Workbook 7
Satellite Positioning and Orbit Determination

Master’s Degree in Aerospace Engineering

Satellite Navigation

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DIPARTIMENTO
DI INGEGNERIA
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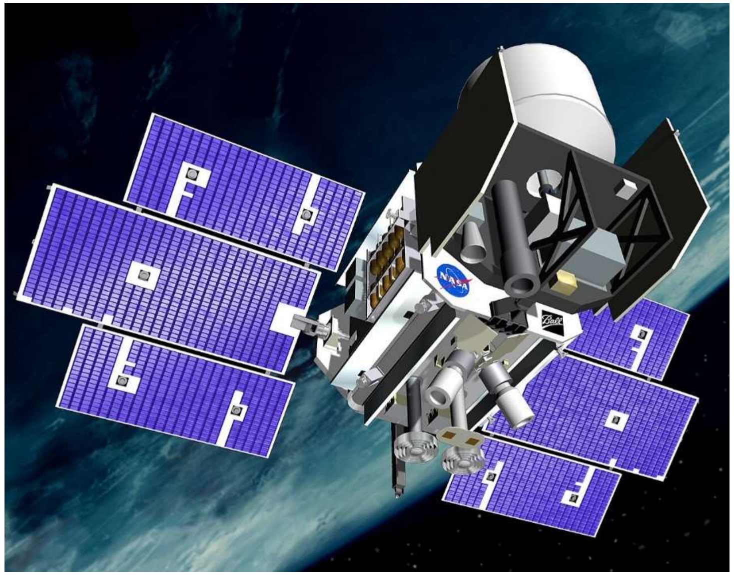
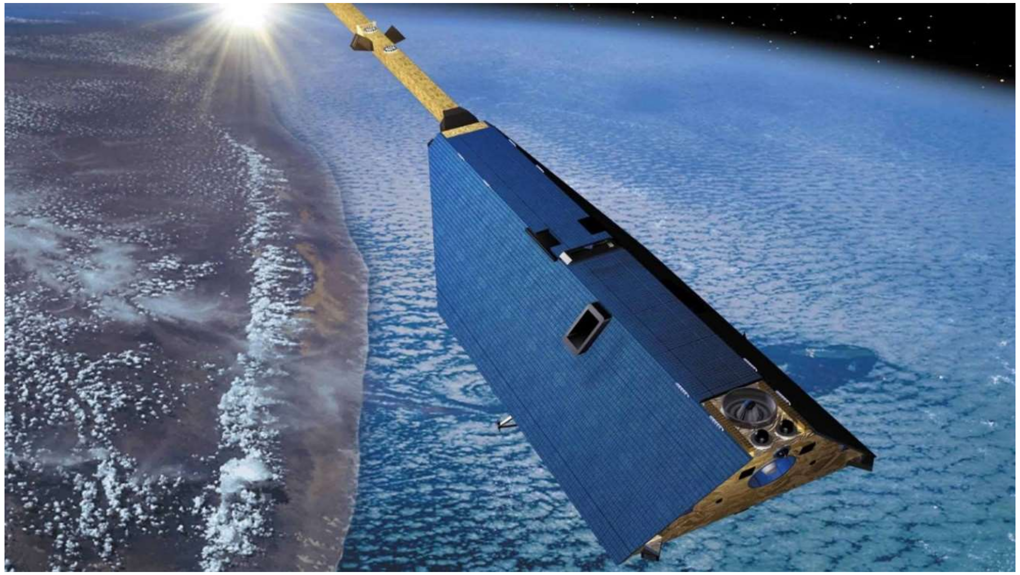
Laboratory experience overview

- **Main objective: to reconstruct the satellite orbits of two missions (ICESat and CHAMP) by parsing real GNSS data**
  - ✓ Develop a Matlab program to reconstruct the trajectory of ICESat (Ice, Clouds, and land Elevation Satellite) and CHAMP (Challenging Minisatellite Payload) during a specific operational phase.
  - ✓ Determine ICESat/CHAMP's Keplerian orbital parameters to characterize and visualize their orbits





ICESat Mission Overview

- **ICESat Mission**.
  - ✓ **Period**: Launched on January 12, 2003, and concluded on August 14, 2010
  - ✓ **Primary Goals**: Monitor the thickness of polar ice sheets and their impact on sea level changes.
  - ✓ **Secondary Goals**: Measure cloud height and properties, and further parameters related to Earth's surface topography, focusing on Greenland and Antarctica.

