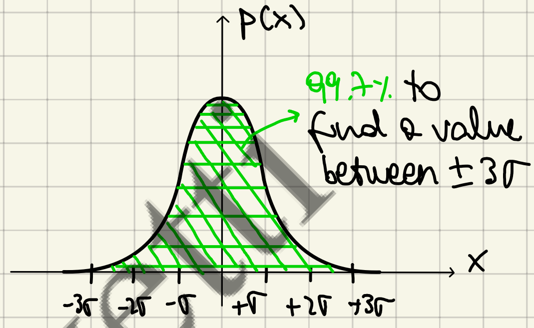
ES05 - NOISE
12/10/2021

NOISE AND POWER SPECTRUM

$$\boxed{x(t) = s(t) + n(t) + d(t)}$$

$$\begin{cases} s(t) = \text{signal} \\ n(t) = \text{noise} \\ d(t) = \text{distortion} \end{cases}$$

Noise is a random process of which we cannot define an instantaneous value.
What we can define is the noise distribution



GAUSSIAN DISTRIBUTION: $\boxed{p(x) = \dfrac{1}{\sqrt{2\pi}\,\sigma_x} e^{-x^2/2\sigma_x^2}}$

where $\displaystyle\int_{-\infty}^{+\infty} p(x)\,dx = 1$

$\boxed{\text{noise peak-to-peak value} = 6\sigma}$

FULL-WIDTH AT HALF MAXIMUM: $\boxed{FWHM = 2.35\,\sigma_x}$

Since noise is a random process, instantaneous value is nonsense, but it's better measuring power:

NOISE POWER: $\boxed{P = \dfrac{1}{T}\displaystyle\int_0^T |x(t)|^2\,dt}$

We are looking only for ergodic processes (time average = sample average) and gaussian, w/ nil mean value.

POWER = VARIANCE = MEAN SQUARED VALUE

$$\boxed{\sigma_x^2 = \langle x^2(t)\rangle = \frac{1}{T}\int_0^t x^2(t)\,dt = \int_0^\infty S(f)\,df}$$

where $S(f)$ = power spectrum / power spectral density

ROOT MEAN SQUARE VALUE (RMS): $\boxed{x_{rms} = \sqrt{\sigma_x^2} = \sigma_x}$

↓

measures the width of the Gaussian distribution that describes the noise process

CORRELATION AMONG NOISE SOURCES:

TWO NOISE SOURCES: $V_t(t) = V_1(t) + V_2(t)$

MEAN TOTAL VALUE: $\langle V_t(t)\rangle = 0$ (ergodic Gaussian process w/ nil mean value)

TOTAL VARIANCE $\langle V_t^2(t)\rangle = \langle [V_1(t) + V_2(t)]^2\rangle = \langle V_1^2(t)\rangle + 2\langle V_1(t)V_2(t)\rangle + \langle V_2^2(t)\rangle$

• in case of no correlation → $\langle V_1(t) \cdot V_2(t) \rangle = 0$ ⇒ $\langle V_t^2(t) \rangle = \langle V_1^2(t) \rangle + \langle V_2^2(t) \rangle$

• in case of total correlation → $V_1(t) = V_2(t)$ ⇒ $\langle V_t^2(t) \rangle = 4 \langle V_1^2(t) \rangle$

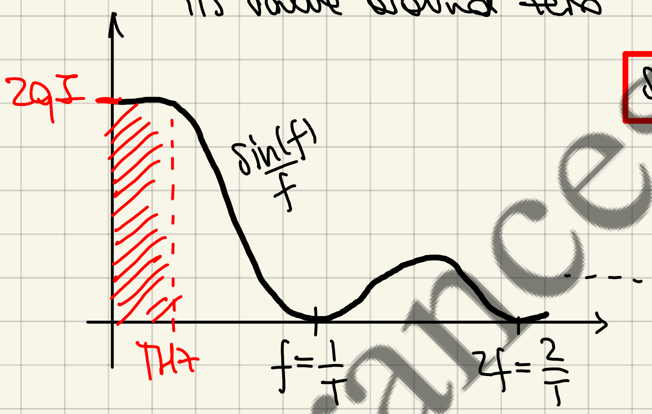
How can we say if two sources are correlated or not ?
If we miss the correlation we cause an error equal to 100% in terms of power which corresponds to an error of 41% in terms of rms
↓
$\boxed{\text{So let's consider all noise sources as uncorrelated !!}}$

⇒ $\sigma_{out}^2 = \sigma_1^2 + \sigma_2^2 + \sigma_3^2 + \sigma_n^2 + \dots = \sum_i \sigma_i^2$ where i = noise sources

TYPES OF NOISE

1 SHOT (GRANULAR) NOISE (or POISSON NOISE)
↓ (quantization)
it's due to "granularity" of charge flowing through a junction

The current flow is given by the electrons transition, but the electrons pass in a randomic way, so the current flow is not a perfect sequence of deltas equally separated, but it's a random sequence of them
↓
The power spectrum is a kind of sinc function which is almost constant up to very high frequencies (~THz) for this reason we can approximate it w/ its value around zero



$\boxed{S(f) \simeq S(0) = 2qI}$ SHOT NOISE POWER SPECTRUM

$\boxed{q = 1.6 \cdot 10^{-19} C}$ ELECTRON'S CHARGE

Notice:
• independent of operating freq. → white noise
• shows up only if there's a current flow

SHOT NOISE POWER (MSV) : $\boxed{\sigma^2 = \langle i^2 \rangle = 2qI \cdot \Delta f}$

where Δf = BANDWIDTH OF THE INSTRUMENTATION USED TO MEASURE THE NOISE

↳ Beware: it's not the operating frequency → σ_{shot} is independent of f

REMEMBER :
• if $I = 50 mA$ → $\sigma = 4 \ pA/\sqrt{Hz}$
• if $I = 5 mA$ → $\sigma = 40 \ pA/\sqrt{Hz}$

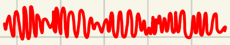
} The rms value increases w/ $\sqrt{I}$

if we increase the current the noise power and rms increase, but also the SNR does

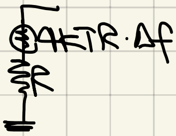
$$SNR = \frac{I}{\sigma} = \frac{I}{\sqrt{2q} \cdot \sqrt{I}} = \frac{\sqrt{I}}{\sqrt{2q}} \div \sqrt{I}$$

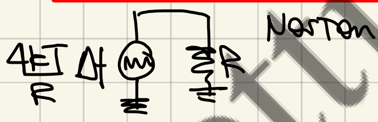
② THERMAL (JOHNSON) NOISE

it's caused by the random thermal motion of electrons w/in the electronic component

- This random motion doesn't depend on the presence of a DC current
- The rms value depends on the squared root of the temperature $\sqrt{T}$ and of the resistive value $\sqrt{R}$
- It's independent of frequency → white noise 

THERMAL NOISE POWER: $\boxed{\sigma_v^2 = \langle v^2 \rangle = 4kTR\,\Delta f}$ or $\boxed{\sigma_i^2 = \langle i^2 \rangle = \frac{4kT}{R}\Delta f}$

Thev. 

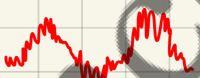
Norton 

where $\boxed{4kT = 1.66 \cdot 10^{-20} \ \frac{V^2}{Hz\,\Omega}}$

THERMAL NOISE RMS: $\boxed{\sigma_v = \sqrt{4kTR\,\Delta f}}$ or $\boxed{\sigma_i = \sqrt{\frac{4kT}{R}\Delta f}}$

REMEMBER:
- if $R = 1k\Omega \rightarrow \sigma = 4nV/\sqrt{Hz}$
- if $R = 100k\Omega \rightarrow \sigma = 40nV/\sqrt{Hz}$

③ FLICKER (1/f) NOISE

- It's proportional to the current flow
- The power spectrum depends on 1/f
- It's a colored noise  (pink noise) → higher @ low freq and lower @ HF

1/f NOISE POWER $\boxed{\sigma_i^2 = \langle i^2 \rangle = k\frac{I^a}{f^b}\Delta f}$

- constant power for each decade: $P = \int_{f_1}^{f_2} \frac{kI}{f}\,df = kI\ln\left(\frac{f_2}{f_1}\right)$

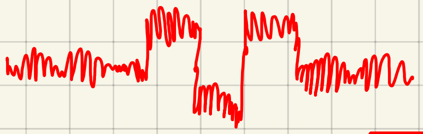
if we consider 1 decade $(f_2 = 10f_1)$ → $\boxed{P_{one\,decade} = kI\ln(10) = kI \cdot 2.3}$

- from a mathematical stand point flicker noise diverges @ 0 Hz, but in reality it doesn't

$f_{(real)\,lower} = \frac{1}{24h \ 60min \ 60sec} \simeq 10^{-5}\,Hz \neq 0$

- it's comparable to white noise @ NOISE CORNER FREQUENCY

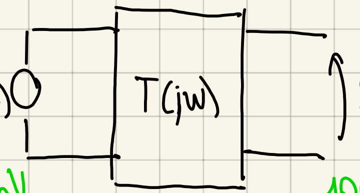
④ BURST (POP-CORN) NOISE



→ it's no more Gaussian

BURST NOISE POWER $\boxed{\sigma^2 = \langle i^2 \rangle = K_1 \dfrac{I^c}{1+\left(\frac{f}{f_c}\right)^2} \Delta f}$

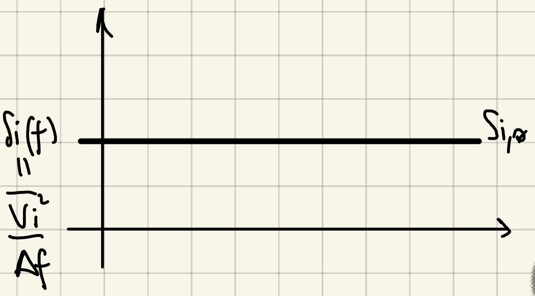
NOISE IN-OUT TRANSFER (FREQUENCY RESPONSE)



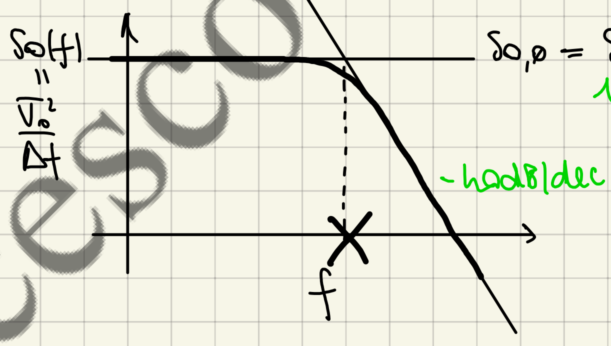
$\Rightarrow S_{out}(f) = S_{in}(f)\,|T(jw)|^2$

For noise only "power" transfer (modulus) matters (independent of phase):

Input noise:



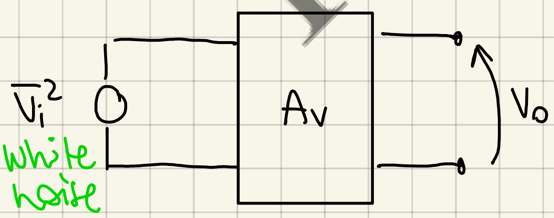
Output noise:



TOTAL OUTPUT NOISE POWER:

$$\langle v_{oT}^2 \rangle = \int_0^\infty S_o(f)\,df = \int_0^\infty |A_v^2(f)| \cdot S_{i,\varnothing}\,df = S_{i,\varnothing} \cdot \int_0^\infty |A_v^2(f)|\,df$$

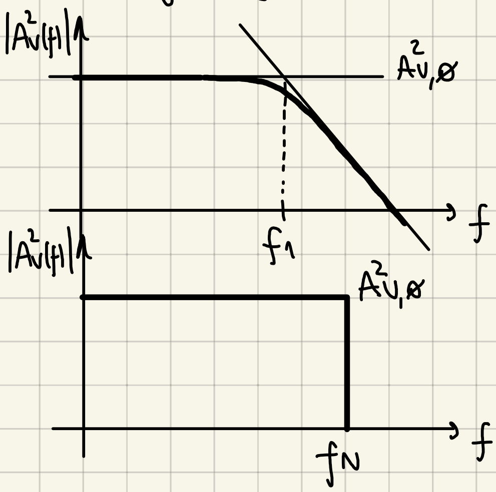
NOISE EQUIVALENT BANDWIDTH



$\boxed{\langle v_{oT}^2 \rangle = S_{i,\varnothing} \cdot A_{v,\varnothing}^2\, f_N}$

constant

where $f_N = \Delta f$ to consider

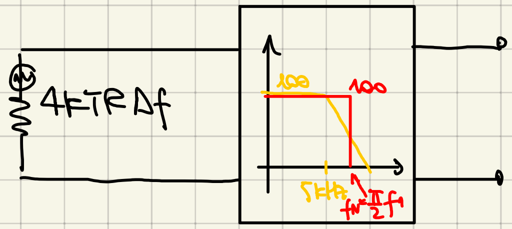


NOISE EQUIVALENT BANDWIDTH: $f_N = \dfrac{1}{A_{v,\varnothing}^2} \displaystyle\int_0^\infty |A_v^2(f)|\,df$

SIMPLE CASES:

- 1 pole: $f_N = \frac{\pi}{2} f_1 = 1.57 f_1$
- 2 poles: $f_N = 1.22 f_1$

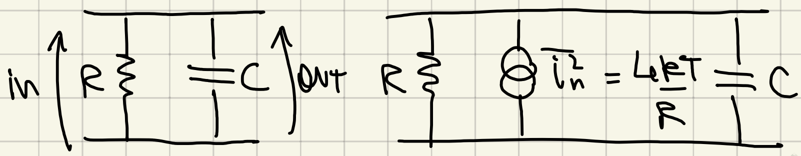
example:



$$\overline{v_{out}^2} = \underbrace{4kTR}_{S_{i,\emptyset}} \cdot \underbrace{\left(5kHz \cdot \frac{\pi}{2}\right)}_{f_N} \cdot \underbrace{100}_{A_{v,\emptyset}^2}$$

