MAGNETOMETERS

MEMS MAGNETOMETERS
PART 1: WORKING PRINCIPLE AND RESONANT OPERATION

MOTIVATIONS & GOALS

Sometimes, magnetometers are referred to as inertial sensors even if they absolutely do not measure an inertial force, bc they measure a true force, not an apparent one, which is the Lorentz force.

Magnetometers are generally used in the so called "9-axis" IMUs (inertial measurement units), so coupled to a 3-axis gyro and a 3-axis axel.

They are used in such navigation systems based on inertial sensors to give an information about the initial orientation w/r+ the Earth surface.

They are also used as:
- current sensors
- Electronic compass
- Vehicle detection
- Automotive features (steering angle measurement)

$B_{earth,min} < 23\,\mu T$ $\quad B_{earth,max} > 66\,\mu T$

LORENTZ FORCE MAGNETOMETERS

The Lorentz force principle
Basic architecture
Sensitivity

FORCE ON A CHARGE MOVING IN AN ELECTROMAGNETIC FIELD:

$$\boxed{\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})}$$

This force is made of two components:

- $\boxed{q\vec{E} = \vec{F}_{elec}}$: electrostatic force on a charge inside an electric field
  ↳ it is responsible of current density $\vec{J}$ flowing through a material of conductivity σ

$$\boxed{\vec{J} = \sigma\vec{E} = q\,n\,\mu_n\,\vec{E} = n\,\mu_n\,\vec{F}_{elec}}$$ GENERALIZED OHM'S LAW

↳ $\boxed{\vec{I} = \vec{J} \cdot A}$

- $\boxed{q(\vec{v} \times \vec{B}) = \vec{F}_{LOR}}$ LORENTZ FORCE: it occurs in a direction orthogonal to the plane including the charged particle velocity and the magnetic field vector

We can now consider a conductive wire biased at its ends w/ different voltage levels ($\Delta V \neq 0$) inside which we have thus electrons flowing in the direction of the electric field

In presence of a magnetic field w/ a component orthogonal to the wire length so to the velocity of the electrons we can calculate the Lorentz force on the wire which is formed by the individual Lorentz forces acting on the individual N_{el} charges

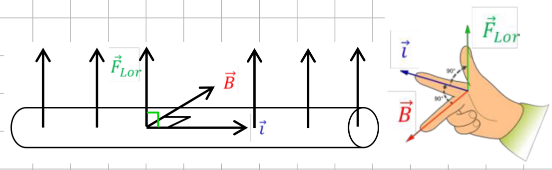
$\vec{F}_{LOR} = N_{el}\, q\, \vec{v} \times \vec{B}$ where $\boxed{N_{el} = nV = nAL}$ NUMBER OF ELECTRONS

$n =$ CARRIERS DENSITY $[1/m^3]$

$\boxed{\vec{v} = \mu_n \vec{E}}$ ELECTRON'S VELOCITY and $\mu_n =$ ELECTRICAL MOBILITY $\left[\frac{m/s}{V}\right]$

$$\rightarrow F_{Lor} = \frac{N_{el}}{AL} AL\, q\, \vec{v} \times \vec{B} = n\, AL\, q\, \vec{v} \times \vec{B} = AL\, q\, n\, \mu_n \vec{E} \times \vec{B} = LA\sigma \vec{E} \times \vec{B} = LA \vec{J} \times \vec{B} = L \vec{i} \times \vec{B}$$

$$\Rightarrow \boxed{\vec{F}_{Lor} = L \vec{i} \times \vec{B}} \rightarrow \boxed{F_{Lor} = BiL}$$



What is the typical intensity of this force that we can reach within reasonable dimensions of MEMS device?

AXEL: $\begin{cases} FSR_{Typical} \sim 20g \\ m_{Typical} = 5nkg \end{cases} \rightarrow F_{Typical} = 1\mu N$

GYRO: $\begin{cases} FSR_{Typical} \sim 2000\,dps \\ V_{typical} \sim 5\mu m\, 2\pi \cdot 20kHz \\ m_{Typical} \sim 5nkg \end{cases} \rightarrow F_{Cor,\,typical} = 200nN$

MAG: $\begin{cases} FSR_{Typical} \sim 5mT \\ L = 1000\mu m \\ i = 0.2mA \end{cases} \rightarrow F_{Lor,\,Typical} = 1nN$

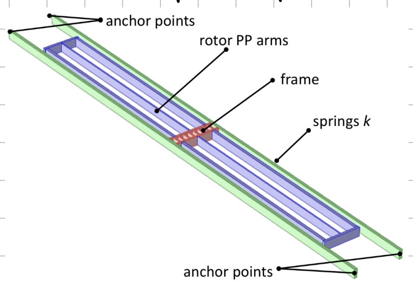
almost one order of magnitude

two orders of magnitude

NOTE:
- As first idea we will study operations at resonance bcz apparently only operations at resonance can give a reasonable amplification to such a tiny force
- We need to design a beam which is anchored at its ends and through which will flow a current and then the Lorentz force that arises will deform the beam causing a capacitance variation that we need to readout.

