta(s)$  must have a pole

$$f_{\text{P},\text{P}} = \frac{1}{2\pi c R_{\parallel} R_{\perp}}$$



How can we compensate the circuit?

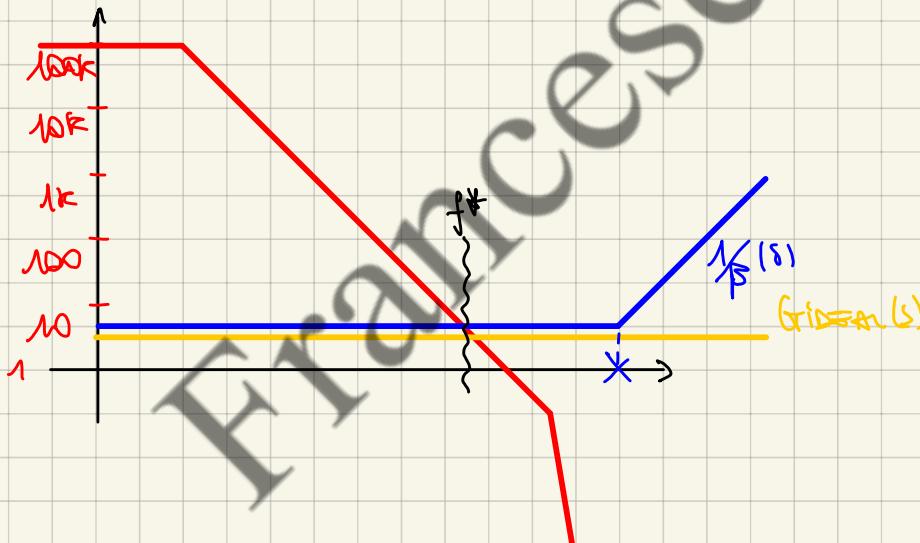
→ NOTICE: After  $f^*$   $A(s)$  decreases by  $-40 \text{ dB/dec}$  w.r.t  $\frac{1}{P}(s)$  so  $G_{\text{loop}}$  will decrease by  $-40 \text{ dB/dec}$  more w.r.t  $G_{\text{loop}}$

Let's move  $f_{P1}$  to a very high frequency by changing the resistors

$$f_{P1} = \frac{1}{2\pi C_{in} (R_1 || R_2)} \quad \text{and} \quad |G_{\text{loop}}| = \left| -\frac{R_2}{R_1} \right| = 2.3$$

Instead of choosing  $R_2 = 470\text{k}$  and  $R_1 = 200\text{k}$ , let's choose:  $\begin{cases} R_2 = 4.7\text{k}\Omega \\ R_1 = 2\text{k}\Omega \end{cases}$

In this way, we keep the same ideal gain but we increase  $\omega$  and the pole frequency  $f_{P1}$  such that now  $\frac{1}{P}(s)$  cuts  $A(s)$  when it is constant



A best way to compensate, as we have previously seen, is having  $\frac{1}{P}(s)$  that cuts  $A(s)$  in such a way that we have 40-20 or 20-40 or closure angle

How can we obtain that in this case?

### SOLUTION 1

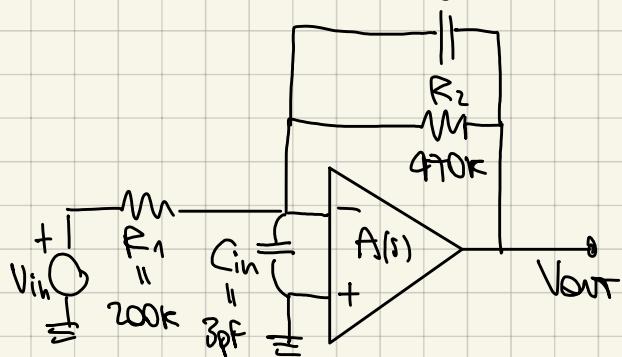
In principle we would like to add a resistor in series to  $C_{in}$ , but we can't since very often  $C_{in}$  is a parasitic term

↳ NOT POSSIBLE !! 😞

## SOLUTION 2

Another way to obtain the desired closure angle (and so to compensate) is to add a COMPENSATION CAPACITOR in parallel to the feedback resistor  $R_2$

$$C_c = 22\text{nF}$$



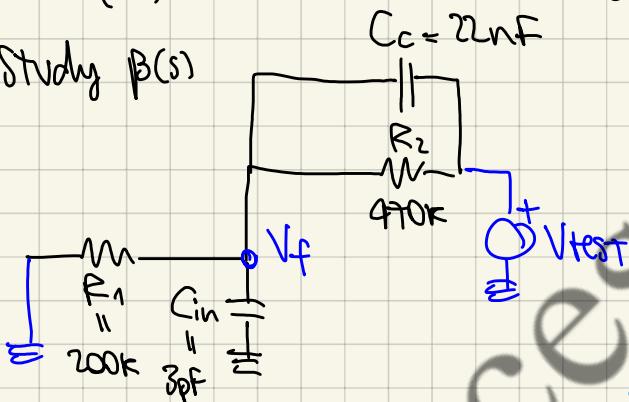
1. Study  $G_{IDEAL}(s)$  REMEMBER: As we told before,  $C_{in}$  has no effects on it since the FB is negative and it is b/w v.f and ground

$$G_{IDEAL}(0) = -R_2/R_1 = -2.3$$

$$G_{IDEAL}(\infty) = 0$$

$$f_{P,i} = \frac{1}{2\pi C_c R_2} =$$

2. Study  $\beta(s)$



$$@DC \rightarrow \beta(0) = \frac{R_1}{R_1 + R_2} \rightarrow \frac{1}{\beta}(0) = 1 + \frac{R_2}{R_1} = 3.3$$

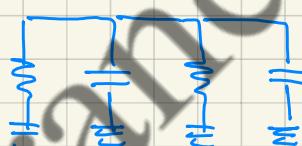
$$@HF \rightarrow \cancel{\beta(\infty) = 0} \rightarrow \cancel{\frac{1}{\beta}(\infty) = \infty} \quad \text{WRONG}$$

$$\beta(\infty) = \dots = \frac{C_c}{C_{in} + C_c} \quad ???$$

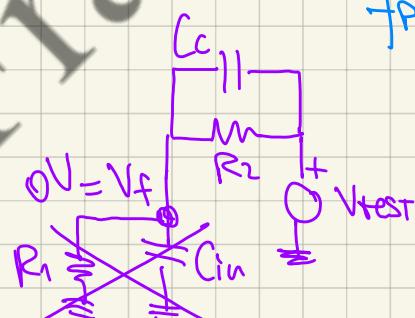
2 dependent capacitors  
↓  
1 pole

$$f_{P,\beta} = \frac{1}{2\pi (C_{in} + C_c) R_1 R_2} =$$

• POLES of  $\beta(s)$



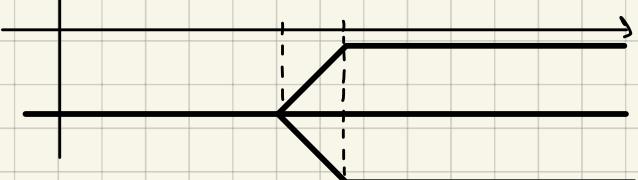
• ZEROS of  $\beta(s)$



$$f_{Z,\beta} = \frac{1}{2\pi C_c R_2} =$$

Depending on the pole and zero frequencies we can have 3 possibilities:

$$\beta(s) \uparrow$$

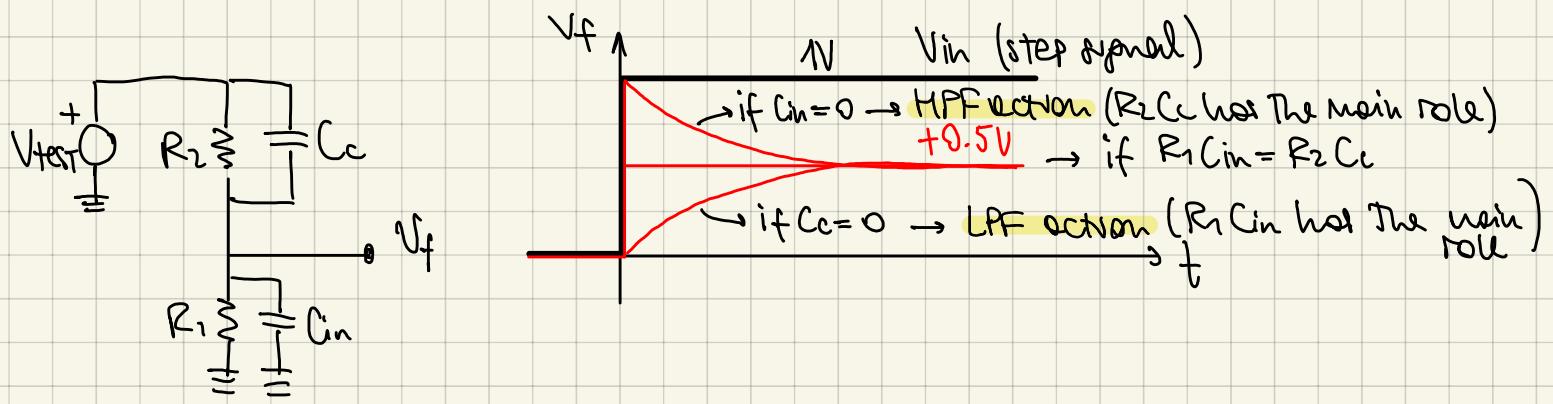


$$f_{Z,\beta} < f_{P,\beta}$$

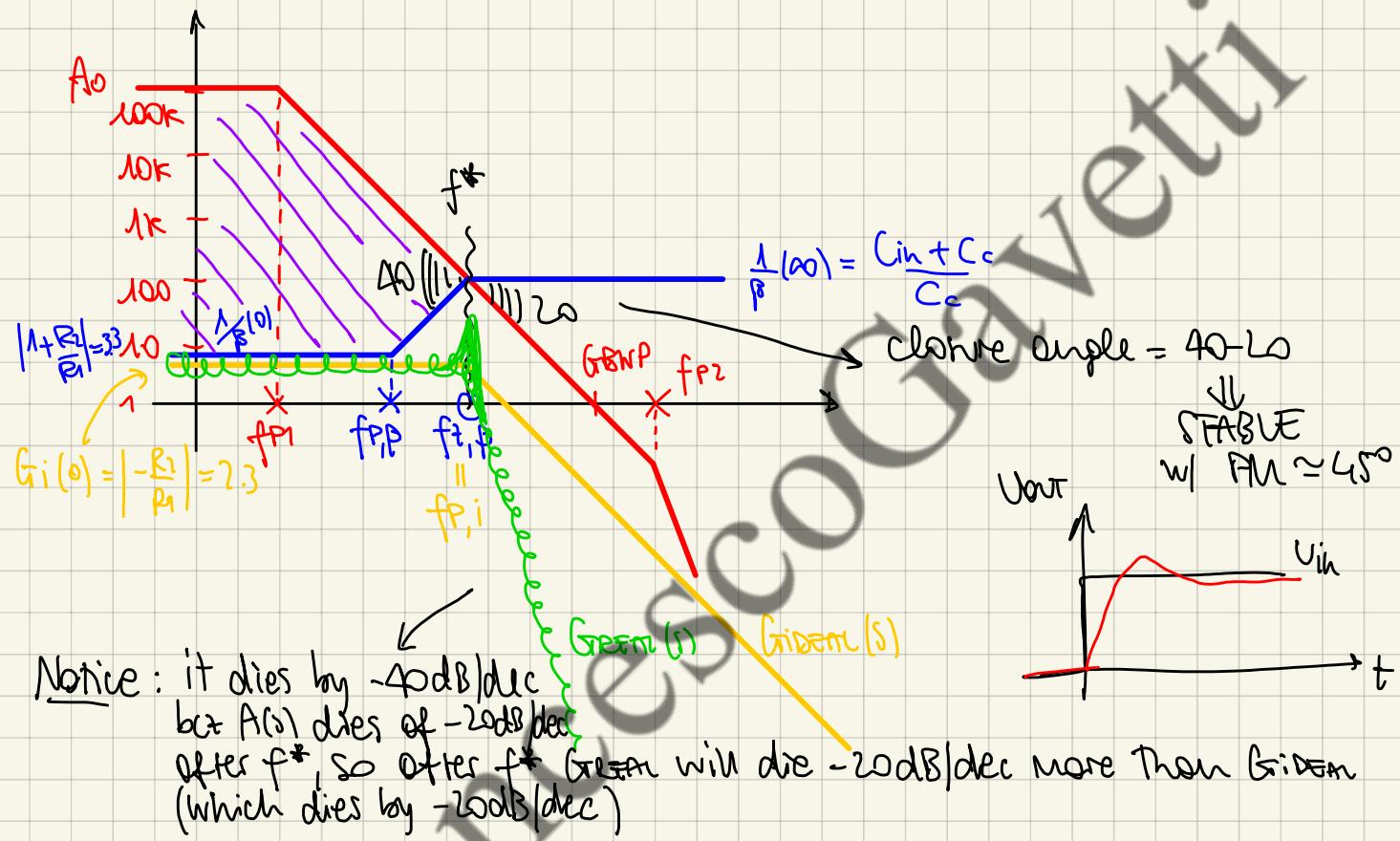
$$f_{Z,\beta} = f_{P,\beta} \quad (\text{POLE-ZERO CANCELLATION})$$

$$f_{Z,\beta} > f_{P,\beta}$$

In the time domain instead we can notice that:



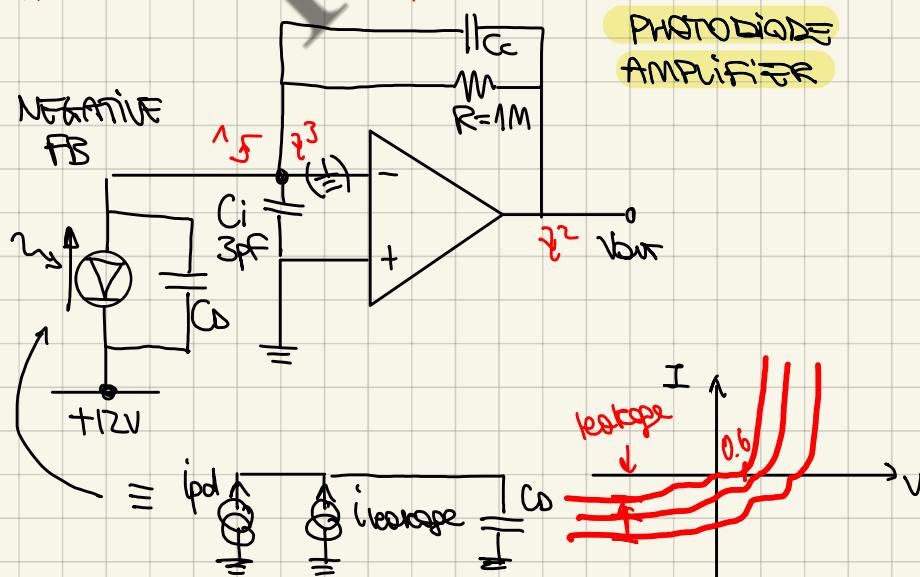
In conclusion, in the freq. domain we'll have



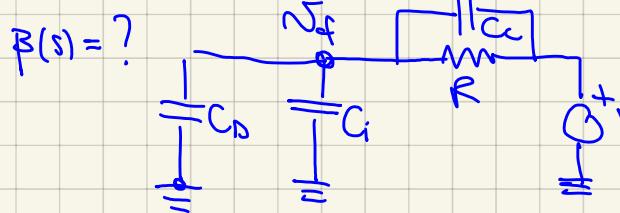
### ES04 - FREQUENCY COMPENSATION (3)

05/10/2021

## TRANSIMPEDANCE AMPLIFIER



$$\text{Gittern} = ? \quad @DC \rightarrow \text{Gittern}(0) = -R \quad \left. \begin{array}{l} \\ @HF \rightarrow \text{Gittern}(\infty) = 0 \end{array} \right\} f_{p,i} = \frac{1}{2\pi C_c R}$$