"WEIRD" RESULT

simple RC network



- Full computation:

$$T(j\omega) = R \parallel C = \frac{R\,\frac{1}{sC}}{R+\frac{1}{sC}} = \frac{R}{1+sCR}$$

$$\Rightarrow S_o(f) = \frac{4kT}{R}\left|T(j\omega)\right|^2 = \frac{4kT}{R}\,\frac{R^2}{1+\omega^2C^2R^2}$$

$$\langle v_{OT}^2\rangle = \int_0^\infty S_o(f)\,df = \frac{4kT}{R}\int_0^\infty \frac{R^2}{1+\omega^2C^2R^2}\,df = \dots = \frac{kT}{C}$$

- Equivalent computation:

$$\langle v_{OT}^2\rangle = \frac{4kT}{R}\cdot \underbrace{R^2}_{|G^2(0)|}\,\underbrace{\frac{\pi}{2}\,\underbrace{\frac{1}{2\pi RC}}_{f_1}}_{f_N=\Delta f} = \frac{kT}{C}$$

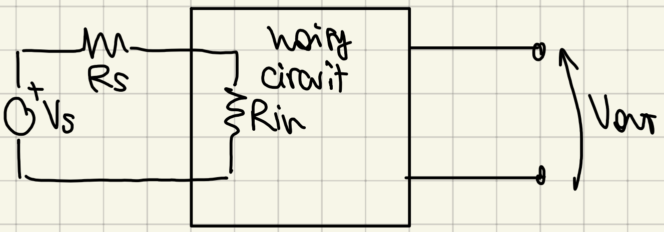
→ The output noise power doesn't depend on the resistor but just on the capacitor

$$\Rightarrow \sigma = \sqrt{\frac{kT}{C}}$$

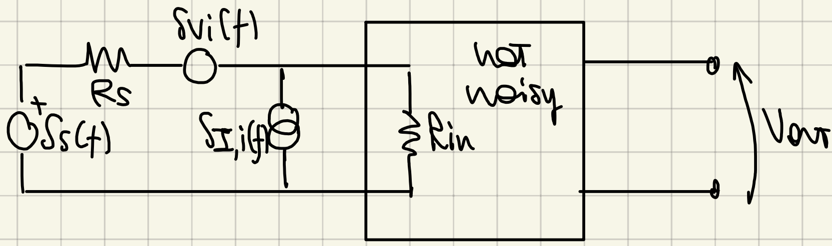
NOISE EQUIVALENT GENERATORS

Since every active electronic component introduces its own sources of noise (white and not), when dealing w/ complex circuits it's often useful to model the noise of each single sub-circuit w/ a couple of "equivalent" noise generators:
- one modelling current noise sources
- one for voltage ones

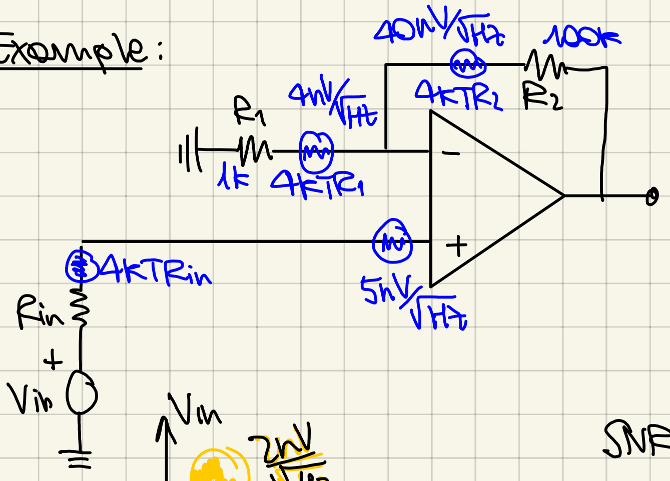
REAL CIRCUIT

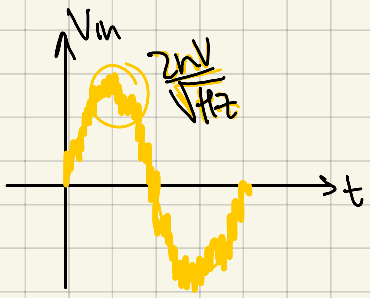


EQUIVALENT CIRCUIT



Example:





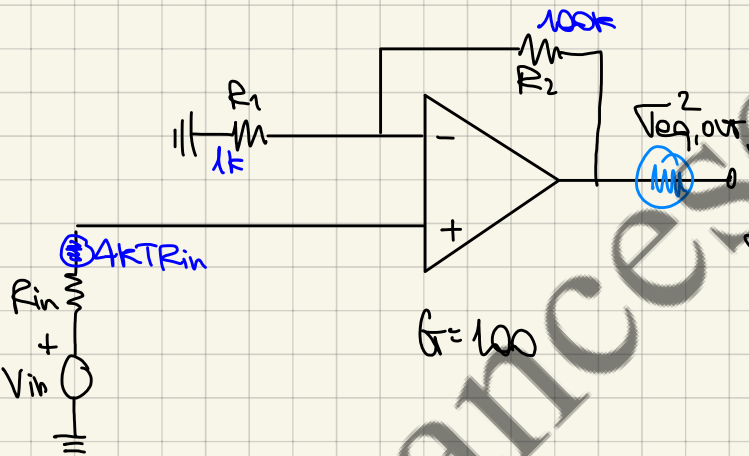
$$SNR_i = \frac{V_{in}}{(2nV/\sqrt{Hz})} = \frac{V_{in}^2}{\sigma_{in}^2} \neq SNR_o = \frac{V_{out}}{\sigma_{out}} = \frac{V_{out}^2}{\sigma_{out}^2}$$

NOISE FIGURE (FACTOR)

$$\boxed{NF = \frac{SNR_i}{SNR_o} \geq 1}$$

(ideally NF = 1)

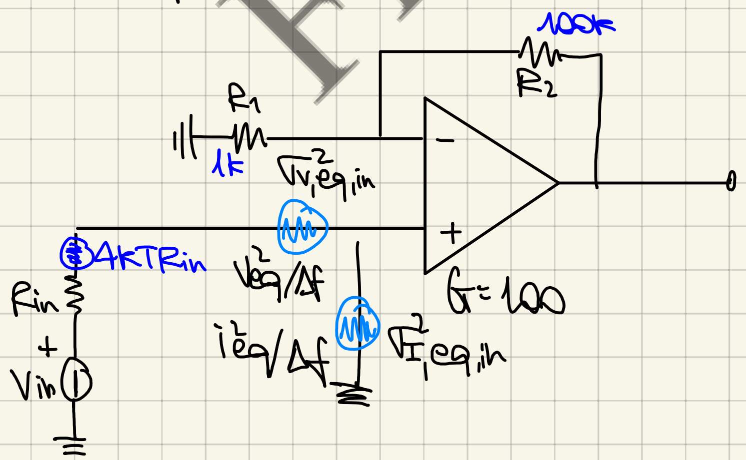
Let's compute the total effect of the noise sources @ the output:



$V_{out} = V_{in} \cdot G$

$\sigma_{out}^2 = \sigma_{in}^2 \cdot G^2 + \sigma_{eq,out}^2$

↓ Now we want to bring the noise generators @ the input



$$V_{in,eq}^2 = 4kTR_s\Delta f + \frac{v_{eq}^2}{\Delta f} \cdot \Delta f + \frac{i_{eq}^2}{\Delta f} \cdot R_s^2 \, \Delta f$$

TOTAL INPUT REFERRED NOISE POWER

NOISE FIGURE (FACTOR)

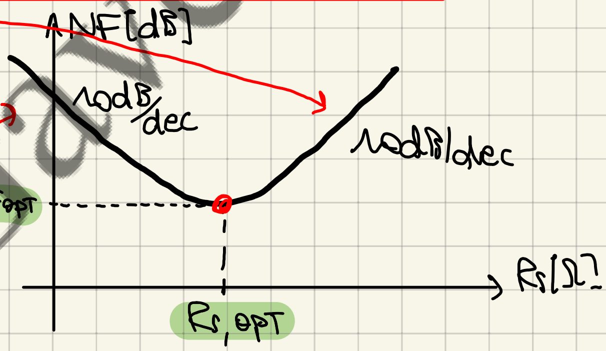
$$\boxed{NF = 10\cdot\log_{10}\left[\frac{\text{total input noise}}{\text{only the source noise}}\right]}$$

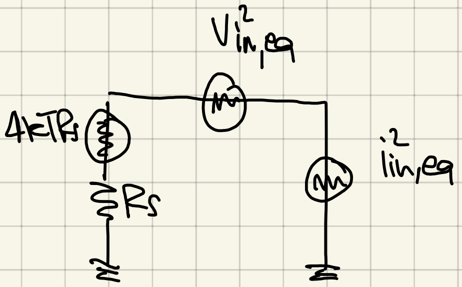
$$NF = 10\log_{10}\left[\frac{4kTR_s + \overline{v_{eq}^2} + \overline{i_{eq}^2}\cdot R_s^2}{4kTR_s}\right] =$$

$$\overset{*}{=} 10\log_{10}\left[1 + \frac{\overline{v_{eq}^2}}{4kTR_s} + \frac{\overline{i_{eq}^2}}{4kT/R_s}\right] \geq 0\,dB \quad (1)$$

$$\boxed{NF = \frac{SNR_i}{SNR_o} = \frac{S_i}{N_i}\cdot\frac{N_o}{S_o} = \frac{\cancel{S_i}}{N_i}\,\frac{N_o}{G\cancel{S_i}} = \frac{N_o}{G N_i} = \frac{\text{total output noise}}{\text{total output noise just due to } R_s} = \frac{\text{Total input noise}}{4kTR_s}}$$

$$* \; NF = 10\log_{10}\left[1 + \frac{\langle v_i^2\rangle}{4kT}\,\frac{1}{R_s} + \frac{\langle i_i^2\rangle}{4kT}\cdot R_s\right]$$





$$NF = 10\log_{10}\left(1 + \frac{\overline{v_{in,eq}^2}}{4kTR_s} + \frac{\overline{i_{in,eq}^2}}{4kTR_s}\,R_s^2\right)$$

OPTIMUM SOURCE RESISTANCE

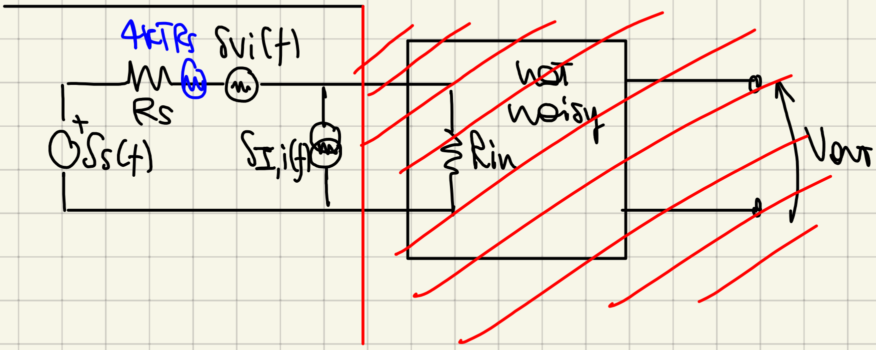
$$\frac{\overline{v_{in,eq}^2}}{4kTR_{s,opt}} = \frac{\overline{i_{in,eq}^2}}{4kT}\,R_{s,opt}$$

$$\boxed{R_{s,opt} = \sqrt{\frac{\overline{v_{in,eq}^2}/\Delta f}{\overline{i_{in,eq}^2}/\Delta f}}}$$

ES05 - NOISE (2)
13/10/2021

$$V_{rms,\,sinusoid} = \frac{V_p}{\sqrt{2}}$$

EQUIVALENT CIRCUIT :



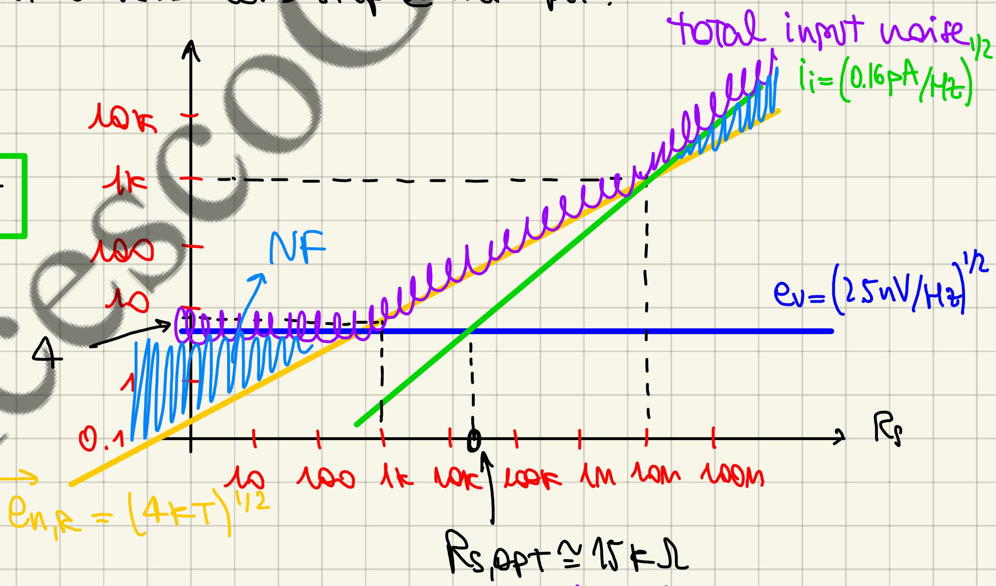
TOTAL INPUT REFERRED NOISE : $S_{in}(f) = 4kTR_s + e_v^2 + i_{in}^2 \cdot R_s^2$
(no Rin)