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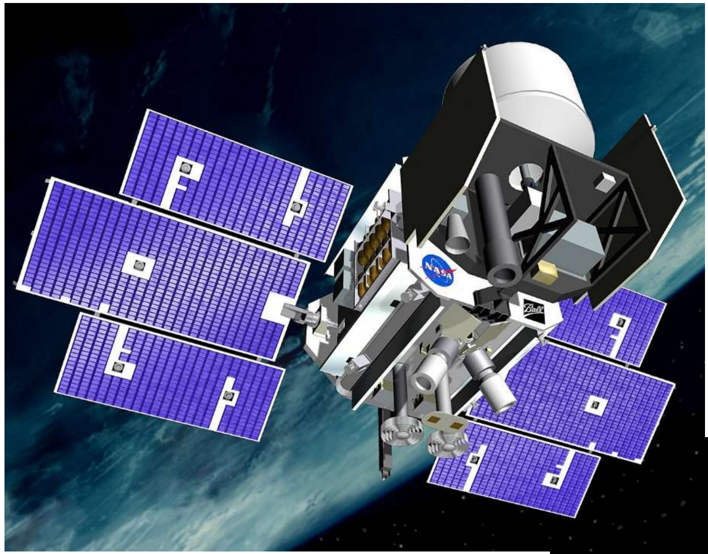
ICESat Mission Overview

