SATELLITE NAVIGATION
Workbook 5: Implementation of a Kalman filter for a low dynamic reciver

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Kalman filter for a low Dynamic receiver

Objective

The exercise propose *the Implementation of an extended Kalman filter for a low dynamic receiver.*

The objectives

- Implement an EKF code for a low dynamic receiver in Matlab.
- Apply the developed code to track the receiver used during the April 10th laboratory session.
- Visualize the reconstructed path.
- Plot the "Innovation" as a function of time.
- Plot the "Kalman Gain" as a function of time.
- Plot the eigenvalues of the covariance matrix as a function of time.

Software

- Matlab (R2023a or later) with Navigation Toolbox and Satellite Communication Toolbox.

Kalman filter for a low Dynamic receiver

Procedure

1. Initialization
- initializing the Kalman filter with an initial estimate of the receiver's position and velocity.
- Initialize the covariance matrix representing the uncertainty in the initial estimate.

2. Measurement Acquisition
- Acquire measurements from GPS satellites in view.
- Measure the pseudo-ranges from the receiver to each satellite.

3. State Prediction
- Predict the state of the receiver (position and velocity) forward in time using the Kalman filter state transition model.
- Update the covariance matrix to account for process noise.

4. Measurement Update
- Calculate the expected pseudo-ranges from the predicted receiver position to each satellite.
- Compare the expected pseudo-ranges with the actual measurements.
- Compute the measurement residuals (the differences between the expected and actual pseudo-ranges).
- Use these residuals to correct the predicted state and update the covariance matrix.

5. Iterative Refinement
- Repeat steps 3 and 4 iteratively for each time step or measurement update.
- Use the updated state and covariance estimates as the new initial conditions for the next iteration.

6. Position Estimation
- After several iterations or when convergence criteria are met, estimate the receiver's final position based on the converged state estimate.

7. Output
- Output the reconstructed path. Plot the "Innovation" as a function of time. Plot the "Kalman Gain" as a function of time. Plot the eigenvalues of the covariance matrix as a function of time.

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Procedure

1. Initialization

- Initial state estimate $\hat{x}_{0,0}$ of $x(0)$

```
X0 = [RecPos0(1) 0 RecPos0(2) 0 RecPos0(3) 0 0 0]';
X(:,1) = X0;
```

$$P(0|0) = \begin{bmatrix} P_{A_{11}} & 0 & P_{A_{12}} & 0 & P_{A_{13}} & 0 & P_{A_{14}} & 0 \\ 0 & \sigma_{\dot{x}}^2(0) & 0 & 0 & 0 & 0 & 0 & 0 \\ P_{A_{21}} & 0 & P_{A_{22}} & 0 & P_{A_{23}} & 0 & P_{A_{24}} & 0 \\ 0 & 0 & 0 & \sigma_{\dot{y}}^2(0) & 0 & 0 & 0 & 0 \\ P_{A_{31}} & 0 & P_{A_{32}} & 0 & P_{A_{33}} & 0 & P_{A_{34}} & 0 \\ 0 & 0 & 0 & 0 & 0 & \sigma_{\dot{z}}^2(0) & 0 & 0 \\ P_{A_{41}} & 0 & P_{A_{42}} & 0 & P_{A_{43}} & 0 & P_{A_{44}} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2Q_{d_{22}} \end{bmatrix}$$

- Covariance of the initial estimate $\hat{x}_{0,0}$

```
R = diag(sigmaPseudo*ones(1,length(psr)));
PA = (J'*R*J)';
P = [PA(1,1)        0 PA(1,2)        0  PA(1,3)        0 PA(1,4)        0;...
           0  sigmaV^2       0        0        0        0       0        0;...
     PA(2,1)        0 PA(2,2)        0  PA(2,3)        0 PA(2,4)        0;...
           0        0       0 sigmaV^2        0        0       0        0;...
     PA(3,1)        0 PA(3,2)        0  PA(3,3)        0 PA(3,4)        0;...
           0        0       0        0        0 sigmaV^2       0        0;...
     PA(4,1)        0 PA(4,2)        0  PA(4,3)        0 PA(4,4)        0;...
           0        0       0        0        0        0       0 2*Qd22];
```

Kalman filter for a low Dynamic receiver

Procedure

2. Measurement Acquisition

- Acquire measurements from four GPS satellites in view.
- Measure the pseudo-ranges from the receiver to each satellite.

```
psr1 = obs_i.C1C;
psr2 = obs_i.C2S;
f1 = 10.23*10^6*154; % Hz
f2 = 10.23*10^6*120; % Hz
```

```
%% IONOSPHERIC CORRECTION through measurements combination

% a1 + a2 = 1                => geometry-preserving
% a1 + f1^2/f2^2*a2 = 0      => ionosphere-free combination
a2 = -(f2^2)/(f1^2-f2^2); % => -1.5457
a1 = 1 - a2; % = f1^2/(f1^2-f2^2) => 2.5457



psr = a1*psr1 + a2*psr2;
```

Add corrections to pseudoranges !!!!