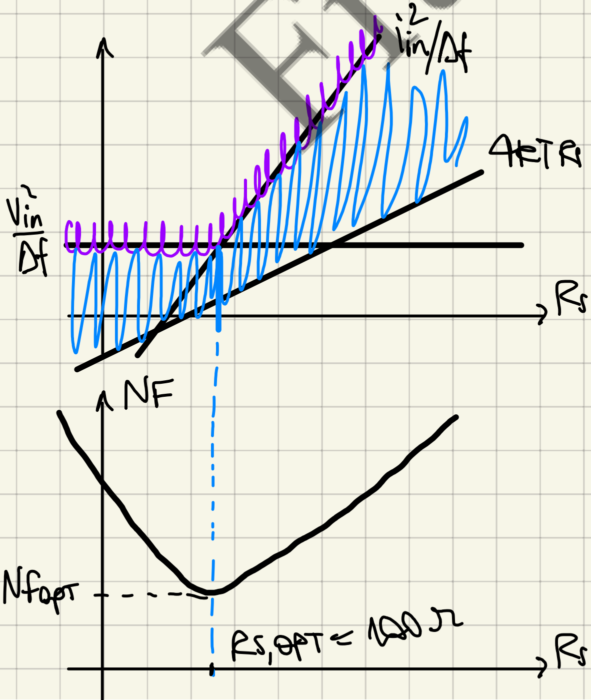
TOTAL OUTPUT NOISE : $S_{out}(f) = |T(j\omega)|^2 \cdot \left\{ S_V(f) \cdot \frac{R_{in}^2}{(R_s+R_{in})^2} + S_I(f) \frac{R_{in}^2 R_s^2}{(R_s+R_{in})^2} \right\}$

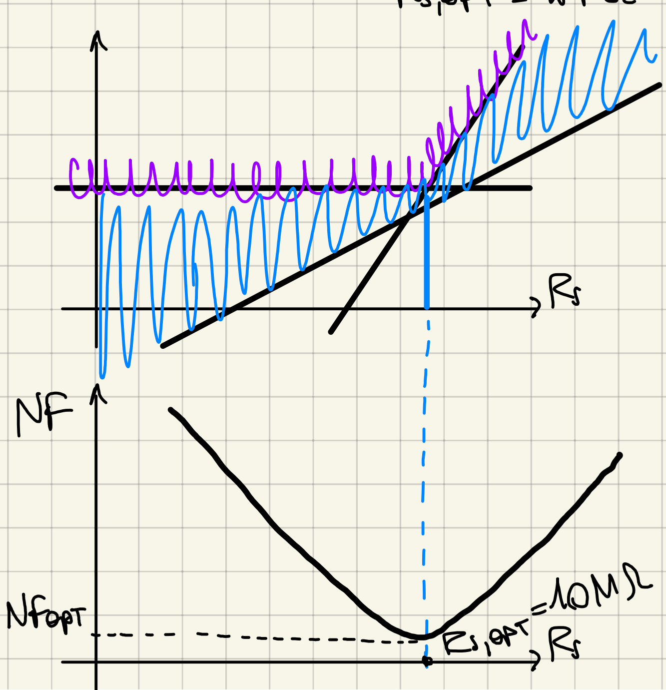
There's no need to compute output noise ... let's stop @ the input!

TOTAL INPUT NOISE

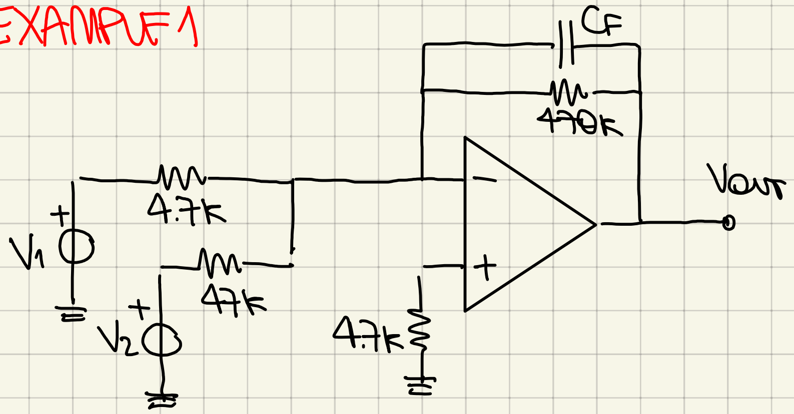
$$S_{in}(f) = \boxed{4kTR_s} + \boxed{e_v^2} + \boxed{i_{in}^2 \cdot R_s^2}$$







EXAMPLE 1



$$V_{in}^2/\Delta f = (8\,nV/\sqrt{Hz})^2$$

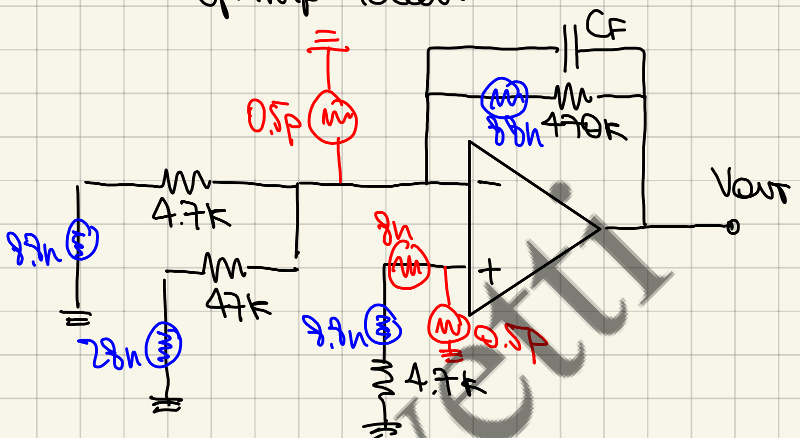
$$i_{in}^2/\Delta f = (0.5\,pA/\sqrt{Hz})^2$$

$$C_F = 1\,nF$$

OpAmp ideal

(B) Compute the output rms value

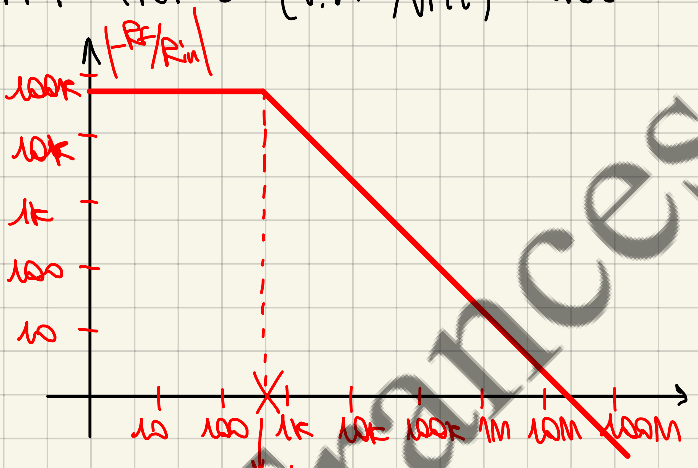
- Switch off the signal generators
- Draw all the noise generators



$$4KT\cdot 4.7k\Omega = 1.66\cdot 10^{-20}\cdot 4.7k = (8.83\,nV/\sqrt{Hz})^2$$

$$4KT\cdot 47k\Omega = (8.8\,nV/\sqrt{Hz})^2\cdot\sqrt{10} \simeq (28\,nV/\sqrt{Hz})^2$$

$$4KT\cdot 470k\Omega = (8.8\,nV/\sqrt{Hz})^2\cdot\sqrt{100} \simeq (88\,nV/\sqrt{Hz})^2$$



$$f_0 = \frac{1}{2\pi C_F R_F} = 339\,Hz$$

$$NEBW = \frac{\pi}{2} f_0 = \frac{\pi}{2}\, 339\,Hz \simeq 532\,Hz$$

$$= (8nV/\sqrt{Hz})^2 + (8.9nV/\sqrt{Hz})^2 = 141.44\,aV^2/Hz = (11.9\,nV/\sqrt{Hz})^2$$

$$\left|\frac{V_{out}}{\sqrt{Hz}}\right|^2 = (8.9nV/\sqrt{Hz})^2\cdot\left(-\frac{470k}{4.7k}\right)^2 + (28nV/\sqrt{Hz})^2\left(-\frac{470k}{47k}\right)^2 + (88nV/\sqrt{Hz})^2\cdot(1)^2 +$$

$$+\left[\sqrt{(8.9nV/\sqrt{Hz})^2 + (8nV/\sqrt{Hz})^2}\right]^2\cdot\left(1+\frac{470k}{4.7k\|47k}\right)^2 +$$

$$+(0.5pA/\sqrt{Hz})^2\cdot(4.7k)^2\cdot\left(1+\frac{470k}{4.7k\|47k}\right)^2 + (0.5pA/\sqrt{Hz})^2(470k)^2$$

$$= (8.9n\cdot 100)^2 + (28n\cdot 10)^2 + (88n)^2 + (11.9n\cdot 111)^2 + (0.5p)^2\{(4.7k)^2(111)^2 + (470k)^2\}$$

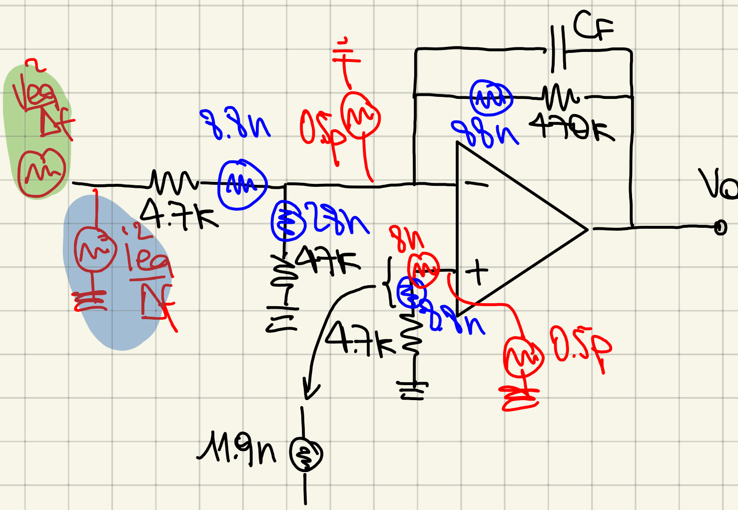
$$\simeq (890n)^2 + (280n)^2 + (88n)^2 + (1.3\mu)^2 + (255n)^2 + (235n)^2$$

$$\doteq \left(1.63\,\mu V/\sqrt{Hz}\right)^2$$

$$V_{out,rms} = \sqrt{\left(1.63\,\mu V/\sqrt{Hz}\right)^2 \cdot \Delta f} = \sqrt{\overline{V_{out}^2}} = \left(1.63\,\mu V/\sqrt{Hz}\right)\sqrt{\Delta f} \simeq 37.7\,\mu V$$

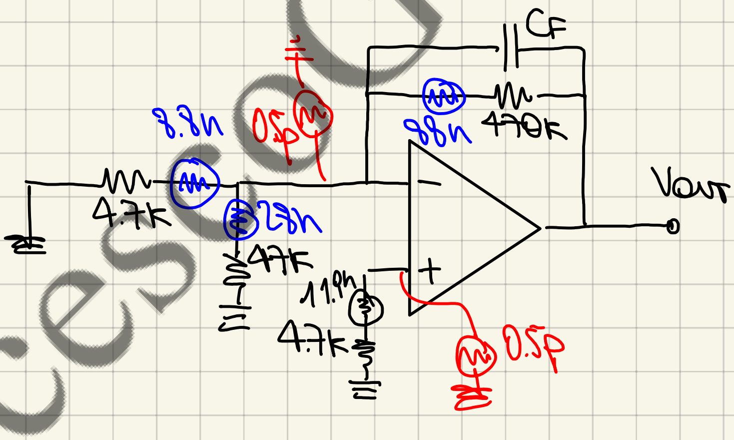
output noise p-t-p value $= 6\,\overline{V_{out}} = 6 \cdot 37.7\,\mu V \simeq 0.226\,mV$

Ⓐ Compute the noise equivalents for input V_1



~~SUPERPOSITION OF EFFECTS~~

- $\frac{V_{in,eq}^2}{\Delta f} = ?$
  - short-circuit the input
  - turn-off $i_{in,eq}^2/\Delta f$



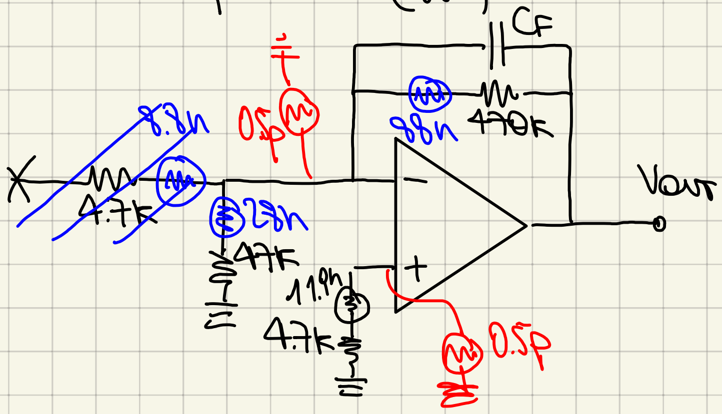
real output noise $= \left(1.63\,\mu V/\sqrt{Hz}\right)^2$

equivalent output noise $= \frac{V_{in,eq}^2}{\Delta f}\left(-\frac{470k}{4.7k}\right)^2$

They must be equal

$$\frac{V_{in,eq}^2}{\Delta f}(100)^2 = \left(1.63\,\mu V/\sqrt{Hz}\right)^2 \Rightarrow \frac{V_{in,eq}^2}{\Delta f} = \frac{\left(1.63\,\mu V/\sqrt{Hz}\right)^2}{(100)^2} = \left(16.3\,nV/\sqrt{Hz}\right)^2$$