A Lorentz-force based MEMS magnetometer in its simplest form is formed by:

- CURRENT-CARRYING SPRINGS of length L
- FRAME
- ROTOR ARMS FOR CAPACITIVE SENSING



NOTE (IMP!!): The frame must be as small as possible bcz we do not need to sense inertial force and we thus need to minimize the mass

Att: The resonance frequency must be set out of the audio and vibration bandwidth (>20kHz), like in gyroscopes.

CAPACITIVE READOUT:

Hp: we first assume to operate the device w/ a current at resonance

Why? Bcz, as we have seen, the Lorentz force is a very tiny force and we need to fully amplify it by the quality factor.

$$\boxed{i(t) = i_0 \sin(\omega_0 t)}$$

As consequence also the Lorentz force will be modulated at resonance:

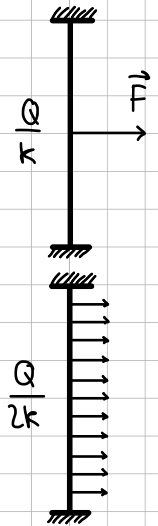
$$\boxed{F_L(t) = BL\, i_0 \sin(\omega_0 t)}$$

REM: $|T_{xF}(j\omega_0)| = \dfrac{Q}{k}$

The displacement x at resonance is thus amplified by the quality factor:

$$\boxed{x = |T_{xF}(j\omega_0)| \cdot F_L = \frac{Q}{2k} B i L = \frac{Q}{2k} BL\, i_0 \sin(\omega_0 t)}$$

NOTE (IMP!!): The factor 2 is due to the fact that the Lorentz force acting on the spring is not concentrated in its central point but it is distributed all along its length.



When we estimated the stiffness values we assumed that the force was applied at the center of the beam.

In our situation, instead, the force is not acting concentrated in the centre of the beam, but it is distributed all along the beam, bc it essentially depends on the electrons motion which we can assume to be uniformly distributed along the wire.

Att: Obviously a force applied at the central point is more effective than a distributed force

→ It's possible to demonstrate that the equivalent stiffness that we have to take into account in this case is $2k$ instead of k

The displacement is readout through a differential capacitive variation ΔC:

$$\boxed{\Delta C = 2C_0 \frac{x}{g} = 2\frac{\varepsilon_0 A N}{g^2} x = \cancel{2}\frac{\varepsilon_0 A N}{g^2}\frac{Q}{\cancel{2}k} B i L = \frac{\varepsilon_0 A N}{g^2}\frac{Q}{k} B i L}$$

CAPACITANCE VARIATION PER UNIT MAGNETIC FIELD

$$\boxed{\frac{\Delta C}{B} = \frac{\varepsilon_0 A N}{g^2}\frac{1}{\cancel{k}}\frac{\cancel{k}}{\omega_0 b} i L = \frac{\varepsilon_0 i L}{\omega_0 g^2}\frac{AN}{b}}$$

As we will see, in PP cells the dominant contribution (squeezed-film damping) is proportional to PP length and number.

$$\boxed{b = 2AN\, b_{area}}$$

$$\rightarrow \boxed{\frac{\Delta C}{B} = \frac{\varepsilon_0 i L}{2\omega_0 g^2 b_{area}}}$$

MECHANICAL SENSITIVITY → NOTE: it is independent on the number of PPs

Now, we want to design a circuit to readout this capacitance variation ΔC.

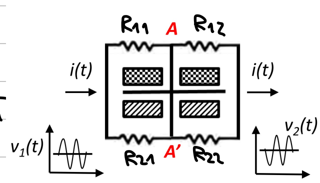
In gyros and axels what we were doing was to apply a voltage at the rotor.

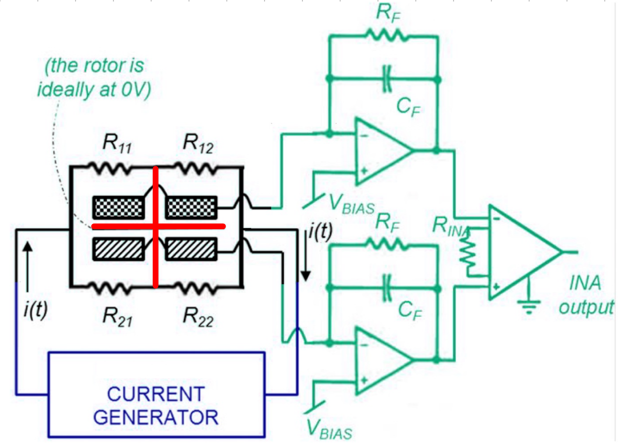
In this case we are not free to do so bc as it is flowing some current inside the rotor its potential will be obviously dependent on the electrical conditions set by the flowing of the current

In other words, the frame cannot be arbitrarily biased as its voltage is determined by the current flowing in the springs.

$\boxed{R_{11} = R_{12},\ R_{21} = R_{22}}$

If we model each spring as a pair of identical resistances, and if we inject a current through a differential voltage, we can reasonably assume that the voltage on the springs central points (AA') is always constant and null. ⇒ RESULT: The rotor is ideally at 0V





We can bias the stators to have a current which flows through the feedback capacitor.

