

RESONATORS

MEMS RESONATORS

PART 1: ACTUATION, READOUT, ELECTRICAL MODEL

The next inertial sensor we would like to study is the gyroscope, however, the working principle of most gyroscopes relies on the stable oscillation of a proof mass at its resonance frequency so to provide the velocity that generates the Coriolis force.

↓
for this purpose, it is convenient to analyze MEMS RESONATORS (the vibrating element) and OSCILLATORS (the complete system w/ the sustaining circuit) before gyros

RESONATOR: mechanical/electrical system that, if excited, resonates at its primary mode frequency (the resonance frequency).

(smallest)

ATT: if it is not sustained by any other mean, its oscillation will damp down

NOTE: we use resonators inside oscillators to provide what is known as SELF-SUSTAINED OSCILLATOR

OSCILLATOR: system that continuously oscillates (typically w/ the same amplitude and frequency) even during the passing of time, bcz the energy lost by the resonator is fed back again by a sustainable electronic circuit

$$\rightarrow \text{OSCILLATOR} = \text{RESONATOR} + \text{SUSTAINING CIRCUIT}$$

COMB RESONATOR OVERVIEW

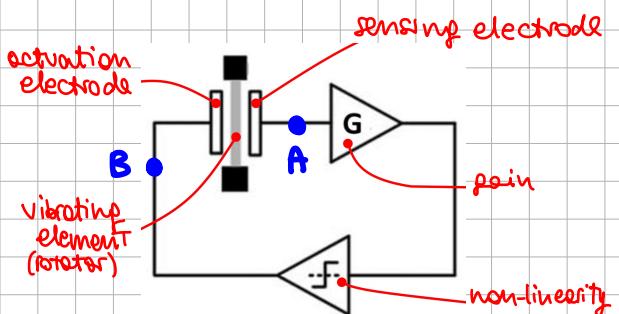
Overview

Electrostatic actuation (and transduction factor)

Motional current detection (and transduction factor)

- one stator to activate (fixed electrode)
- 2 suspended resonant element
- one stator to sense the motion (fixed electrode)

ATT: it's not an inertial sensor
→ mass should be minimized



We need to use an electronic circuit that processes the signal at the output of the oscillator and brings it back to the activation part in order to provide again the same signal that we had at the beginning.

⇒ The role of the electronic circuit is to compensate the losses of the mechanical resonator using a loop

$$\left. \begin{array}{l} \omega_0 = \sqrt{\frac{k}{m}} \\ Q = \frac{\omega_0 m}{b} = \frac{k}{\omega_0 b} \end{array} \right\}$$

What does self-sustained oscillation mean?

If the processed signal at point B has exactly the same amplitude and phase of the sensed one at point A, it means that the oscillation is SELF-SUSTAINED

The conditions to be satisfied in order to have a SELF-SUSTAINED oscillation are called BARKHAUSEN CRITERIA:

$$\left. \begin{array}{l} \text{condition on amplitude: } |G_{loop}(j\omega_0)| = 1 \\ \text{condition on phase: } \angle G_{loop}(j\omega_0) = 0^\circ \end{array} \right\}$$

NOTE: as we already know, the larger the quality factor Q, the longer the ring down time ($T = Q\pi f_0$) and so the smaller is the energy loss that we have at any cycle
→ This also means that we have to inject less energy from our battery inside the system to maintain the oscillation

$$\Rightarrow Q \uparrow \Rightarrow T \uparrow \Rightarrow E \downarrow \Rightarrow PC \downarrow$$

Why are we interested in comb-finger based resonators?

- The goal is studying gyroscopes and we will see that pyros require a relative large motion to get good performances and we already know that if we have PP rotors the motion of the rotor w/in the space surrounded by the PPs, in order to be linear, is constrained by the small displacement approximation

Furthermore, in CF config.s, F_{elec} is independent of the position and so we don't have strong nonlinearity effects

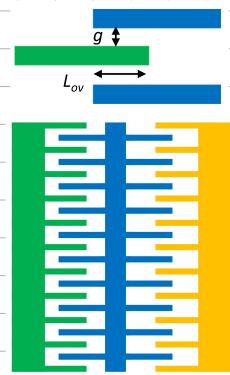
CF CONFIG = LINEARITY FOR LARGE DISPLACEMENTS

- we need an high Q to minimize the energy loss and so the power consumption

CF CONFIG = ABSENCE OF SQUEEZED-FILM DAMPING = LARGE Q

SQUEEZED-FILM DAMPING: if you have a PP, during its motion it is essentially compressing and stretching the gas layer that is b/w the two plates
 → This is a huge source of damping, b/c we cannot compress arbitrary a volume of gas

Otherwise in a CF config. you are not compressing significantly any gas film, so the effect of the squeezed-film damping is much smaller for a CF configuration



Parameters:

N_{CF} = # of rotor fingers per side

g = gap b/w rotor and stator fingers

L_{ov} = fingers overlap at rest

m = effective mass of the resonant element

k = effective stiffness

b = damping coefficient

h = process height

$A = h \cdot L_{ov}$

V_p = rotor voltage

V_a = activation voltage

V_s = sensing voltage

- We apply a voltage to the activation plate in order to excite the suspended rotor

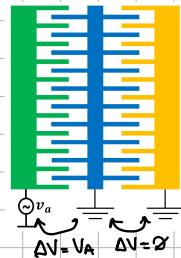
- The rotor will start to oscillate

- The oscillation will give a capacitance variation on the sensing plate

↳ This capacitance variation will be used as signal that is fed to oscillator which will process it and will deliver it again to the activation part

As we have to excite the system at the resonance frequency, the first idea is to apply to the activation part a signal at the ω_0 of our rotor and let's see what happens in terms of F_{elec}

$$\left\{ \begin{array}{l} V_a = V_a \sin(\omega_0 t) \\ V_p = 0 \\ V_s = 0 \end{array} \right.$$



$$\rightarrow \frac{dC_A}{dx} = \frac{2 \epsilon_0 h N_{CF}}{g}$$

$$\Rightarrow |F_{elec}| = \left| \frac{V_a}{2} \frac{dC_A}{dx} \right| = \frac{\epsilon_0^2 \epsilon_0 h N_{CF}}{g} \sin^2(\omega_0 t) = \frac{\epsilon_0^2 \epsilon_0 h N_{CF}}{g} \frac{1 - \cos(2\omega_0 t)}{2}$$

Generally the expression of F_{elec} is:

$$F_{elec} = \frac{\epsilon_0^2}{2} \frac{dC}{dx}$$

$$C_A = \frac{2 \epsilon_0 h (x + L_{ov}) N_{CF}}{g}$$

ACTUATOR CAPACITANCE

NOTE: 2 is the differential factor, b/c for each finger we have essentially two capacitances one facing upwards and the other one facing downwards

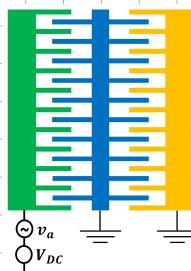
Att: In this way we end up w/ an electrostatic force which is not at ω_0 but at $2\omega_0$, so we will not excite the rotor at resonance.

→ we need an alternative approach

Which solution can solve this problem?

- If you apply a voltage at $\omega_0/2$ you will excite the MEMS at ω_0 , but you'll end up w/ a spiral at the end of the loop of the oscillator that is at ω_0' , so the activation signal and the precessional one are not in phase
→ Barkhausen criterion NOT satisfied
- Another technique to linearize the activation force is to superimpose a small AC signal to a large DC value

$$\begin{cases} V_A = V_a \sin(\omega_0 t) + V_{DC} \\ V_P = 0 \\ V_S = 0 \end{cases}$$



$$\Rightarrow |F_{elec}| = \left| \frac{V_A^2}{2} \frac{dC_A}{dx} \right| = [V_a \sin(\omega_0 t) + V_{DC}] \frac{\epsilon_0 h N_{CF}}{g} = \frac{\epsilon_0 h N_{CF}}{g} [(V_a \sin(\omega_0 t))^2 + 2V_a V_{DC} \sin(\omega_0 t) + V_{DC}^2]$$

(aperto che)

Provided that SMALL-SIGNAL CONDITION is met, we obtain:

* $\frac{V_a^2}{2} \ll 2V_{DC}V_a \rightarrow V_a \ll 4V_{DC}$

* This comes from:

$$(V_a \sin(\omega_0 t))^2 = V_a^2 \frac{1 - \cos(2\omega_0 t)}{2}$$

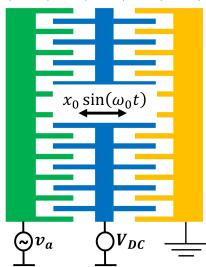
$$|F_{elec}| = \frac{\epsilon_0 h N_{CF}}{g} [2V_{DC}V_a \sin(\omega_0 t) + V_{DC}^2]$$

Att: The force has still a DC term that breaks the resonator symmetry, but we can easily solve it

NOTE: The DC contribution is caused by the DC difference b/w the rotor and the activation stator

If we have the same DC force on the right sensing side we can compensate the DC shift. In order to have it, let's apply the DC voltage directly to the rotor:

$$\begin{cases} V_A = V_a \sin(\omega_0 t) \\ V_P = V_{DC} \\ V_S = 0 \end{cases}$$



$$\Rightarrow |F_{elec}| = \left| \frac{(V_A - V_P)^2 - (V_P - V_S)^2}{2} \frac{dC_A}{dx} \right| = \frac{(V_a \sin(\omega_0 t) - V_{DC})^2 - (V_{DC} - 0)^2}{2} \frac{\epsilon_0 h N_{CF}}{g} = \frac{\epsilon_0 h N_{CF}}{g} [(V_a \sin(\omega_0 t))^2 - 2V_a V_{DC} \sin(\omega_0 t) + V_{DC}^2 - V_{DC}^2]$$

RESONATOR DISPLACEMENT

$$x = x_0 \sin(\omega_0 t)$$

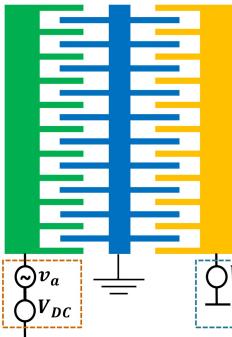
Small signal condition

\downarrow
 $V_a \ll 4V_{DC} \rightarrow |F_{elec}| \approx \frac{\epsilon_0 h N_{CF}}{g} 2V_{DC}V_a \sin(\omega_0 t) = C_0 2V_{DC}V_a \sin(\omega_0 t)$

ELECTROSTATIC FORCE EXCITING THE RESONATOR

Now we have an F_{elec} that excites the MEMS at ω_0 , so w/ the maximum gain in terms of transfer function b/w force and displacement, w/o any displacement offset.

