CreditRisk+ Model

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In the framework of the CreditRisk+ model developed by Credit Suisse in the late 90's I'm going to replicate the results obtained analitically through the Monte Carlo approach. In the original example is given a portfolio with 25 risks classified in 8 different classes with corresponding exposures and default rate probabilities, so let's just start by importing them:

```
library(readxl)
cr2 <- read_excel("cr2.xls", sheet = "Example2",
                  range = "C10:J35")
N = 25
K = 3
PTF = data.frame(Exposures = cr2$Exposure,
                 CreditRating = cr2$Rating,
                 DefaultRate = cr2$`Default rate`,
                 sd = cr2$Deviation)
```

Following "Example 2" each obligor is allocated to only one systematic factor: the country in which its located, it is responsible for all of the uncertainty of the default rate. So let's import those market data:

```
MKT = data.frame(US = cr2$US,
                 JPN = cr2$Japan,
                 EU = cr2$Europe)
```

Now that all the necessary data is imported we can compute the credit loss distribution. All the technical details are explained in the Credit Suisse paper, here I just give a tiny summary of the Loss distribution:

$$L = \sum_{i=1}^{N} Y_i \cdot E_i, \; where \; Y_i \sim Poisson(\lambda)$$

$$\lambda_i = p_i \cdot \left(\omega_{i0} + \sum_{k=1}^{K} \omega_{ik} \cdot \Gamma_k\right)$$

The Gamma variable represents the latent variable which describes the market and its parameters are calculated below:

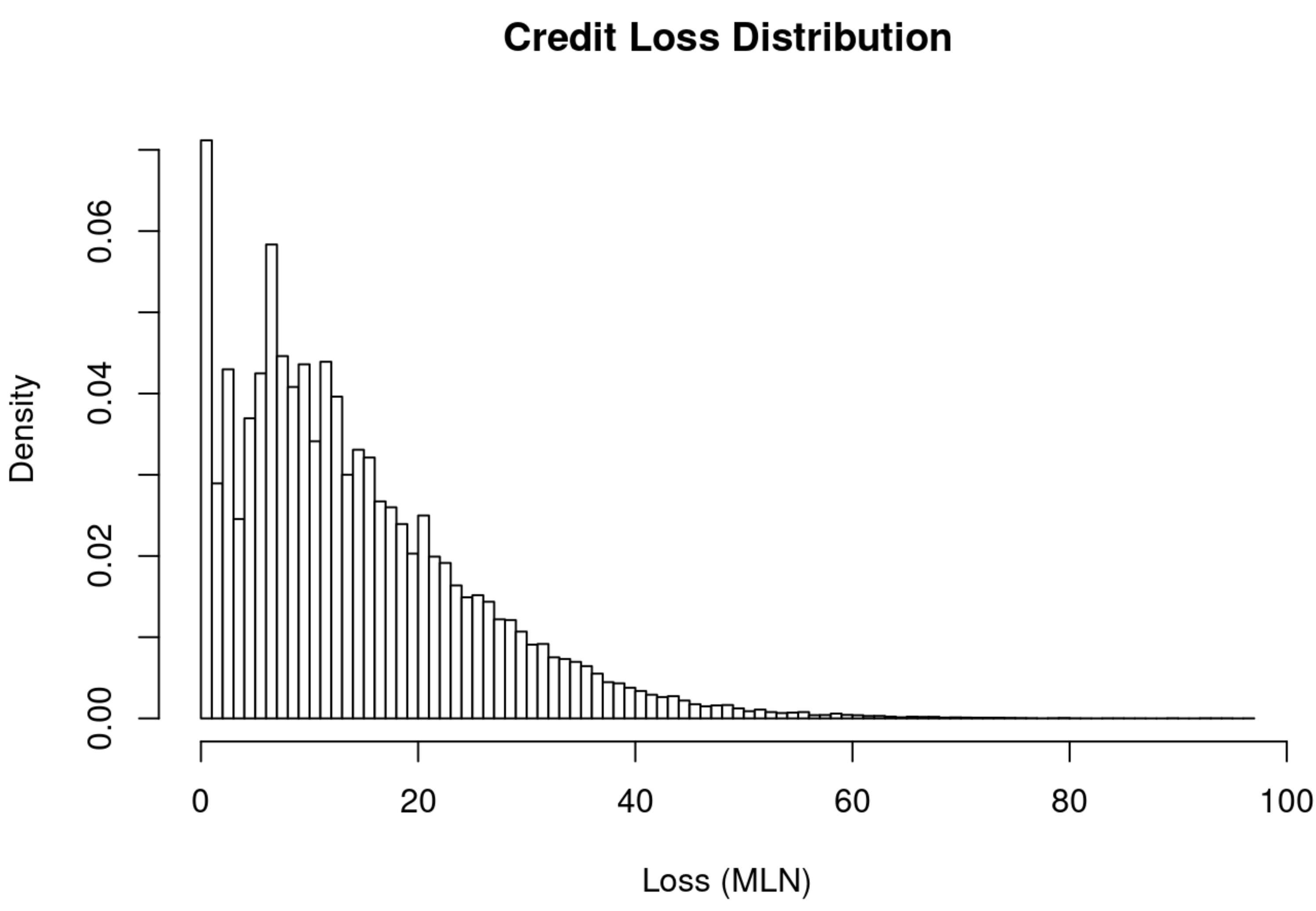
```
omega = matrix(c(MKT$US,MKT$JPN,MKT$EU),nrow = N,ncol = K,byrow = F)

mu = PTF$DefaultRate %*% omega;
sd = PTF$sd %*% omega;

alfa = (mu/sd)^2
beta = sd^2/mu
```

