Workbook 3
GNSS Standard Point Positioning (SPP)

Master's Degree in Aerospace Engineering

Satellite Navigation

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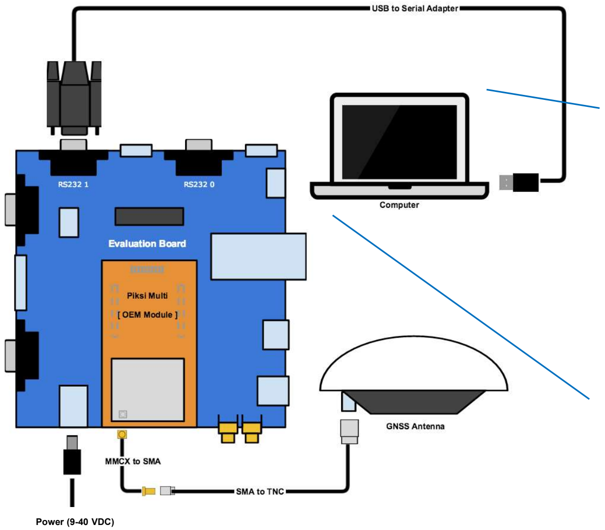
Laboratory experience overview

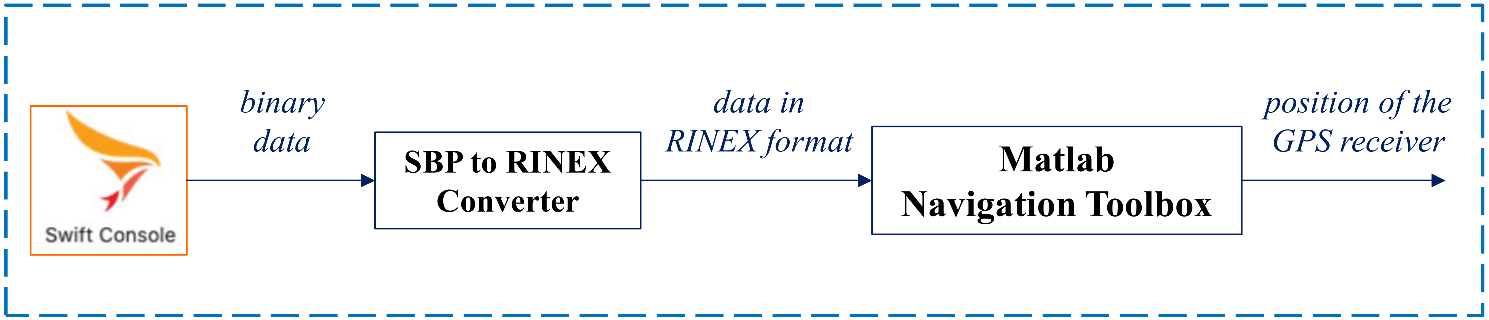
➢ Main objective

- ✓ compute the position of the receiver using <u>pseudorange measurements</u> and <u>satellite broadcast and precise ephemeris</u>.

➢ Instrumentation and software

- ✓ professional GNSS receiver[1] by Swift Navigation: Piksi Multi GNSS Module (https://www.swiftnav.com/piksi-multi)
- ✓ Swift Console for data recording and visualization
- ✓ SBP[2] to RINEX Converter for conversion of binary data to data in RINEX format
- ✓ Matlab (R2022b or later) with Navigation Toolbox for computing the position of the GPS receiver (SPP)

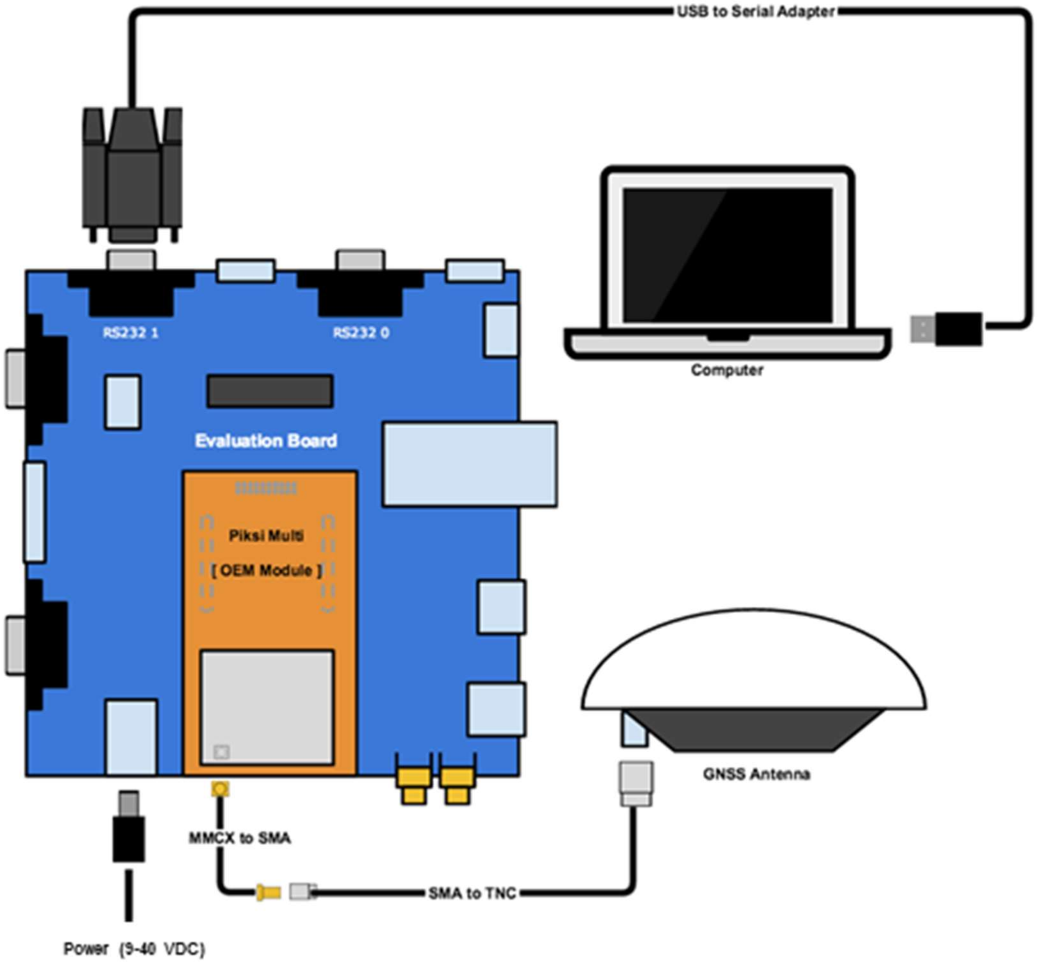




(1) The GNSS receiver and related accessories were bought with DII founds for the improvement of didactic laboratories
(2) Swift Binary Protocol

GNSS receiver main features

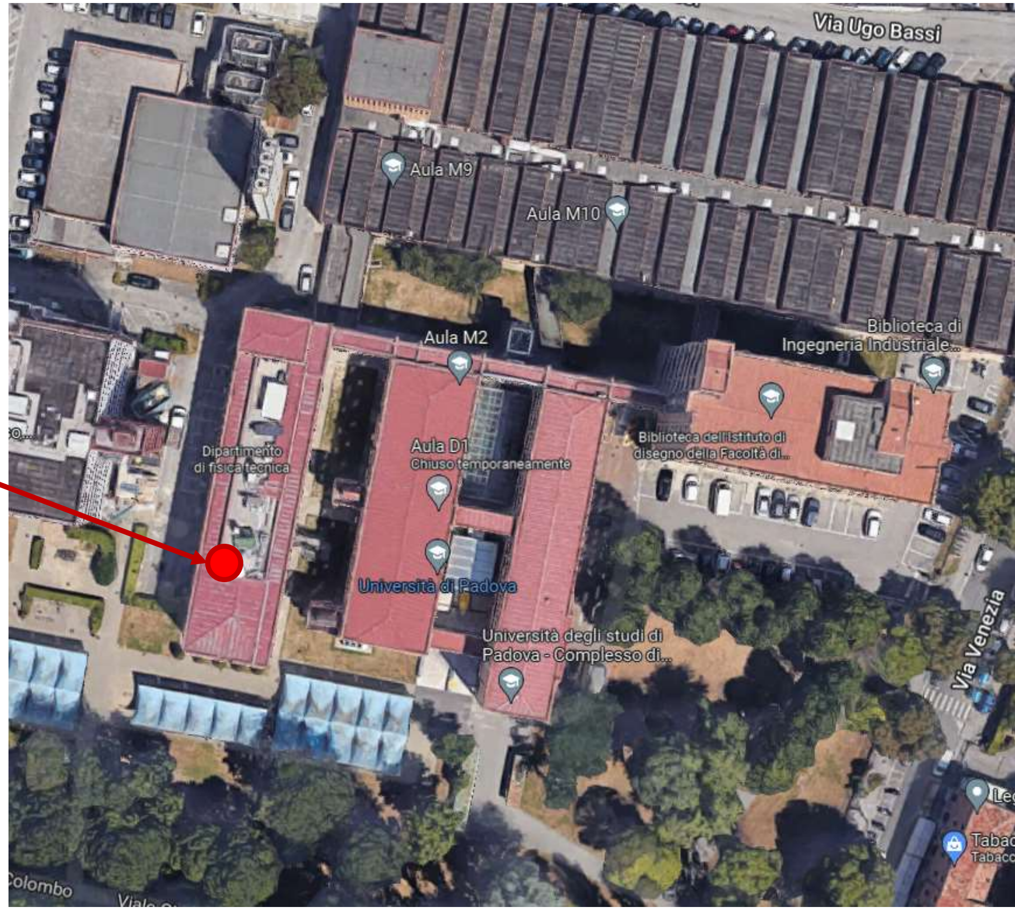
- **Piksi Multi GNSS receiver**
  - ✓ Multi-Frequency, Multi-Constellation, Centimeter-Accurate GNSS
  - ✓ supports GPS L1/L2, GLONASS G1/G2, BeiDou B1/B2, Galileo E1/E5b and SBAS
  - ✓ product summary: https://www.swiftnav.com/sites/default/files/piksi_multi_product_summary.pdf
- **Swift Console**
  - ✓ data recording and visualization
  - ✓ Observation and navigation data saved in Swift Binary Protocol (SBP) format $\Rightarrow$ conversion to RINEX format