In principle we would obtain the same result using this equivalent configuration:



ATT: Anyway it's preferable the former solution (thus applying the DC voltage directly to the seismic mass) bcz in that case, since the rotor is suspended and so it is not connected to other actuating or amplifying circuits, it will draw almost zero current from the voltage source, so it will dissipate a very low power

→ Otherwise in this config. we have a power consumption issue!!

We have found the expression:

$$|F_{\text{elec}}| = \frac{\epsilon_0 h NCF}{f} 2V_{\text{dc}} V_a = V_{\text{dc}} \frac{dC_A}{dx} V_a \quad \text{where } V_a = V_a \sin(\omega t)$$

We can define the ELECTROMECHANICAL TRANSDUCTION FACTOR FOR THE ACTUATION PORT

$$\eta_A = \frac{F_{\text{elec}}(s)}{V_a} = V_{\text{dc}} \frac{dC_A}{dx} = V_{\text{dc}} \frac{2 \epsilon_0 h NCF}{f}$$

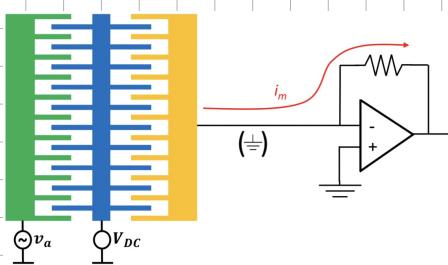
NOTE:

- It tells us how much force we get on our rotor per applied oscillating voltage at the activation port
- The higher η_A , the better the "ACTUATION" capability of my driving states (the activation one)
- η_A is a function of the resonator geometry ($1/dx$) and polarization (V_{dc})

$$\Rightarrow |F_{\text{elec}}| = \eta_A V_a \quad \text{ELECTROSTATIC FORCE AS A FUNCTION OF } \eta_A$$

what kind of signal do we have at the sensing port?

If we keep the sensing node at GND we can readout nothing from it, so we have to keep it not at GND but at VIRTUAL GROUND



i_m = OUTPUT MOTIONAL CURRENT

In principle: $(Q = C \Delta V)$

$$i_m = \frac{d(CV)}{dt} = V_{\text{dc}} \frac{dC_s}{dt} + C_s \frac{dV_{\text{dc}}}{dt}$$

~~\rightarrow~~

$$\Rightarrow i_m = V_{\text{dc}} \frac{dC_s}{dt}$$

C_s = SENSING CAPACITANCE

⇒ To readout the rotor displacement we need to sense the current induced by the capacitance variation, while keeping the virtual ground at the sense node → to do it we can use a transimpedance amplifier (like in the figure) or a transconductance amplifier as well (a charge amplifier)

Now we want to find a single parameter that links the motion of the resonator to the current that flows at its output

$$x = x_0 \sin(\omega t) \rightarrow \dot{x} = \frac{d}{dt}(x_0 \sin(\omega t)) = \omega x_0 \cos(\omega t)$$

$$\Rightarrow i_m = V_{\text{dc}} \frac{dC_s}{dt} = V_{\text{dc}} \frac{dC_s}{dx} \frac{dx}{dt} = V_{\text{dc}} \frac{dC_s}{dx} \dot{x}$$

NOTE: $V_{DC} \frac{dC_s}{dx}$ has the same form of η_A , thus it means that we can define the electromechanical transduction factor also for the sensing port

ELECTROMECHANICAL TRANSDUCTION FACTOR FOR THE SENSE PORT

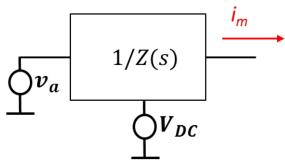
$$\eta_s = \frac{i_m(s)}{s X(s)} = V_{DC} \frac{dC_s}{dx}$$

NOTE: • IT tells us how much current we get at the output per displacement's velocity
⇒ The higher η_s , the better the "DETECTION" CAPABILITY of our resonant states

$$\Rightarrow i_m = \eta_s \dot{x} \quad \text{OUTPUT MOTATIONAL CURRENT AS A FUNCTION OF } \eta_s$$

ELECTRICAL ADMITTANCE

AT resonance
In The Laplace domain



we can define the ELECTRICAL ADMITTANCE $1/Z(s)$ b/w the applied voltage V_a and the output current i_m

NOTE: The system has 3 parts

We first evaluate the force-displacement law at ω_0 .

$$\begin{aligned} \frac{F_{elec}(j\omega_0)}{V_a(j\omega_0)} &= \eta_A \\ \frac{X(j\omega_0)}{F_{elec}(j\omega_0)} &= \frac{Q}{jK} \\ \frac{i_m(j\omega_0)}{j\omega_0 X(j\omega_0)} &= \eta_s \end{aligned}$$

GOAL: $\frac{i_m(s)}{V_a} = \frac{1}{Z(s)} =$ (we can split this fraction into 3 sub-fractions)

$$= \frac{F_{elec}(s)}{V_a} \cdot \frac{X(s)}{F_{elec}} \cdot \frac{i_m(s)}{X} = \left(\text{where } \frac{X(s)}{F_{elec}} = \text{force-displacement TF} \right)$$

$$= \eta_A \cdot \frac{1}{m} \frac{1}{s^2 + \frac{\omega_0}{Q}s + \omega_0^2} \cdot s \eta_s = \left(\text{Hyp: assuming the resonator perfectly symmetric } \eta_A = \eta_s = \eta \right)$$

$$= \frac{1}{m} \frac{s \eta^2}{s^2 + \frac{\omega_0}{Q}s + \omega_0^2}$$

ELECTRICAL ADMITTANCE

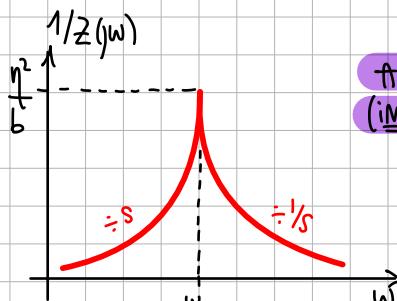
$$\frac{i_m(s)}{V_a} = \frac{1}{Z(s)} = Y(s) = \eta^2 s \frac{1/m}{s^2 + s \frac{\omega_0}{Q} + \omega_0^2}$$

$$\frac{i_m(s)}{V_a} = \frac{1}{Z(s)} = \eta^2 s \frac{1/m}{s^2 + s \frac{\omega_0}{Q} + \omega_0^2} = \frac{s}{\frac{m}{\eta^2} s^2 + \frac{1}{\eta^2} \frac{m \omega_0}{Q} s + \frac{m \omega_0^2}{\eta^2}} = \frac{1}{\left(\frac{m}{\eta^2} s + \frac{b}{\eta^2} + \frac{k}{m \eta^2} \right)}$$

- $W \ll \omega_0 \rightarrow \frac{1}{Z(j\omega)}|_{W \ll \omega_0} \sim s \frac{\eta^2}{k}$

- $W = \omega_0 \text{ at resonance} \rightarrow \frac{1}{Z(j\omega)}|_{W=\omega_0} \sim \frac{\eta^2}{b}$

- $W \gg \omega_0 \rightarrow \frac{1}{Z(j\omega)}|_{W \gg \omega_0} \sim \frac{1}{s} \frac{\eta^2}{m}$



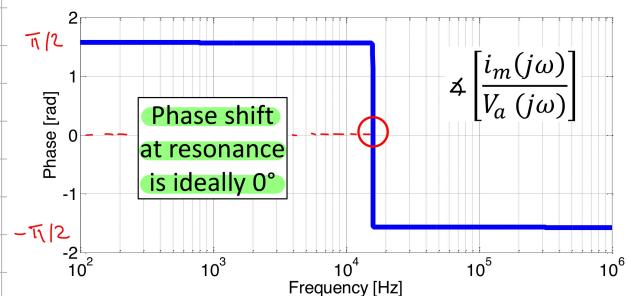
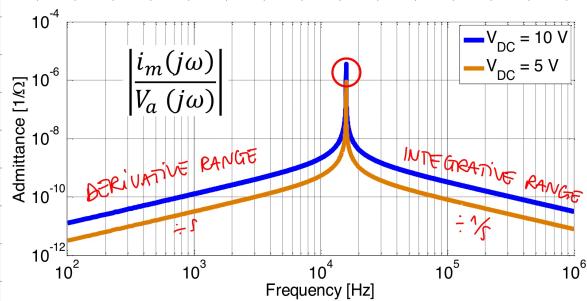
At: we are discussing the relationship (IMP!) b/w the current and the voltage b/w 2 ports, but actually we have also a 3rd port which affect the behavior of the transfer function but when we set V_{DC} it sets the value of η^2

NOTE (IMP!): The admittance at resonance is real and depends only on the damping coefficient (no dependence on m or k) \Rightarrow THE MOTIONAL CURRENT i_m IS IN PHASE WITH THE APPLIED VOLTAGE V_a

The capacitive readout introduces a zero in the origin, followed by two complex conjugate poles at ω_0 (but for resonators we always assume large quality factor)

NOTE:

The admittance modulus is maximized at resonance and depends strongly on V_{DC}
The phase is substantially independent of V_{DC}



EQUIVALENT ELECTRICAL MODEL

RLC equivalent circuit

Dissipation: quality factor and equivalent resistance

Can we find a dipole formed by electrical components that yields exactly the same shape in terms of electrical admittance?

