ENGR 65 Extra Credit Project

Section: 8L

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Objectives:

This project is designed in order to determine the maximum energy exerted by any circuit element, while minimizing the current or voltage in the source. Specifically, this project aims to discover if there is any particular circuit element that leads to a generally higher than expected output of energy. This may be either by the individual element itself, or if multiple of that element in series or parallel, or both, may lead to an increase overall in the energy exerted by individual elements, or in the overall theoretical equivalent element. This is important because if there is some way to obtain the largest quantity of energy for the lowest current or voltage value, then that would be quite beneficial. As a result, this project will explore circuits with single or multiple elements, including resistors, capacitors, or inductors, in a circuit. This project will explore simple circuit analysis, primarily discovering the equivalency of resistors, capacitors, or inductors, then will proceed to examine more complex circuits, specifically circuits that would consist of circuit elements in both series and parallel. One thing to note is that there will be a few circuits in this demonstration. They were created using Matlab and Simulink, and the .slx file will be made available at the end of this writeup.

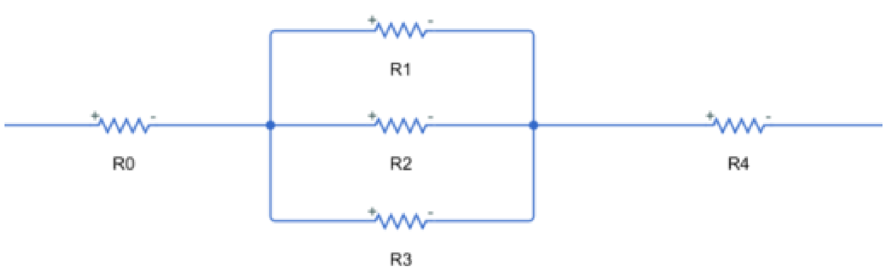
Theoretical Background:

For any given closed circuit, the total energy exerted by all elements in that circuit is 0J. This does not mean that every circuit element needs to have 0J, which is very much not the case. Instead, it means that elements may have negative energy, so that the summation of the energy exerted by all elements equal 0J.

The energy of individual elements is instead determined by mathematical laws. Looking at the definition of energy, energy is the power over time. Formally, it is the integral from time point a to b, of the power function p(t)dt, added to initial energy at time a. Simplified, energy is power times time, or w = pt. For resistors, current sources, and voltage sources, calculating the energy is trivial. This can be seen by obtaining the law $p = vi = v^2/R = Ri^2$. That law multiplied by time will yield the result of energy for either a resistor, current source, or voltage source. For capacitors and inductors, power is a little more complex. For inductors, the energy exerted by that inductor is $w = 0.5Li^2$, while it is $w = 0.5Cv^2$ for capacitors.

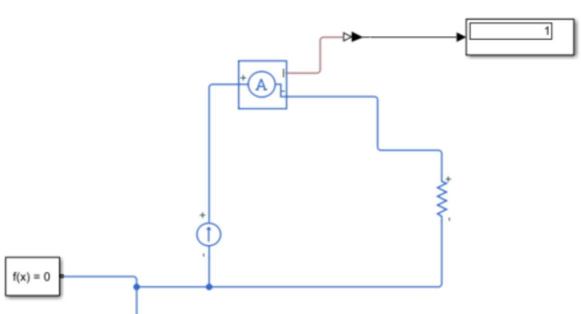
This project will also be looking at circuits which will contain more than one circuit element. The most common way that multiple circuit elements are dealt with in the same circuit are series and parallel. Two circuit elements are considered to be in series if, for the wire connecting those two elements, those are the only two elements connected to that wire. Conversely, two circuit elements are considered to be in parallel if the node where the positive terminal for both circuit elements is the same, as well as the node where both circuit elements have their negative terminal is the same. Importantly, it does not need to only be two elements in series or in parallel. It is possible to have three, four, five, or more elements in either series or parallel. For that to be the case, for series, the circuit elements would need to be one after the other, consecutively. For parallel circuit elements, all of them will need to have their positive terminal at the same node, and their negative terminal at the same node.