Data Acquisition

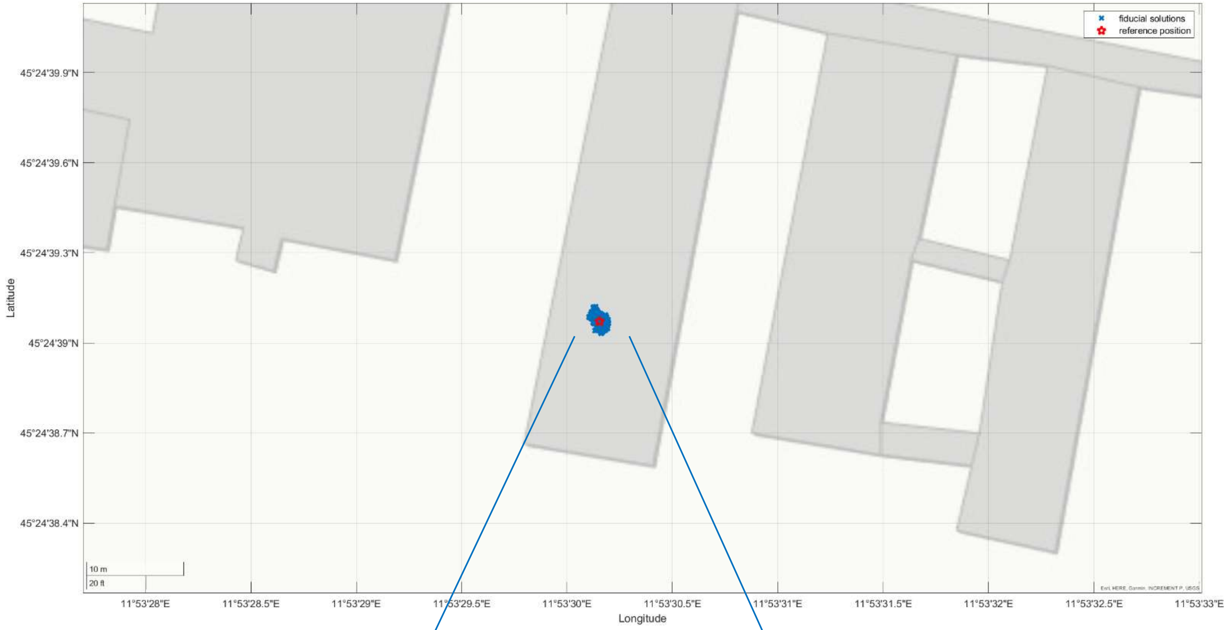
- **Data acquisition:**
  - ✓ Date: 6/12/2022
  - ✓ Time: from 11:45:22 to 12:01:40 (GPS time)
  - ✓ Location: roof of "Fisica Tecnica" building (DII, via Venezia 1, Padova)
  - ✓ Fiducial receiver position computed with RTKLIB (SPP):

$$\text{latitude} = 45.41085340 \text{ deg}$$
$$\text{longitude} = 11.89170949 \text{ deg}$$
$$\text{altitude} = 70.041 \text{ m}$$

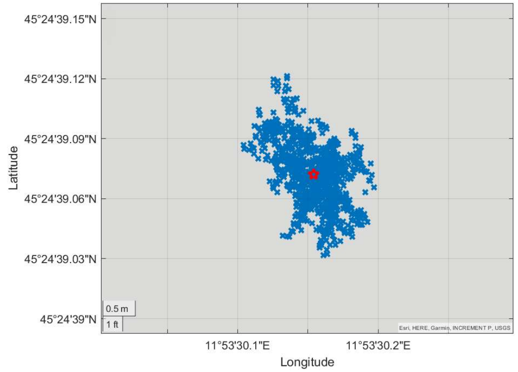


Laboratory experience main objectives and tools

- **Main Objectives:**
  1. display the fiducial receiver position computed with RTKLIB[3] (SPP) on a map; the RTKLIB uses the code pseudorange measurements of all the constellations (GPS, GLONASS, Galileo and BeiDou)
  2. Covariance analysis to compute the uncertainty ellipse for the fiducial position estimations
  3. Skyplot and GNSS performance metrics
  4. Analysis of error sources effects

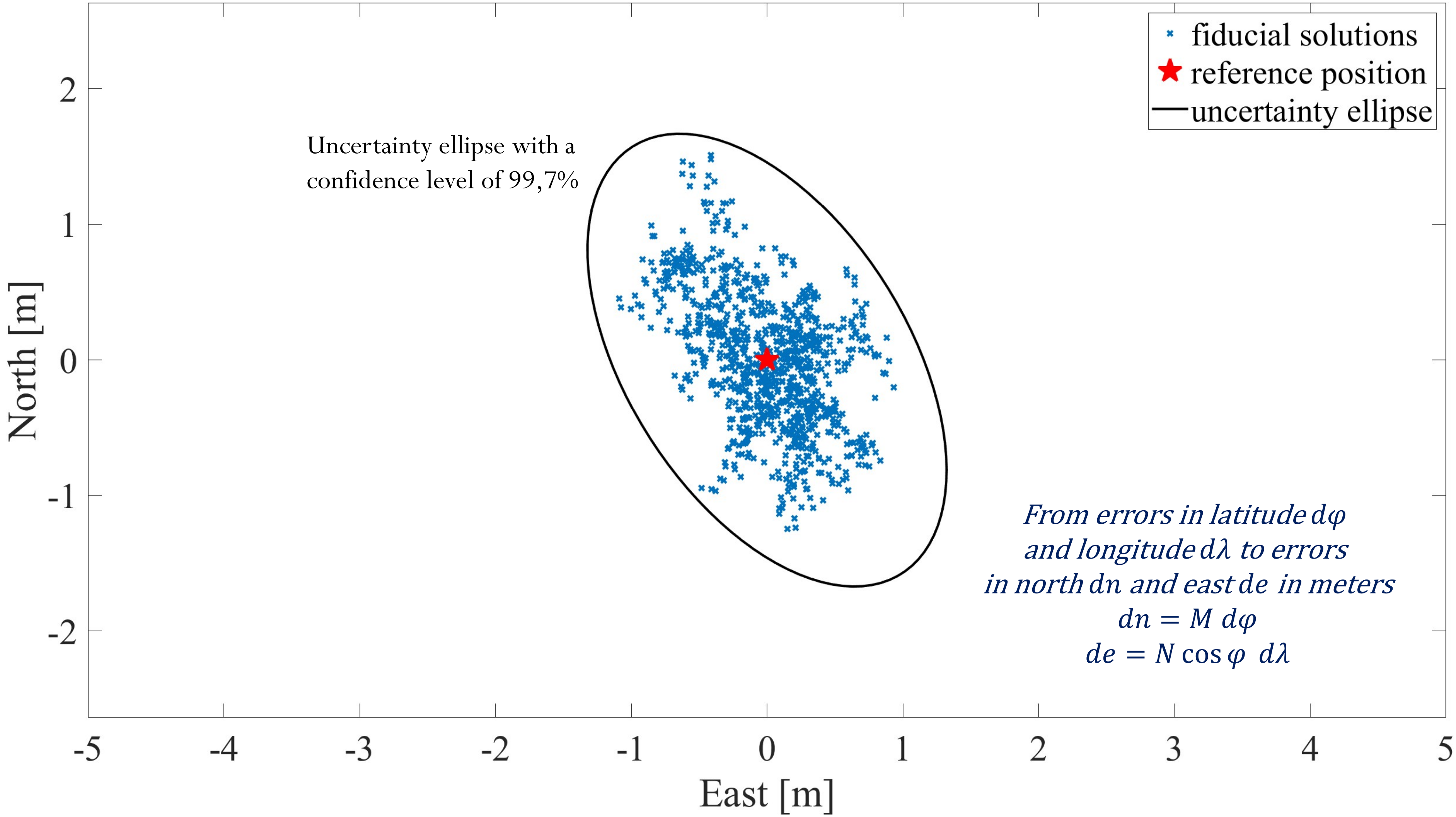


- **Software and tools:**
  - ✓ RTKLIB used to parse the complete acquired data and compute the fiducial position of the receiver (SPP)
  - ✓ Matlab with Aerospace Toolbox and Navigation Toolbox specific functions



(3) https://www.rtklib.com/

Uncertainty ellipse



From errors in latitude $d\varphi$ and longitude $d\lambda$ to errors in north dn and east de in meters