FREQUENCY:

ADMITTANCE:

- $\omega \ll \omega_0 \rightarrow \frac{i_m(s)}{V_a} = \frac{1}{Z(s)} = \frac{\eta^2}{K} s = s C_{eq}$
- $\omega = \omega_0 \rightarrow \frac{i_m(s)}{V_a} = \frac{1}{Z(s)} = \frac{\eta^2}{b} = \frac{1}{R_{eq}}$
- $\omega \gg \omega_0 \rightarrow \frac{i_m(s)}{V_a} = \frac{1}{Z(s)} = \frac{1}{s} \frac{\eta^2}{m} = \frac{1}{s L_{eq}}$

IMPEDANCE:

$$Z(s) = \frac{1}{s C_{eq}}$$

ELECTRICAL EQUIVALENT:

$$C_{eq} = \frac{\eta^2}{K}$$

$$C_{eq} \propto \frac{1}{K}$$

$$R_{eq} = \frac{b}{\eta^2}$$

$$R_{eq} \propto b$$

$$L_{eq} = \frac{m}{\eta^2}$$

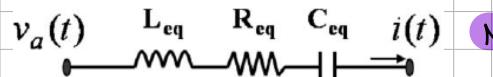
$$L_{eq} \propto m$$

NOTE: The term that represents dissipation in the mechanical domain (b) is related to the term that represents dissipation in the electrical domain (R)

$$\frac{i_m(s)}{V_a} = \frac{\eta^2}{m} \frac{s}{s^2 + \frac{b}{m}s + \frac{K}{m}} = \frac{1}{\frac{\eta^2}{m} s + \frac{b}{\eta^2} + \frac{K}{s \eta^2}} = \frac{1}{L_{eq} s + R_{eq} + \frac{1}{s C_{eq}}}$$

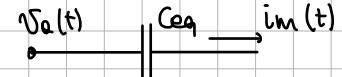
ELECTRICAL ADMITTANCE (RESONATOR TF)

\Rightarrow The 3-part resonator can be fully modeled by an electrical equivalent 2-part model (series RLC)



NOTE: all the parameters are a function of V_{DC} which represents the third hidden part

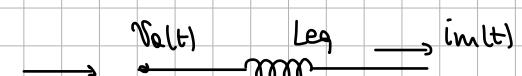
- $\omega \ll \omega_0 \rightarrow L_{eq}$ is shorted and R_{eq} is negligible w/r/t $C_{eq} \rightarrow$



- $\omega = \omega_0 \rightarrow L_{eq}$ and C_{eq} perfectly balance one another \rightarrow



- $\omega \gg \omega_0 \rightarrow C_{eq}$ is shorted and R_{eq} is negligible w/r/t $L_{eq} \rightarrow$



NOTE: what we need to compensate to build up an oscillator is represented by its losses, i.e. its dissipation, i.e. its R_{eq} .

(IMP!) \Rightarrow The sustaining circuit will need a resistive gain equal to $1/R_{eq}$ in the regime condition (The circuit must have a resistive gain!!)

→ Thus it's relevant an accurate modeling of the damping coefficient b , and thus of the Q factor, in order to correctly design the resonator.

NOTE: In the end, the 3-part resonator seems not that different from an accelerometer from the point of view of the structure: two fixed electrodes and a moving frame.
Huge differences are:

1. in the design parameters:

- resonator → no need for large mass, higher f_0 , very large Q
- accelerometer → large mass, low Q

2. in the operation

Numerical example:

$$NCF = 50$$

$$g = 2 \mu\text{m}$$

$L_{ov} = \text{no influence}$ (NOT avoid fringe effects)

$$M = 1 \text{ nkg}$$

$$k = 50 \text{ N/m}$$

$$b = 10^{-7} \text{ kg/s}$$

$$h = 20 \mu\text{m}$$

$$\frac{dC}{dx} = \frac{2\pi h NCF}{g} = 8.95 \text{ fF}/\mu\text{m} \approx 10 \text{ fF}/\mu\text{m}$$

CONCLUSIONS:

- The equivalent electrical model is very useful when we need to couple the MEMS w/ an oscillating circuit in a simulation phase.

The 3-part resonator seems not that different from an accelerometer from the structure pov: two fixed electrodes and a moving frame.

Differences are in:

1. DESIGN PARAMETERS : for resonators we do not need large masses, we need higher resonance frequencies and very large quality factors *

2. OPERATION

- There exists a parallelism b/w:
 - a. resistance and damping coefficient $R_{eq} = b$
 - b. capacitance and spring stiffness $C_{eq} = 1/k$

* The larger the Q factor ($Q = \frac{\omega_0 M}{b}$) → the lower b ⇒ the lower the losses ($R_{eq} = \frac{b}{\eta^2}$)

MEMS RESONATORS

PART 2: MEMS-BASED OSCILLATOR CIRCUIT

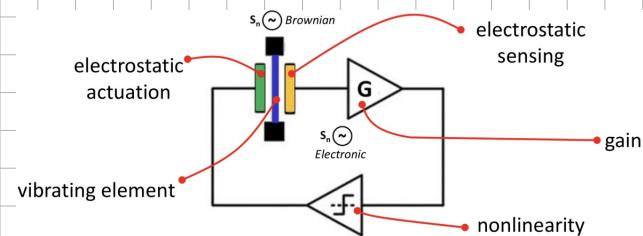
OSCILLATOR CONDITIONS

Barkhausen criterion

Compensation of the equivalent resistance

We need a circuit to compensate for the resonator losses

REM: CF config. allows large displacements w/o issues of linearity



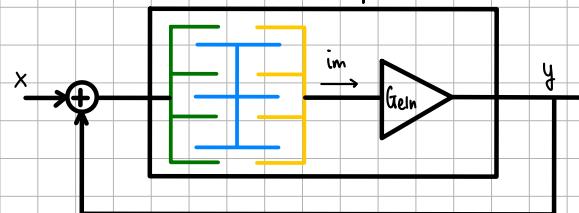
$$W_0 = \sqrt{\frac{k}{m}}$$

$$Q = W_0 \frac{m}{b}$$

$\rightarrow Q \uparrow \Rightarrow b \downarrow \Rightarrow R_{eq} \downarrow \Rightarrow \text{lower The losses}$

Attn: The loop will oscillate provided that it satisfies the Barkhausen criteria:

$$H(s) = G_{loop}(s)$$



$$y = H(s)[x + g]$$

To be an oscillator, it must be self-sustained w/o any input signals x

$$\rightarrow x = 0 \rightarrow y = y H(s)$$

*1
 $\Rightarrow H(s) = 1$ any freq. w for which $H(jw) = 1$ will be self sustained, but we need that this condition is satisfied at least at resonance (at w_0) bcz it represents the regime operation of our resonator (*1, it means unitary modulus and no phase shift)

In regime operation, the loop gain shall satisfy the conditions:

$$\Rightarrow \begin{cases} |G_{loop}(s)| = 1 = 0 \text{dB} \\ \angle G_{loop}(s) = 0^\circ \end{cases} \quad \text{BARKHAUSEN CRITERIA}$$

NOTE: Gain can be computed by opening the loop at the activation port, for instance, applying a signal and checking the signal that comes back there after one loop turnaround

$$G_{loop}(s) = \frac{i_m(s)}{v_a} G_{in}(s)$$

*2 This requirement is needed when we are in regime conditions, but when we switch on our circuit not necessary we have immediately a signal w/ the correct amplitude and phase. The harmonic oscillator needs a STARTUP (BUILDUP) PHASE which is a transient interval where the oscillation grows from zero up to the steady state (regime) amplitude condition for which the Barkhausen criteria should be satisfied.

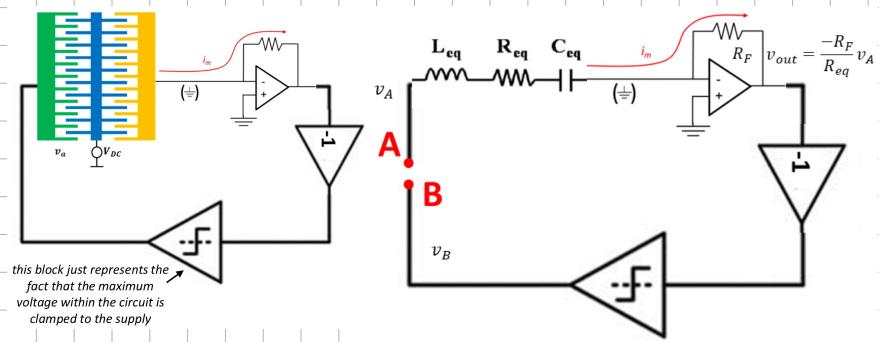
Obviously, as the oscillation needs to grow at the beginning, the loop gain during the build-up phase should not be equal to 1, but it must be lesser, otherwise the oscillation wouldn't grow.

In the initial phase, to build up the oscillation, the conditions should be:

$$\begin{cases} |G_{loop}(jw_0)| > 1 > 0 \text{dB} \\ \angle G_{loop}(jw_0) = 0^\circ \end{cases} \quad \text{BUILD-UP PHASE CONDITIONS}$$

At $w=w_0$ R_{eq} is the dominant term and the gain b/w nodes A and B is:

$$G_{loop}(jw_0) = \frac{v_B}{v_A} = \frac{R_F}{R_{eq}}$$



$|G_{loop}(j\omega_0)| > 1$ at startup is satisfied only if the overall circuit resistive gain is initially larger than R_{eq}

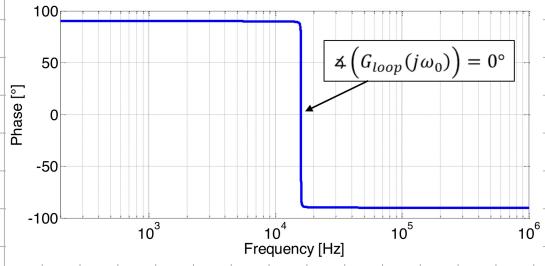
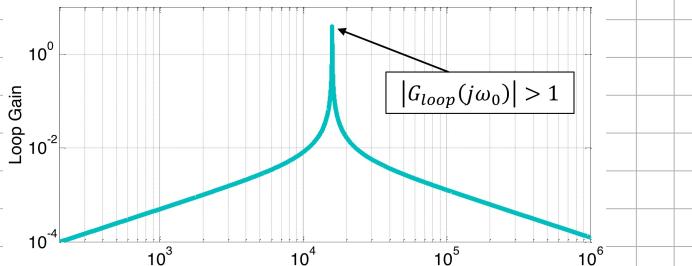
$|G_{loop}(j\omega_0)| = \frac{V_B}{I_A} = \frac{R_F}{R_{eq}}$ → If I want $|G_{loop}(j\omega_0)| = 1$, I have to choose $R_F = R_{eq}$, but it's impossible to precisely know how much R_{eq} is

⇒ it is not possible to exactly match $|G_{loop}(j\omega_0)| = 1$ in a linear circuit

- w/ $|G_{loop}(j\omega_0)| < 1$ the oscillation never starts

- w/ $|G_{loop}(j\omega_0)| > 1$ the oscillation diverges

IDEA: The idea is thus to have $|G_{loop}(j\omega_0)| > 1$ at the startup then to add a NONLINEARITY (i.e. a change in gain as function of signal amplitude) in the electronics, so to hold $|G_{loop}(j\omega_0)| = 1$



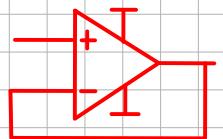
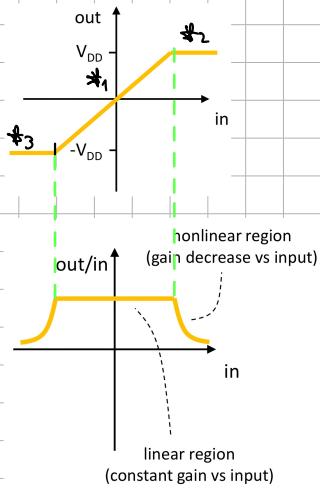
How does the nonlinearity fix the loop gain at 1?