The differential output voltage at the INA input can be calculated as for the gyroscope readout:

$$i_{C_F,i} = \frac{dQ}{dt} = \cancel{C_{S,i}\frac{dV_i}{dt}}^{\,DC = 0} + V_{BIAS}\frac{dC_{S,i}}{dt} =$$

$$= V_{BIAS}\frac{dC_{S,i}}{dt} = V_{BIAS}\frac{dC_{S,i}}{dx}\frac{dx}{dt} =$$

$$= V_{BIAS}\frac{dC_{S,i}}{dx}\omega_0 x = V_{BIAS}\,\Delta C_{SE}\,\omega_0$$

$$\Delta V_{OUT} = 2\frac{i_{C_F,i}}{\omega_0 C_F} = \frac{V_{BIAS}\Delta C \cancel{\omega_0}}{\cancel{\omega_0} C_F} \rightarrow \boxed{\frac{\Delta V_{OUT}}{\Delta C} = \frac{V_{BIAS}}{C_F}}$$

$$\boxed{S = \frac{\Delta V_{OUT}}{B} = \frac{\Delta V_{OUT}}{\Delta C}\frac{\Delta C}{B} = \frac{V_{BIAS}}{C_F}\frac{\varepsilon_0 i L}{2\omega_0 g^2 b_{area}}}$$ SENSITIVITY

RESONANT OPERATION ISSUES

Bandwidth
Brownian noise

Like for gyroscope, the information on the value to be measured is modulated around the drive frequency, so we expect that the bandwidth of a system excited exactly at the peak of the transfer function is given by what we called $\Delta\omega_{BW}$

The maximum sensing bandwidth is again given by the half width at half maximum (i.e. −3dB) of the resonance peak:

$$\boxed{\Delta\omega_{BW} = \frac{\omega_0}{2Q} = \frac{b}{2m}} \equiv \boxed{\Delta f_{BW} = \frac{f_0}{2Q} = \frac{b}{4\pi m}}$$ MAXIMUM SENSING BANDWIDTH

Att (imp!): As both the sensing bandwidth and the sensitivity depend on the damping coefficient in an opposite way

$\downarrow$

we still have a: SENSITIVITY vs BW TRADE-OFF

To calculate the thermomechanical (Brownian) noise we start as usual from the power spectral density of the fluctuation force and we turn it in terms of displacement and then into magnetic field

$$S_{F_n} = 4k_BTb$$

$$x = F\frac{Q}{2k} = BiL\frac{Q}{2k} \rightarrow \frac{x}{B} = \frac{iL}{2k}Q \rightarrow \sqrt{S_{x_n}} = \frac{iL}{2k}Q\sqrt{S_{B_n}}$$

$$\rightarrow \boxed{NEMD = \sqrt{S_{B_n}} = \frac{\sqrt{4k_BTb\left(\frac{Q}{k}\right)^2}}{\frac{iL}{2k}Q} = \frac{4}{iL}\sqrt{k_BTb}} \quad \left[\frac{T_{rms}}{\sqrt{Hz}}\right]$$ NOISE EQUIVALENT MAGNETIC FIELD DENSITY

To minimize the NEMD, so improving the minimum detectable field, we can:

- increase the length (paying in area and cost)
- increase the injected current (paying in power consumption)
- decrease the damping coefficient → ATT: BANDWIDTH-RESOLUTION TRADE-OFF

GENERAL RESULT:

$S_{Fn} = 4k_BTb$

AXEL: $a = \frac{F_{inertial}}{m}$ $\quad$ $NEAD = \frac{\sqrt{S_{Fn}}}{m} = \frac{\sqrt{4k_BTb}}{m}$

GYRO: $\Omega = \frac{F_{coriolis}}{2m_sW_0X_{D,0}}$ $\quad$ $NERD = \frac{\sqrt{S_{Fn}}}{2m_sW_0X_{D,0}} = \frac{\sqrt{\cancel{4}k_BTb}}{\cancel{2}m_sW_0X_{D,0}}$

MAG: $B = \frac{F_{Lorentz}}{iL/2}$ $\quad$ $NEMD = \frac{\sqrt{S_{Fn}}}{iL} = \frac{4}{iL}\sqrt{k_BTb}$

ELECTRONIC NOISE

Power dissipation

Sources of electronic noise:

- Feedback resistance R_F
- OpAmp

NOTE: R_{11}, R_{12}, R_{21} and R_{22} are such small resistances that we can neglect their noise contributions

FEEDBACK RESISTANCE NOISE:

$\sqrt{S_{n,RF}} = \sqrt{\frac{4k_BT}{R_F}}$ $\quad$ $\sqrt{S_{n,RF,OUT}} = \sqrt{2\,\frac{4k_BT}{R_F}\left(\frac{1}{W_0C_F}\right)^2}$

$$\boxed{\rightarrow \sqrt{S_{Bn,RF}} = \frac{\sqrt{S_{n,RF,OUT}}}{S} = \frac{\sqrt{2\,\frac{4k_BT}{R_F}\left(\frac{1}{W_0C_F}\right)^2}}{\frac{\Delta V_{OUT}}{B}} = \frac{\sqrt{\frac{8k_BT}{R_F}}\;2g^2b_{area}}{\varepsilon_0 iLV_{BIAS}}}$$

$$\boxed{\sqrt{B_{n,RF}} = \sqrt{S_{Bn,RF}}\cdot\sqrt{BW}}$$

OPAMP NOISE:

$S_{Vn} = \frac{4k_BT\gamma}{g_m}$ $\quad$ $\sqrt{S_{Vn,OUT}} = \sqrt{2S_{Vn}\left(1+\frac{C_P}{C_F}\right)^2}$

$$\boxed{\sqrt{S_{Bn,OA}} = \frac{\sqrt{S_{Vn,OUT}}}{S} = \frac{\sqrt{2S_{Vn}\left(1+\frac{C_P}{C_F}\right)^2}\;2W_0g^2b_{area}C_F}{\varepsilon_0 iLV_{BIAS}} \approx \sqrt{2S_{Vn}}\;C_P\,\frac{2W_0g^2b_{area}}{\varepsilon_0 iLV_{BIAS}}}$$

$\sqrt{B_{,OA}} = \sqrt{S_{Bn,OA}}\cdot\sqrt{BW}$

ATT: here the constant voltage V_{BIAS} at the stators cannot be as high as it was for the rotor in gyros, because the OpAmps operate between the power supply.

OVERALL INPUT-REFFERRED NOISE IN TERMS OF MAGNETIC FIELD DENSITY:

$$\boxed{\sqrt{S_{Bn,TOT}} = \sqrt{S_{Bn,RF} + S_{Bn,OA} + NEMD^2} = \frac{1}{iL}\sqrt{\left[\frac{\sqrt{\frac{8k_BT}{R_F}}\,2g^2 b_{area}}{\varepsilon_0 V_{BIAS}}\right]^2 + \left[\sqrt{2S_{vn}}\,C_p\,\frac{2W_0 g^2 b_{area}}{\varepsilon_0 V_{BIAS}}\right]^2 + \left[4\sqrt{k_BTb}\right]^2}}$$

MINIMUMUM MEASURABLE MAGNETIC FIELD

$$\boxed{B_{min} = \sqrt{\sigma_{B,TOT}} = \sqrt{S_{Bn,TOT}} \cdot \sqrt{BW}}$$

NOTE: In order to optimize noise, we can:

- Increase i and L (pay in PC and area)
- If the electronic noise dominates
  - you can act on any parameter related to the sensitivity
  - you should minimize C_p and maximize R_F
- If device noise dominates:
  - you have not so many options

MEMS MAGNETOMETERS
PART 2: ADVANCED ARCHITECTURES AND OPERATION MODES

MOTIVATIONS & GOALS

First analyzed MEMS magnetometers do not prove to be so compentitive against other state-of-the-art technologies, as they have a low bandwidth

However, several advantages of the MEMS approach push the research towards solutions to simultaneously improve:

ROBUSTNESS

Since F_{LOR} is tiny we have to design a structure which is robust against accelerations, temperature changes, acoustic vibrations etc. which would displace the mass by an amount much larger than that one caused by F_{LOR}

NOISE DENSITY vs BW TRADE-OFF

- AREA

<u>ISSUES IN LORENTZ-FORCE BASED MEMS MAGNETOMETERS:</u>

- ACCELERATION SENSITIVITY
- NOISE DENSITY vs BW TRADE-OFF
  → This issue lead us to operate off-resonance

  Att: if we operate off-resonance we need to generate a resonant frequency reference
- AREA OCCUPATION
- POWER CONSUMPTION

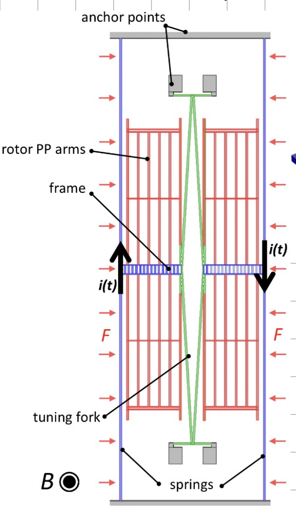
ADVANCED ARCHITECTURES

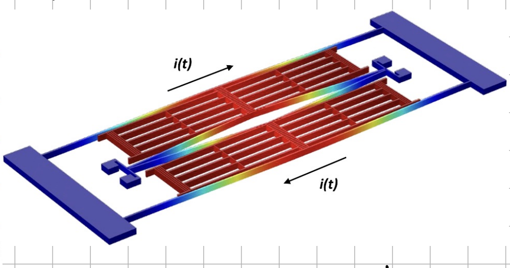
Differential architectures for OOP magnetometers
Differential architectures for IP magnetometers

① DIFFERENTIAL ARCHITECTURE FOR OOP FIELDS:

A DIFFERENTIAL ARCHITECTURE is obtained if the device is splitted into two halves, w/ the current circulating in opposite directions

TUNING-FORK BASED LORENTZ FORCE MAGNETOMETER



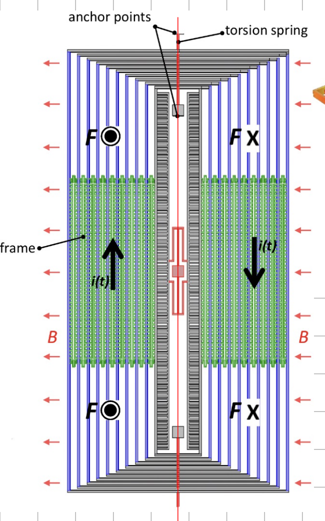


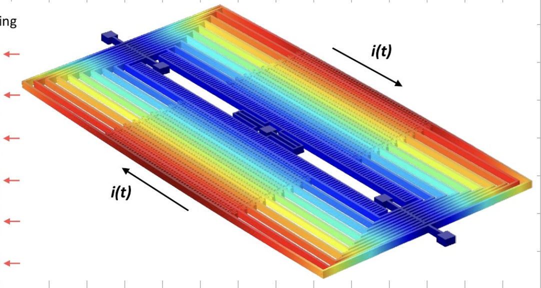
The Lorentz force will itself have different directions on the two device halves