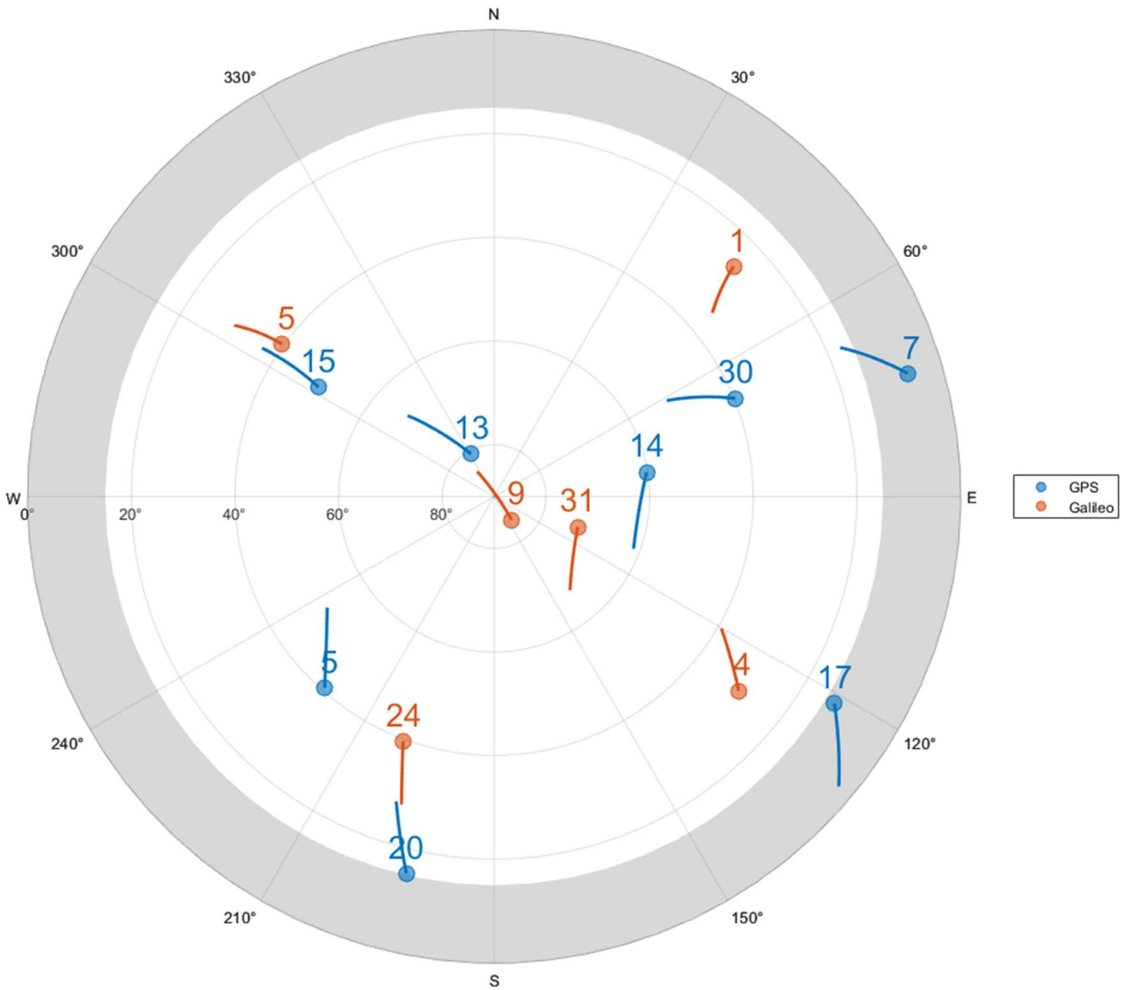
$$dn = M\, d\varphi$$

$$de = N \cos\varphi\ d\lambda$$

Skyplot

- **Objectives:**
  - ✓ read observation and navigation data in RINEX format
  - ✓ display the skyplot for the GPS and Galileo satellites in view
- **Software and tools:**
  - ✓ RTKLIB used to parse the complete acquired data and compute the fiducial position of the receiver (SPP)
  - ✓ Matlab with the Navigation Toolbox:

    `rinexread, lookangles, skyplot`
- **Skyplot of GPS and Galileo satellites in view**
  - ✓ from 11:45:22 (GPS time) for 30 minutes
  - ✓ masking angle: 15 deg
  - ✓ 8 GPS satellites, 6 Galileo satellites



Observation data in RINEX format

```
     3.04           OBSERVATION DATA    M: Mixed            RINEX VERSION / TYPE
sbp2rinex v2.3 (RTKL                    20221206 123528 UTC PGM / RUN BY / DATE
format: Swift Navigation SBP-JSON, option:                  COMMENT
log: swift-gnss-20221206-124503.sbp.json                    COMMENT
                                                            OBSERVER / AGENCY
                                                            REC # / TYPE / VERS
                                                            ANT # / TYPE
  4388984.1634   924238.9471  4519578.2828                  APPROX POSITION XYZ
        0.0000        0.0000        0.0000                  ANTENNA: DELTA H/E/N
G    8 C1C L1C D1C S1C C2S L2S D2S S2S                      SYS / # / OBS TYPES
R    8 C1C L1C D1C S1C C2C L2C D2C S2C                      SYS / # / OBS TYPES
E    8 C1B L1B D1B S1B C7I L7I D7I S7I                      SYS / # / OBS TYPES
C    8 C2I L2I D2I S2I C7I L7I D7I S7I                      SYS / # / OBS TYPES
  2022     12     06     11     45    22.0000000     GPS    TIME OF FIRST OBS
  2022     12     06     12     01    40.0000000     GPS    TIME OF LAST OBS
G L1C                                                       SYS / PHASE SHIFT
G L2S  0.00000
R L1C
R L2C
E L1B
E L7I
C L2I
C L7I                                                       SYS / PHASE SHIFT
  6 R01  1 R02 -4 R03  5 R11  0 R12 -1 R13 -2               GLONASS SLOT / FRQ #
 C1C    0.000 C1P    0.000 C2C    0.000 C2P    0.000        GLONASS COD/PHS/BIS
                                                            END OF HEADER
> 2022 12 06 11 45 22.0000000  0 23       0.000000000000
G13  20547689.680   107978834.387       1564.031        53.500
G30  21160484.820   111199096.051      -1638.336        53.000    21160486.660
```

`t` :observation type	`C` = pseudorange,	`L` = carrier phase,	`D` = doppler,	`S` = signal strength
`n` :band / frequency	`1, 2,...,9`			
`a` : attribute	tracking mode or channel, e.g., `I`, `Q`, etc			

Constellation identifier:

- ✓ G: GPS
- ✓ R: GLONASS
- ✓ E: GALILEO
- ✓ C: BEIDOU

Observation data in RINEX format

GNSS System	Freq. Band /Frequency	Channel or Code	Observation Codes			
			Pseudo Range	Carrier Phase	Doppler	Signal Strength
GPS	L1/1575.42	C/A	C1C	L1C	D1C	S1C
		L1C (D)	C1S	L1S	D1S	S1S
		L1C (P)	C1L	L1L	D1L	S1L
		L1C (D+P)	C1X	L1X	D1X	S1X
		P (AS off)	C1P	L1P	D1P	S1P
		Z-tracking and similar (AS on)	C1W	L1W	D1W	S1W
		Y	C1Y	L1Y	D1Y	S1Y
		M	C1M	L1M	D1M	S1M
		codeless		L1N	D1N	S1N
	L2/1227.60	C/A	C2C	L2C	D2C	S2C
		L1(C/A)+(P2-P1) (semi-codeless)	C2D	L2D	D2D	S2D
		L2C (M)	C2S	L2S	D2S	S2S
		L2C (L)	C2L	L2L	D2L	S2L
		L2C (M+L)	C2X	L2X	D2X	S2X
		P (AS off)	C2P	L2P	D2P	S2P
		Z-tracking and similar (AS on)	C2W	L2W	D2W	S2W
		Y	C2Y	L2Y	D2Y	S2Y
		M	C2M	L2M	D2M	S2M
		codeless		L2N	D2N	S2N
	L5/1176.45	I	C5I	L5I	D5I	S5I
		Q	C5Q	L5Q	D5Q	S5Q
		I+Q	C5X	L5X	D5X	S5X

GNSS System	Freq. Band /Frequency	Channel or Code	Observation Codes			
			Pseudo Range	Carrier Phase	Doppler	Signal Strength
Galileo	E1 / 1575.42	A PRS	C1A	L1A	D1A	S1A
		B I/NAV OS/CS/SoL	C1B	L1B	D1B	S1B
		C no data	C1C	L1C	D1C	S1C
		B+C	C1X	L1X	D1X	S1X
		A+B+C	C1Z	L1Z	D1Z	S1Z
	E5a / 1176.45	I F/NAV OS	C5I	L5I	D5I	S5I
		Q no data	C5Q	L5Q	D5Q	S5Q
		I+Q	C5X	L5X	D5X	S5X
	E5b / 1207.140	I I/NAV OS/CS/SoL	C7I	L7I	D7I	S7I
		Q no data	C7Q	L7Q	D7Q	S7Q
		I+Q	C7X	L7X	D7X	S7X
	E5(E5a+E5b) / 1191.795	I	C8I	L8I	D8I	S8I
		Q	C8Q	L8Q	D8Q	S8Q
		I+Q	C8X	L8X	D8X	S8X
	E6 / 1278.75	A PRS	C6A	L6A	D6A	S6A
		B C/NAV CS	C6B	L6B	D6B	S6B
		C no data	C6C	L6C	D6C	S6C
		B+C	C6X	L6X	D6X	S6X
		A+B+C	C6Z	L6Z	D6Z	S6Z

- See the reference document for RINEX v 3,04 available at https://files.igs.org/pub/data/format/rinex304.pdf

Observation data in RINEX format

epoch (date and time)

number of satellites observed in current epoch

```
                                                                                   ADER
> 2022 12 06 11 45 22.0000000  0 23       0.000000000000
G13  20547689.680   107978834.387        1564.031          53.500
G30  21160484.820   111199096.051       -1638.336          53.000    21160486.660    86648656.613    -1277.535    50.250
G05  21076687.280   110758746.348        -988.043          52.750    21076688.740    86305514.109     -769.500    49.250
G07  23480536.820   123391049.770       -2691.906          49.000    23480537.840    96148865.234    -2097.445    44.500
G15  22365355.480   117530731.3321       3061.910          49.000    22365356.880    91582411.4061    2385.984    44.750
G14  20999258.640   110351843.535        1056.121          51.250
G20  22121125.200   116247302.082       -3047.793          50.000
R02  19164129.800   102263509.164         149.855          57.500    19164135.080    79538279.191      116.734    53.000
R13  21047700.620   112393576.574        3324.789          45.750    21047705.200    87417224.848     2584.512    43.500
R12  19147430.540   102282172.258        -140.566          57.500    19147435.660    79552799.258     -108.340    53.250
R11  21917540.820   117120721.859       -3718.656          53.000    21917546.640    91093878.516    -2893.035    51.250
R01  21755379.300   116294980.414       -3578.578          49.250    21755388.680    90451674.035    -2783.359    47.000
R03  21475325.940   114959129.504        3517.676          51.250    21475334.380    89412658.418     2736.273    49.500
C37  24787029.760   129072569.941        2915.266          46.750
C07  38966890.300   202910826.723          73.230          44.000    38966889.000   156903511.586       57.039    44.750
C23  21904153.160   114060675.750         699.590          53.750
C25  22909530.340   119295939.250       -2110.156          51.750
C32  22066183.440   114904409.172         592.625          53.500
C10  38632282.440   201168418.230         474.586          43.250    38632285.040   155556184.320      366.527    48.000
E04  24811506.400   130385338.652       -1844.063          48.500    24811507.960    99905644.820    -1414.051    51.250
E09  23556414.140   123789789.488         551.559          51.250    23556416.080    94851910.641      422.758    51.750
E24  26859153.760   141145832.289        3093.977          41.250    26859157.620   108150694.121     2370.414    45.500
E31  23981624.220   126024298.344        1324.520          49.750    23981628.180    96564067.168     1015.828    51.750
> 2022 12 06 11 45 23.0000000  0 23       0.000000000000
G13  20547392.760   107977273.449        1563.500          53.750
```