Mathematically, each of the 3 circuit elements have different identities for calculating an individual theoretical equivalent for series and parallel. Multiple resistors in series can be treated as an equivalent resistor with the resistance equal to the sum of all the resistors in series, or $R_{eq} = R_1 + R_2 + \ldots + R_n$. For resistors in parallel, the mathematical identity is instead that the equivalent resistor is equal to the inverse of the sum of the inverse of all the resistors, or $R_{eq} = (1/R_1 + 1/R_2 + \ldots + 1/R_n)^{-1}$. This is mathematically identical to the equivalency identity for inductors. Capacitors instead are the inverse, with a capacitor equivalency in parallel being the sum of capacitance of all the capacitors in parallel, of $C_{eq} = C_1 + C_2 + \ldots. + C_n$, and the equivalent capacitor to multiple capacitors in series is the inverse of the sum of the inverses of the capacitors in series, or $C_{eq} = (1/C_1 + 1/C_2 + \ldots + 1/C_n)^{-1}$. Some circuits are complex enough that they contain circuit elements in both parallel and in series. If that is the case, then to solve, the correct solution is to find equivalencies in such a way that the equivalencies found can be used to find larger equivalencies. For example, if there is a resistor in parallel with three resistors in series, first, R_{eq1} will be found to be the sum of the three resistors in parallel, then R_{eq1} will be used to find R_{eq}, along with the other resistor it is in parallel with. For a visual example, the circuit shown shows five resistors, R0, R1, R2, R3, and R4. R1, R2, and R3 are in parallel with each other, and R0 and R4 in series with each other and the set of resistors R1, R2, and R3.

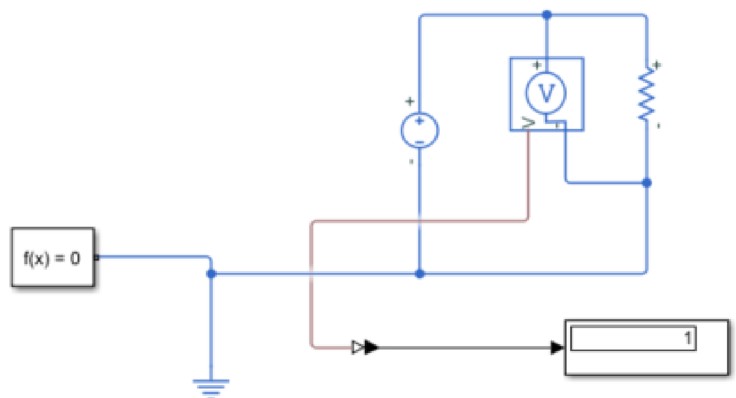


Basic Circuits:

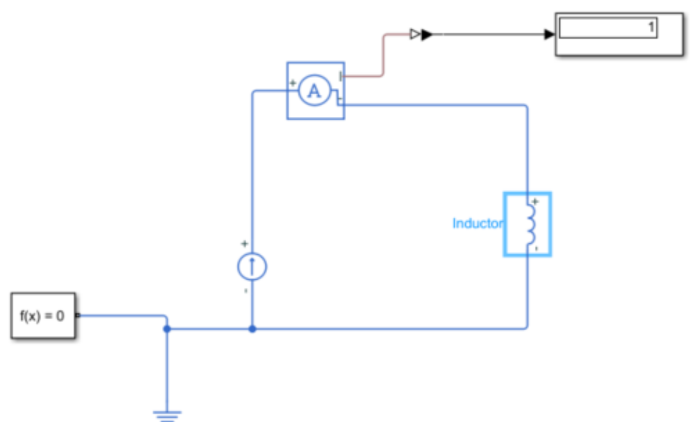
The first type of circuits that must be examined are the simplest type of circuits. They involve only an individual type of circuit element, along with a source. The reason that these circuits need to be examined is because these simple circuits act as controls and baselines to compare the more complex circuits off of. First is a circuit consisting only of a current source and resistor, as shown in the accompanying image. The current source is a DC current source with a value of 1A, while the resistor



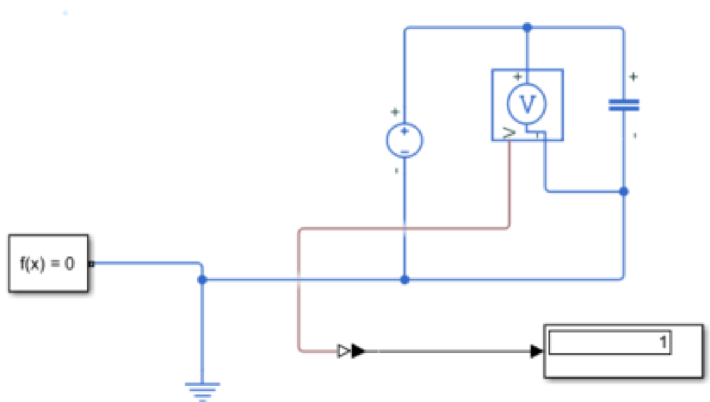
has a value of 1Ω. While it may seem unnecessary to have a current sensor in this case, due to the fact that there is a single circuit element other than the source, it is still useful to have because it will provide uniformity to the remainder of the circuits, which will be more complex, and contain more circuit elements. This circuit will have 1A of current entering the 1Ω resistor, which will lead that resistor to exerting 1J of energy ($W = i^2R = 1A^2*1\Omega = 1J$). This is similar to the next circuit, which is a single voltage source with a value of 1V, and a single resistor with a value of 1Ω. Calculating the energy exerted by the resistor is quite similar to the previous circuit. The energy of this resistor is 1J ($W = v^2/R = 1V^2/1\Omega = 1J$). The next circuit is similar to this circuit, with the difference being that it will have a current source and an individual inductor. Again, the current source will have a value of 1A, and the inductor will have a value of 1μH. Unlike the resistors, the following two control circuits will be using μH and μF, which are the more common values available. Following the formula for energy exerted by an inductor, it is possible to determine that this inductor exerted, in total, 0.5μJ ($W = Li^2/2 = 1\mu H*1A^2/2 = 1*10^{-6}H*1A^2/2 = 0.5*10^{-6}J = 0.5\mu J$). This is noticeably less than the energy exerted by the resistor with a current source.



