ACCELEROMETERS

THE CAPACITIVE MEMS ACCELEROMETER
PART 1: OVERVIEW, SENSITIVITY, PULL-IN ISSUES

MOTIVATIONS & GOALS

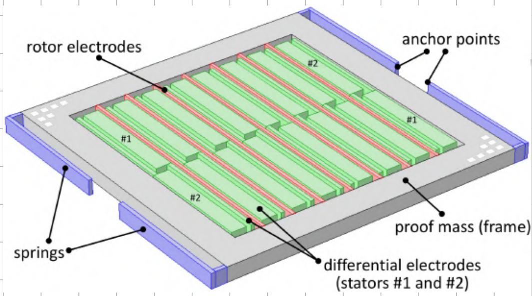
ACCELERATION $\longrightarrow$ INERTIAL FORCE $\longrightarrow$ DISPLACEMENT $\longrightarrow$ CAPACITIVE VARIATION

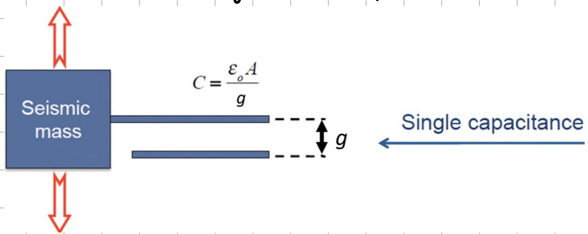
GOALS:
- to understand the working principle w/ a sample front-end circuit
- to derive an expression for the sensitivity (output voltage vs acceleration)
- to analyze the pull-in issues that affects the parallel-plates devices

GENERAL ARCHITECTURE OF A PP CONFIGURATION

Coupling to a sample front-end
Charge transfer
Output voltage vs displacement

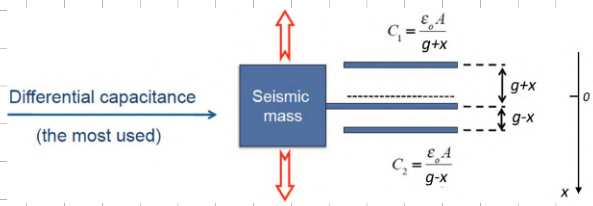
We know that the sensor should be formed by:
- a suspended mass
- some fixed electrodes which face some parts of the suspended mass to form capacitive sensing plates
- suspending springs





: The capacitance changes non-linearly w/ the displacement

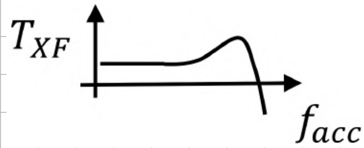
The difference btw the two capacitances is still non-linear, but it is easier to linearize



How do we measure such this differential capacitive sensing? We need an electronic interface but before chosing what interface we want, we need to understand which is the current generated in such a type of capacitor.

In order to readout a signal from an accelerometer, we need to capacitively measure its displacement to recover the information on accelerations.

Hp: For the sake of simplicity, let's suppose for the moment that the acceleration occurs always at frequencies which are much lower than the resonance frequency of our system ω_0 (QUASI-STATIONARY APPROXIMATION)



$$\omega \ll \omega_0 \longrightarrow \cancel{m\ddot{x}} \cancel{+ b\dot{x}} + kx = m a_{ext} + F_{elec}$$

Current through a GENERIC VARIABLE CAPACITOR biased by a generic voltage V

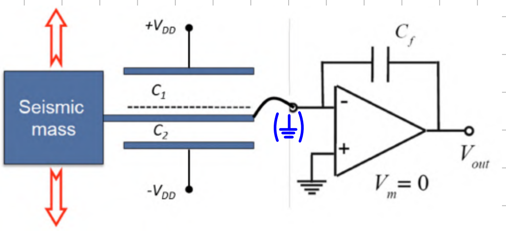
$$i_c = \frac{dQ}{dt} = \frac{d}{dt}(CV) = C\frac{dV}{dt} + V\frac{dC}{dt}$$

$C\frac{dV}{dt}$: same term that we have in a fixed capacitor

$V\frac{dC}{dt}$: additional term due to the fact that the capacitor is variable

There are two different PARALLEL-PLATE (PP) READOUT CONFIGURATIONS :

① Application of a constant DC voltage $\pm V_{DD}$ b/w each stator and the rotor

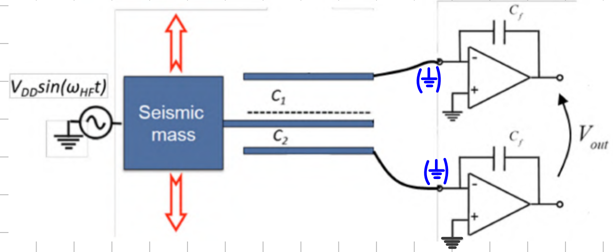


The rotor is maintained to a fixed known potential (∅V) "generated" by the virtual ground mechanism of the input node of the opamp.
So b/w the rotor and the two stators we have a fixed constant voltage V_i ($= +V_{DD}$ for C_1, $= -V_{DD}$ for C_2)

$$i = 1,2 \;:\; V_i = \text{const} \longrightarrow i_{C_i} = \frac{dQ_i}{dt} = C_i \cancelto{\varnothing}{\frac{dV_i}{dt}} + V_i \frac{dC_i}{dt} = V_i \frac{dC_i}{dt}$$

This term is proportional to the capacitive derivative: not to lose the stationary (DC) value of the acceleration, we need to integrate the signal.
The feedback impedance is capacitive.

② Application of a modulated AC voltage b/w each stator and the rotor



This time we keep the two stators at virtual ground and we apply an AC voltage to the rotor

$$V_i = V_{DD}\sin(\omega t)$$

$$\frac{dV_i}{dt} = \omega V_{DD}\cos(\omega t) \quad (\text{linear w/ } \omega)$$

if $\omega = \omega_{HF}$ (very high frequency) $\longrightarrow C_i \frac{dV_i}{dt} \gg V_i \frac{dC_i}{dt}$ ⟶ becomes negligible

$\rightarrow i_{C_i} \simeq C_i \dfrac{dV_i}{dt}$: This term dominates for very-high-frequency AC signals

Hp: for the sake of simplicity we will proceed studying the 1st configuration

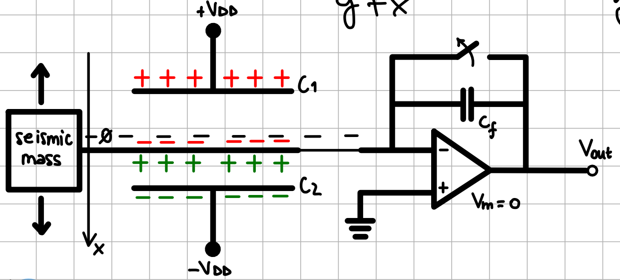
NOTE: In both configurations, the same sensitivity is obtained

Hp: initial approximation: no bias current of the opamp (otherwise this is continuously integrated by C_f, leading to output voltage saturation).

example: Differential capacitive sensing

Hp: Accelerometer is initially in the rest position and C_f is initially discharged

$\rightarrow V_m = \varnothing \qquad C_1 = \dfrac{\varepsilon_0 A N}{g+x} \qquad C_2 = \dfrac{\varepsilon_0 A N}{g-x} \qquad Q_F = 0$



As C_1 is positive biased, on its top electrode will arise positive charges while the top face of the rotor will show negative charges. We have the opposite situation on the bottom side, since C_2 is inversely biased w/r/t C_1.

GOAL: to calculate the output voltage per unit displacement
→ The corresponding electrostatic force
→ The output voltage per unit acceleration

Hp: assume to be in the rest position ($x=0$)
⟶ The charges on the faces of the rotor are : $Q_1 = -C_1 V_{DD} \quad Q_2 = C_2 V_{DD}$
$\Rightarrow$ @ $x=0$: $Q_1 = Q_2$ (charge neutrality)

If the device displaces: $x > 0 \longrightarrow C_1$ decreases, C_2 increases

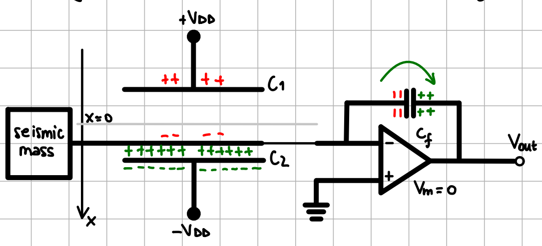
This means that for the same bias voltage, C_1 will have a lower current, while C_2 a larger one $\longrightarrow$ a net charge on the rotor appears !!

NET CHARGE DIFFERENCE WHICH ARISES ON THE ROTOR

$$\boxed{\Delta Q_m = Q_2 + Q_1 = C_2 V_{DD} - C_1 V_{DD} = V_{DD}(C_2 - C_1) = V_{DD}\,\Delta C}$$

Where does this charge come from? It's well known that charge cannot pass through a capacitor bc there is the dielectric in between, so the only path the charge can come from is the virtual ground
$\Rightarrow$ the charge is delivered by the feedback branch of the amplifier

As we need to guarantee that the charge balance is preserved, this means that some negative charge should be delivered by the feedback branch



we need "4" negative charges on the left plate of C_f
this means that this 4 negative charges will recall 4 positive charges on the right plate, so coming from the low impedance of the amplifier

Across C_f we'll have a voltage difference given by : $V_{C_f} = \frac{\Delta Q_m}{C_f}$

$$\Rightarrow \boxed{\Delta V_{OUT} = \Delta V_{C_f} = \frac{\Delta Q_m}{C_f} = \frac{V_{DD}}{C_f}\,\Delta C} \qquad \text{where} \quad \boxed{\frac{V_{DD}}{C_f} = G_{AMP}}$$

This amplifier having a v.g. that picks up the charge from our MEMS and gives out a voltage through the feedback capacitance C_f is known as CHARGE AMPLIFIER
It takes the charge coming from the v.g. and amplifies it to the output dividing it by the value of C_f itself.

Calculation of ΔQ_m as a function of x: (Hp: positive displacement $x>0$)

$$\Delta Q_m = Q_1 + Q_2 = -C_1 V_{DD} + C_2 V_{DD} = V_{DD}\,\Delta C_{diff} = V_{DD}\left(-\frac{\varepsilon_0 AN}{g+x} + \frac{\varepsilon_0 AN}{g-x}\right) = V_{DD}\frac{\varepsilon_0 AN}{g}\left(\frac{-1}{1+x/g} + \frac{1}{1-x/g}\right)$$

$$= V_{DD} C_0 \left[\frac{-\cancel{1} + x/g + \cancel{1} + x/g}{1-(x/g)^2}\right] = V_{DD} C_0 \left[\frac{2x/g}{1-(x/g)^2}\right]$$

Hp: SMALL DISPLACEMENT APPROXIMATION: small displacement x of the suspended mass w/r+ the air gap g b/w the parallel plates.

$$\boxed{x \ll g \longrightarrow \frac{x}{g} \ll 1} \longrightarrow \boxed{\Delta Q_m \approx 2 V_{DD} C_0 \frac{x}{g}} \longrightarrow \boxed{\Delta C_{diff} = 2 C_0 \frac{x}{g}}$$

NET CHARGE ON THE ROTOR AS A FUNCTION OF THE DISPLACEMENT x

NOTE:
- Under small displacement approximation we obtain that the net charge that arises on the rotor is a LINEAR function of the displacement x
- By using a differential capacitive sensing (and under the assumption of small displacement approximation) we have linearized our configuration

Now we can calculate:

OUTPUT VOLTAGE AS FUNCTION OF THE DISPLACEMENT x (generated by the external acceleration)

$$\boxed{\Delta V_{OUT} = \frac{\Delta Q_m}{C_f} = \frac{V_{DD}\,\Delta C_{diff}}{C_f} = 2\,\frac{V_{DD}}{C_f}\,\frac{C_0}{g}\,x}$$

where V_{DD}/C_f = CAPACITANCE TO VOLTAGE GAIN (C2V)

C_0/g = DISPLACEMENT TO CAPACITANCE GAIN (x2C)

RESULT: The charge amplifier output voltage is, under the approximation of small displacement, a linear function of the displacement x.