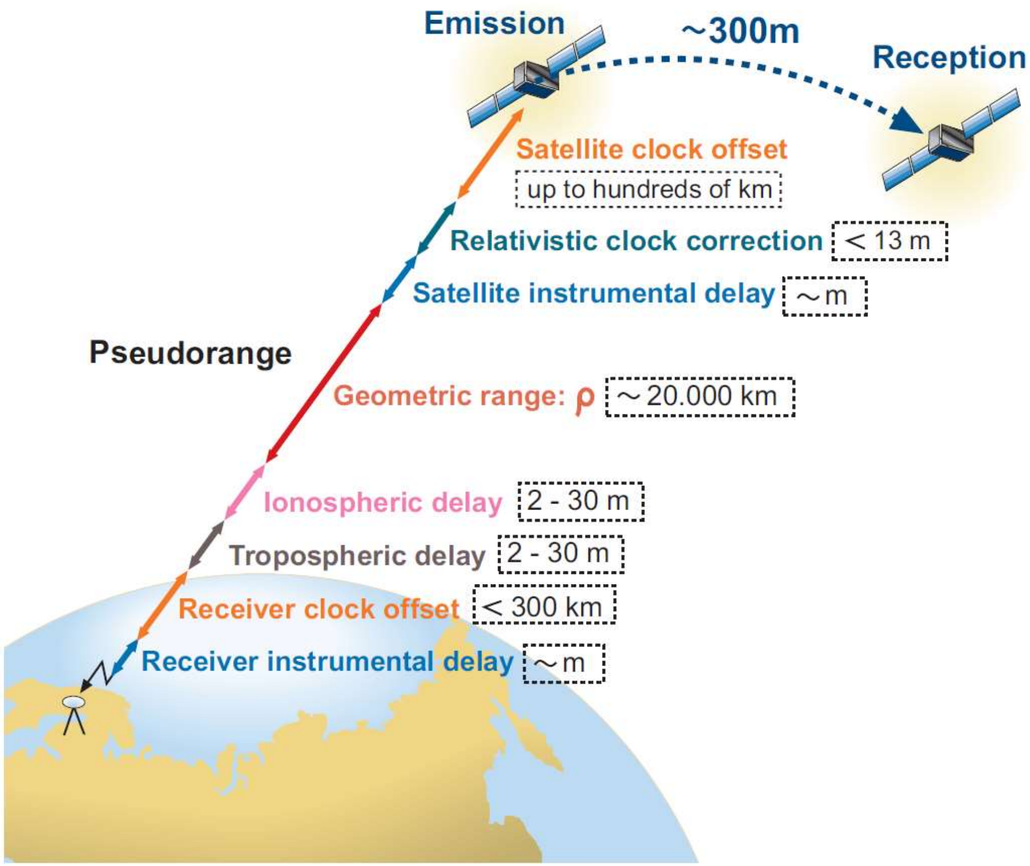
pseudoranges on L1 and L2 for GPS satellite with ID 30

pseudoranges on E1 and E5b for Galileo satellite with ID 04

Analysis of error sources effects

- **Objectives:**
  - ✓ Implement a Single Point Positioning (SPP) algorithm to estimate the GNSS receiver position, using code pseudorange measurements and considering only the GPS constellation
  - ✓ compute Dilution Of Precision (DOP) parameters
  - ✓ compare the implemented SPP solution with the fiducial SPP solution
  - ✓ evaluate the effect of the following error sources:
    - "leap seconds"
    - satellite clock drift
    - satellite positions
    - relativistic effects
    - Segnac effect
    - ionospheric delay
- **Software and tools:**
  - ✓ Matlab with the Aerospace and Navigation Toolboxes:

    `rinexread, gnssconstellation`

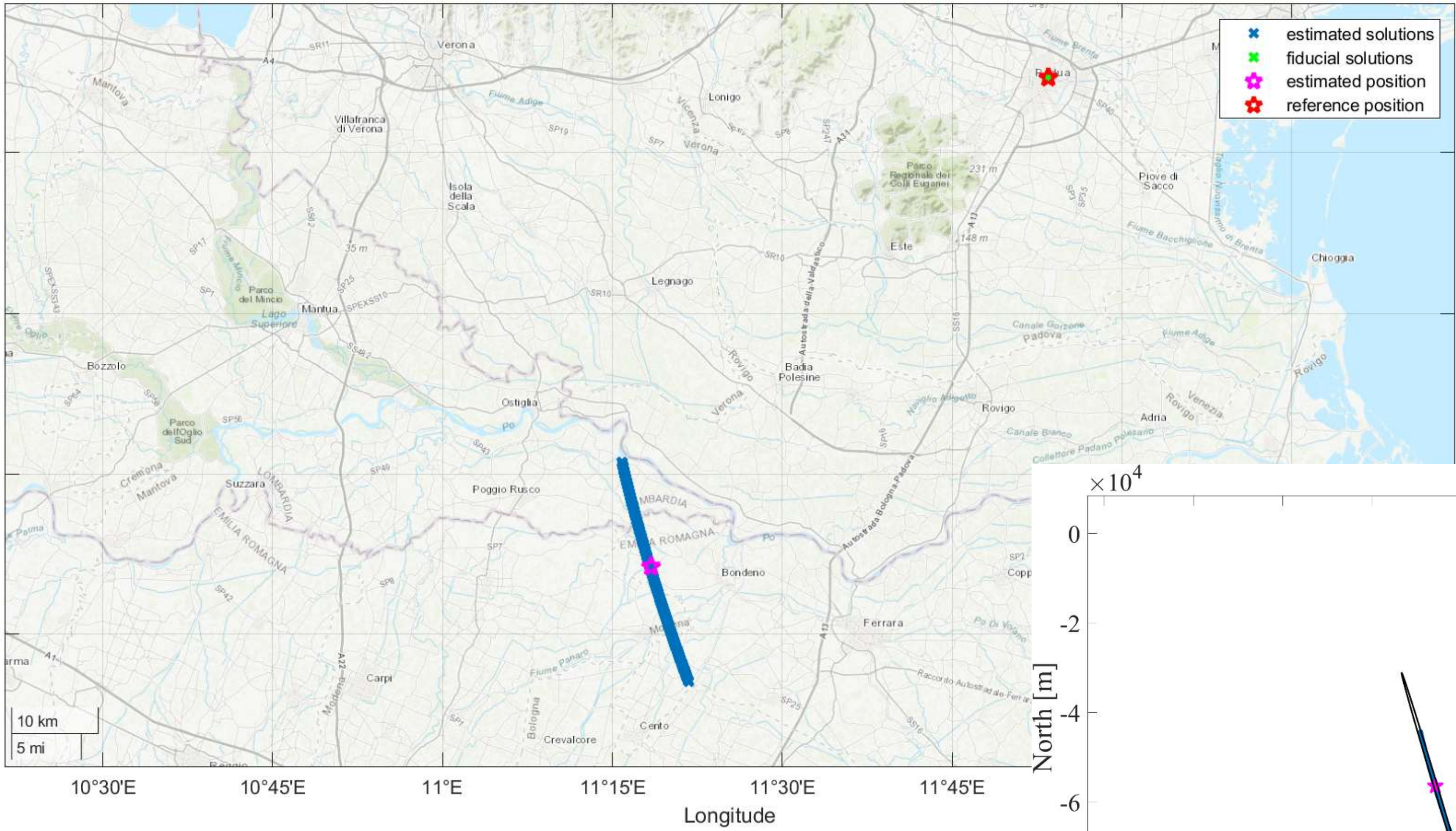


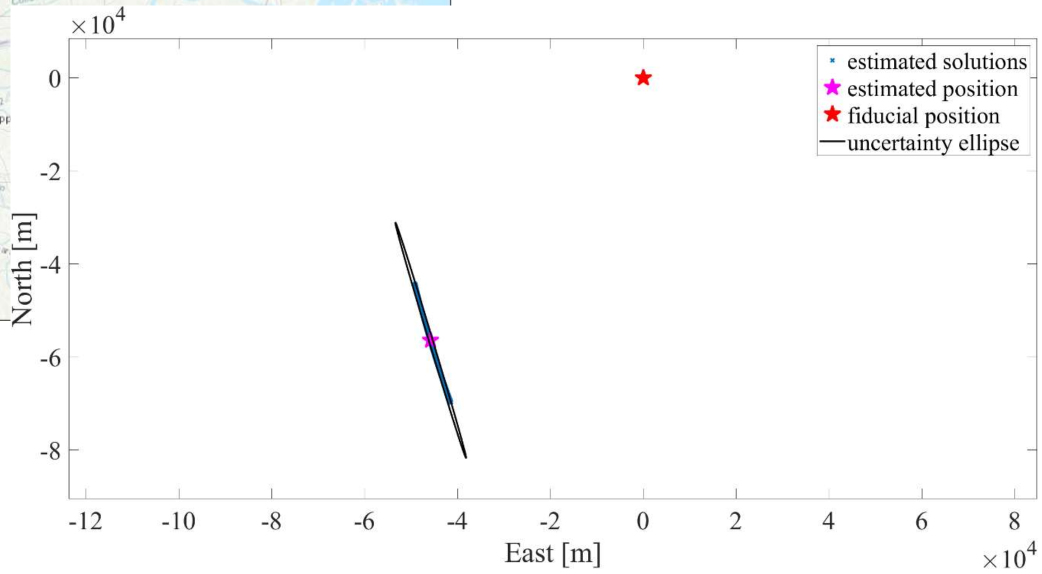
https://gssc.esa.int/navipedia/GNSS_Book/ESA_GNSS-Book_TM-23_Vol_I.pdf

Analysis of error sources effects

			Uncertainty ellipse		
Error source	North error [m]	East error [m]	Semi-major axis [m]	Semi-minor axis [m]	Orientation of semi-major axis [deg]
Sat. clock offset	-56381	-45771	26476	418	106
«Leap seconds»	9977	7081	1166	22,46	84,7
Sat. position	-39,38	29,53	5,94	5,01	132,8
Segnac	1,66	23,98	5,64	5,14	185,8
Relativistic	2,75	6,12	5,67	5,01	185,3
Ionosperic delay	-0,03	-1,21	2,64	1,86	102,5
All error source compensated	1,14	-0,12	5,66	4.95	182,7

