GYROSCOPES

THE CAPACITIVE MEMS GYROSCOPE
PART 1: CORIOLIS FORCE, DRIVE MODE, SENSITIVITY

MOTIVATIONS AND GOALS

Gyroscopes represent the category of inertial sensors of rotational motion.

NOTE: Though there exist gyroscopes that measure directly an angle, most MEMS gyros measure the ANGULAR RATE (or angular velocity) relying on the Coriolis acceleration (angular accelerations, instead, give rise to too small signals)

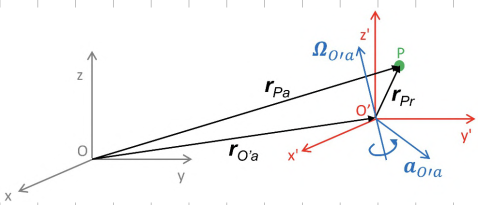
GOAL: understand the working principle of MEMS gyroscopes and to derive the expression of the motion induced by the Coriolis force

STATE OF THE ART FOR CONSUMER APPLICATIONS

- Angular rate measurement range: ±125 – ±2000 dps
  $\left(\frac{\pm 2000}{360} = 5{,}55 \text{ turns/s}\right)$
- Angular rate sensitivity:
  Angular rate sensitivity change vs. temperature: ± 1.5 %
- Angular rate typical zero-Rate level (Offset): ± 10 dps
- Angular rate typical zero-Rate level vs. temperature: ± 0.05 dps/°C
  Rate noise density: 7 mdps/$\sqrt{Hz}$
- Supply voltage: 1.8 V
- IddNM: 0.9 mA

THE CORIOLIS FORCE

- Kinematics of relative motion in presence of rotation
- Expression of the Coriolis force
- Measurement of angular rates



We want to measure the Coriolis acceleration by measuring the relative motion of the mass P in the non-inertial (relative) reference system ($\vec{a}_{Pr}$) and by knowing other parameters of our structure (the "true" Newtonian forces that are acting on our mass)

REM: translational acceleration and angular acceleration are negligible w/rt the Coriolis acceleration

$$\rightarrow \vec{a}_{Pa} = \vec{a}_{Pr} + \vec{a}_{O'a} + \cancel{\vec{\Omega}_{O'a} \times (\vec{\Omega}_{O'a} \times \vec{r}_{Pr})} + \cancel{(\dot{\vec{\Omega}}_{O'a} \times \vec{r}_{Pr})} + 2(\vec{\Omega}_{O'a} \times \vec{v}_{Pr})$$

$\vec{a}_{Pa}$ = "true" Newtonian acceleration of the mass P w/rt the absolute reference
↳ given by known forces acting on the system

$\vec{a}_{Pr}$ = acceleration of the mass P w/rt the relative reference
↳ what we exploit for the measurement

$\vec{a}_{O'a}$ = "apparent" acceleration of the relative reference w/rt the absolute one
↳ what we want to reject (the AXEL will measure it)

$\vec{\Omega}_{O'a} \times \vec{v}_{Pr}$ = Coriolis acceleration
↳ what we want to measure

Hp: Assume a system which is rotating only ($\vec{a}_{o'a} = \emptyset$)

Newton's law in our inertial frame: $\vec{F}_{Pa} = m\vec{a}_{Pa}$

$$\Rightarrow \vec{F}_{Pr} = m\vec{a}_{Pr} = m\vec{a}_{Pa} - m\vec{a}_{o'a} - 2m(\vec{\Omega}_{o'a} \times \vec{v}_{Pr}) = \vec{F}_{Pa} + \vec{F}_{Acc} + \vec{F}_{cor}$$

Known "true" Newtonian forces:
- Elastic force: $F_{elastic} = -k r_{Pr}$
- Damping force: $F_{damping} = -b v_{Pr}$

} $*_1$

$*_1$ given by the fact that our MEMS (our mass P) is not floating inside the package, but it is suspended through springs and packaged in a gas encapsulation

NOTE: A gyro that measures a z-axis angular rate moves in 2 directions: along the x-axis bcz it needs to have a velocity (v_{Pr}) and along the y-axis bcz the force acts in that direction

$$m\ddot{y} + b\dot{y} + ky = F_{inertial} \longrightarrow m\ddot{\vec{y}}_{Pr} + b\dot{\vec{y}}_{Pr} + k\vec{y}_{Pr} = \cancel{-m\vec{a}_{o'a}} - 2m(\vec{\Omega}_{o'a} \times \vec{v}_{x,Pr})$$

↓

Hp: Assume that effects of acceleration are rejected in a gyro

PROCEDURE TO MEASURE Z-AXIS ANGULAR RATE:

1. Hp: assume a velocity $\vec{v}_x$
2. Hp: assume for the sake of simplicity to have rotational motion only: $\vec{a}_{o'a} = \emptyset$

   ↳ NOTE: we will effectively design our gyro to be immune to acceleration which will be measured by an accelerometer
3. we measure the Coriolis force along the y-direction

   $$\vec{F}_{y,cor} = -2m(\Omega_z \times \vec{v}_x)$$

   if the value of the velocity $\vec{v}_x$ in the relative system is known

   $$\rightarrow m\ddot{y} + b\dot{y} + ky = -2m\Omega_z v_x$$

   $\Rightarrow$ NOTE: v_x needs to be controlled and Ω_z is what we want to measure

RESULT: Since the system will feature motion along two directions, it will need two spring-mass-damper systems (two modes of interest), one per motion direction

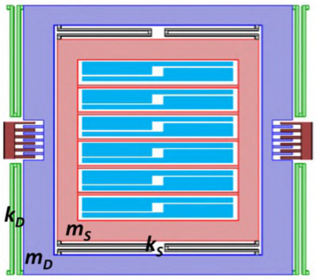
NOTE (IMP!): we measure the angular rate through the Coriolis acceleration bcz it is the largest, so the easiest to measure.
Dragging contributions, as well as linear accelerations, will be rejected by using a differential architecture

RESUME: we exploit the Coriolis force to measure the rotation angle and in order to exploit it we need to set a velocity along one direction and we need to measure motion along one of its orthogonal directions
→ we need to design a device that can displace along two different directions
For example, if we want to measure the rotation angle (and so the angular rate) along the z-axis, we can set a velocity along the x-axis and we measure the Coriolis force along the y-one.

MEMS GYROSCOPES AT A GLANCE

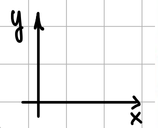
- Architecture and working principle overview
- The drive resonator: drive motion and velocity
- The sense frame: capacitive detection

How do we design a device that can displace along two different directions?



We can exploit the technique of coupling two nested masses

Hp: assume we want to measure a z-axis angular rate



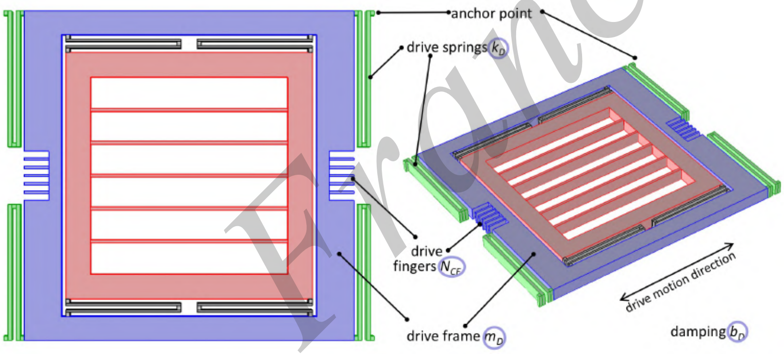
DESIGN: The external DRIVE SPRINGS (k_D) are rigid along the y-direction, so the outer mass (the purple one) which is called DRIVE MASS/FRAME (m_D) can displace only along the x-direction.
Viceversa, the inner SENSE SPRINGS (k_S) are rigid along the x-direction, so the inner mass (the orange one), which is called SENSE MASS/FRAME (m_S) can move along the y-direction, but it can also move along the x-one bcz it is attached to the externall mass which can displace along this direction.

=> There are 2 MOTION DIRECTIONS OF INTEREST in a MEMS gyroscope:

1. DRIVE MODE DIRECTION: along which the device is excited by an electrostatic oscillation to create the desired $\vec{v}_x$ (which will be sinusoidal)
2. SENSE MODE DIRECTION: along which only the sense (inner) mass of the device moves, under the presence of an angular rate, to sense the Coriolis force (which also will be sinusoidal)

NOTE:
- both directions are orthogonal to the angular rate sensing direction
- if there not exist any rotation (and so any angular rate to be measured) the device will displace only along the drive mode direction
- Since the velocity should be known, the drive motion, which is the mode that provides such velocity, is fixed in amplitude
- The sense springs are also called DECOUPLING SPRINGS bcz they decouple the 2 motions

The drive mode is kept in resonance oscillation, usually via COMB FINGERS



NOTE:
- Since both the drive and sense masses take part to the drive motion, when we study it we can consider the overall mass given by the sum of m_d and m_s
- As this device is driven along the drive-direction using comb fingers which usually feature a low damping coefficient, we would expect to have:
  - low b_D → high Q_D

- The drive springs are attached directly to the substrate and they need to be designed in order to be very rigid in the sense-direction
- The decoupling sense springs are attached to the drive frame and they need to be designed in order to be quite rigid in the drive-direction

WORKING PRINCIPLE AT A GLANCE:

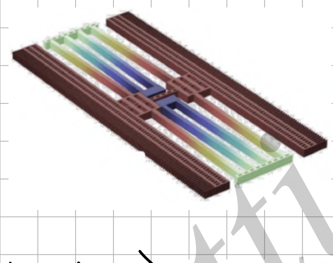
1. The drive mode is kept in oscillation along the x-direction: both frames ($m_D + m_S$) move together w/ a sinusoidal velocity $\vec{v}_{Dr}$

② In presence of an angular rate in the z-direction, both frames experience a Coriolis force in the y-direction. However the drive frame is rigid in that direction and will not move, while the sense frame displacement will be detected via capacitive sensing and will provide the information on the Coriolis force.

Where would you put the largest part of your mass? In the sense or in the drive frame?

A gyroscope can be seen as a combination of:

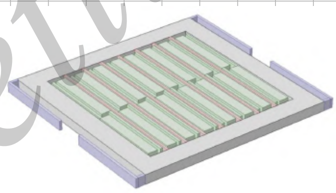
COMB-DRIVEN RESONATOR



- $m = m_D + m_S$
- $k = k_D$
- $b = b_D$

$$\boxed{(m_D+m_S)\ddot{x} + b_D\dot{x} + k_D x = F_D}$$

- PARALLEL-PLATE ACCELEROMETER (of the Coriolis acceleration)



- $m = m_S$
- $k = k_S$
- $b = b_S$

$$\boxed{m_S\ddot{y} + b_S\dot{y} + k_S y = F_{cor}}$$

PROCEDURE TO COMPUTE THE GYROSCOPE' SENSITIVITY:

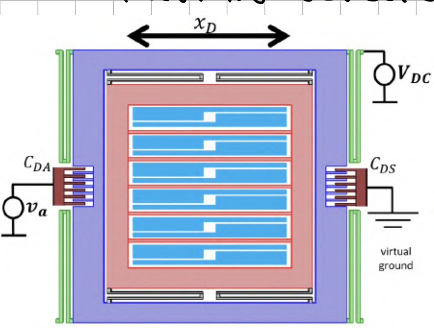
① Calculation of the DRIVE ELECTROSTATIC FORCE $\boxed{F_{elec}}$
② Calculation of the DRIVE DISPLACEMENT AT RESONANCE $\boxed{X_D}$ — DRIVE RESONATOR
③ Calculation of the DRIVE VELOCITY $\boxed{v_D}$
④ Calculation of the CORIOLIS FORCE $\boxed{F_{cor}}$
⑤ Calculation of the SENSE MODE DISPLACEMENT $\boxed{y_S}$
⑥ Calculation of the SENSE CAPACITANCE VARIATION $\boxed{\Delta C_S}$
⑦ Calculation of the OUTPUT VOLTAGE PER UNIT RATE (SENSITIVITY) $\boxed{\frac{\Delta V_{out}}{\Omega}}$

CALCULATION OF THE SENSITIVITY { Sensitivity as a function of drive displacement and bandwidth

- MOTION IN THE DRIVE DIRECTION: DRIVE RESONATOR

① Calculation of the DRIVE ELECTROSTATIC FORCE $\boxed{F_{elec}}$

NOTE: The calculation of the electrostatic force follows exactly what we have seen for a comb-driven, comb-sensed resonator, biased w/ a rotor DC voltage much larger than the actuation AC voltage → Hp: small signal approximation: $\boxed{\frac{v_{ac}}{4} \ll V_{DC}}$



$$\boxed{C_{DA} = \frac{2\varepsilon_0 h (L_{ov}-x) N_{CF}}{g}}$$ DRIVE-MODE ACTUATION CAPACITANCE

$$\boxed{C_{DS} = \frac{2\varepsilon_0 h (L_{ov}+x) N_{CF}}{g}}$$ DRIVE-MODE SENSING CAPACITANCE

The electrostatic force depends on the capacitance variation per unit displacement:

$$F_{elec} = \frac{\Delta V^2}{2}\frac{dC}{dx}$$

$$\frac{dC_{DA}}{dx} = \frac{2\varepsilon_0 h N_{CF}}{g} = \frac{dC_{DS}}{dx}$$ (NOTE: independent of the displacement itself)

Hp: Assume to have the rotor biased at V_{DC}, that the closed loop sustaining circuit applies an AC signal $V_A = v_a \sin(\omega_0 t)$ to the actuation port and that the detection (sensing) port is kept at ground

$$\boxed{\begin{cases} V_A = v_a \sin(\omega_0 t) \\ V_P = V_{DC} \\ V_S = \emptyset \end{cases}}$$ BIASING CONDITIONS

$$\Rightarrow |F_{elec}| = \left|\frac{(V_A - V_P)^2 - (V_P - V_S)^2}{2}\cdot\frac{dC_A}{dx}\right| = \frac{(v_a\sin(\omega_0 t) - V_{DC})^2 - (V_{DC} - 0)^2}{2}\,\frac{2\varepsilon_0 h N_{CF}}{g}$$

$$= \frac{\varepsilon_0 h N_{CF}}{g}\left[(v_a\sin(\omega_0 t))^2 - 2 v_a V_{DC}\sin(\omega_0 t) + \cancel{V_{DC}^2} - \cancel{V_{DC}^2}\right]$$

Hp:

$$\boxed{v_a \ll 4V_{DC}} \longrightarrow |F_{elec}| \sim \frac{\varepsilon_0 h N_{CF}}{g} 2 V_{DC} v_a \sin(\omega_0 t)$$

$$\longrightarrow \boxed{F_{elec,0} = \frac{\varepsilon_0 h N_{CF}}{g} 2 V_{DC} v_a}$$ AMPLITUDE OF THE DRIVE ELECTROSTATIC FORCE

2 Calculation of the DRIVE DISPLACEMENT AT RESONANCE $\boxed{X_D}$

Hp:
- The resonator is self-sustained at resonance through an oscillator circuit designed around the drive mode
- The transfer function b/w force on the drive mode and drive displacement is fully amplified by the quality factor Q_D:

$$|T_D(j\omega_0)| = \left|\frac{X_D(j\omega_0)}{F_D(j\omega_0)}\right| = \frac{Q_D}{k_D}$$

$$\Rightarrow \boxed{X_{D,0} = F_{elec,0}\cdot\frac{Q_D}{k_D} = \frac{Q_D}{k_D}\frac{\varepsilon_0 h N_{CF}}{g} 2 V_{DC} v_a}$$ AMPLITUDE OF THE DRIVE DISPLACEMENT AT RESONANCE

3 Calculation of the DRIVE VELOCITY $\boxed{v_D}$

In general we can write:

$$X_D = X_{D,0}\,\mathrm{trig}(\omega_0 t) \rightarrow \dot{x}_D = v_D = \omega_0 X_{D,0}\,\mathrm{trig}(\omega_0 t) \rightarrow v_{D,0} = \omega_0 X_{D,0}$$

$$\Rightarrow \boxed{v_{D,0} = \omega_0 X_{D,0} = \omega_0 \frac{Q_D}{k_D}\frac{\varepsilon_0 h N_{CF}}{g} 2 V_{DC} v_a}$$ AMPLITUDE OF THE DRIVE VELOCITY

4 Calculation of the CORIOLIS FORCE $\boxed{F_{cor}}$

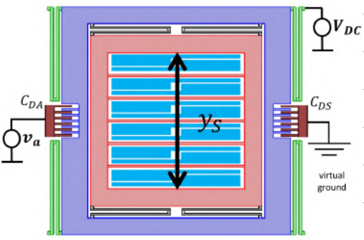
$F_{cor} = m\, a_{cor} = -2 m_s v_D \cdot \Omega_z$ NOTE: we simplify the cross-product as we are considering the angular rate component orthogonal to drive motion

$$\Rightarrow \boxed{F_{cor,0} = 2 m_s \omega_0 \frac{Q_D}{k_D} \frac{\varepsilon_0 h N_{CF}}{g} 2 V_{DC} v_a \Omega_z}$$ AMPLITUDE OF THE CORIOLIS FORCE

Att: So far we have always written ω_0, but writing ω_D instead of ω_0 would have been more correct.