Accelerations, which have the same effect on the two halves, will be seen as a common mode and automatically cancelled out by the capacitive readout

REMEMBER: A key factor in the designing of a tuning-fork structure is that the motion of the two halves must be in anti-phase

② DIFFERENTIAL ARCHITECTURE FOR IP FIELDS





↳ rotation axis

Devices sensitive to IP fields move in the OOP direction and correctly show a differential readout and reject accelerations provided that:

- They are balanced in terms of gravity w/rt the rotation axis

The current flows in opposite directions

How can we effectively guide the current through this polysilicon in order to have exactly the path that we define?

Metal paths can be deposited on the top of the polysilicon layer, to guide the current through a low resistivity path.

NOTE: This can be done exploiting the same metal deposition used for pads in the process

OFF-RESONANCE (MODE-SPLIT) OPERATION

Sensitivity
Bandwidth and noise density trade off
Drive signal generation and stability against drifts

What are the differences btw the resonance and the off-resonance operation?

If we work at resonance we know that the amplification is given by the Q-factor and it's obviously larger the larger is Q, but at the same time we also know that large Q narrows the -3dB BW which represents our sensing BW.

Also if we work at f_0, not only our signal (F_{LOR}), but also the thermomechanical noise (NEMD), which is a fluctuation force, are both amplified by the same value Q.

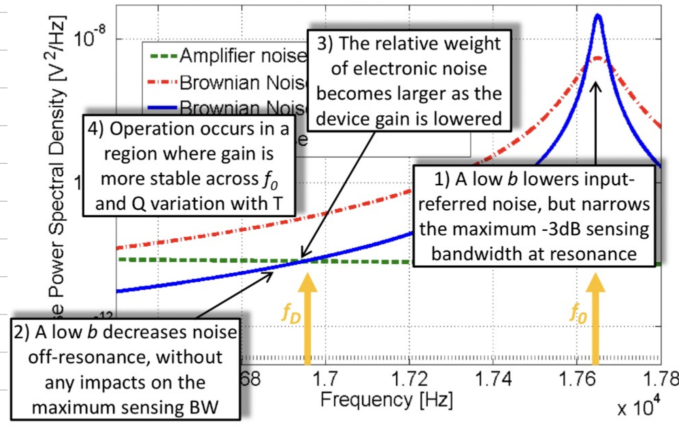
↓

Unlike axels and gyros, mags suffer from a non-negligible thermomechanical noise. Lowering the damping coefficient would imply a consistent reduction in the maximum achievable bandwidth

Additionally we know that working at f_0 is critical bcz if the temperature changes, the frequency changes, the Q-factor changes and, in turn, this will yield to a change in the scale factor.

AIM: solve the BW vs RESOLUTION (NOISE) TRADE-OFF

IDEA: So the idea is to moving into a region that is more flat, but it has a gain which is anyway larger than that one we would have working at DC



If we operate off-resonance (f_0), not only our signal F_{LOR} but also noise around that region will be amplified by a factor $Q_{eff} < Q$ so we expect that our NEMD does not change.

At the same time we expect that, as the electronic noise doesn't change in terms of output value, if we want to input refer it in terms of NEMD we have to divide it by Q_{eff}. As $Q_{eff} < Q$ we expect that the effect of the electronic noise will be larger.

So, which is the advantage of the off-resonance operation?

The advantage lies in the fact that we work in a region which is much more stable against temperature effects.

Additionally, as the BW is now related to the distance btw f_0 and f_D we expect to solve the BW vs NEMD TRADE-OFF that we have seen in resonance operation.

REMEMBER: if we work at resonance and we lower the damping coefficient b, in order to lower NEMD we are also lowering the BW.

Otherwise, if we work off-resonance, lowering b, we are still lowering our noise, but this time w/o any impact on the maximum sensing bandwidth which in off-resonance operation is related essentially to the difference btw f_0 and f_D.

$$\boxed{Q_{eff} = \frac{f_0}{2\Delta f} = \frac{\omega_0}{2\Delta\omega}} \qquad \boxed{\Delta f = f_0 - f_D}$$

$$\boxed{k_{1/2} = k_{1\text{-beam}} + k_{1/2\,TF}} \qquad \boxed{x = F_{LOR}\frac{Q_{eff}}{2k_{1/2}} = BiL\frac{Q_{eff}}{2k_{1/2}}}$$

$$\Delta C = 2\,C_0\frac{x}{g} = \frac{C_0}{g\,k_{1/2}}\,BiL\,\frac{f_0}{2\Delta f}$$

$$\boxed{\frac{\Delta V_{out}}{B} = \frac{dV}{dC}\frac{dC}{dB} = \frac{V_{BIAS}}{C_F}\frac{C_0}{g}\frac{iL}{k_{1/2}}\frac{f_0}{2\Delta f} = \frac{V_{DC}}{C_F}\frac{\varepsilon_0 A N_{PP}}{g^2}\frac{iL}{k_{1/2}}\frac{f_0}{2\Delta f}}$$ SENSITIVITY OF A TUNING-FORK MAGNETOMETER

NOTE:

This time S is dependent on N_{PP}. Indeed Q_{eff} is not dependent on b.
→ Increase as much (w/ available area) N_{PP} and lower the pressure to lower b.