Analysis of error sources effects: satellite clock offset effect





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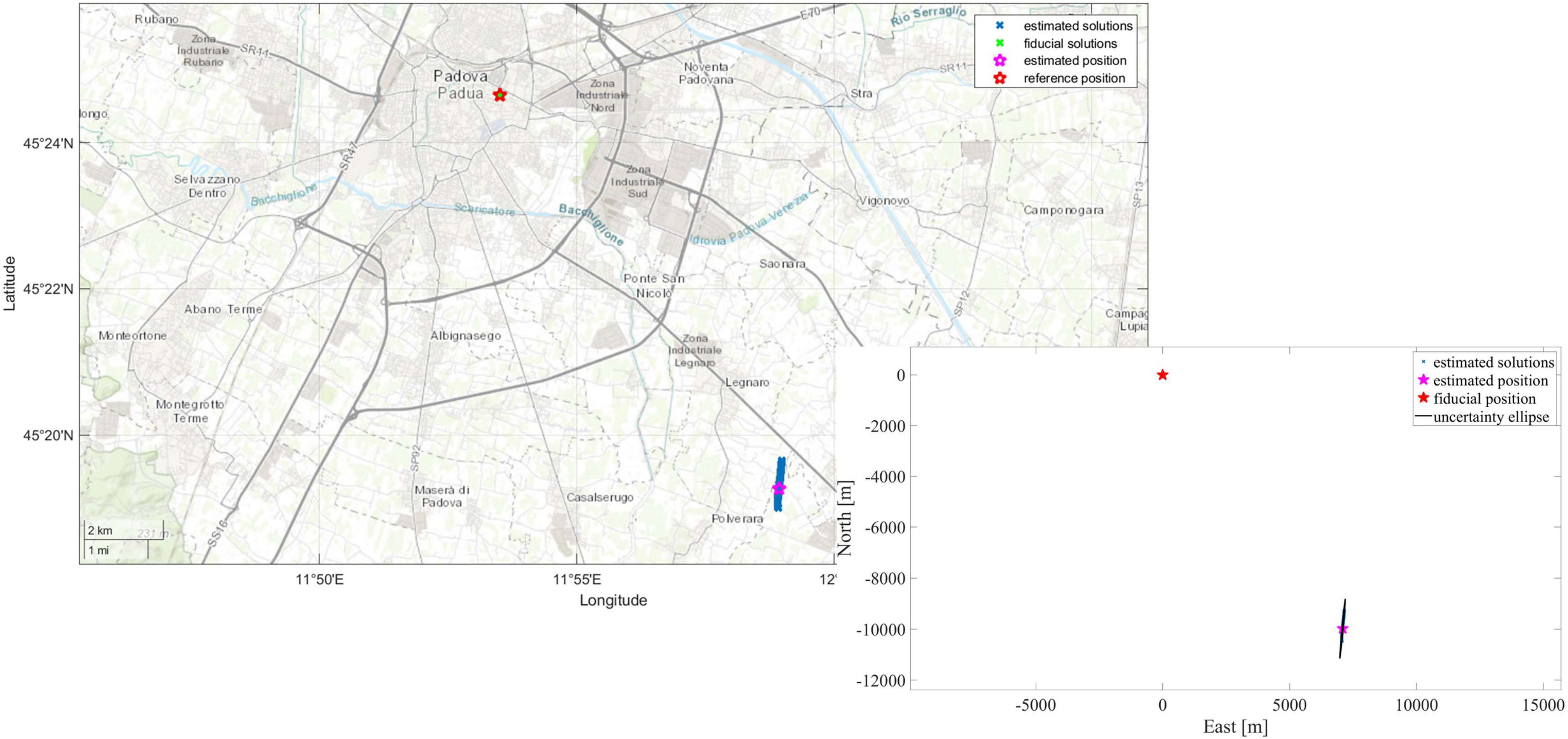
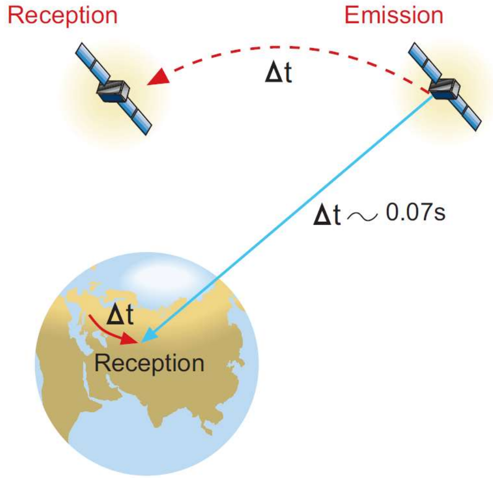
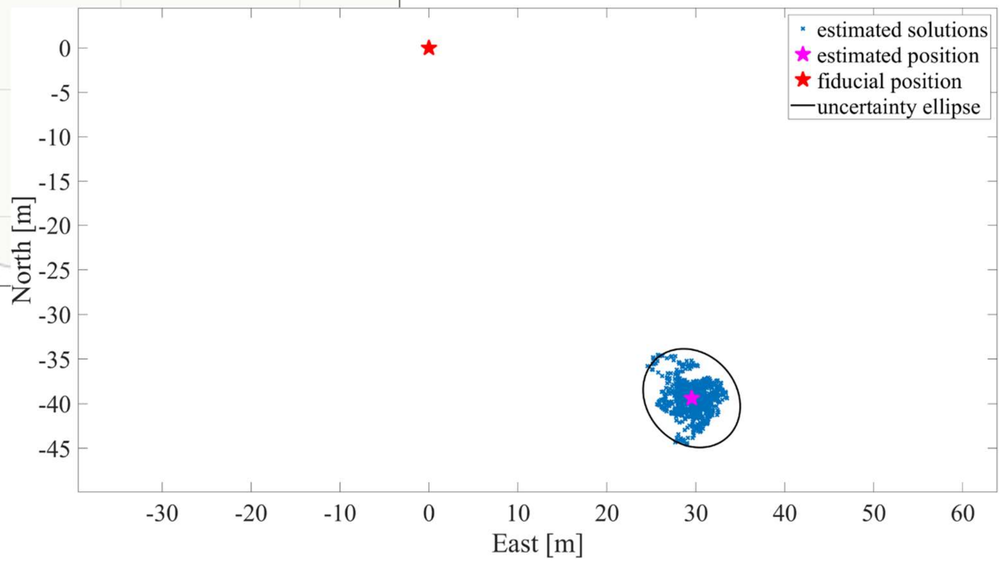
Analysis of error sources effects: "leap seconds" effect



Analysis of error sources effects: satellite position effect





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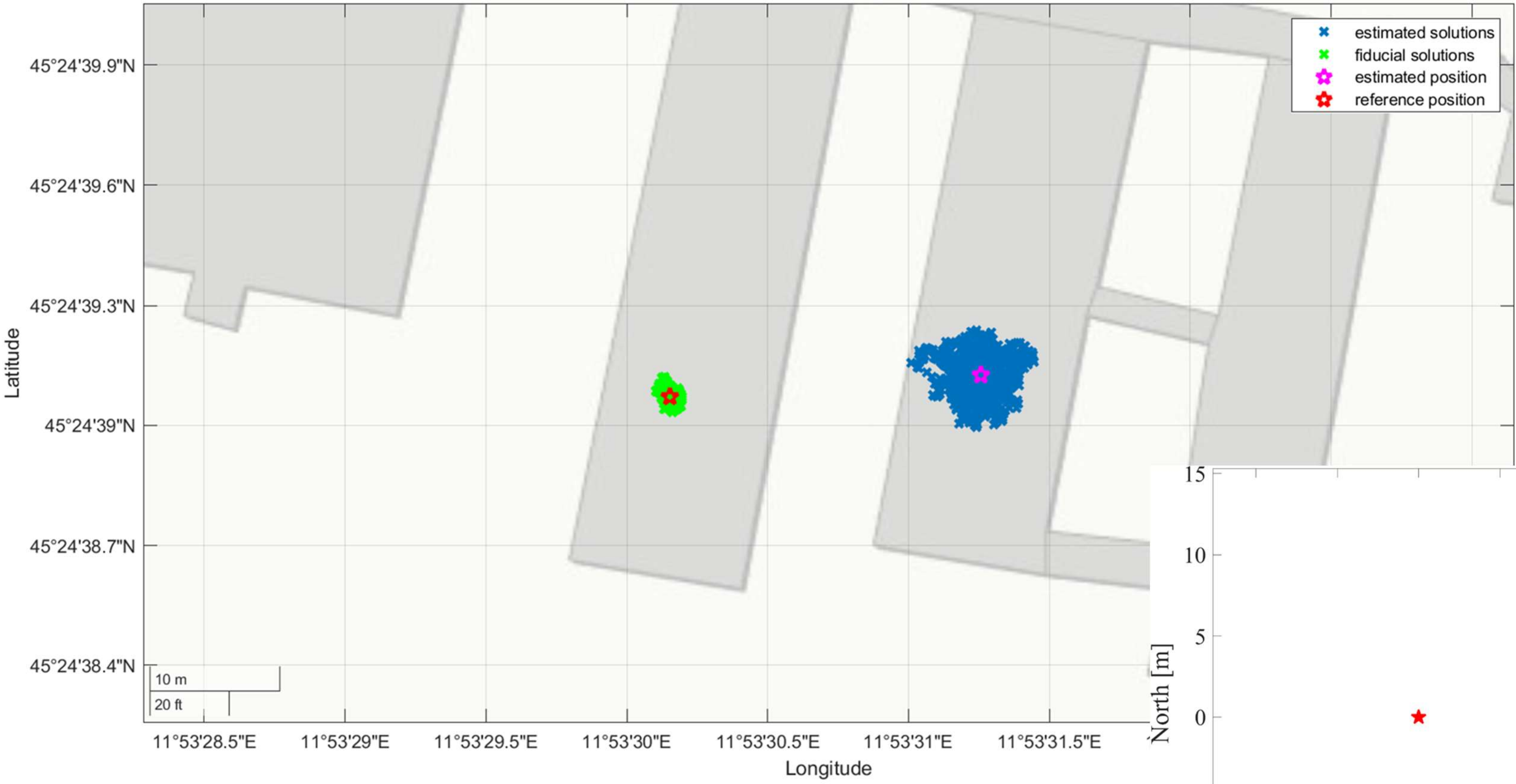
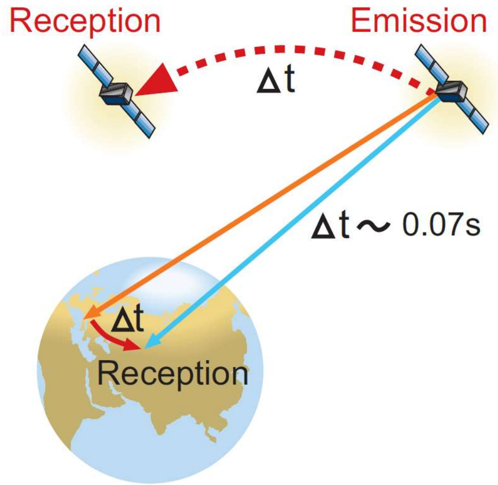
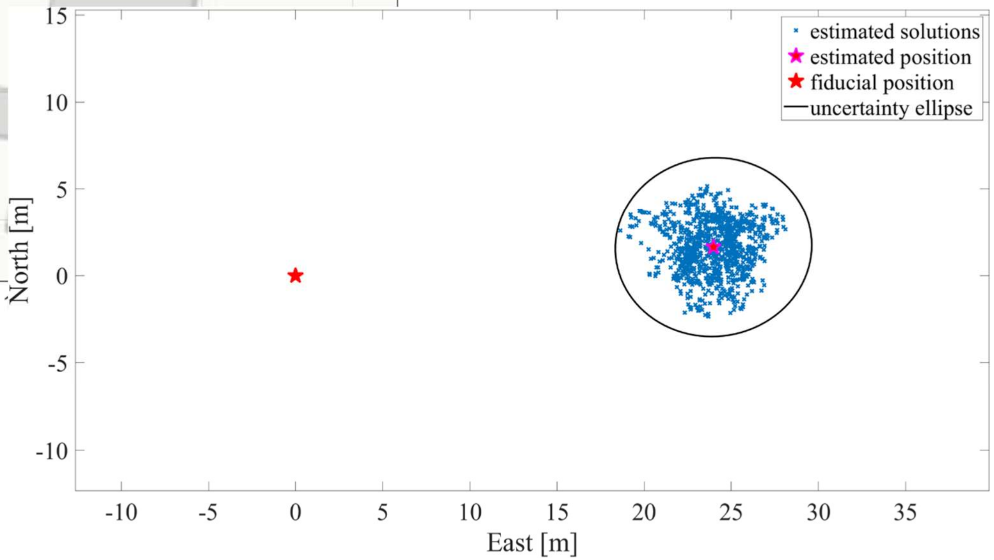
Analysis of error sources effects: Segnac effect