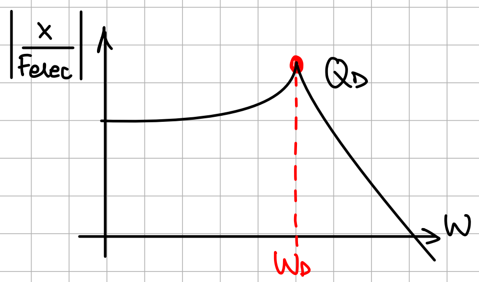
- **MOTION IN THE SENSING DIRECTION : RESONANT SENSING**

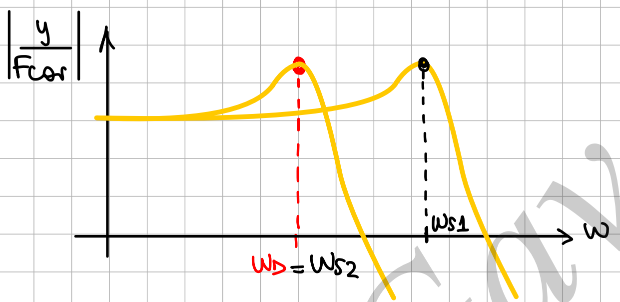
⑤ Calculation of the SENSE MODE DISPLACEMENT $\boxed{y_s}$



Now we are interested in calculating the motion in the y-direction on the sense frame caused by the Coriolis force

NOTE: There are few different options that we can have for a gyro design, depending on whether the sense-mode resonance frequency is matched or not w/ the drive-mode one





NOTE: 1. $\boxed{\omega_S \gg \omega_D}$: if the sense-mode resonance frequency falls much beyond the drive-mode one, the Coriolis force will excite the sense-mode not at the resonance frequency and therefore we will have

2. $\boxed{\omega_S = \omega_D}$: if the sense resonator is designed in such a way that its sense-mode resonance frequency ω_S falls exactly where ω_D is, we are maximizing the transduction factor $|y/F_{cor}|$

Hp: we assume the sense mode to be matched to the drive mode at ω_0

↳ MODE MATCH CONDITION $\boxed{\omega_D = \omega_S = \omega_0}$

$$\Rightarrow \boxed{y_{s,0} = F_{cor,0} \frac{Q_s}{k_s} = 2 m_s \omega_0 x_{D,0} \Omega_z \frac{Q_s}{k_s}}$$ AMPLITUDE OF THE SENSE DISPLACEMENT (MODE MATCHED)

NOTE: Typically $Q_S < Q_D$ →

$$\boxed{Q_s = \frac{k_s}{\omega_0 b_s} = \frac{\omega_d m_s}{b_s}}$$ SENSE-MODE QUALITY FACTOR

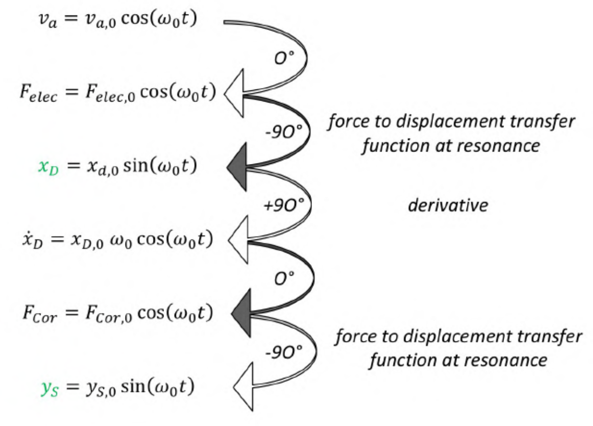
$$\Rightarrow \frac{y_{s,0}}{\Omega_z} = 2 m_s \cancel{\omega_0} x_{D,0} \frac{\cancel{k_s}}{\cancel{\omega_0} b_s} \frac{1}{\cancel{k_s}} = \frac{2 m_s x_{D,0}}{b_s} = \frac{x_{D,0}}{b_s / 2m_s}$$

NOTE: $$\boxed{\frac{\omega_d}{2Q} = \frac{\omega_d}{2 \frac{\omega_d m}{b}} = \frac{b}{2m} = \Delta\omega_{BW}}$$ SENSING BANDWIDTH AT RESONANCE

$$\Rightarrow \boxed{\frac{y_{s,0}}{\Omega_z} = \frac{x_{D,0}}{\Delta\omega_{BW}}}$$ SCALE FACTOR ⇒ $\boxed{\text{TRADE-OFF SCALE FACTOR vs SENSING BW}}$

NOTE: SCALE FACTOR = GAIN OF THE SENSE MODE

PHASES :



CONCLUSIONS

- The drive motion amplitude needs to be stable against Q factor changes → an ACC - AMPLITUDE CONTROL CIRCUIT is needed

  The sense mode needs to be readout → TIA / CA FRONT-END needed
- The signal is modulated at the drive frequency → DEMODULATION needed

THE CAPACITIVE MEMS GYROSCOPE
PART 2: OVERALL SENSITIVITY AND ADVANCED ARCHITECTURES

MOTIVATIONS AND GOALS

GOALS:
- to define the complete block scheme of a gyroscope (device + electronics)
- to define advanced architectures for the device and electronics implementations, in order to reject undesired environmental effects

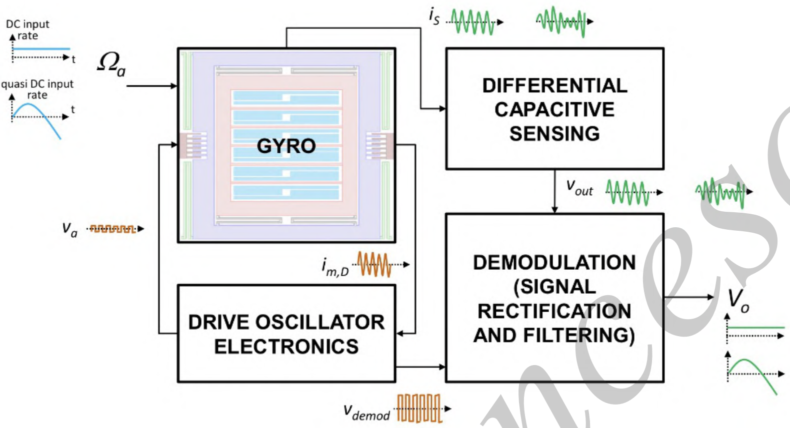
We have seen that, under the presence of an angular rate, the drive velocity v_D at ω_D generates a Coriolis force which is itself a sinusoidal signal at ω_D. Such a force acts on both the drive and sense frames, but only the sense frame moves along the y-direction. Therefore, the Coriolis force generates a sinusoidal displacement y_S at ω_D which we need to readout → we need a differential capacitive readout

↳ **NOTE**: assuming mode-matched condition $\omega_S = \omega_D$ → The Coriolis force excites the sense mode at a freq. for which its response is maximized

SENSE-MODE RESONANCE FREQUENCY $\boxed{\omega_S = \sqrt{\dfrac{k_S}{m_S}}}$

DRIVE-MODE RESONANCE FREQUENCY $\boxed{\omega_D = \sqrt{\dfrac{k_D}{(m_D + m_S)}}}$

NOTE (IMP!): since $m_D + m_S > m_S$, in order to have $\omega_D = \omega_S$ → $k_D > k_S$



- **DIFFERENTIAL CAPACITIVE SENSING**: its purpose is to readout the sense displacement y_S induced by the Coriolis force
- **DEMODULATOR (RECTIFICATOR + FILTER)**: its purpose is to bring back signals which are modulated around ω_0 to baseband (to around DC)
- **DRIVE OSCILLATOR ELECTRONICS**: its purpose is to keep the gyro in continuous oscillation at a desired displacement amplitude

DRIVE LOOP: AMPLITUDE CONTROL IN MEMS GYROSCOPES

- Summary of basic working principle and sensitivity
- Need for AGC circuits
- AGC examples

$$\begin{cases} x_{D,0} = F_{elec,0}\,\dfrac{Q_D}{k_D} = \dfrac{Q_D}{k_D}\,\dfrac{\varepsilon_0 h N_{cf}}{g}\,2V_{DC}v_a \quad \div\ Q_D \\ \\ Q_D = Q_D(T) \end{cases}$$

→ The Q factor is a strong function of the absolute temperature

CONSEQUENCES:
- $x_{D,0}$ changes w/ the absolute temperature
- The scale factor $\left(\dfrac{y_{S,0}}{\Omega_z} = \dfrac{x_{D,0}}{\Delta\omega_{BW}}\right)$ changes w/ the temperature as well

⇒ we need an AMPLITUDE CONTROL CIRCUIT that keeps the motion amplitude $x_{D,0}$ stable against Q factor changes (due to T and from part to part as well)

REPEATABILITY ISSUE: actually The Q factor changes not only as a function of The temperature but also from part to part. This means That if we don't use an amplitude control circuit the sensitivity will be widely different among different parts of The device.

Which topology among The ones studied for the oscillator would you chose to control The amplitude?

As The CA topology gives an output proportional to the displacement x_D we choose This one

What do we use in electronics to stabilize? → Negative feedback loops!!

CA-BASED FRONT-END AGC (AUTOMATIC-GAIN-CONTROL) CIRCUIT

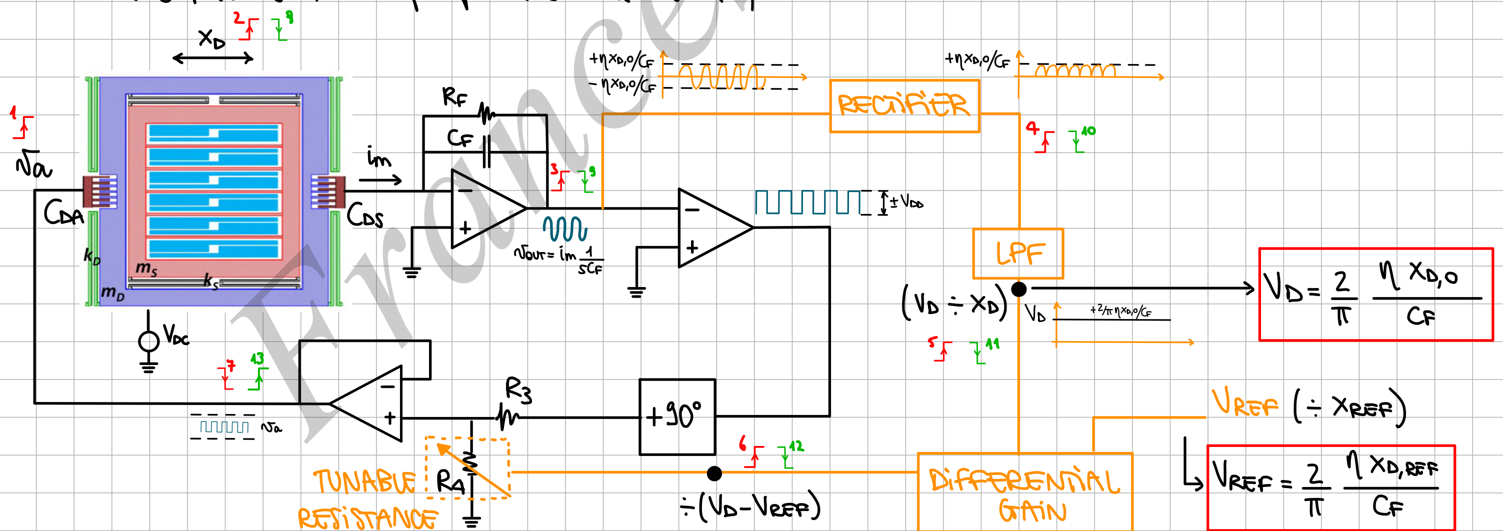
Working principle of The AGC loop:

1. take The AC signal at The CA output proportional to $x_{D,0}$
2. rectify + low pass filter → we now have a DC signal proportional to $x_{D,0}$
3. compare it w/ a reference V_{REF}, related To The motion x_{REF} we want to set

↳ REM: we want to control The drive displacement amplitude $x_{D,0}$

As in any circuit w/ the feedback mechanism we need a reference to which compare our signal, we can imagine That our reference is an external DC signal which will be essentially compared to The signal which contains information about the real time amplitude of our drive displacement.

NOTE: The easiest way to have a DC signal whose content is proportional to The drive displacement output is to rectify and low pass filter The CA output signal which we know to be proportional To $x_{D,0}$



We have two loops:

- OSCILLATOR POSITIVE LOOP (PRIMARY) Group1
  - CHARGE-AMPLIFIER *
  - COMPARATOR w/ PS
  - +90°
  - RESISTIVE DIVIDER
  - BUFFER *
- CONTROL NEGATIVE LOOP (SECONDARY) Group2
  - RECTIFIER
  - LOW-PASS FILTER
  - COMPARATOR w/ V_{REF}
  - TUNABLE RESISTANCE (R_4) *

NOTE:
- The MEMS, the CA, R_4 and the BUFFER are part of both loops (*)
- The factor $2/\pi$ gives the mean value of the rectified waveform

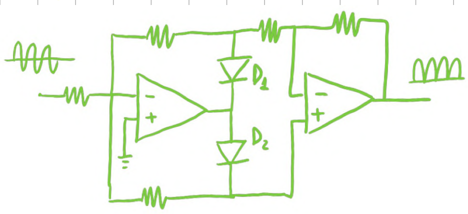
Depending on the sign of the difference $(V_D - V_{REF})$ we need to react on our primary loop through the tunable resistance (R_4):

- $\boxed{V_D - V_{REF} > 0 \equiv V_D > V_{REF}}$ → we need to decrease R_4 in order to decrease $\hat{V}_a$ and so $x_{D,0}$
- $\boxed{V_D - V_{REF} < 0 \equiv V_{REF} > V_D}$ → we need to increase R_4 in order to increase $\hat{V}_a$ and so $x_{D,0}$

⇒ The feedback (G_{loop2}) tends to cancel the variation of x_D keeping it to x_{REF} by changing the drive signal amplitude $\hat{V}_a$

NOTE: The negative feedback brings $x_{D,0}$ to the "virtual ground" x_{REF} through G_{loop2}

① **RECTIFIER** ⟶ it makes the absolute value



② **LOW-PASS FILTER** (active RC or SALLEN-KEY LPF)

Which frequency does the LPF operate at?