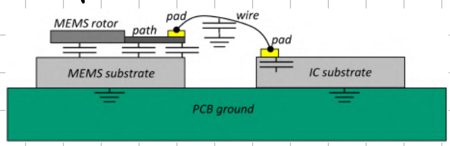
NOTE: In the real case we have to take into account also the presence of a PARASITIC CAPACITANCE btw the rotor contact and the ground

Which are the physical sources of such a parasitic capacitance?

Several are the sources of parasitic capacitance:

direct facing towards the substrate

-
-



$\boxed{C_P \sim 1\text{-}5\ pF}$

Even if C_P is very large, since it is continuously kept btw ground and v.g., there is no charge flow through it and thus no loss in signal.

EFFECTS OF ELECTROSTATIC FORCES IN PP ACCELEROMETERS

Equivalent electrostatic negative stiffness

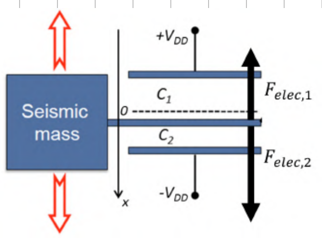
Pull-in instability

Mechanical stoppers

GOAL: to find the SENSITIVITY → once $\Delta V_{out}(x)$ is found, we need to relate x to the external acceleration

We saw that the application of a voltage for the readout generates unavoidable electrostatic forces that should be considered in the force balance of the motion equation

$$\boxed{F_{elec} = \frac{\Delta V^2}{2}\frac{dC}{dx} = \frac{V_{DD}^2}{2}\frac{dC}{dx}} \rightarrow F_{elec1} = -\frac{V_{DD}^2}{2}\frac{\varepsilon_0 A N}{(g+x)^2},\ F_{elec2} = +\frac{V_{DD}^2}{2}\frac{\varepsilon_0 A N}{(g-x)^2}$$



$$\rightarrow F_{elec} = F_{elec1} + F_{elec2} = \frac{V_{DD}^2}{2}\frac{\varepsilon_0 A N}{(g-x)^2} - \frac{V_{DD}^2}{2}\frac{\varepsilon_0 A N}{(g+x)^2}$$

Hp:
- no external acceleration;
- quasi-stationary behavior → voltage is constant → F_{elec} is slowly varying in time

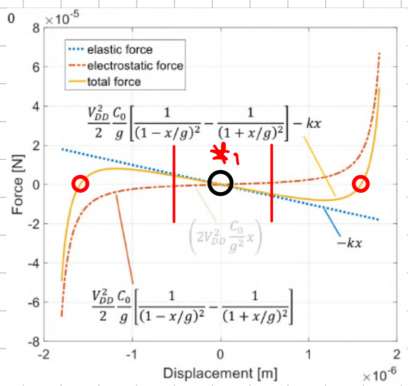
$$\rightarrow \cancel{m\ddot{x}} + \cancel{b\dot{x}} + kx = \cancel{m\,a_{ext}} + F_{elec}$$

RESTORING STIFFNESS FORCE:

$$\boxed{kx = F_{elec} = \frac{V_{DD}^2}{2}\frac{\varepsilon_0 A N}{(g-x)^2} - \frac{V_{DD}^2}{2}\frac{\varepsilon_0 A N}{(g+x)^2}}$$

Let's solve this equation w/ a graphical approach:

$$\rightarrow \frac{V_{DD}^2}{2}\frac{C_0}{g}\left[\frac{1}{(1-x/g)^2} - \frac{1}{(1+x/g)^2}\right] - kx = 0$$



*1 In the region close to zero (so for small displacements) we see that in principle we can linearize the expression of the force to: $2\,V_{DD}^2\,C_0/g^2\,x$

We observe that there are ONE STABLE and TWO UNSTABLE EQUILIBRIUM POINTS

What happens if we perturb the mass from the central position?

- slightly positive displacement → the total force is negative
- slightly negative displacement → the total force is positive

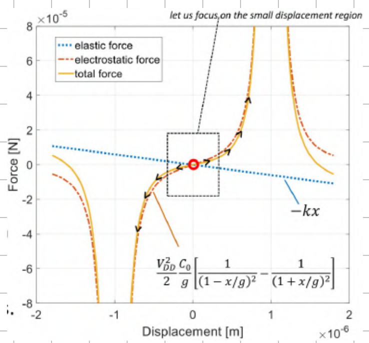
⇒ This means that if the mass slightly moves away from the central position, the force will always tend to put the mass back to that central initial position
⇒ The central point is a STABLE EQUILIBRIUM POINT

NOTE: we should work around the stable equilibrium point!!

What happens if we perturb the mass from the external positions?
- slightly positive displacement ⟶ the total force is positive
- slightly negative displacement ⟶ the total force is negative

⇒ The force will tend to take the mass away from these points
⇒ The external points are UNSTABLE EQUILIBRIUM POINTS

Can we change anyway the equilibrium conditions of the suspended mass?
The equilibrium conditions can significantly change if one tries to:
- increase the facing area
- decrease the gap btw plates
- decrease the stiffness
- increase the bias voltage



Now, if we slightly move away the mass from the origin what we obtain is that the force increases its value up to such a condition that the sign of the function around the rest point is changed

⟶ we have no longer any equilibrium point

What does it mean? It means that as soon as I switch on the biasing voltage of my circuit, unless the mass is perfectly centered btw the 2 stators, as soon as there is a perturbation, it will go indefinitely leftwards or rightwards
Obviously it's impossible for the mass to stay exactly in the center bcs the process nonuniformities, vibration, noise, etc.
So, as soon as I switch on my accelerometer the mass will go to crash against one of the two parallel plates
⟶ obviously, under these conditions the device cannot work

This phenomenon of electromechanical instability is known as PULL-IN INSTABILITY

Not always the device goes to crash against one of the two plates, so under certain conditions the device properly works

How do we determine the limit conditions btw a working and a non-working device?
As we have seen that things change as soon as the slope of the total force around the rest position changes sign, we essentially need to equate the elastic restoring force to the electrostatic force around the rest position (small displacement approximation)

$$kx = \frac{V_{DD}^2}{2}\frac{\varepsilon_0 AN}{(g-x)^2} - \frac{V_{DD}^2}{2}\frac{\varepsilon_0 AN}{(g+x)^2}$$

$$= \frac{V_{DD}^2}{2}\frac{\varepsilon_0 AN}{g^2}\left[\frac{1}{\left(1-\frac{x}{g}\right)^2} - \frac{1}{\left(1+\frac{x}{g}\right)^2}\right]$$

$$= \frac{V_{DD}^2}{2}\frac{C_0}{g}\left[\frac{1}{1+(x/g)^2-2(x/g)} - \frac{1}{1+(x/g)^2+2(x/g)}\right]$$

Hp: small displacement approximation ⟶ $x \ll g$ ⟶ $(x/g)^2$ is negligible

$$= \frac{V_{DD}^2}{2}\frac{C_0}{g}\left[\frac{1}{1-2(x/g)} - \frac{1}{1+2(x/g)}\right]$$

NOTE: $(a-b)(a+b) = a^2-b^2$

$$= \frac{V_{DD}^2}{2}\frac{C_0}{g}\left[\frac{\cancel{1}+2(x/g)-\cancel{1}+2(x/g)}{1-(2x/g)^2}\right]$$

$$= \frac{V_{DD}^2}{2}\frac{C_0}{g}\left[\frac{4(x/g)}{1-\cancel{(2x/g)^2}}\right]$$

$$= \frac{V_{DD}^2}{2}\frac{C_0}{g}\,4\,\frac{x}{g}$$

$$\Rightarrow \boxed{kx = F_{elec} = 2V_{DD}^2\frac{C_0}{g^2}x}$$ **ELECTROSTATIC FORCE** where k = slope of the elastic force

$2V_{DD}^2\frac{C_0}{g^2}$ = slope of the electrostatic force

$$k - 2V_{DD}^2\frac{C_0}{g^2} = 0 \longrightarrow \boxed{k_{elec} = -2V_{DD}^2\frac{C_0}{g^2}}$$ **ELECTROSTATIC STIFFNESS**

Since we have define an electrostatic stiffness, now we can refer to what we have called simply k so far as MECHANICAL STIFFNESS k_m

$$k_m + k_{elec} = 0 \longrightarrow \boxed{V_{DD,PI} = \sqrt{\frac{k_m g^2}{2C_0}} = \sqrt{\frac{g^3 k_m}{2\varepsilon_0 AN}}}$$

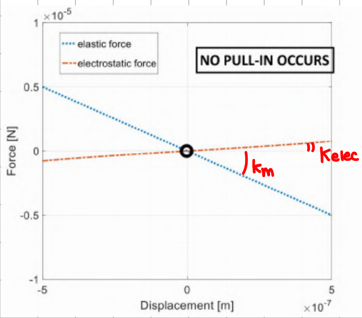
PULL-IN VOLTAGE = maximum voltage that we can apply to our rotor in order to avoid pull-in instability

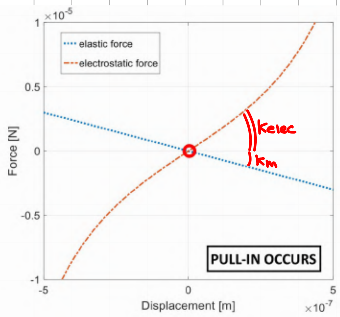
PULL-IN CONDITION: $\boxed{k_{TOT} = 0 \longrightarrow |k_m| = |k_{elec}|}$

CONDITION TO AVOID PULL-IN INSTABILITY

$$\boxed{k_m > k_{elec} \longrightarrow k_m > 2V_{DD}^2\frac{C_0}{g^2} \longrightarrow V_{DD} < V_{DD,PI} = \sqrt{\frac{g^3 k_m}{2\varepsilon_0 AN}}}$$

NOTE: from a graphical point of view, we see that a stable equilibrium point exists only if the elastic force slope around zero is larger than the electrostatic force slope





NOTE: Accelerometers should be designed such that the elastic stiffness is >> than the electrostatic stiffness
$k_m >> k_{elec}$

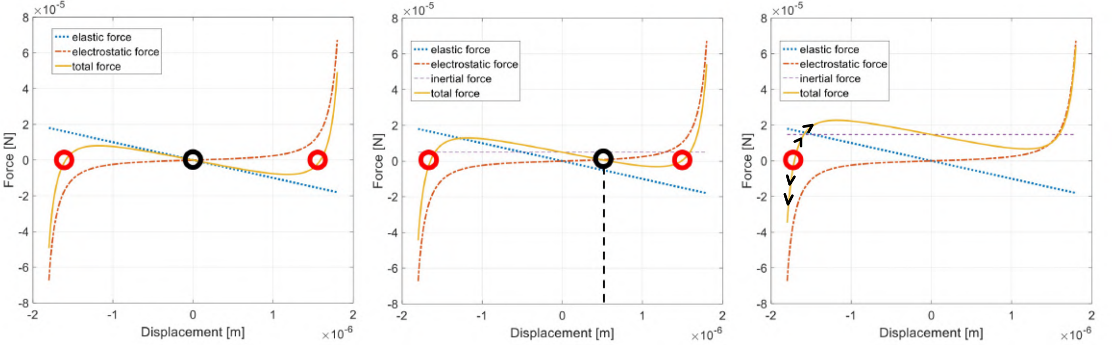
Att: This result has been obtained assuming the mass in its rest position (no acceleration)

GOAL: to calculate the acceleration

If we consider a non-zero acceleration:

- for small accelerations: shift of the equilibrium point $\longrightarrow$ This is the displacement we want to measure in order to recover the acceleration value
- after a certain acceleration value however, no more stable point exists!