Now we can compute the Loss Distribution through Monte Carlo, in particular using 10^5 simulations and a fixed seed in order to replicate the following results:

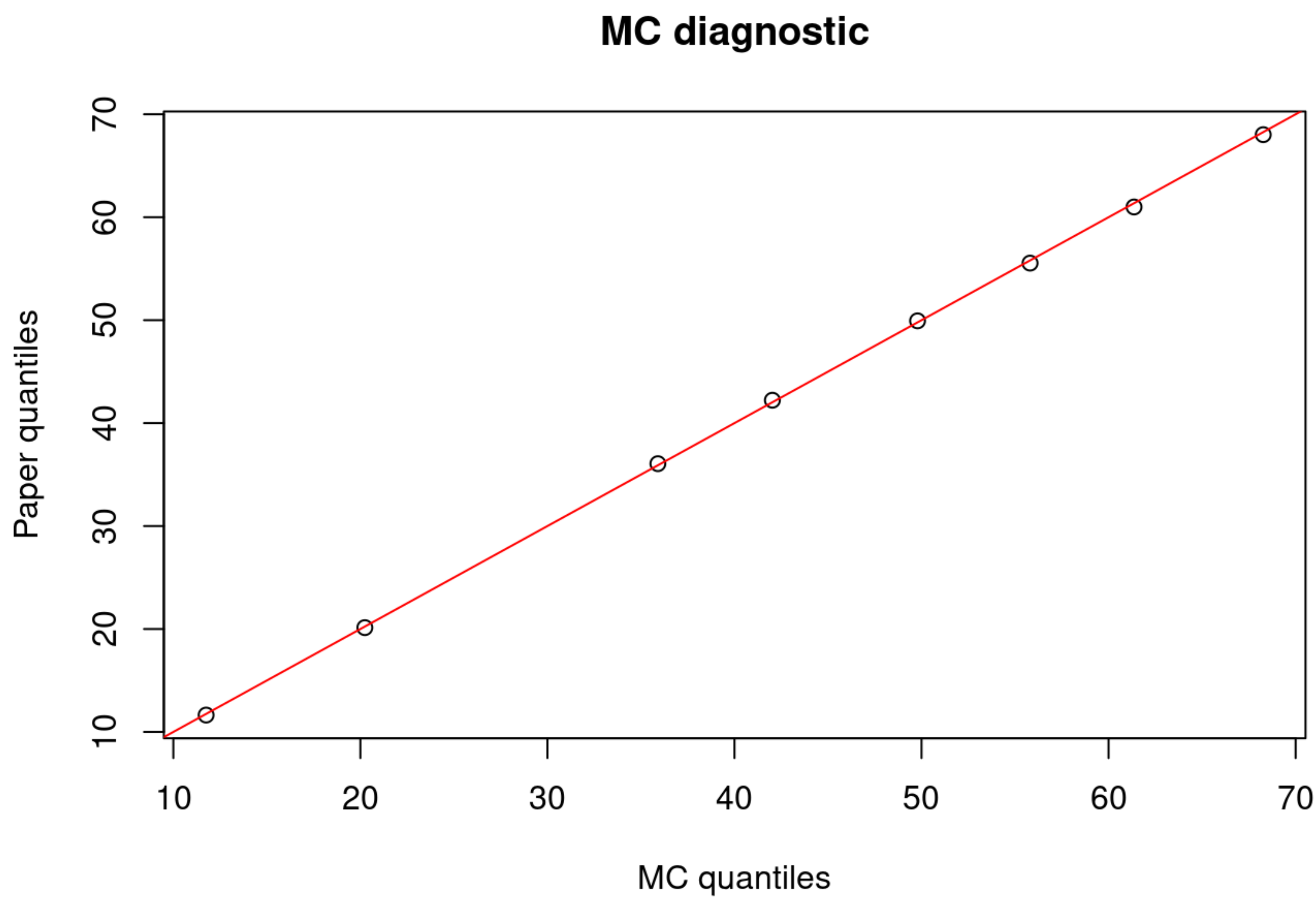
```
set.seed(07112022)
nsim = 10^5
L = array(data = NA, dim = nsim)
lambda = array(data = NA, dim = N)
for (j in 1:nsim) {
  gamma = rgamma(n = K, shape = alfa ,scale = beta)/(alfa*beta)
  for (i in 1:N) {
    lambda[i] = PTF$DefaultRate[i]*(sum(omega[i,]*gamma))
  }
  L[j] = PTF$Exposures%*%rpois(n = N, lambda = lambda)
}
hist(L/10^6,breaks = 100,xlab = "Loss (MLN)",probability = T, main = "Credit Loss Distribution",
col = "white")
```