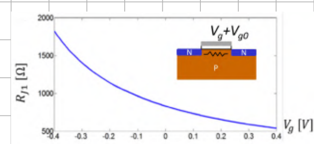
The rectified waveform has components at DC that we want save and components at $2\omega_0$ that we want cancel out

⇒ we have to set the frequency of the poles at $f_P < \omega_0/5$

③ **TUNABLE GAIN** → is a n-FET resistance in Ohmic region

Acting on the gate voltage we can change the drain-source resistance

↳ VGA - VARIABLE GAIN AMPLIFIER



OVERALL SENSITIVITY CALCULATION ⟨ Capacitive readout / Charge amplifier differential readout / Overall system view

Why did we choose to use PPs for gyroscope sensing?

REM: PPs sensing is preferable to CF if displacements $y_{s,0}$ are small compared w/ the gap

$$y_{s,0} = \frac{x_{D,0}}{\Delta\omega_{BW}}\,\Omega_z$$

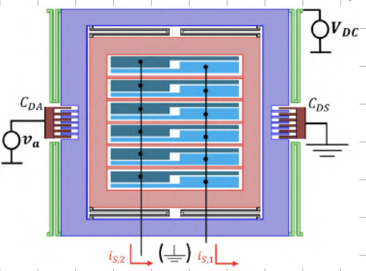
NOTE: $x_{D,0}|_{Typ} = 5\,\mu m$ $\quad \Delta\omega_{BW}|_{min} = 200\,Hz$ $\quad \Omega_z \sim 30\ rad/s$

→ $y_{s,0} \sim 120\,nm$ → The small displacement approximation holds and so we can effectively use PPs ▨

NOTE: In the former class we have interrupted our sensitivity computation procedure finding the sense-mode displacement amplitude $y_{s,0}$ (step 5); we have thus continue computing the sense capacitance variation ΔC_S

NOTE: The results we'll found now on are under small displacement approximation

⑥ Calculation of the SENSE CAPACITANCE VARIATION $\boxed{\Delta C_S}$



As the entire rotor (including drive springs, drive frame, sense springs and sense frame) is kept at V_{DC}, we keep the virtual ground of the readout stages at 0V

$$\rightarrow \boxed{i_{S,0i} = V_{DC}\frac{dC_{Si}}{dt} = V_{DC}\frac{dC_{Si}}{dy}\frac{dy}{dt} = V_{DC}\,\omega_0\, y_{S,0}\frac{dC_{Si}}{dy}}$$

CURRENT FLOWING THROUG EACH SENSE PORT ($i = 1, 2$)

$$C_{S1} = \frac{\varepsilon_0 A_{PP} N_{PP}}{g+y}\;,\quad C_{S1} = \frac{\varepsilon_0 A_{PP} N_{PP}}{g-y} \rightarrow \boxed{\left|\frac{dC_{Si}}{dy}\right| \approx \frac{\varepsilon_0 A_{PP} N_{PP}}{g^2} = \frac{C_{S0}}{g}}$$

SINGLE-ENDED SENSE CAPACITANCE VARIATION FOR SMALL DISPLACEMENTS

⑦ Calculation of the OUTPUT VOLTAGE PER UNIT RATE (SENSITIVITY) $\boxed{\frac{\Delta V_{OUT}}{\Omega}}$

The current is integrated by each charge amplifier:

$$\boxed{V_{OUT,0i} = \left|\frac{i_{S,0i}}{j\omega_0 C_{FS}}\right| = V_{DC}\frac{dC_{S,i}}{dy}\frac{y_{S,0}}{C_{FS}} = \frac{V_{DC}}{C_{FS}}\frac{C_{S0}}{g}\,y_{S,0}}$$

VOLTAGE AT THE OUTPUT OF THE CA

$$\Rightarrow \Delta V_{OUT,0} = 2\frac{V_{DC}}{C_{FS}}\frac{C_{S0}}{g}\,y_{S,0} = 2\frac{V_{DC}}{C_{FS}}\frac{C_{S0}}{g}\frac{x_{D,0}}{\Delta\omega_{BW}}\,\Omega_z$$

$$\boxed{C_{S0} = \frac{\varepsilon_0 A_{PP} N_{PP}}{g}}$$

$$\Rightarrow \boxed{\frac{\Delta V_{OUT,0}}{\Omega_z} = 2\frac{V_{DC}}{C_{FS}}\frac{C_{S0}}{g}\frac{x_{D,0}}{\Delta\omega_{BW}}}$$

OVERALL GYROSCOPE SENSITIVITY UNDER SMALL DISPLACEMENTS AND MODE-MATCHED CONDITIONS

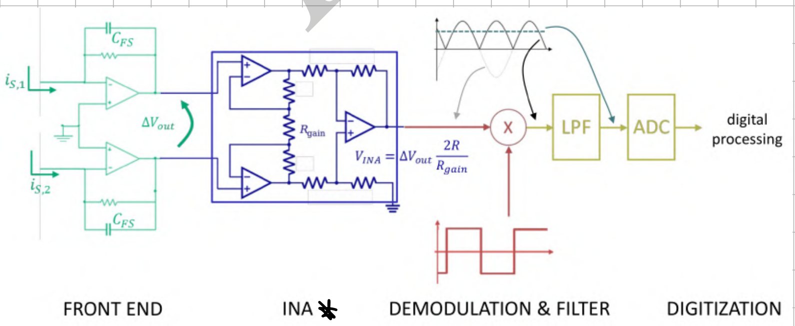
Att: $\Delta V_{OUT,0}$ is still modulated around $\omega_0 = \omega_D$ (mode matched condition)

↳ before being processed by the software that uses the angular rate information, the signal needs to return to a DC value which will be digitized by an ADC.

↳ PROCEDURE:

1. Turn the differential AC signal into a single-ended AC signal → INA
2. Multiply the AC signal by a square wave at the same frequency and phase
3. Filter the high frequency component to save the DC value → LPF
4. Acquired the so obtained DC value w/ the ADC

   ↳ NOTE: $N_{bits} = \frac{\pm FSR}{\text{resolution}}$



$$* \boxed{G_{INA} = 1 + \frac{2R}{R_{gain}} \approx \frac{2R}{R_{gain}}}$$

INA - INSTRUMENTATION AMPLIFIER GAIN

Why synchronous demodulation?
(Why do we multiply for a square wave and we don't rectify?) → to take the absolute value

DEMODULATION = bringing an AC signal modulated at ω_0 back to baseband (around DC), holding information about its amplitude

There exist two types of demodulation:

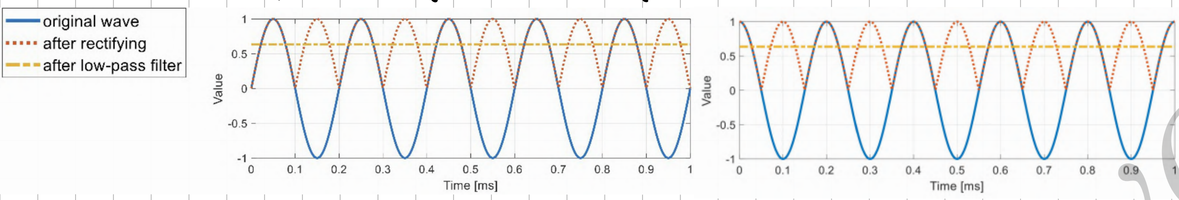
ASYNCHRONOUS DEMODULATION (bc it doesn't care about the phase of our signal)

↳ RECTIFYING + FILTERING

It does not take into account the phase of the signal to demodulate: we thus obtain the same result for different phase signals

NOTE: in addition, if there are any spurious contents in our output which are not related to the angular rate, these signals would be rectified and kept in the output as undesired information

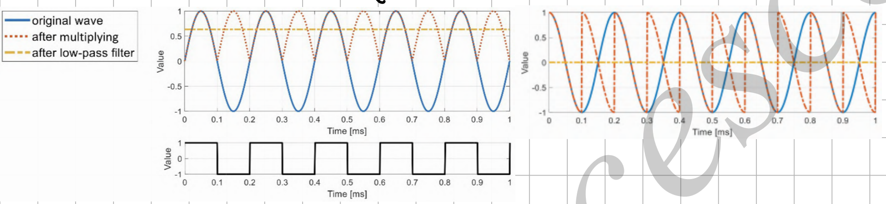
ISSUE: using an asynchronous demodulation we loose the information about the sign of the angular rate (about the direction in which our device is rotating)



- **SYNCHRONOUS DEMODULATION**

↳ MULTIPLICATION BY A SQUARE WAVE + FILTERING

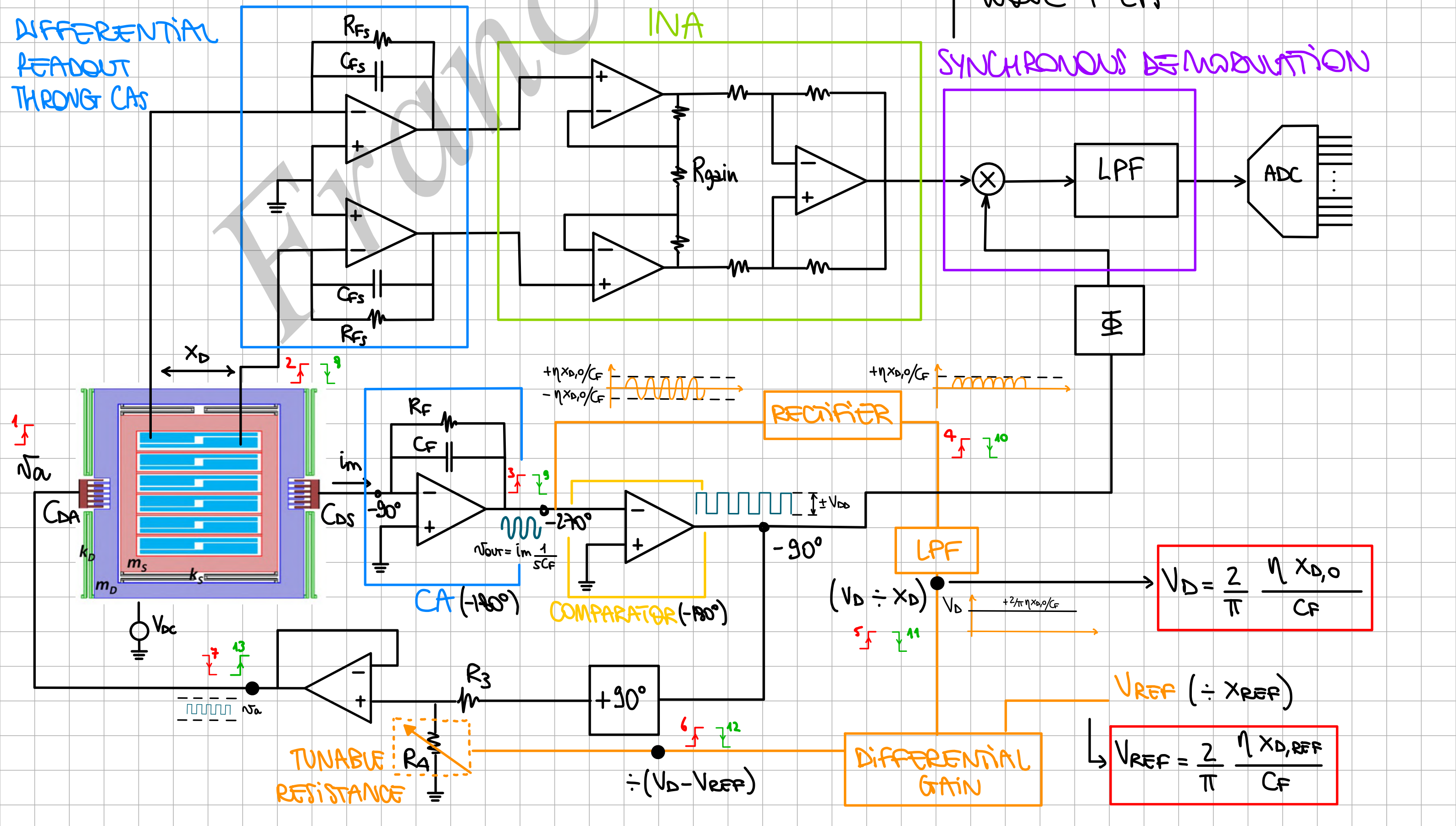
It takes into account the phase of the signal to demodulate: we can distinguish the amplitude of different phase components, so the sign of the angular rate, and we are also able to reject all the components that are in quadrature w/rt the signal that I want to measure (FUNDAMENTAL FEATURE)



in order to not loose the sign of the angular rate

↑

→ multiplication by a square wave + LPF



4 MAIN ELECTRONIC BLOCKS :
1. OSCILLATOR (primary loop)
2. AGC - AMPLITUDE GAIN CONTROL (secondary loop)
3. SENSE READOUT and DEMODULATION CHAIN
4. ADC CONVERTION

ADVANCED GYROSCOPES ARCHITECTURES
- Effects of accelerations and vibrations
- Dual-mass, Tuning force gyroscope
- Levered sense mode

NOTE (IMP!): we have initially assumed to just have rotational motion : $\vec{a}_{o'a} = 0$ (neglected linear accelerations)
→ or in other words to have a device designed in such a way that the accelerations in the y-direction (which represent disturbances) are canceled out

How does the gyroscope sense mass reject the accelerations in the y-direction?
How disturbing these accelerations on the suspended mass be?

example : (consumer application)

$FSR = 18\,g$, $f_0 = 5\,kHz$ → $y_{s,MAX} = \frac{FSR}{\omega_0^2} = \frac{18 \cdot 9.8\, m/s^2}{(2\pi \cdot 5000)^2} = 176\ nm$ (effect of a consumer FSR acceleration)

$x_{D,0} = 5\mu m$, $\Delta f_{BW} = 200Hz$, $\Omega_z = 1000\, dps$ → $y_{s,0} = \frac{5\mu m}{2\pi \cdot 200 \frac{rad}{s}} \, 2\, \frac{mrad}{s} = 8\, pm$ (effect of a Coriolis force)

→ we conclude that the effects of acceleration are effectively disturbing

NOTE: even if the acceleration is not modulated at ω_0 and could be thus partially filtered, its effects are huge and it would be better to avoid such signals in the readout chain

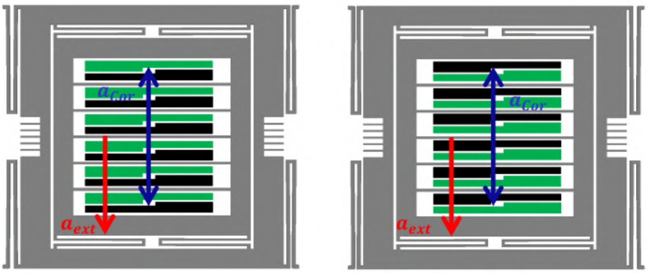
Att: In this example we have also assumed that the acceleration is not at resonance w/ the gyro modes, otherwise the unwanted acceleration would be fully amplified by the Q factor and its effects would be even more critical

RESULT: The frequencies of the gyro modes do not have to fall in a range where other disturbances, like sounds and vibrations, are common!!
→ gyro modes are designed to be :
- at 15-20kHz for consumer applications → above the audio range
- at 30-50 kHz for automotive, aerospace and military applications → above vibrations range

Att: Often this is not enough and more advanced gyroscope architecture should be design in order to reject or at least mitigate these undesired disturbances

- DUAL-MASS GYROSCOPE

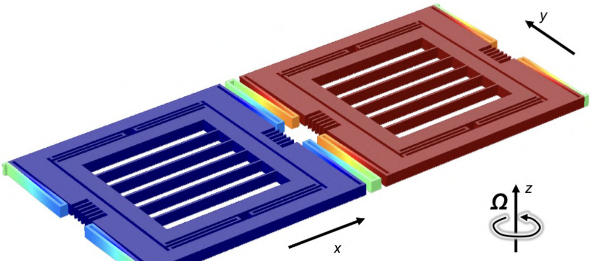
We use two gyroscopes instead of one, which are activated w/ the drive modes in opposite direction.
→ As a consequence, the Coriolis accelerations have opposite directions, as $v_{x_l} = -v_{x_r}$.