Kalman filter for a low Dynamic receiver

Procedure

3. State Prediction (of the receiver)

The state vector $\boldsymbol{x}$ consists of the receiver's position $[x, y, z]^T$ and velocity $[\dot{x}, \dot{y}, \dot{z}]^T$.
At each time step k, the state transition model predicts the state at the next time step:

$$\boldsymbol{x}_{k+1} = \boldsymbol{F}(k)\boldsymbol{x}_k + \boldsymbol{\Gamma}_{k+1}\,\boldsymbol{v}_k$$

$$F(k) = \begin{bmatrix} 1 & T & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & T & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & T & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & T \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

```
% state transition matrix
F = [1 T 0 0 0 0 0 0;...
     0 1 0 0 0 0 0 0;...
     0 0 1 T 0 0 0 0;...
     0 0 0 1 0 0 0 0;...
     0 0 0 0 1 T 0 0;...
     0 0 0 0 0 1 0 0;...
     0 0 0 0 0 0 1 T;...
     0 0 0 0 0 0 0 1];
```

$$\Gamma(k) = \begin{bmatrix} 0.5T^2 & 0 & 0 & 0 & 0 \\ T & 0 & 0 & 0 & 0 \\ 0 & 0.5T^2 & 0 & 0 & 0 \\ 0 & T & 0 & 0 & 0 \\ 0 & 0 & 0.5T^2 & 0 & 0 \\ 0 & 0 & T & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

```
Gamma = [0.5*T^2       0       0 0 0;...
         T             0       0 0 0;...
         0 0.5*T^2             0 0 0;...
         0             T       0 0 0;...
         0             0 0.5*T^2 0 0;...
         0             0       T 0 0;...
         0             0       0 1 0;...
         0             0       0 0 1];
```

$$\mathbf{v}(k) = \begin{bmatrix} \ddot{x} & \ddot{y} & \ddot{z} & c\omega_{d\phi} & c\omega_{df} \end{bmatrix}_k^T$$

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Procedure

4. Measurement Update

During the measurement update step, we compare the predicted pseudo-ranges $\hat{\rho}_{k,i}$ (distance from receiver to satellite i) with the actual measured pseudo-ranges $\rho_{k,i}$. The measurement residual $\Delta\rho_{k+1}$ is calculated as:

$$\Delta\rho_{k,i} = \hat{\rho}_{k,i} + \rho_{k,i}$$

```
% Measurement Prediction and residual
satPos(idNaN,:) = [];
rho_k = [];
v = [];
for i = 1 : length(psr)
    rho_k(i) =  ((satPos(i,1)-X_(1,k))^2+(satPos(i,2)-X_(3,k))^2+(satPos(i,3)-X_(5,k))^2)^0.5 + X_(7,k);
    v(i) = psr(i) - rho_k(i);
end
```

Kalman filter for a low Dynamic receiver

Procedure

5. Iterative Refinement

We then update the state estimate $\boldsymbol{x}_{k+1}$ and its covariance matrix $\boldsymbol{P}_{k+1}$ as follow:

$$\boldsymbol{x}_{k+1} = \boldsymbol{x}_{k+1} + \boldsymbol{\Gamma}_{k+1}\,\Delta\boldsymbol{z}_{k+1}$$
$$\boldsymbol{P}_{k+1} = (\boldsymbol{I} - \boldsymbol{\Gamma}_{k+1}\,\boldsymbol{H}_{k+1})\boldsymbol{P}_{k+1|k}$$

- $\boldsymbol{\Gamma}_{k+1}$ is the Kalman gain
- $\Delta\boldsymbol{z}_{k+1}$ is the innovation vector where $\Delta\boldsymbol{z}_{k+1} = \boldsymbol{z}_k - \boldsymbol{H}_k\boldsymbol{x}_k$ ($\boldsymbol{z}_k$ is the actual measurement at time step k)
- $\boldsymbol{H}_{k+1}$ is the measurement matrix:

$$H(k+1) = \left.\begin{bmatrix} \frac{\partial\rho_1}{\partial x} & 0 & \frac{\partial\rho_1}{\partial y} & 0 & \frac{\partial\rho_1}{\partial z} & 0 & \frac{\partial\rho_1}{\partial r_{t_r}} & 0 \\ \frac{\partial\rho_2}{\partial x} & 0 & \frac{\partial\rho_2}{\partial y} & 0 & \frac{\partial\rho_2}{\partial z} & 0 & \frac{\partial\rho_2}{\partial r_{t_r}} & 0 \\ . & . & . & . & . & . & . & . \\ . & . & . & . & . & . & . & . \\ \frac{\partial\rho_{N_s}}{\partial x} & 0 & \frac{\partial\rho_{N_s}}{\partial y} & 0 & \frac{\partial\rho_{N_s}}{\partial z} & 0 & \frac{\partial\rho_{N_s}}{\partial r_{t_r}} & 0 \end{bmatrix}\right|_{\mathbf{x}=\hat{\mathbf{x}}(k+1|k)}$$

```
% Measurement Noise Matrix
H = [];
for i = 1 : length(psr)

    H(i,1) = J(i,1);
    H(i,2) = 0;
    H(i,3) = J(i,2);
    H(i,4) = 0;
    H(i,5) = J(i,3);
    H(i,6) = 0;
    H(i,7) = J(i,4);
    H(i,8) = 0;

end
```

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Procedure: results

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6. Position Estimation and Output

- Visualize the reconstructed path.
- Plot the "Innovation" as a function of time.
- Plot the "Kalman Gain" as a function of time.
- Plot the eigenvalues of the covariance matrix as a function of time