Att: Too large accelerations can cause instability in PP accelerometers even at biasing values lower than the pull-in voltage



RESULT: Accelerometer should be designed in such a way that also under FSR acceleration the pull-in condition remain far

MECHANICAL STOPPERS = are rigid elements used to avoid the complete snap btw rotor and stator plates which can cause adhesion

→ stoppers are placed at a distance from the rotor which is smaller than the gap btw plates

NOTE: stoppers are biased at the same voltage as the suspended mass to avoid electrostatic forces

ACCELEROMETER OVERALL SENSITIVITY

GOAL: to find out the output voltage per unit acceleration = SENSITIVITY

Hp: small displacement assumption

We now solve the stationary condition:

$$k_m x = m a_{ext} + F_{elec} \rightarrow k_m x = m a_{ext} + 2 V_{DD}^2 \frac{C_0}{g^2} x$$

$$x = \frac{m}{k_m - 2V_{DD}^2 \frac{C_0}{g^2}} a_{ext} = \frac{m}{k_m + k_{elec}} a_{ext} = \frac{m}{k_{TOT}} a_{ext} = \frac{1}{\omega_0^2} a_{ext} \longrightarrow \boxed{x = \frac{a_{ext}}{\omega_0^2}}$$

$\boxed{k_{TOT} = k_m + k_{elec}}$ **OVERALL STIFFNESS** **NOTE**: $\boxed{k_{elec} < 0 \longrightarrow k_m > k_{TOT}}$ $*_1$

$*_1$ $\sqrt{\frac{k_{TOT}}{m}} < \sqrt{\frac{k_m}{m}}$ => **NOTE**:
- The MEMS resonance frequency changes as the electrostatic stiffness changes
  ↓
  ELECTROSTATIC SOFTENING (TUNING) EFFECT
- a MEMS accelerometer is well-designed only if it effectively undergoes small displacements even for FSR accelerations

$$\Delta V_{out} = 2 \frac{V_{DD}}{C_f} \frac{C_0}{g} x = 2 \frac{V_{DD}}{C_f} \frac{C_0}{g} \frac{1}{\omega_0^2} a_{ext}$$

$$\boxed{\frac{\Delta V_{out}}{a_{ext}} = 2 \frac{V_{DD}}{C_f} \frac{C_0}{g} \frac{1}{\omega_0^2} = 2 \frac{V_{DD}}{C_f} \frac{C_0}{g} \frac{m}{(k_m - 2V_{DD}^2 C_0/g^2)}}$$

OVERALL SENSITIVITY OF A DIFFERENTIAL PP MEMS AXEL READOUT THROUGH A CHARGE AMPLIFIER $\left[\frac{V}{m/s^2}\right]$

- 2 = differential gain
- $\boxed{\frac{dV}{dC} = \frac{V_{DD}}{C_f}}$: CAPACITANCE TO VOLTAGE GAIN
- $\boxed{\frac{dC}{dx} = \frac{C_0}{g}}$: DISPLACEMENT TO CAPACITANCE GAIN
- $\boxed{\frac{dx}{da} = \frac{1}{\omega_0^2}}$: PHYSICAL QUANTITY TO DISPLACEMENT GAIN

MECHANICAL SENSITIVITY OF A DIFFERENTIAL PP MEMS AXEL

$$\boxed{S_m = 2 \frac{dC}{dx} \frac{dx}{da} = 2 \frac{C_0}{g} \frac{1}{\omega_0^2}}$$

THE CAPACITIVE MEMS ACCELEROMETER
PART 2: BANDWIDTH AND NOISE CONSIDERATIONS

MOTIVATIONS & GOALS

TRADE-OFFS:
- LINEARITY (PP vs CF)
- FSR
- BANDWIDTH
- NOISE

CONSIDERATIONS ON THE SENSITIVITY
- Linearity
- Comparison with a CF approach

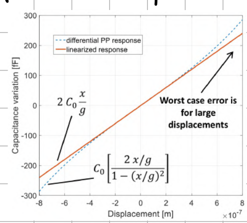
$$\boxed{\frac{\Delta V_{OUT}}{a_{EXT}} = 2\frac{dV}{dC}\frac{dC}{dx}\frac{dx}{da} = 2\frac{V_{DD}}{C_f}\frac{C_0}{g}\frac{1}{\omega_0^2} = 2\frac{V_{DD}}{C_f}\frac{C_0}{g}\frac{m}{(k_m - 2V_{DD}^2\, C_0/g^2)}}\ ; \quad \boxed{V_{DD} \ll V_{DD,PI} = \sqrt{\frac{g^3 k}{2\varepsilon_0 A N}}}\ ; \quad \boxed{\omega_0 = \sqrt{\frac{k_{TOT}}{m}}}$$

- if $g \downarrow$ ⇒ sensitivity ↑ but $V_{DD,PI} \downarrow$
- if $m \uparrow$ ⇒ sensitivity ↑ but BW↓ ($\omega_0 \downarrow$)
- if $k_{TOT} \downarrow$ ⇒ sensitivity ↑ but $V_{DD,PI}\downarrow$ and BW↓
- if $V_{DD} \uparrow$ ⇒ sensitivity ↑ but it facilitates PI
- if $h \uparrow$ ⇒ sensitivity ↑

A lot of TRADE-OFFS!!!

Which is the maximum error that we can tollerate in terms of deviation of the sensitivity w/r.t the linearized curve?

The maximum error that we can tollerate depends on the application, not by the MEMS design itself



There are 2 SOURCES OF NON LINEARITY IN THE AXEL RESPONSE:

1. The NONLINEAR RESPONSE of differential PPs
2. NONLINEAR EFFECTS from the electrostatic force ⟶ Hp: we assume This term negligible (NO PULL-IN INSTABILITY: $k_m \gg k_{elec}$)

% LINEARITY ERROR OF THE SENSITIVITY (relative to the FSR)

$$\boxed{\varepsilon_{\%}(x) = \frac{\Delta C_{REAL} - \Delta C_{LIN}}{\Delta C_{REAL,FSR}}\,100 = \frac{2C_0\frac{x}{g}\left[\frac{1}{1-(x/g)^2}\right] - 2C_0\frac{x}{g}}{2C_0\frac{x_{MAX}}{g}\left[\frac{1}{1-(x_{MAX}/g)^2}\right]}\cdot 100}$$

where:

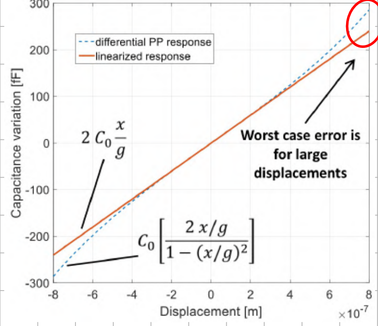
$$\boxed{\Delta C_{REAL} = 2C_0\frac{x}{g}\left[\frac{1}{1-(x/g)^2}\right]}$$ TRUE CAPACITANCE VARIATION

$$\boxed{\Delta C_{LIN} = 2C_0\frac{x}{g}}$$ LINEARIZED CAPACITANCE VARIATION

$$\boxed{\Delta C_{REAL,FSR} = 2C_0\frac{x_{MAX}}{g}\left[\frac{1}{1-(x_{MAX}/g)^2}\right]}$$ TRUE CAPACITANCE VARIATION AT THE FSR

NOTE: setting a limit TO $\varepsilon_{\%}$ we basically quantify the small displacement assumption

$$\varepsilon_{\%}(x) = \frac{\cancel{2C_0}\frac{x}{\cancel{g}}\left[\frac{1}{1-(x/g)^2}\right] - \cancel{2C_0}\frac{x}{\cancel{g}}}{\cancel{2C_0}\frac{x_{MAX}}{\cancel{g}}\left[\frac{1}{1-(x_{MAX}/g)^2}\right]}\cdot 100 = \frac{x\left[\frac{1}{1-(x/g)^2}\right] - x}{x_{MAX}\left[\frac{1}{1-(x_{MAX}/g)^2}\right]}\cdot 100$$



→ largest error at the largest displacement

NOTE: In the datasheet, instead of giving the entire graph representing the non-linearity as function of the displacement, it is common to define the WORST CASE ERROR

Which is the worst case error? it's the error corresponding to the FSR so for $x = x_{MAX}$

$$\varepsilon_{\%}(x)\Big|_{x=x_{MAX}} = \frac{\cancel{x_{MAX}}\left[\frac{1}{1-(x_{MAX}/g)^2}\right]-\cancel{x_{MAX}}}{\cancel{x_{MAX}}\left[\frac{1}{1-(x_{MAX}/g)^2}\right]}\cdot 100 = \cancel{1}-\cancel{1}+\left(\frac{x_{MAX}}{g}\right)^2\cdot 100$$

$$\boxed{\varepsilon_{\%}(x)\Big|_{x=x_{MAX}} = \frac{\Delta C_{REAL,FSR}-\Delta C_{LIN,FSR}}{\Delta C_{REAL,FSR}}\cdot 100 = \dots = \left(\frac{x_{MAX}}{g}\right)^2\cdot 100}$$

WORST CASE ERROR (NON LINEARITY ERROR @FSR)

$$\boxed{x_{MAX} = \frac{\sqrt{\varepsilon_{\%}}}{10}\cdot g}$$ **MAXIMUM DISPLACEMENT**

$$\boxed{a_{FSR} = \frac{x_{MAX}}{dx/da_{ext}} = x_{MAX}\,\omega_0^2 = \frac{\sqrt{\varepsilon_{\%}}}{10}\cdot g\cdot \omega_0^2}$$

$$\boxed{\omega_0\Big|_{FSR} = \sqrt{\frac{a_{FSR}}{x_{MAX}}}}$$

FULL-SCALE RANGE (FSR)
maximum acceleration for which the response remains within linearity limits set by the application

NOTE: Once x_{MAX} is fixed to a fraction of the gap, if I need to increase the maximum acceleration that we have to sense (if I need the FSR to be larger), I need to lower the term $dx/da_{ext} = 1/\omega_0^2$ and thus the overall sensitivity (since $\Delta V_{OUT}/a_{ext} = 2\cdot V_{DD}/C_f\cdot C_0/g\cdot 1/\omega_0^2$).

Essentially for a larger acceleration to be sensed, we need to guarantee the same displacement which means that we have to lower the displacement per unit acceleration, but lowering displacement per unit acceleration (dx/da) means basically lowering the scale factor, so lowering the sensitivity

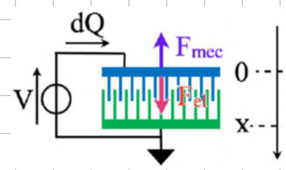
RESULT: **SENSITIVITY VS FSR TRADE-OFF**: if you want to increase the FSR you need to lower the sensitivity

→ **NOTE**: The parameter that puts these two quantities in trade-off is the resonance frequency

⇒ if you want to increase the FSR you typically need to decrease the sensitivity by increasing the resonance frequency ω_0

Is there any alternative configuration to avoid pull-in and nonlinearity?

→ COMB-FINGER CONFIGURATION → **REM**: Comb-finger capacitors are area-varying type *,



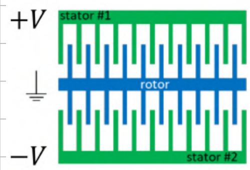
$$*, \boxed{C = \frac{2\varepsilon_0 A N}{g} = \frac{2\varepsilon_0 h(x+x_0)N}{g}} \longrightarrow \boxed{\frac{dC}{dx} = \frac{2\varepsilon_0 h N}{g}}$$

CAPACITANCE VARIATION PER UNIT DISPLACEMENT

NOTE: It's independent of the displacement

$$\boxed{F_{elec} = \frac{V^2}{2}\frac{dC}{dx} = \frac{V^2}{2}\frac{2\varepsilon_0 h N}{g} = \frac{V^2\varepsilon_0 h N}{g}}$$

ELECTROSTATIC FORCE IN A COMB-FINGER CONFIGURATION

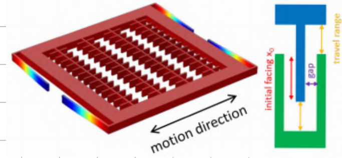


In a DIFFERENTIAL CONFIGURATION: $|F_{elec1}| = |F_{elec2}| = \frac{V^2\varepsilon_0 h N}{g}$

$$\Rightarrow \boxed{F_{elec,TOT} = \emptyset} \Rightarrow m\ddot{x}+b\dot{x}+kx = m\,a_{ext}+\cancel{F_{elec}}$$

RESULT: In a DIFFERENTIAL CONFIGURATION the net overall electrostatic force is null bc, since F_{elec} is independent of the displacement, F_{elec1} and F_{elec2} are always equal and opposite.