Linear accelerations have instead the same direction on both sense masses

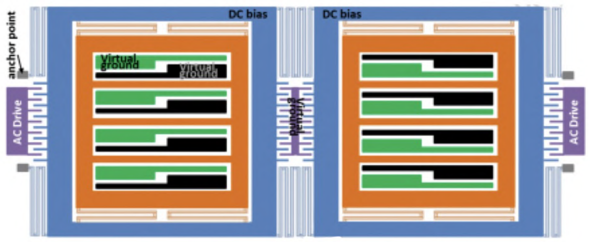
PROBLEM: This structure works properly as long the two masses move exactly at the same resonance frequency.

DIFFERENTIAL TUNING FORK (TF) GYROSCOPE



Rather than designing two separated devices, it is useful to couple them through a spring called TUNING FORK (TF)

TUNING FORK (TF): is the spring in the middle that couples the two part of the gyro.
This mechanical coupling is such that there is only a single resonance frequency.
In other words, this ensures a single frequency for the ANTI-PHASE DRIVE MODE and avoids the chance that two separate drive modes have different frequencies due to process nonuniformities.

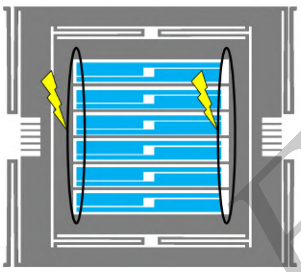


→ The Coriolis force and the sense mode are in anti-phase too: w/ a suitable arrangement of PP stators, acceleration will be rejected as a common mode signal

NOTE:
- each half undergoes the same displacement y_s
- the overall capacitance variation is doubled
- the overall sensitivity is doubled
- the area is doubled as well

ADVANTAGES:
- linear accelerations are rejected
- doubled sensitivity

DRAWBACKS:
- doubled area
- more PC



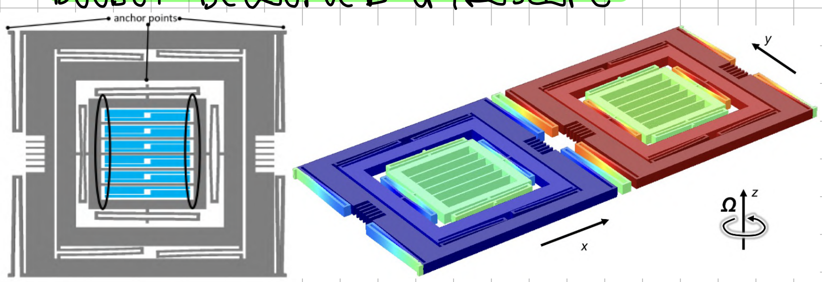
ANOTHER ISSUE: the sense mode, though moving parallel to the stators during drive motion, can see small, but not negligible, capacitance variation because of FRINGE EFFECTS

FRINGE EFFECT =

SOLUTION: to adopt a further decoupling by an intermediate frame, called CORIOLIS / DECOUPLING FRAME

↓

- DOUBLY-DECOUPLED GYROSCOPE

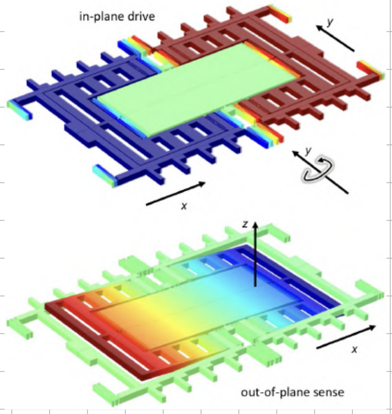


Now we have 3 frames:
1. DRIVE FRAME: move along the x-direction
2. CORIOLIS / DECOUPLING FRAME: move along both directions
3. SENSE FRAME: moves only along the y-direction

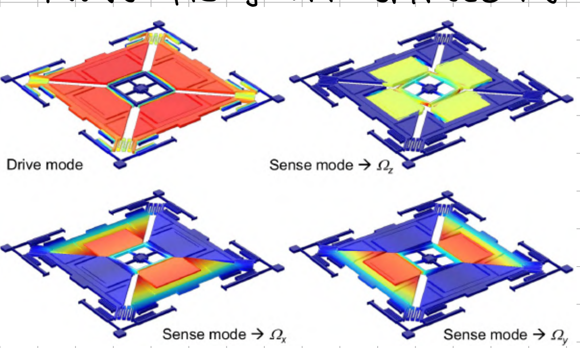
In this case, where would you put the largest part of your mass?

Other gyroscopes' architectures:

- 1-AXIS GYROSCOPE



- MONOLITIC 3-AXIS GYROSCOPE



ADVANTAGE: As only one drive circuit is needed, it allows to save power consumption (good for consumer applications)

CONCLUSIONS

- GYROSCOPE = RESONATOR + ACCELEROMETER OF F_{cor}

THE CAPACITIVE MEMS GYROSCOPE
PART 3: BANDWIDTH AND NOISE

MOTIVATIONS AND GOALS

In this class we analyze two further relevant parameters for a gyroscope operating in mode-matched conditions:

BANDWIDTH

- NOISE DENSITY

MODE-MATCHED GYROSCOPE BANDWIDTH

- Quality factor and -3dB bandwidth
- Trade-offs btw bandwidth and sensitivity

SENSITIVITY: $\frac{\Delta V_{OUT,0}}{\Omega_z} = 2 \frac{V_{DC}}{C_{FS}} \frac{C_{S0}}{g} \frac{x_{D,0}}{\Delta \omega_{BW}}$

$\rightarrow S \div x_{D,0}$ and $x_{D,0} = F_{elec,0} \frac{Q_D}{k_D} \div \begin{cases} v_a & \text{(applied AC actuator voltage)} \\ V_{DC} & \text{(applied DC rotor voltage)} \\ Q_D & \text{(quality factor)} \end{cases}$

$F_{elec,0} = \frac{2\varepsilon_0 h N_{CF}}{g} V_{DC} v_a$

It follows that:

- if you want to maximize the sensitivity while keeping relatively low voltages, it is useful to exploit high Q factor for the drive mode (Q_D) to reach large drive displacements $x_{D,0}$.
  - NOTE: This is why it's preferable to use CF actuation and sensing w/rt PPs for the gyro drive mode
    $\rightarrow$ PPs would be too nonlinear at large displacements

$CF \equiv \begin{cases} F_{elec} \div x_D \rightarrow \text{small displacement approx. not needed} \\ \text{large Q factor} \end{cases}$

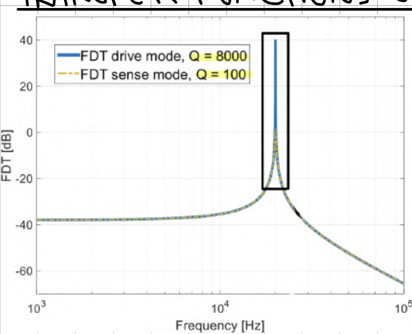
- The sense Q factor (Q_S) does not explicitly appear in the sensitivity expression, but it is hidden inside the sensing bandwidth:

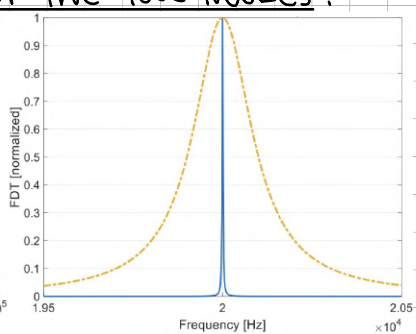
$$\Delta\omega_{BW} = \frac{b_S}{2m_S} = \frac{\omega_0}{2Q_S} \div \frac{1}{Q_S} \implies S = \frac{\Delta V_{OUT,0}}{\Omega_z} \div Q_S$$

NOTE: Even the largest angular rate that we need to measure in several applications ($\sim$2000dps) usually induces a displacement in the order of $y_S \div 100/150$ nm which is much smaller than the gap and it is acceptable also using PPs sensing.
$\rightarrow$ So we are fine in using PPs sensing bcz we have no linearity issues.

BUT, what about the Q factor of the PPs?

TRANSFER FUNCTIONS OF THE TWO MODES:





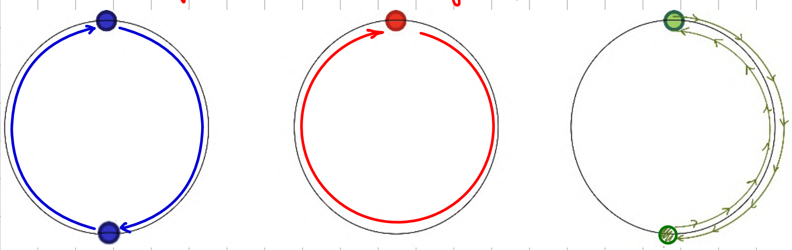
The SYSTEM RESPONSE is given by the sense mode transfer function (yellow) excited by the Coriolis force (blue) at a freq. corresponding to the drive mode frequency ω_0 times the frequency of the angular rate ω_Ω:

$$\boxed{F_{cor,0} \propto v_{D,0}(\omega_0) \cdot \Omega_z(\omega_\Omega)}$$

Therefore the above figure is valid for $\begin{cases} \text{perfect mode matching: } \omega_s = \omega_D = \omega_0 \\ \text{DC angular rate: } \omega_\Omega = 0 \end{cases}$

What if we had an AC angular rate? $*_1$

Which of the following signals has the highest angular rate frequency?



- The blue ball rotates clock wise w/ a 180 dps velocity
- The red ball rotates clock wise w/ a 360 dps velocity

Att: The difference btw the blue and the red signals is the amplitude of the angular rate, not the frequency → they both are DC angular rates

→ an AC angular rate is a signal for which the direction periodically changes, as it happen for the green ball (●)

$*_1$ **What does it happen around the peak of the sense mode transfer function when we have an AC angular rate?**

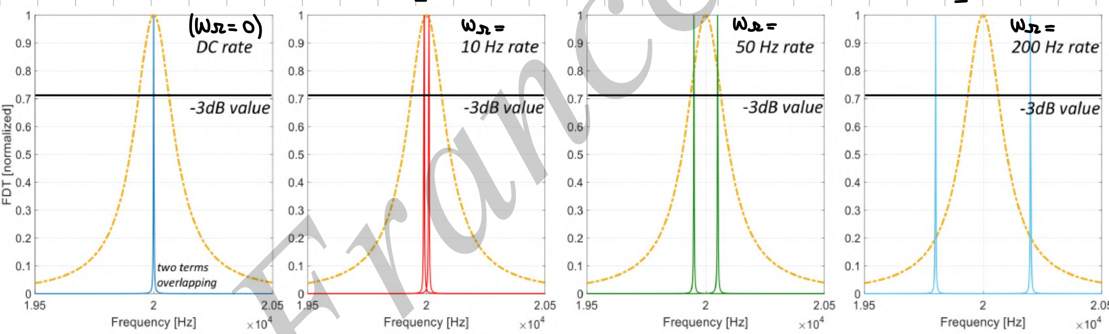
SENSING BANDWIDTH OF A GYROSCOPE ($\Delta\omega_{BW}$) = the maximum frequency of the angular rate that a gyroscope can measure. In other words, it is the frequency of the angular rate for which the response of our system in terms of amplitude is lower by $-3dB$. ($*_2$)

Hp: Assume that we have an AC cosinusoidal angular rate: $\boxed{\Omega_z(t) = \Omega_0 \cos(\omega_\Omega t)}$

$$x_D = x_{D,0} \sin(\omega_0 t) \rightarrow v_D = \dot{x}_D = -v_{D,0} \cos(\omega_0 t) = -\omega_0 x_{D,0} \cos(\omega_0 t)$$

$$F_{cor} = -2m_s \vec{v}_D \cdot \vec{\Omega}_z = 2m_s v_{D,0} \cos(\omega_0 t) \Omega_z(t) = 2m_s v_{D,0} \cos(\omega_0 t) \Omega_0 \cos(\omega_\Omega t) =$$

$$= 2m_s v_{D,0} \Omega_0 \frac{1}{2} \left[\cos[(\omega_0 + \omega_\Omega)t] + \cos[(\omega_0 - \omega_\Omega)t] \right]$$



NOTE:
- if $\omega_\Omega = 0$ → F_{cor} falls exactly on the peak of the sense mode transfer function → we have the maximum sensitivity
- by increasing ω_Ω progressively moves away the peak of the sense mode TF, so the sensitivity decreases

$*_2$ **$\Delta\omega_{BW}$ DEMONSTRATION:**

$$\frac{y_s}{F_{cor}}(s) = \frac{1}{m_s} \frac{1}{s^2 + \frac{\omega_0}{Q_s}s + \omega_0^2} \rightarrow \frac{y_s}{F_{cor}}(j\omega) = \frac{1}{m_s} \frac{1}{-\omega^2 + \frac{\omega_0}{Q_s}j\omega + \omega_0^2}$$

$$\rightarrow \boxed{\left| \frac{y_s}{F_{cor}}(j\omega) \right| = \frac{1}{m_s} \frac{1}{\sqrt{(\omega_0^2 - \omega^2)^2 + \left(\frac{\omega\omega_0}{Q_s}\right)^2}}}$$

MODULUS OF THE SENSE-MODE TRANSFER FUNCTION

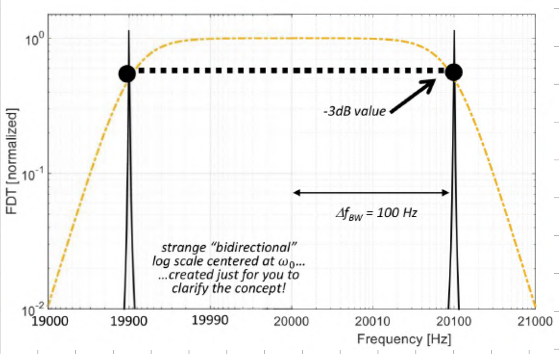
@ $\omega=\omega_0$: $\boxed{\left|\frac{y_s}{F_{cor}}(j\omega_0)\right| = \frac{1}{m_s}\frac{1}{\sqrt{(\cancel{\omega_0^2-\omega_0^2})^2+\left(\frac{\omega_0^2}{Q_s}\right)^2}} = \frac{1}{m_s}\frac{1}{\frac{\omega_0^2}{Q_s}} = \frac{Q_s}{k_s}}$ AMPLIFICATION FOR A DC ANGULAR RATE $(\omega_\Omega = 0)$

Hp: assume now $\omega_\Omega = \Delta\omega_{BW} = \frac{\omega_0}{2Q}$

Calculate the TF at $\omega_0 \pm \Delta\omega_{BW}$ $(\omega_0(1 \pm 1/2Q))$: AMPLIFICATION FOR AN Ω w/ $\omega_\Omega = \Delta\omega_{BW}$

$$\boxed{\left|\frac{y_s}{F_{cor}}(j\omega_0(1\pm 1/2Q))\right| = \frac{1}{m_s}\frac{1}{\sqrt{\left(\cancel{\omega_0^2}-\cancel{\omega_0^2}-\cancel{\frac{\omega_0^2}{4Q_s^2}}\pm\frac{\omega_0^2}{Q_s}\right)^2+\left(\frac{\omega_0^2}{Q_s}\pm\cancel{\frac{\omega_0^2}{2Q_s^2}}\right)^2}} = \frac{1}{m_s}\frac{1}{\sqrt{2\left(\frac{\omega_0^2}{Q_s}\right)^2}} = \frac{1}{\sqrt{2}}\frac{Q_s}{k_s}}$$