➢ **ICESat Spacecraft**

✓ **Launch mass**: 970 kg

✓ **Dimensions**: 2 × 2 × 3.1 m

✓ **Power**: 640 W

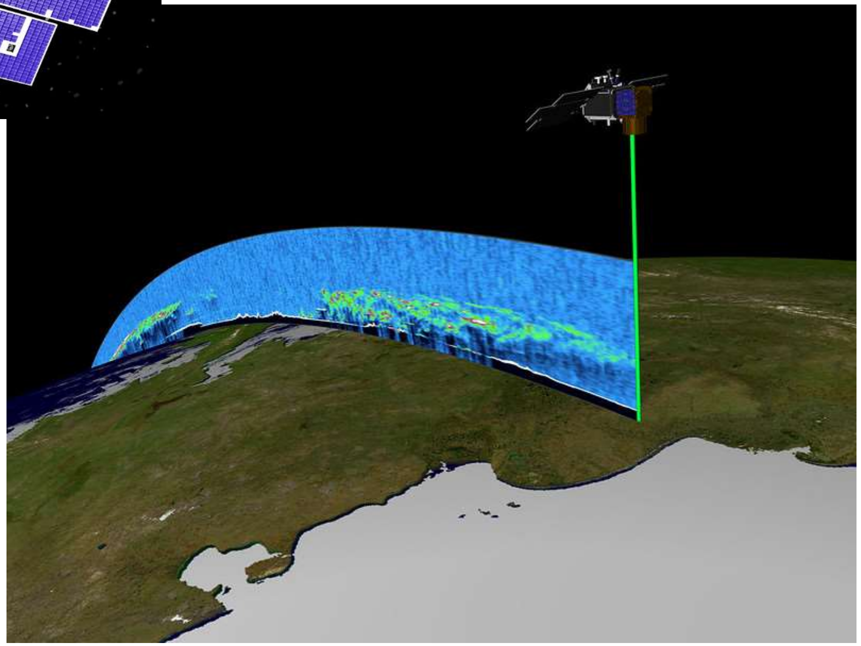


➢ **Nominal Orbital Parameters**

✓ Near-polar orbit with an inclination of 94 degrees

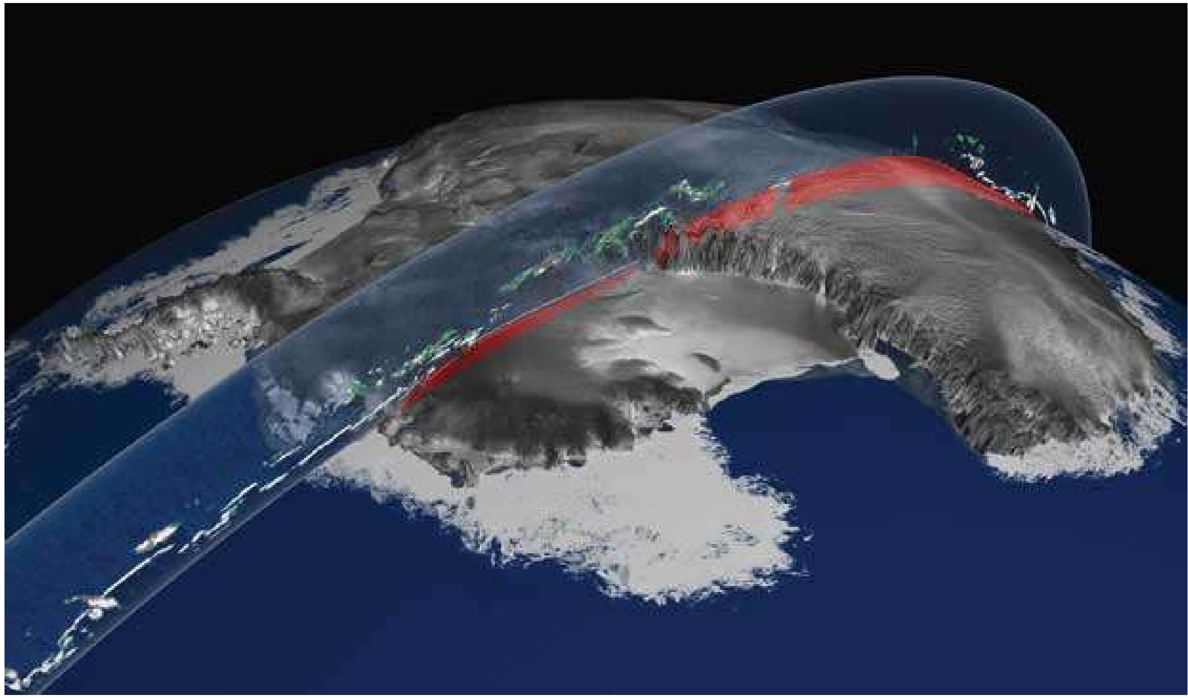
✓ Orbital altitude of 600 km (LEO)