However that is due to the fact that the resistor was in the scale of Ω, at 10^1, while the inductor was at the scale of μH, which is 10^{-6}. Normalizing for this, it is possible to see that the energy exerted would have been six factors of 10 larger, to be 0.5J, which is half as much as the resistor in either case. Returning to the final control circuit, the control circuit will contain a voltage source of 1V, and a capacitor with capacitance of 1μC. This circuit will have



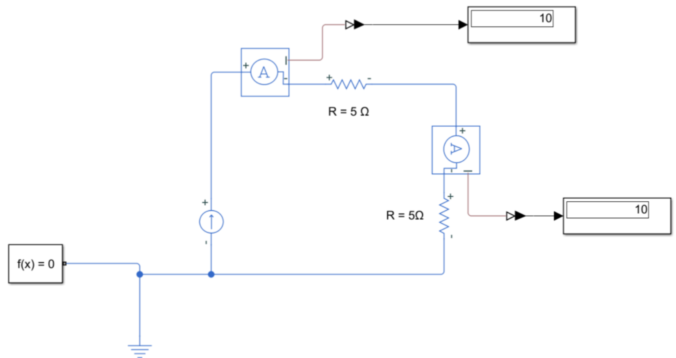
the capacitor exert a total of 0.5µJ ($W = Cv^2/2 = 1\mu F*1V^2/2 = 1*10^{-6}F*1V^2/2 = 0.5*10^{-6}J = 0.5\mu J$). For very similar reasons to the previous inductor circuit, this circuit has substantially smaller energy exerted than either of the two control resistor circuits, and for similar reasons as the control inductor circuit, when normalized for the correct power of 10, this circuit ends up exerting a total of 0.5J of energy, which is on par with the control inductor circuit, and is justvhalf of either resistor circuit.

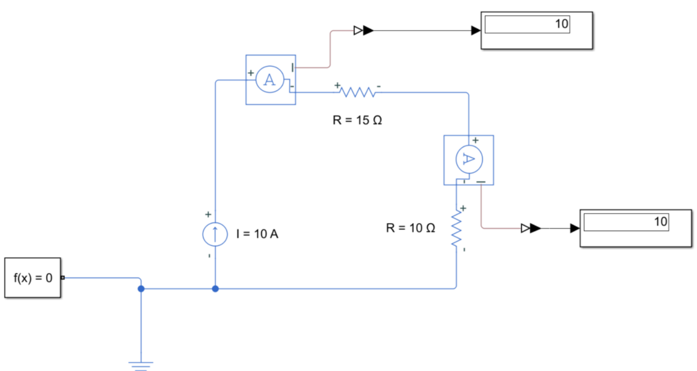


However, all of these four circuits are very simple, and will only contain simple numbers, which are unrealistic. This was done on purpose, to use these simple circuits to find simple conclusions so that they may be expanded to find the energy exerted for more complicated circuits. Using the same circuits as the control circuits, but modifying the values of either the sources, resistors, inductors, or capacitors, it is possible to reveal if there is any pattern. Returning to the current source with an individual resistor, it is possible to extrapolate based on an increase in the current or voltage of the source, an increase in the resistance, inductance, or capacitance, or both. This can be seen in math. For the first circuit, the one where there is a singular current source and a resistor, increasing either the current or the resistance would lead to a larger amount of energy exerted by the resistor. This is because the energy is directly related to the resistance, but quadratically related to the current. This means that a theoretical increase of the current to 10A, and a theoretical increase of the resistance to 10Ω would lead to an energy exertion of 1000J, not 100J, because the current is squared, for example. However, this is outside the scope of this specific project, and will be discussed more in the Conclusions and Next Steps section. For this point forward, for the sake of consistency, each source will provide either 10V or 10A. This is done so that there can be a fairer comparison between the circuits. Additionally, each circuit element will be measured in the same denomination, again, for fairer comparison. This means that each resistor will be measured in the scale of Ω instead of any kΩ, mΩ, or the like. Similarly, any and all inductors will be measured in the scale of H and any capacitors will be measured in the scale of F