RESULT: It is evident that $\Delta\omega_{BW}$ is effectively the value for which the sense-mode transfer function looses $-3dB$, i.e. a factor $\sqrt{2}$ in amplitude (or a factor 2 in power)



$$\boxed{\Delta\omega_{BW} = \frac{\omega_0}{2Q_s} = \frac{\omega_0 b_s}{2\omega_0 m_s} = \frac{b_s}{2m_s}} \quad [rad/s]$$

$$\boxed{\Delta f_{BW,s} = \frac{f_0}{2Q_s}} \quad [Hz]$$

$$\boxed{Q_s = \frac{f_0}{2\Delta f_{BW,s}}}$$

$$\frac{\Delta V_{out,0}}{\Omega_z} = 2\frac{V_{DC}}{C_{FS}}\frac{C_{S0}}{g}\frac{x_{D,0}}{\Delta\omega_{BW}} \longrightarrow \frac{\Delta V_{out,0}}{\Omega_z}\cdot\Delta\omega_{BW} = const \rightarrow \boxed{GBWP = const}$$

$\rightarrow$ $\boxed{\text{TRADE-OFF SENSITIVITY vs BW}}$

NOTE (imp!): we can easily see that a too small damping coefficient b_s in the sense mode (too large Q_s) is not suitable bc it would determine high sensitivity but a too low bandwidth

IMPORTANT REMARK: Both the sensitivity and the -3dB bandwidth are functions of the damping coefficient in mode-matched gyroscopes
↓
This means that they both are functions of temperature

MODE-MATCHED GYROSCOPE NOISE ←
- Thermomechanical noise of the gyro frames
- Trade-offs between noise density and bandwidth
- Noise equivalent angular rate density (NERD)

• GYROSCOPE THERMOMECHANICAL NOISE

We have two decoupled modes, so we should consider thermomechanical noise contribution from each of them:

① THERMOMECHANICAL NOISE OF THE DRIVE FRAME

$\boxed{S_{Fn} = 4k_BTb_D}$ $\left[\frac{N^2}{Hz}\right]$ BROWNIAN NOISE IN TERMS OF FORCE ON THE DRIVE FRAME

↳ This noise is transferred into a drive displacement noise (trembling) through the drive-mode transfer function. In turns, it becomes a "SENSITIVITY NOISE".

Hp: Assume that this contribution is constant over the drive-mode peak width $\Delta f_{BW,D}$

$\rightarrow$ $\boxed{S_{xn} = 4k_BTb_D\left(\frac{Q_D}{k_D}\right)^2}$ $\left[\frac{m^2}{Hz}\right]$ BROWNIAN NOISE IN TERMS OF DISPLACEMENT OF THE DRIVE FRAME

* Around the peak, the force noise spectral density is amplified by $(Q_D/k_D)^2$

$$\sqrt{S_{x_n}\cdot\Delta f_{BW_D}} = \sqrt{4k_BTb_D\left(\frac{Q_D}{k_D^{\cancel{2}}}\right)^{\cancel{2}}\frac{f_0}{2\cancel{Q_D}}\frac{\cancel{2\pi}}{\cancel{2\pi}}} = \sqrt{\cancel{4}k_BTb_D\frac{Q_D}{k_D^2}\frac{\omega_0}{\cancel{4}\pi}} = \sqrt{k_BT\frac{\cancel{k_D}}{\cancel{\omega_0}\cancel{Q_D}}\frac{\cancel{Q_D}}{k_D^{\cancel{2}}}\frac{\cancel{\omega_0}}{\pi}}$$

$$\rightarrow \boxed{\sigma_{x_n} = \sqrt{S_{x_n}\cdot\Delta f_{BW_D}} = \sqrt{\frac{k_BT}{\pi k_D}}} \quad [m_{rms}]$$

RMS VALUE OF NOISE IN TERMS OF DISPLACEMENT OF THE DRIVE FRAME

NOTE: To get the rms value of the noise in terms of displacement we integrate the power spectral density of the noise in terms of displacement over the resonance peak, i.e. over the -3dB bandwidth of the drive mode transfer function.

② THERMOMECHANICAL NOISE OF THE SENSE FRAME

$$\boxed{S_{F_n} = 4k_BTb_s} \quad \left[\frac{N^2}{Hz}\right]$$

BROWNIAN NOISE IN TERMS OF FORCE ON THE SENSE FRAME

Hp: Assume that this contribution is constant over the sense-mode peak width $\Delta f_{BW,S}$

$$\rightarrow \boxed{S_{y_n} = 4k_BTb_s\left(\frac{Q_s}{k_s}\right)^2} \quad \left[\frac{m^2}{Hz}\right]$$

BROWNIAN NOISE IN TERMS OF DISPLACEMENT OF THE SENSE FRAME

Integrating it over the -3dB bandwidth, we obtain:

$$\sqrt{S_{y_n}\cdot\Delta f_{BW,S}} = \sqrt{4k_BTb_s\left(\frac{Q_s}{k_s^{\cancel{2}}}\right)^{\cancel{2}}\frac{f_0}{2\cancel{Q_s}}\frac{\cancel{2\pi}}{\cancel{2\pi}}} = \sqrt{\cancel{4}k_BTb_s\frac{Q_s}{k_s^2}\frac{\omega_0}{\cancel{4}\pi}} = \sqrt{k_BT\frac{\cancel{k_s}}{\cancel{\omega_0}\cancel{Q_s}}\frac{\cancel{Q_s}}{k_s^{\cancel{2}}}\frac{\cancel{\omega_0}}{\pi}}$$

$$\rightarrow \boxed{\sigma_{y_n} = \sqrt{S_{y_n}\cdot\Delta f_{BW,S}} = \sqrt{\frac{k_BT}{\pi k_s}}} \quad [m_{rms}]$$

RMS VALUE OF NOISE IN TERMS OF DISPLACEMENT OF THE SENSE FRAME

NOTE: As the sense frame is typically smaller (lower mass, only m_s) than the driven frames (the drive frames itself and the Coriolis/decoupling frame, eventually) its stiffness (k_s) is usually a bit smaller too in order to get the same resonance (mode-matched condition)

$$\frac{y_s}{\Omega_z} = \frac{x_{D,0}}{\Delta\omega_{BW,S}} \rightarrow \Omega = \frac{y_s}{x_{D,0}}\Delta\omega_{BW,S} \rightarrow \boxed{\sqrt{S_{\Omega_n}} = \sqrt{S_{y_n}}\frac{\Delta\omega_{BW,S}}{x_{D,0}}}$$

EQUIVALENT ANGULAR RATE NOISE USING THE SENSITIVITY

$$\rightarrow \sqrt{S_{\Omega_n}} = NERD = \frac{\sqrt{4k_BTb_s\left(\frac{Q_s}{k_s}\right)^2}}{x_{D,0}/\Delta\omega_{BW,S}} = \frac{\sqrt{4k_BTb_s\left(\frac{Q_s}{k_s}\right)^2\Delta\omega_{BW,S}^2}}{x_{D,0}} = \frac{\sqrt{4k_BTb_s\left(\frac{Q_s}{k_s}\right)^2\frac{\omega_0^2}{4Q_s^2}}}{x_{D,0}}$$

$$= \frac{1}{x_{D,0}}\sqrt{k_BTb_s\frac{1}{\omega_0^2m_s^2}} = \frac{\sqrt{k_BTb_s}}{x_{D,0}\,\omega_0\,m_s} \quad \left[\frac{rad/s}{\sqrt{Hz}}\right]$$

$$\boxed{NERD = \frac{180}{\pi}\underbrace{\frac{1}{x_{D,0}\,\omega_0\,m_s}}_{\text{What do these terms represent?}}\sqrt{k_BTb_s}} \quad \left[\frac{dps}{\sqrt{Hz}}\right]$$

NOISE EQUIVALENT (ANGULAR) RATE DENSITY

NOTE (IMP!!): This term essentially represents the minimum rate density that you can measure

SNR = 1 $\left(\frac{S}{N}=1\right)$

$$S = N \Longrightarrow \boxed{\Omega_{z,min} = NERD\cdot\sqrt{BW} = NERD\cdot\sqrt{\Delta\omega_{BW}}} \quad [dps]$$

RESOLUTION = minimum angular rate we are able to measure

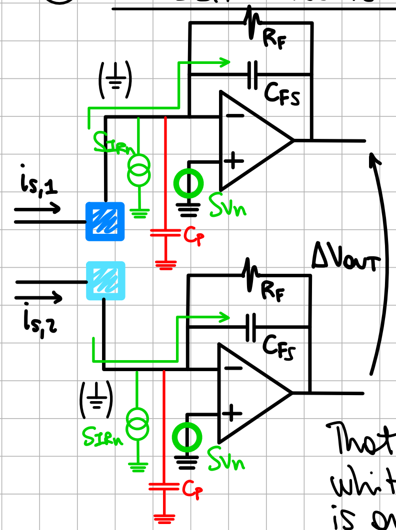
NOTE: The fact that we have to keep a relatively large damping coefficient for the sense mode in order to keep relative large bandwidth implies that noise is not that negligible

ELECTRONIC NOISE CONTRIBUTIONS

- Feedback resistance noise
- Operational amplifier noise
- Phase noise

• SENSE CHAIN ELECTRONIC NOISE

① FEEDBACK RESISTORS NOISE CONTRIBUTION



$$S_{IR_n} = \frac{4k_BT}{R_F}$$

The S_{IR_n} current noise is white, but as we have to apply demodulation around the resonance frequency and as we have to filter w/ a LPF, it's equivalent to say that we essentially work around the resonance frequency.
This freq. will be brought back to baseband (DC) and filtered w/ a LPF, so it is equivalent to directly work w/ a BPF around ω_0

That's why we can consider that the noise component, though it is white and generally amplified by the TF of the feedback network, is only transferred through the dominant of the parallel btw R_F and C_F around ω_0. As we have sized the front-end stage to be a charge amplifier, this current will essentially pass to the output just through $1/sC_{FS}$ which in modulus corresponds to $1/\omega_0 C_{FS}$

$$\rightarrow \boxed{\sqrt{S_{\Omega n,R_F}} = \frac{\sqrt{2\,\frac{4k_BT}{R_F}\left(\frac{1}{\omega_0 C_{FS}}\right)^2}}{\frac{\Delta V_{OUT,0}}{\Omega}}\,\frac{180}{\pi} = \sqrt{\frac{4k_BT}{2R_F}}\,\frac{g}{C_S V_{DC}\,\omega_0}\,\frac{\Delta\omega_{BW}}{x_{D,0}}\,\frac{180}{\pi}}$$

FEEDBACK RESISTANCES NOISE CONTRIBUTION IN TERMS OF INPUT ANGULAR RATE

↳ NOTE:
- we multiply for the TF $(1/\omega_0 C_{FS})$ bcz we starts w/ the input-referred feedback resistance noise contribution S_{IR_n} and we want output-referring it
- we divide for the sensitivity bcz we want the feedback resistances noise contribution in terms of input angular rate
- The factor 2 is due to the fact that the front-end of the sense chain consists of two CAs and so we have 2 feedback resistances, each of ones gives its own noise contribution
- we multiply for $180/\pi$ bcz we want the result in $dps/\sqrt{Hz}$ instead of $rad/s/\sqrt{Hz}$

① OPERATIONAL AMPLIFIER NOISE CONTRIBUTION

We can make the same reasoning, so we multiply the input-referred opamp noise for the corresponding transfer function in order to have the opamp noise contribution at the output of the front-end stage and then we divide it for the sensitivity in order to have the opamp noise contribution in terms of input angular rate

$$\rightarrow \boxed{\sqrt{S_{\Omega n,CA}} = \frac{\sqrt{2}\cdot S_{n,OA}\left(1+\frac{C_P}{C_{FS}}\right)}{\frac{\Delta V_{OUT,0}}{\Omega}}\,\frac{180}{\pi} = \sqrt{\frac{S_{n,OA}}{2}}\left(1+\frac{C_P}{C_{FS}}\right)\frac{C_{FS}}{C_{S0}}\,\frac{g}{V_{DC}}\,\frac{\Delta\omega_{BW,S}}{x_{D,0}}\,\frac{180}{\pi}}$$

OPAMP NOISE CONTRIBUTION IN TERMS OF INPUT ANGULAR RATE

FINAL RESULT:

$$\boxed{\sqrt{S_{\Omega n,TOT}} = NERD + \sqrt{S_{\Omega n,R_F}} + \sqrt{S_{\Omega n,CA}} = \sqrt{NERD^2 + S^2_{\Omega n,R_F} + S^2_{\Omega n,CA}}}$$

OVERALL NOISE DENSITY IN TERMS OF INPUT ANGULAR RATE

$$\sqrt{S_{\Omega n,TOT}} = \frac{190}{\pi}\sqrt{\left(\sqrt{\frac{4k_BT}{2R_F}}\,\frac{g}{C_S V_{DC}\,\omega_0}\,\frac{\Delta\omega_{BW}}{X_{D,0}}\right)^2 + \left(\sqrt{\frac{S_{n,OA}}{2}}\left(1+\frac{C_P}{C_{FS}}\right)\frac{C_{FS}}{C_{S0}}\,\frac{g}{V_{DC}}\,\frac{\Delta\omega_{BW,S}}{X_{D,0}}\right)^2 + \left(\frac{\sqrt{k_BT\,b_S}}{X_{D,0}\,\omega_0\, m_S}\right)^2} =$$

$$\approx \frac{190}{\pi}\,\frac{1}{X_{D,0}}\sqrt{\left(\sqrt{2\,\frac{4k_BT}{R_F}}\,\frac{g\,\Delta\omega_{BW}}{2C_{S0}V_{DC}\,\omega_0}\right)^2 + \left(\sqrt{\frac{S_{n,OA}}{2}}\,\frac{C_P}{C_{S0}}\,\frac{g\,\Delta\omega_{BW}}{V_{DC}}\right)^2 + \left(\frac{1}{\omega_0 m_S}\sqrt{k_BT\,b_S}\right)^2}$$

NOTE. • it's undoubtely useful to increase $X_{D,0}$ which is the only one parameter that improves all the noise contributions, and if possible also ω_0

Att: increasing ω_0, so having a larger cut-off frequency in any electronic stage means that we are increasing also the current we have to inject in and so the power consumption increases too
→ In low power consumption applications we can't do it!!

- if the electronic noise dominates $\boxed{\sqrt{S_{\Omega n,RF}} + \sqrt{S_{\Omega n,CA}} \gg NERD}$ we could:
  - increase R_F, i.e. by using off-MOS in integrated implementations;
  - lower C_P;
  - minimize g;
  - increase V_{DC} as much as we can → Att: we increase also the dissipation and so the power consumption

- if the thermomechanical noise dominates: $\boxed{NERD \gg \sqrt{S_{\Omega n,RF}} + \sqrt{S_{\Omega n,CA}}}$
  - increase m_S (either thicken your process - very good option - or pay in area);
  - decrease b_S (you loose your maximum sensing bandwidth)

• DRIVE CHAIN ELECTRONIC NOISE

The drive chain is formed by an oscillator and noise in oscillators is described by the PHASE NOISE THEORY: noise in harmonic signals is seen as the combination (50% each) of amplitude noise and phase noise.