- $\frac{i_{in,eq}^2}{\Delta f} = ?$
  - open-circuit the input
  - turn-off $V_{in,eq}^2/\Delta f$



real output noise = ? ✱

equivalent output noise $= \left(i_{in,eq}^2/\Delta f\right)\left(470k\right)^2$

$$✱\left|\frac{V_{out}}{\sqrt{Hz}}\right|^2 = \cancel{\left(8.8nV/\sqrt{Hz}\right)^2\left(-\frac{470k}{4.7k}\right)^2} + \left(28nV/\sqrt{Hz}\right)^2\left(-\frac{470k}{47k}\right)^2 + \left(88nV/\sqrt{Hz}\right)^2\cdot(1)^2 +$$

$$+\left(11.9nV/\sqrt{Hz}\right)^2\cdot\left(1+\frac{470k}{\cancel{4.7k}\|47k}\right)^2 + \left(0.5pA/\sqrt{Hz}\right)^2\cdot(4.7k)^2\cdot\left(1+\frac{470k}{\cancel{4.7k}\|47k}\right)^2 +$$

$$+\left(0.5pA/\sqrt{Hz}\right)^2(470k)^2$$

$$= \left(280nV/\sqrt{Hz}\right)^2 + \left(88nV/\sqrt{Hz}\right)^2 + \left(11.9nV/\sqrt{Hz}\right)^2(11)^2 + \left(2.3nV/\sqrt{Hz}\right)^2(11)^2 + \left(235nV/\sqrt{Hz}\right)^2$$

$$\simeq \left(400nV/\sqrt{Hz}\right)^2$$

$$\left(i_{in,eq}^2/\Delta f\right)(470k)^2 = \left(400nV/\sqrt{Hz}\right)^2 \Rightarrow \left(\frac{i_{in,eq}}{\Delta f}\right)^2 \simeq \left(850pA/\sqrt{Hz}\right)^2$$

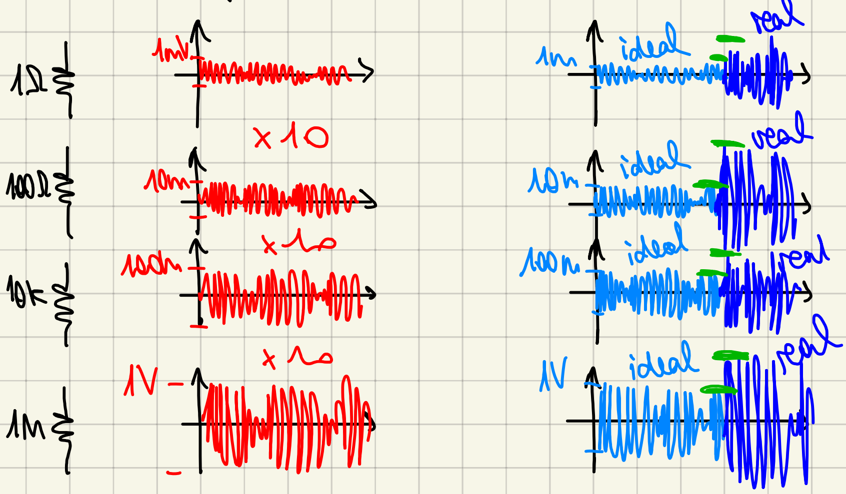
$$R_{S,opt} = \sqrt{\frac{\left(V_{in,eq}^2/\Delta f\right)}{\left(i_{in,eq}^2/\Delta f\right)}} = \sqrt{\frac{\left(16.3nV/\sqrt{Hz}\right)^2}{\left(850pA/\sqrt{Hz}\right)^2}} \simeq 19.7\,\Omega$$

$$NF_{opt} = 10\log_{10}\left(1+\frac{V_{in,eq}^2/\Delta f}{4kTR_{S,opt}} + \frac{i_{in,eq}^2/\Delta f}{4kT}R_{S,opt}\right) =$$

$$= 10\log_{10}\left(1+2\frac{\sqrt{\left(V_{in,eq}^2/\Delta f\right)\cdot\left(i_{in,eq}^2/\Delta f\right)}}{4kT}\right)$$

$R_{S,opt}$ is not the resistance for which we have the minimum noise @ the output
→ let's see what it is:

INPUT $4kTR_S\cdot\Delta f$ OUTPUT Hp: $G=1$

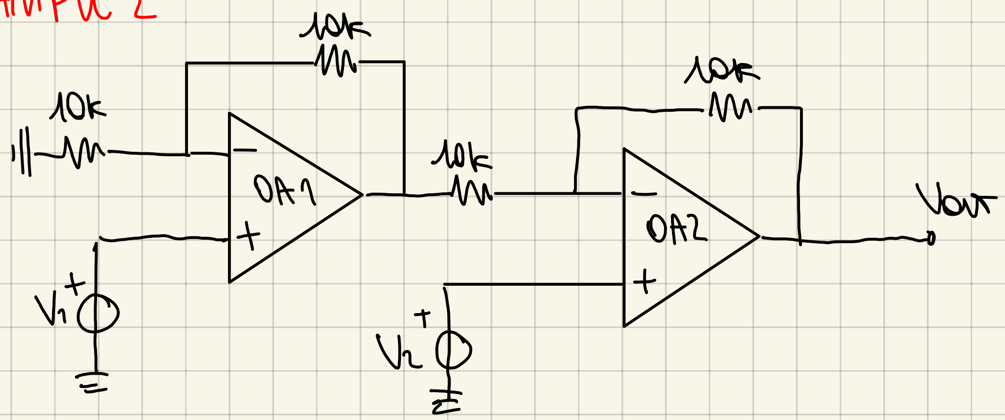


$NF = \frac{real}{ideal}$

⇓ (IMP!!)

$R_{S,opt}$ is the input resistance for which at the output the real noise differs as less as possible from the ideal one, so the resistance for which we have the NF_{opt}

EXAMPLE 2



$A_0 = 100\,dB = 10^5$, $GBWP = 10\,MHz$ $\quad \overline{v_{in}^2}/\Delta f = (1\,nV/\sqrt{Hz})^2$ $\quad \overline{i_{in}^2}/\Delta f = (1\,pA/\sqrt{Hz})^2$

Ⓐ Compute output spectral density neglecting 4kTR contributions

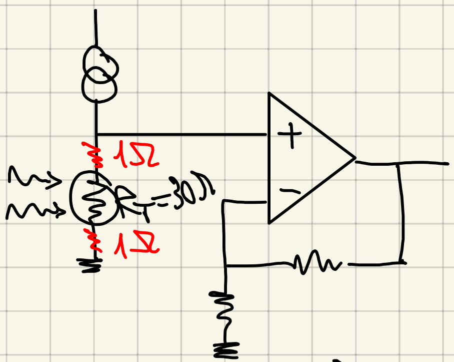
$f_0 \cdot A_0 = GBWP \rightarrow f_0 = \frac{10\,MHz}{10^5} = 100\,Hz$

$\rightarrow \Delta f = f_N = NEBW = \frac{\pi}{2} f_0 \cong 157\,Hz$

ES06 - INA
19/10/2021

GOAL: we want to amplify a voltage difference btw two nodes of the circuit

For example we need to amplify a voltage across a photoresistor, so we have to pump a current through it:



Any possible parasitism in series w/ that PR will cause an error in the readout

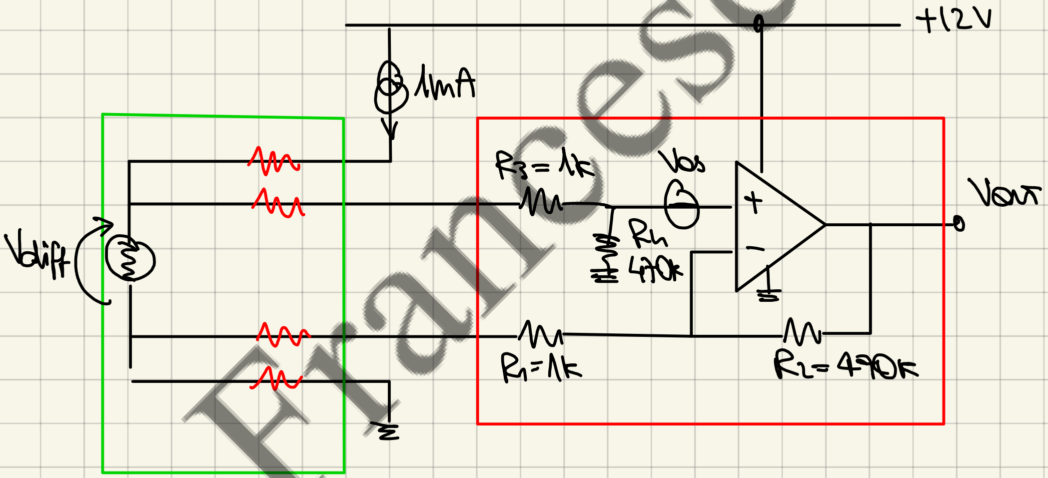
let's suppose to have two parasitic resistors equal to 1Ω
↓
So what we read is the voltage across a resistor equal to 32Ω

Since $R_T = 30\Omega\ (@\,0°C) + 0.15\,\Omega/°C$

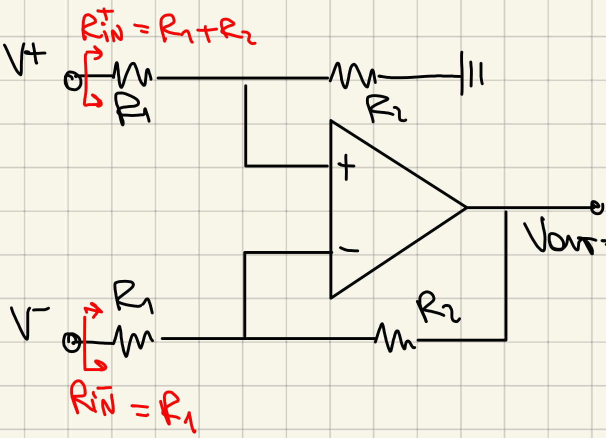
A parasitic term of 2Ω corresponds to a misreading of $\frac{2}{0.15} \cong 14°C$

How can we improve the configuration in order to decrease the dependence on the parasitisms?

SOLUTION ①: KELVIN CONNECTION → we use 4 wires instead of 2



it's a voltage difference amplifier

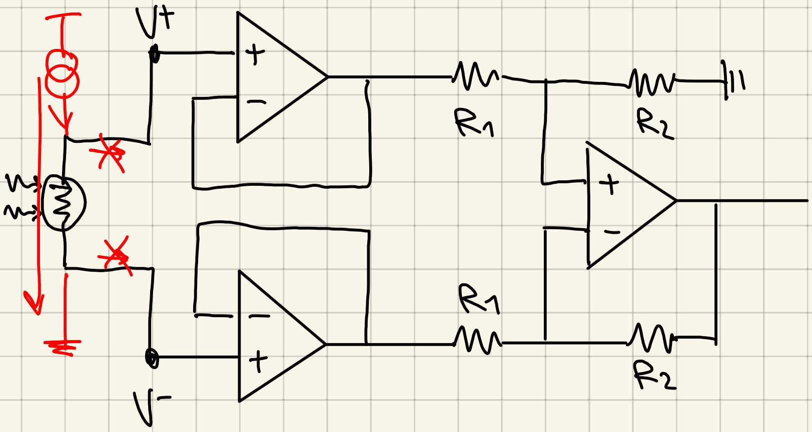


$V_{OUT} = (V^+ - V^-)\,\frac{R_2}{R_1}$

Since $R_{IN}^+ \neq R_{IN}^-$, there will be an unwelcome current flow through the amplifier's input connections and the two currents will differ from eachother so the voltage drops across the parasitic resistances will not compensate each other

How can we introduce a perfect symmetry of the stage?
↳ By introducing two buffers:

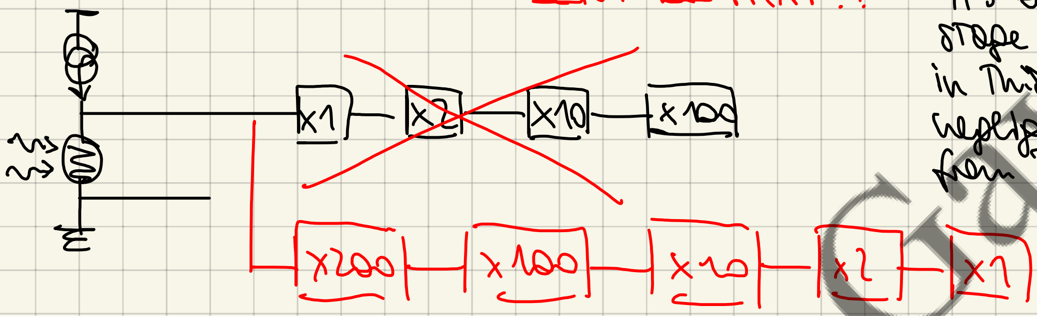
SOLUTION ②



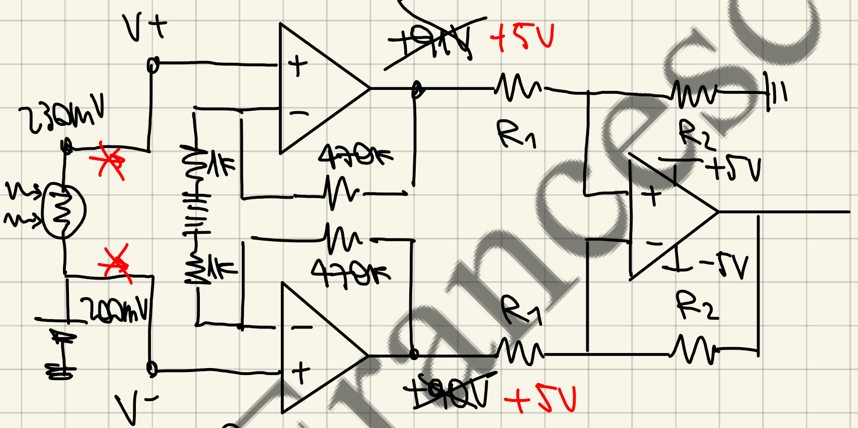
This solution is not suitable too bc we are introducing too many noisy components (the buffers are noise sources and have their own offsets)

SOLUTION ③

DON'T DO THAT!! → it's better to place first the stage w/ the highest gain in this way we make somehow negligible the noise coming from the following stages



→ Amplify as soon as possible!!