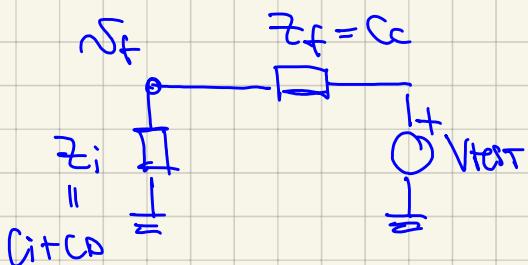
$$\text{If } I_R = 0 \rightarrow V_R = 0 \rightarrow V_f = V_{\text{rest}}$$

$\Rightarrow$

$$@ DC \rightarrow \beta(0) = 1 \rightarrow \frac{1}{\beta}(0) = 1$$

$$@ HF \rightarrow \beta(\infty) = ?$$

$$\begin{cases} V_{test} = V_f \\ V_f = 0 \end{cases} \rightarrow \beta(\alpha) = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \text{ F.I.}$$

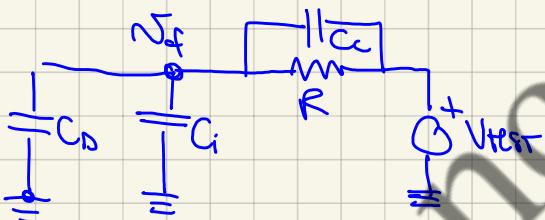


$$\beta(\infty) = \frac{z_i}{z_i + z_f} = \frac{\frac{1}{\gamma(C_i + C_D)}}{\frac{1}{\gamma(C_i + C_D)} + \frac{1}{\gamma C_c}} =$$

$$\approx \frac{\cancel{\gamma C_c (C_i + C_D)}}{[\cancel{\gamma C_c} + \cancel{\gamma (C_i + C_D)}] \cancel{\gamma (C_i + C_D)}} = \frac{C_c}{C_c + (C_i + C_D)}$$

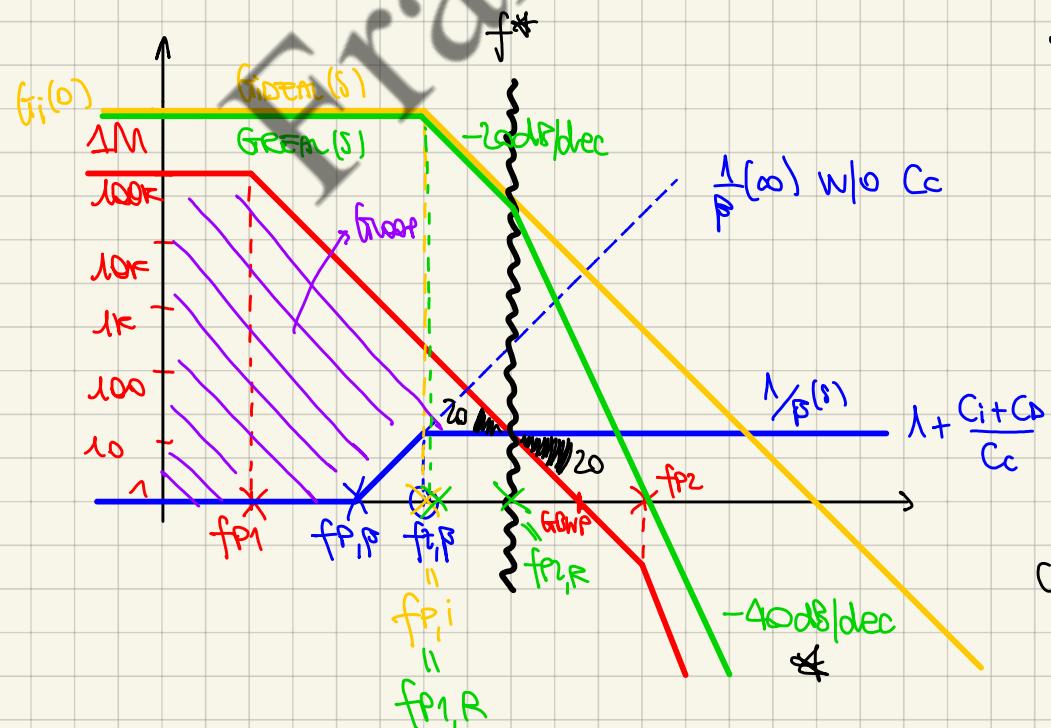
$$\Rightarrow \frac{1}{P}(\infty) = 1 + \frac{C_i + C_0}{C_0}$$

$$\left\{ \begin{array}{l} \frac{1}{1-\beta}(0) = 1 \\ \frac{1}{1-\beta}(\infty) = 1 + \frac{c_i + c_o}{c_o} \end{array} \right.$$



$$f_{P,P} = \frac{\lambda}{2\pi (C_C + C_I + C_D) R}$$

$$f_{\pi, p} = \frac{1}{2\pi C_c R}$$



\* After  $f^*$   $A(s)$  slides by  
 $-2\alpha \partial B/\partial C$  w/r/t  $1/\beta(s)$

↓  
This means that after  $\text{GLEAN}(S)$  will do by  
 $-20\%$  less than  
what  $\text{GLEAN}(S)$  does

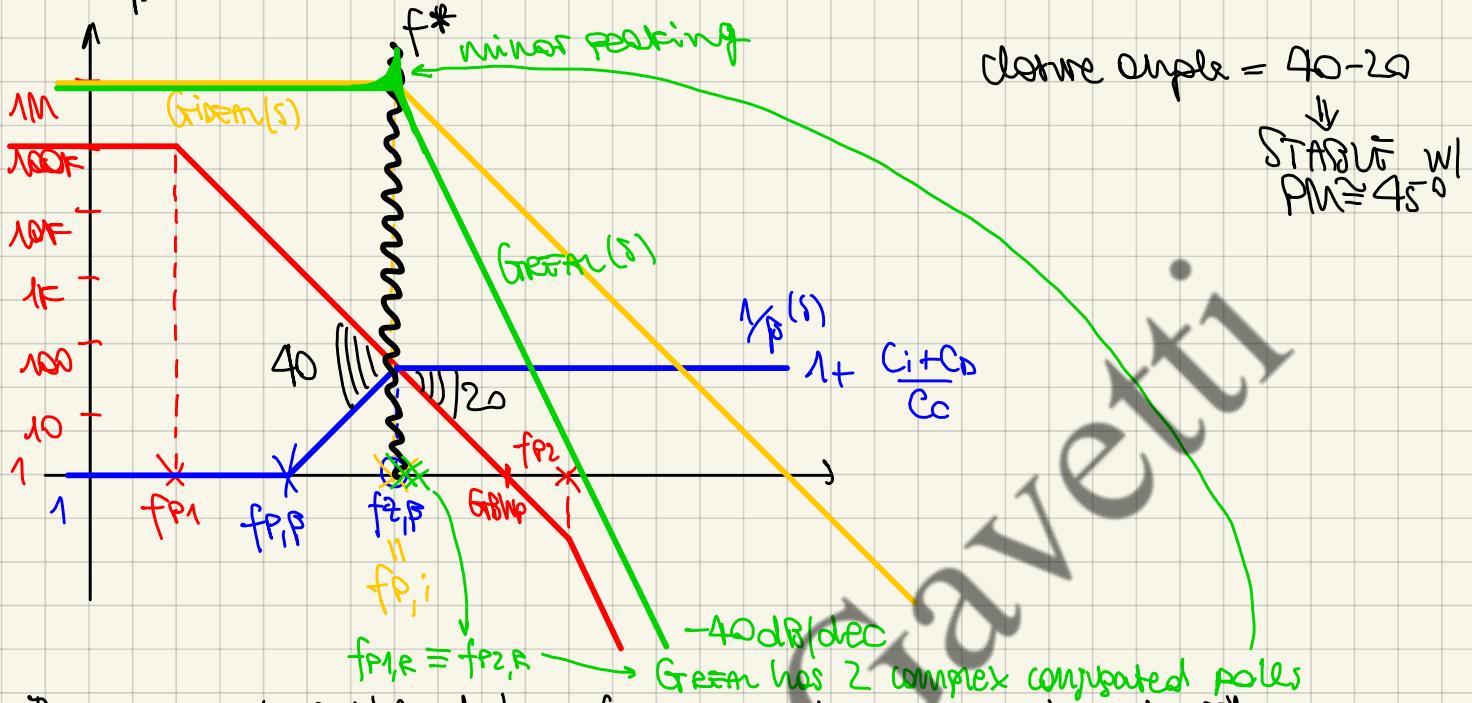
so after f\* Green (s) will  
die by - And B/dec

$$\text{closure angle} = 20^\circ - 20^\circ$$

The circuit is stable, but is the customer satisfied?

Not at all, bct the transimpedance amplifier doesn't show a constant gain up to  $f^*$

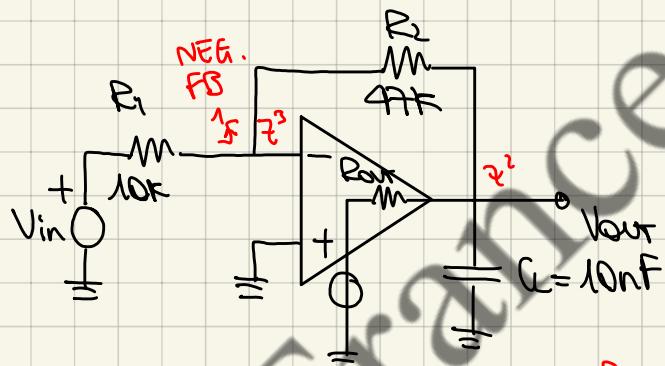
As we have already seen the best choice is to place  $f_{2,p}$  (and so  $f_{p,i}$ ) where  $\frac{1}{Y_p(s)}$  cuts  $A(s)$



Now the customer is satisfied bct Green remains constant up to  $f^*$

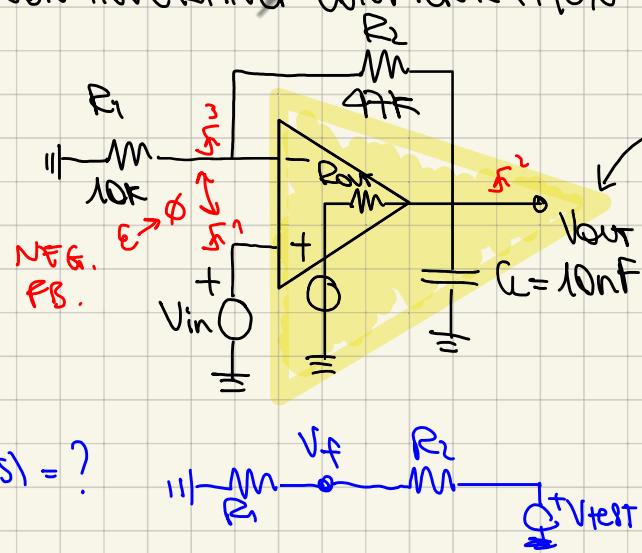
### EFFECT OF $C_o$

- INVERTING CONFIGURATION



But, what happens if  $R_o \neq 0$ ?

- NON-INVERTING CONFIGURATION



In case of absolutely ideal OpAmp ( $R_o = 0$ )  
 $C_o$  has no effects

it has no poles at all bct it's in parallel w/ a voltage generator w/ zero impedance (an ideal one), so once we turn it off  $C_o$  will be b/w 2 grounds  
 $\rightarrow C_o$  causes no instability

### METHOD 1

Considering  $C_o$  inside A

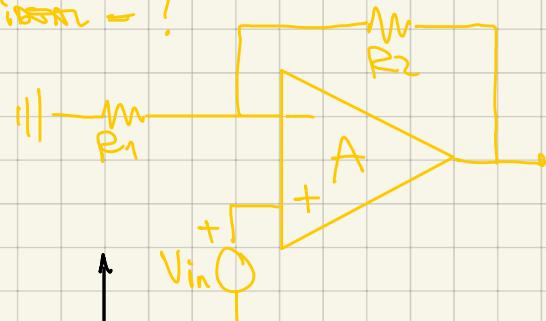
$$\tilde{A}(s) \approx A(s) \cdot \frac{\frac{1}{sC_o}}{R_o + \frac{1}{sC_o}} = A(s) \cdot \frac{1}{1 + sC_o R_o}$$

$$f_{o\pi} = \frac{1}{2\pi C_o R_o} \quad (\text{additional pole})$$

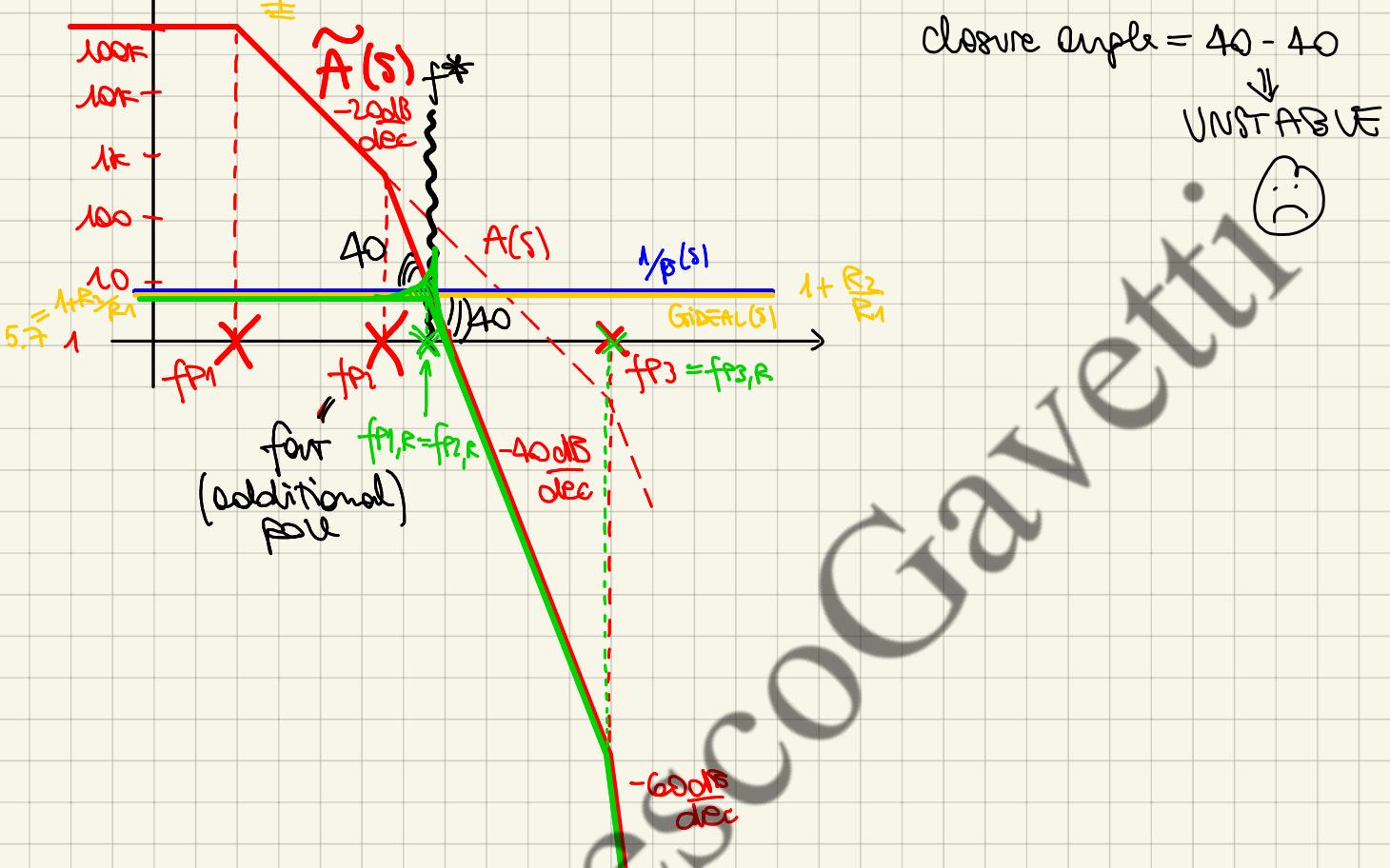
$$\beta(0) = \beta(\infty) = \frac{R_1}{R_1 + R_2}$$

$$\rightarrow \frac{1}{\beta(s)} = 1 + \frac{R_2}{R_1} \quad (\text{constant})$$

Gideal = ?