Att: Amplitude noise and phase noise are uncorrelated and therefore they cannot be distinguished from a frequency domain analysis

HARMONIC SIGNAL: $\boxed{u(t) = A_{err} + A\,[1+\alpha(t)]\cos(\omega_0 t + \varphi_{err} + \varphi_{noise})}$

where
A = average AC amplitude
$\alpha(t)$ = fractional amplitude fluctuations → AMPLITUDE NOISE
A_{err} = offset
$\omega_0 t$ = instantaneous phase
φ_{err} = PHASE OFFSET
φ_{noise} = phase fluctuations → PHASE NOISE

NOTE (IMP!): The amplitude noise can be cancelled out by saturation and this is exactly what we do
↓
The only noise contribution that remains to take into account from the drive chain is the phase noise
↓
$u(t) = A\cos(\omega_0 t + \varphi_{err} + \varphi_{noise})$

NOTE:
- Effects of drive noise on the output white noise are not negligible, but they are very hard to predict as they depend on the so called QUADRATURE ERROR
- Effects of phase offset drift, coupled to quadrature error, are a very critical point for some applications like INDOOR INERTIAL NAVIGATION

CONCLUSIONS

- GYRO = RESONATOR + ACCELEROMETER OF THE CORIOLIS FORCE

$\Rightarrow$ Gyro needs OSCILLATING CIRCUIT + CAPACITIVE SENSING AT AN AC FREQUENCY ω_D

- The gyro sensitivity strongly depends on $\begin{cases} x_{D,0} \\ \Delta\omega_{BW} \text{ (sensing bandwidth)} \end{cases}$
- The gyro signals are small so we have to use advanced architectures to reject linear/rotational accelerations
- Mode-matched gyros suffer from
  - SENSITIVITY (GAIN) vs BW TRADE-OFF
  - BW vs BROWNIAN NOISE TRADE-OFF

NOTE: PPs of the sense mode are in principle subject to pull-in issue. However, gyros frequency (and thus stiffness) is usually much higher than in axels, therefore pull-in issues are generally unlikely to occur.

THE CAPACITIVE MEMS GYROSCOPE
PART 4: MODE-SPLIT OPERATION

MOTIVATIONS AND GOALS

In real life, devices operate in changing environmental conditions (temperature) which we need to take into account.
↓
We will introduce an operation mode, called MODE-SPLIT OPERATION, which is more tolerant to these effects.

BRIEF SUMMARY OF GYROS IN MODE-MATCHED OPERATION

- Sensitivity and trade-offs
- Frequency mismatch and f vs. T
- Q vs T

Sensitivity of a mode-matched gyro: $\frac{\Delta V_{out,0}}{\Omega_z} = 2 \frac{V_{DC}}{C_{FS}} \frac{C_{S0}}{g} \frac{X_{D,0}}{\Delta \omega_{BW,S}}$

Issues in mode-matched gyroscopes:

① RELATIVELY LIMITED BANDWIDTH and ITS RELATIONSHIP w/ THE DAMPING COEFFICIENT (b_S) and w/ THE THERMOMECHANICAL NOISE

$$\frac{\Delta V_{out,0}}{\Omega_z} = 2 \frac{V_{DC}}{C_{FS}} \frac{C_{S0}}{g} \frac{X_{D,0}}{\Delta \omega_{BW,S}}$$

$$\Delta \omega_{BW,S} = \frac{b_S}{2 m_S}$$

$$NERD = \frac{1}{X_{D,0}\, \omega_D\, m_S} \sqrt{k_B T\, b_S}$$

Marked trade-off b/w best achievable noise density (assuming electronic noise negligible) and maximum sensing bandwidth →

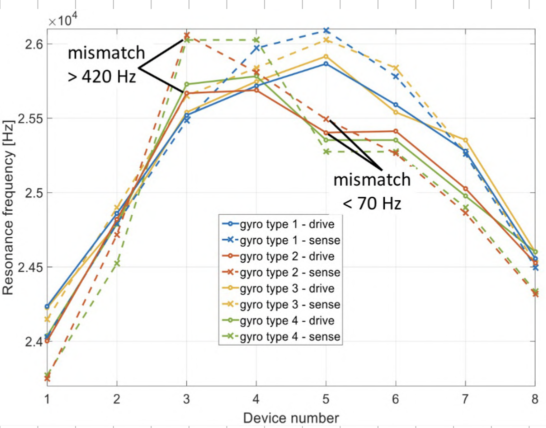
$\boxed{\text{TRADE-OFF NERD vs } \Delta \omega_{BW,S} \text{ (Through } b_S)}$

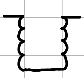
② So far we assumed to operate in mode-matched conditions $\omega_D = \omega_S = \omega_0$, but this is obviously very challenging to obtain, mainly for 2 reasons:

- In any case (even if the process has no spreads) the real resonance frequencies and the ones predicted by the simulation will never be identical
- We'll always have process spreads which affect exactly in the same way both ω_D and ω_S

RESULT: even w/ a very good design the two frequencies will never exactly match one another

$\boxed{f_S|_{typical} = f_D \pm 600\ Hz\ (\pm 3\sigma) \quad \text{where } f_D|_{typical} \sim 20\ kHz}$





MODES DISTRIBUTION ON A Si WAFER

Nonuniformities are mostly due to local differences in DRIE of springs (cubic dependence on w). Therefore, nonuniformities affect similarly the two modes of the same gyroscope, as their springs (k_D and k_S) lie very close one another.

However, a residual difference b/w the frequencies of the two modes ($f_D - f_S$) still remains and varies b/w few 10s of Hz and few 100s of Hz.

→ $\boxed{(f_D - f_S)|_{typical} \sim 10s - 100s\ Hz}$

We learned how frequencies can be tuned via electrostatic force (TUNING PLATES / ELECTROSTATIC SOFTENING). However, even if we use tuning plates on the sense mode to match the frequency at a certain T, there still is an issue associated to the temperature dependence of the Young's modulus:

- The mechanical stiffness k is proportional to the Young's modulus ($k = \beta E$), this means that its relative percentage variation (dk/k) is proportional to the relative percentage variation of the Young's modulus (dE/E)

$$\rightarrow \frac{dk}{k} \propto \frac{dE}{E}$$ **NOTE:** β is a geometrical factor $\left(= \frac{h w^3}{L^3}\right)$

- The resonance frequency goes w/ the square root of the stiffness:

$$f_0 = \frac{1}{2\pi}\sqrt{\frac{k}{m}} \rightarrow \frac{df_0}{f_0} = \frac{1}{2}\frac{dk}{k}$$

$\Rightarrow$ **TEMPERATURE COEFFICIENT OF FREQUENCY**

$$\boxed{TC_f = \frac{\frac{df_0}{f_0}}{\Delta T} = \frac{\frac{1}{2}\frac{dk}{k}}{\Delta T} = \frac{1}{2}\frac{\frac{dE}{E}}{\Delta T} = -30\ \frac{ppm}{K}}$$

↳ **NOTE:** TC_f tells us that for any mode of MEMS device, the resonance frequency f_0 will change by -30 ppm/K if you operate w/ the typical polySi (or w/ a relative low doped one).

Assuming: $f_D = 20$ kHz and $f_S = 20.6$ kHz $T = -45 \rightarrow +125\ °C \rightarrow \Delta T = 170K = \pm 85K$

we obtain:
- MAXIMUM SHIFT OF THE DRIVE FREQUENCY: $df_D = TC_f \cdot f_D \cdot \pm\Delta T = \pm 51$ Hz
- MAXIMUM SHIFT OF THE SENSE FREQUENCY: $df_S = TC_f \cdot f_S \cdot \pm\Delta T = \pm 52.53$ Hz
- MAXIMUM SHIFT OF THE MODES DIFFERENCE: $d(\Delta f) = TC_f \cdot \Delta f \cdot \pm\Delta T = \pm 1.53$ Hz

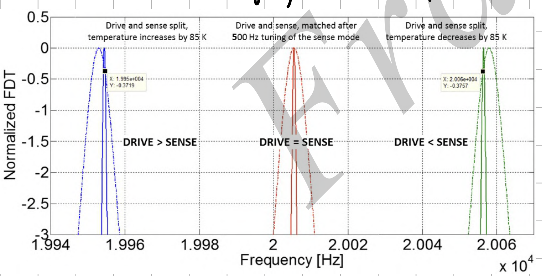
$*_1$

RESULT: even if we use tuning to match frequencies at one temperature, there will be a variation of the split of the frequencies larger than ± 1 Hz, indeed the electrostatic stiffness k_{elec} that we use for tuning is basically independent of T changes.

Example: $Q_D = 10'000$, $Q_S = 500$, $f_D = 20$ kHz, $f_S = 20.6$ kHz, $BW_{-3dB} = \Delta w_{BW,S} = 20$ Hz

Initial tuning makes the frequencies matched.

Under T changes, the frequencies are no longer matched



- $\boxed{T\downarrow\ :\ f_S > f_D}$

The drive mode will no longer excite the sense-mode transfer function at its peak, so we expect to have a drop in sensitivity.

Only due to the fact that the temperature is changed we have a non-negligible loss in the sensitivity of about 5% (-0.4 dB).

→ This is a relevant issue when operating in mode-matched conditions

- $\boxed{T\uparrow\ :\ f_D > f_S}$ → same considerations

CONCLUSION: Temperature changes coupled to the fact that in Si the Young's modulus is a function of temperature, can induce sensitivity changes when operating in mode-matched conditions even if we tune our frequencies to be perfectly matched at a certain T.

③ We already observed that the drive Q factor (Q_D) changes w/ T due to b_D changes and so does the sensitivity which goes linearly w/ $x_{D,0}$

What about Q_S? Also the sense Q_S factor is affected by temperature changes (following also the same law: $Q_S \propto 1/\sqrt{T}$), but the sensing damping coefficient b_S determines simultaneously changes in the sensitivity and in the sensing bandwidth

Example Automotive temperature range: $\%T = \pm 85K$ around $315K$

$$Q_S = \frac{\alpha}{\sqrt{T}} \rightarrow dQ_S = -\frac{1}{2}\alpha T^{-3/2} dT = -\frac{1}{2T}\frac{\alpha}{\sqrt{T}} dT \rightarrow \left|\frac{dQ_S}{Q_S}\right| = \left|\frac{db_S}{b_S}\right| = \frac{1}{2}\left|\frac{dT}{T}\right| = 0.5\,\frac{85}{315} = 13.5\,\%$$

Absolutely not negligible and absolutely far from our target of having something in the order of 1% in terms of scale factor scalability.

Such percentage Q_S changes directly translates in corresponding bandwidth and sensitivity changes

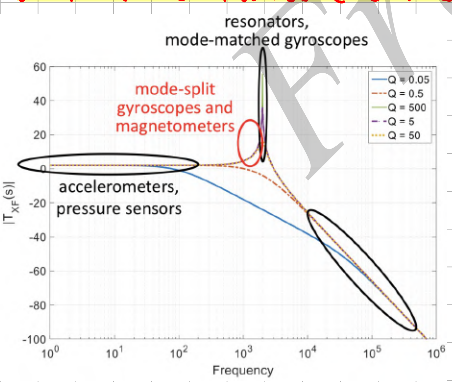
- BANDWIDTH DECREASES AT LOW TEMPERATURES: $\Delta\omega_{BW,S} = \dfrac{\omega_0}{2\,Q_S} \propto \sqrt{T}$
- SCALE-FACTOR DECREASES AT HIGH TEMPERATURES. $\dfrac{y_S}{\Omega} \propto \dfrac{1}{\sqrt{T}}$

SUMMARY OF ISSUES IN MODE-MATCHED OPERATION

- ISSUE ①: Q_D DEPENDENCE ON TEMPERATURE
  → SOLUTION: using an AGC we stabilize our displacement vs T whatever the changes of Q_D
- ISSUE ②: TRADE-OFF NERD vs $\Delta\omega_{BW,S}$ through b_S
  → low noise density is achievable only at low maximum BW
- ISSUE ③: IMPOSSIBILITY OF HAVING PERFECT MODE MATCHING DUE TO PROCESS NON UNIFORMITIES AND ASSOCIATED DIFFERENT FREQUENCY DRIFTS OF THE MODES
- ISSUE ④: Q_S DEPENDENCE ON TEMPERATURE and consequent effects on sensitivity and BW

MODE-SPLIT OPERATION
- Analysis of the transfer function in mode-split
- Sensitivity in mode-split operation

What happens if we make operate our device in a region which is close to the resonance frequency, so that we still have an amplification, but it's not exactly at the peak?



- Axels work in the flat region, far before the resonance and they usually have a low Q factor
- Mode-matched gyros work around resonance and they usually have a relatively high Q factor
- In an alternative topology, gyros and other sensors, like magnetometers, can operate measuring a force that occurs slightly before resonance
  ↓
  This is called OFF-RESONANCE / MODE-SPLIT OPERATION

REM: no device operate beyond the resonance frequency

Let's now compute the transfer function response $T_{YF}(s) = \frac{y_s}{F_{COR}}(s) = \frac{1/m_s}{s^2 + \frac{b_s}{m_s}s + \frac{k_s}{m_s}}$

Hp: $\omega_D \ll \omega_s \rightarrow \Delta\omega_{MS} = \omega_s - \omega_D$ where $\Delta\omega_{MS} \ll \omega_D, \omega_s$ and $\Delta\omega_{MS} \gg \Delta\omega_{BW}$ $(Q_s \gg 1)$

Since the x-axis displacement is set by the drive mode resonance frequency, we compute the modulus of the transfer function of the sense mode at the drive mode frequency:

$$|T_{YF}(j\omega_D)| = \left|\frac{1/m_s}{-\omega_D^2 + j\frac{\omega_D\omega_s}{Q_s} + \omega_s^2}\right| = \frac{1/m_s}{\sqrt{(\omega_s^2-\omega_D^2)^2 + \left(\frac{\omega_D\omega_s}{Q_s}\right)^2}} =$$

NOTE: $\omega_D = \omega_s - \Delta\omega_{MS}$
$\rightarrow \omega_D^2 = \omega_s^2 + \Delta\omega_{MS}^2 - 2\omega_s\Delta\omega_{MS}$

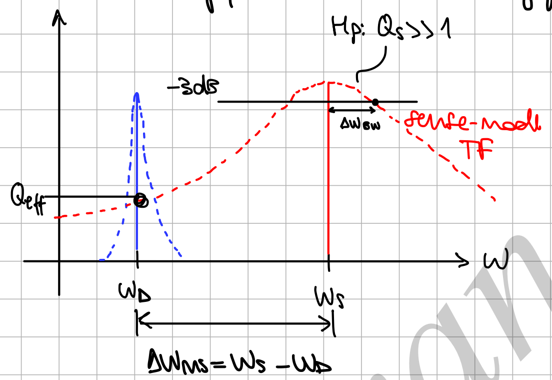
$$= \frac{1/m_s}{\sqrt{(\cancel{\omega_s^2} - \cancel{\omega_s^2} + 2\omega_s\Delta\omega_{MS} - \Delta\omega_{MS}^2)^2 + \left(\frac{\omega_s(\omega_s - \Delta\omega_{MS})}{Q_s}\right)^2}} =$$

$$= \frac{1/m_s}{\sqrt{(2\omega_s\Delta\omega_{MS} - \cancel{\Delta\omega_{MS}^2})^2 + \left(\frac{\omega_s^2 - \cancel{\omega_s\Delta\omega_{MS}}}{Q_s}\right)^2}} = \frac{1/m_s}{\sqrt{4\omega_s^2\Delta\omega_{MS}^2 + \frac{\omega_s^4}{Q_s^2}}} = \frac{1/m_s}{2\omega_s\sqrt{\Delta\omega_{MS}^2 + \frac{\omega_s^2}{4Q_s^2}}}$$

$$= \frac{1/m_s}{2\omega_s\sqrt{\Delta\omega_{MS}^2 + \cancel{\Delta\omega_{BW}^2}}} = \frac{1/m_s}{2\omega_s\Delta\omega_{MS}} = \frac{1}{k_s}\frac{\omega_s}{2\Delta\omega_{MS}} = \frac{1}{k_s}Q_{eff}$$

where $\boxed{Q_{eff} = \frac{\omega_s}{2\Delta\omega_{MS}} = \frac{\omega_s}{2(\omega_s - \omega_D)}}$ **EFFECTIVE QUALITY FACTOR** $\rightarrow \boxed{\Delta\omega_{MS} = \frac{\omega_s}{2Q_{eff}}}$