CONCLUSION: A solution to LINEARITY and FSR ISSUES could be the implementation of a CF-based readout



SENSITIVITY CALCULATION:

1. evaluate the displacement per unit acceleration. dx/da
2. evaluate the single-ended capacitance variation per unit displacement. dC/dx
3. evaluate the output voltage per unit differential capacitance variation. dV/dC

$$\frac{\Delta V_{out}}{a_{ext}} = 2\,\frac{dV}{dC}\,\frac{dC}{dx}\,\frac{dx}{da} \quad w/ \quad \frac{dC}{dx} = \frac{\varepsilon_0 h N}{g}\cdot\frac{x_0}{x_0} = \frac{C_0}{x_0} \implies \boxed{\frac{\Delta V_{out}}{a_{ext}} = 2\,\frac{V_{DD}}{C_F}\cdot\frac{C_0}{x_0}\cdot\frac{1}{\omega_0^2}}$$

SENSITIVITY FOR A COMB-FINGER CONFIGURATION

NOTE: The sensitivity is linear even if we have done no approximations

ADVANTAGES:

- F_{elec} is not a function of the displacement
  $\Rightarrow k_{elec} = 0 \quad (k_{TOT} = k_m)$
  NO ELECTROSTATIC SOFTENING $\left(\cancel{\sqrt{k_{TOT}/m} < \sqrt{k_m/m}}\right)$
  $k_m \gg k_{elec}$ for sure $\rightarrow$ NO PULL-IN
- There is no nonlinearity in the capacitive readout
  $\Rightarrow \nexists$ TRADE OFF SENSITIVITY vs. FSR

DRAWBACKS:

- Given W_0, g and V_{DD}, The # of fingers that one can fit in a given area does not allow to reach The same dC/dx as for PP devices
  - $x_0 \gg g$ $*_1$
  - $C_{0,CF} < C_{0,PP}$

  $\boxed{\left(\frac{dC}{dx}\right)_{CF} < \left(\frac{dC}{dx}\right)_{PP}}$

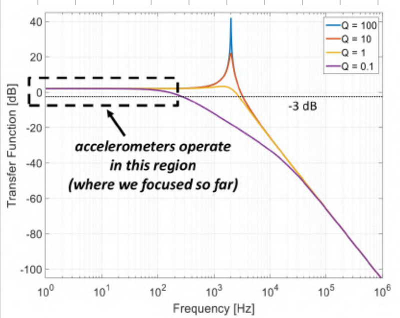
$*_1$ to avoid FRINGE EFFECT

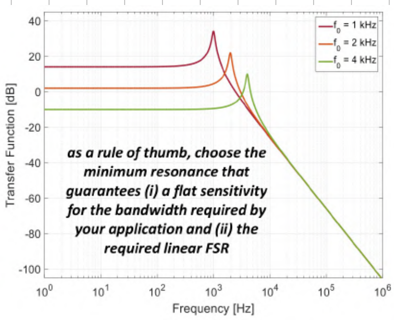
NOTE: to have the same C_0 we have to increase A

ACCELEROMETER BANDWIDTH AND OPERATING REGION { transfer function vs Q

NOTE: The TF btw acceleration and output voltage is exactly The same of That one btw The external force and The displacement found a few classes ago, since:

- The external acceleration is linear w/ The applied force $(F_{ext} = m\,a_{ext})$
- The output voltage is linear w/ x in CF-config and in PP-config, but w/ small displacement approximation





RULE OF THUMB: choose The minimum resonance frequency ω_{0min} That guarantees:

- a flat sensitivity for the BW required by The application
- the required linear FSR

NOTE:

- All other parameters being equal, The Q does not affect The low-frequency response and Thus The sensitivity
- All other parameters being equal, low-frequency response (and Thus the sensitivity) goes w/ $1/f_0^2$

Using The formulas of previous classes, we can write:

$$|T_{VA}(j\omega)| = \left|\frac{X(j\omega)}{F_{ext}(j\omega)}\right| \left|\frac{V(j\omega)}{X(j\omega)}\right| \left|\frac{F_{ext}(j\omega)}{A(j\omega)}\right| = m\,\frac{2V_{DD}}{C_F}\cdot\frac{C_0}{g}\cdot\frac{1/m}{\sqrt{(\omega_0^2-\omega^2)^2+\left(\frac{\omega\omega_0}{Q}\right)^2}}$$

$$\Rightarrow \boxed{|T_{VA}(j\omega)| = \frac{2V_{DD}}{C_F}\cdot\frac{C_0}{g}\cdot\frac{1}{\sqrt{(\omega_0^2-\omega^2)^2+\left(\frac{\omega\omega_0}{Q}\right)^2}}}$$

TRANSFER FUNCTION BTW EXTERNAL ACCELERATION AND OUTPUT VOLTAGE

for low-frequency values $(\omega \ll \omega_0)$: $\quad |T_{VA}| = \frac{\Delta V_{out}}{a_{ext}} = 2\,\frac{V_{DD}}{C_F}\cdot\frac{C_0}{g}\cdot\frac{1}{\omega_0^2}$

How does the Q factor effect the response?

NOTE:

- CHANGING Q ≡ CHANGING THE PACKAGE PRESSURE (since $Q = \frac{\omega_0 m}{b}$)
- As the quality factor Q has no effect on the stationary response, going towards too low Q values ($Q<0.5$) has the only effect of splitting the poles and lowering the -3dB bandwidth
  ⇒ In principle we desire to have $Q \geq 0.5$
- On the other side, when there is an unexpected shock, the larger Q the longer it takes for my suspended mass to stabilize again into the rest position
  ↳ This time is known as: RINGDOWN TIME: $\boxed{\tau = \frac{Q}{\pi f_0}}$

How do we define the accelerometer bandwidth?

- if $Q \leq 0.5$ → just take the -3dB BW (1st pole) ⇒ $Q=0.5$ seems the best choice to maximize the BW (all the other things being equal)
- if $Q \geq 0.5$ → take the minimum btw the -3dB BW (1st pole) and $\frac{1}{2\pi\tau} = \frac{f_0}{2Q}$
  ⇒ $Q = 0.5$ seems the best choice to maximize the BW (all the other things being equal)

⇒ OPTIMUM QUALITY FACTOR $\boxed{Q_{opt} = 0.5}$

Att: This result does not take into account two facts:
1. Q is dependent on b, but also noise depends on b;
2. oscillations at ω_0 can be always filtered out using electronic LPF.

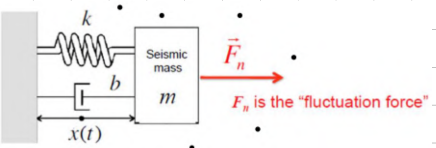
THERMO-MECHANICAL NOISE IN MEMS DEVICES

- Brownian noise origin
- NEAD = Noise Equivalent Acceleration Density
- Optimum Q

NOTE: Noise is a fundamental quantity bcz it determines the minimum change in the quantity that we want to measure that we can appreciate w/ our system

GOAL: we want to design MEMS devices in such a way the different noise contributions which belong to different physical domain are well balanced

We have seen how the response of a MEMS to steps or pulses shows a ring-down behavior, during which its kinetic energy decreases
↳ How do we explain this behavior from a physical pov?



FLUCTUATION-DISSIPATION THEOREM = any dissipative mechanism that results in mechanical damping must be balanced by a FLUCTUATION FORCE to maintain macroscopic energy balance, hence thermal equilibrium
in other words: to prevent the unphysical result that the system goes below the temperature of the surroundings, we include in the motion equation a term which we name FLUCTUATION FORCE F_n.
This is the force through which the energy passes from the MEMS device to the environment
→ This force maintains the energy balance

The gas molecules continuously and randomly hit the suspended mass generating a sort of trembling on the mass. This kind of trembling can be seen as noise that continuously keep the mass not in a perfect stationary state

GOAL: Find an expression of the POWER SPECTRAL DENSITY S_{F_n} of F_n in order to quantify the noise

PROCEDURE TO CALCULATE MECHANICAL / BROWNIAN NOISE:

1. Starting from the spring-mass-damper equation in the Laplace domain
$$m s^2 X(s) + b s X(s) + k X(s) = F_n(s)$$

2. Write it as a function of the velocity: $V(s) = s X(s) \rightarrow m s V(s) + b V(s) + \frac{k}{s} V(s) = F_n(s)$

3. We define the TF btw the velocity and the applied force
$$V(s)\left[m s + b + \frac{k}{s} \right] = F_n(s)$$
$$\rightarrow \boxed{T_{VF}(s) = \frac{V(s)}{F_n(s)} = \frac{1}{m s + b + \frac{k}{s}}}$$

4. We pass to the Fourier domain: $s = j\omega \longrightarrow V(j\omega) = \frac{1}{m j\omega + b + \frac{k}{j\omega}} F_n(j\omega) = \frac{1}{b + j\left(m\omega - \frac{k}{\omega}\right)} F_n(j\omega)$

5. We write the power spectral density of the velocity, $S_{V_n}(\omega)$:
$$S_{V_n}(\omega) = |T_{VF}(j\omega)|^2 S_{F_n}(\omega) = \frac{1}{b^2 + \left(m\omega - \frac{k}{\omega}\right)^2} S_{F_n}(\omega) =$$
$$= \frac{1}{b^2} \frac{1}{1 + \left(\frac{m\omega}{b} - \frac{k}{b\omega}\right)^2} S_{F_n}(\omega)$$

6. We rearrange the found expression: using Q and ω_0
$$\omega_0 = \sqrt{\frac{k}{m}} \qquad Q = \frac{\omega_0 m}{b} = \frac{k}{\omega_0 b}$$

POWER SPECTRAL DENSITY OF THE VELOCITY
$$\boxed{S_{V_n}(\omega) = \frac{1}{b^2} \frac{1}{1 + \left(\frac{\omega}{\omega_0} Q - \frac{\omega_0}{\omega} Q\right)^2} S_{F_n}(\omega) = \frac{1}{b^2} \frac{1}{1 + Q^2\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right)^2} S_{F_n}(\omega)}$$

7. To evaluate the rms velocity noise we integrate the velocity spectrum over the entire frequency range ($df = d\omega / 2\pi$)
$$\overline{V^2} = \frac{1}{2\pi} \int_0^\infty S_{V_n}(\omega)\, d\omega = \frac{1}{2\pi b^2} \int_0^\infty \frac{1}{1 + Q^2\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right)^2} S_{F_n}(\omega)\, d\omega =$$
$$= \frac{\omega_0}{2\pi b^2} \int_0^\infty \frac{1}{1 + Q^2\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right)^2} S_{F_n}(\omega)\, d\frac{\omega}{\omega_0}$$

8. Hp: Assume that the force noise spectrum is white: $S_{F_n}(j\omega) = S_{F_n}$
$$\Rightarrow \overline{V^2} = \frac{\omega_0 S_{F_n}}{2\pi b^2} \int_0^\infty \frac{1}{1 + Q^2\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right)^2} d\frac{\omega}{\omega_0}$$

9. We change the variable term: $\frac{\omega}{\omega_0} = \tilde{\omega}$
$$\rightarrow \int_0^\infty \frac{1}{1 + Q^2\left(\tilde{\omega} - \frac{1}{\tilde{\omega}}\right)^2} d\tilde{\omega} = \frac{1}{Q} \int_0^\infty \frac{Q}{1 + Q^2\left(\tilde{\omega} - \frac{1}{\tilde{\omega}}\right)^2} d\tilde{\omega} = \frac{1}{Q} \frac{\pi}{2}$$

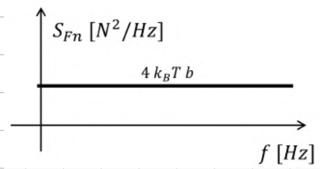
$$\Rightarrow \boxed{\overline{V^2} = \frac{\omega_0 S_{F_n}}{2\pi b^2} \frac{1}{Q} \frac{\pi}{2} = \frac{\omega_0 S_{F_n}}{4 b^2 Q} = \frac{S_{F_n}}{4 m b}}$$ rms VELOCITY NOISE

$$\Rightarrow E_{kin,n} = \frac{1}{2} m \overline{V^2} = \frac{1}{2} m \frac{S_{F_n}}{4 m b}$$ rms NOISE IN TERMS OF KINETIC ENERGY OF 1-DOF SYSTEM

NOTE: we also know that the energy of 1-DOF system is related to the temperature through the Boltzmann constant k_B

$$\Rightarrow E_{kin,n} = \frac{1}{2} m \overline{V^2} = \frac{1}{2} k_B T = \frac{1}{2} m \frac{S_{F_n}}{4 m b} \Rightarrow \boxed{S_{F_n} = 4 k_B T b}$$ POWER SPECTRAL DENSITY OF THE FLUCTUATION FORCE $\left[\frac{N^2}{Hz}\right]$

NOTE:



- This expression is independent of the frequency as we assumed white noise ($S_{Fn}(j\omega) = S_{Fn}$)
- This expression is valid not just for accelerometers, but it's valid for a generic spring-mass-damper system
- You can notice the similarity w/ the expression of the voltage noise in a resistor, but here we have the damping coefficient b instead of the resistance value R.
  In both cases the elements that generate noise are dissipative elements: in the mechanical domain the dissipative element is the damping, while in the electronic domain the dissipative element is the resistor.
- This expression is in terms of noise, so to evaluate the noise in terms of displacement or other quantities we need to pass through the corresponding (squared) transfer function

How can we turn this force contribution into an equivalent acceleration in order to be able to quantify which is the minimum acceleration that we can measure?