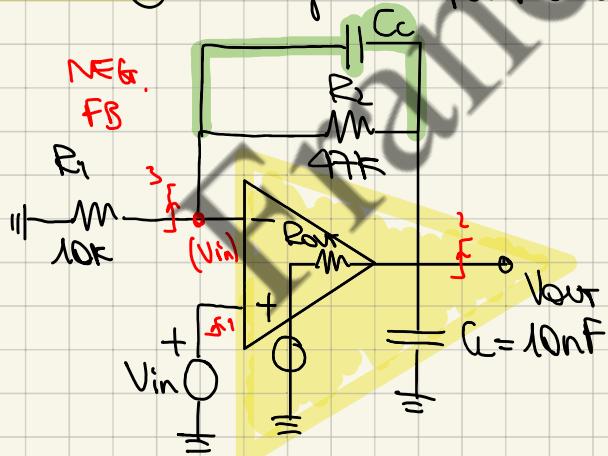


$$G_{\text{ideal}}(s) = 1 + \frac{R_2}{R_1} = 5.7$$



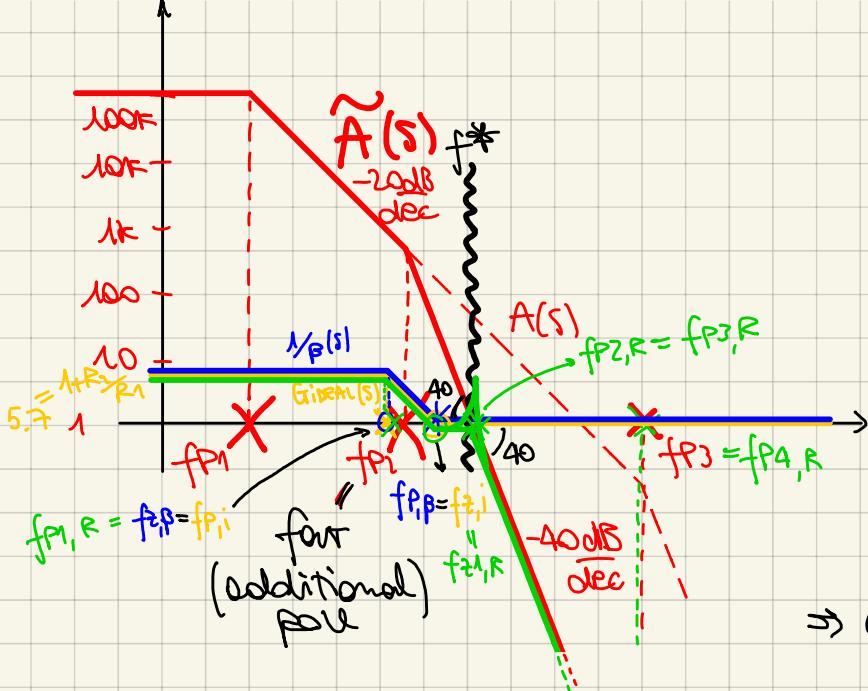
How can we compensate it?

Solution ①: adding a compensation capacitor in parallel to the feedback resistor



$$\left. \begin{array}{l} G_{\text{ideal}}(0) = 1 + R_2/R_1 \\ G_{\text{ideal}}(\infty) = 1 \end{array} \right\} f_{P,i} = \frac{1}{2\pi C_c R_2}$$

$$f_{Z,i} = \frac{1}{2\pi C_c (R_1 || R_2)}$$



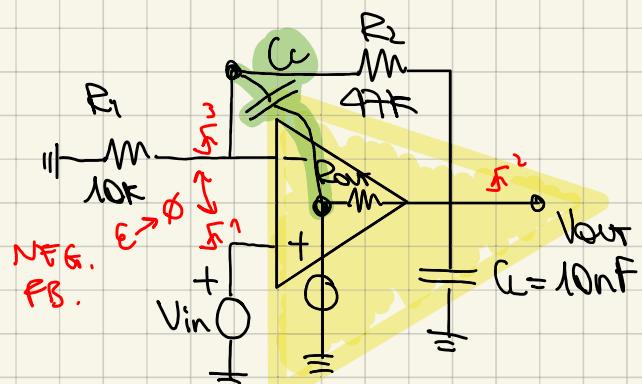
closure angle = 40 - 40

UNSTABLE  $\ominus$

This solution doesn't work!

$\Rightarrow C_c$  in parallel to  $R_2$  is not enough!

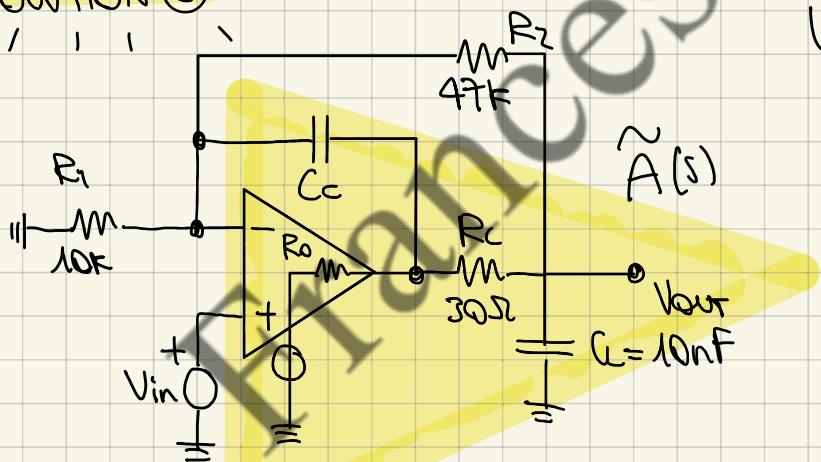
SOLUTION ②



BUT WE CANNOT ACCESS TO THE OPAMP IN THIS WAY!!  $\ominus$

This solution doesn't work too!

SOLUTION ③



Let's set  $\{R_c \approx R_o\}$

$$\left\{ C_c = C_L \cdot 2 \frac{R_o}{R_2} = 13 \text{ pF} \right.$$

Considering  $R_o$  and  $R_c$  much lower than  $R_1$  and  $R_2$  we get:

$$\tilde{A}(s) = A(s) \cdot \frac{1}{1 + s C_L (R_o + R_c)} \cdot \frac{1}{1 + s C_c (R_1 || R_2)}$$

$$f_{\text{NEW}} = \frac{1}{2\pi C_L (R_o + R_c)} \\ = 265 \text{ kHz}$$

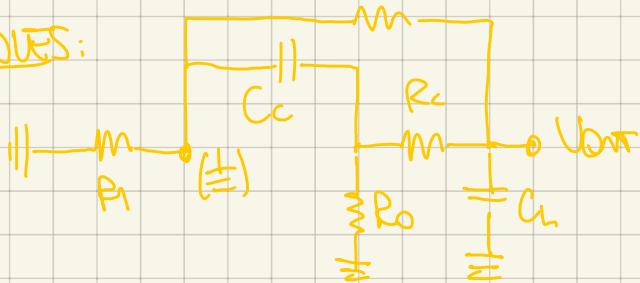
$$f_{\text{NEW}} = \frac{1}{2\pi C_c (R_1 || R_2)}$$

$$G_{DESR}(s) = ? \rightarrow G_{DESR}(0) = 1 + R_2/R_1$$

$$G_{DESR}(\infty) = ? \\ = 1$$



POLES:



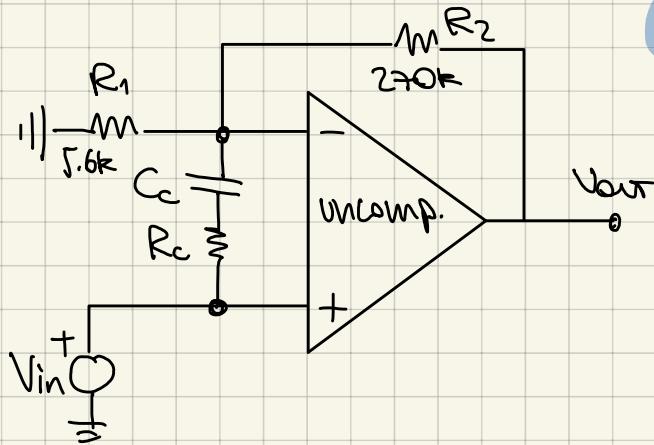
$$f_{P1,i} = \frac{1}{2\pi C_c (R_2 + R_0)}$$

$$f_{P2,i} = \frac{1}{2\pi C_1 (R_0 || R_2)}$$

??  
..

TO  
Francesco Gavetti  
COMPLETATE!

## SOLUTION 4 → NEGATIVE FEEDBACK COMPENSATION

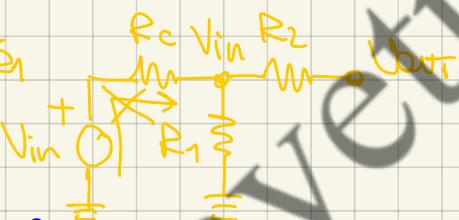


Hp: let's suppose  $A(s)$  is an uncompensated OpAmp

$$f_{P2} < G_{SWP}$$

$$G_{IDEAL}(s) = ? \quad @DC \rightarrow G_i(0) = 1 + R_2/R_1 = 5.8$$

$$@HF \rightarrow G_i(\infty) = 1 + R_2/R_1$$



IDEAL has neither poles nor zeros, it's constant

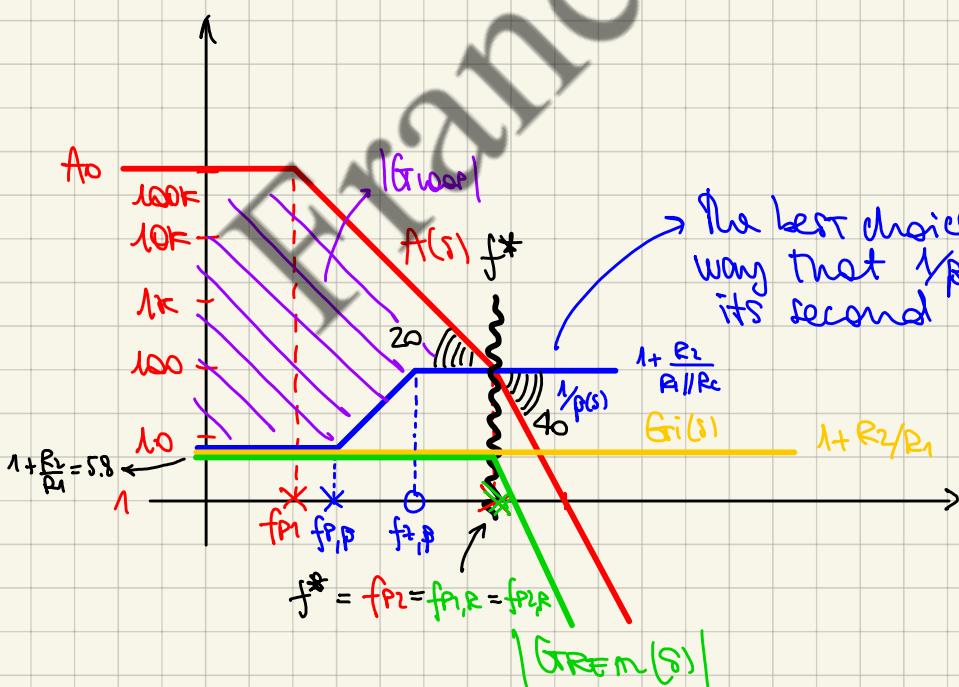
$$\beta(s) = ? \quad @DC \rightarrow \beta(0) = 1 + R_2/R_1 = 5.8$$

$$@HF \rightarrow \beta(\infty) = \frac{R_1||R_c}{R_1||R_c + R_2}$$

$$\Rightarrow \frac{1}{\beta}(\infty) = 1 + \frac{R_2}{R_1||R_c} =$$

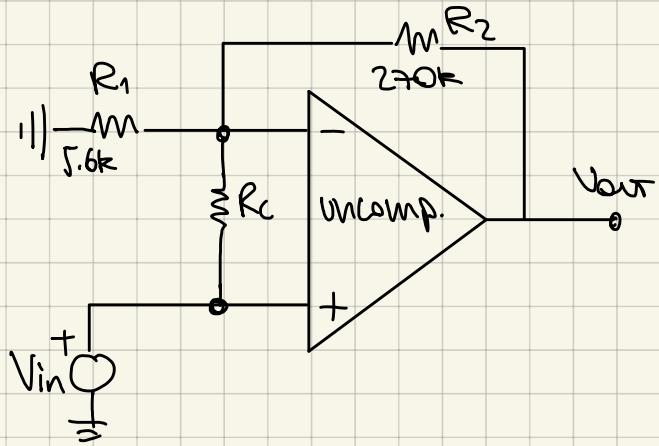
$$f_{P,\beta} = \frac{1}{2\pi C_c (R_c + R_1||R_2)} =$$

$$< f_{z,\beta} = \frac{1}{2\pi C_c R_c} =$$

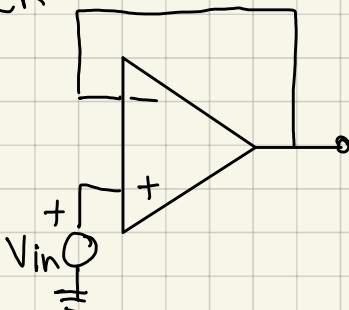


The best choice is to size the circuit in such a way that  $1/\beta(s)$  intersects  $A(s)$  where its second pole happens  
 $\Rightarrow f^* = f_{P2}$

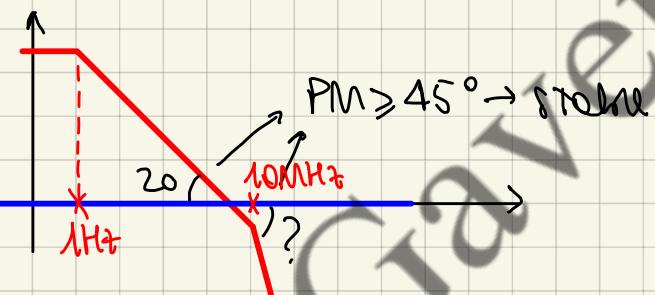
Possible simplification → REMOVE  $C_c$



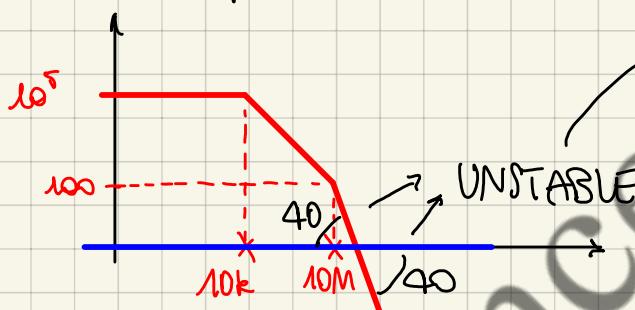
BUFFER



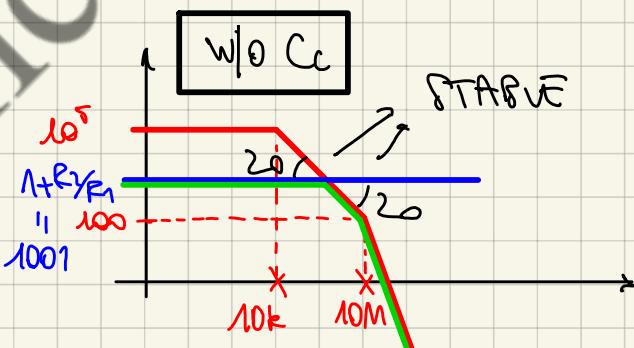
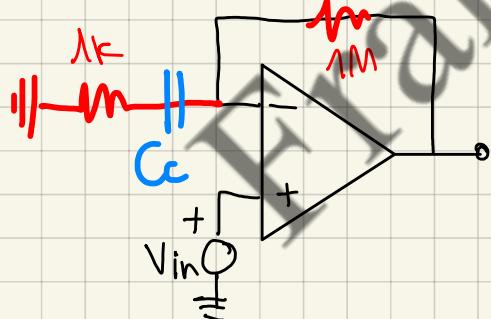
• compensated OpAmp