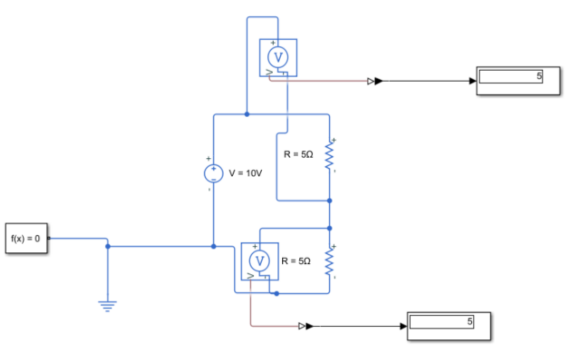
<u>Circuits in Series:</u>

One of the simplest ways that a circuit can be made more complex is by putting multiple of one type of circuit element in series. This is because circuits with circuit elements in series act very similarly to circuits with only an individual element, but instead have multiple elements. This is because there is a possibility to theorize a theoretical equivalent circuit element, as discussed in the previous section Theoretical Background. The first example that will be analyzed is a circuit that consists of a current source that generates 10A, and two resistors in series, each with a resistance of 5Ω. As seen in the diagram, each of the resistors in the circuit has the same amount of current passing through them, which means that this will simplify the math a little bit, since the values for the current and the values for the resistance in both cases are the same. This leads to the fact that the energy exerted by both resistors is the same, at ($W = A^2R = 10A^2{*}5\Omega = 500J$) 500J. That is quite a lot of energy, but that was also

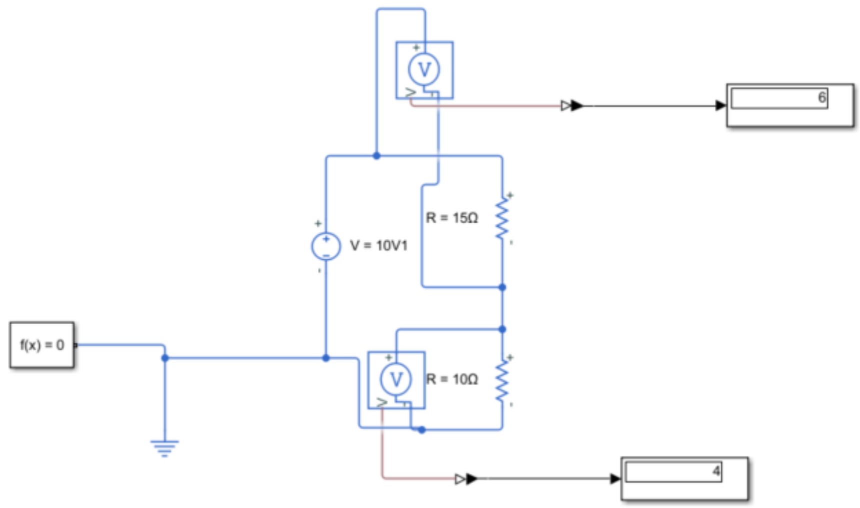


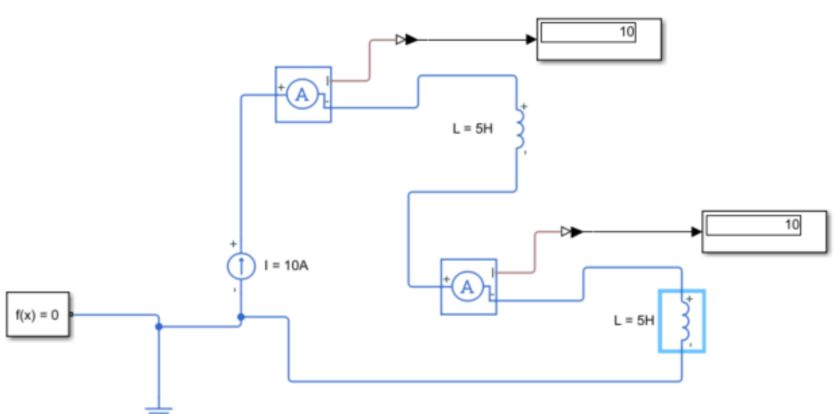


simplified by the fact that both of the resistors had the same resistance, so it was not the most optimal circuit to compare resistors to. Instead, let the circuit be constructed similarly, but give the resistors a different resistance. This second circuit, instead, has two resistors with different values. The current in the source is identical, 10A, in both circuits. However, the second circuit has resistors with R = 15Ω and R=10Ω, in that order. However, the current for both of these cases is identical, 10A, and runs through both resistors. This means that the first resistor is exerting ($W = 10A^2{*}15\Omega = 1500J$) 1500J of energy, while the second resistor is only exerting ($W = 10A^2{*}10\Omega = 1000J$) 1000J of energy. This disparity is entirely due to the increased resistance. Proceeding to testing voltage-resistor circuits in series, it is possible to see that the sum of the voltage drop across resistors is equal to the total voltage of the source. The drop is proportional to the resistance of the resistor compared to the total equivalent resistance. In this case, it makes the math simple again. The resistors will both exert ($W = v^2/R =$