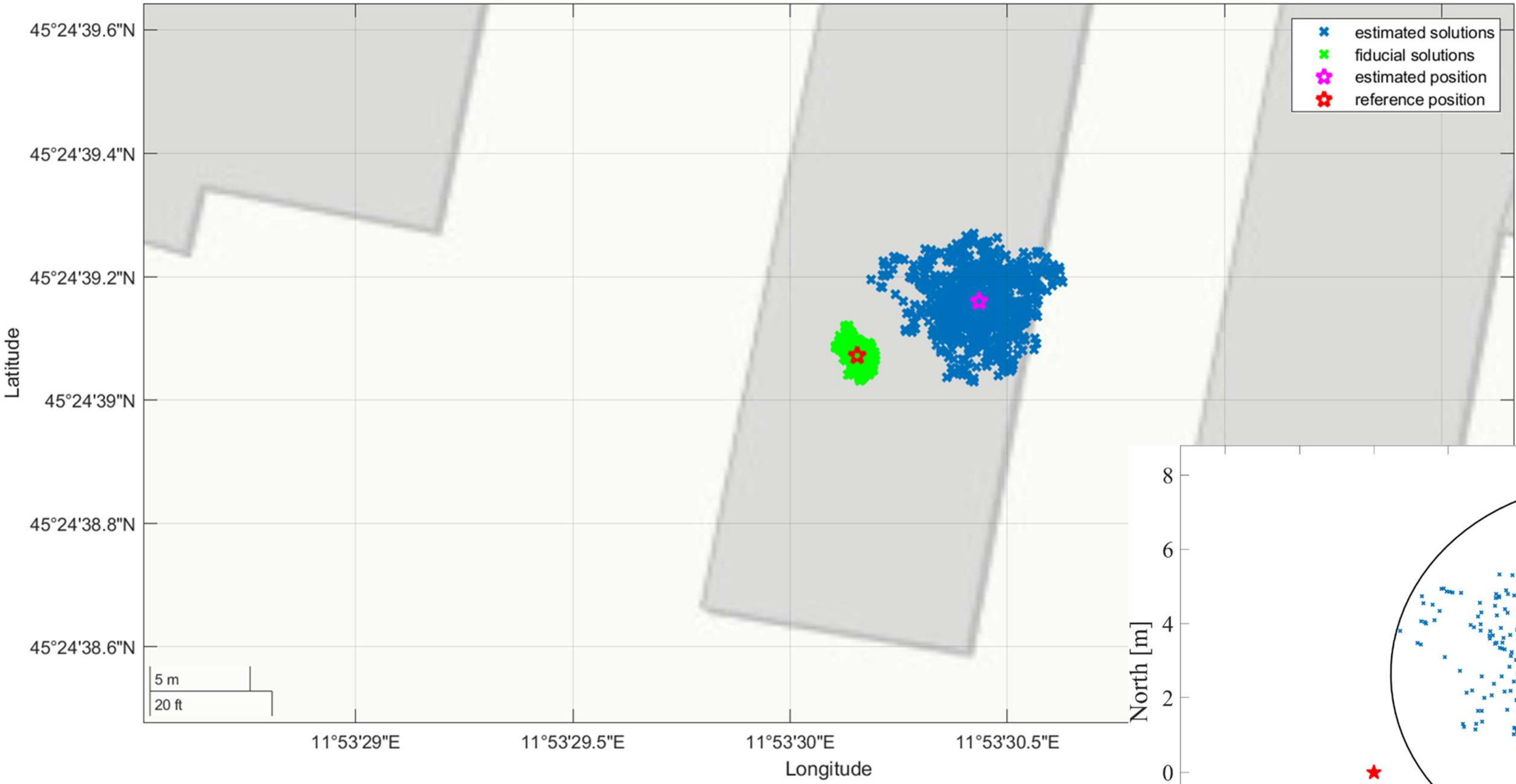


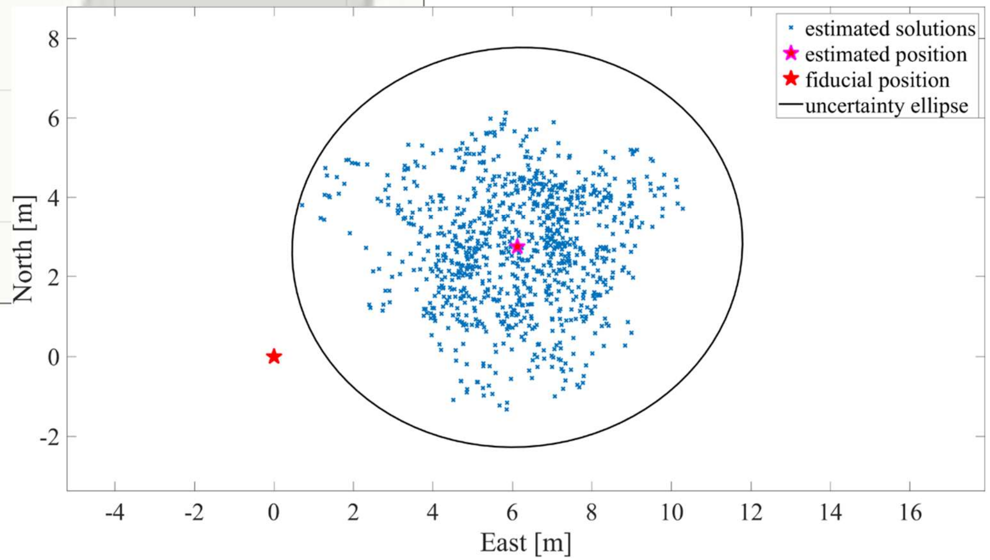


$$\mathbf{r}^{sat} = \mathbf{R}_3\left(\omega_E \Delta t\right) \cdot \widetilde{\mathbf{r}}^{sat}$$

$$\mathbf{R}_3[\theta] = \begin{bmatrix} \cos(\theta) & \sin(\theta) & 0 \\ -\sin(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

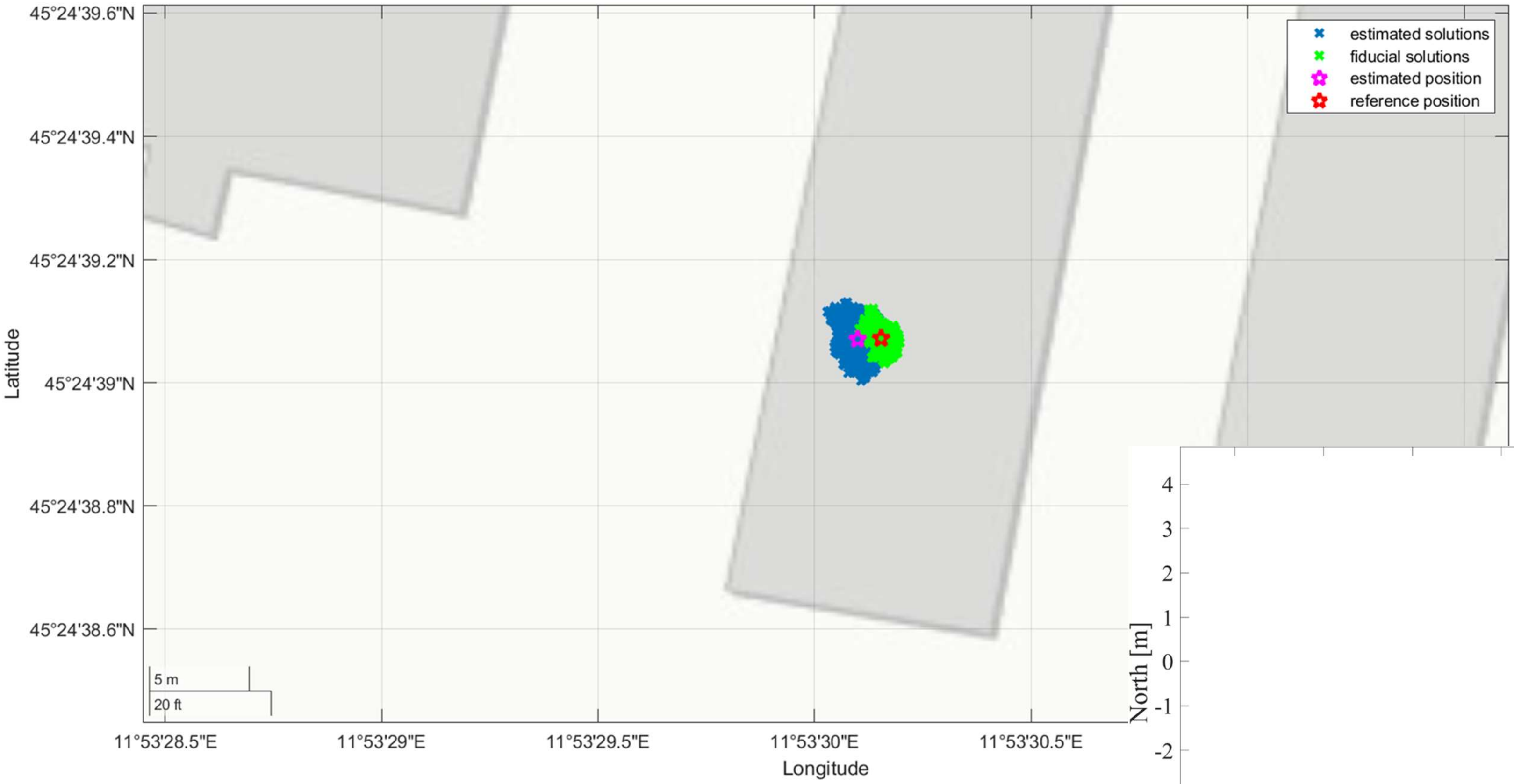
Analysis of error sources effects: relativistic effect