Before showing some risk measures let's check the paper results with the Monte Carlo ones by looking at the quantiles of the two distributions:

```
qq_MC = quantile(L, probs = c(0.5, 0.75, 0.95, 0.975, 0.99, 0.995, 0.9975, 0.999))
qq_Paper = c(11638427, 20130908, 36057338, 42221594, 49931502,
             55540041, 60993272, 68017542) #Extracted from xls files

plot(qq_MC/10^6,qq_Paper/10^6,xlab = "MC quantiles",ylab = "Paper quantiles")
abline(0,1,col="red")
title(main = "MC diagnostic")
```



From this last graph we can observe a good behaviour of the MC method. In the fourth chapter of the CreditRisk+ paper is given a broad explanation about the utility of Economic capital as a measure of risk, since it takes into account the benefits of diversification, credit quality and exposures. Moreover, it is a dynamic measure which can be used for portfolio optimization.

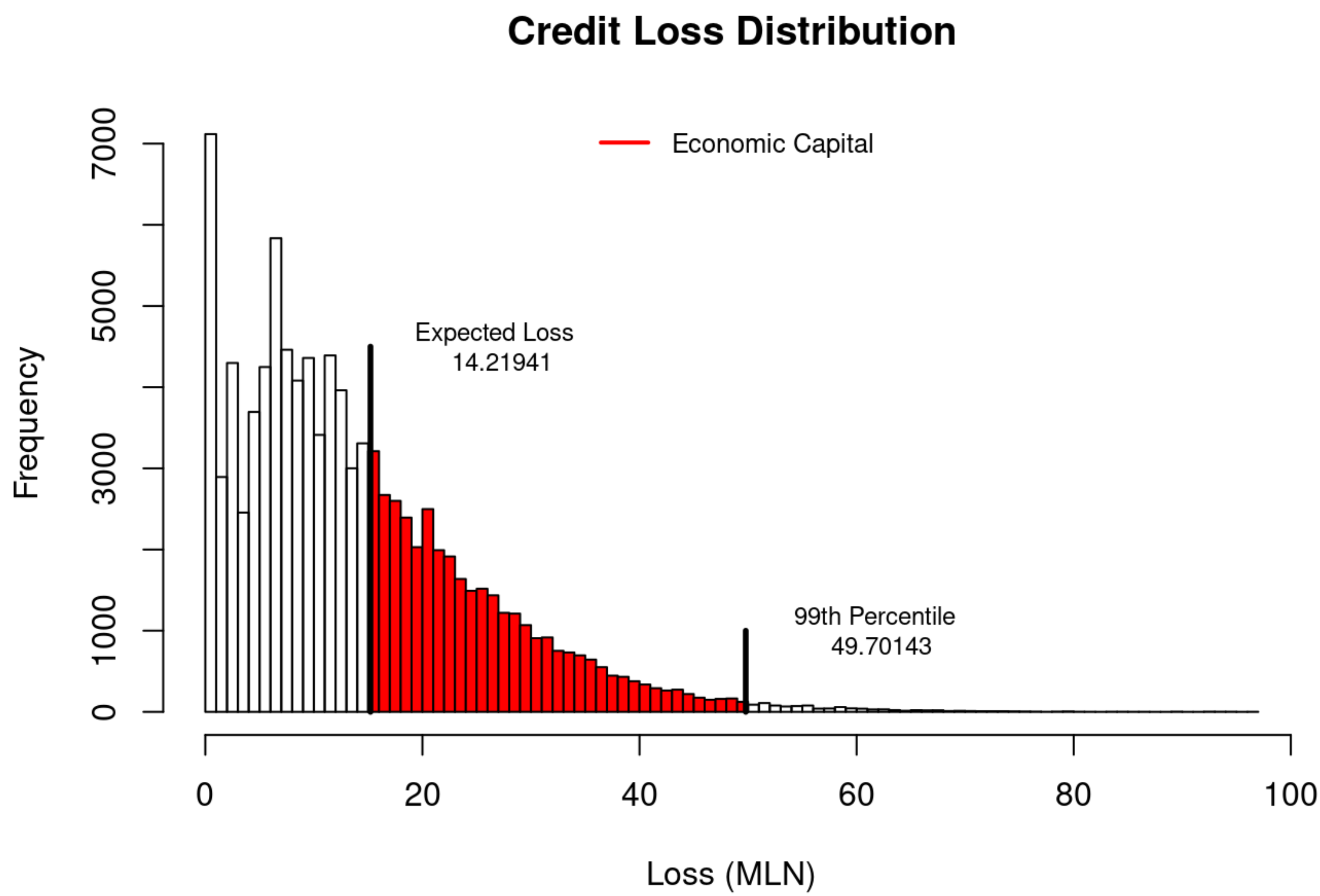
```
#Statistics
ExpLoss = mean(L)/10^6
Perc_99 = quantile(L/10^6,probs = 0.99)

#Histogram
h <-hist(L/10^6,breaks = 100,plot = F)