NOTE: it's "effective" not bc+ it has some relevant physical meaning, but bc+ the analogy w/ $Q_s = \frac{\omega_s}{2\Delta\omega_{BW}}$



The amplification that we have w/rt the DC operation is called effective quality factor Q_{eff} which, since $\Delta\omega_{MS} \gg \Delta\omega_{BW}$, is obviously lower than Q_s and it is independent of Q_s itself

$$\left.\begin{array}{l} Q_s = \frac{\omega_s}{2\Delta\omega_{BW}} \\ Q_{eff} = \frac{\omega_s}{2\Delta\omega_{MS}} \\ \Delta\omega_{MS} \gg \Delta\omega_{BW} \end{array}\right\} \Rightarrow \boxed{Q_{eff} \ll Q_s}$$

NOTE: $\Delta\omega_{MS}|_{typical} = 100s - 1000$ Hz , $Q_{eff}|_{typical} = 10s$

NOTE: when we work in mode-split operation we don't change the electronics (so the displacement is still controlled by the AGC) and the GBWP is still constant

RESULT: mode-split operation sacrificies sensitivity for bandwidth

DIM :

- Drive mode is identical to mode-matched operation.

$$F_{elec} = \frac{\Delta V^2}{2}\frac{dC}{dx} = \frac{\varepsilon_0 h N_{CF}}{g} 2V_{DC} v_a \sin(\omega_D t) = F_{elec,0}\sin(\omega_D t)$$

$$x_{D,0} = F_{elec,0}\frac{Q_D}{k_D} \qquad v_{D,0} = F_{elec,0}\frac{Q_D}{k_D}\omega_D$$

- For the sense mode we will use Q_{eff} instead of Q_S, since this time F_{cor} is not acting on the peak of the sense-mode transfer function, but it is acting on the so called off-resonance region

$$F_{cor,0} = 2 m_S v_{D,0}\, \Omega$$

$$y_{S,0} = F_{cor,0} \frac{Q_{eff}}{k_S} = 2 m_S x_{D,0}\, \omega_D \frac{Q_{eff}}{k_S}\, \Omega$$

$$\Rightarrow \frac{y_{S,0}}{\Omega} = 2 x_{D,0}\, \omega_D m_S \frac{Q_{eff}}{k_S} = x_{D,0} \frac{2 Q_{eff}\, \omega_D}{\omega_S^2} = x_{D,0} \frac{2\omega_D}{\omega_S^2} \frac{\omega_S}{2\Delta\omega_{ms}} = \frac{\omega_D}{\omega_S} \frac{x_{D,0}}{\Delta\omega_{ms}}$$

$$\Rightarrow \boxed{\frac{y_{S,0}}{\Omega} \approx \frac{x_{D,0}}{\Delta\omega_{ms}}}$$ SCALE FACTOR IN MODE-SPLIT OPERATION

→ ATT. This is valid for a DC Ω

NOTE: SCALE FACTOR = GAIN OF THE SENSE MODE

NOTE: $\Delta\omega_{ms}$ is not exactly the BW of our gyro in mode-split operation, but it is very close to it so we can continue to say that we still have the TRADE-OFF SENSITIVITY vs BW

NOTE: since $\Delta\omega_{ms} \gg \Delta\omega_{BW}$ we are effectively sacrifying sensitivity for BW

↳ Is this sacrifice effectively an advantage? we are going to see it later

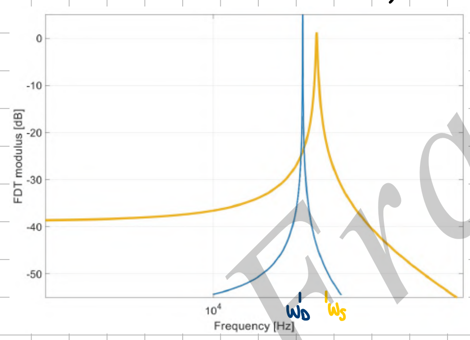
What happens for a non-constant angular rate ($\Omega = \Omega(t)$)?

BANDWIDTH IN MODE-SPLIT OPERATION { ±3dB definition; Effect of LPF electronics }

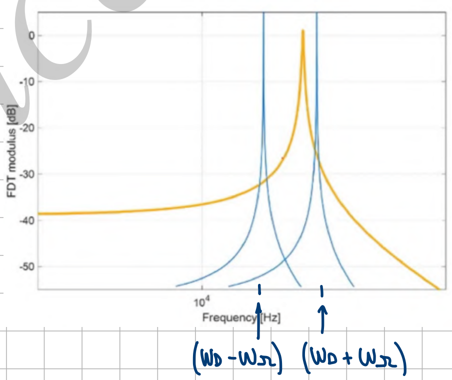
As we have seen for the mode-matched gyro, also in this case F_{cor} acts exactly at $\omega_S = \omega_D + \Delta\omega_{ms}$ (Hp: $\omega_S > \omega_D$) if and only if Ω is a constant (DC) angular rate, i.e. $\omega_\Omega = 0$

If Ω is not constant we'll have a split in F_{cor} and its AC components will lie exactly at $\omega_{D,0} \pm \omega_\Omega$

- Ω = constant ($\omega_\Omega = 0$)



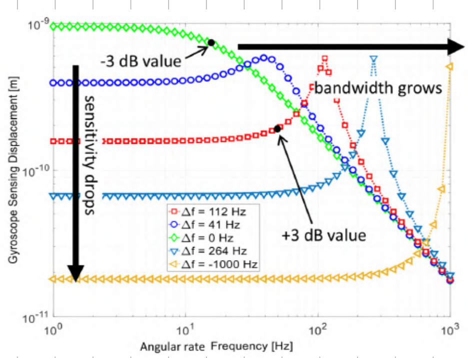
- $\Omega = \Omega(t)$ ($\omega_\Omega \neq 0$)



NOTE: When ω_Ω increases, the sum of these two components initially remains constant. Then, at larger increases of ω_Ω, the component $\omega_{D,0} + \omega_\Omega$ approaches the sense mode peak and generates an increase in the overall gyroscope sensitivity

We are used to define -3dB bandwidth as would be the case for an overdamped accel or for a mode-matched gyro.

For such an underdamped overall response, we define the USEFUL BANDWIDTH as the ±3dB value.



RESULT: The overall system bandwidth will be given by the combination of the shown MEMS mechanical BW and an electronic filtering BW in the sense chain that will try to flatten the peak and further extend the overall bandwidth.

⇒ RULE OF THUMB: if $\Delta\omega_{ms}$ is a certain value, we can assume that the BW can extend up to 1/2 of this value.

THE CAPACITIVE MEMS GYROSCOPE
PART 5: NOISE & ROBUSTNESS IN MODE-SPLIT OPERATION

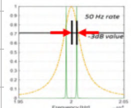
MOTIVATIONS and GOALS

We have seen that mode-split operation sacrifices gain of the sense-mode (so in the end the sensitivity) for a wider bandwidth

We still need to verify:
- solution of the trade-off BW vs NERD
- Immunity to temperature changes
- immunity to Qs changes

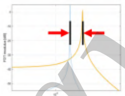
BRIEF SUMMARY OF GYROSCOPES IN MODE-SPLIT OPERATION
- Sensitivity
- Bandwidth

MODE-MATCHED : $\frac{y_{S,0}}{\Omega} = \frac{x_{D,0}}{\Delta\omega_{BW}}$ $\quad \Delta\omega_{BW} = \frac{b_S}{2m_S} = \frac{\omega_S}{2Q_S}$



$\Delta\omega_{BW}$ → **NOTE:** it is the half width at -3dB of the resonance peak

MODE-SPLIT : $\frac{y_{S,0}}{\Omega} = \frac{x_{D,0}}{\Delta\omega_{MS}}$ $\quad \Delta\omega_{MS} = \omega_S - \omega_D = \frac{\omega_S}{2Q_{eff}}$



$\Delta\omega_{MS}$

NOTE: $\Delta\omega_{BW}$ is the maximum achievable mechanical bandwidth in mode-matched conditions
↳ Is $\Delta\omega_{MS}$ the maximum achievable mechanical bandwidth in mode-split conditions?

MODE-SPLIT GYROSCOPE NOISE
- Calculation of Thermomechanical noise
- Effects of electronic noise

THERMOMECHANICAL (BROWNIAN) NOISE FOR THE SENSE FRAME IN MODE-SPLIT OPERATION

MODE-MATCHED $\quad \sqrt{S_{\Omega n}} = \sqrt{\frac{S_{yn}}{(y_{S,0}/\Omega)^2}} = \sqrt{\frac{4k_BTb_S\left(\frac{Q_S}{k_S}\right)^2}{\left(x_{D,0}/\Delta\omega_{BW}\right)^2}} \qquad Q_S = \frac{\omega_S}{2\Delta\omega_{BW}}$

MODE-SPLIT $\quad \sqrt{S_{\Omega n}} = \sqrt{\frac{S_{yn}}{(y_{S,0}/\Omega)^2}} = \sqrt{\frac{4k_BTb_S\left(\frac{Q_{eff}}{k_S}\right)^2}{\left(x_{D,0}/\Delta\omega_{MS}\right)^2}} \qquad Q_{eff} = \frac{\omega_S}{2\Delta\omega_{MS}}$

→ $\boxed{NERD = \frac{180}{\pi}\frac{1}{x_{D0}\,\omega_0\,m_S}\sqrt{k_BT\,b_S}}$ $\left[\frac{dps}{\sqrt{Hz}}\right]$ NOISE EQUIVALENT RATE DENSITY IN MODE-SPLIT CONDITION

SNR=1 ⇒ $\boxed{\Omega_{z,min} = NERD\cdot\sqrt{BW} = NERD\cdot\sqrt{\Delta\omega_{MS}}}$ $[dps]$ RESOLUTION

RESULT: The Thermomechanical (Brownian) noise, ie The NERD, does not change, bc it's true That we are amplifying The signal less, but if we look at the origin of This noise, This is essentially a force noise so as the Coriolis force Fcor is amplified less also The Brownian noise S_{Fn} is amplified less
Why can we assume That The noise is amplified exactly like The signal?

Actually, white Thermomechanical noise should be amplified by the entire sense-mode TF.

Due to demodulation any generic frequency f is shifted to $(f-f_D)$ and to $(f+f_D)$, since we are multiplying by The drive frequency f_D.
After LPFing we see That The contribution of noise That is fully amplified by The peak of the sense-mode TF is cancelled out by the LPF itself

CONCLUSION: As we know That in principle we have to demodulate and low-pass filter our signal, we can state That The noise is amplified just like The signal.

NOTE (IMP!): we can easily see that the effective quality factor Q_{eff} is not a function of the damping coefficient

RESULT: Therefore the useful bandwidth (assumed as 1/4 to 1/2 of $\Delta\omega_{MS}$) is itself not related to the damping coefficient

↓

Bandwidth and NERD are no longer linked through the damping coefficient: it is thus possible to lower the NERD w/o affecting the BW

↓

The trade-off NERD vs BW is thus solved

↓

we no longer need to keep a low value of Q_s

In other words, now we can arbitrary reduce the ultimate achievable noise density by reducing the package pressure (so b_s) w/o sacrifying the BW

What's about the electronic noise instead?

The noise contributions from the electronics are the same when computed at the circuit output, but when calculated as equivalent input rate, they are now divided by a sensitivity which is lower by a factor $\Delta\omega_{EW}/\Delta\omega_{MS}$

↓

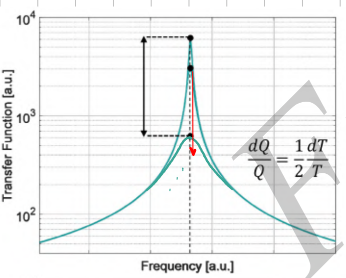
so the impact of electronic noise on the system grows. Thus we should design low-noise electronics which unavoidably require high power consumption

$$\sqrt{S_{\Omega n,TOT}} = NERD + \sqrt{S_{\Omega n,RF}} + \sqrt{S_{\Omega n,CA}} = \sqrt{NERD^2 + S_{\Omega n,RF}^2 + S_{\Omega n,CA}^2}$$

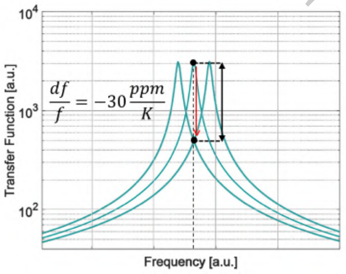
$$\boxed{\sqrt{S_{\Omega n,TOT}} \approx \frac{190}{\pi}\frac{1}{X_{D,0}}\sqrt{\left(\sqrt{2\frac{4k_BT}{R_F}}\frac{g\,\Delta\omega_{MS}}{2C_{S0}V_{DC}\omega_0}\right)^2+\left(\sqrt{\frac{S_{n,OA}}{2}}\frac{C_F}{C_{S0}}\frac{g\,\Delta\omega_{MS}}{V_{DC}}\right)^2+\left(\frac{1}{\omega_0 m_s}\sqrt{k_BT b_s}\right)^2}}$$

OVERALL EQUIVALENT INPUT RATE NOISE DENSITY IN MODE-SPLIT OPERATION

ADVANTAGES OF MODE-SPLIT OPERATION
- Immunity to T changes
- Immunity to Q changes



NOTE: if we operate in mode-matched conditions if Q changes and we are operating at resonance, obviously also the gain at the transfer function peak will change



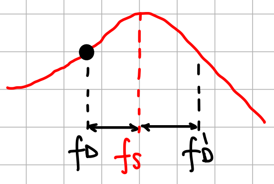
NOTE: if we operate in mode-matched conditions if the frequency f_s changes relative to f_D, also the gain changes

MODE-SPLIT OPERATION

PROS:
- sensitivity is very well independent of the Q value (under certain conditions Q_{eff} is independent of Q_s)
- sensitivity is tolerant to Δf vs T: the changes of our working point due to f_s changes are very small

CONS:
- gain reduction
- consequent need of high performance electronics

NOTE: we have assumed to locate f_D on the rising edge of the sense-mode TF



The distance $(f_S - f_D)$ is not so different from $(f_D' - f_S)$, so in principle we can get the same results if we put the drive mode frequency at a frequency slightly larger than f_S

(limit: $f_D = f_S + 500 \div 1000\,Hz$)

CONCLUSIONS

THE CAPACITIVE MEMS GYROSCOPE
PART 6: QUADRATURE ERROR AND ITS COMPENSATION

MOTIVATIONS AND GOALS

We have seen that for an harmonic signal circulating in a self-sustained circuit we have an amplitude and a phase noise contributions, but if in a certain point of the circuit we make the signal saturate to the supply we completely cancel out the amplitude noise contribution. We still have the phase noise, so instead of having an ideal phase $\omega_0 t$ we will have an additional term φ_{noise} which represents a random fluctuation due to non-idealities in terms of phase transfer through the different electronic stages both in the drive loop and in the sense chain.