The nonlinearity stage can be seen as a gain which is not constant, but changes as a function of the input signal amplitude

→ A **NONLINEAR ELECTRONIC STAGE** is a stage for which the response is linear up to a certain input amplitude and then after the input amplitude reaches a determinate value, the gain of the stage starts to decrease as a function of the amplitude in such a way the loop gain of the entire oscillator stabilizes at 1.

Possible solutions:

- **BUFFER**: saturates after the oscillation reaches a specific amplitude

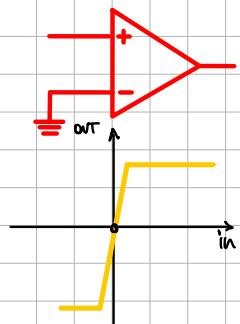


$*_1$ in this range the gain: $out/in = \text{const}$ (linear region)

$*_2$ input increases but the output remains constant ⇒ out/in decreases (non linear region)

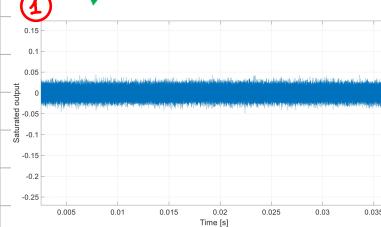
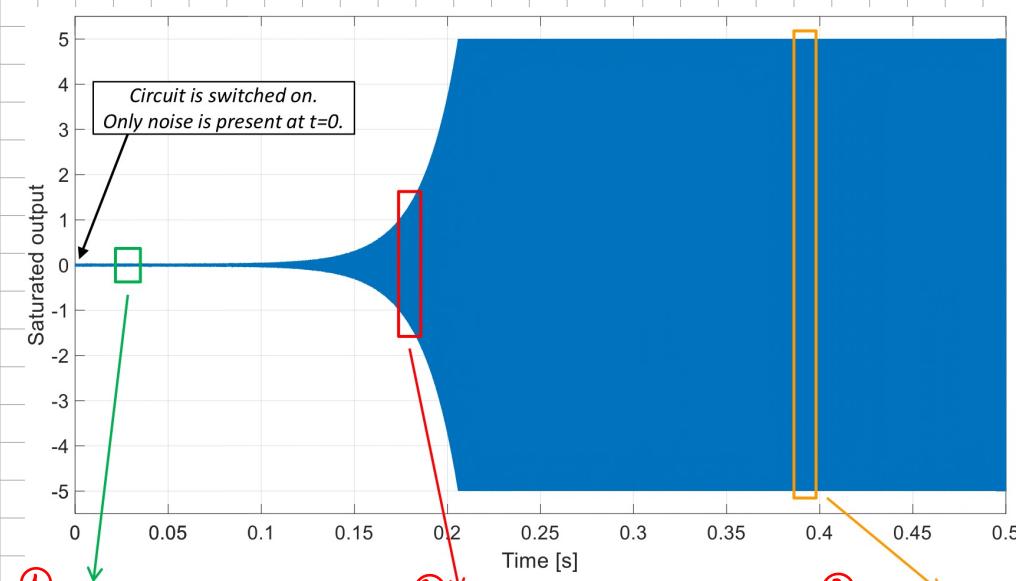
$*_3$ input decreases but the output remains constant ⇒ out/in increases (non linear region.)

- **COMPARATOR** (e.g. implemented as an open-loop opamp) or a **HIGH-GAIN AMPLIFIER** have as well the behavior described by the graphs, but w/ a very high gain in a narrow linear region

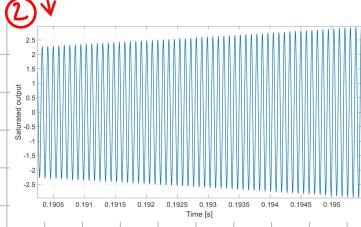


How do these nonlinearities help us to stabilize gain at 1?

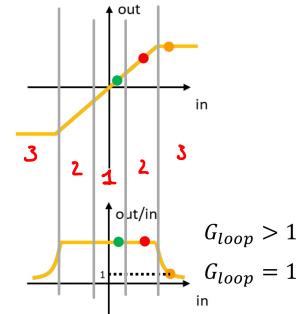
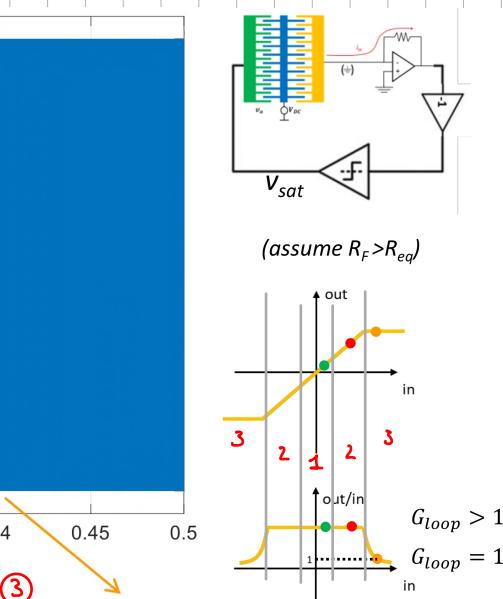
NOTE: the white noise has all the frequency components including the lucky one which is exactly at ω_0



Noise is dominant. Within all the frequency components, the one corresponding to ω_0 begins to grow. Here it is still hidden in the noise.



The component @ ω_0 begins to dominate over the others (it is the only one satisfying the start-up condition $G_{loop} > 1$)
Gloop is still > 1 .



The circuit reaches the saturation at one node
 $G_{loop} > 1 \Rightarrow G_{loop} = 1$
the oscillation stabilizes

NOTE: turn-on time depends on Q

The damping coefficient b and thus the Q factor are functions of the absolute temperature

$$\text{In the most common situation: } Q \propto \frac{1}{\sqrt{T}} \rightarrow b = \frac{\omega_0 m}{Q} \propto \sqrt{T}, \quad R_{eq} = \frac{b}{\eta^2} \propto \sqrt{T}$$

CONSEQUENCE: R_{eq} must be compensated for every T value of the T operating range

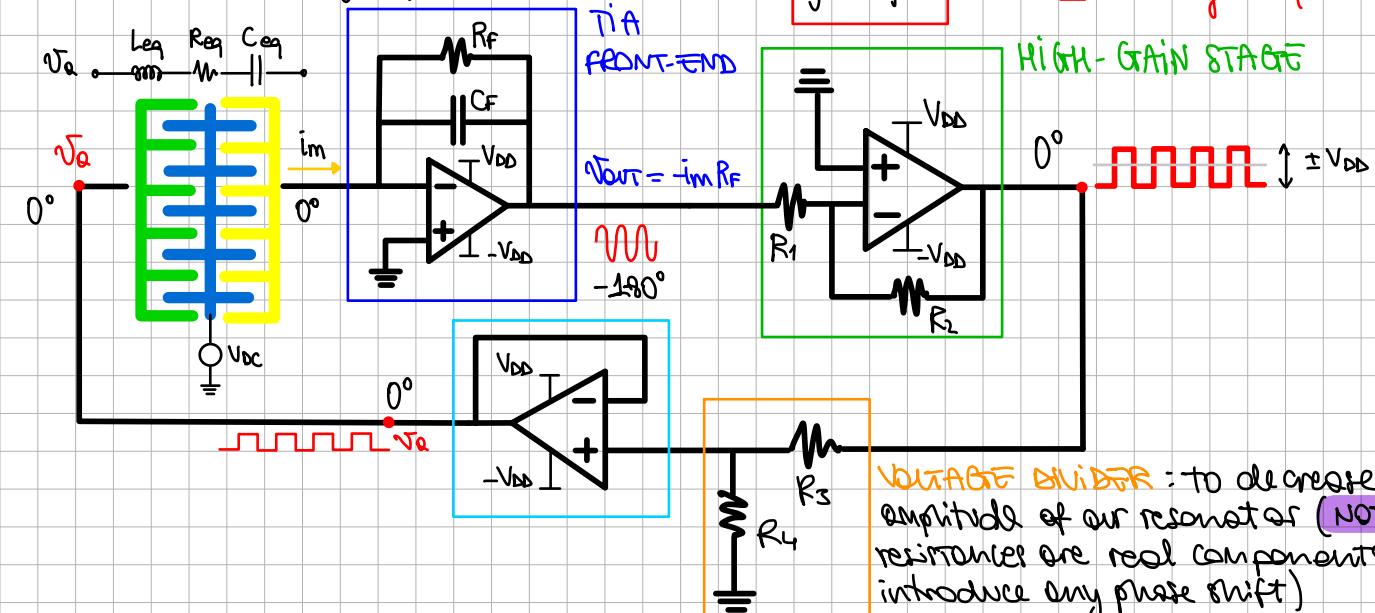
WORST CONDITION: highest T \equiv lowest Q, highest b, highest R_{eq}

CIRCUITS BASED ON LINEAR AMPLIFICATION AND SATURATION

- Trans-resistance front-end
- nonlinearity
- driving amplitude decrease

TRANS-IMPEDANCE (TIA) FRONT-END SOLUTION

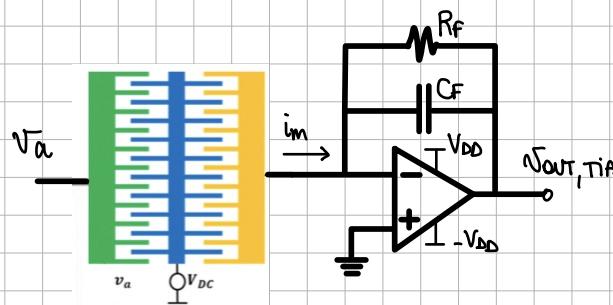
$$f_0 < f_{p1}$$



ADVANTAGE: The TIA has the advantage of introducing a 180° phase shift from V_a to $V_{out,TIA}$

NOTE: if the gain given by R_F is not enough to compensate R_{eq} , an HIGH-GAIN STAGE is introduced in order to allow the oscillation to rise up from noise.