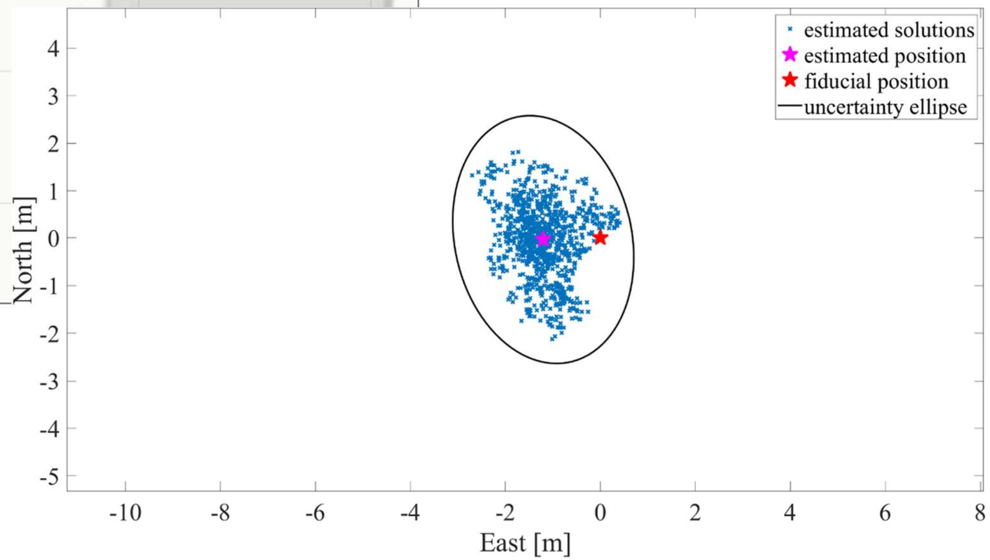




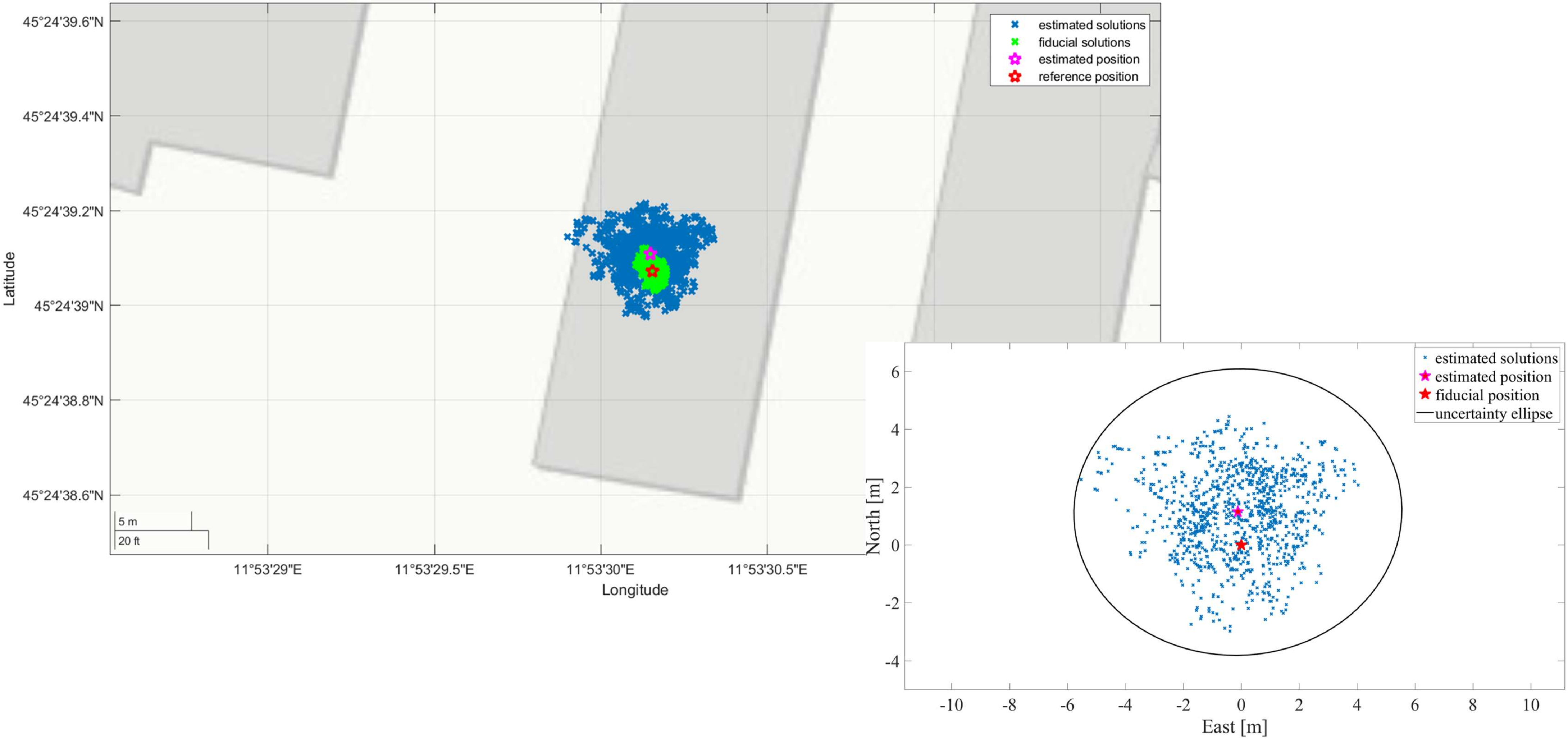
$$\Delta_{rel} = -2\,\mathbf{r}\cdot\mathbf{v}/c^2 = -2\left(\sqrt{\mu\,a}/c^2\right)e\sin E$$

Analysis of error sources effects: ionospheric delay effect





Analysis of error sources effects: full corrections



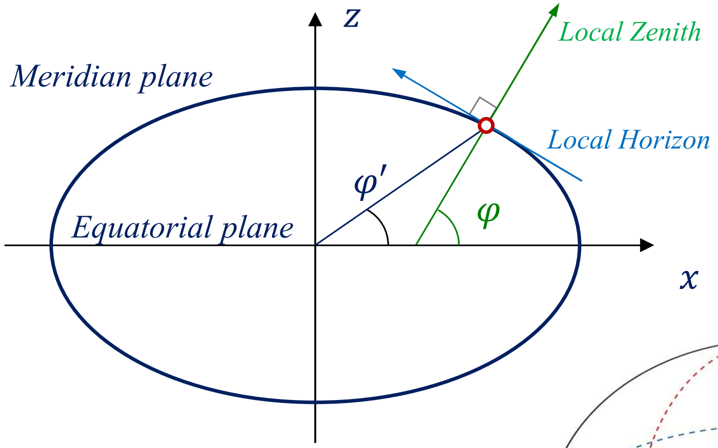
Additional slides with useful resources and algorithms

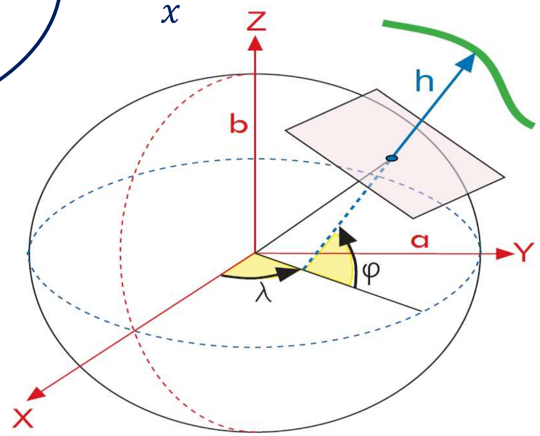
Position in Cartesian Coordinates and Geodetic Coordinates

Cartesian coordinates: x, y, z in ECEF reference frame

$\Updownarrow$

Geodetic coordinates: φ, λ, h in Wolrld Geodetic System WGS84 reference frame





φ: geodetic latitude

λ: longitude

h: altitude

a: equatorial radius

b: polar radius

$f = \frac{a-b}{a}$: flattening

$$e = \sqrt{1 - \left(\frac{b}{a}\right)^2}$$

$N = \frac{a}{\sqrt{1-e^2 \sin^2 \varphi}}$: normal radius

$(\varphi, \lambda, h) \rightarrow (x, y, z)$: analytical solution

$$x = (N + h) \cos \varphi \cos \lambda$$

$$y = (N + h) \cos \varphi \sin \lambda$$

$$z = [N(1 - e^2) + h] \sin \varphi$$

$(x, y, z) \rightarrow (\varphi, \lambda, h)$: numeric solution

✓ in Matlab: function `ecef2lla` of the Aerospace Toolbox

Estimation Accuracy in the Geodetic Reference Frame

- Covariance matrix of the estimated geodetic parameters φ, λ, h

$$\boldsymbol{C}_G = \boldsymbol{J}^{-1}_{\varphi\lambda h \to xyz}\, \boldsymbol{C}_{xyz} \left(\boldsymbol{J}^{-1}_{\varphi\lambda h \to xyz}\right)^T$$