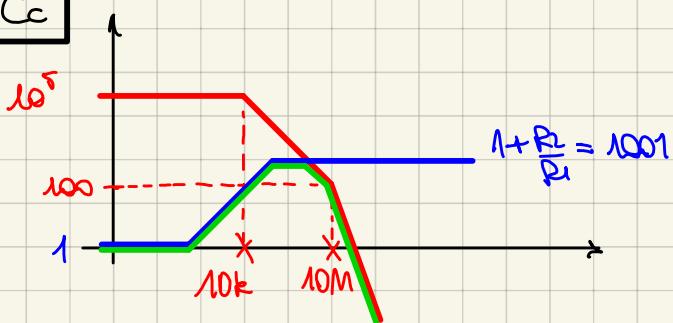
• uncompensated OpAmp



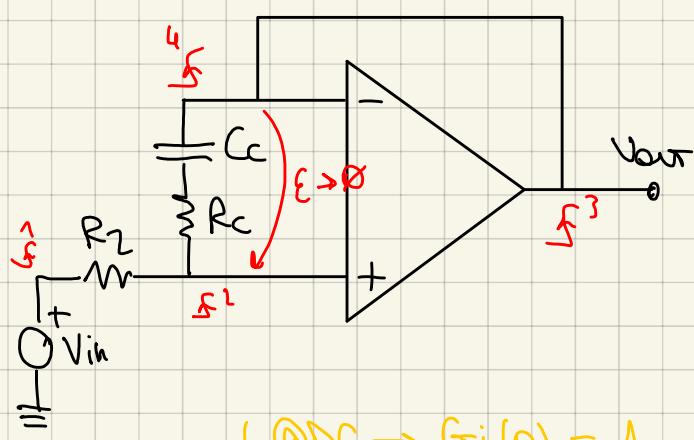
if we want to use an uncompensated OpAmp we have to regain stability



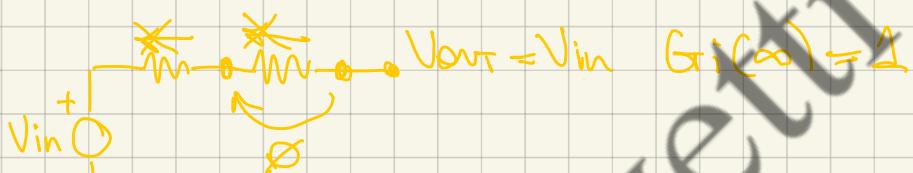
w/  $C_c$



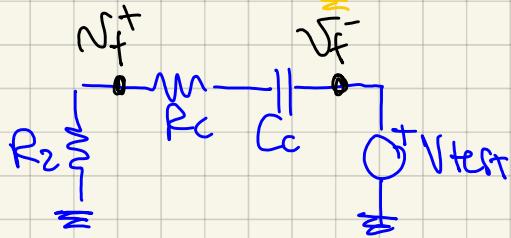
## SOLUTION ⑤ → POSITIVE FEEDBACK COMPENSATION



$$G_{\text{feed}}(s) = \begin{cases} @\text{DC} \rightarrow G_i(0) = 1 \\ @\text{HF} \rightarrow \end{cases}$$



$$\beta(s) = ?$$



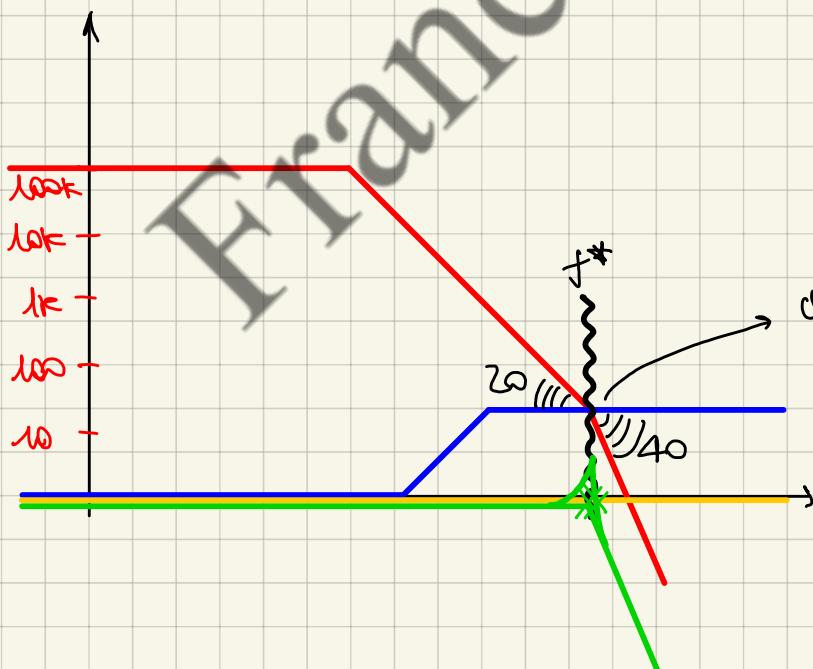
$$@\text{DC} \rightarrow \beta(0) = \beta^-(0) - \beta^+(0) =$$

$$= 1 - 0 = 1$$

$$\Rightarrow \frac{1}{\beta}(0) = 1$$

$$@\text{HF} \rightarrow \beta(\infty) = \beta^-(\infty) - \beta^+(\infty) = 1 - \frac{R_2}{R_2 + R_c} = \frac{R_c}{R_2 + R_c}$$

$$\rightarrow \frac{1}{\beta}(\infty) = 1 + \frac{R_2}{R_c}$$

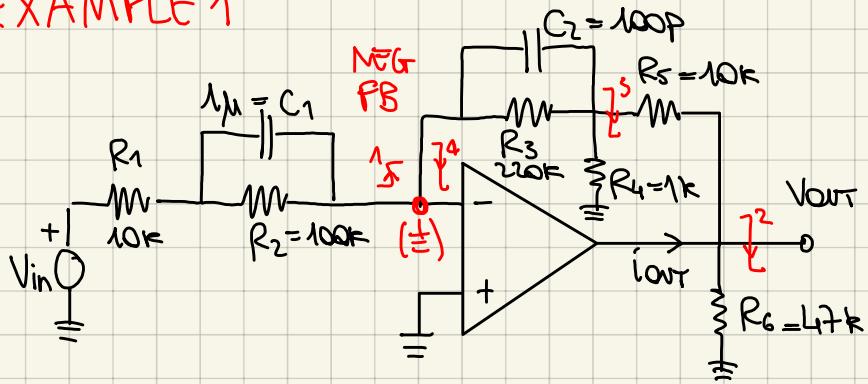


crossover angle =  $20 - 40$

↓  
STABLE  
and highest  $f^*$

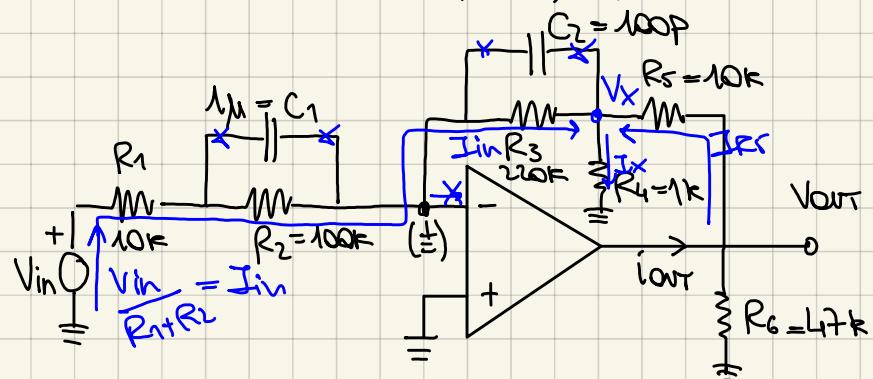
$\text{PM} \approx 45^\circ$

# EXAMPLE 1



$$\begin{aligned}
 R_1 &= 10\text{k}\Omega & C_1 &= 1\mu\text{F} \\
 R_2 &= 100\text{k}\Omega & C_2 &= 100\text{pF} \\
 R_3 &= 220\text{k}\Omega & \\
 R_4 &= 1\text{k}\Omega & \\
 R_h &= 1\text{k}\Omega & \\
 R_f &= 10\text{k}\Omega & \\
 R_6 &= 47\text{k}\Omega &
 \end{aligned}$$

(A) Plot The ideal  $V_{out}(f)/V_{in}(f)$  gain



$$I_{in} = \frac{V_{in}}{R_1 + R_2}$$

$$V_x = -I_{in}R_3 = -V_{in} \frac{R_3}{R_1 + R_2}$$

$$I_x = \frac{V_x}{R_h}$$

$$I_{bx} = I_x - I_{in}$$

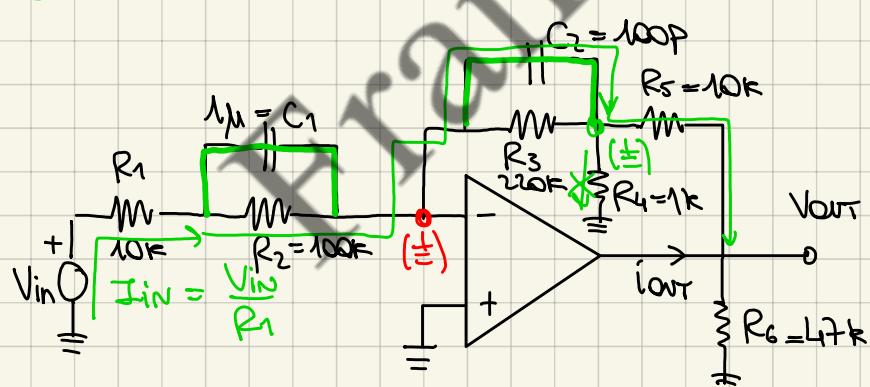
@ DC  $\rightarrow$  C open

$$V_{out} - V_x = I_{R5} \cdot R_5$$

$$\begin{aligned}
 V_{out} &= V_x + (I_x - I_{in})R_5 = V_x + \left( \frac{V_x}{R_h} - \frac{V_x}{R_3} \right) R_5 \\
 &= V_x \left[ 1 + \frac{R_5(R_h + R_3)}{R_3 R_h} \right] = -V_{in} \frac{R_3}{R_1 + R_2} \left[ 1 + \frac{R_5(R_3 + R_4)}{R_3 R_4} \right]
 \end{aligned}$$

$$G_i(0) = -\frac{R_3}{R_1 + R_2} \left[ 1 + \frac{R_5(R_3 + R_4)}{R_3 R_4} \right] \approx -22.1$$

@ HF  $\rightarrow$  C close



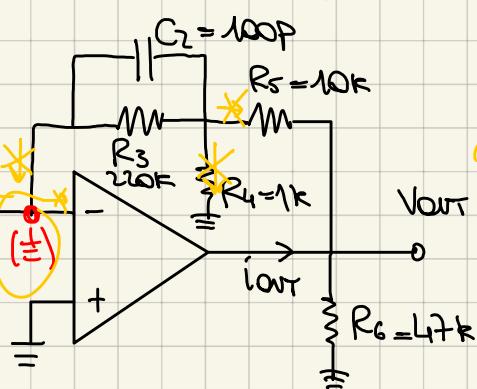
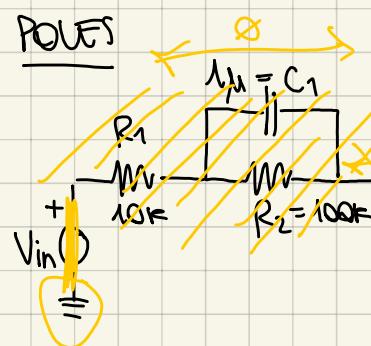
$$V_{out} = -I_{in}R_5$$

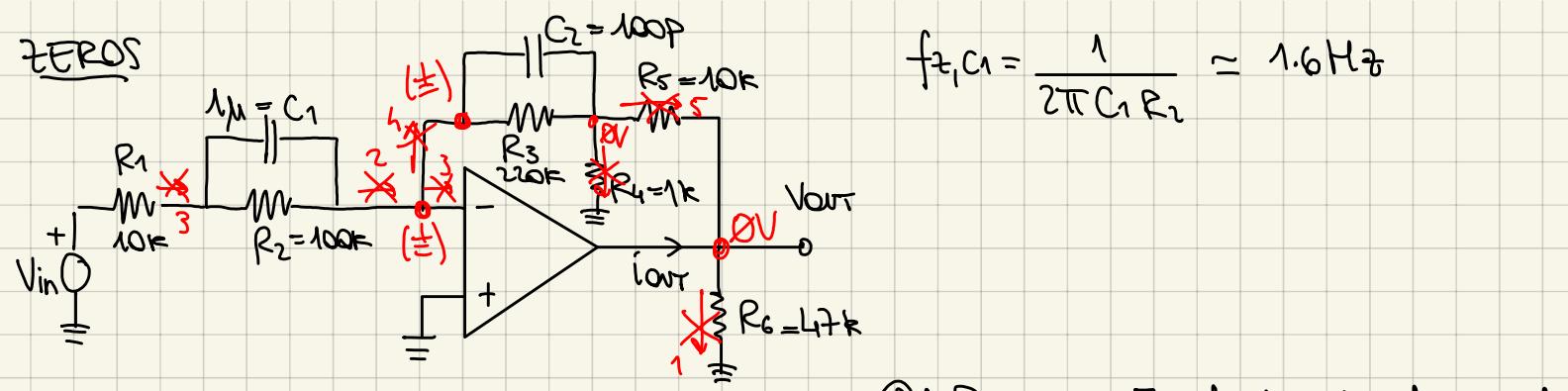
$$I_{in} = -V_{in} \frac{R_5}{R_1}$$

$$G_i(\infty) = -\frac{R_5}{R_1} = -1$$

$$f_{P,C_1} = \frac{1}{2\pi C_1 (R_1 || R_2)} = 17.5\text{Hz}$$

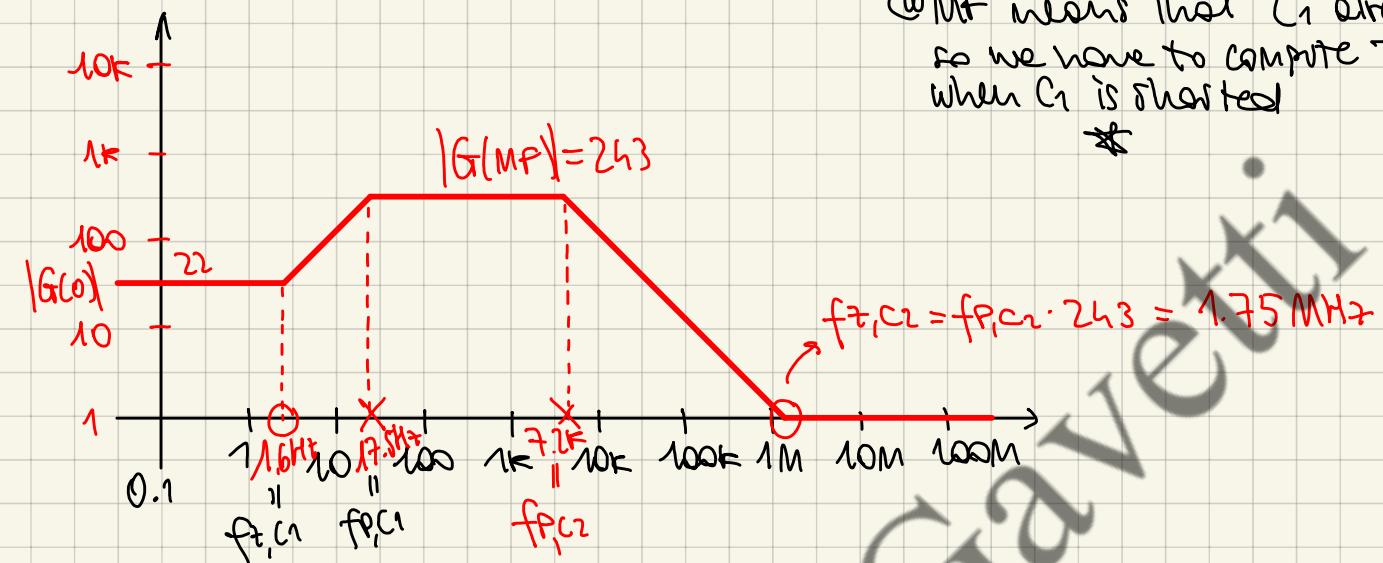
$$f_{P,C_2} = \frac{1}{2\pi C_2 R_3} = 7.2\text{kHz}$$





$$f_z, C_1 = \frac{1}{2\pi C_1 R_2} \approx 1.6 \text{ Hz}$$

@MF means that  $C_1$  already acted so we have to compute the gain when  $C_1$  is shorted