① TIA - TRANSIMPEDANCE AMPLIFIER STAGE $\Rightarrow f_p \gg f_0$ #1 TIA OPERATING REGION



R_F = FEEDBACK RESISTANCE

\rightarrow it represents the gain of this stage

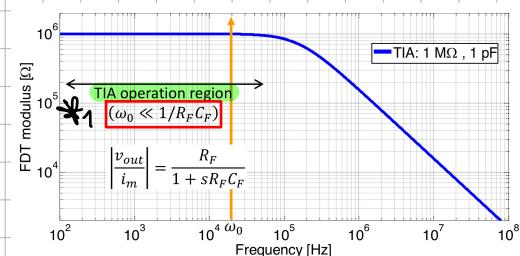
NOTE: in principle a large R_F is convenient to minimize the Johnson noise and the impact of noise of the following stages

$$\frac{V_{out,1}(s)}{i_m} = \frac{-R_F}{1 + sR_F C_F}$$

$$\frac{V_{out,1}(w_0)}{i_m} \approx -\frac{R_F}{R_{eq}}$$

- when s , and so w_0 , is low $\rightarrow \frac{V_{out,1}(s)}{i_m} \approx -R_F$

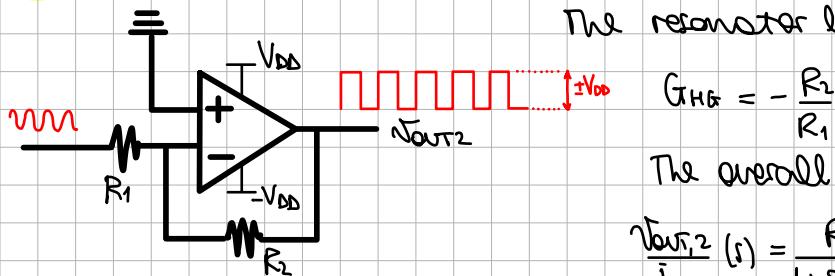
- when s , and so w_0 , increases $\rightarrow \frac{V_{out,1}(s)}{i_m} \approx \frac{-R_F}{sR_F C_F} = -\frac{1}{sC_F}$



NOTE: R_F typical $< 5-10 \text{ M}\Omega$ \rightarrow it does not provide $G_{loop} > 1$

\hookrightarrow implies the need for a further gain stage

② HIGH-GAIN STAGE \longrightarrow we introduce it b/c R_F only may not be enough to compensate the resonator losses



$$G_{HG} = -\frac{R_2}{R_1}$$

The overall startup loop gain is now set by:

$$\frac{V_{out,2}(s)}{i_m} = \frac{R_F}{1 + sR_F C_F} \frac{R_2}{R_1} \quad \frac{V_{out,2}(w_0)}{i_m} \approx \frac{R_F}{R_{eq}} \frac{R_2}{R_1} > 1$$

Assuming that this condition is met, when we switch on the circuit at time $t=0$ we have only noise, i.e. harmonic components at every frequency.

The harmonic corresponding to w_0 is the only one which is amplified by a positive loop gain larger than 1.

The oscillation at w_0 starts to increase. At a certain point, the high-gain stage saturates its output (square wave at $\pm V_{DD}$).

The loop thus decreases, stabilizing to 1.

NOTE (impl!): The first harmonic of a square wave at w_0 has an amplitude increased by $4/\pi$

REM: we have seen how the MEMS resonator can be linearized only when $V_a \ll 4V_{DC}$

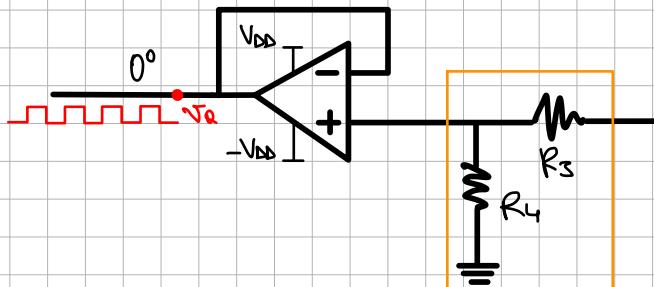
if $V_{DC} = 5V$ and we bias the circuit at $V_{DD} = \pm 1.5V$, the condition on the 1st harmonic becomes:

$$\frac{4}{\pi} \frac{V_{DD}}{4} \ll V_{DC} \rightarrow \left(\frac{1.5}{\pi} \right)^{0.5} \ll 5 \quad (\text{roughly one order of magnitude}) \#2$$

#2 ISSUE: we are at the limits of the small signal condition

SOLUTION: we need to lower the driving amplitude \rightarrow e.g. by simply using a RESISTIVE DIVIDER

③ RESISTIVE DIVIDER w/ THE OUTPUT BUFFER → in order to provide an AMPLITUDE LIMITATION



$$\text{Gain} = \frac{R_4}{R_3 + R_4}$$

This de-fain is useful to limit amplitude during nonlinear operation

$$\rightarrow \text{Gloop}(W_0) = \frac{R_F}{R_{\text{req}}} \frac{R_2}{R_1} \frac{R_4}{R_3 + R_4} > 1 \quad \text{OVERALL LOOP GAIN AT RESONANCE}$$

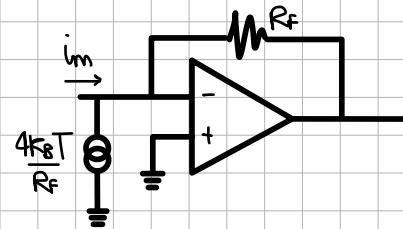
ATT: even w/ this gain lowering, the overall loop gain must remain larger than 1 to guarantee the oscillation building up

NOTE: The buffer is used to drive the MEMS w/ a low output impedance

OVERALL LOOP GAIN
w/ A TIA CONFIGURATION

$$\text{Gloop}(s) = \frac{1}{sL_{\text{eq}} + R_{\text{req}} + \frac{1}{sC_{\text{eq}}}} \frac{R_F}{1 + sR_F C_F} \frac{R_2}{R_1} \frac{R_4}{R_3 + R_4}$$

For power and noise constraints, it would be nice to have large R_F



if we want a large SNR → we need a large R_F in order to minimize the $4k_B T / R_F$ noise contribution

However, large integrated resistances are not feasible (too large area)

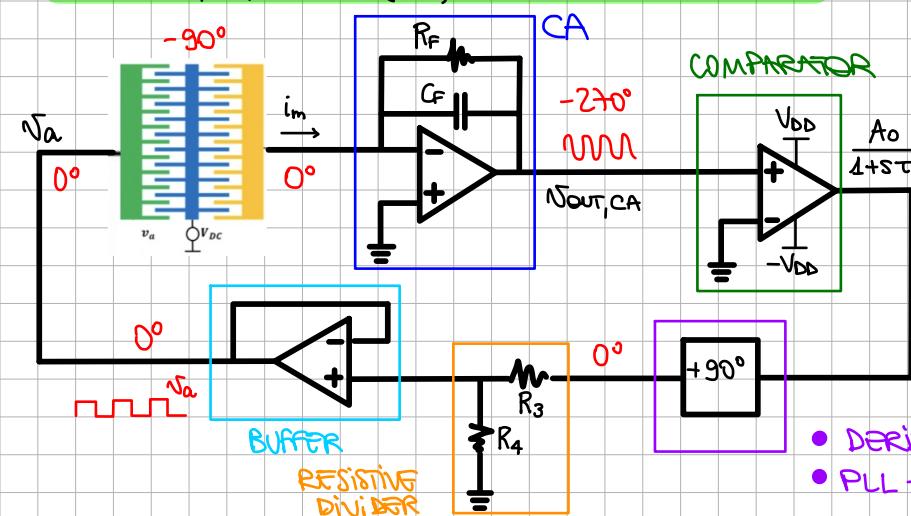
IDEA: use a MOS switched off and exploit the large resistance of the channel

ISSUE: its value is hard to predict and highly depends on the bias voltage which may fluctuate (exponential relationship)

This is a problem for TIA-BASED OSCILLATORS → often a CHARGE AMPLIFIER APPROACH is preferred

CIRCUITS BASED ON COMPARATORS ← charge amplifier front-end
comparator
need for a start-up circuit

• CHARGE AMPLIFIER (CA) FRONT-END SOLUTION

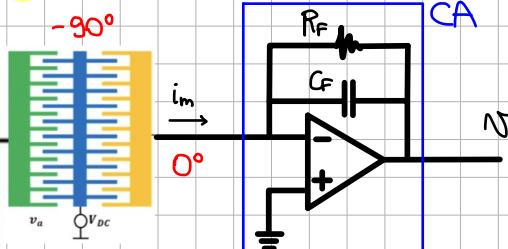


NOTE: how the high-gain stage is substituted by a comparator

NOTE: since we are adopting the approach based on a charge amplifier (integrator), we thus need to recover a 90° phase shift to satisfy the Barkhausen condition on the phase

- DERIVATOR (INTEGRATOR+INVERTER)
- PLL-PHASE WALKED LOOP

① CHARGE AMPLIFIER (CA) STAGE \Rightarrow $f_{p1} \ll f_0$ *1, CA OPERATING REGION

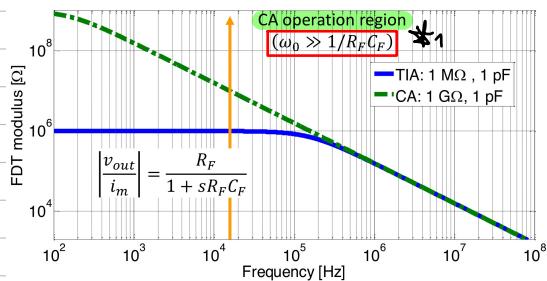


$$V_{out, CA} = -i_m \frac{1}{sC_F}$$

$$\frac{V_{out, CA}}{i_m} (s) = - \frac{R_F}{1 + sR_F C_F}$$

$$\frac{V_{out, CA}}{i_m} (\omega_0) \approx - \frac{1}{sC_F R_{eq}}$$

NOTE: The feedback resistance R_F is still needed to bias the opamp and avoid saturation due to integration of its bias currents.