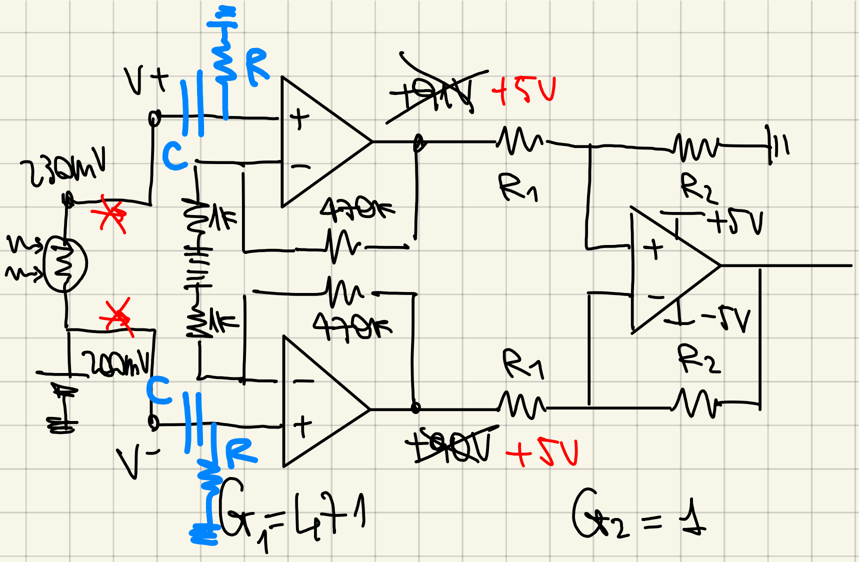
$G_T = 471$

$G_1 = 471$ $G_2 = 1$

ISSUE: The common mode voltage could cause saturation of the input stage

SOLUTION ④ → We'd like to have $G_{cm} = 1$ (very low) $G_{diff} = 471$ (very high)

How can we amplify the signal and not the common mode signal?
↳ we can introduce a DC stop component → a capacitor

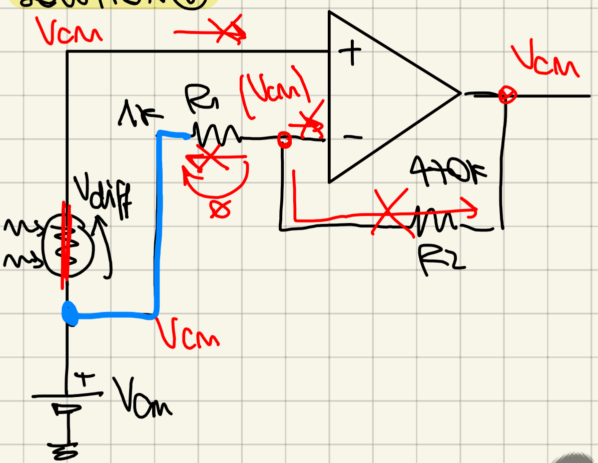


$G_T = 471$

ISSUE: in our example the input signal can be also a DC signal and in this way it would be stopped by the capacitor

⇒ This solution is not suitable too! ☹

SOLUTION ⑤

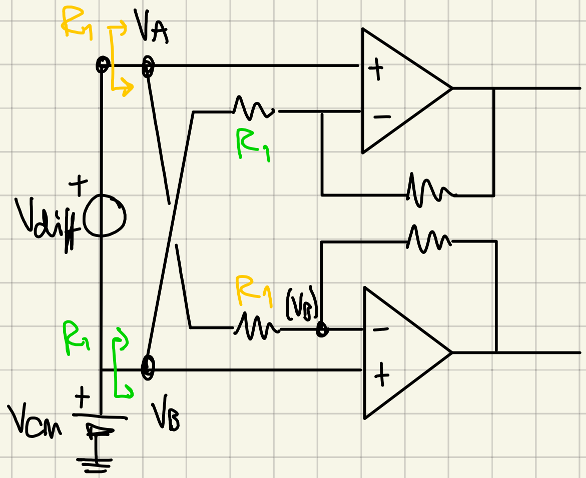


- if $V_{diff} = 0$ we have V_{cm} at both inputs of the OpAmp

⇒ $G_{cm} = 1$ ☺

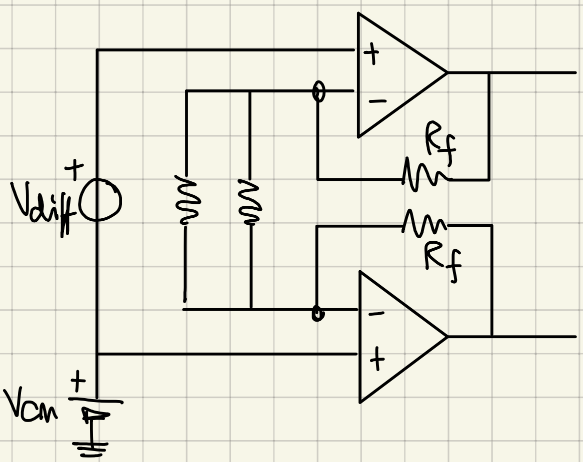
- if $V_{cm} = 0$ → $V_{out} = V_{diff}\left(1 + \frac{R_2}{R_1}\right)$

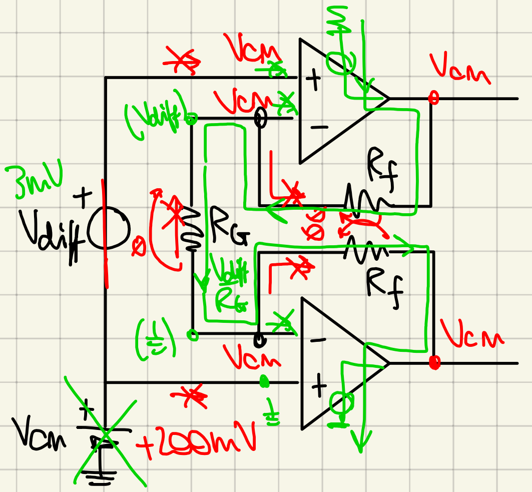
⇒ $G_{diff} = 471$



ISSUE: too low input impedance ☹

SOLUTION ⑥





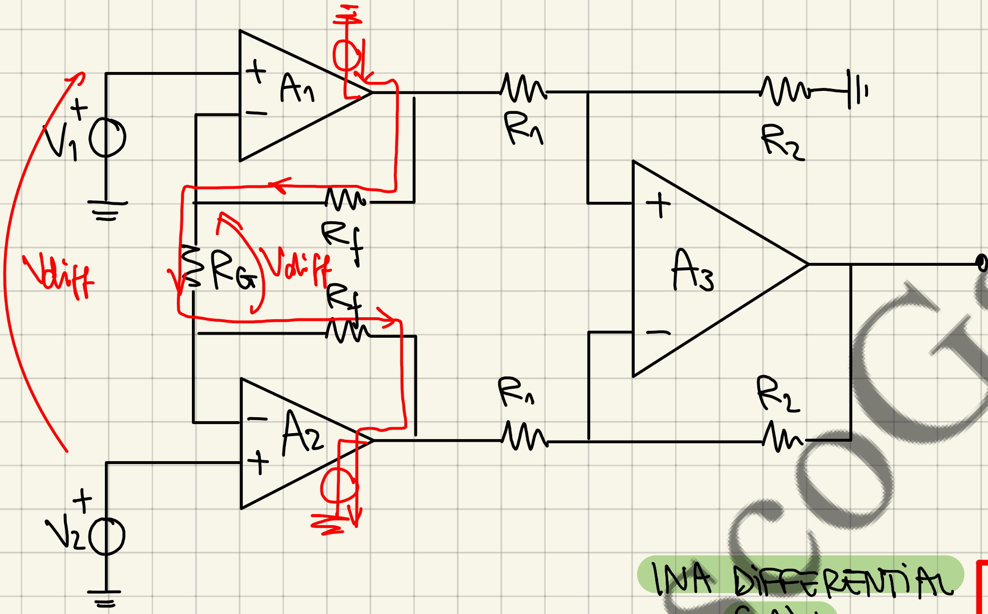
$G_{cm} = 1$ (not high)

$G_{diff} = ?$

$$V_{out,diff} = \left(\frac{V_{diff}}{R_G}\right)(R_f + R_G + R_f)$$

$$G_{diff} = \left(1 + \frac{2R_f}{R_G}\right)$$

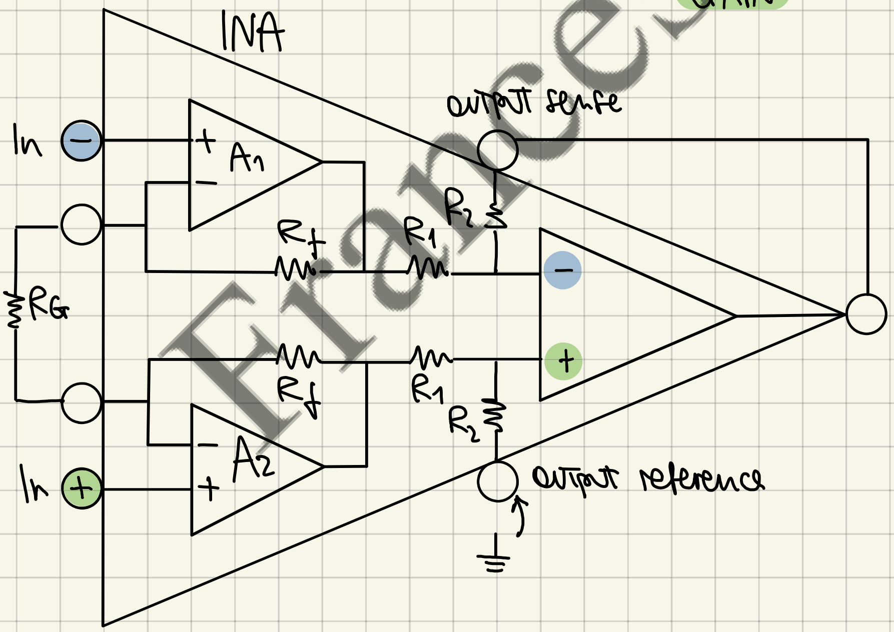
INA - INSTRUMENTATION AMPLIFIER



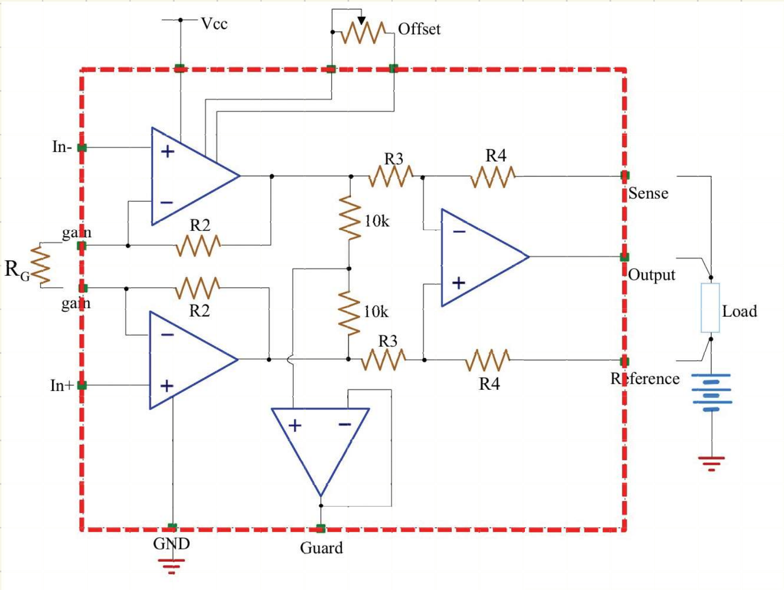
INA DIFFERENTIAL GAIN

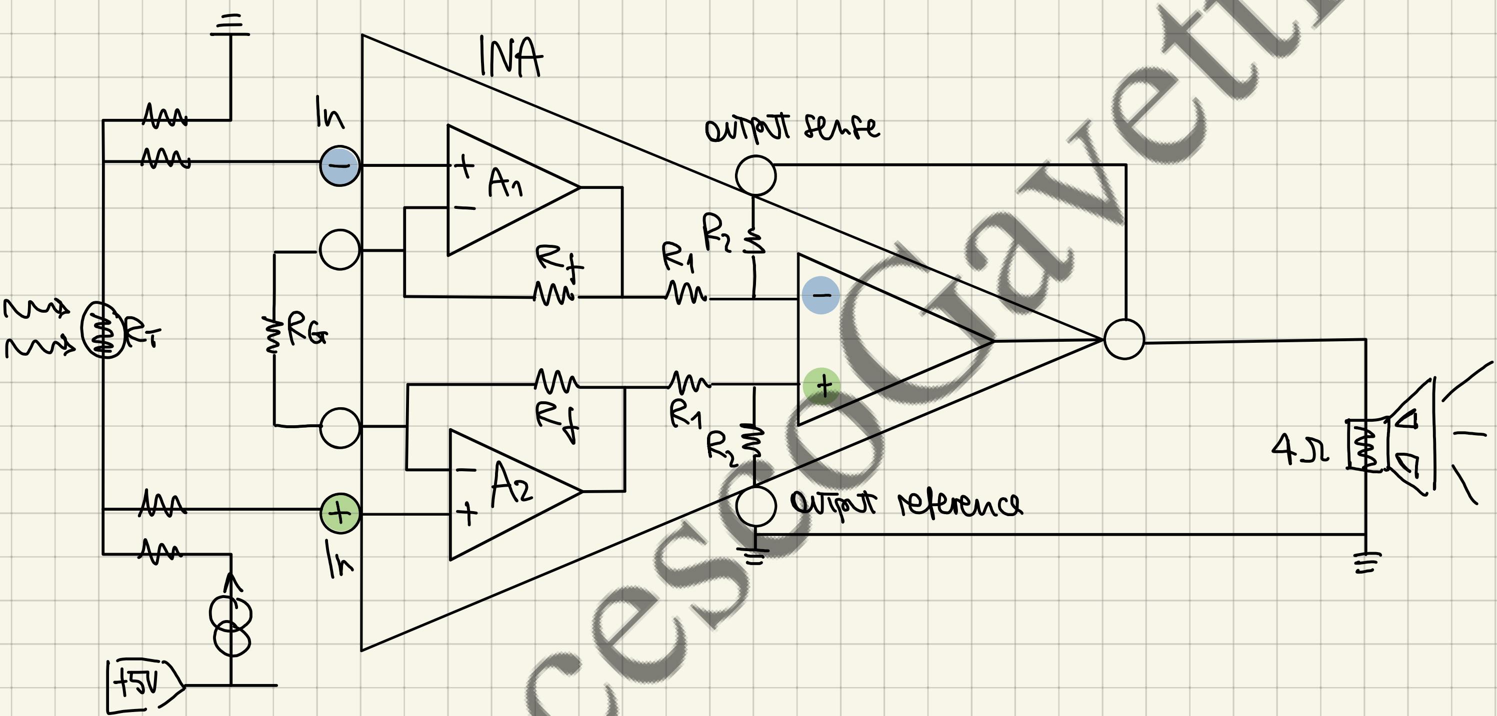
$$\boxed{G_{diff} = \left(1 + \frac{2R_f}{R_G}\right)\left(\frac{R_2}{R_1}\right)}$$