✓ Orbital period of 97 minutes

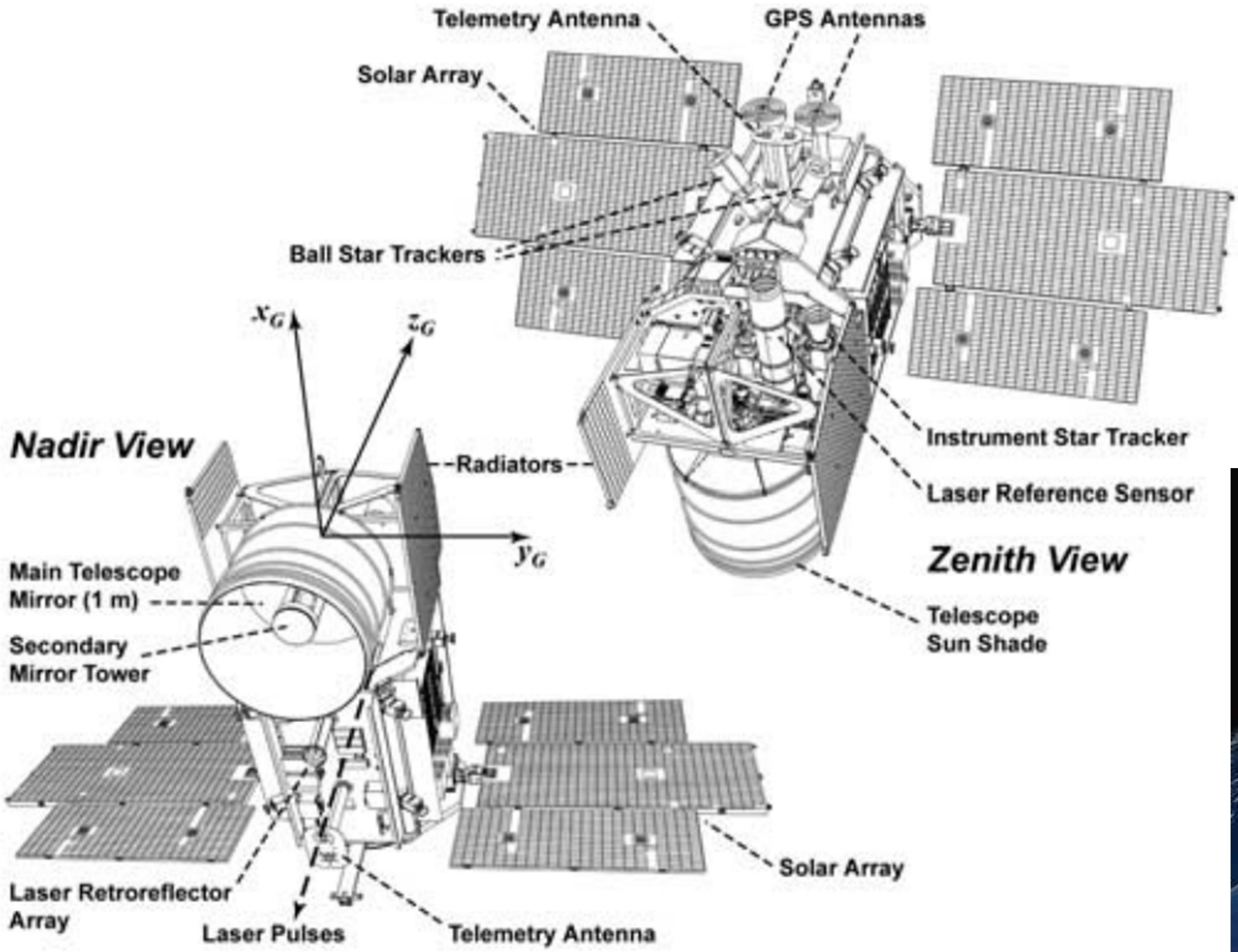


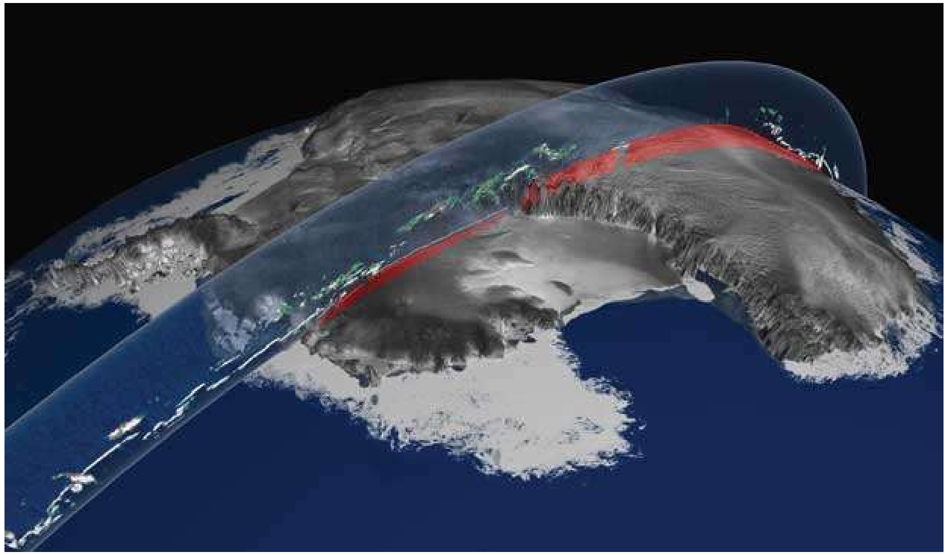
ICESat Onboard Instrumentation

- **Technologies Used:** A laser altimeter (**GLAS**) measured the altitude from the ice surface, while a high-resolution **GPS** receiver tracked satellite height relative to the WGS84 spheroid**.**
- **Data Integration:** The comparison of altimetry and GPS data provided information on ice sheet thickness.
- **Geoscience Laser Altimeter System (GLAS)**
  - ✓ Space-based lidar system with dual-wavelength capability, emitting infrared and visible laser pulses at 1064 nm and 532 nm
  - ✓ Precision surface lidar integrated with a sensitive cloud and aerosol lidar