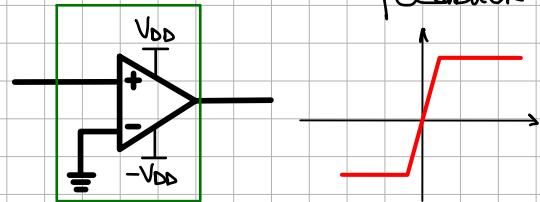
NOTE: Using the charge amplifier the gain is set by the capacitance which is very very repeatable (the resistance only affects the position of the pole)

Resistances implemented through NOS are not easily predictable nor repeatable, however, this has no impact on the CA gain at resonance

$$|G_{CA}(\omega_0)| = \left| \frac{V_{out, CA}(\omega_0)}{i_m} \right| = \frac{1}{\omega_0 C_F}$$

The only condition to match is: $f_p \ll f_0 \rightarrow \frac{1}{2\pi R_F C_F} \ll f_0$

② COMPARATOR \rightarrow it can be easily implemented as an operational amplifier w/ no feedback



- NOTE:**
- The rise and fall times of the square wave will be determined by the opamp slew rate
 - offset can cause the saturation of the comparator before it has set up our circuit
 - comparator can be implemented w/ cascaded inverters

PRO: • very low PC wrt the high-gain slope **CON:** • comparator needs a low input offset to correctly square an input sinewave

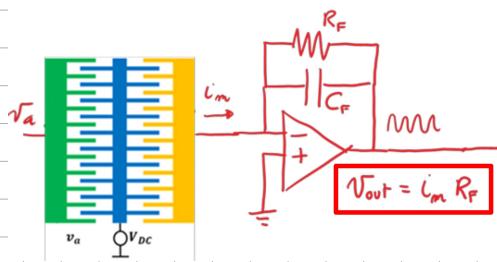
ATT: an amplitude limitation is still required in order to satisfy the small signal condition for the 1st harmonic

③ RESISTIVE DIVIDER

Additionally, a $+90^\circ$ PHASE SHIFT is required to match the Barkhausen condition on the phase, as the CA introduces a -90° shift

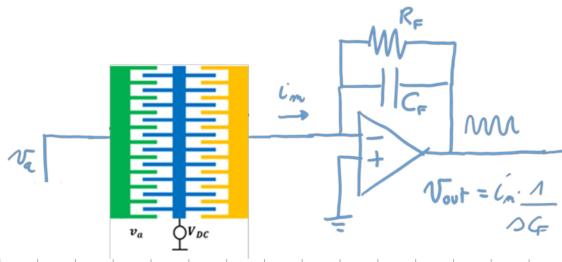
④ DERIVATOR (or INTEGRATOR + INVERTING STAGE) or a PLL-PHASE LOCKED LOOP

TIA - BASED FRONT-END SOLUTION



$$\text{TIA : } i_m = \eta_s \dot{x}_d \rightarrow V_{out} = \eta_s \dot{x}_d$$

CA - BASED FRONT-END SOLUTION



$$\text{CA : } i_m = \eta_s \dot{x}_d \rightarrow V_{out} = \eta_s \int \dot{x}_d dt = \eta_s x_d$$

NOTE: In a CA based front-end, since we are integrating i_m , V_{out} is no more proportional to the velocity \dot{x}_d , but it is directly proportional to the displacement x_d .

CONCLUSIONS

- TRADE-OFFS :
 1. A high Q would be desirable as the electronics gain could be lower (power dissipation would correspondingly decrease)
However, a high Q usually demands for low pressures which are poorly repeatable
 2. A high bias voltage on the rotor would increase the transduction factors (η_A , η_s), but it also makes increase the power consumption

MEMS RESONATORS

PART 3: EFFECTS OF FEEDTHROUGH CAPACITANCE

MOTIVATIONS AND GOALS

PROBLEM: The resonator model we have used so far does not take into account possible parasitic electrical components affecting the nodes where the MEMS is electrically connected to the sustaining circuit, i.e. the drive port node and the sense port node.

ATT: These nodes see large parasitic capacitances and share to ground and, above all, may see also a direct coupling in b/w them.

ELECTRICAL MODEL W/ PARASITICS

Feedthrough capacitance

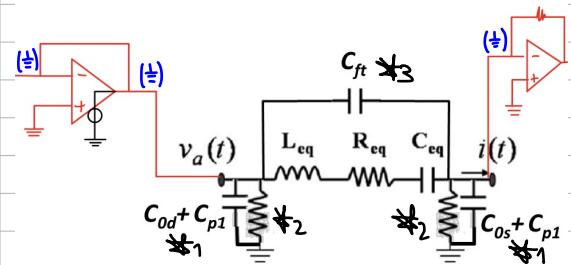
Effects of the electrical admittance

IDEAL LINEARIZED ELECTRICAL MODEL

$$\frac{i_m(s)}{V_a} = \frac{1}{\left(\frac{m}{\eta^2} s^2 + \frac{b}{\eta^2} + \frac{k}{s\eta^2} \right)} = \frac{1}{sL_{eq} + R_{eq} + \frac{1}{sC_{eq}}} \quad \begin{cases} C_{eq} = \frac{\eta^2}{k} \\ R_{eq} = \frac{b}{\eta^2} \\ L_{eq} = \frac{m}{\eta^2} \end{cases}$$

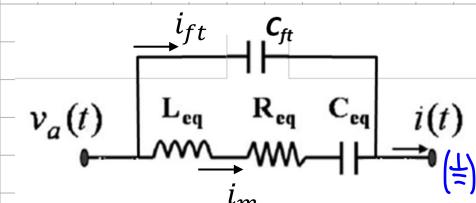
To make this ideal model more realistic, we take into account parasitic electrical elements that affect the resonator:

- CAPACITANCES TO GROUND (parasitic and drive/sense capacitances at rest) $\cancel{*}_1$
→ They are b/w ground and a low impedance node → They can be neglected
- PARASITIC RESISTANCE TO GROUND $\cancel{*}_2$
large value ($\gg R_{eq}$) → They can be neglected
- PARASITIC CAPACITANCE DIRECTLY COUPLING THE ACTUATION PORT TO THE SENSE PORT $\cancel{*}_3$
↓
it's called **FEEDTHROUGH CAPACITANCE** but a portion of the signal applied to the drive port is fed directly to the sense port through this parasitic capacitance
→ even small values (0.1 - 10 fF) can be critical for the resonator behavior.



NOTE: C_{ft} is due to the fact that the distance b/w the two electrodes is short

ATT: We need to modify the expression of the electrical admittance previously found, taking into account the presence of the feedthrough capacitance



NOTE: Admittances in // can be summed

$$\begin{cases} i_{ft}(s) = s C_{ft} V_a(s) \\ i(s) = i_m(s) + i_{ft}(s) \\ i_m(s) = \frac{V_a(s)}{sL_{eq} + R_{eq} + \frac{1}{sC_{eq}}} \end{cases}$$

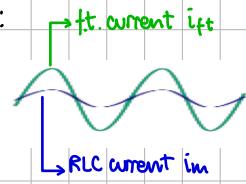
$$\Rightarrow \frac{i(s)}{V_a(s)} = \frac{i_m(s) + i_{ft}(s)}{V_a(s)} = \frac{1}{sL_{eq} + R_{eq} + \frac{1}{sC_{eq}}} + sC_{ft}$$

**ELECTRICAL ADMITTANCE (RESONATOR TF)
W/ FEEDTHROUGH EFFECT**

NOTE (IMP!): the feedthrough capacitance C_{ft} adds a growing contribution w/ frequency that becomes eventually dominant for large frequencies

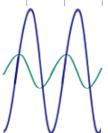
→ depending on f this contribution can be negligible or relevant:

- $\omega \ll \omega_0$ (LOW FREQUENCIES) →
$$\frac{i(s)}{V_{in}(s)} = sC_{eq} + sC_{ft} = s(C_{eq} + C_{ft})$$



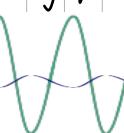
- both contributions (C_{eq} and C_{ft}) go linearly w/ s , so the phase doesn't change (it remains at $+90^\circ$)
- in terms of modulus, usually $C_{ft} > C_{eq}$ so the modulus will slightly increase

- $\omega \approx \omega_0$ (AROUND RESONANCE) →
$$\frac{i(s)}{V_{in}(s)} = \frac{1}{R_{eq}} + sC_{ft}$$

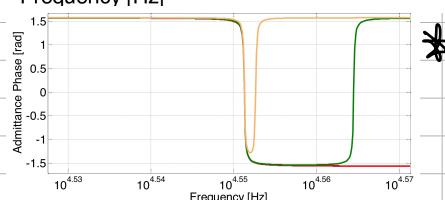
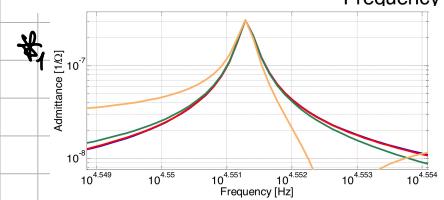
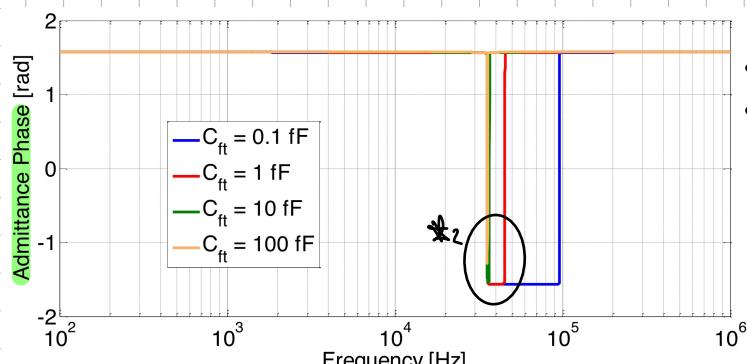
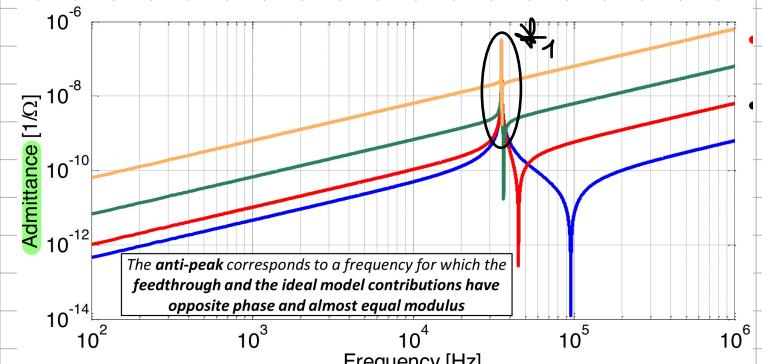


- $\omega \lesssim \omega_0 \rightarrow |sC_{ft}| < \frac{1}{R_{eq}} \rightarrow$ The phase doesn't change, remains at $+90^\circ$
- $\omega \gtrsim \omega_0 \rightarrow |sC_{ft}| > \frac{1}{R_{eq}} \rightarrow$ the phase is dominated by the feedthrough

- $\omega \gg \omega_0$ (HIGH FREQUENCIES) →
$$\frac{i(s)}{V_{in}(s)} = \frac{1}{sL_{eq}} + sC_{ft}$$



→ The phase is dominated by the feedthrough as its contribution is much larger than $1/sL_{eq}$
→ the two signals have opposite phase



NOTE: there is a strange effect: right after the resonance freq. ω_0 (right after the peak) while the phase will have a rapid change for the contribution given by the motional current i_m back to -90° , the modulus may be slowly decreasing.
So, we may find a point right after ω_0 where the phase of the two signals (i_m and i_{ft}) is already opposite, but the amplitude is comparable.