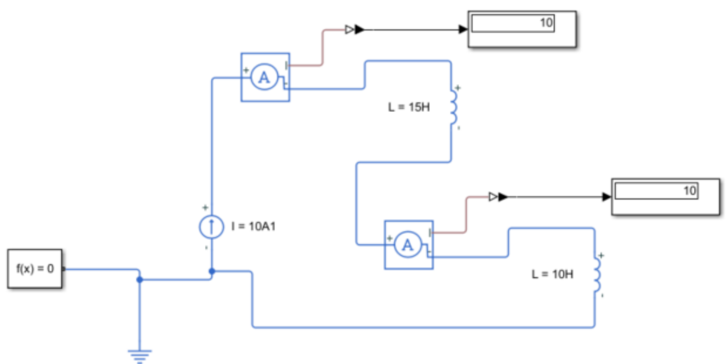


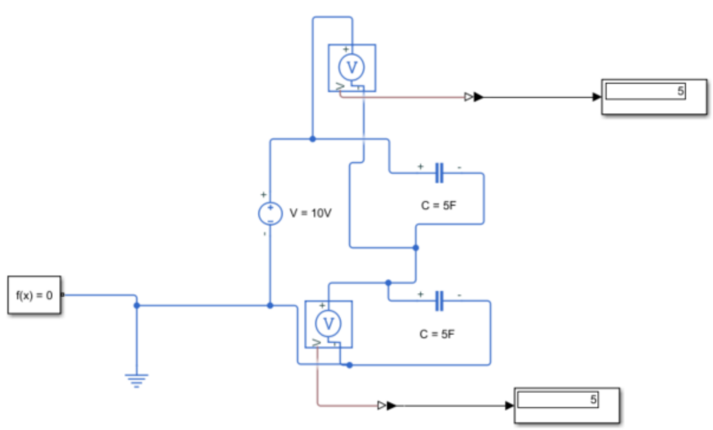
$10W^2/5\Omega = 20J$) 20J of energy. This is quite noticeably lower than the similar amount of current for the same resistance, but that is due to the fact that energy has a proportional relationship with resistance in the current example, but an inverse relationship with resistance in the voltage example. However, in this case, like the first current-resistor case, it is important to note that both of the resistors had the same amount of resistance, which might have an adverse reaction on the circuit. The experiment will be run again, this time using the same resistances as the second current-resistor circuit, 15Ω and 10Ω, in that order. This changes the voltage drop across each of the resistors. The first resistor is 15Ω, or 60% of the total resistance of the equivalent resistor, and has a drop of 6V, or 60% of the total 10V. The second resistor is 10Ω and drops 4V, to complete the 10V drop of the circuit. The energy exerted by the different resistors will therefore be different. For the first resistor, the energy exerted is ($W = v^2/R = 6V^2/15\Omega = 36/15 = 2.4J$) 2.4J, however, the second resistor will exert ($W = v^2/R = 4V^2/10\Omega = 40/10 = 4J$) 4J. In this case, there is a noticeable, and importantly, nonlinear, difference. Hopping over to current-inductor circuits, there is an initial similarity to current-resistor circuits. This is because when the source is 10A, and the inductors are 5H each, the current passing through both inductors is 10A. This leads, again, to simplified math. The energy exerted by both of the inductors is ($W = 0.5*L*i^2 = 0.5*5H*10A^2 = 250J$) 250J. This is quite similar to the results of the current-resistor circuit with the 10A current source and two 5Ω resistors. This is due to the fact that mathematically, the energy exerted by an inductor is just half of the