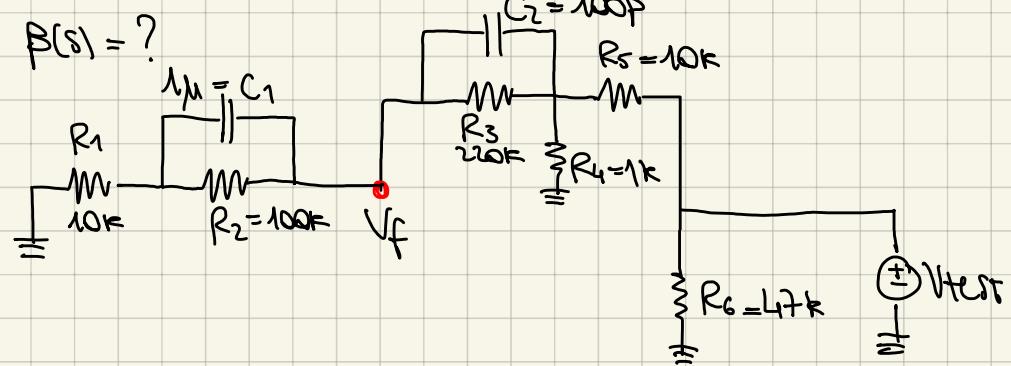
\*  $G(MF) = ?$

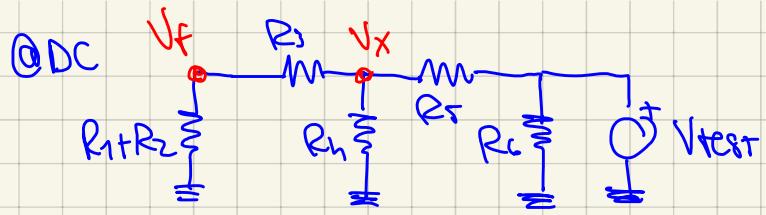
$$\begin{cases} V_{out} - V_x = -\left(\frac{V_x}{R_h} + \frac{V_{in}}{R_1}\right) R_5 \\ V_x = -\frac{V_{in}}{R_1} R_3 \end{cases}$$

$$V_{out} = -V_{in} \frac{R_3}{R_1} + V_x \frac{R_5}{R_4} - V_{in} \frac{R_5}{R_1} =$$

$$= -V_{in} \frac{R_3}{R_1} - V_{in} \frac{R_3 R_5}{R_1 R_4} - V_{in} \frac{R_5}{R_1}$$

$$= -V_{in} \left( \frac{R_3 R_h + R_3 R_5 + R_h R_5}{R_1 R_4} \right) \Rightarrow G(MF) = -\frac{R_3 R_h + R_3 R_5 + R_h R_5}{R_1 R_4} = -243$$

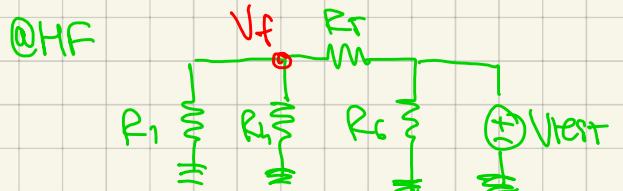




$$V_f = V_x \frac{R_1 + R_2}{(R_1 + R_2) + R_3}$$

$$V_x = V_{\text{test}} \frac{(R_1 + R_2 + R_3) \parallel R_h}{(R_1 + R_2 + R_3) \parallel R_h + R_f}$$

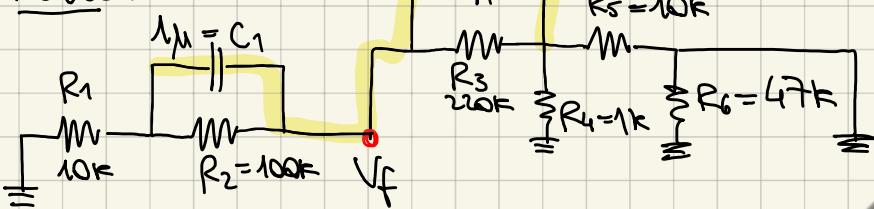
$$\beta(0) = \frac{R_h \parallel (R_1 + R_2 + R_3)}{R_h \parallel (R_1 + R_2 + R_3) + R_f} \cdot \frac{R_1 + R_2}{R_1 + R_2 + R_3} \approx \frac{1}{33} \Rightarrow \frac{1}{\beta}(0) = 33$$



$$V_f = V_{\text{test}} \frac{R_1 \parallel R_h}{R_1 \parallel R_h + R_f}$$

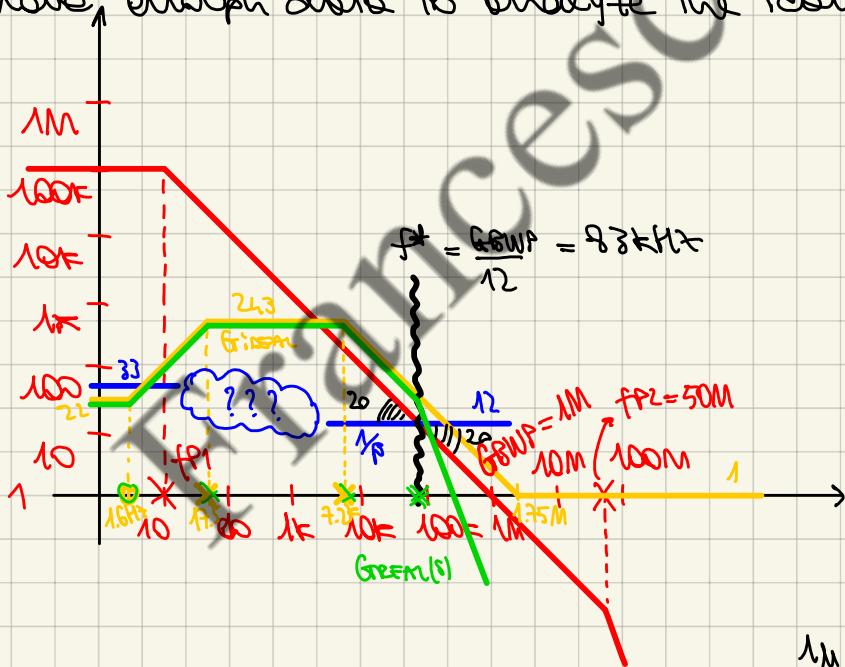
$$\beta(\infty) = \frac{R_1 \parallel R_h}{R_1 \parallel R_h + R_f} \approx \frac{1}{12} \Rightarrow \frac{1}{\beta}(\infty) \approx 12$$

POLIES:



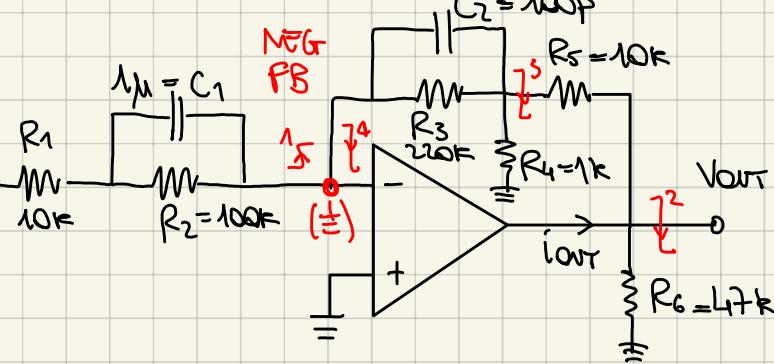
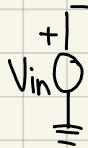
We can see that  $C_1$  and  $C_2$  are dependent to each other  $\Rightarrow$  JUST 1 pole

We should use the To, Too METHOD, but we don't need to do so  
we have enough data to analyze the real behavior of the circuit in an  
intuitive way

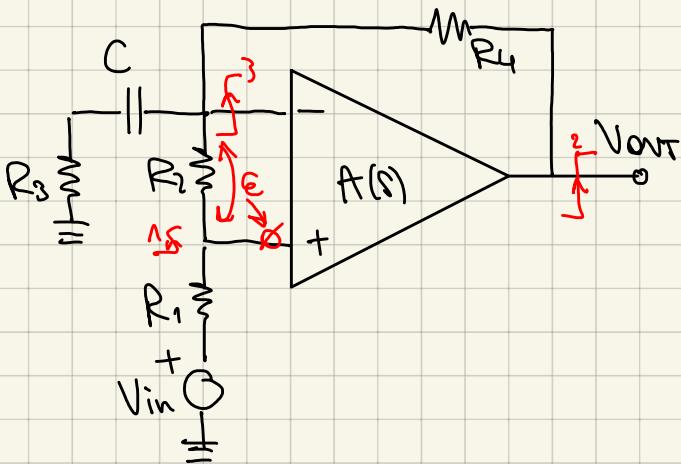


(B) Compute  $i_{\text{out}}$  when  $V_{\text{in}} = -100 \mu\text{V}$

$V_{\text{in}} = -100 \mu\text{V} \rightarrow$  it means that we are @DC



## EXAMPLE 2



$$A_0 = 1000 \text{dB} = 10^5$$

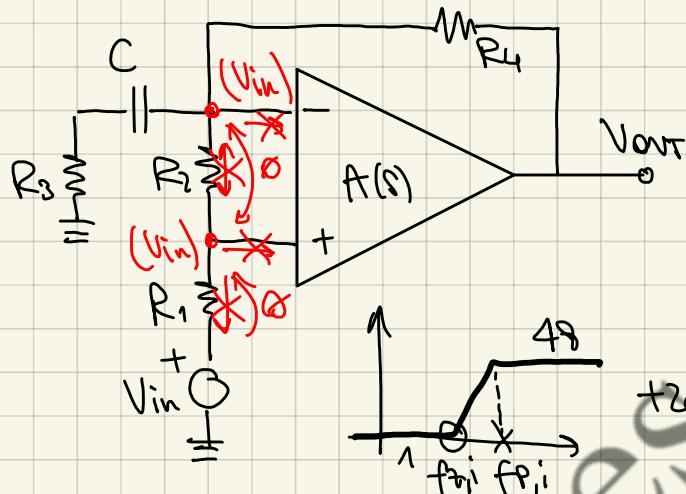
$$GAINP = 100 \text{MHz}$$

$$R_1 = 22 \text{k}\Omega \quad R_2 = R_3 = 1 \text{k}\Omega \quad R_4 = 47 \text{k}\Omega$$

$$C = 1 \text{nF}$$

Ⓐ Plot The Bode diagram of The ideal and real  $V_{out}(f) / V_{in}(f)$

$$GIDEAR = ?$$



ideally if  $GAINP \rightarrow \infty \Rightarrow E \rightarrow 0$   
 $\Rightarrow$  no current flows through  $R_2$

$$@DC \rightarrow GIDEAR(0) = 1$$

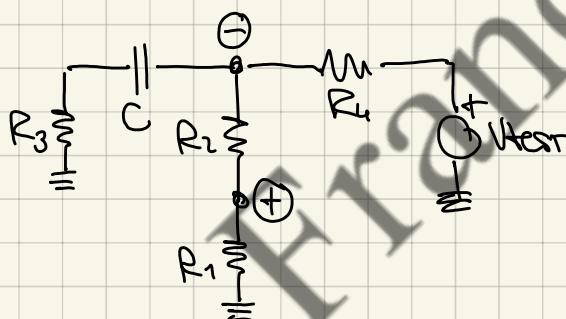
$$@HF \rightarrow GIDEAR(\infty) = \frac{1 + R_4}{R_3} = 48$$

$$f_{P,i} = \frac{1}{2\pi C R_3} = 160 \text{kHz}$$

$$+20 \text{dB/dec} \rightarrow f_{T,i} = \frac{f_{P,i}}{48} \rightarrow f_{T,i} = \frac{160 \text{kHz}}{48} = 3.3 \text{kHz}$$

$$\tilde{\beta} = \tilde{\beta}_f^- - \tilde{\beta}_f^+$$

$$\beta(s) = ?$$



$$@DC \rightarrow \beta(0) = \beta^-(0) - \beta^+(0) =$$

$$= \frac{R_1 + R_2}{R_1 + R_2 + R_h} - \frac{R_1}{R_1 + R_2 + R_h} =$$

$$= \frac{R_2}{R_1 + R_2 + R_h} = \frac{1}{70}$$

NEG. FB

$$\Rightarrow \frac{1}{\beta}(0) = -70$$

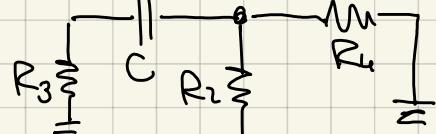
$$@HF \rightarrow \beta(\infty) = \beta^-(\infty) - \beta^+(\infty) = \frac{R_3 || (R_1 + R_2)}{R_3 || (R_1 + R_2) + R_h} \left( 1 - \frac{R_1}{R_1 + R_2} \right) =$$

$$= \frac{R_3 || (R_1 + R_2)}{R_3 || (R_1 + R_2) + R_h} \cdot \frac{R_2}{R_1 + R_2} \approx \frac{1}{48} \cdot \frac{1}{23} =$$

$$R_3 \ll R_1 + R_2 = 23 \text{k}\Omega$$

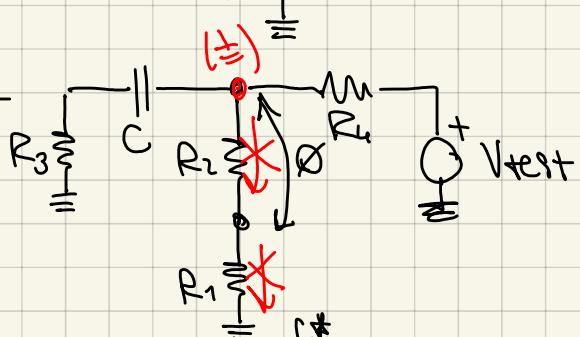
$$\Rightarrow \frac{1}{\beta}(\infty) = -48 \cdot 23 = -1104$$