- At a given f_0, the sensitivity grows w/ a lower k (and a lower m)
→ minimize the frame mass and the stiffness accordingly.

- GBWP = constant $\left(\Delta V_{out}/B \cdot \Delta f = \text{constant}\right) \Rightarrow$ The lower Δf, the larger the sensitivity
→ choose Δf to be about twice or three times the desired sensing BW

- $\Delta V_{out}/B \propto 1/g^2$
→ use the minimum gap allowed by the process

$$x = F_{LOR}\frac{Q_{eff}}{2k_{1/2}}\cdot 2 \quad\rightarrow\quad \sqrt{S_{x_n}} = \sqrt{4k_BTb\left(\frac{Q_{eff}}{2k_{1/2}}\right)^2}\cdot\sqrt{2}$$

$$\rightarrow NEMD = \frac{\sqrt{S_{x_n}}}{x/B} = \frac{4}{iL}\sqrt{k_BTb}\cdot\frac{1}{\sqrt{2}}$$ NOTE: NEMD doesn't change

* if we use a TF structure

The damping coefficient b can be lowered by decreasing the package pressure to improve the NEMD, while the maximum sensing BW is chosen through an electronic LPF w/ a cut-off at $\sim \Delta f/2$ to $\Delta f/3$

$$\Delta f_{eln} < \frac{\Delta f}{2}$$

If the electronic filter is made programmable, the sensor can achieve high-performance or high-bandwidth in a dual-mode operation:

- low LPF pole → low overall rms noise, but small BW

  high LPF pole → large sensing BW, but bad rms noise performance

The working principle, as assumed so far, implies the injection of a current exactly at the desired frequency f_0

How to provide the reference drive frequency?

The idea is to use an oscillator at the chosen frequency split

We want to build the oscillator w/ a MEMS resonator in the same package of the mag. In this way, in case of temperature changes, the resonator and the mag will drift almost in the same way, w/o consistent changes in Δf and sensitivity

MULTI-LOOP ARCHITECTURES **Improvements in sensitivity and noise**

Like in the case of gyros and axels, electronic noise is due to two major contributions:

- FEEDBACK RESISTANCE NOISE. $\sqrt{S_{Bn,RF}} = \dfrac{\sqrt{2\,\frac{4k_BT}{R_F}\left(\frac{1}{\omega_0 C_F}\right)^2}}{\frac{\Delta V_{out}}{B}}$

- OPAMP NOISE: $\sqrt{S_{Bn,CA}} = \dfrac{\sqrt{2\,S_{Vn}\left(1+\frac{C_P}{C_F}\right)^2}}{\frac{\Delta V_{out}}{B}}$

As the BW-resolution trade-off is solved, one can in principle decrease b to have a better SNR in case the thermomechanical noise dominates

However, as we decreased the device gain (bc we now work off-resonance) and thus the overall sensitivity $\Delta V_{out}/B$, likely electronic noise will dominate so that advantages of off-resonance mode will be somewhat reduced

$\Rightarrow$ if $\Delta V_{out}/B \downarrow \;\rightarrow\; S_{Bn,RF}\uparrow$ and $S_{Bn,OA}\uparrow$

We thus need to find a way to re-boost the device gain (the sensitivity)

↓

We have to act on the Lorentz force, but:

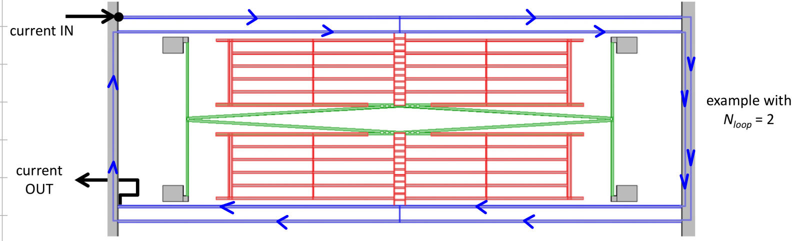
- we cannot act on the length, bc of the area constraint

  we cannot act on the current intensity, bc of the power consumption constraint

BUT: we can reuse the same current and make it re-circulate several times (N_{loop}) to boost the sensitivity by the same factor.

$$\boxed{\frac{\Delta V_{OUT}}{B} = \frac{V_{DC}}{C_F}\frac{C_0}{g}\, i_L\, \frac{Q_{eff}}{k_{1/2}}\, N_{loop}}$$

$$\boxed{NEMD = \frac{4}{N_{loop}\, i_L}\sqrt{k_B T b}}$$



As the sensitivity is boosted by N_{loop}, the electronic noise contributions are reduced by a factor N_{loop}.