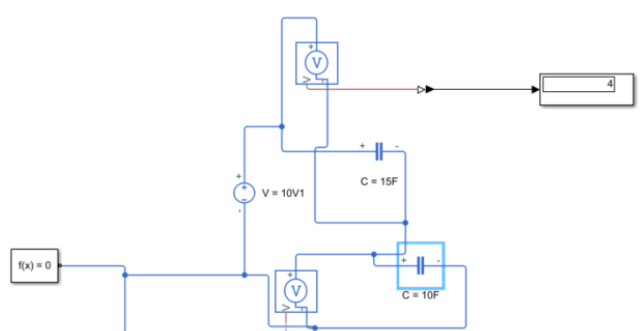


energy, assuming that the current is the same, and the inductor and resistor have the same amount of inductance as resistance. However, this was tested with two inductors which have the same inductance value. Testing this again, only with different values of inductance, it can be seen that there is little change and deviation from both the first current-inductor circuit, and the progression from the first to the second current-inductor circuit resembles a lot the progression from the first to the second current-resistor circuit. Replacing the inductors to two inductors with different values (the first being 15H and the second being 10H), it can be seen that the current does not differ, similarly to the second current-resistor circuit. Calculating the energy exerted for each inductor yields ($W = 0.5*L*i^2 = 0.5*15H*10A^2 = 750J$) 750J for the 15H inductor, and ($W = 0.5*L*i^2 = 0.5*10H*10A^2 = 500J$) 500J for the 10H inductor. This relationship is linear, just like the current-resistor circuits, and in fact is just half, due to the fact that there is a 0.5 constant in the equation for energy exerted by an inductor, as opposed to a constant of 1 for the resistor, which means that assuming the same current, and a one to one translation of resistance to inductance, a resistor will exert a larger energy each time. For capacitance, given a 10V source, and two 5F capacitors in series, there is a 5V voltage drop at each capacitor. This leads to the energy exerted by each of the capacitors to be ($W = 0.5*C*v^2 = 0.5*5F*5C^2 = 62.5J$) 62.5J. This uniformity is very easy to see, since the voltage drop and capacitance of both of the capacitors is the same. However, changing the value of both capacitors so that they are not equal to each other may yield different results. Oppositely to the voltage-resistor circuit, the voltage capacitor circuit sees the larger valued capacitor have a lower voltage drop than the lower valued capacitor. This does yield different energies exerted by





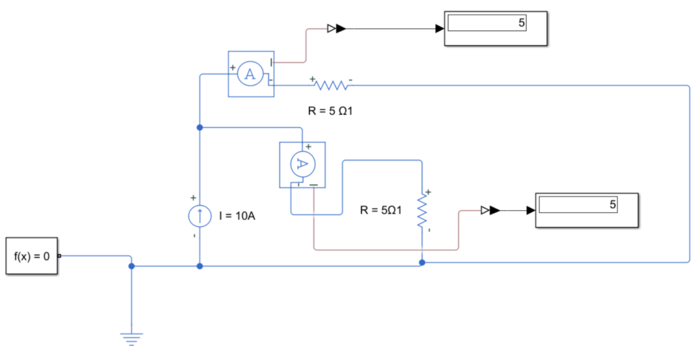
the capacitors. The 15F capacitor exerts ($W = 0.5*v^2*C = 0.5*4V^2*15C = 120J$) 120J, while

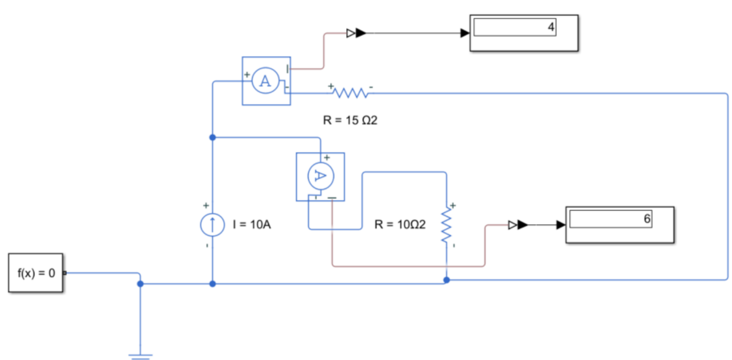


the 10F capacitor exerts ($W = 0.5*v^2*C = 0.5*6V^2*10C = 180J$) 180J. This difference, similarly to the current-inductor circuit, is noticeable and nonlinear. This leads to the 10F capacitor to yield a larger energy than the 15F capacitor, but both are larger than when the capacitors were both 5F.