ICESat Onboard Instrumentation



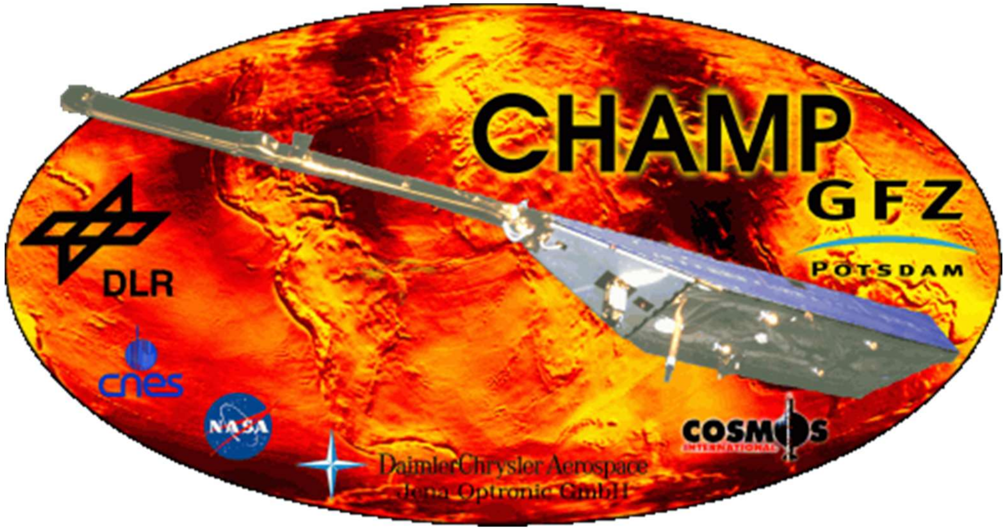


https://icesat.gsfc.nasa.gov/icesat/publications/GRL/schutz-1.pdf

CHAMP Mission Overview

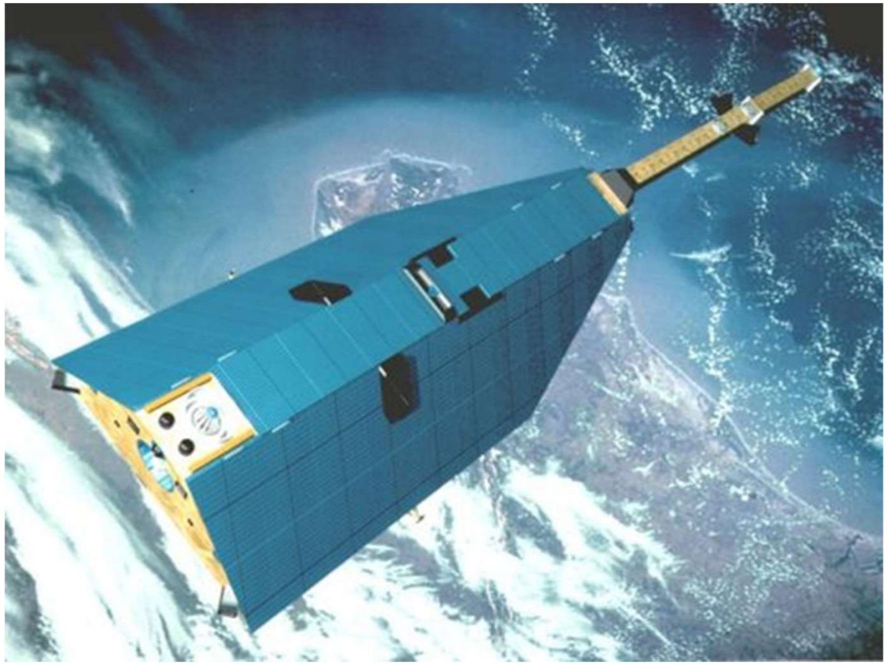
- **Period**: Launched on July 15, 2000, and concluded on September 19, 2010
- **Overall science objectives**:
  - ✓ Global recovery of the static and time-variable Earth gravity field from orbit perturbation analyses for use in geophysics (solid Earth), geodesy (reference surface), and oceanography (ocean currents and climate), supported by a feasibility test of GPS altimetry for ocean and ice surface monitoring
  - ✓ Global Earth magnetic field recovery (solid Earth and solar-terrestrial physics)
  - ✓ Atmosphere/ionosphere sounding by GPS radio occultation with applications in weather forecasting, navigation, space weather, and global climate change.