$F_n = m\,a_n \;\rightarrow\; S_{Fn} = m^2\,S_{An} \;\Rightarrow$ ACCELERATION NOISE DENSITY $\boxed{S_{An} = \dfrac{S_{Fn}}{m^2} = \dfrac{4k_BT\,b}{m^2}}$ $\left[\dfrac{(m/s)^2}{Hz}\right]$

NOISE EQUIVALENT ACCELERATION DENSITY $\boxed{NEAD = \sqrt{S_{An}} = \sqrt{\dfrac{4k_BT\,b}{m^2}} = \sqrt{\dfrac{4k_BT\,\omega_0}{mQ}}}$ $\left[\dfrac{(m/s)}{\sqrt{Hz}}\right] / \left[\dfrac{\hat{g}}{\sqrt{Hz}}\right]$

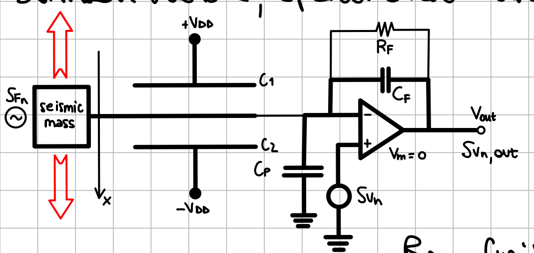
Noise improves:
- w/ low b (or high Q) $\rightarrow$ low pressure, fewer molecules, lower damping
- w/ large m $\rightarrow$ $F_n = m\,a_n$: if a_n is given, if $m\uparrow \Rightarrow F_n\uparrow$

NOTE: we have seen that $Q_{opt} = 0.5$, but now we also see that for low Q factor we have poor noise performance, so the optimum Q factor is not necessary 0.5, but for consumer applications it can be of the order of few units

(RESOLUTION)
$\Rightarrow$ MINIMUM DETECTABLE MECHANICAL ACCELERATION $\boxed{\sqrt{\overline{a^2}} = NEAD \cdot \sqrt{BW}}$ $\left[\dfrac{m}{s^2}\right] / \left[\hat{g}\right]$ (BROWNIAN NOISE)

PROCEDURE TO CALCULATE ELECTRONIC NOISE:

Electronic noise should be also taken into account, typically deriving it from resistance Johnson noise, operational amplifier white and $1/f$ noise.



For $C_P \gg C_0 = \dfrac{\varepsilon_0 A N}{g}$

$$\Rightarrow \boxed{S_{Vn,out} = S_{Vn}\left(1 + \frac{sR_fC_P}{1+sR_fC_f}\right)^2 \approx S_{Vn}\left(1+\frac{C_P}{C_f}\right)^2 \approx S_{Vn}\left(\frac{C_P}{C_f}\right)^2}$$

R_f = finite parasitic resistance of the switch across C_f

$$\sqrt{S_{Vn,out}} \sim \sqrt{S_{Vn}\left(\frac{C_P}{C_f}\right)^2} \Rightarrow \boxed{\sqrt{S_{a,eln}} = \frac{\sqrt{S_{Vn,out}}}{S} = \frac{\sqrt{S_{Vn}}\,(C_P/\cancel{C_f})}{2\,V_{DD}\,\frac{C_0}{\cancel{C_f}}\,\frac{1}{g}\,\frac{1}{\omega_0^2}} = \frac{\sqrt{S_{Vn}}\;C_P}{2\,V_{DD}\,\frac{C_0}{g}\,\frac{1}{\omega_0^2}}}$$

$$\Rightarrow \boxed{\sqrt{\overline{a^2_{eln}}} = \sqrt{S_{a,eln}} \cdot \sqrt{BW}}$$ INPUT-REFERRED ELECTRONIC NOISE

NOTE:
- At low frequencies (< 1-10kHz) noise may be limited by $1/f$ noise issues

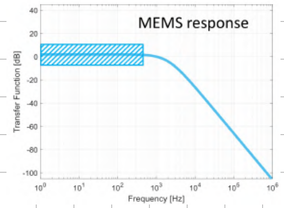
⇒ For this reason the MEMS is often switched (w/ a certain frequency) btw full bias ($\pm V_{DD}$) and no bias, in order to modulate the signal to higher frequencies and so to high-pass filter the 1/f noise contributions. Demodulation is then applied to bring back the signal to the baseband (@ DC)

In the end, which is the optimum Q factor? → it depends on the application and so on what we want to achive:

- OPTIMIZATION OF THE BANDWIDTH

→ $Q_{opt} = 0.5$

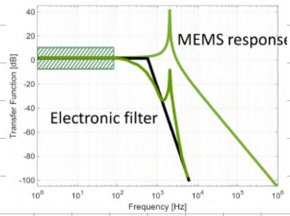
As far as electronic noise still dominates, you can lower Q down to 0.5 w/o affecting the overall system noise



- NOISE OPTIMIZATION

→ requires high Q

This should be pursued as far as the device noise still dominates. This typically occurs for high-performance applications, where power dissipation is raised and electronic noise is low. Electronic LPF is required to filter long ring-downs.



In this case the BW is reduced bcz it is determined by the combining of the electronic filter and the max response. If you increase Q, the response of your MEMS will become the one in green and you will have ringing in the case the MEMS is subject to shocks which somewhat corrupt the response. This is why, in these cases, we need to implement also an electronic filter in order to filter out those ringings

NOTE: we still have the peak but it is widely mitigated by the filtering.

RESULT: TRADE-OFF BANDWIDTH vs NOISE
→ it changes depending on the impact of electronic noise

THE CAPACITIVE MEMS ACCELEROMETER
PART 3: SPRINGS AND TYPICAL ARCHITECTURES

MOTIVATIONS AND GOALS

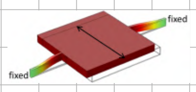
The stiffness k directly or indirectly related to:
- ω_0 (bandwidth)
- pull-in
- sensitivity

} TRADE-OFFS

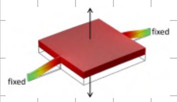
GOAL: study the different springs configurations and find which topologies are more immune to process tolerances.

There are 3 main categories of springs in MEMS:

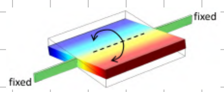
1. SPRINGS FOR IN-PLANE (IP) TRANSLATIONAL MOTION



2. SPRINGS FOR OUT-OF-PLANE (OOP) TRANSLATIONAL MOTION



3. SPRINGS FOR OUT-OF-PLANE (OOP) TORSIONAL (ROTATIONAL) MOTION



NOTE: the same sensing structure can have different degrees of freedom, and accordingly, different resonant modes.

GOAL: to have the stiffness and mass distribution such that the mode of interest falls at the desired frequency, while other modes fall far from this value, in order to not perturb the device operation if occidentally excited.

FLEXURAL SPRINGS

- Young's modulus and axial stiffness
- In-plane stiffness
- Out-of-plane stiffness
- free-end vs guided-end springs

In case of springs design we're able to choose the spring length L_s and the width W_s, while the height h is fixed by the process height (by the epitaxial polySi growth).

The ELASTIC STIFFNESS k_{el} of a spring depends both on its material and its geometry:

YOUNG'S MODULUS E (MODULUS OF ELASTICITY)

The Young's modulus is the ratio btw the stress σ applied orthogonally to a material surface A (force per unit area) and the reversible strain ε shown by the material (relative elongation)

$$\boxed{E = \frac{\sigma}{\varepsilon}} \quad [GPa]$$

$$\Rightarrow E = \frac{\sigma}{\varepsilon} \rightarrow \sigma = E \cdot \varepsilon \rightarrow \frac{F}{A} = E\frac{x}{L}$$

NOTE:
- E defines the material resistance to elastic (non permanent) deformations
- Silicon and Polysilicon have $E \sim 150\text{-}180$ GPa

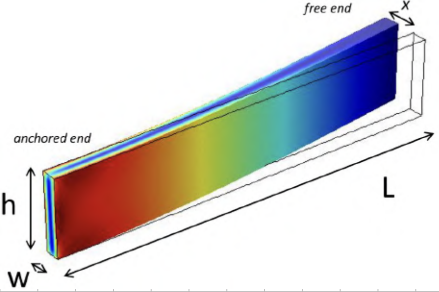
$$\Rightarrow F = \frac{AE}{L} \cdot x \rightarrow k_{el} = \frac{F}{x} = \frac{AE}{L} = \frac{Ehw}{L}$$

$$\left[\frac{N}{m}\right] \quad \boxed{k_{el} = \frac{Ehw}{L}}$$

ELASTIC STIFFNESS
- GEOMETRY $\leftrightarrow$ w, L
- MATERIAL $\leftrightarrow$ E

NOTE: heavy calculations lead to the generation of the so-called STIFFNESS MATRIX indicating the stiffness of a spring in one direction as a function of the applied force direction

The FREE-END BEAM (or IN-PLANE CLAMPED BEAM) is a beam that is anchored from one side to the substrate while can completely displace freely on the other side



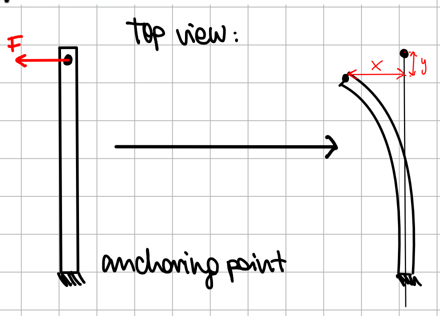
NOTE: As one intuitively can expect, The stiffness decreases w/ The beam length and increases w/ The width and The height

FREE-END BEAM STIFFNESS (IP MOTION)

$$\boxed{k_x = \frac{F_x}{x} = \frac{E w^3 h}{4L^3}}$$

NOTE:
- if $L\uparrow \Rightarrow k_x\downarrow$
- if $w, h\uparrow \Rightarrow k_x\uparrow$
- if $E\uparrow \Rightarrow k_x\uparrow$

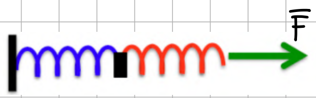
Top view:



I would expect To have a displacement both in The x-and in y-direction

In reality, as The AXEL is constrained by 4 different springs The free end of each beam actually is not free to move up and down in The y-direction, but it is forced to move just along The x-direction.
For This reason we actually talk about: **GUIDED-END BEAM**
NOTE: it is The case of any device suspended by an even number of symmetric springs.