cuts <- cut(h$breaks, c(ExpLoss,Perc_99))
plot(h, col = c("red")[cuts],xlab = "Loss (MLN)", main = "Credit Loss Distribution")

text(x = 27,y = 4500, "Expected Loss \n 14.21941",col="black",cex=0.75,lwd=2)
xy_EL = matrix(c(ExpLoss+1,0,ExpLoss+1,4500),nrow = 2,ncol = 2,byrow = T)
lines(x = xy_EL,col="black",lwd=2.8,type = "l")

xy_99 = matrix(c(Perc_99,0,Perc_99,1000),nrow = 2,ncol = 2,byrow = T)
lines(x = xy_99,col="black",lwd=2.8,type = "l")
text(x = 62,y = 1000, "99th Percentile \n 49.70143",col="black",cex=0.75,lwd=2)
legend(col = "red",legend = "Economic Capital",x = "top",cex=0.8,
       box.lwd = 0,lwd=2)
```



Scenario Analysis

We can now observe what would happen to the given portfolio in a period of global economic recession: starting from the assumption that the unconditional probabilities of default in this framework are 30% higher we can now do the former computations in order to observe the changes in the risk measures:

```
PTF$DefaultRate = PTF$DefaultRate*1.3
PTF$sd = PTF$sd*1.3
```