CHAMP: CHAllenging Minisatellite Payload

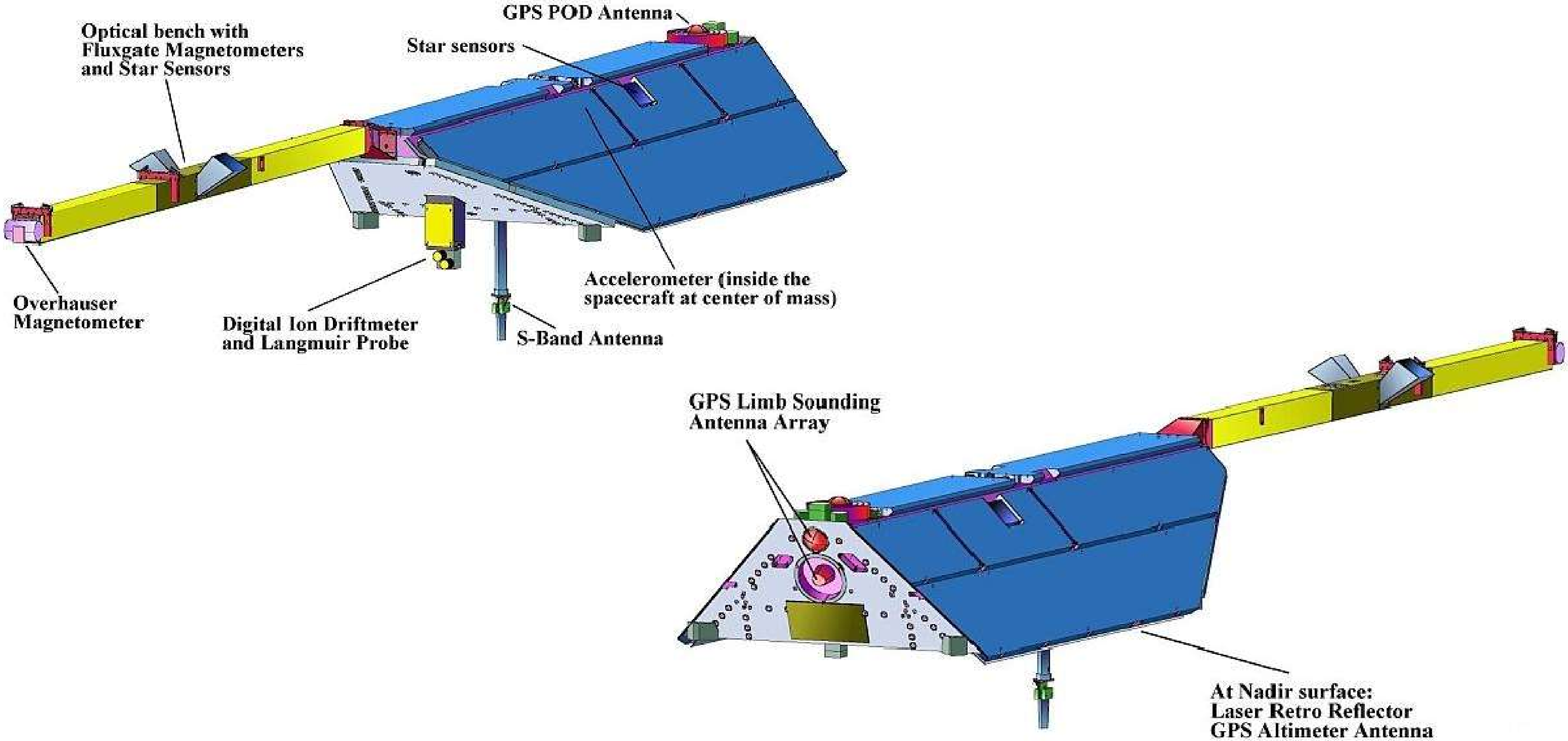


CHAMP Mission Overview

- **CHAMP Spacecraft**
  - **Launch mass**: 522 kg
  - **Dimensions**: 4.3 × 0.75 × 1.6 m
  - **Power**: 150 W
  - **Stabilization**: Earth pointing, 3 magnetorquers and cold gas propulsion for attitude and orbit change maneuvers
- **Nominal Orbital Parameters**
  - Near-polar orbit with an inclination of 87.3 degrees
  - Orbital altitude of 454 km (LEO)
  - Orbital period of 93.55 minutes