SPRINGS IN SERIES: the end of one spring is rigidly connected to one end of another one



$$x_1 = \frac{F}{k_1} \quad x_2 = \frac{F}{k_2} \quad \rightarrow x = x_1 + x_2 = F\left(\frac{1}{k_1} + \frac{1}{k_2}\right) = \frac{F}{k_x}$$

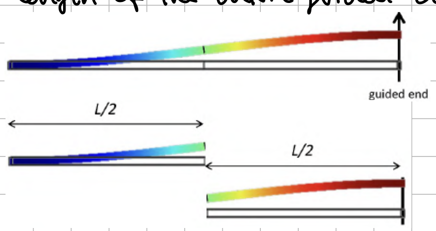
$$\rightarrow \frac{1}{k_x} = \frac{1}{k_1} + \frac{1}{k_2}$$

EQUIVALENT STIFFNESS FOR THE SERIES OF TWO SPRINGS (IP MOTION)

$$\boxed{k_x = \frac{k_1 k_2}{k_1 + k_2} = \frac{1}{2}k = \frac{1}{2}\, 2\frac{E w^3 h}{L^3} = \frac{E w^3 h}{L^3}}$$

Hp: $k_1 = k_2 = k$

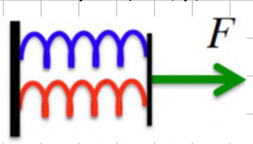
Why did we introduce The concept of springs in series?
If you look at The guided-end spring, if you imagine to cut it exactly in the central point, you can consider it as The series of two identical free-end beams w/ length equal to one half of The original length of The entire guided-end beam.



$$k_1 = k_2 = \frac{E w^3 h}{4(L/2)^3} = 2\frac{E w^3 h}{L^3}$$

$$k_x = \frac{k_1 k_2}{k_1 + k_2} = \frac{1}{2}k = \frac{E w^3 h}{L^3} \longrightarrow$$ **GUIDED-END BEAM STIFFNESS (IP MOTION)** $$\boxed{k_x = \frac{E w^3 h}{L^3}}$$

SPRINGS IN PARALLEL: one end of The springs is attached to the same rigid body (i.e. The substrate) and The other end of both springs is attached to another rigid body (i.e. The suspended mass)



If you apply a force to displace these springs, the force need to be the sum of The two forces required to elongate each spring of The same quantity.

$$F_1 = k_1 x, \quad F_2 = k_2 x, \quad F = F_1 + F_2 = (k_1 + k_2)x = k_x x \quad \Rightarrow k_x = k_1 + k_2$$

NOTE: The overall stiffness of an accelerometer suspended by 4 springs is The parallel of 4 springs, each of which is of The guided-end type $\longrightarrow k_{ix} = \frac{E w^3 h}{L^3}$

$$\Rightarrow K_x = N k_{ix} = 4\frac{E w^3 h}{L^3}$$

NOTE: In AXELS $k_{IP,\,typical} < 10\ \frac{N}{m}$

For the calculation of the stiffness for a force acting along the vertical direction, identical considerations hold, but we should exchange w and h

Att: while w can be made as narrow as 1/2 μm, the height h is fixed by the process and cannot be made as narrow as few μm.
As consequence, we will unavoidably fall towards values of the final elastic stiffness for OOP motion which are very larger w/r/t the values obtainable for the IP motion
→ This configuration is not very suited to design AXELS to sense OOP accelerations, therefore we need to find an alternative approach

$$\boxed{k_z = \frac{F_z}{z} = \frac{E h^3 w}{4 L^3}}$$
FREE-END BEAM STIFFNESS (OOP MOTION)

NOTE: $k_{OOP,\,typical} < 1000\ N/m$

$$\boxed{k_z = \frac{E h^3 w}{L^3}}$$
GUIDED-END BEAM STIFFNESS (OOP MOTION)

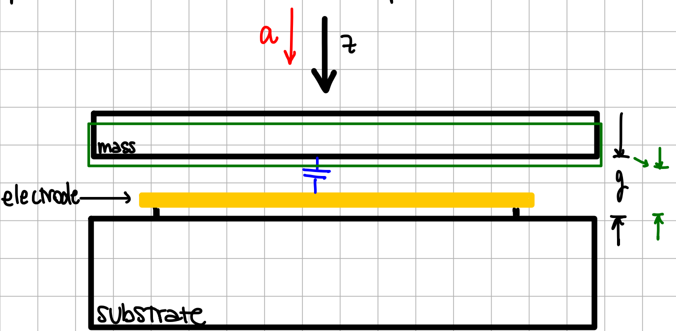
ISSUES w/ THE CONFIGURATIONS SEEN SO FAR:

- Too large k for z-axis translational motion (we need to lower it by about 2 orders of magn.) (too large k_z)
  NOTE: by the way this is good to reject undesired OOP motion in IP sensing devices
- too thin beams for x-axis translational motion

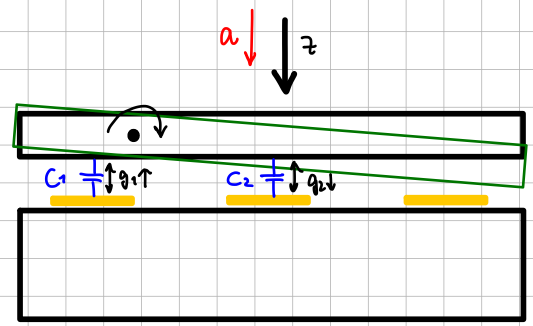
TORSIONAL SPRINGS { Simple configuration and stiffness calculation

SOLUTION: To solve the issue related to the vertical motion (too large k_z), we can use torsional springs to allow OOP rotations

Furthermore, OOP ROTATIONS allow for a DIFFERENTIAL READOUT through a pair of electrodes placed beneath the suspended mass



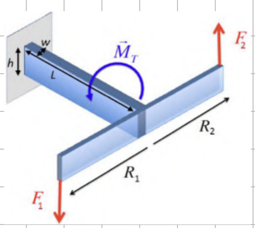
ISSUES:
① too large values for k_z;
② w/ this config. we do not have a differential capacitance variation, we get only a single ended cap. variation



In this case the mass is not free to translate along the z-axis, but it is free to rotate around a rotation center which is not coincident w/ the center of gravity. Due to the rotation: $g_1 \uparrow \Rightarrow C_1 \downarrow$
$g_2 \downarrow \Rightarrow C_2 \uparrow$

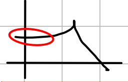
In this case we effectively get a differential capacitive sensing and so: we minimize the effects of F_{elec}, we improve the linearity in the response and we remove all undesired cm disturbances.

An OOP rotation is caused by a TORQUE



TOTAL TORQUE: $\boxed{M_T = M_1 + M_2 = R_1 F_1 + R_2 F_2}$

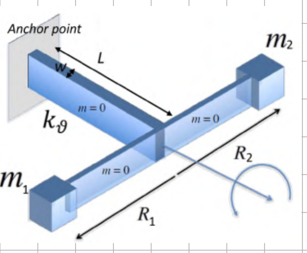
TORSIONAL MOTION EQUATION: $\boxed{I\ddot{\theta} + b_\theta \dot{\theta} + k_\theta \theta = M_T}$

Hp: we assume to be in quasi-stationary conditions 

$\Rightarrow \cancel{I\ddot{\theta}} + \cancel{b_\theta \dot{\theta}} + k_\theta \theta = M_T \Rightarrow \boxed{k_\theta \theta = M_T} \longrightarrow \boxed{\theta = \frac{M_T}{k_\theta}}$ ROTATION ANGLE , $\boxed{k_\theta = \frac{M_T}{\theta}}$ TORSIONAL STIFFNESS

TORSIONAL STIFFNESS: $\boxed{k_\theta = \underbrace{G}_{\text{material}} \underbrace{\frac{hw^3}{3L}}_{\text{geometry}}}$ where $\boxed{G|_{PolySi} \sim 65\,GPa}$ SHEAR MODULUS (The twin of E used) for torsional motion

Hp: for the computation of I, the torsional bar itself can be neglected compared to the contributions from other parts of the MEMS $\rightarrow m = \varnothing$

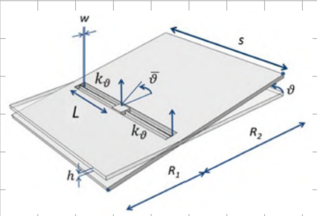


The MOMENT OF INERTIA measures the capability of a body to maintain its status of rotational motion

NOTE: The farther the masses are from the rotation center, the higher is I

MOMENT OF INERTIA: $\boxed{I = \int_m r\,dm = \int_0^R r^2 s(r)\,h\,\rho\,dr = \frac{R^3 s \rho h}{3} = \frac{R^2 m}{3}}$

$$\boxed{I = \frac{R^3 s \rho h}{3} = \frac{R^2 m}{3}}$$

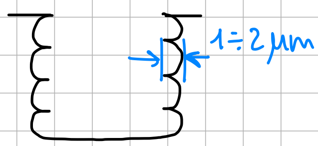


$$\rightarrow I_1 = \frac{R_1^2 m_1}{3} \quad , \quad I_2 = \frac{R_2^2 m_2}{3} \longrightarrow I_{TOT} = I_1 + I_2$$

COMMON SPRINGS ARCHITECTURES
- Process nonuniformities and folded springs
- Example of single-axis, 2-axis and 3-axis accelerometers

PROBLEM TO SOLVE: too narrow width of IP springs for translation

deep reactive ion etching:



The capability of this process typically gives gaps which are in the order of 1÷2 μm, but also the typical maximum width of the structure we are going to design is around 1÷2 μm

example: $w_s = 1.5\,\mu m$ Hp: max variability in the width definition: $dw = \pm 0.15\,\mu m$

$$k = E\frac{hw^3}{L^3} \rightarrow dk = E\frac{h}{L^3} 3w^2\,dw \longrightarrow \frac{dk}{k} = 3\frac{dw}{w}$$

$$f_0 = \frac{1}{2\pi}\sqrt{\frac{k}{m}} \longrightarrow df_0 = \frac{1}{2\pi\sqrt{m}} \frac{-1}{2\sqrt{k}} dk \longrightarrow \left|\frac{df_0}{f_0}\right| = \frac{1}{2}\frac{dk}{k} = \frac{3}{2}\frac{dw}{w}$$

$$S = \frac{\Delta V_{out}}{a_{ext}} = \alpha \frac{1}{f_0^2} \longrightarrow dS = -2\alpha \frac{1}{f_0^3} df_0 \longrightarrow \left|\frac{dS}{S}\right| = 2\frac{df_0}{f_0} = 3\frac{dw}{w}$$

↳ a ±10% uncertainty on the width ($dw/w = 0.1$) turns directly into a ±30% uncertainty on k and on sensitivity. (and a ±15% on bandwidth uncertainty)

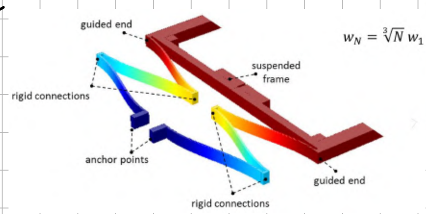
ATT: if $w\uparrow \Rightarrow \frac{dw}{w}\downarrow$ $\left(\frac{dk}{k}\downarrow \text{ and } \frac{dS}{S}\downarrow\right)$ but $k\uparrow$ and $S\downarrow$ → NOT A CONVENIENT APPROACH

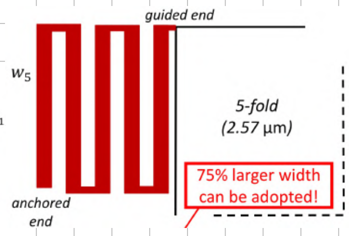
GOAL: To obtain a low stiffness without : – increasing L (so the area)
– narrowing too much w

SOLUTION: ARRANGE IN A SERIES MORE SPRINGS OF RELATIVELY LARGE STIFFNESS : **SPRING FOLDING**

→ instead of a single-fold, narrow spring, we put in series more fold w/ a larger w

→ $k_{1\text{-fold}} = E \frac{h w_1^3}{L^3}$, $k_{N\text{-fold}} = \frac{1}{N} E \frac{h w_N^3}{L^3} \longrightarrow \boxed{w_N = w_1 \sqrt[3]{N}}$



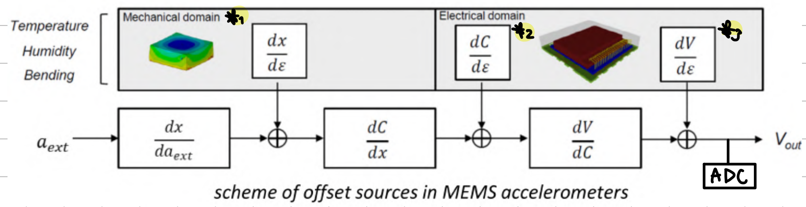


CONCLUSIONS

- fixed process height → TORSIONAL STIFFNESS FOR Z-AXIS ACCELEROMETER $\quad k_\theta = G \frac{h w^3}{3L} N_{springs}$
- process spread in DRIE → STIFFNESS FOR FOLDED SPRINGS IN X- AND Y-AXIS ACCELEROMETERS $\quad k_x = \frac{E w^3 h}{L^3} \frac{N_{springs}}{N_{folds}}$

THE CAPACITIVE MEMS ACCELEROMETER
PART 4: OFFSET AND OTHER ELECTRONIC READOUT

MOTIVATIONS AND GOALS



scheme of offset sources in MEMS accelerometers

*1 deformation of the substrate: the mass is not perfectly centered (undesired fixed displacement);
*2 The mass can be perfectly centered, but due to nonuniformities in etching, the gap btw stator 1 and the rotor (g_1) can differ from the gap btw stator 2 and the rotor, so even if the device is centered, there is a differential capacitive output which doesn't correspond to any input acceleration;
*3 Assuming the device perfectly centered and $g_1 = g_2$, we can have offset due to the electronic stages.