$$\boxed{G_{cm} \approx \emptyset}$$

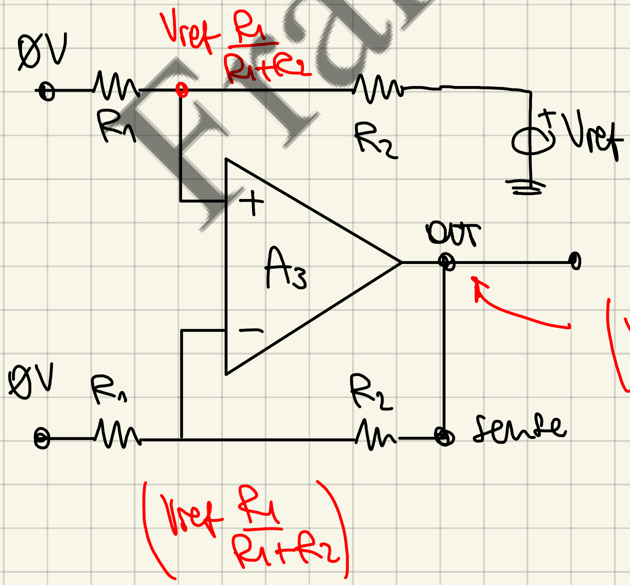


In principle if the offset were perfectly deterministic the INA would cancel it out, but since it's not, but it presents a mismatch, even an INA will have its offset





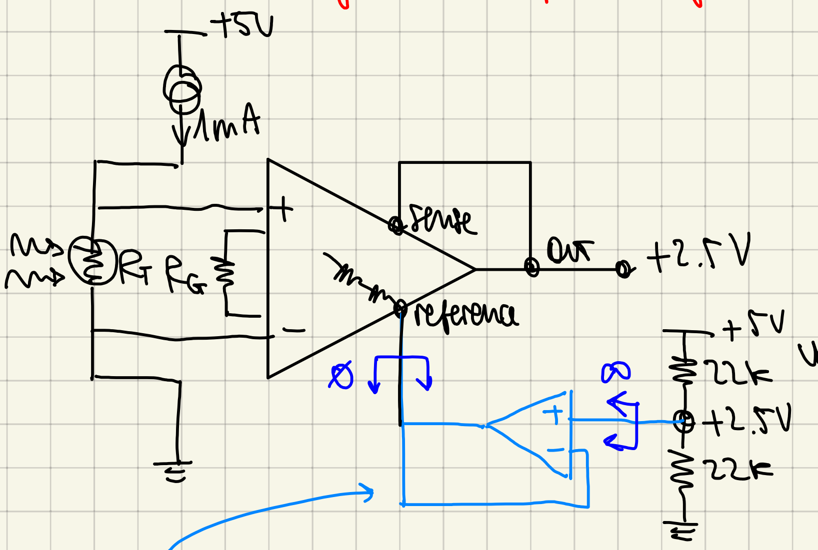
What's the poin btw the reference pin and the output pin?



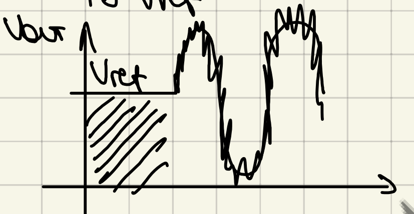
$$\left(V_{ref}\frac{\cancel{R_1}}{\cancel{R_1+R_2}}\right)\left(\cancel{1+\frac{R_2}{R_1}}\right)=V_{out}$$

$$\Rightarrow V_{out}=V_{ref} \quad \Rightarrow \quad G=1$$

How can we change the output voltage w/rt Vref?

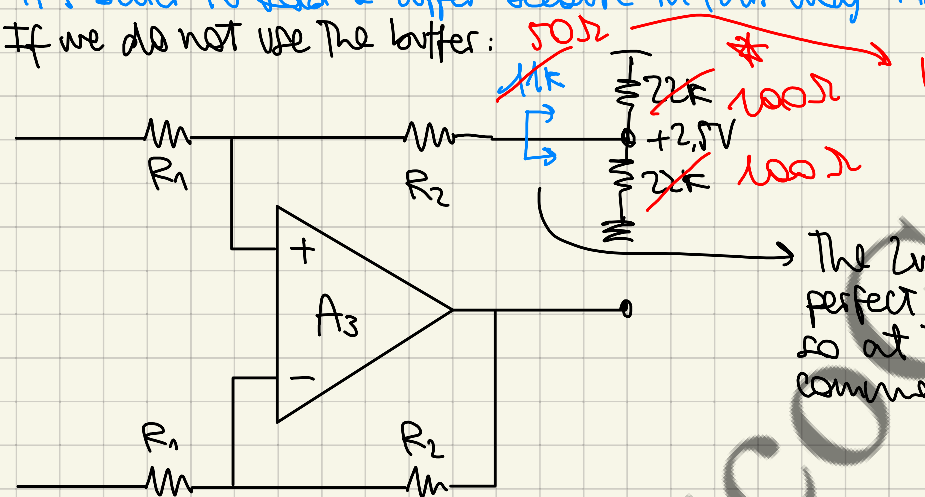


→ The output will not start from ØV but it will have an offset equal to Vref



it's better to add a buffer because in this way $R_{IN} \sim \infty$ $R_{OUT} \sim \emptyset$

If we do not use the buffer:

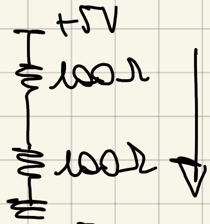


→ The 2nd stage is no longer a perfect voltage difference amplifier so at the output we'll have a common mode signal and so on...

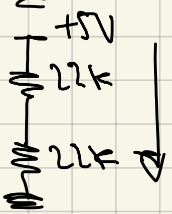
So if we want to spear money and not use the buffer let's ensure that we use small resistances *

↓

There's still a big issue → in this way we have a direct path from PS to ground, so we discharge our battery very quickly as the two resistors are low



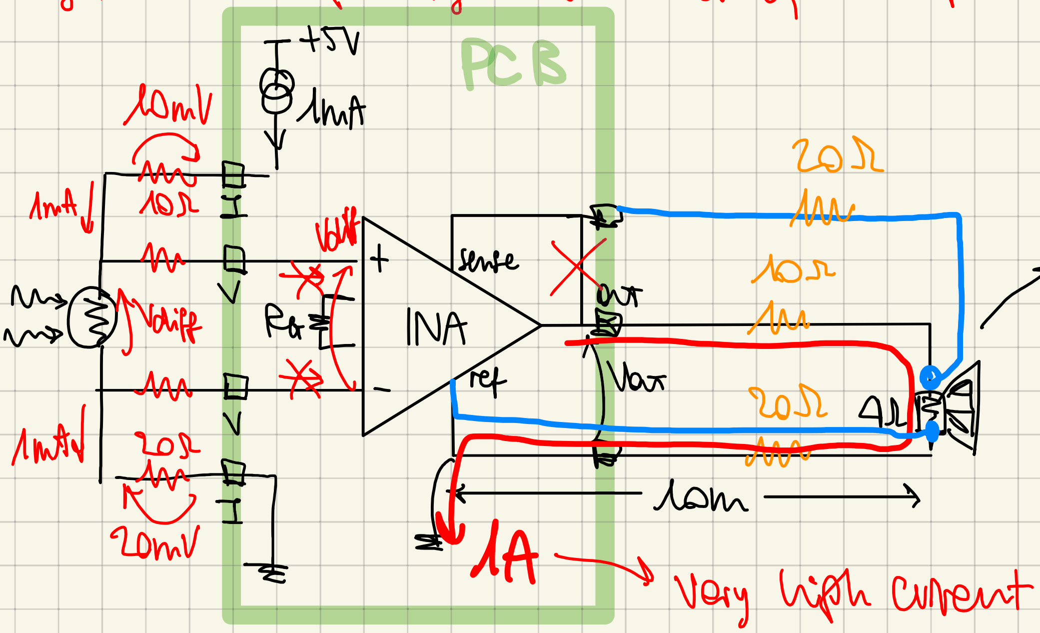
$I = \frac{5V}{200\Omega} = 0{,}025 = 25mA$



$I = \frac{5V}{44k} = 114\,\mu A$

→ let's choose a resistors' value in the middle which is high enough to determine a low leakage current and low enough to be negligible w/rt R_2 or (if you have enough money) let's use a buffer

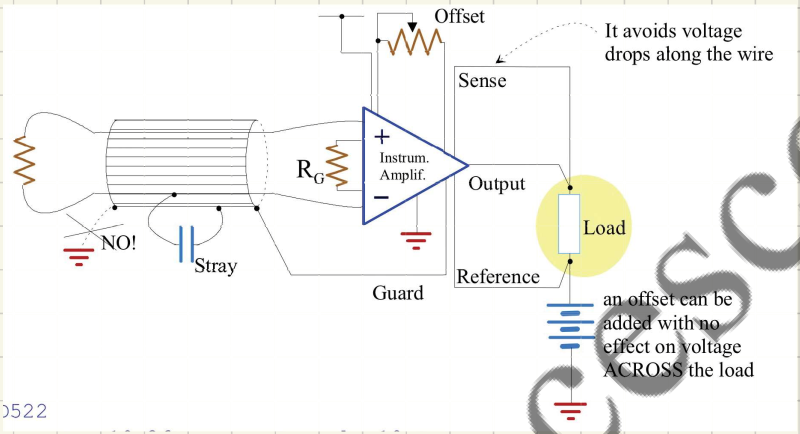
Why did the manufacturer give us the sense, the output and the reference pins?

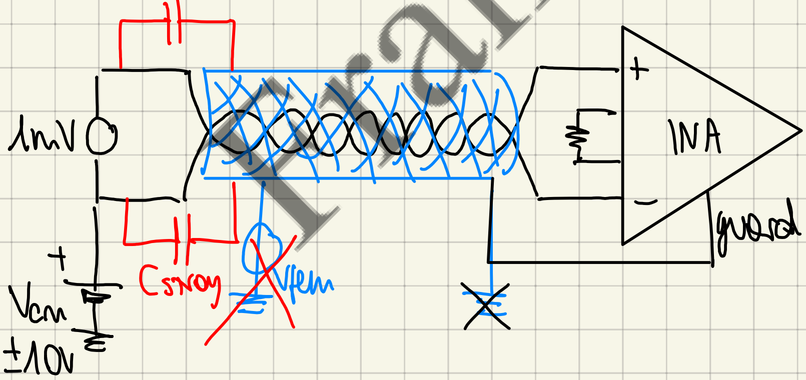


let's connect the sense pin as close as possible to the load

There's also another pin called GUARD PIN

What's the guard pin useful for?

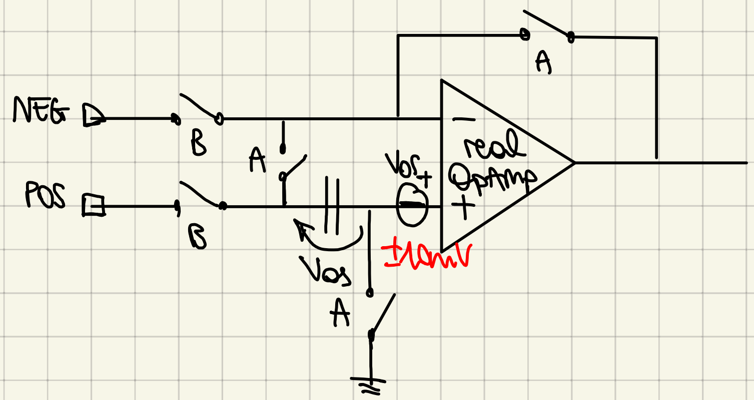




The guard pin is useful to provide a constant voltage to the coaxial cable or even better to provide a voltage that is independent of Vdiff, but depends only on Vcm in order to reject the detrimental effects of Cstray

How can we design an INA w/ a very low residual offset?

