SATELLITE NAVIGATION
Workbook 6: Implementation of a Direct Kalman filter for GNSS/INS navigation

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Direct Kalman filter for GNSS/INS navigation

Objective

The exercise propose *the Implementation of a direct Kalman filter for GNSS/INS navigation* using the approach proposed in Honghui Qi and J. B. Moore, "Direct Kalman filtering approach for GPS/INS integration," in IEEE Transactions on Aerospace and Electronic Systems, vol. 38, no. 2, pp. 687-693, April 2002, doi: 10.1109/TAES.2002.1008998.

The objectives

- Implement an GNSS/INS integration code in Matlab.
- Apply the developed code to track the trajectory of a Pixhawk® 4 Mini placed on-board a bicycle.
- Compare the trajectory obtained with the INS system only, the GPS trajectory and the trajectory obtained by means of the GPS/INS sensor fusion.
- Study the effect of measurement and model covariance matrices on the reconstructed trajectory.

Software

- Matlab (R2023a or later) with Navigation Toolbox and Satellite Communication Toolbox.

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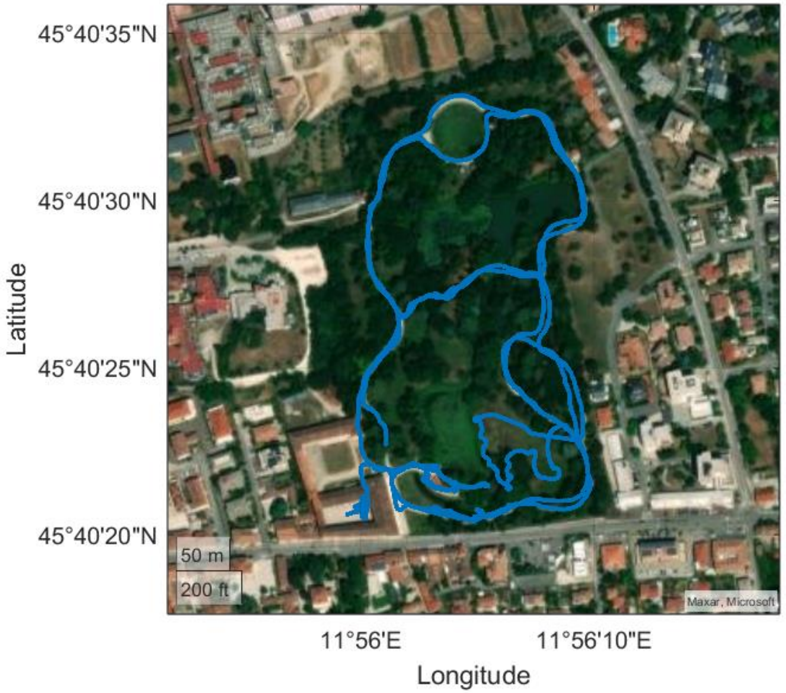
Dataset

Pixhawk 4 mini	
IMU	
Model	BMI055[1]
Digital resolution	Accelerometer (A): 16-bit Gyroscope (G): 16-bit
Resolution	(A): 0.09 mg (G): 0.004°/s
Measurement range and sensitivity	(A) ± 3 g: 10920 LSB/g (G): ± 125°/s: 262.144 LSB/°/s
Zero offset	(A): ± 70 mg (G): ± 1°/s
Noise density (typ.)	(A): 150 µg/√Hz (G): 0.014 °/s/√Hz
Acquisition Rate	20 Hz
GNSS	
Model	HolyBro Pixhawk 4 Neo-M8N GPS
Satellite systems	GPS/QZSS; GLONASS; Galileo; BeiDou
Max nav update rate	5 Hz (Glonass/BeiDou) 10 Hz (GPS)
Velocity accuracy	0.05 m/s
Heading accuracy	0.3°
Horizontal accuracy	Autonomous 2.5 m SBAS 2 m



Detailed description of messages:
https://ardupilot.org/plane/docs/logmessages.html