CHAMP Onboard Instrumentation



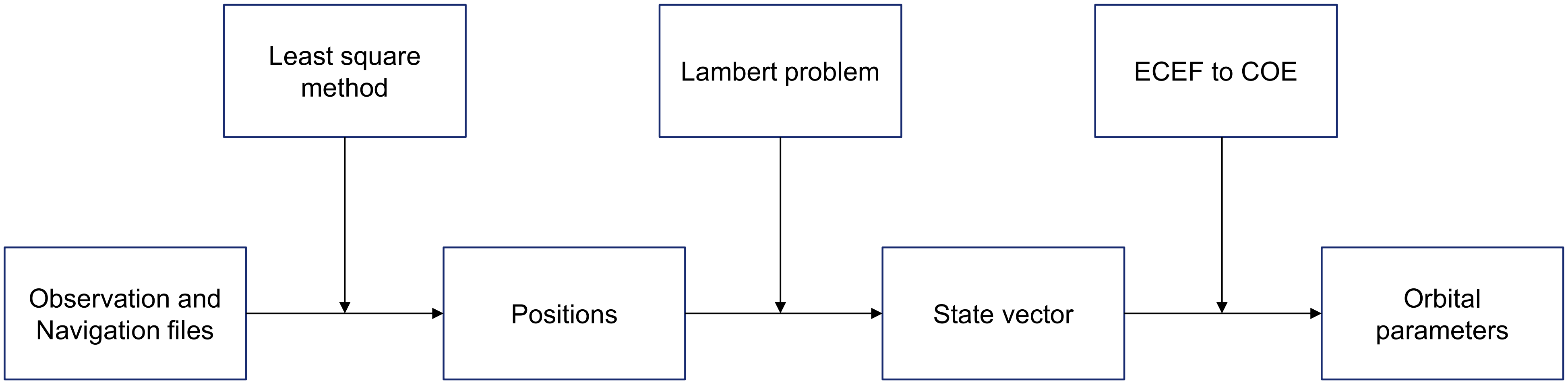
Data Processing in Matlab

- **Reference Epoch:**
  - ✓ ICESat: October 27, 2007, 02:00:00 (GPS week 1450, day 7)
  - ✓ CHAMP: May 20, 2001, 10:00:00 UTC

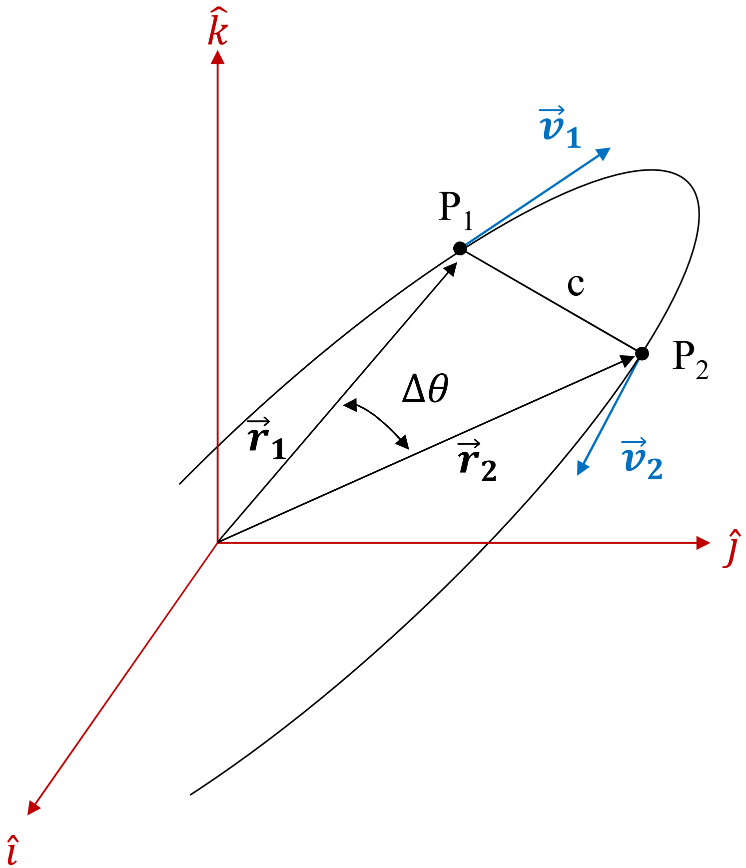
- **Data Files Used:**
  - ✓ RINEX observation files for ICESat/CHAMP
  - ✓ Navigation and SP3 files for GNSS satellites position and clock offset data
  - ✓ Data available at: *https://cddis.nasa.gov/Data_and_Derived_Products/GNSS/onboard_data.html*