Basically, in that point, we are summing two signals that have similar amplitude but opposite phase and this will essentially yield to a zero in the transfer function



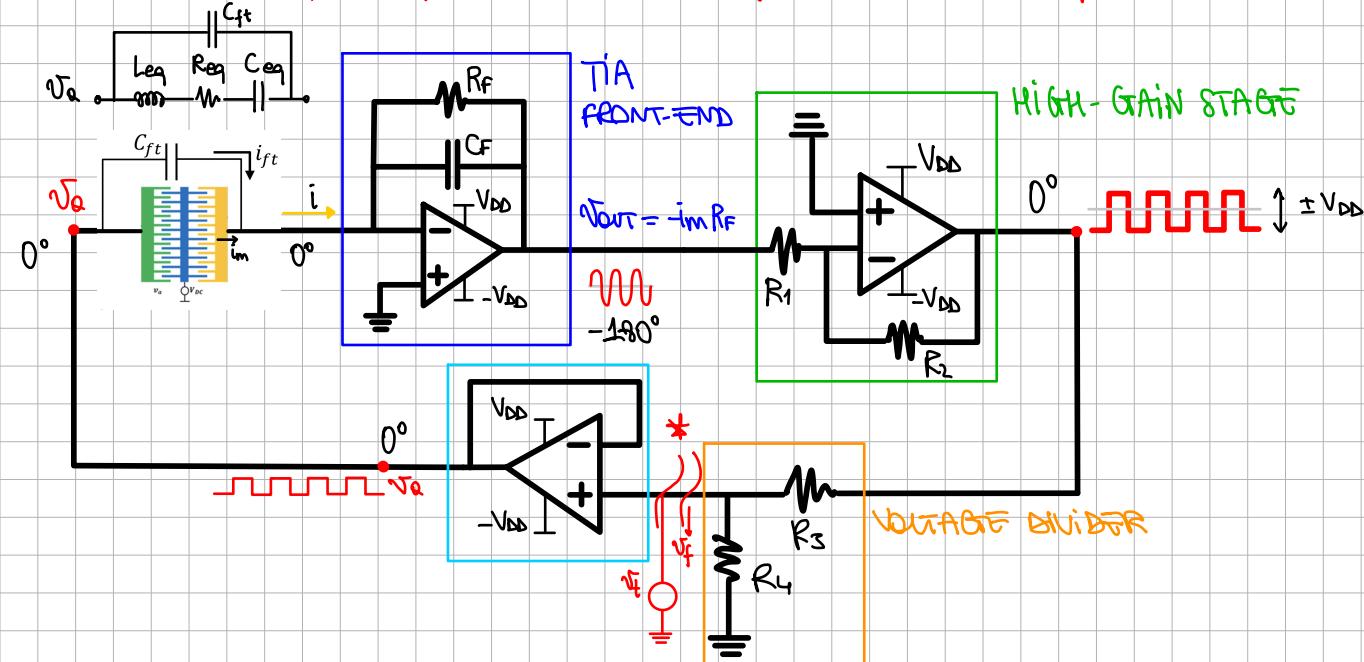
presence of an **ANTI-PEAK**

EFFECTS OF FEEDTHROUGH ON ELECTRONIC CIRCUITS

Loop gain w/ feedthrough
Compensation through poles
Compensation through auxiliary circuits

Is our self-sustained oscillator still working even in presence of this parasitic capacitance that generate feedthrough effect?

What are the effects of C_{ft} on the loop gain and phase of the whole oscillator?



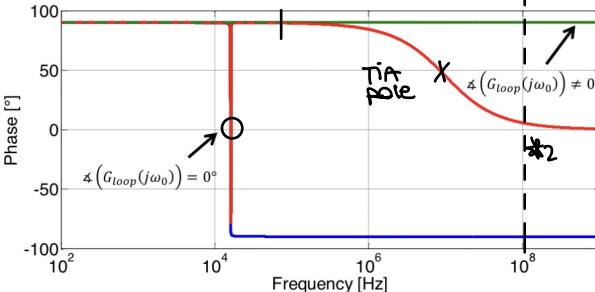
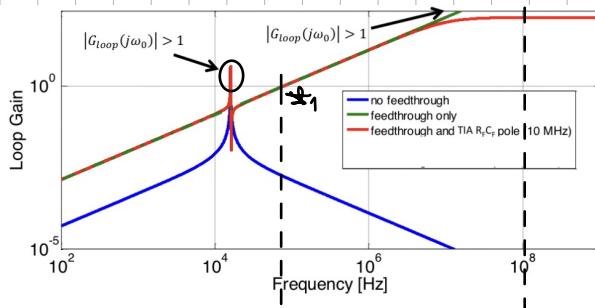
LOOP CALCULATION PROCEDURE :

1. Cut the loop in a high impedance node * (to avoid the need to reconstruct the infa)
2. Probe the circuit w/ a test generator \mathcal{V}_t
3. Compute the transfer function $\mathcal{V}_f/\mathcal{V}_t$

$$\Rightarrow G_{loop}(s) = \left(\frac{n^2}{m} \frac{s}{s^2 + \frac{b}{m}s + \frac{k}{m}} + sC_{ft} \right) \frac{-R_F}{1 + sR_F C_F} \frac{-R_2}{R_1} \frac{R_4}{R_3 + R_4}$$

LOOP GAIN

MEMS ADMITTANCE + FEEDTHROUGH TIA TF HIGH-GAIN STAGE TF RESISTIVE DIVIDER TF



NOTE: the term sC_{ft} generates an anti-peaks in the modulus and a return to $\pi/2$ of the phase
 \rightarrow as a consequence the $|G_{loop}|$ is >1 for several frequencies other than w_0 .

*1, In a range of frequencies above the resonance we have $|G_{loop}| > 1$, but in this range the phase condition is not met, so there is no risk of spurious oscillations

*2, The circuit will have at least the TIA pole that generates a -90° phase shift which leads the phase to 0° in a region where the loop gain is much larger than 1

PROBLEM: a lot of frequencies other than w_0 can oscillate
 \rightarrow it's an undesired effect: the circuit will not work as desired

\Rightarrow we want that our circuit oscillates only at w_0

SOLUTION ①: a way to limit this effect is the introduction of additional poles in the circuit

IDEA: to lower the loop gain right after the resonance peak to avoid chances that it grows larger than 1 for other frequencies

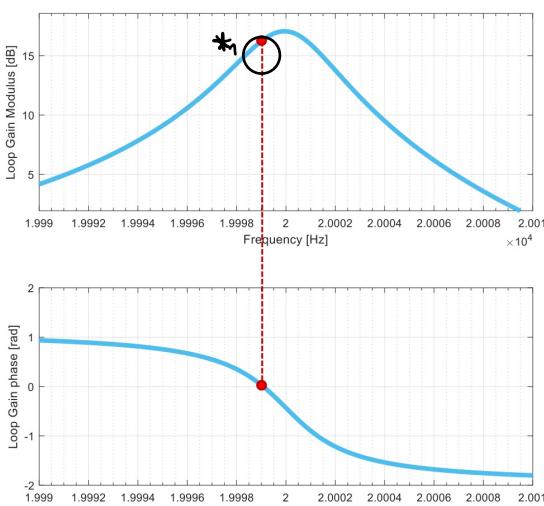
Att: as the phase shift introduced by an RC pole (-90°) begins well before the pole frequency, in order to avoid EFFECTS ON THE PHASE LAG AT RESONANCE, we should ideally place the pole at least one decade beyond resonance

PROBLEM: if you place the pole too close to resonance, since the phase starts changing one decade before the pole freq. The risk is that we may introduce an additional phase shift at resonance due to this new pole and The Barkhausen criteria would not be satisfied anymore (we wouldn't have $\angle G_{loop}(W_0) = 0^\circ$).

$$\Rightarrow G_{loop}(s) = \left(\frac{h^2}{m} \frac{s}{s^2 + \frac{b}{m}s + \frac{k}{m}} + sC_{ft} \right) \frac{-R_F}{1 + sR_F C_F} \frac{-R_2}{R_1} \frac{R_4}{R_3 + R_4} \frac{1}{1 + sT_{op}}$$

optimp pole

zoom around the resonance frequency



* The Barkhausen condition on the modulus is still satisfied though is not satisfied exactly at resonance b/c the phase crosses the 0° point at a frequency different from resonance

→ This means that the circuit will be able to oscillate, but it will not oscillate at the resonant peak of our transfer function

→ The working point of the resonator instead of being in correspondence of the peak will move slightly leftwards

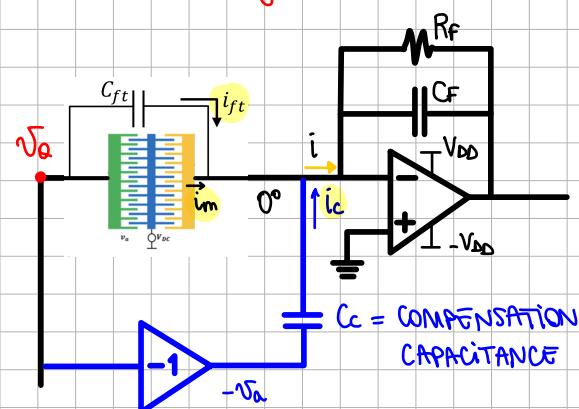
ISSUE: The working point will continue to move away from the resonance on the passing of time, therefore the resonator will become less and less efficient until the working point will cross the 0dB and at that point the resonator will have no longer any oscillation

→ we have to find another solution

SOLUTION ②: an alternative and definitive solution for the feedthrough is to compensate it at the origin i.e. to sum at the sense node a current opposite to the feedthrough current in order to cancel out the feedthrough contribution

→ we have to add to the sense port an additional circuit branch which is called FEEDTHROUGH COMPENSATION CIRCUIT

How can we generate a current which is equal and opposite to i_{ft} ?