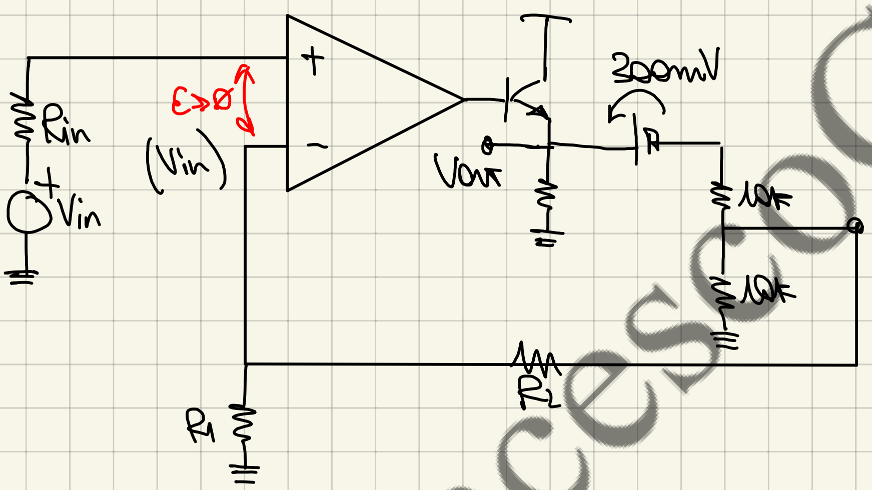
CAZ, COMMUTATING AUTO ZEROING



$I_B = C \frac{dV}{dt}$

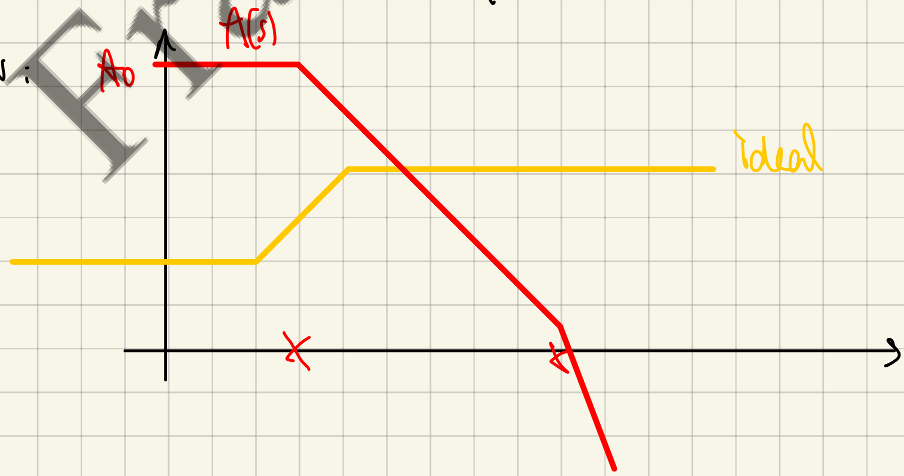
$C = \frac{Q}{V}$

We should do computations in anticlockwise direction:



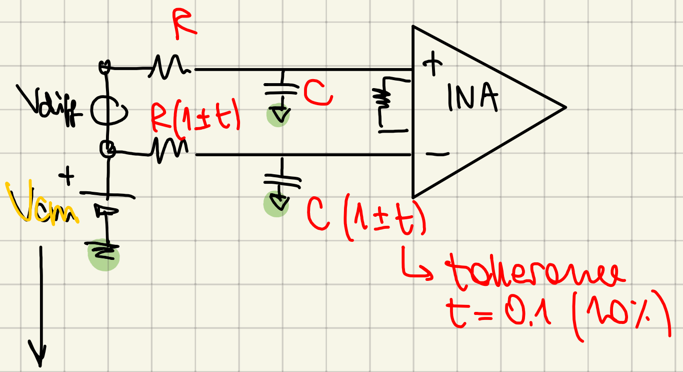
IDEAL GAIN: $V_{out} = V_{in}\left(1 + \frac{R_2}{R_1}\right)\left(\frac{10k + 10k}{10k}\right) + 300mV$

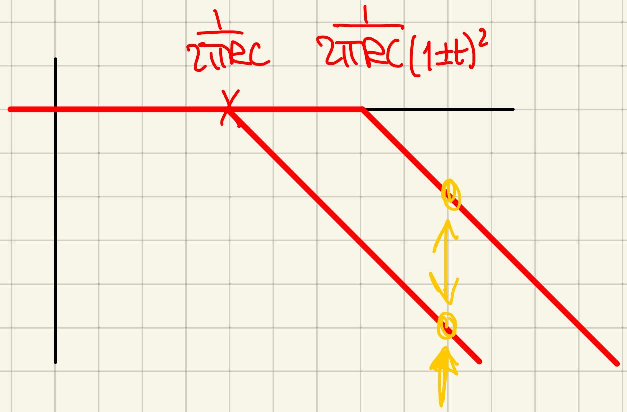
REAL GAIN:



INA - POSSIBLE CAUSES OF MISMATCHES AND ERRORS

Hp: let's assume we want to low pass our signal





if Vcm is not a DC term, but it's applied at a given freq higher than those ones of the two poles, it will get differently amplified due to the mismatch

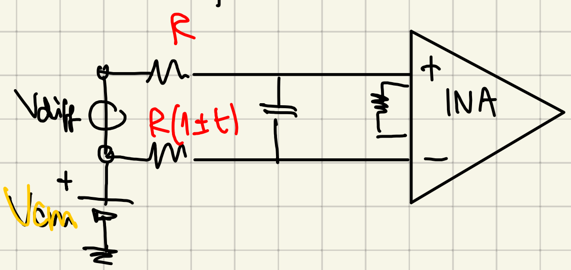
↓

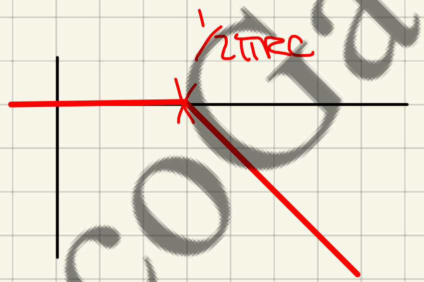
So this disturbance will appear at the output due to the mismatch even if $V_{diff} = 0$

How can we solve this issue? How can we remove this residual value at the output?

↓

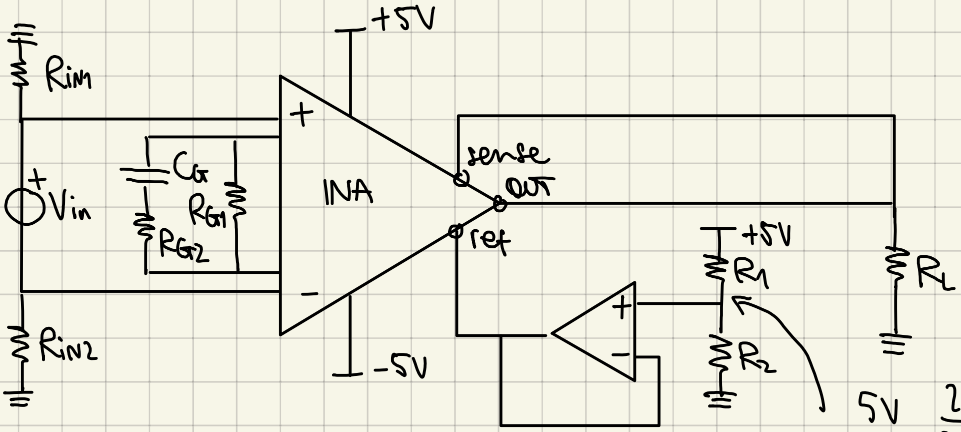
we can use just one capacitor





20/10/2021

EXAMPLE 1



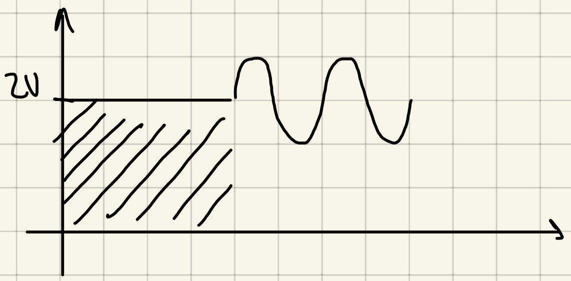
$R_{IN1} = R_{IN2} = 100k\Omega$
$R_{G1} = 10k\Omega \quad R_{G2} = 1k\Omega$
$C_G = 1.6nF$
$R_1 = 3.3k\Omega \quad R_2 = 2.2k\Omega$
$R_L = 1k\Omega$

All the resistors inside the INA are $100k\Omega$

$5V \cdot \dfrac{2.2k}{5.5k} = 2V \Rightarrow V_{ref} = 2V$

(A) Compute Vout for Vin = 0

Vout is equal to Vref @ the beginning $\quad V_{ref} = 5V \cdot 2.2k/5.5k = 2V$

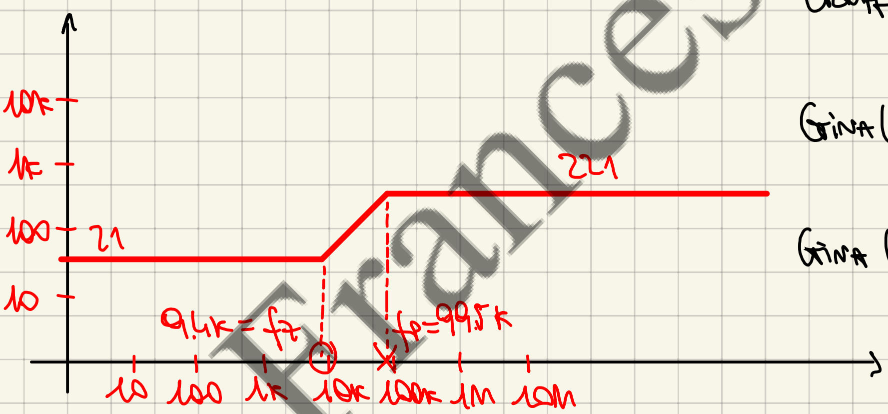


(B) Compute the ideal gain Vout(f)/Vin(f)

$$G_{diff} = \left(1 + \frac{2R_f}{R_G}\right)\left(\frac{R}{R}\right) = 1 + \frac{200k}{R_G}$$

$$G_{INA}(0) = 1 + \frac{200k}{10k} = 21$$

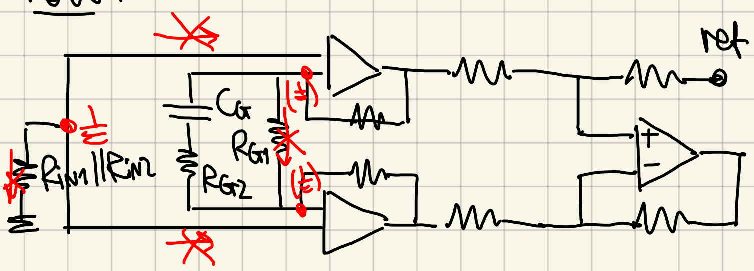
$$G_{INA}(\infty) = 1 + \frac{200k}{10k \| 1k} = 1 + \frac{200k}{909.1} = 221$$



There must be a zero and a pole

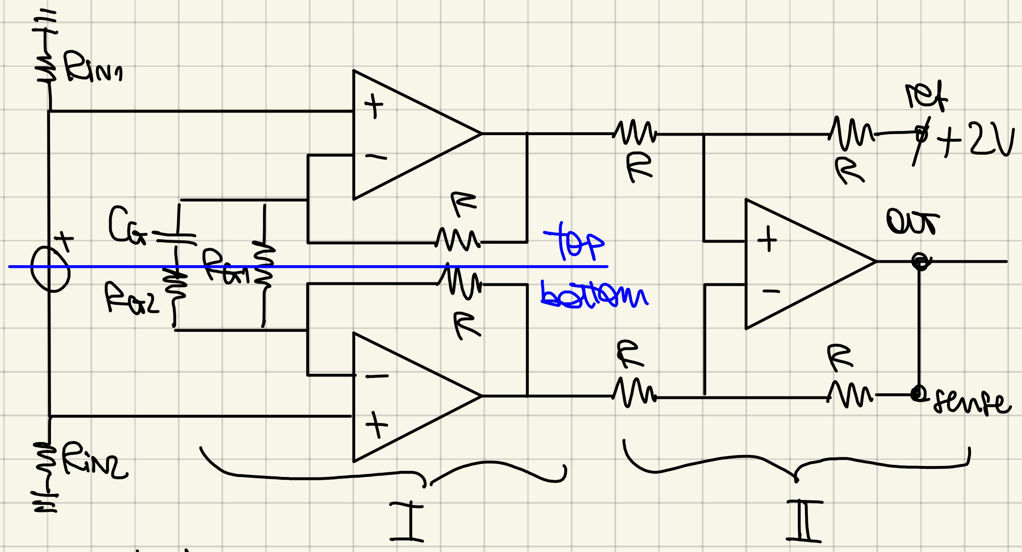
$$f_p = \frac{1}{2\pi C_G R_{G2}} = 99.5kHz$$

POLE:

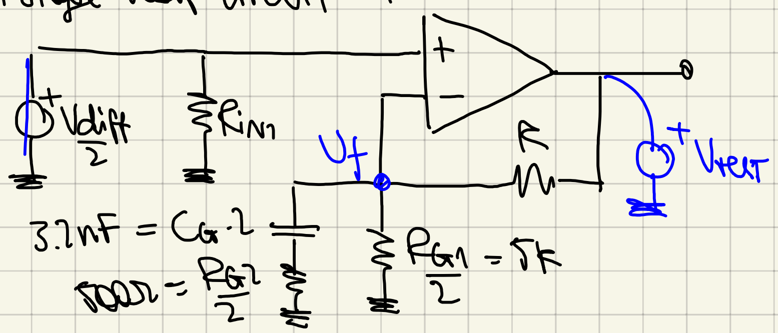


$$f_z/21 = f_p/221 \Rightarrow f_z = f_p \frac{21}{221} = 9.4\ kHz$$

To compute the real gain we have to open the INA :