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Data to Use - IMU

IMU

112134x16 double

	t_{IMU}		$\omega^b_{x,ib}$	$\omega^b_{y,ib}$	$\omega^b_{z,ib}$	a^b_x	a^b_y	a^b_z

	1	2	3	4	5	6	7	8	9
1	73907	291746482	1	0.0176	2.4973e-04	-0.0297	-0.0783	0.6194	-9.7349
2	73929	291786214	0	0.0473	0.0082	0.0060	-0.0843	0.3614	-9.8272
3	73930	291786214	1	0.0288	0.0017	-0.0012	0.0029	0.3459	-9.8496
4	73931	291825194	0	0.0623	-2.8381e-05	0.0603	0.0933	0.3794	-9.8134
5	73932	291825194	1	0.0422	-0.0059	0.0535	0.1723	0.3555	-9.8416
6	73955	291865255	0	0.0633	0.0114	0.0751	0.1607	0.3200	-9.8536
7	73956	291865255	1	0.0455	0.0053	0.0686	0.2874	0.2767	-9.8635
8	73957	291905243	0	0.0617	0.0053	0.0677	-0.1707	0.0266	-9.7454
9	73958	291905243	1	0.0422	-9.3845e-04	0.0619	-0.0866	-0.0216	-9.7695
10	73964	291945995	0	0.0518	0.0031	0.0474	-0.3594	-0.0323	-9.7410
11	73965	291945995	1	0.0297	-0.0012	0.0425	-0.3029	-0.0943	-9.7825
12	73985	291986155	0	0.0352	0.0080	0.0082	-0.2802	0.0973	-9.7871
13	73986	291986155	1	0.0129	0.0037	0.0022	-0.2007	0.0917	-9.8017
14	73987	292025231	0	0.0304	0.0116	-0.0294	-0.1578	0.2302	-9.8087

- ω^b_{ib}: gyro measured angular velocities with respective to the inertial frame coordinated in the body frame [rad/s].
- a^b specific force vector in the body frame (b-frame) [m/s^2]

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Data to Use - GNSS

IMU | GPS | GPS_label | XKF1_label

9948x16 double

	t_{GPS}						HDOP	lat	lon	alt	$\|\|v^e_{hor}\|\|$	Ψ	$\|\|v^e_{ver}\|\|$

	1	2	3	4	5	6	7	8	9	10	11	12	13	14
1	146818	544945885	0	3	228564000	2205	13	0.8500	45.6730	11.9336	43.7100	2.4730	353.5366	-0.0890
2	146874	545145797	0	3	228564200	2205	13	0.8500	45.6730	11.9336	43.5300	2.0620	2.3066	-0.0940
3	146934	545345796	0	3	228564400	2205	13	0.8500	45.6730	11.9336	43.7100	2.9170	3.6172	0.4920
4	146990	545545932	0	3	228564600	2205	13	0.8500	45.6730	11.9336	43.4900	2.5960	358.0868	0.8980
5	147046	545745796	0	3	228564800	2205	13	0.8500	45.6730	11.9336	43.4200	3.0600	354.7573	0.4370
6	147106	545945798	0	3	228565000	2205	13	0.8500	45.6730	11.9336	43.2400	2.2880	356.7679	0.1450
7	147163	546145940	0	3	228565200	2205	13	0.8500	45.6730	11.9336	43.3600	2.4430	8.9191	0.4580
8	147221	546345713	0	3	228565400	2205	13	0.8500	45.6730	11.9336	43.6100	2.9830	10.2032	-0.2380
9	147277	546545920	0	3	228565600	2205	13	0.8500	45.6730	11.9336	43.7100	2.9840	0.5350	0.2800
10	147333	546745926	0	3	228565800	2205	13	0.8500	45.6730	11.9336	43.8500	2.7920	0.4555	-0.1380
11	147393	546945804	0	3	228566000	2205	13	0.8500	45.6730	11.9336	44	3.1710	359.4482	-0.1370
12	147450	547145924	0	3	228566200	2205	13	0.8500	45.6730	11.9336	43.8500	2.6960	355.7806	0.1830

- (lat, lon, alt): latitude [deg], longitude [deg] and altitude [m] of the reciever.
- **HDOP**: Horizontal Dilution of Precision, to use to compute position covariance [m]
- $\|\|v^e_{hor}\|\|$ ground speed [m/s]
- $\|\|v^e_{ver}\|\|$ vertical speed [m/s]
- Ψ ground course [deg]