## POLIES OF B

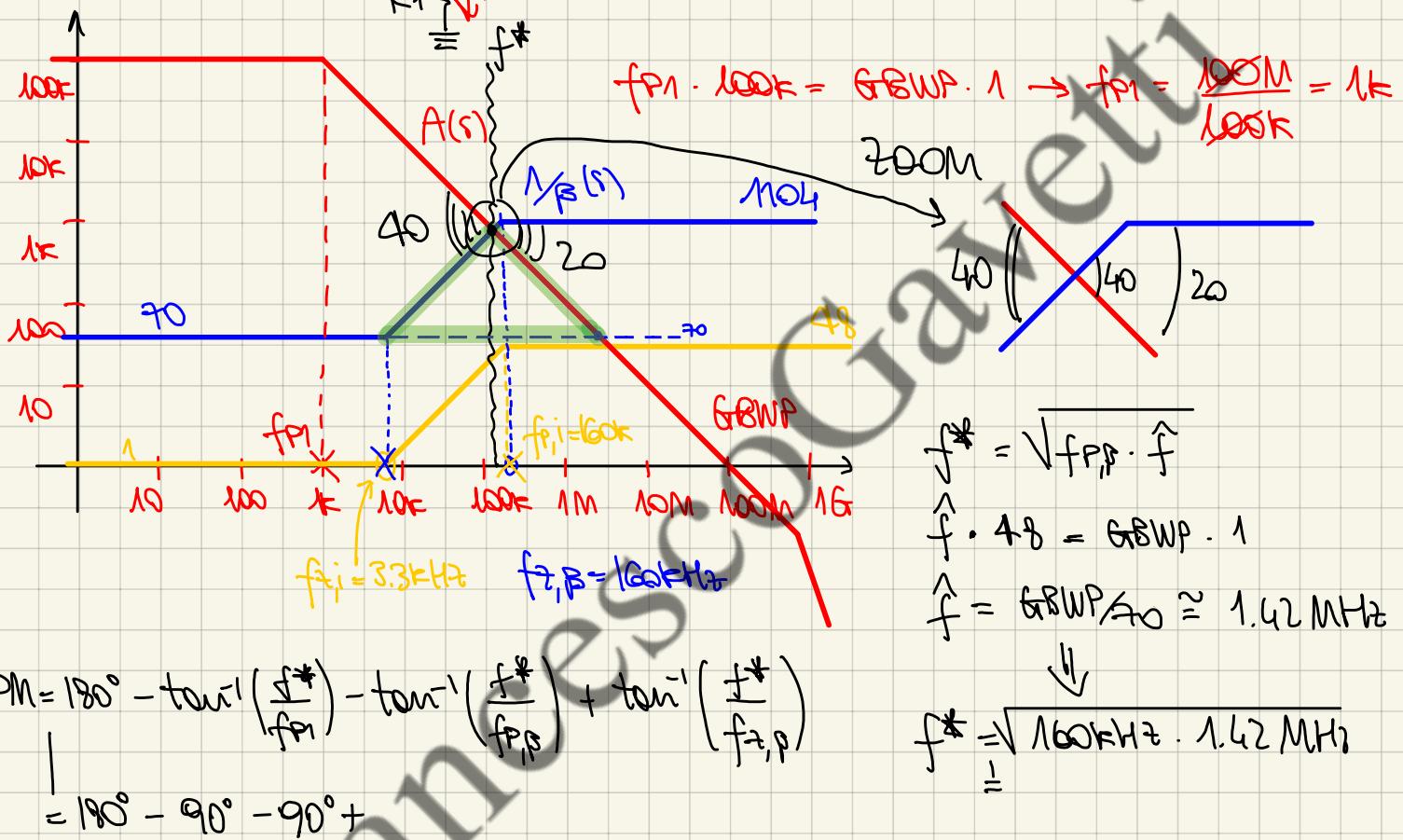


$$f_{P,Q} = \frac{1}{2\pi C [R_3 + (R_2 + R_1) // R_h]} =$$

## ZEROS of $\beta$



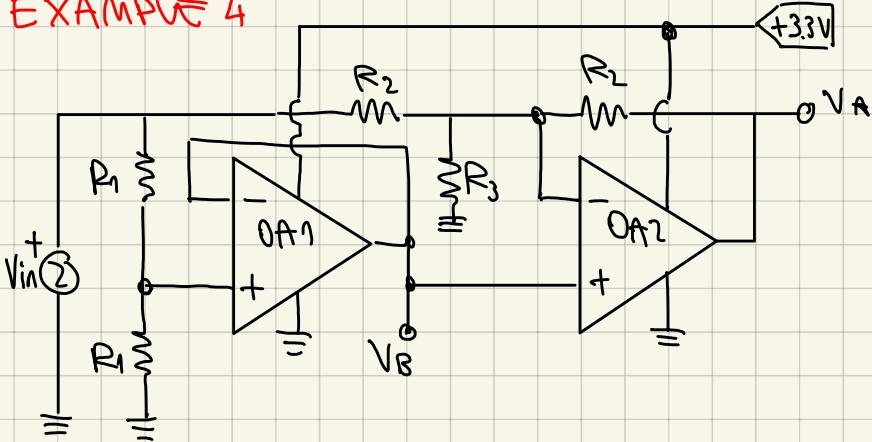
$$f_{T,P} = \frac{1}{2\pi C R_3} = 160 \text{ kHz}$$



## ES04 - FREQUENCY COMPENSATION (4)

06/10/2021

EXAMPLE 4



$$f_0 = 100 \text{ dB} = 10^5$$

$$G_{FBNP} = 100 \text{ MHz}$$

$$R_1 = 67 \text{ k}\Omega$$

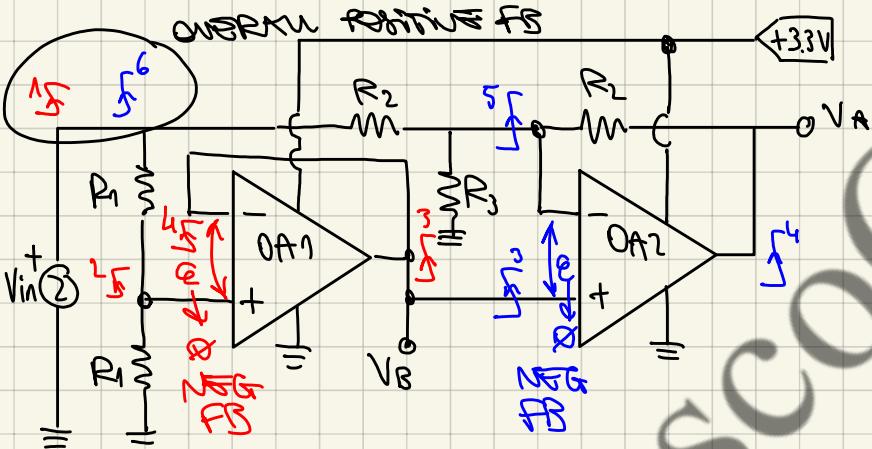
$$R_2 = 220 \text{ k}\Omega$$

$$R_3 = 110 \text{ k}\Omega$$

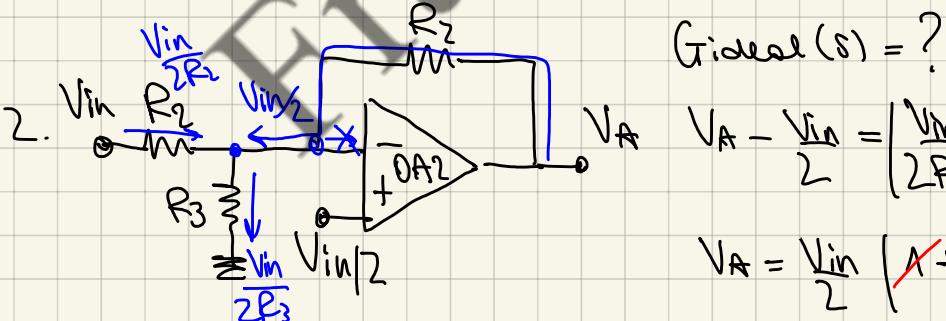
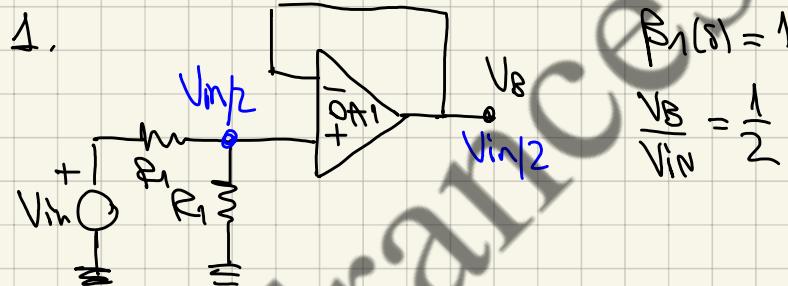
- (A) Compute the real  $V_A(f)/V_{in}(f)$  and  $V_B(f)/V_{in}(f)$  gain and the input impedance.

### FEEDBACK ASSESSMENT

OVERALL FEEDBACK FS



Two negative feedback loops that forms a global positive feedback loop

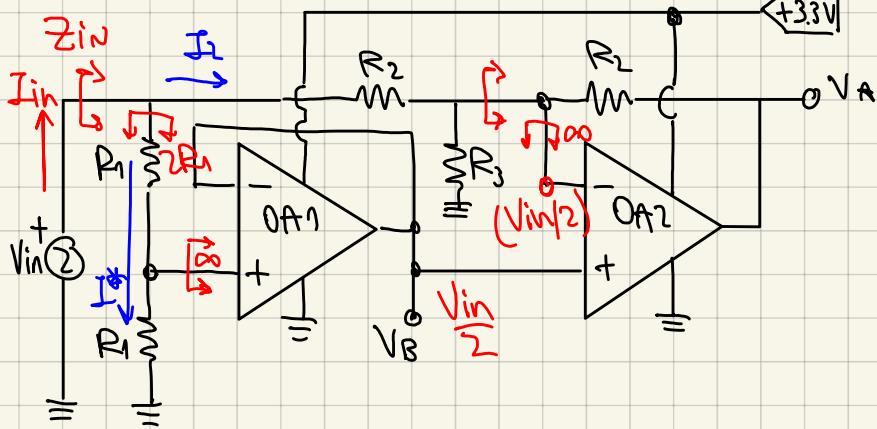


$$G_{ideal}(s) = ?$$

$$V_A - \frac{V_{in}}{2} = \left(\frac{V_{in}}{2R_3} - \frac{V_{in}}{2R_2}\right) \cdot R_2$$

$$V_A = \frac{V_{in}}{2} \left(1 + \frac{R_1}{R_3} - 1\right)$$

$$\frac{V_A}{V_{in}} = \frac{1}{2} \frac{R_1}{R_3}$$



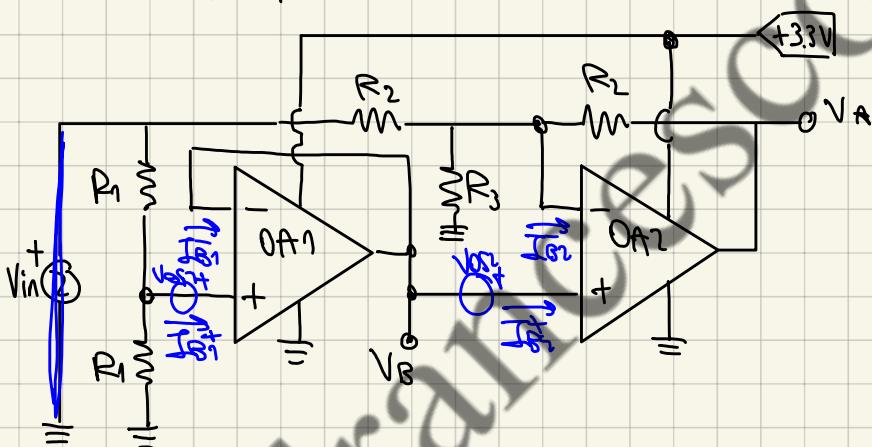
$$Z_{in} = \frac{V_{in}}{I_{in}} =$$

$$I_{in} = I^* + I_2 \quad \text{where} \quad \begin{cases} I^* = \frac{V_{in}}{2R_1} \\ I_2 = \frac{(V_{in} - V_{in}/2)}{R_2} = \frac{V_{in}}{2R_2} \end{cases}$$

$$\Rightarrow \frac{I_{in}}{V_{in}} = \frac{1}{2R_1} + \frac{1}{2R_2} = \frac{1}{2(R_1||R_2)}$$

$$\Rightarrow Z_{in} = \frac{V_{in}}{I_{in}} = 2(R_1||R_2) = 2R_1||2R_2$$

⑤ Compute the output static error on  $V_A$ , due to  $I_{BS} = 10\text{nA}$  and  $V_{OS} = 5\text{mV}$  of both OpAmps



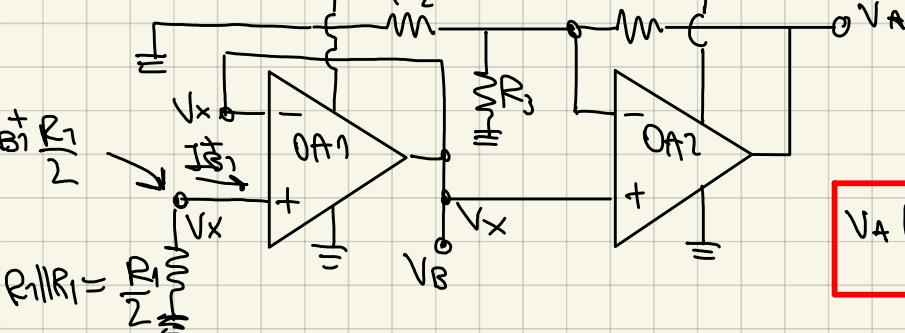
$$V_A(\epsilon) = V_{A1}(V_{OS1}) + V_{A1}(V_{OS2}) + V_A(I_{BS1}^-) + V_A(I_{BS1}^+) + V_A(I_{BS2}^-) + V_A(I_{BS2}^+)$$

$$V_A(V_{OS1}) = ? \quad V_{OS1} = V_A \frac{R_2||R_3}{R_1||R_2 + R_3} \rightarrow V_A(V_{OS1}) = V_{OS} \left( 1 + \frac{R_2}{R_1||R_3} \right) = V_A(V_{OS2})$$