2 topics:

1. OFFSET SOURCES (in mechanical and electronics domain) and THEIR TEMPERATURE DRIFTS
   ↳ critical in contexts where high offset stability is needed
2. MINIATURIZATION → critical bcs it may cause nonlinearities and pull-in issue

MECHANICAL OFFSET

- Offset sources
- Effects on input-referred acceleration
- Effects on sensitivity
- Trade-offs w/ FSR

DEF. The OFFSET (or ZGO, ZERO-g-OUTPUT) is defined as the output value that you measure on an accelerometer axis when no accelerations occur in that direction

MECHANICAL OFFSET: The mass can be offset wrt the nominal centered rest position, due to process tolerance:
- UNIFORMITY OF GAPS: btw the two sets of differential parallel plates;
- RESIDUAL MECHANICAL STRESSES: induced by wafer bending (piegatura del wafer)

BIG ISSUE: The offset can vary w/ temperature therefore a simple initial calibration does not solve the offset drift problem.

data-sheet: ZGO temperature drift: $TCO = \pm 1.0\ \text{m}\hat{g}/K$ } 125 m$\hat{g}$ over the whole T range
Operating temperature: $T_A = -40 \div +85°C$

$$\rightarrow C_1 = \frac{\varepsilon_0 A N}{g - x_{os}}, \quad C_2 = \frac{\varepsilon_0 A N}{g + x_{os}}$$

$C_0 = 200\,fF$, $g = 2\,\mu m$, $m = 5\,nkg$, $k = 3\ N/m$
$f_0 = 3.8\ kHz$, $x_{os} = 10\,nm$

$$\frac{x_{os}}{a_{os}} = \frac{1}{\omega_0^2} \longrightarrow a_{os} = x_{os}\,\omega_0^2 = 10\,nm\,(2\pi\ 3.8\ 10^3\ Hz)^2 = 5.7\,\frac{m}{s^2} = 582\ m\hat{g}$$

which is about 600 times the 1m$\hat{g}$ resolution and about one 14^{th} of the FSR:

$$a_{FSR} = x_{FSR}\,\omega_0^2 = 140\,nm\,(2\pi\ 3.8\ 10^3\ Hz)^2 \simeq 78\,\frac{m}{s^2} = 8\,\hat{g}$$

Att: As mechanical offset can be much larger than 10nm, typical native values range w/in $\pm 10\,\hat{g}$, even larger than the FSR

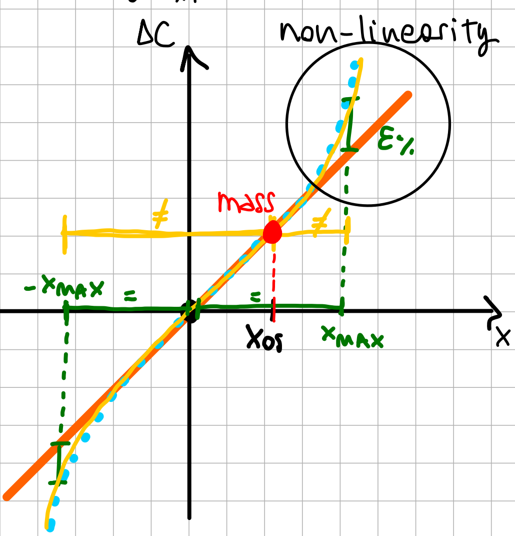
$$S = 2\,\frac{dx}{da_{ext}} \cdot \frac{dC}{dx}\,\frac{dV}{dC}, \quad x_{MAX} = FSR\,\frac{dx}{da_{ext}} \longrightarrow \boxed{FSR = \frac{x_{MAX}}{dx/da_{ext}}}$$

$$\boxed{ZGO(T) = \frac{x_{os}(T)}{dx/da_{ext}}}$$

} TRADE-OFF: FSR vs ZGO drift * (due to the transaction factor $dx/da_{ext} = 1/\omega_0^2$)

* NOTE:
- if you want to extend the FSR you need to lower the transaction factor dx/da_{ext}
- if you want to reduce the ZGO you need to increase the transaction factor dx/da_{ext}

NOTE: $\frac{dx}{da_{ext}} = \frac{1}{\omega_0^2}$ → if we lower ω_0, we get a better ZGO, but also a lower FSR



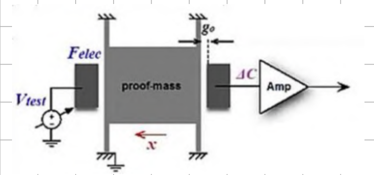
The mass is no more centered
↓
The FSR will not be anymore symmetric btw the two signs of acceleration and obviously, as we need to guarantee linearity whatever the sign of the acceleration, our FSR has to be limited to a much smaller value bc+ due to mechanical offset we would incur more easily into non-linearity when moving rightwards

CAPACITANCE VARIATION IN PRESENCE OF A MECHANICAL OFFSET:

$$\Delta C_{diff} = C_0\left(\frac{-1}{1+x/(g-x_{os})} + \frac{1}{1-x/(g+x_{os})}\right) \approx 2C_0\frac{x}{g}\left[\frac{1}{1-\left(\frac{x-x_{os}}{g}\right)^2}\right]$$

There are 2 possible ways to compensate offset:

① CALIBRATION OF OFFSET AND LINEARITY ERRORS AT WAFER LEVEL



- sweep the wafer into a temperature chamber and for every temperature value:
  - measure the device offset by measuring the cap. on the two sides;
  - measure the device sensitivity:
    since it is not possible to apply accelerations to the whole wafer w/ the probes landing on it, one stator is used as a self-test electronic actuator that can apply a known force which, due to the other stator that works as a sensor, is converted into an equivalent applied acceleration.
    → in this way the nonlinearity can be calibrated!

ADVANTAGES:
- fast
- low cost

DISADVANTAGES:
- not accurate

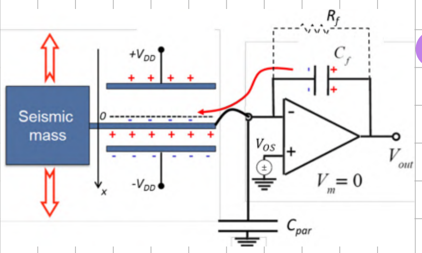
② IN-OPERATION COMPENSATION

use values derived in the calibration phase to compensate offset and sensitivity as a function of input and temperature measured by an auxiliary T sensor on the ASIC

ADVANTAGES:
- very accurate

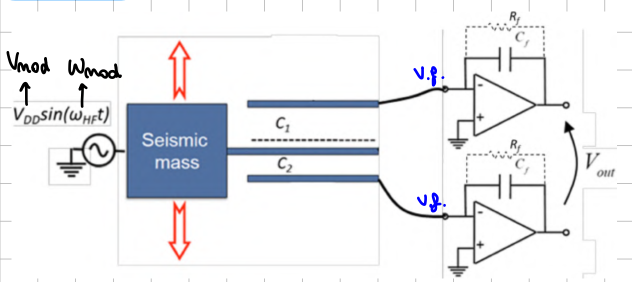
DISADVANTAGES:
- slow
- high cost

ELECTRONIC OFFSET:
- Modulation to bypass amplifier offsets
- Drift of the feedback capacitance



ISSUE: W/ this topology we cannot readout DC accelerations like gravity
→ bc+ at low frequency (@DC) the feedback is dominated by R_f
→ the low freq pole introduced to avoid the saturation of the CA stops the integration of DC signals (of signals whose freq is lower than f_{pole}).
Furthermore, an additional issue of this topology is that the offset of the opamp will be unavoidably seen at the output

SOLUTION: to readout also DC accelerations we can use a high-frequency modulation of the suspended mass, w/ each of the sensing stators kept to the virtual ground of the CA



The current flowing in each capacitor is:

$$i_C = \frac{dQ}{dt} = \frac{d(CV)}{dt} = C\frac{dV}{dt} + V\frac{dC}{dt}$$

where $C\frac{dV}{dt} \neq \emptyset$ and $V = V_{DD}\sin(\omega_{HF}t)$

we begin by assuming a generic sinusoidal capacitance variation at w_a:

$C = C_0 + C_a \cos(w_a t)$ $\qquad$ w_a = frequency of the capacitance variation = = frequency of the acceleration

$\rightarrow i_C = C\frac{dV}{dt} + V\frac{dC}{dt} = [C_0 + C_a\cos(w_a t)]\, w_{HF} V_{DD} \cos(w_{HF} t) - C_a w_a V_{DD} \sin(w_a t)\sin(w_{HF} t)$

we thus have two contributions both modulated around w_{HF}

Their amplitude ratio is roughly: $\frac{C\, dV/dt}{V\, dC/dt} \approx \frac{w_{HF}}{w_a}$

Considering that accelerometers typically measure signals up to a max acceleration frequency of few 100 Hz, using a $f_{HF} = 100\,kHz$ the $C\,dV/dt$ is 1k times larger than $V\,dC/dt$ and so it will dominate in the sum

$\rightarrow i_C \sim C\frac{dV}{dt}$

The calculation leads to a sensitivity similar to what we have seen so far:

$$\frac{\Delta V_{out}}{a_{ext}} = 2\frac{V_{DD}}{C_f}\frac{C_0}{g}\frac{1}{w_0^2}\sin(w_{HF} t)$$

offset given by opAmp bias current

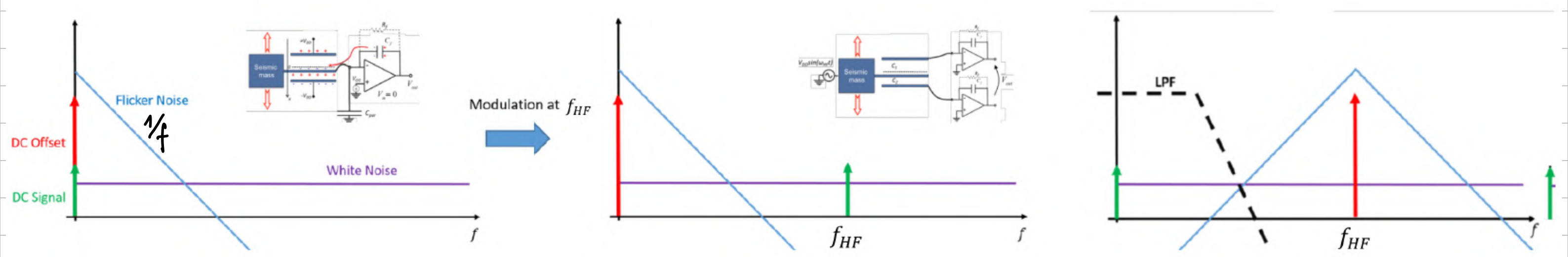
$$\Delta V_{out} = 2\frac{V_{DD}}{C_f}\frac{C_0}{g}\frac{1}{w_0^2}\sin(w_{HF} t)\, a_{ext} + V_{os}(T)$$

NOTE: since the temperature changes are very slow we can consider it as a DC component

NOTE: it is also important to note that the modulation of the rotor voltage is useful to move the signal away from electronics flicker noise (neglected in the former calculation) and from the DC offset

The flicker noise and the offset voltage are introduced by the opAmp after the modulation and so they remain at low frequencies

$\rightarrow$ In other words, we are modulating the rotor voltage that modulates the current but does not change the transfer function of the electronic noise to the output



it's very difficult to discriminate b/w the DC signal and the offset bc they're essentially at the same frequency

After modulation, offset remains at DC while the contribution related to the signal is modulated around a tone at the modulation frequency

*

* After DEMODULATION (multiplying the output signal by a tone w/ same frequency and phase) and LOW-PASS FILTERING, the signal is moved back to baseband, the flicker noise and the offset are moved at high frequency and then cut-off.

$$\Delta V_{demod} = \left[2\frac{V_{DD}}{C_f}\frac{C_0}{g}\frac{1}{w_0^2}\sin(w_{HF} t)\, a_{ext} + V_{os}(T)\right]\sin(w_{HF} t) =$$

$$= \frac{1}{2}\left[2\frac{V_{DD}}{C_f}\frac{C_0}{g}\frac{1}{w_0^2}a_{ext} + \cancel{2\frac{V_{DD}}{C_f}\frac{C_0}{g}\frac{1}{w_0^2}a_{ext}\sin(2w_{HF} t)}\right] + \cancel{V_{os}(T)\sin(w_{HF} t)}$$

LPF

$$\Rightarrow \boxed{\frac{\Delta V_{demod}}{a_{ext}} = 2\,\frac{V_{DD}}{C_f}\,\frac{C_0}{g}\,\frac{1}{\omega_0^2}}$$

→ Hp: assuming that the LPF has a gain of 2

Are we completely free from effects of drifts in the IC?
NO, there still is one contribution which is not related to the amplifier and may cause some drifts.