1st stage half-circuit :



Hp: $A_0 = 100\,dB$ $GBWP = 10\,MHz$

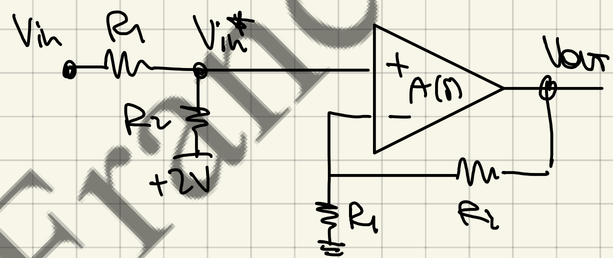
$$f_0 = \frac{GBWP}{A_0} = \frac{10 \cdot 10^6}{10^5} = 100\,Hz$$

$\beta_{HC}(s) = ?$

$$\begin{cases} \beta_{HC}(0) = \dfrac{5k}{105k} = \dfrac{5}{105} \Rightarrow \dfrac{1}{\beta}_{HC}(0) = 21 \\ \beta_{HC}(\infty) = \dfrac{5k \parallel 500}{100k + 5k \parallel 500} = \dfrac{454.54}{100k + 454.54} = 5.52 \cdot 10^{-3} \Rightarrow \dfrac{1}{\beta}_{HC}(\infty) = 221 \end{cases}$$

$$f_{z,p} = \frac{1}{2\pi\, C_G \cdot 2 \cdot R_{G2}/2} = 99.5\,kHz \qquad f_{p,p} = 9.4\,kHz$$

2nd stage:

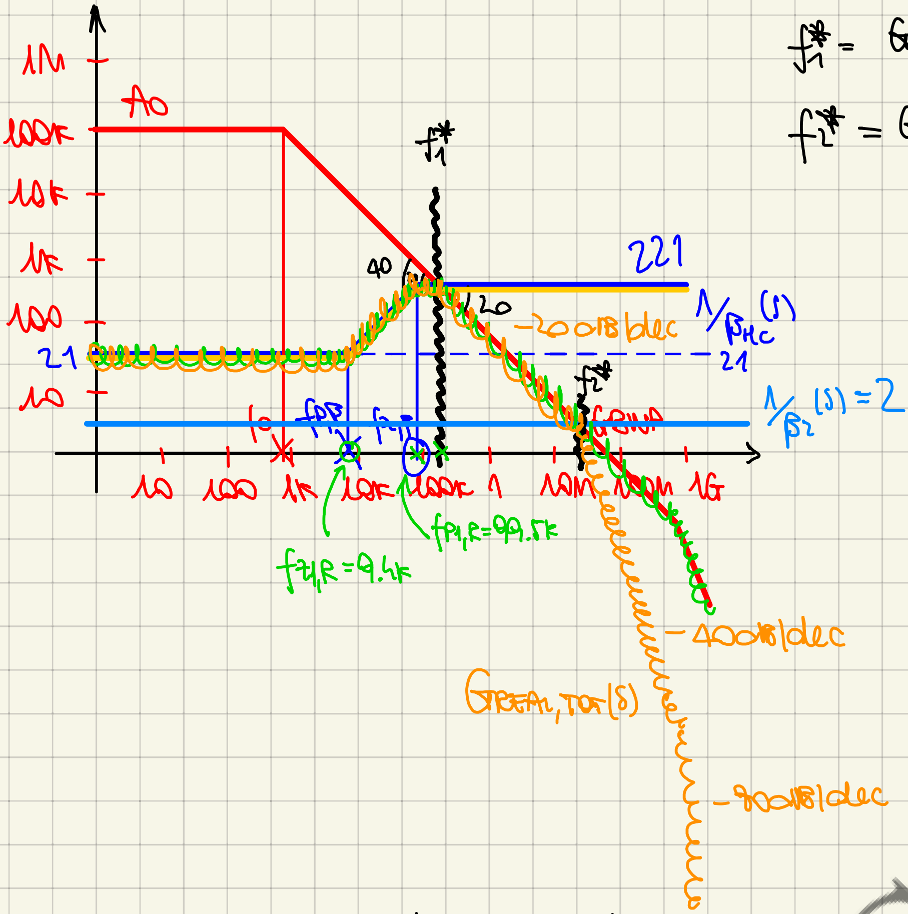


ideal gain: $V_{in}^{+} = V_{in} \dfrac{R_2}{R_1 + R_2} = \dfrac{1}{2} V_{in}$

$$V_{out} = \frac{1}{2} V_{in} \left(1 + \frac{R_2}{R_1}\right) = \frac{1}{2} V_{in} \cdot 2 = V_{in}$$

$$\Rightarrow G_{i,2}(s) = 1$$

$$\beta_2(s) = \frac{1}{2} \Rightarrow \frac{1}{\beta_2}(s) = 2$$



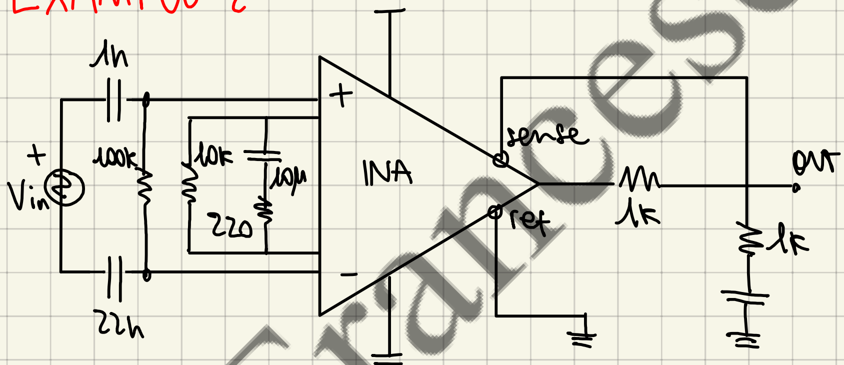
$$f_1^* = \frac{GBWP}{221} \cong 226\,kHz$$

$$f_2^* = \frac{GBWP}{2} = 25\,MHz$$

$$PM_{REAL1} = 180^\circ - \tan^{-1}\left(\frac{f_1^*}{f_0}\right) - \tan^{-1}\left(\frac{f_1^*}{f_{p,\beta}}\right) + \tan^{-1}\left(\frac{f_1^*}{f_{z,\beta}}\right)$$

$$\simeq 180^\circ - 90^\circ - 90^\circ + \tan^{-1}\left(\frac{f_1^*}{f_{z,\beta}}\right) \cong 66^\circ$$

EXAMPLE 2

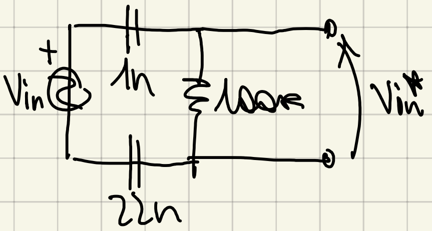


$R_f = 100k\Omega$

Ⓐ Plot the Bode diagram for Vout(f)/Vin(f) ideal gain

Let's divide the circuit in 2 stages

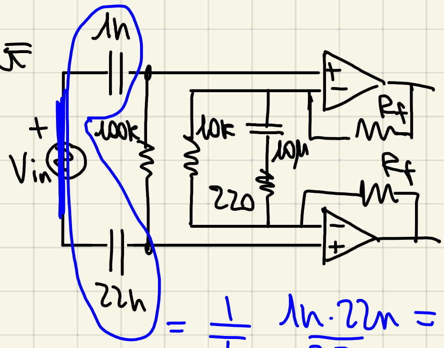
1ST stage:



$$\frac{V_{in}^*}{V_{in}}(0) = 0$$

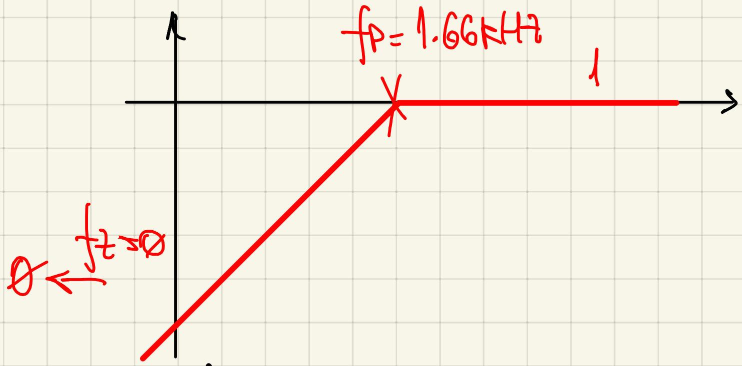
$$\frac{V_{in}^*}{V_{in}}(\infty) = 1$$

POLE



$= \frac{1n \cdot 22n}{23n} = 956pF$

$$f_P = \frac{1}{2\pi\, 956p \cdot 100k} = 1.66kHz$$



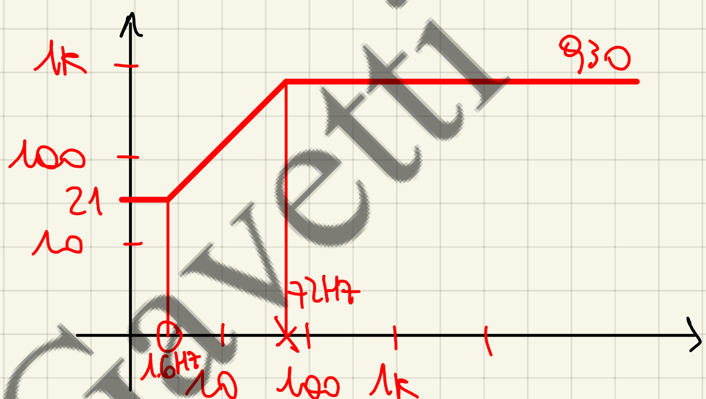
2nd stage → INA:

Hp: internal INA's resistances are all equal

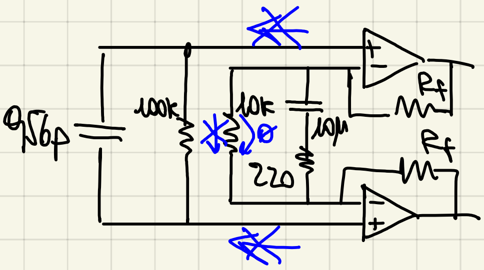
$$G_{i,INA}(s) = G_{diff} = \left(1 + \frac{2R_f}{R_G}\right)\left(\frac{R_2}{R_1}\right)$$

$$G_{i,INA}(0) = 1 + \frac{2R_f}{10k} = 1 + \frac{200k}{10k} = 21$$

$$G_{i,INA}(\infty) = 1 + \frac{2R_f}{10k \| 220} \simeq 930$$



There must be a zero and a pole, at least



$$f_{P,INA} = \frac{1}{2\pi\, 10\mu\, 220} = 72\,Hz$$

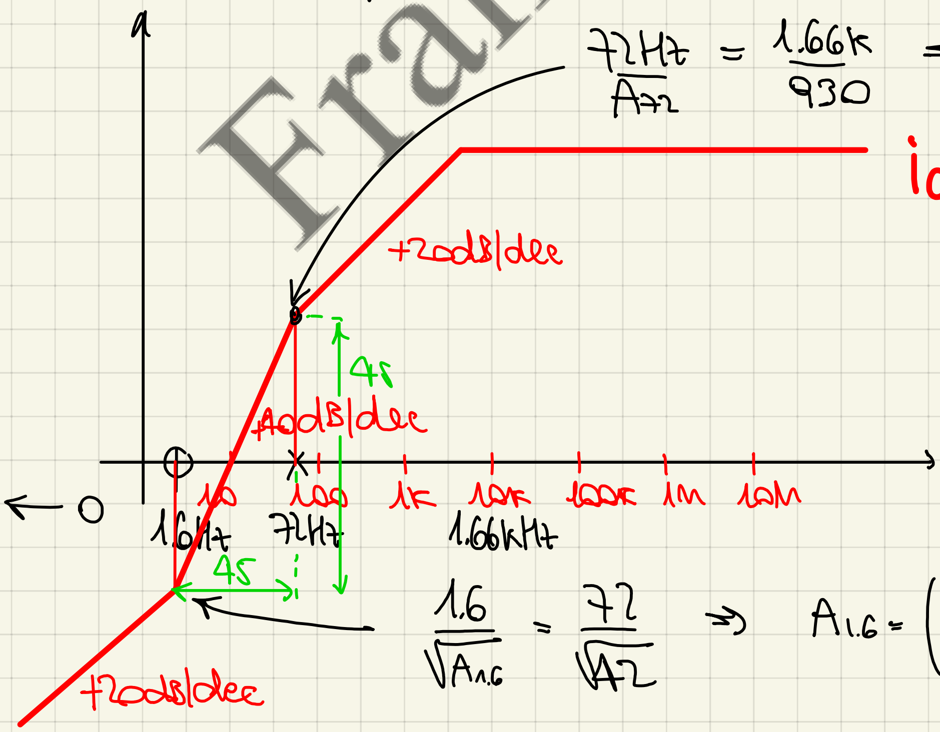
$$\Downarrow$$

$$f_{z,INA} = \frac{f_{P,INA}}{930} \cdot 21 = 1.6\,Hz$$

We have to multiply these two poles:

$$\frac{72Hz}{A_{72}} = \frac{1.66k}{930} \Rightarrow A_{72} = \frac{72}{1.66k}\, 930 \simeq 42$$

ideal poin



$$\frac{1.6}{\sqrt{A_{1.6}}} = \frac{72}{\sqrt{A_{72}}} \Rightarrow A_{1.6} = \left(\frac{1.6}{72}\right)^2 42 = 0.02$$