$$V_A(I_{BS1}^+) = ?$$

$$V_x = V_A \frac{R_2||R_3}{R_2||R_3 + R_1}$$

$$V_x = -I_{BS1} \frac{R_1}{2}$$

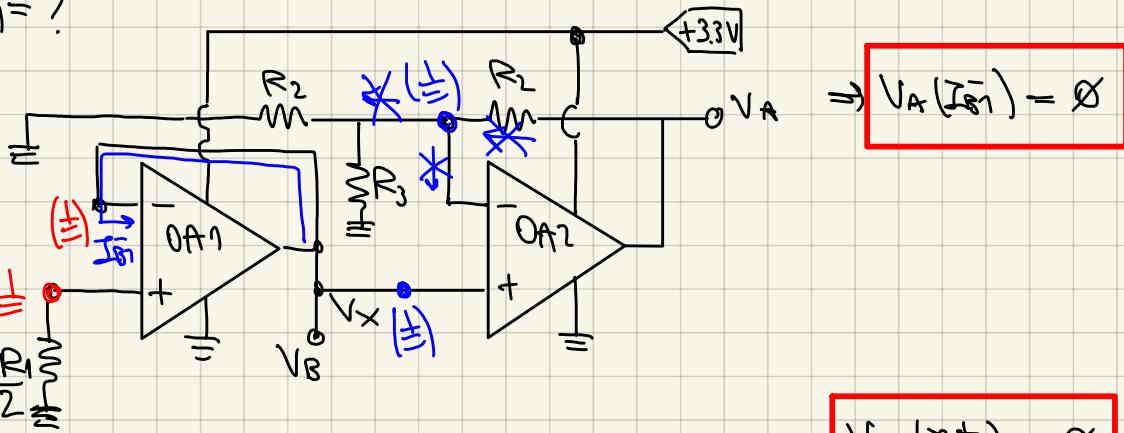


$$R_1||R_1 = \frac{1}{2} \frac{R_1}{R_2}$$

$$V_A(I_{BS1}^+) = V_A \left( 1 + \frac{R_2}{R_2||R_3} \right)$$

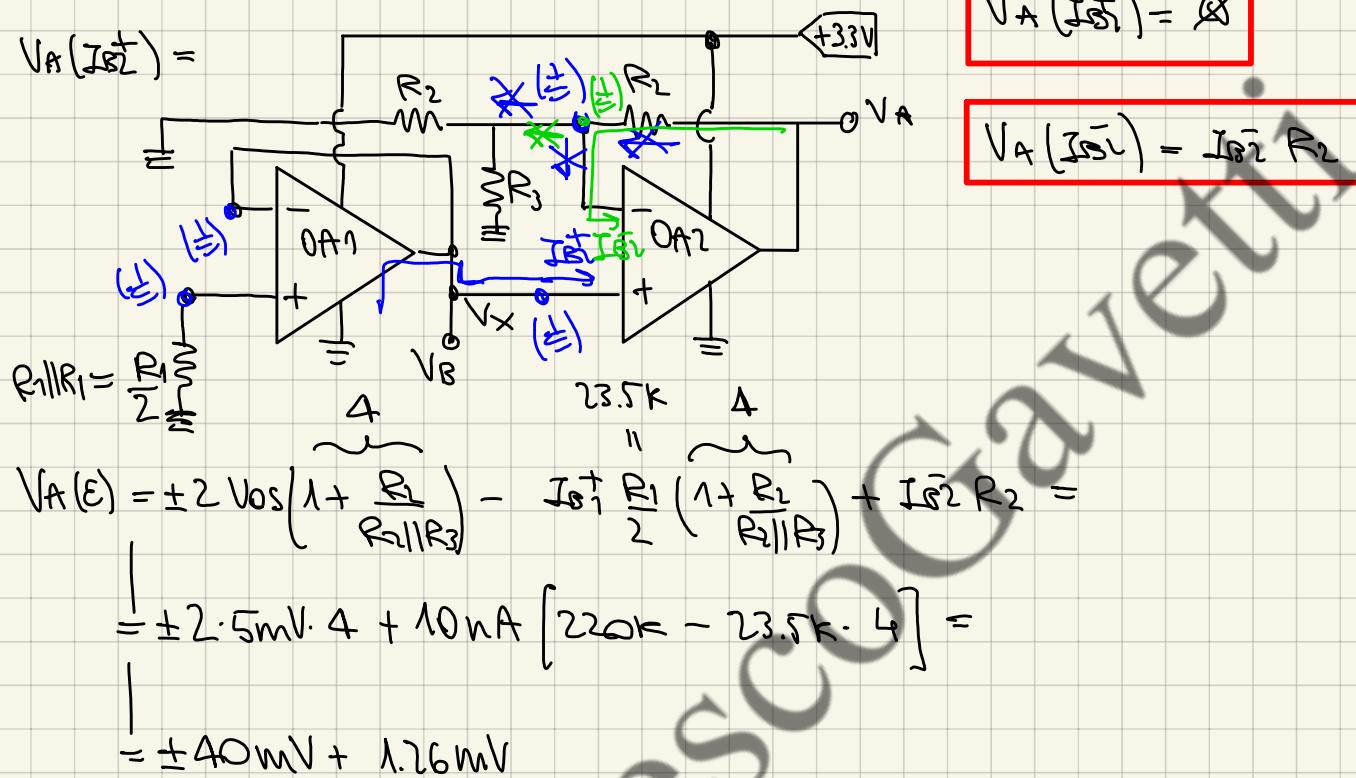
$$V_A(I_{BS1}^+) = I_{BS1}^+ \left[ -\frac{R_1}{2} \left( 1 + \frac{R_2}{R_2||R_3} \right) \right]$$

$$V_A(I_{S1}^-) = ?$$



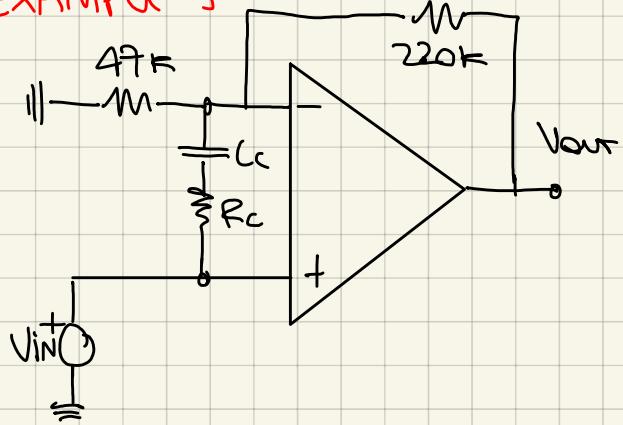
$$R_1 \parallel R_1 = \frac{1}{2} R_1$$

$$V_A(I_{S1}^-) =$$



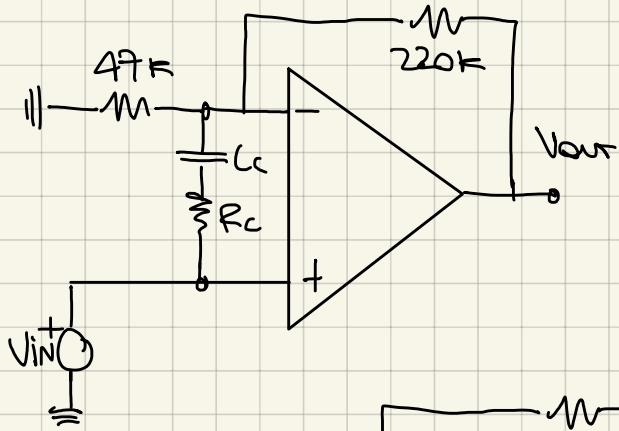
$$\begin{aligned} V_A(E) &= \pm 2 V_{OS} \left( 1 + \frac{R_1}{R_1 \parallel R_3} \right) - I_{S1}^+ \frac{R_1}{2} \left( 1 + \frac{R_2}{R_2 \parallel R_3} \right) + I_{S1}^- R_2 = \\ &= \pm 2 \cdot 5 \text{mV} \cdot 4 + 10 \text{nA} \left[ 220 \text{k} - 23.5 \text{k} \cdot 4 \right] = \\ &= \pm 40 \text{mV} + 1.26 \text{mV} \end{aligned}$$

### EXAMPLE 5



Uncompensated Opamp w/  
 $A_0 = 12 \text{dB} = 10^{\frac{A_0}{20}} = 1M$   
 $f_0 = 5 \text{kHz}$   
 $f_1 = 50 \text{MHz}$   
 $I_B = 1 \text{nA}, V_{os} = 3 \text{mV}$

(A) Without  $C_c$  and  $R_c$  compute stability and PM

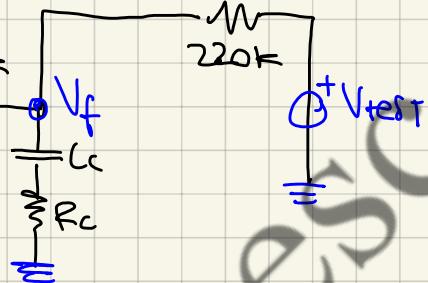


$$G_i(s) = ?$$

$$G_i(0) = 1 + \frac{220k}{47k} \approx 5.68$$

$$G(\infty) = 1 + \frac{220k}{47k} \approx 5.68$$

$$\beta(s) = ?$$



$$\beta(0) = \frac{47k}{47k + 220k}$$

$$\Rightarrow \frac{1}{\beta}(0) = 1 + \frac{220k}{47k} \approx 5.68$$

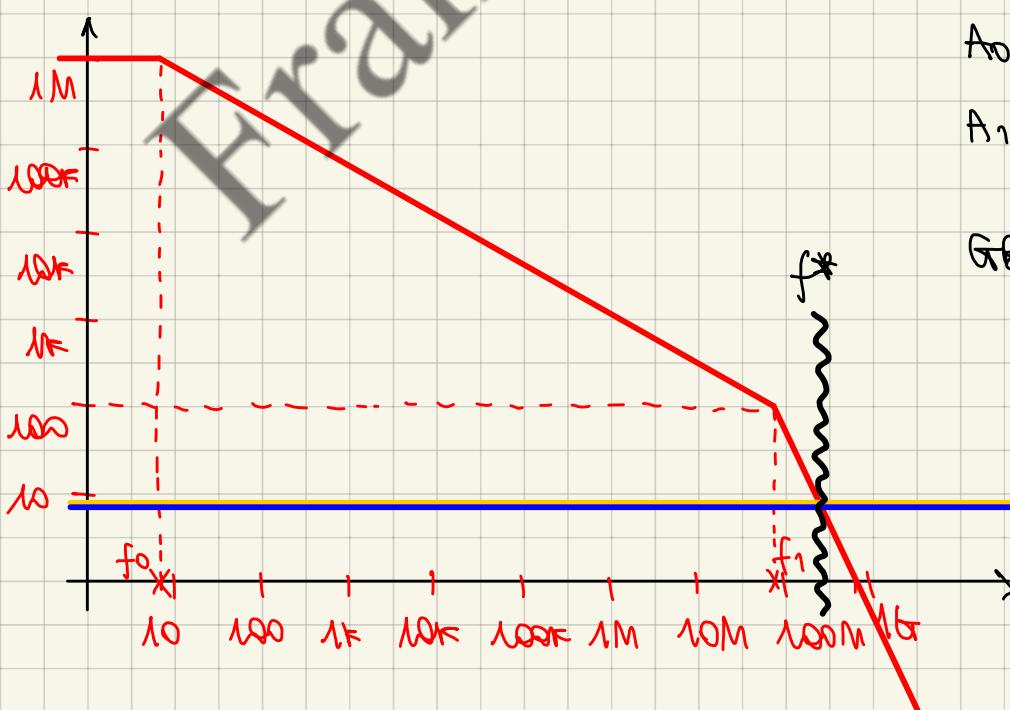
↓

$$\text{w/o } C_c \text{ and } R_c \\ \Rightarrow \frac{1}{\beta}(s) = 5.68$$

$$A_0 \cdot f_0 = A_1 \cdot f_1$$

$$A_1 = \frac{A_0 f_0}{f_1} = \frac{10^{\frac{12}{20}} \cdot 5k}{50M} = 100$$

$$\text{GBWP} = f_1 \sqrt{A_1} = \omega f_1 = \\ = 500 \text{MHz}$$



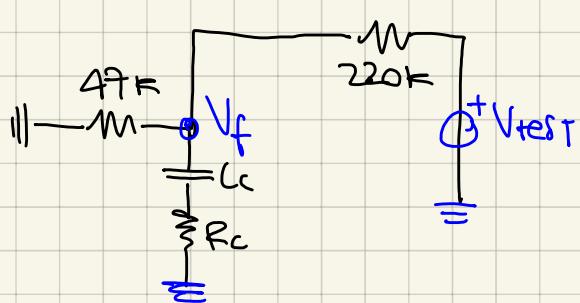
$$G_i(s) = 5.68$$

$$\frac{1}{\beta}(s) = 5.68 \text{ w/o } C_c, R_c$$

$$f^* = \frac{\sqrt{A_1} \cdot f_1}{\sqrt{5.68}} = \frac{10f_1}{\sqrt{5.68}} = \frac{500M}{\sqrt{5.68}} \cong 210 \text{ MHz}$$

$$\text{PM} = 180^\circ - \tan^{-1}\left(\frac{f^*}{f_0}\right) - \tan^{-1}\left(\frac{f^*}{f_1}\right) = 180 - 90^\circ - 77^\circ \cong 13^\circ$$

(B) Properly size  $C_c$  and  $R_c$  to obtain  $\text{PM} = 90^\circ$



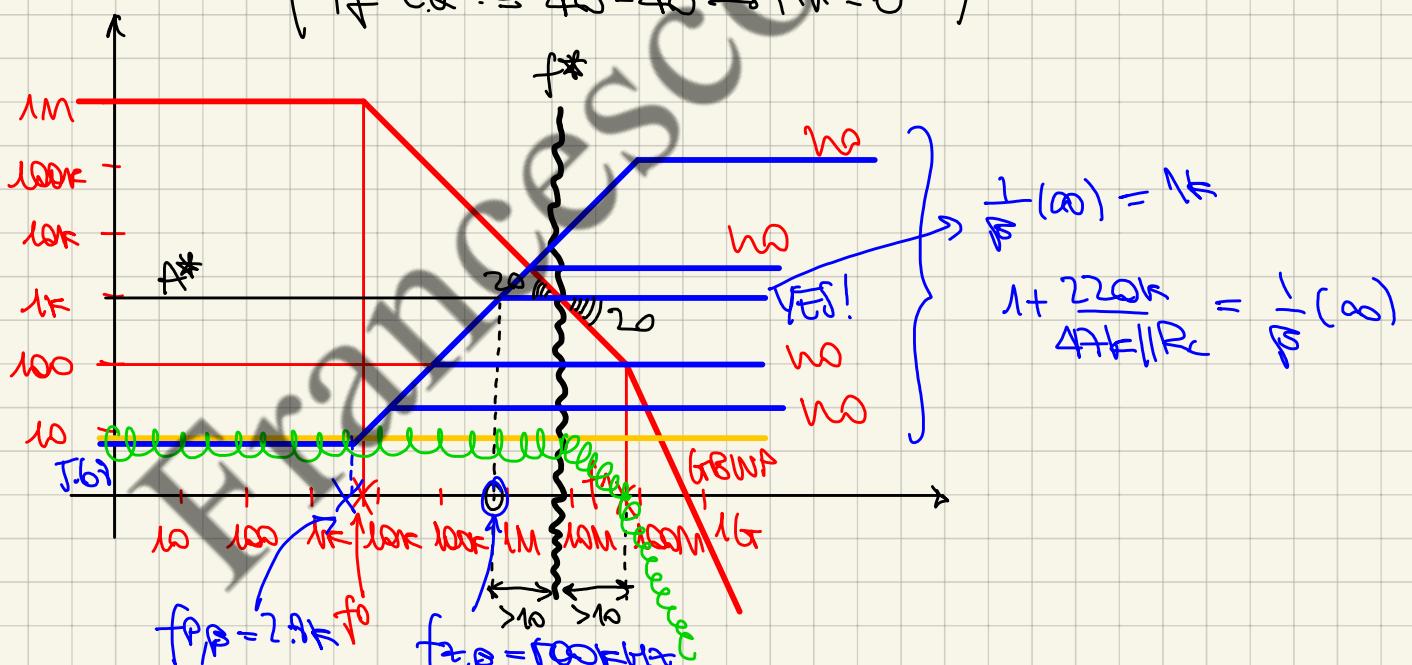
$$f_{P,P} = \frac{1}{2\pi C_c [R_c + 47k || 220k]}$$

$$f_{Z,P} = \frac{1}{2\pi C_c R_c}$$

$$\text{PM} = 180^\circ - 90^\circ - 77^\circ - \tan^{-1}\left(\frac{f^*}{f_{P,P}}\right) + \tan^{-1}\left(\frac{f^*}{f_{Z,P}}\right) = 90^\circ$$

$f^* = ?$  Notice: to have  $\text{PM} = 90^\circ \rightarrow$  we need a 20-20 closure angle

$$\begin{cases} \text{if } \text{C.Q.} = 20-40 \rightarrow \text{PM} \cong 45^\circ \\ \text{if } \text{C.Q.} = 40-40 \rightarrow \text{PM} \cong 0^\circ \end{cases}$$



In order to have a closure angle of 20-20

$$f^* = \frac{f_1}{10} = 5 \text{ MHz}$$

$$f_{P,P} = \frac{f^*}{10} = \frac{f_1}{100} = \frac{50M}{100} = 500 \text{ kHz} = \frac{1}{2\pi C_c R_c}$$

$$f^* \cdot A^* = f_1 \cdot A_1 \Rightarrow A^* = \frac{f_1 A_1}{f^*} = \frac{50M \cdot 100}{5M} = 1000 = 1k$$

$$\Rightarrow \frac{1}{R}(\infty) = 1 + \frac{220k}{47k \parallel R_C} = 1k$$

$$\Rightarrow \frac{220k}{47k \parallel R_C} \approx 1k \rightarrow \frac{47k \cdot R_C}{47k + R_C} = 220$$