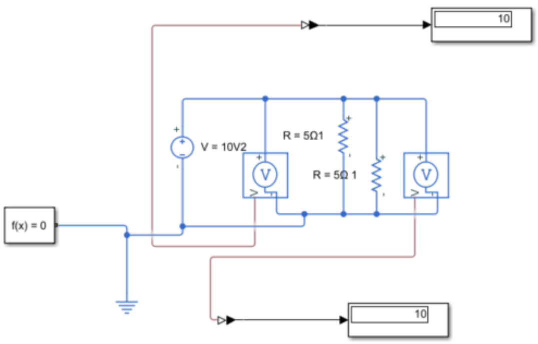
<u>Circuits in Parallel:</u>

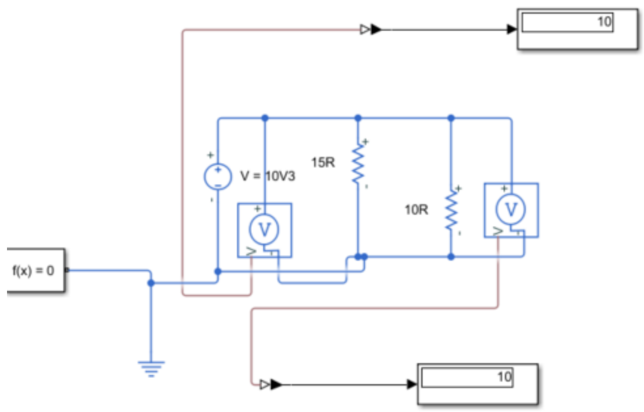
The second way to make more complex circuits, and the final one that this project will explore, is to put circuit elements in parallel. For circuits that consist of current-resistor circuits, such as the example provided, where there is a single current source worth 10A, and two resistors in parallel, each with 5Ω of resistance. This leads to 5A of current, or the 10 current split evenly since each resistor has the same resistance. This, like the multiple times before where the circuit elements have the same value, simplifies and expedited the math by a lot. In this circuit, both resistors will exert a total of ($W = i^2R = 5A^2*5\Omega = 125J$) 125J. This is substantially less than that of either and both of the resistors when they were in series. This is because the current, which gets squared in the calculation, decreases here by a factor of 2, which decreases the energy by a factor of 2^2, or 4. Instead, if both of the resistors were to have a different resistance, then the current will be split non evenly among the resistors, inversely proportional to what percentage of the total equivalent resistance they are. In this case, the 15Ω resistor has 4A of current, which leads it to have ($W = i^2R = 4A^2*15\Omega = 240J$) 240J of energy, while the 10Ω resistor has 6A of current, and as such ($W = i^2R = 6A^2*10\Omega = 360J$) 360J of energy exerted. This difference is notable, and, similar to the current-capacitor circuit in series, the larger current exerts the larger energy.



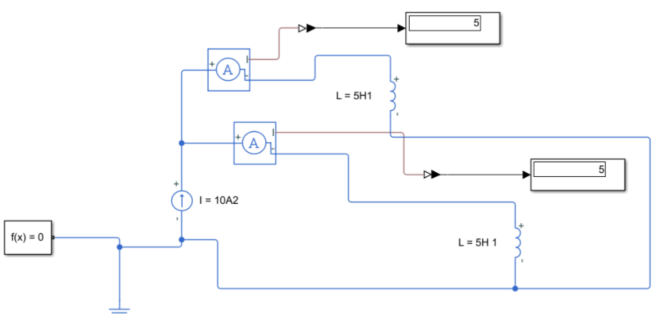


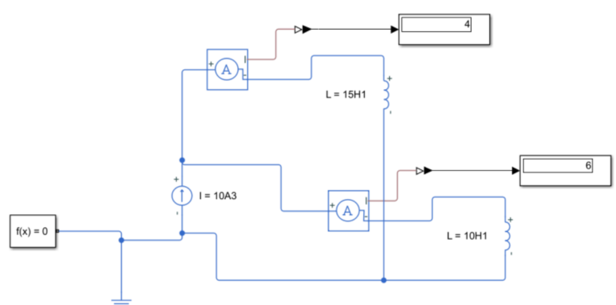
Regarding voltage-resistor circuits, when the resistors are put in parallel, both yield the same voltage. This leads to both of the resistors exerting the same amount of energy, at an exertion of ($W = v^2/R = 10V^2/10\Omega = 10J$) 10J. Because voltage in parallel is constant, both of the resistors will also maintain the same voltage, despite having different resistances, when they are different, such as 15Ω and 10Ω, seen in the following circuit. This leads to the resistors having a different energy exertion, with the 15Ω resistor exerting ($W = v^2/R = 10V^2/15\Omega = 6.67J$) 6.67J, while the 10Ω resistor instead exerts ($W = v^2/R = 10V^2/10\Omega = 10J$) 10J. This difference can be accounted for when it is taken into account that the voltage stays constant, yet the resistance is relatively large, which results in a constant numerator with an ever growing denominator, which leads to a lesser and lesser overall energy.