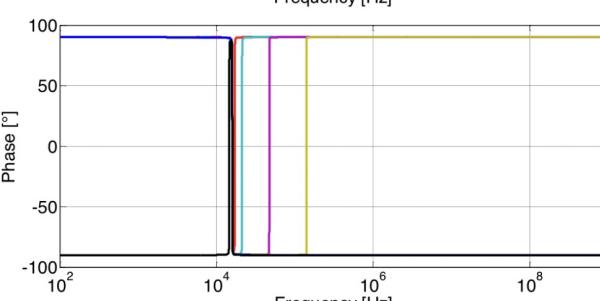
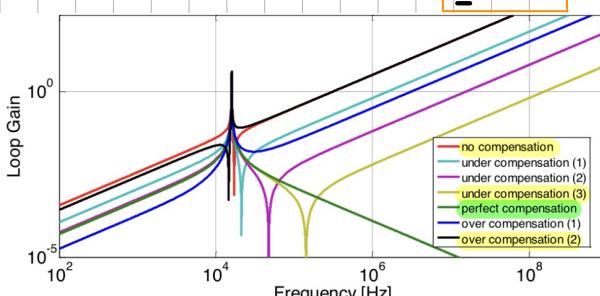
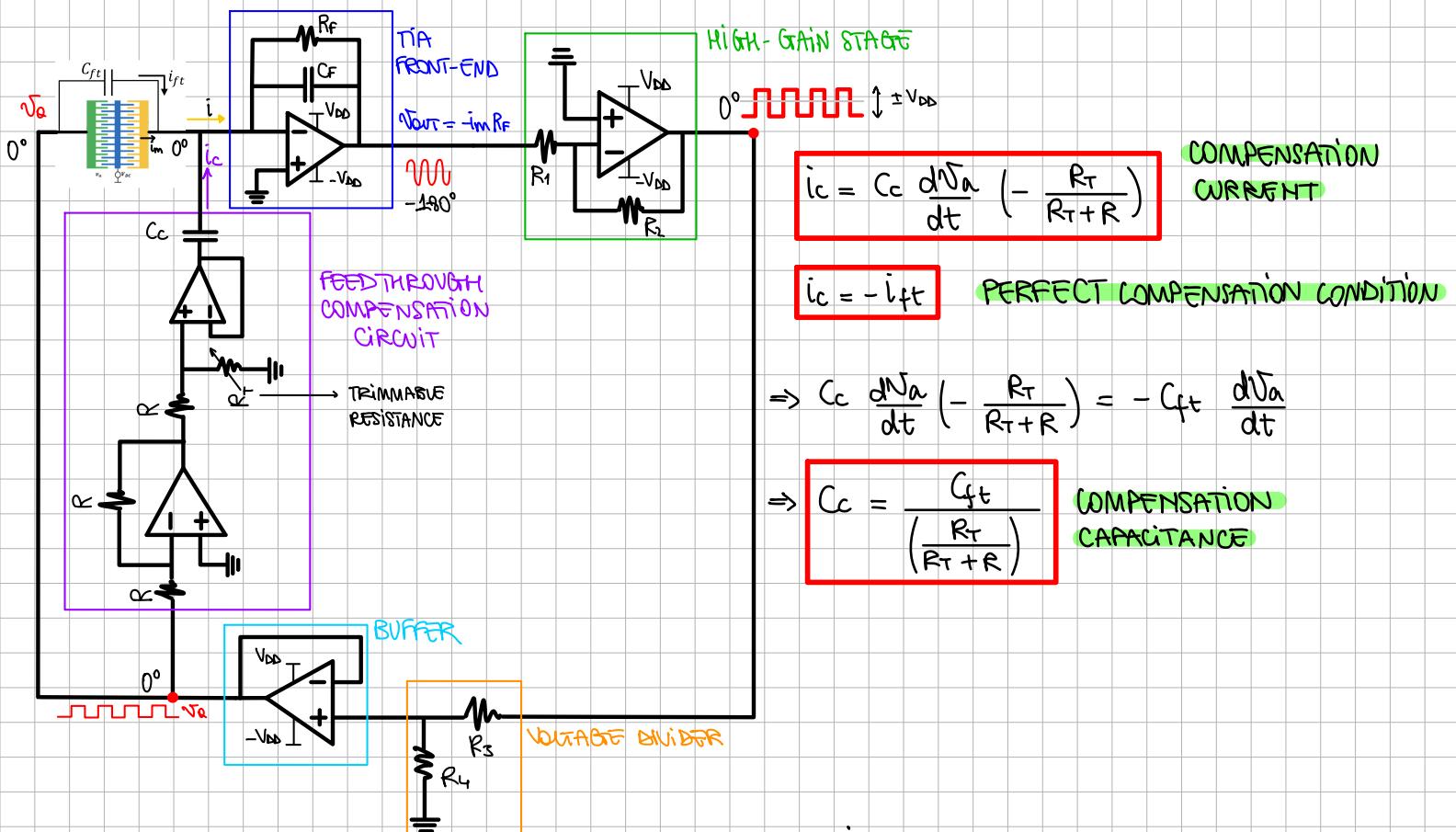
$$\left. \begin{aligned} i_{ft} &= C_{ft} \frac{dV_a}{dt} \\ i_c &= -C_c \frac{dV_a}{dt} \end{aligned} \right\} \text{if } C_c = C_{ft} \rightarrow i_c = -i_{ft}$$

Att: The compensation circuit is different from this one essentially for 2 reasons:

1. it's not possible to know C_{ft} a priori but it is a parasitic term (it's hard to predict it)
2. C_{ft} value is very low and so it's difficult to implement it

→ we have to digitally tune a capacitive gain, excited by a current which is in antiphase w.r.t. the feedthrough signal

FEEDTHROUGH COMPENSATION CIRCUIT:



- NO COMPENSATION:

- PERFECT COMPENSATION:

- UNDER COMPENSATION: still on antipeak after ω_0

OVER COMPENSATION: The antipeak is before the resonance peak at ω_0

$$\Rightarrow \frac{i(s)}{\sqrt{a(s)}} = \left(\frac{r^2}{m} \frac{s}{s^2 + \frac{b}{m}s + \frac{k}{m}} + sC_{ft} - sC_C \frac{R_T}{R_T + R} \right)$$

ELECTRICAL ADMITTANCE OF THE RESONATOR
CONSIDERING THE FT CAPACITANCE AND THE
FT COMPENSATION CIRCUIT

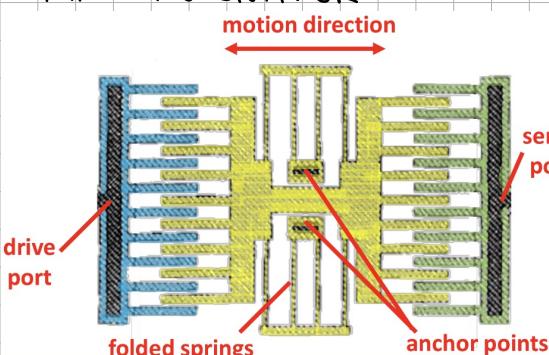
DRAWBACK: additional circuit branch = additional power consumption

1. Is a high Q factor an advantage or a disadvantage in presence of C_{ft} ?
2. Is a high rotor voltage an advantage or a disadvantage in presence of C_{ft} ?
3. How does the situation change if we adopt a CHARGE AMPLIFIER APPROACH instead of the TIA approach? (remember that we need the extra $+90^\circ$ stage)

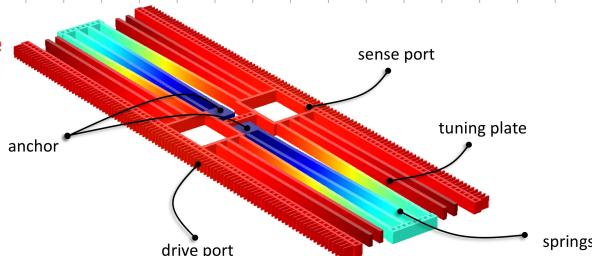
COMMON RESONATOR TOPOLOGIES

Tang resonator
Tunable Tang resonator
Clamped-clamped beam resonator

TANG RESONATOR



The springs are designed as four 2-fold beams
→ This allows a large capability for relief of built-in residual stress and temperature stress in the structural film



tuning the frequency = changing the frequency

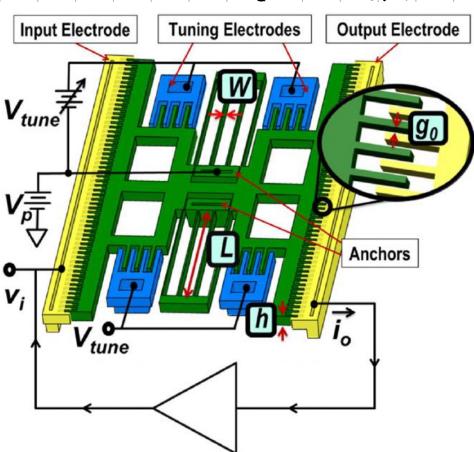
it can be done in 2 ways: • changing the mass
• changing the stiffness *

* REM: we have already seen that PPs in every MEMS device imply an electrostatic softening effect that corresponds to a downshift of the frequency.

→ if we embed in our resonator some suitable PPs formed by the rotor and suitable stators and we apply a voltage difference b/w these PPs we will have an electrostatic softening which will tune our resonance frequency

⇒ a resonator which exploits the electrostatic softening effect to tune its resonance frequency is the:

TUNABLE TANG RESONATOR



At: exploiting electrostatic softening to tune the resonance frequency, only downshift is possible

for this reason we have to target larger frequencies

→ This tuning is obtained by means of PPs (as we already said)
→ These are not used for activation and sensing, but just to tune the frequency

NOTE: The larger the "frequency displacement", the larger the bias voltage we need to apply to the tuning electrodes to provide the tuning

⇒ TRADE-OFF BTW OBTAINABLE Q FACTOR VS DISPLACEMENT

OBTAINABLE FREQUENCY SHIFT

$$\Delta\omega = \sqrt{\frac{k_{TOT}}{m}} - \sqrt{\frac{k_m - 2(V_{DC} - V_{tune})^2 C_{PP}/g_0^2}{m}}$$

CONCLUSIONS

The presence of the parasitic feedthrough capacitance may lead to the generation of frequencies other than the MEMS resonance one which satisfy the Barkhausen criterion

• Feedthrough effects can be:

- MINIMIZED BY ACCURATE DESIGN → keep distant the drive and sense pads;
- COMPENSATED THROUGH POLES → There is the risk that the circuit will oscillate at a freq. slightly different from the maximum one
- COMPLETELY COMPENSATED BY SUITABLE CIRCUITS → you pay in terms of power consumption
- MADE NEGIGIBLE BY VERY LARGE Qs