$$\sqrt{S_{Bn,TOT}} = \sqrt{\left[\sqrt{2S_{Vn}}\frac{C_P}{C_F}\frac{C_F}{V_{BIAS}}\frac{g^2 k_{1/2}}{C_0 A N_{RP}}\frac{2\Delta\omega}{i_L \omega_0 N_{loop}}\right]^2 + \left[\sqrt{\frac{8k_BT}{R_F}}\frac{1}{\omega_0 C_F}\frac{C_F}{V_{BIAS}}\frac{g^2 k_{1/2}}{C_0 A N_{RP}}\frac{2\Delta\omega}{i_L\omega_0 N_{loop}}\right]^2 + \left[\frac{4}{i_L N_{loop}}\sqrt{k_BT b}\right]^2}$$

$$= \frac{1}{N_{loop}\, i_L}\sqrt{\left[\sqrt{2S_{Vn}}\frac{C_P}{V_{BIAS}}\frac{g^2 k_{1/2}}{C_0 A N_{RP}}\frac{2\Delta\omega}{\omega_0}\right]^2 + \left[\sqrt{\frac{8k_BT}{R_F}}\frac{1}{V_{BIAS}}\frac{g^2 k_{1/2}}{C_0 A N_{RP}}\frac{2\Delta\omega}{\omega_0^2}\right]^2 + \left[4\sqrt{k_BTb}\right]^2}$$

NOTE:

If the electronic noise dominates.
- you can act on any parameter related to the sensitivity
- you should minimize C_P and maximize R_F

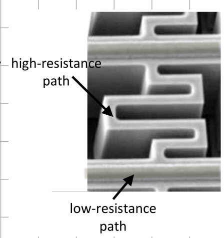
- If device noise dominates:
  - you can lower the damping coefficient

CONCLUSION: Adding recirculating loops decreases all input-referred noise contributions as it directly increases the Lorentz force.

How many loops?

The number of loops should be compatible w/ maximum area and should not increase $k_{1/2}$

Inter-loop resistance should be made very high to avoid current leakage through unwanted paths
↓
links btw current strips are made w/ serpentines to maximize inter-loop resistance



CONCLUSIONS

We have solved three of the five initial issues:

1. REJECTION OF ACCELERATIONS → use of differential architectures
2. NOISE vs BW TRADE-OFF → use of off-resonance operation and multi-loop architectures
3. GENERATION OF THE DRIVE FREQUENCY

MEMS MAGNETOMETERS
PART 3: MULTI-LOOP ARCHITECTURES

MOTIVATIONS & GOALS

NOTE: MEMS mags operating off-resonance essentially show the same advantages of mode-split gyros.
Likewise, we saw that the MEMS does not change btw resonance and off-resonance operation modes, but we expect electronic noise to be more relevant as the sensitivity decreases.

→ Multi-loop architectures were thus introduced to re-boost the sensitivity, keeping electronic noise down to acceptable values

GOALS:

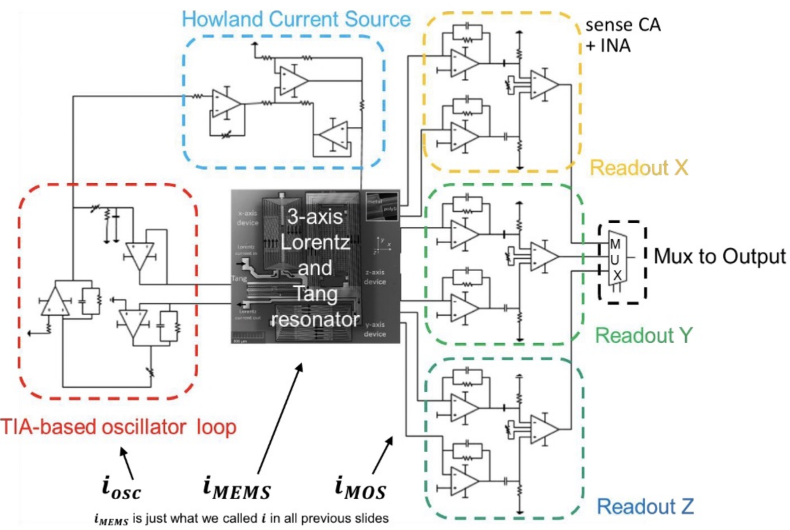
- Optimization of the current budget allocation btw the device and the electronics
- Monolithic multi-loop 3-axis sensors to decrease area occupation

SYSTEM-LEVEL CONSIDERATIONS ON POWER CONSUMPTION

Front-end electronic noise vs current

If you operate in off-resonance mode, you can even afford to use the same current and make it re-circulate through the 3 different devices

Att: This means that the three devices must have the same resonance frequency but at resonance is impossible to have the 3 frequencies perfectly matched!



NOTE: A single current recirculating in all the 3 devices implies a 3x saving in power dissipation

Given a current budget i_{TOT}, how much would you give to the sensor (i_{MEMS}) and how much to the electronics ($N_{MOS}\, i_{MOS} + i_{osc}$), to optimize noise?