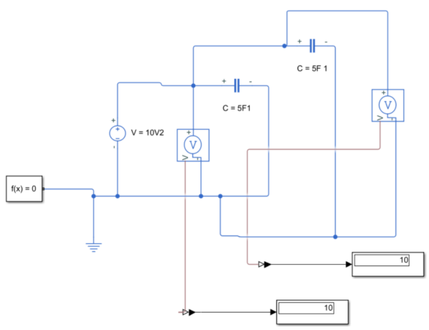
Current-inductor circuits, when dealing with inductors in parallel, lead to similar results as voltage-capacitor circuits with capacitors in series. This leads to an even distribution of current between the two paths when the inductors have the same inductance, while an inversely proportional distribution of current when the inductors have a different inductance. In the two examples provided, for the circuit with two 5L inductors, the energy exerted by each inductor is ($W = 0.5Li^2 = 0.5*5L*5A^2 = 62.5J$) 62.5 J. This is the same as when two capacitors are in series. Inversely, the energy exerted by each inductor in parallel will vary when the inductors have different inductance values. In the circuit provided, there is a 10A source with two inductors in parallel, one with 15H and the other with 10H. In this circuit, the 15H inductor exerts ($W = 0.5Li^2 = 0.5*15L*4A^2 = 120J$) 120J, while the 10H inductor exerts ($W = 0.5Li^2 = 0.5*10H*6A^2 = 180J$)



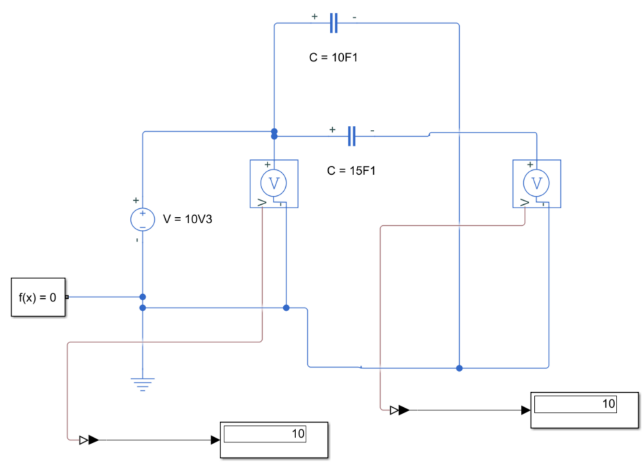


180J. Similarly to the previous circuit being similar to the energy exerted by two equivalent capacitors in series, two uneven inductors in parallel will have the same energy exerted as two uneven capacitors in series, as long as there is a one to one equivalency between the inductors and capacitors.

The last circuits that need to be analyzed are capacitors in parallel. When run through the simulation, two equivalent capacitors with 5F of capacitance each had 10V drop through them. This leads, as for all of the circuits where the circuit elements were equivalent, to simplified calculations. In this case, the energy exerted by both of the capacitors is ($W = 0.5Cv^2 = 0.5*5F*10V^2 = 250J$) 250J. This is similar to equivalent inductors in series. However, the capacitors are not equal, but still in parallel, the energy exertion for each capacitor is different. This is entirely due to the fact the capacitance



value is different for each of the capacitors, because the voltage does not change, since the voltage is still in parallel, which means it stays constant. Given the shown example, the energy exerted by the 15F capacitor is ($W = 0.5Cv^2 = 0.5*15F*10V^2 = 750J$) 750J, while the energy exerted by the 10F capacitor is ($W = 0.5Cv^2 = 0.5*10F*10V^2 = 500J$) 500J. This, like the previous circuit, is identical energy to the energy exerted by inductors in series with uneven inductance values.



<u>Conclusions and Next Steps:</u>

Different circuits will lead to different energy excitations that can be obtained. That being said, there are some patterns that can be derived from this project. Firstly, by far the largest energy output comes from current-resistor circuits when there are multiple resistors in series, specifically from the greatest value resistor, at 1500J, twice the next largest energy value. This is because, mathematically, current-resistor circuits are the only circuits to grow exponentially and unconditionally, and lack any coefficient less than one. Secondly, by far, voltage-resistor circuits in series offer the lowest energy, at just around 2.5J. This is because, mathematically, the voltage and the resistance grow inversely to each other. Thirdly, voltage-capacitor and current-inductor circuits are opposites, meaning that voltage-capacitor circuits in series are similar to current-inductor in parallel, and voltage-capacitor circuits in parallel are similar to current-inductor circuits in series. This is due to the fact that mathematically, the energy exerted by capacitors and inductors is similar, with the only difference being due to the identities overseeing the equivalency between capacitors and inductors in series and parallel.

This experiment was very limited in scope. There are multiple ways that it might be able to be extended. A follow up experiment might be conducted to experiment with circuits that contain circuit elements that are both in parallel as well as circuits, such as a circuit like that depicted in the theoretical background. Alternatively, an experiment might be performed to test circuits with multiple circuit elements, in different orders, to see if that might impact the energy exerted by any individual circuit element.

Finally, for reference, the full file used to create the visual aides can be found at: https://github.com/FrancescoGnerre/ENGR-65-Energy-Exerted-Project