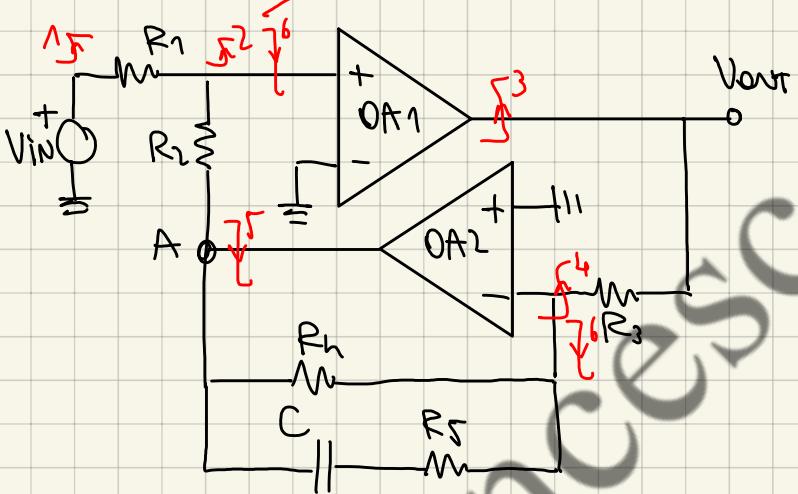
$$47k \cdot R_C - 220 \cdot R_C = 47k \cdot 220$$

$$R_C = \frac{47k \cdot 220}{47k - 220} \approx 220\text{k}$$

$$\frac{1}{2\pi C_c R_C} = 500\text{kHz} \rightarrow C_c = \frac{1}{2\pi \cdot 220 \cdot 500\text{k}} \approx 1.45\text{nF}$$

$$f_{P,B} = \frac{1}{2\pi C_c [R_C + 47k \parallel 220k]} \approx 2.8\text{Hz}$$

### EXAMPLE 6



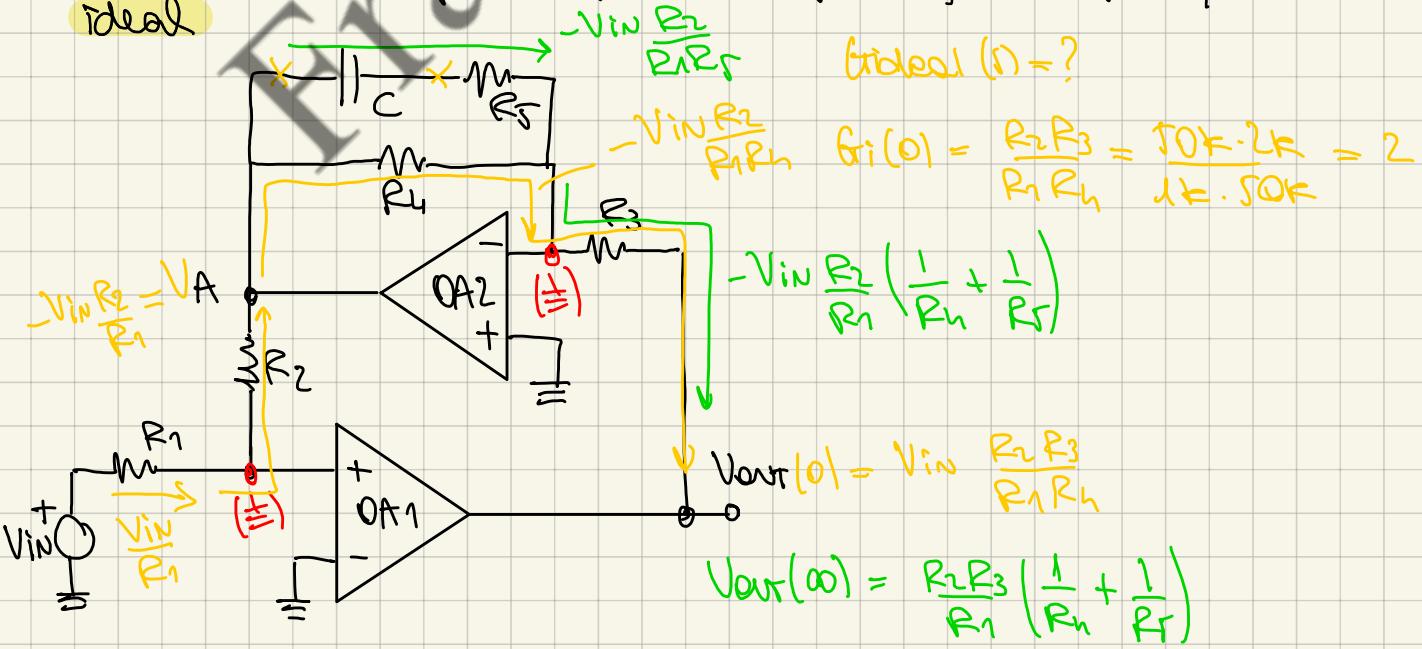
$$A_0 = 120 \text{dB} = 10^4$$

$$GBWP = 20\text{MHz}$$

$$I_B = 10\text{nA}, V_{os} = 5\text{mV}$$

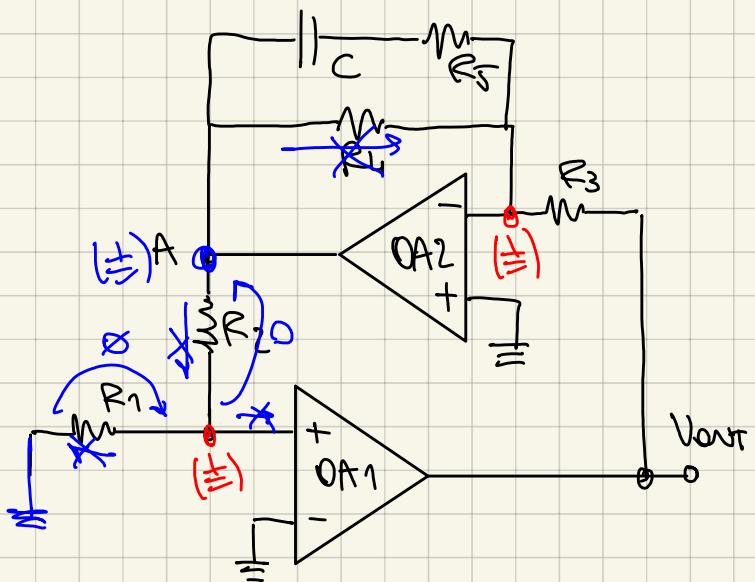
$$\begin{aligned} R_1 &= 1k \\ R_2 &= 50k \\ R_3 &= 2k \\ R_4 &= 50k \\ R_5 &= 1k \end{aligned}$$

④ Plot The Bode diagram of The  $V_{out}(f) / V_{in}(f)$  real gain, when OA2 is  $\approx$  ideal



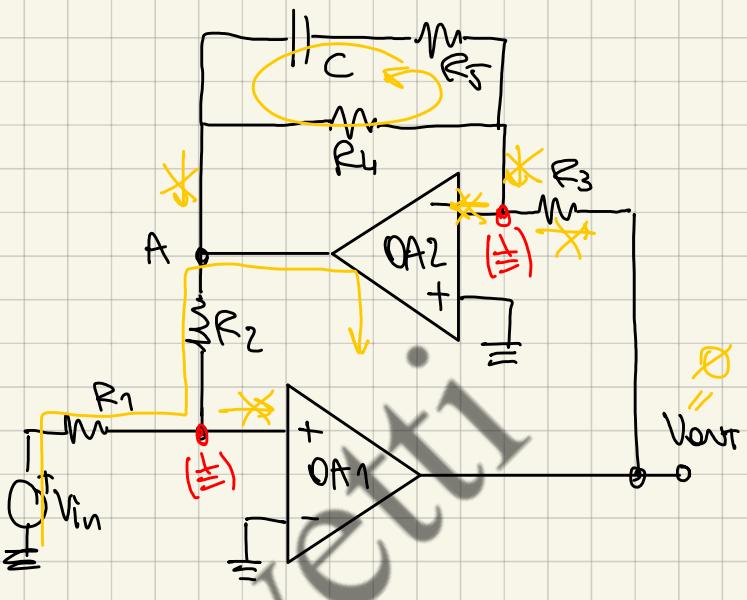
$$G_i(\infty) = \frac{R_2 R_3}{R_1} \left( \frac{1}{R_h} + \frac{1}{R_f} \right) = \frac{50k \cdot 2k}{1k} \left( \frac{1}{50k} + \frac{1}{1k} \right) = 102$$

POLE

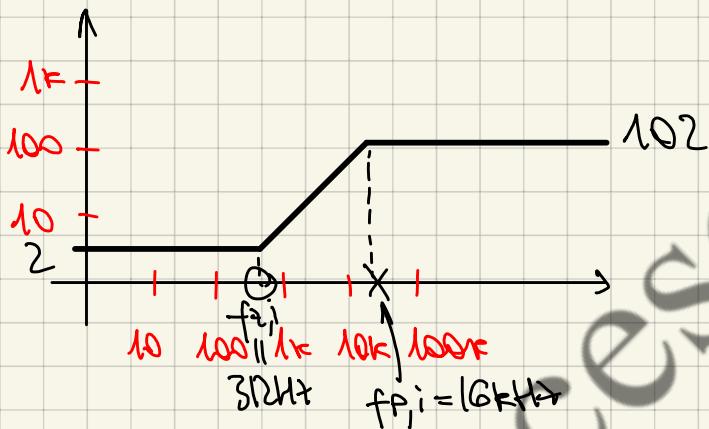


$$f_{P,i} = \frac{1}{2\pi C R_f} \approx 16 \text{ kHz}$$

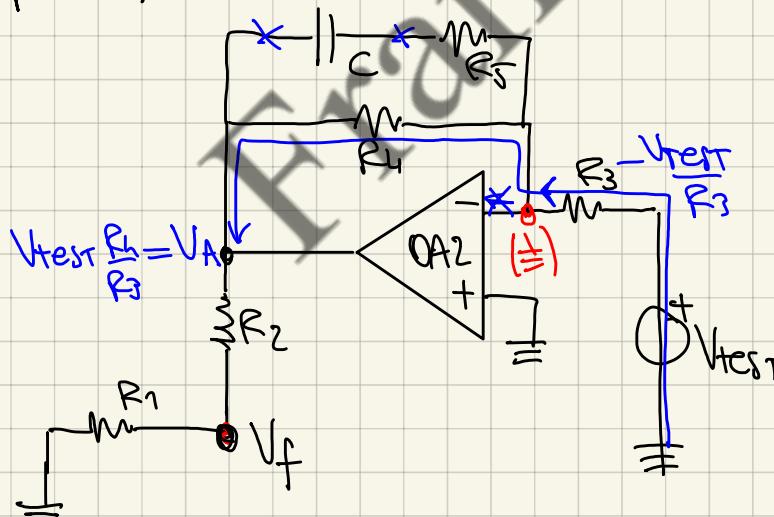
ZERO



$$f_{Z,i} = \frac{1}{2\pi (R_h + R_f)} = 812 \text{ Hz}$$

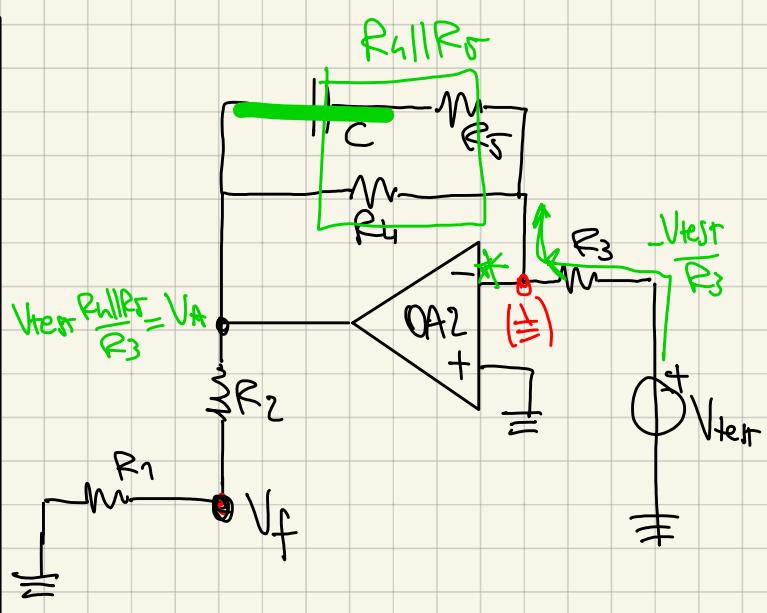


$$\beta(s) = ?$$



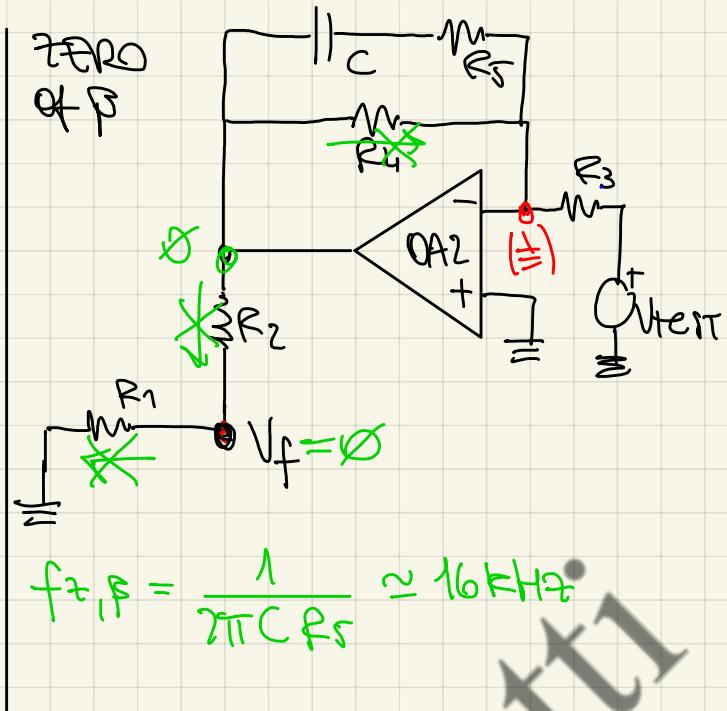
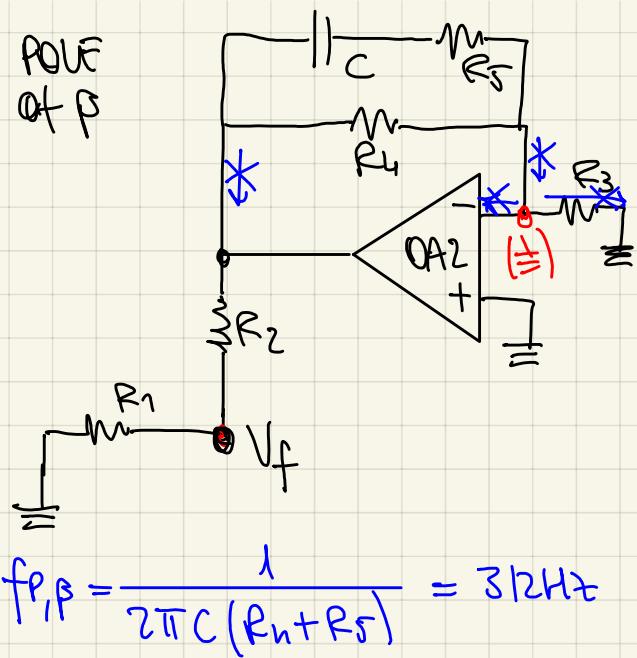
$$\beta(0) = ? = \frac{R_h}{R_3} \cdot \frac{R_1}{R_1 + R_2} = 0.5$$

$$\Rightarrow \frac{1}{\beta}(0) = 2$$



$$\beta(\infty) = \frac{R_h || R_f}{R_3} \cdot \frac{R_1}{R_1 + R_2} = 9.8 \text{ M}$$

$$\Rightarrow \frac{1}{\beta}(\infty) = 102$$



$$\Rightarrow \frac{1}{\beta}(s) = G_i(s)$$