- **ICESat/CHAMP orbit reconstruction**
  - ✓ Satellite position and its uncertainty calculated over a 100-minute arc to visualize at least one complete orbit.
  - ✓ ICESat/CHAMP positions to estimate its Keplerian orbital parameters

Data analysis process



Lambert problem



- **Data:** $\vec{r}_1, \vec{r}_2$ e Δt
- **Target**: find the trajectory.

The trajectory is determined once we find $\vec{v}_1$.

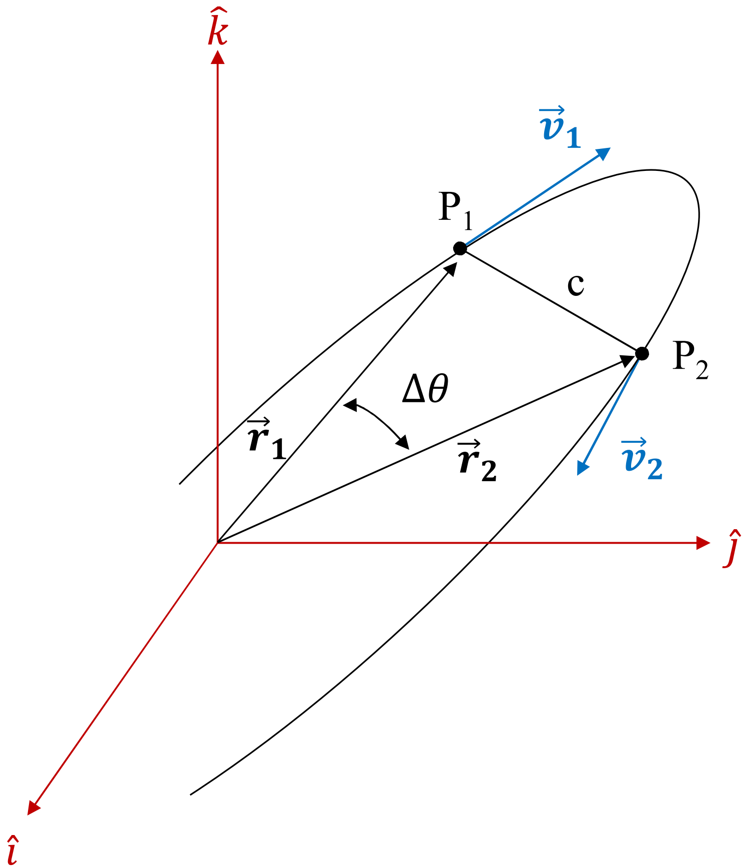
From the **Lagrange formulation** we have

$$\vec{r}_2 = f\,\vec{r}_1 + g\,\vec{v}_1 \quad \Longrightarrow \quad \vec{v}_1 = \frac{1}{g}(\vec{r}_2 - f\,\vec{r}_1)$$

Lagrange coefficients f, g are expressed as function of $z = \frac{\chi^2}{a}$, where χ is the universal anomaly and a the semimajor axis.

$$f(z) = 1 - \frac{\gamma(z)}{r_1} \qquad g(z) = A\sqrt{\frac{\gamma(z)}{\mu}}$$

Lambert problem



$$\gamma(z) = r_1 + r_2 + A\frac{zS(z)-1}{\sqrt{C(z)}}$$

$$A = \sin\Delta\theta\sqrt{\frac{r_1 r_2}{1-\cos}} \quad \text{with } \Delta\theta = \arccos\left(\frac{\vec{r}_1\cdot\vec{r}_2}{r_1 r_2}\right)$$

$S(z)$ and $C(z)$ are the Stumpff functions of the universal variables.

The variable z is computed iteratively with a Newton-Raphson Method that evaluate the zero of the function

$$F(z) = \left[\frac{\gamma(z)}{C(z)}\right]^{\frac{3}{2}} S(z) + A\sqrt{\gamma(z)} - \sqrt{\mu}\Delta t$$