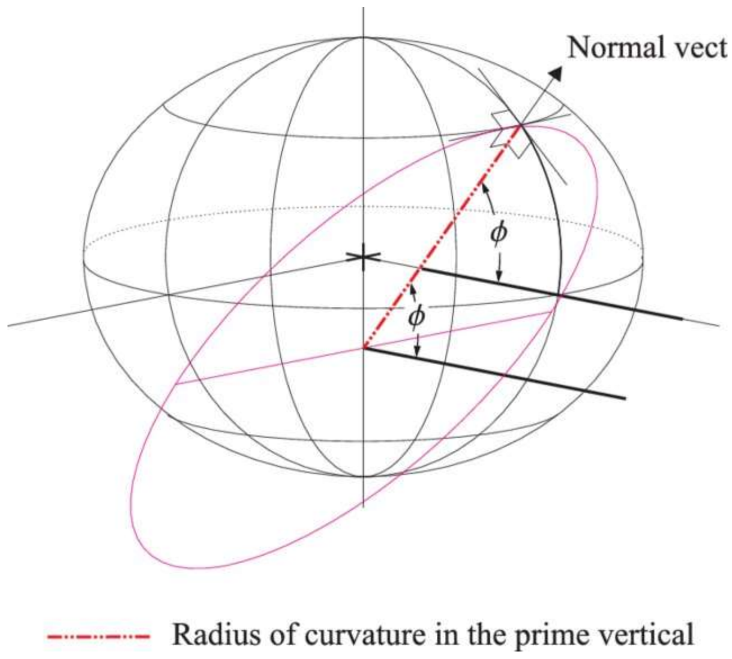
- ✓ $\boldsymbol{J}_{\varphi\lambda h \to xyz}$ is the Jacobian Matrix of the affine transformation $(\varphi, \lambda, h) \rightarrow (x, y, z)$

$$\boldsymbol{J}_{\varphi\lambda h \to xyz} = \left[\frac{\partial x_i}{\partial \varphi_j}\right] \; ; \; i = 1{,}2{,}3 \; ; j = 1{,}2{,}3$$

$$\boldsymbol{J}_{\varphi\lambda h \to xyz} = \begin{bmatrix} -(M+h)\sin\varphi\cos\lambda & -(N+h)\cos\varphi\sin\lambda & \cos\varphi\cos\lambda \\ -(M+h)\sin\varphi\sin\lambda & (N+h)\cos\varphi\cos\lambda & \cos\varphi\sin\lambda \\ -(M+h)\cos\varphi & 0 & \sin\lambda \end{bmatrix}$$

$N = \frac{a}{\sqrt{1-e^2\sin^2\varphi}}$: normal radius

$M = \frac{a(1-e^2)}{\sqrt{(1-e^2\sin^2\varphi)^3}}$: meridian radius

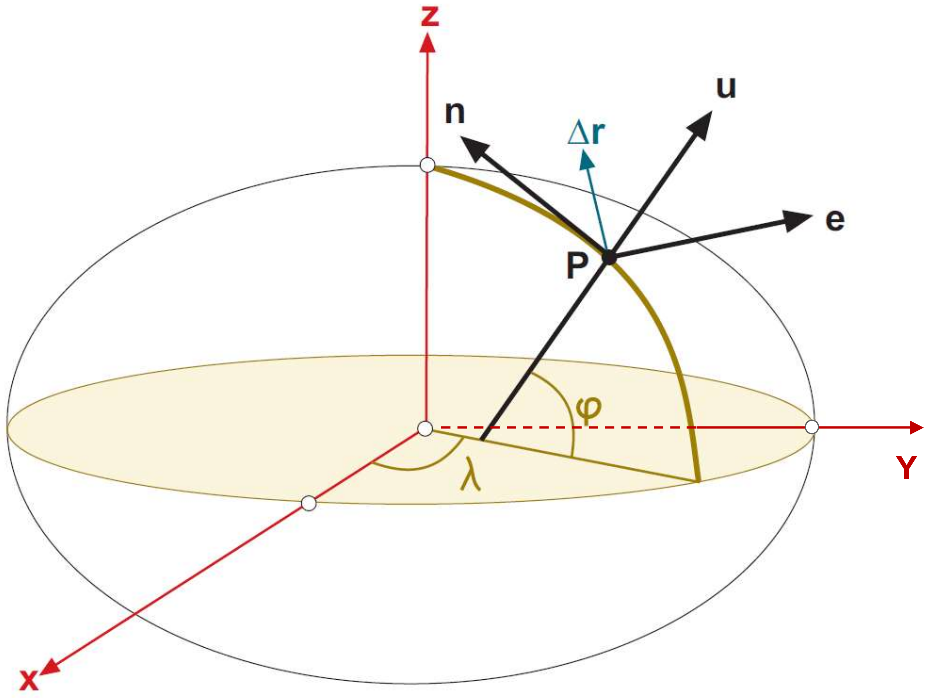


Estimation Accuracy in the Local Topocentric Reference Frame

- Local Topocentric Reference Frame: East – North – Up (ENU)

- Transformation from (dx, dy, dz) to (dn, de, du)

$$\begin{bmatrix} dn \\ de \\ du \end{bmatrix} = \underbrace{\begin{bmatrix} -\sin\varphi\cos\lambda & -\sin\lambda & \cos\varphi\cos\lambda \\ -\sin\varphi\sin\lambda & \cos\lambda & \cos\varphi\sin\lambda \\ \cos\varphi & 0 & \sin\varphi \end{bmatrix}^{-1}}_{\boldsymbol{J}_{xyz\to neu}} \begin{bmatrix} dx \\ dy \\ dz \end{bmatrix}$$



- Covariance matrix of the estimated topocentric parameters n, e, u

$$\boldsymbol{C}_{neu} = \boldsymbol{J}_{xyz\to neu}\ \boldsymbol{C}_{xyz}\ \boldsymbol{J}_{xyz\to neu}^{T} = \sigma^2 \boldsymbol{J}_{xyz\to neu}\ (\boldsymbol{H}^T\boldsymbol{H})^{-1}\ \boldsymbol{J}_{xyz\to neu}^{T} = \sigma^2 \begin{bmatrix} \sigma_n^2 & \sigma_{ne} & \sigma_{nu} \\ \sigma_{en} & \sigma_e^2 & \sigma_{eu} \\ \sigma_{un} & \sigma_{ue} & \sigma_u^2 \end{bmatrix}$$

Dilution Of Precision

- Dilution Of Precision (DOP) parameters:

Vertical Dilution of Precision: $VDOP = \sigma_u$

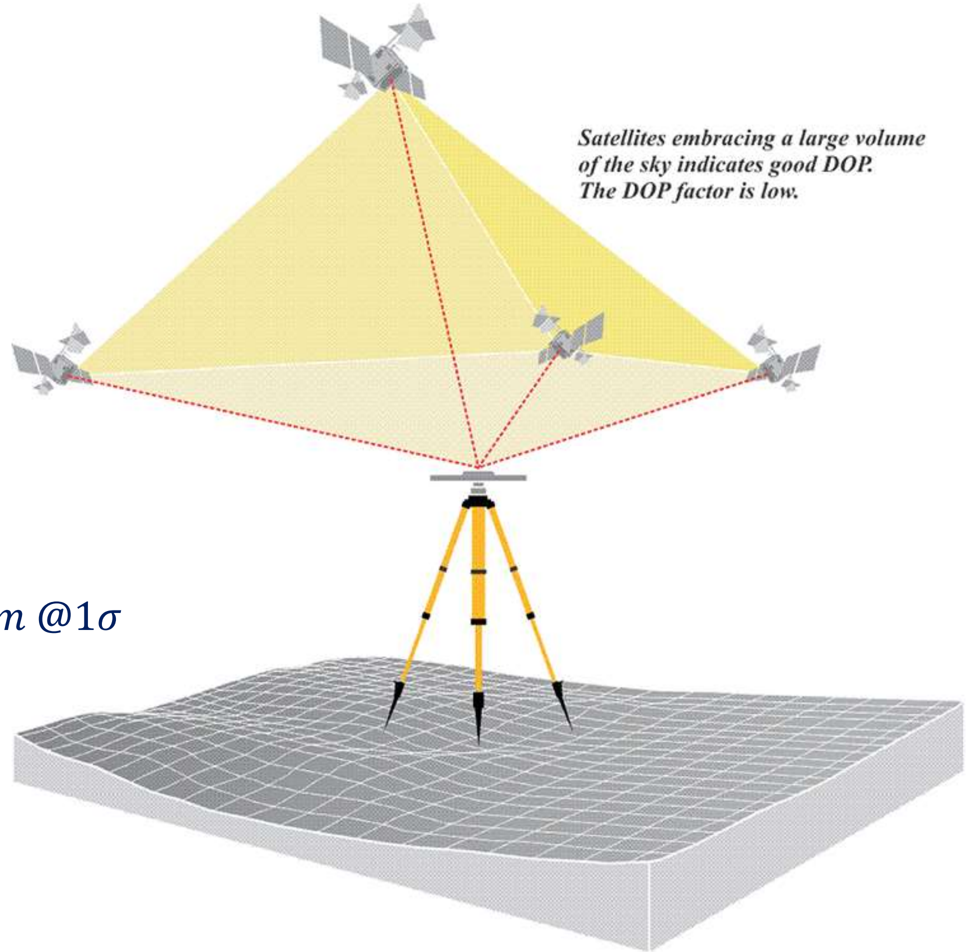
Horizontal Dilution of Precision: $HDOP = \sqrt{\sigma_n^2 + \sigma_e^2}$

Position Dilution of Precision: $PDOP = \sqrt{\sigma_n^2 + \sigma_e^2 + \sigma_u^2}$

Time Dilution of Precision: $TDOP = \sigma_\tau$

Geometric Dilution of Precision: $GDOP = \sqrt{\sigma_n^2 + \sigma_e^2 + \sigma_u^2 + c^2\sigma_\tau^2}$

  - ✓ Example: if PDOP = 2 and $\sigma = 3m$, then the position accuracy is $6m\ @1\sigma$



- DOP parameters:
  - ✓ are purely a function of satellite geometry as observed by the receiver
  - ✓ A "good geometry" gives low DOP values