- At 1st order approximation the current needed by the oscillator to sustain the resonator (i_{osc}) has no relevant role in setting noise.
  ↓
  we can assume that i_{osc} is a minimum constant term : i_{osc} = constant ~ 50μA

- On the other hand, the current consumed by the sense CAs determines to a large extent the electronic noise S_{vn}

Hp: assume to operate in saturation: $i_{MOS} = k_n V_{ov}^2$ where $k_n = \frac{1}{2}\mu_n C'_{ox}\left(\frac{W}{L}\right)$

$S_{in,MOS} = 4k_BT\gamma g_m \rightarrow S_{vn,MOS} = \frac{4k_BT\gamma}{g_m}$ where $g_m = \frac{2 i_{MOS}}{V_{ov}} = 2\sqrt{k_n i_{MOS}}$

$$\Rightarrow \boxed{S_{v_n,mos} = \frac{4k_BT\gamma}{g_m} = \frac{4k_BT\gamma V_{ov}}{2 i_{mos}} = \frac{2k_BT\gamma}{\sqrt{k_n i_{mos}}}}$$ MOSFET VOLTAGE NOISE

How many transistors shall we take into account in our calculation?

A typical differential operational amplifier has a pair of input transistors

$$\boxed{S_{v_n} = 2\, S_{v_n,mos} = \frac{4k_BT\gamma V_{ov}}{i_{mos}} = \frac{8k_BT\gamma}{g_m} = \frac{4k_BT\gamma}{\sqrt{k_n i_{mos}}}}$$ OPAMP VOLTAGE NOISE

NOTE (IMP!!): if you have two amplifiers for differential sensing, then noise power spectral density should be multiplied by another factor

OVERALL NOISE EQUATION AS A FUNCTION OF THE CURRENT TERMS

$$\boxed{\sqrt{S_{B_n,TOT}} = \frac{1}{N_{loop}\, i_{MEMS}\, L}\sqrt{\left[\sqrt{2\frac{4k_BT\gamma V_{ov}}{i_{mos}}}\,\frac{C_P}{V_{BIAS}}\,\frac{g^2 k_{1/2}}{\varepsilon_0 A N_{PP}}\,\frac{2\Delta W}{W_0}\right]^2 + \left[\sqrt{\frac{8k_BT}{R_F}}\,\frac{1}{V_{BIAS}}\,\frac{g^2 k_{1/2}}{\varepsilon_0 A N_{PP}}\,\frac{2\Delta W}{W_0^2}\right]^2 + \left[4\sqrt{k_BTb}\right]^2}}$$

MONOLITHIC MULTI-LOOP ARCHITECTURES

Area occupation

Examples of monolithic 3-axis sensors

PROBLEM:

The solution presented so far takes up an overall area of about $(5mm)^2$ which is too big compared to $(1mm)^2$ areas of non-MEMS solution

How can we reduce the area by a factor of about 5?

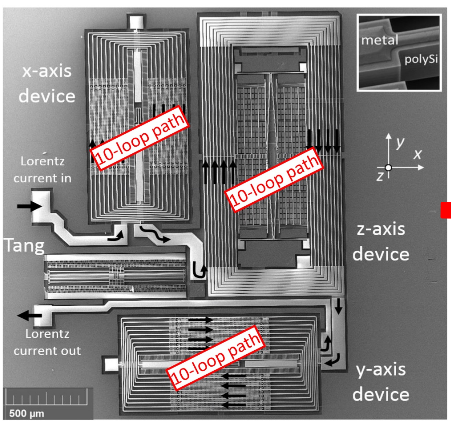
IDEA: instead of designing 3 times 10 recirculating loops, we can use the same 10-loop path for all the three devices

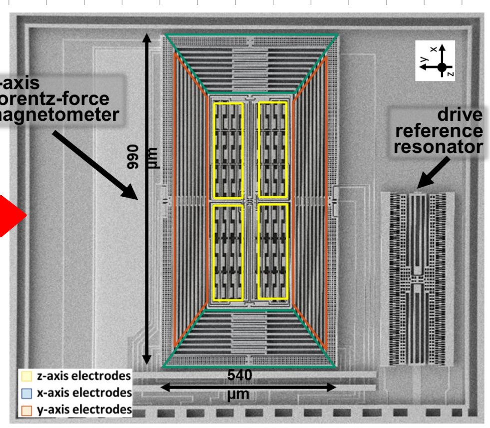
When you design a monolithic structure there are at least two fundamental considerations you have to do:

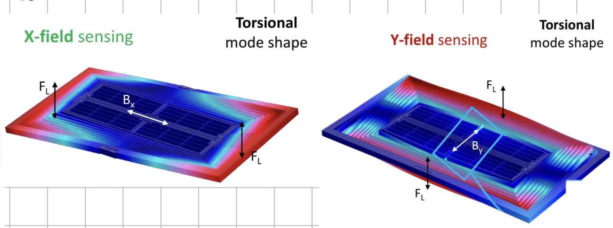
1. The resonant modes of the structure for the different sensing directions should not be at the same resonant frequency, otherwise it is very probable that if you excite one mode at a certain frequency, then, due to cross-coupling, the other modes will be excited as well
   ↓
   we have to design the three different modes to be at three different frequencies

2. In designing your structure one mode will tend to deform the structure and this deformation should not excite the other modes.
   We have to avoid the CROSS-TALK ISSUE btw different sensing modes.

   This cross-talk can be either mechanical or electrical







EXAMPLES OF OTHER CAPACITIVE MEMS SENSORS

Pressure sensors
Microphones

$$S = \frac{\Delta V_{OUT}}{P} = \frac{V_{DD}}{C_F}\frac{C_0}{g}\frac{dy}{dP} = \frac{V_{DD}}{C_F}\frac{C_0}{g}\frac{A}{2k}$$