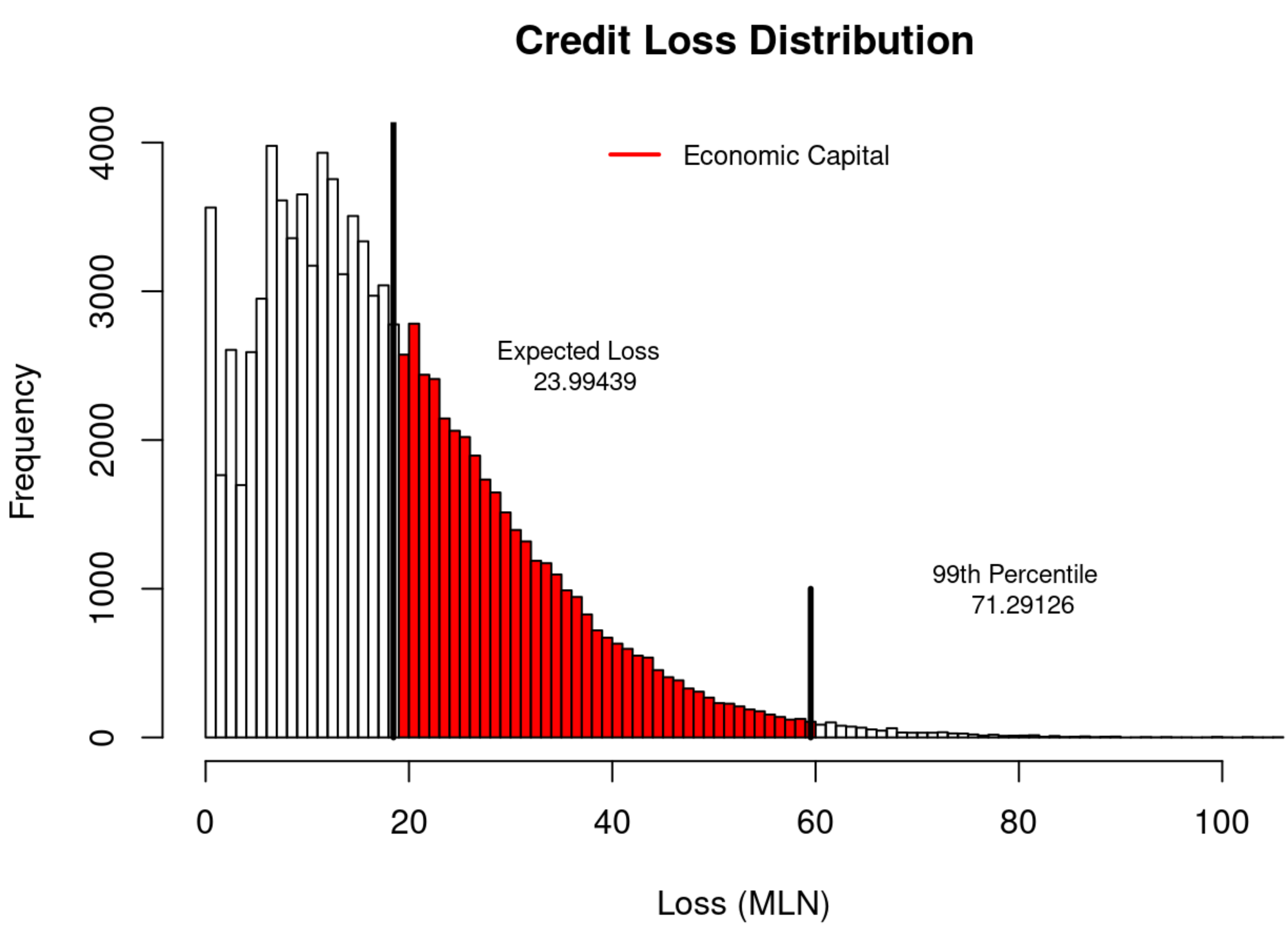
```
#Statistics
ExpLoss = mean(L)/10^6
Perc_99 = quantile(L/10^6,probs = 0.99)

#Histogram
h <-hist(L/10^6,breaks = 100,plot = F)

cuts <- cut(h$breaks, c(ExpLoss,Perc_99))
plot(h, col = c("red")[cuts],xlab = "Loss (MLN)", main = "Credit Loss Distribution")

text(x = 37,y = 2500, "Expected Loss \n 23.99439",col="black",cex=0.75,lwd=2)
xy_EL = matrix(c(ExpLoss,0,ExpLoss,4500),nrow = 2,ncol = 2,byrow = T)
lines(x = xy_EL,col="black",lwd=2.8,type = "l")

xy_99 = matrix(c(Perc_99,0,Perc_99,1000),nrow = 2,ncol = 2,byrow = T)
lines(x = xy_99,col="black",lwd=2.8,type = "l")
text(x = 80,y = 1000, "99th Percentile \n 71.29126",col="black",cex=0.75,lwd=2)
legend(col = "red",legend = "Economic Capital",x = "top",cex=0.8,
       box.lwd = 0,lwd=2)
```



As we can observe there has been an increase in the risk measures calculated above due to the unstable economic situation that we have assumed.