Hp:
- assume to have a mechanical offset a_{os} that ideally (BRUTE APPROX.) is not drifting w/ T
- amplifier offset and its drifts are bypassed by demodulation

What if the passive components and in particular C_f are subject to drifts in temperature?
Even if we assume that the two capacitances change exactly in the same way and a_{os} is constant as a function of T, if T changes also C_f changes and so also our output changes

$$\Rightarrow \boxed{\Delta V_{demod} = 2\,\frac{V_{DD}}{C_f(T)}\,\frac{C_0}{g}\,\frac{1}{\omega_0^2}\,a_{os}}$$

Att: if the two capacitances drift w/ different coefficients, the situation becomes even more critical

ALTERNATIVE READOUT TOPOLOGIES FOR NEXT-GEN AXELS
- Force feedback
- Charge control

VOLTAGE-CONTROLLED READOUT: applying a known voltage to a MEMS capacitor, to readout the generated charge

MINIATURIZATION = LOWER AREA = LOWER MASS → since $\omega_0 = \sqrt{\frac{k}{m}}$ a lower k is needed to keep the same resonance frequency

$$\frac{\Delta V_{demod}}{a_{ext}} = 2\,\frac{V_{DD}}{C_f}\,\frac{C_0}{g}\,\frac{1}{\omega_0^2}$$

$$V_{DD,PI} = \sqrt{\frac{g^3 k}{2\varepsilon_0 A N}}$$

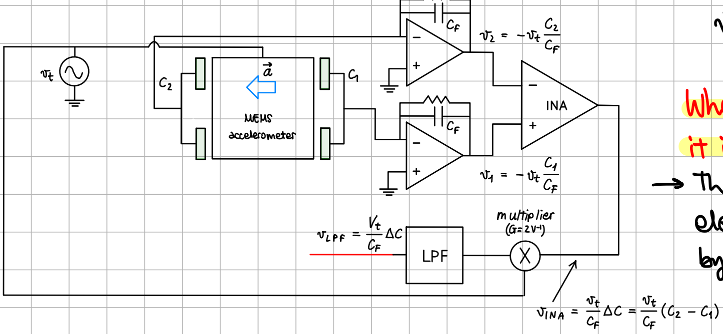
- if k is left as it is, ω_0 increases and the sensitivity decreases
- I cannot decrease V_{DD} otherwise, again, I'm lowering the sensitivity
- if $g\downarrow \Rightarrow \frac{\Delta V_{demod}}{a_{ext}}\uparrow$ but $V_{DD,PI}\downarrow$ and so it would lead to pull-in instability and to heavier nonlinear effects

CONCLUSION: we have to change readout physics or working principle

1 IDEA 1: **FORCE-FEEDBACK WORKING PRINCIPLE**

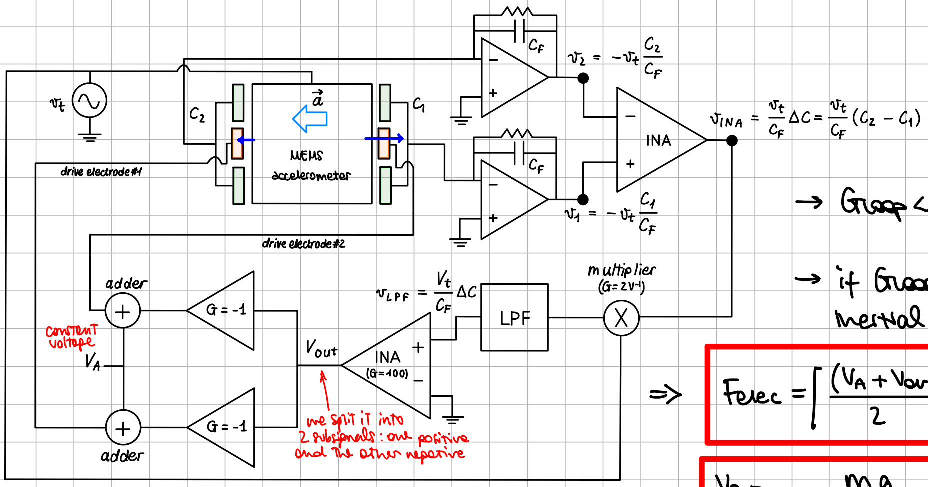
instead of having a device that displaces and we simply readout the capacitance variation, let's use a device that can displace, but we don't simply readout the capacitance variation, we also react in such a way to keep the device in the central position

→ so the force that we need to apply to the device in order to maintain it in the central position is in the end our output quantity.



$v_t = V_t \sin(\omega_{HF} t)$

What if we exploit this voltage to react on the mass motion and keep it in the central position?
→ The LPF output is duplicated w/ opposite sign and sent to the new electrodes which apply a force that reacts on the motion induced by a_{ext}.



The electrostatic force applied by the right electrode is larger than that one applied by the left electrode

→ $G_{loop} < 0$: it reacts to the mass motion, tending to keep it in the central position

→ if G_{loop} is large enough, F_{elec} will balance the inertial action (F_{ext}):

$$\Rightarrow \boxed{F_{elec} = \left[\frac{(V_A + V_{out})^2}{2} - \frac{(V_A - V_{out})^2}{2}\right]\frac{C_{od}}{g} = \frac{2 V_A V_{out} C_{od}}{x_0} = m\, a_{ext}}$$

$$\Rightarrow \boxed{\frac{V_{out}}{a_{ext}} = \frac{m g}{2 V_A C_{od}} = \frac{k g}{\omega_0^2 \, 2 V_A C_{od}}}$$

ADVANTAGE: as the MEMS is always close to the central position: LINEARITY AND FSR ARE EXTENDED!

DRAWBACK: additional circuit blocks = LARGER POWER CONSUMPTION!

② IDEA 2 : **CHARGE - CONTROLLED READOUT** Why don't we apply a known charge amount Q_1 and readout the corresponding voltage?

→ readout quantity

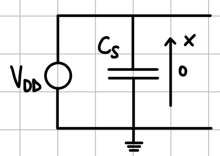
$$\boxed{\Delta V = \frac{Q_1}{\Delta C}} \longrightarrow \text{controlled quantity}$$

In this case we don't keep the rotor at virtual ground anymore : we bias the stators w/ a certain voltage and we control the quantity of charge that we inject in the stators

→ leaving the rotor floating, its voltage value will depend on the charge deposited on the stators

single-ended MEMS capacitor.

VOLTAGE - CONTROLLED READOUT

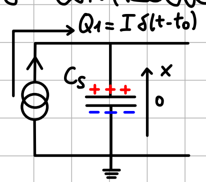


$Q_1 = V_{DD} C_S$

$$\boxed{F_{elec} = \frac{V_{DD}^2}{2}\frac{\varepsilon_0 A}{(g-x)^2}}$$

position dependent → This is the source of our problems : the fact that the electrostatic force is not linear w/ the displacement generate, even in a differential configuration, residual electrostatic force that in the end yields to pull-in instability

CHARGE - CONTROLLED READOUT



- $t < t_0$: we have discharged C_S
- $t = t_0$: we inject a charge packet corresponding to a current pulse w/ an area: $I \cdot \delta(t - t_0)$

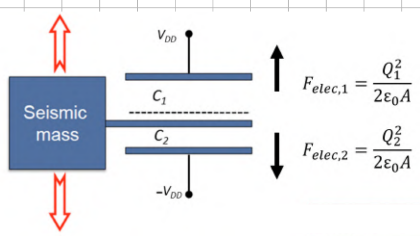
$V_{out} = \frac{Q_1}{C_S}$ where $C_S = \frac{\varepsilon_0 A}{g - x}$

$$\boxed{F_{elec} = \frac{V_{out}^2}{2}\frac{\varepsilon_0 A}{(g-x)^2} = \frac{Q_1^2}{2 C_S^2}\frac{\varepsilon_0 A}{(g-x)^2} = \underbrace{\frac{Q_1}{2}\frac{1}{\varepsilon_0 A}}_{*}}$$

* position independent : in the charge-controlled readout there is no longer any dependence of the electrostatic force on the position

controlling the charge instead of controlling the voltage gives you the capability of controlling the electrostatic force: also from a physics point of view the ultimate quantity that controls the force is the charge.

if we use a DIFFERENTIAL CHARGE-CONTROLLED SYSTEM, due to charge neutrality on the mass, we always have $Q_1 = Q_2$



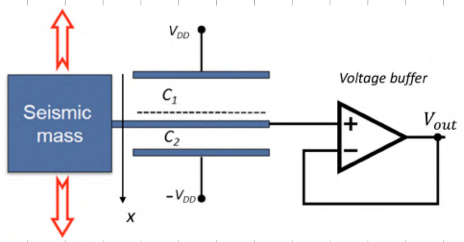
$$\boxed{\text{if } Q_1 = Q_2 \rightarrow F_{elec,TOT} = \varnothing \Rightarrow \text{NO PULL-IN PHENOMENA}}$$

NOTE: also C_F capacitors led to this advantage, but w/ a sensitivity decrease

How do we compute the voltage on the rotor that, in the end, is our readout quantity?

Let's begin imposing charge neutrality

$$Q_1 + Q_2 = 0$$



$$-(V_{DD} - V_m)C_1 + (V_m + V_{DD})C_2 = 0$$

$$-\frac{(V_{DD} - V_m)}{g + x} + \frac{(V_m + V_{DD})}{g - x} = 0$$

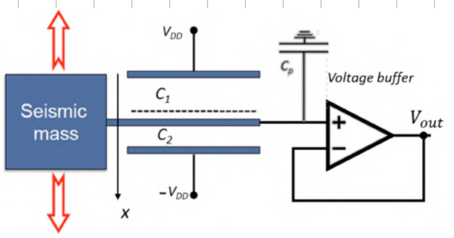
$$\frac{V_m}{g+x} + \frac{V_m}{g-x} = \frac{V_{DD}}{g+x} - \frac{V_{DD}}{g-x} \rightarrow V_m\left(\frac{2g}{g^2 - x^2}\right) = V_{DD}\left(\frac{-2x}{g^2 - x^2}\right)$$

$$\rightarrow \boxed{V_{OUT} = V_m = -V_{DD}\frac{x}{g}}$$

→ linear w/ no approximations (no small displacement assumption)

NOTE: to satisfy charge neutrality conditions, V_m changes w/ position x

ATT: up to now we have neglected the presence of parasitic capacitances (which are much larger than the MEMS capacitance itself, at least for consumer applications)



$$Q_1 + Q_2 + Q_P = 0$$

$$\rightarrow \boxed{V_{OUT} = V_m = -V_{DD}\frac{x}{g}\left(\frac{1}{1 + \frac{C_P}{C_1 + C_2}}\right) = -V_{DD}\frac{x}{g}\left(\frac{1}{1 + \frac{C_P(g^2 - x^2)}{2\varepsilon_0 A g}}\right)}$$

ATT: The gain is dependent on the parasitic
→ The sensitivity is nonlinear again!!

Through a feedback mechanism the circuit action is such that the voltage V_m is not held at a constant value: it is adjusted so that the voltage difference $(V_{DD} - V_m)$ is diminished across the capacitance whose value increased, and the voltage difference $(V_m - V_{DD})$ is increased across the capacitance whose value is diminished.
This keeps the charge on C_1 and C_2 equal and opposite.

The quantity which ultimately generates the feedback is the rotor charge

The result is a minimization of the overall electrostatic force on the seismic mass