There is also a phase difference which is called PHASE ERROR/OFFSET φ_{err} btw the ideal reference phase that we would need for a perfect synchronous demodulation and the real reference that we have at the output of the drive loop

$\rightarrow$ $u(t) = A \cos(\omega_0 t + \varphi_{noise} + \varphi_{err})$

GOALS:
- to introduce and model the largest gyro's issue/nonideality from a mechanical point of view which is the so called QUADRATURE ERROR
- to show how quadrature error (QE) generate a worsening in the performance because of its coupling w/ non-idealities in the electronics
- to indicate possible ways to compensate this issue

NOTE (IMP!!): All the following considerations are independent of the operation mode

QUADRATURE ERROR ISSUES FROM SPRING DEFECTS

- Preliminary considerations
- Quadrature error modeling
- Spring mismatch

Example: $FSR = 2000°/s$, resolution $= NERD \cdot \sqrt{BW} = 5\,mdps/\sqrt{Hz} \cdot \sqrt{300} = 100\,mdps$

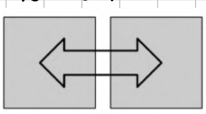
if $x_{REF} = 5\mu m$ $\rightarrow$ $y_{s,0}\big|_{MAX} = \frac{x_{REF}}{\Delta\omega_{MS}}\Omega = \frac{5\mu m}{300Hz} 2000\,dps \sim 30\,nm$ $\left(\cdot\frac{\pi}{180} \approx 35\,rad/s\right)$

$y_{s,0}\big|_{min} = \frac{5\mu m}{300Hz} 100\,dps \approx 1.5\,pm$ $\left(\cdot\frac{\pi}{180} \approx 1.7\,mrad/s\right)$

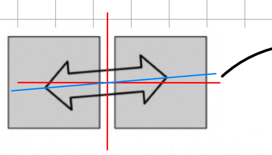
Att: This is an ultra small displacement, it's more than 3 million times smaller than the drive displacement

$\rightarrow$ Does it generate any issue?

This enormous displacement inequality btw the two modes can cause some issues if the drive motion is not perfectly orthogonal to the sense one due to the presence of electromechanical nonidealities. $*_1$



top view: ideal in-plane drive motion only along the x-axis



top view: in-plane motion not only along the x-axis

NOTE: This angle is typically very small, but due the fact that $x_{D,0}$ is very huge compared w/ $y_{s,0}|_{min}$, it can generate a very large effect on our sensor.

$*_1$ for this reason the sense mode will see a y-displacement at ω_D which is not caused by any angular rate (which is present even in absence of angular rates)

$\downarrow$

This error, which is in first approximation a DC term independent of Ω, may represent a huge OFFSET.

NOTE (imp!): fortunately we have one thing that can save us, which is the fact that the max displacement in the y-direction occurs exactly when we have the max displacement in the x-direction, which means that this offset is in quadrature w/rt the Coriolis force, as it is proportional to x and not to $\dot{x}$

We model this offset w/ an **EQUIVALENT QUADRATURE FORCE F_q** and for what we have just said we can write:

- $|\vec{F_q}| \propto x$
- $|\vec{F_{cor}}| \propto \dot{x}$

} **NOTE:** for this reason this offset term is called **QUADRATURE OFFSET/ERROR**

QUADRATURE ERROR SOURCES (possible sources of misalignment in the drive mode):

1. INHOMOGENEITY IN THE STIFFNESS OF THE DIFFERENT SPRINGS
   - local etching (DRIE) differences
   - skew-angle issue

NOTE: This is generally the main source of quadrature error

2. INHOMOGENEITY IN THE COMB-DRIVE GAPS
   - local etching (DRIE) differences
3. DESIGN IMPERFECTIONS OR MASKS MISALIGNMENTS

Which is the effect of this offset/error on the output of our system?
Which is a modeling that we can do in order to predict the amount of this quadrature error?
Which parameters is this quadrature error related to?

$$\Delta V_{out,0} = 2\frac{V_{DC}}{C_{FS}}\frac{C_0}{g}\frac{x_{D,0}}{\Delta\omega_{MS}}\left[\Omega\cos(\omega_D t)+\Omega_q\sin(\omega_D t)\right] = S\left[\Omega\cos(\omega_D t)+\Omega_q\sin(\omega_D t)\right]$$

Where **Ω_q = EQUIVALENT INPUT ANGULAR RATE OF THE QUADRATURE ERROR**

Hp: Assume to apply an IDEAL DEMODULATION which consists on two steps:

1. multiply the signal above by $\cos(\omega_D t)$
2. filtering at the bandwidth BW

Hp: Assuming Ω in DC ($\omega_\Omega = 0$) and $G_{LPF} = 2$ in order to compensate the demodulation loss:

$$V_{dem} = S\left[\Omega\cos(\omega_D t)+\Omega_q\sin(\omega_D t)\right]\cos(\omega_D t) * LPF =$$

$$= S\frac{1}{2}\left\{\Omega\left[\cos(0)+\cos(2\omega_D t)\right]+\Omega_q\left[\sin(0)+\sin(2\omega_D t)\right]\right\} =$$

$$= \frac{S}{\cancel{2}}\cancel{G_{LPF}}\left[\Omega\cos(0)+\cancel{\Omega_q\sin(0)}\right] = S\Omega = 2\frac{V_{DC}}{C_{FS}}\frac{C_0}{g}\frac{x_{D,0}}{\Delta\omega_{MS}}\cdot\Omega$$

RESULT: quadrature error is completely erased.

↓

In this ideal condition, the care that one should take in the design is just the need for an EXTENDED ELECTRONIC FULL-SCALE (for the supply before the demodulation stage) defined by:

$$V_{DD,min} = S\sqrt{\Omega^2+\Omega_q^2}\;G_{INA}$$

Unfortunately, in real systems, the demodulation will unavoidably have a phase error ϕ_{err} and a phase noise ϕ_{noise} components.
This means that the sense output:

$$\Delta V_{out,0} = 2 \frac{V_{DC}}{C_{FS}} \frac{C_0}{g} \frac{x_{D,0}}{\Delta \omega_{MS}} \left[\Omega \cos(\omega_D t) + B_q \sin(\omega_D t) \right] = S \left[\Omega \cos(\omega_D t) + B_q \sin(\omega_D t) \right]$$

will not simply multiplied by the ideal $\cos(\omega_D t)$, but by $\cos(\omega_D t + \phi_{err} + \phi_{noise})$

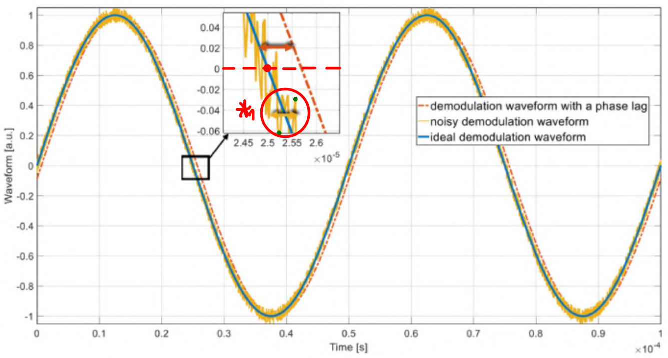
The signal after demodulation and filtering will be:

$$V_{dem} = S \left[\Omega \cos(\omega_D t) + B_q \sin(\omega_D t) \right] \cos(\omega_D t + \phi_{err} + \phi_{noise}) * LPF$$

$$\approx \frac{S}{2} G_{LPF} \left[\Omega \cos(\phi_{err} + \phi_{noise}) + B_q \sin(\phi_{err} + \phi_{noise}) \right] =$$

$$\approx S \left[\Omega + B_q \phi_{err} + B_q \phi_{noise} \right] =$$

$$\approx \underbrace{S\Omega}_{\text{SIGNAL}} + \underbrace{S B_q \phi_{err}}_{\text{OFFSET}} + \underbrace{S B_q \phi_{noise}}_{\text{NEW NOISE CONTRIBUTION}}$$



*1 if you look at the yellow curve you can see that sometimes it is delayed w/rt the blue one and other times it anticipates the blue curve
↓
This means that the phase of the yellow curve is continuously fluctuating w/rt the blue curve
↓
This behavior is described by ϕ_{noise}

NOTE: As we already seen, if we assume that at a certain point the signal saturates, the amplitude noise disappears

So if we take our demodulation reference out of the saturated square-wave for which the 1st harmonic is obviously $\cos(\omega_D t)$, we can assume that noise is just given by phase noise, while amplitude noise is essentially erased by saturation effect.

Can we find out an expression for the equivalent input-referred angular rate B_q?

To estimate the magnitude of B_q, we add to the model a further cross-axis term due to the cross-axis stiffness.
This term represents the quadrature force F_q in the y-direction due to a displacement occuring in the x-direction:

$$|\vec{F_q}| = k_{DS} x \quad \text{where} \quad k_{DS} = \text{CROSS-AXIS STIFFNESS}$$

The motion equation for the gyroscope coupled modes becomes:

$$\begin{cases} \boxed{m_S \ddot{y} + b_S \dot{y} + k_S y - k_{DS} x = -2 m_S \Omega \dot{x}} \\ (m_D + m_S) \ddot{x} + b_D \dot{x} + k_D x - \cancel{k_{DS} y} = F_{elec} - \cancel{2 (m_S + m_D) \Omega \dot{y}} \end{cases}$$

since $\begin{cases} y \ll x \rightarrow \dot{y} \ll \dot{x} \\ k_{DS} \ll k_D \end{cases}$

Att: The correct way to analyze this coupled system would be to write a system of equations that couples the drive and sense modes, so through the Coriolis and the cross-axis effects. However, while the coupling from the drive- into the sense-mode is huge, the reverse effect is typically negligible, so for sake of simplicity we can consider just the drive-axis equation

$\rightarrow \; s^2 m_s Y(s) + s b_s Y(s) + k_s Y(s) = -2 m_s \Omega s X(s) + k_{DS} X(s)$

$$= -(2 m_s \Omega s - k_{DS}) X(s)$$

For the sense-mode the offset term can be written in terms of equivalent input-referred angular rate B_q, phase shifted by 90° (j)

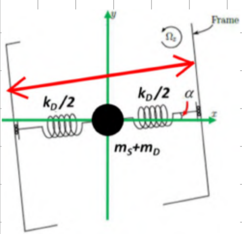
$\boxed{k_{DS} = 2 m_s B_q \omega_D} \rightarrow \boxed{B_q = \frac{k_{DS}}{2 m_s \omega_D}}$ EQUIVALENT INPUT-REFERRED ANGULAR RATE OF THE QUADRATURE ERROR

$\rightarrow \; s^2 m_s Y(s) + s b_s Y(s) + k_s Y(s) = -2 m_s X(s) \, [\Omega s + B_q \omega_D]$

For a given process, do we exactly know which is the cross-axis stiffness coupling term k_{DS}?

No we don't!!! k_{DS} depends essentially on the geometry and it's not something that we can simply deduce by knowing the process parameters.

GOAL: determine k_{DS} in order to determine B_q as well



y-direction displacement will be: $\frac{y}{x} = \tan(\alpha)\, Q_{eff} \approx \alpha \, Q_{eff}$

NOTE: we should use Q_s instead of Q_{eff} in case of mode-matched conditions

NOTE: angle α is small

α = DEFLECTION ANGLE FOR QUASI DC MOTION

The corresponding force in the y-direction is related to y through k_{DS}:

$$y = \frac{F_q}{k_s} \cdot Q_{eff} = \frac{k_{DS} x}{k_s} Q_{eff} \rightarrow \begin{cases} \frac{y}{x} = \frac{k_{DS}}{k_s} Q_{eff} \\ \frac{y}{x} \approx \alpha \, Q_{eff} \end{cases} \rightarrow \boxed{\frac{k_{DS}}{k_s} \approx \alpha}$$

We can thus find an expression for B_q as a function of the nonideality α:

$$\boxed{B_q = \frac{k_{DS}}{2 m_s \omega_D} \approx \frac{\alpha k_s}{2 m_s \omega_D} \approx \frac{\alpha}{2} \omega_s \approx \frac{\alpha}{2} \omega_D}$$

EQUIVALENT INPUT-REFERRED ANGULAR RATE OF THE QUADRATURE ERROR

NOTE: even if we don't work in mode-matched operation ω_s is very close to ω_D

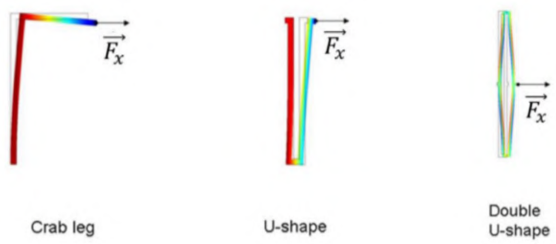
NOTE (imp!): if we increase the resonance frequency we have a worsening of the quadrature effect and as consequence a worsening of the phase noise ϕ_{noise}

↓

That's why it is safe to keep gyros modes above the audio bandwidth but not higher than that

One critical parameter for quadrature in gyros is the width of the spring folds w (and its non-uniformities), as the total in-plane stiffness is proportional to w^3

As:
- There are springs that ideally do not deflect in the y-direction under x-direction forces
- There are usually 4 symmetric springs for a single frame

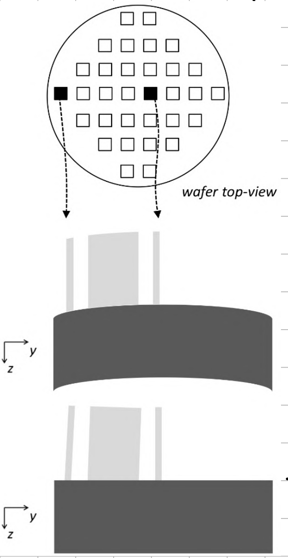


=> Nonuniformities in spring width give rise to non-null k_{DS}, but w/ a very low α angle.

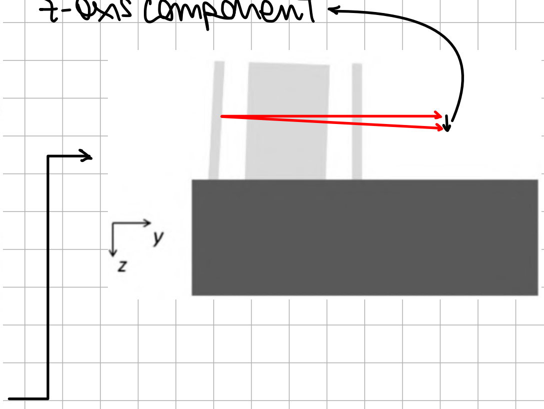
As previously announced, the main misalignment source is given by the so called SKEW ANGLE EFFECT.

SKEW ANGLE EFFECT = is an effect that usually occurs in gyros for in-plane rate detection where usually the sense frame has a low out-of-plane stiffness

Process nonuniformities make the etching non orthogonal to the substrate at wafer edges.



An attempt to bend the beam in the x-direction results in a force w/ a z-axis component



How can we minimize the quadrature error?

$$B_q = \frac{k_{xs}}{2 m_s \omega_D} \approx \frac{\alpha k_s}{2 m_s \omega_D} \approx \frac{\alpha}{2} \omega_s$$

We essentially have 3 possibilities:

1. MINIMIZE k_{xs} (SO THE ANGLE α)

   - improving the process uniformity across the wafer
   - increasing the beam width
   - choosing springs w/ the lowest cross-axis term (folded or double-U)
   - choosing only gyroscopes from wafer center

2. DECREASE THE GYROSCOPE RESONANCE

   Att: This is limited by acoustic disturbances (up to >20 kHz) and presence of vibrations (up to 1kHz in consumer, 10 kHz in automotive and 50 kHz in military applications)

3. INCREASE THE INERTIAL MASS

   Att: it's limited by the maximum area

ELECTRONIC COMPENSATION OF QUADRATURE