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Available Data - EKF

IMU | GPS | GPS_label | XKF1_label | XKF1

44844x17 double

		t_{EKF}		ϕ	θ	ψ	v_N^e	v_E^e	v_D^e		p_N^e	p_E^e	p_D^e			
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
1	74050	292249576	0	-0.3400	-2.6200	216.4100	-0.3067	-0.5265	-0.0081	0.0107	7.3215	-6.5522	0.2327	1.1400	0.2900	1.1
2	74059	292249576	1	0.0400	-1.9700	218.8500	-0.2670	-0.5565	-0.0173	0.0219	7.3071	-6.5278	0.1912	-0.0300	-0.0400	0.7
3	74083	292349570	0	-0.4300	-2.6000	216.2500	-0.3245	-0.4883	-0.0037	0.0180	7.3016	-6.5895	0.2343	1.1400	0.2900	1.1
4	74090	292349570	1	-0.0400	-1.9500	218.6900	-0.2804	-0.5226	-0.0099	0.0314	7.2927	-6.5639	0.1941	-0.0200	-0.0400	0.8
5	74109	292449555	0	-0.6000	-2.5900	216.2400	-0.3299	-0.4788	-0.0034	0.0258	7.2965	-6.6053	0.2371	1.1400	0.2800	1.1
6	74116	292449555	1	-0.2000	-1.9300	218.6900	-0.2841	-0.5141	-0.0099	0.0386	7.2884	-6.5827	0.1983	-0.0200	-0.0400	0.8
7	74139	292549619	0	-0.7500	-2.6200	215.8000	-0.3265	-0.4790	-0.0047	0.0304	7.2887	-6.6254	0.2400	1.1400	0.2800	1.1
8	74146	292549619	1	-0.3600	-1.9600	218.2400	-0.2793	-0.5174	-0.0106	0.0427	7.2821	-6.6056	0.2023	-0.0200	-0.0400	0.8
9	74165	292649608	0	-0.8900	-2.6300	215.4400	-0.3104	-0.4509	9.1627e-05	0.0235	7.2895	-6.6398	0.2415	1.1400	0.2800	1.1
10	74172	292649608	1	-0.4900	-1.9600	217.8800	-0.2621	-0.4880	-0.0055	0.0363	7.2840	-6.6222	0.2050	-0.0200	-0.0400	0.8
11	74197	292749553	0	-1.1000	-2.6100	215.2900	-0.3227	-0.4326	-0.0012	0.0233	7.2803	-6.6603	0.2443	1.1400	0.2800	1.1
12	74206	292749553	1	-0.7000	-1.9400	217.7300	-0.2710	-0.4746	-0.0043	0.0369	7.2806	-6.6426	0.2091	-0.0200	-0.0400	0.8

Pixhawk® 4 Mini Extended Kalman filter to use for comparison and initialization

- (ϕ, θ, ψ): Estimated roll, pitch and yaw angles [deg]
- (v_N^e, v_E^e, v_D^e): Estimated velocity in the NED frame [m/s]
- (p_N^e, p_E^e, p_D^e): Estimated position in the NED frame [m/s]

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State space model

- Let the state space model for the design of the data fusion Kalman filter be:

$$\hat{\boldsymbol{x}} = [\boldsymbol{p}, \dot{\boldsymbol{p}}, \Delta\boldsymbol{a}_1^e, \Delta\boldsymbol{a}_2^e]^{\boldsymbol{T}}$$

where $\boldsymbol{p}$ is the GPS receiver's position coordinates in the NED frame, and $\Delta\boldsymbol{a}_1^e, \Delta\boldsymbol{a}_2^e$ the accelerometer bias and drift.

- Consider a discrete time signal model for data fusion, operating at a fast sample rate with sampling period δT, as

$$\hat{\boldsymbol{x}}_{\boldsymbol{k+1}}^{-} = \boldsymbol{A}_{\boldsymbol{k}}\hat{\boldsymbol{x}}_{\boldsymbol{k}} + \boldsymbol{B}_{\boldsymbol{k}}\boldsymbol{u}_{\boldsymbol{k}} + \boldsymbol{w}_{\boldsymbol{k}}$$

- The model take into account the motion euations:

$$\boldsymbol{v}^e(t+\delta t) = \boldsymbol{v}^e(t) + \left(\boldsymbol{R}_b^e(t)\boldsymbol{a}^b(t) - \boldsymbol{g}^e\right)\delta t$$
$$\boldsymbol{p}^e(t+\delta t) = \boldsymbol{p}^e(t) + \boldsymbol{v}^e(t)\delta t$$

Kalman filter for a low Dynamic receiver

Procedure

1. **Initialization**
   - Initializing the Kalman filter with an initial estimate of the receiver's position and velocity from GPS. Attitude form the Pixhawk
   - Initialize the covariance matrix representing the uncertainty in the initial estimate.
2. **State Prediction**
   - Predict the attitude of the IMU relative to the NED frame using the gyroscope data

$$\hat{\mathbf{R}}^{e}_{b}(l+1) = \hat{\mathbf{R}}^{e}_{b}(l)\overline{\exp(\hat{\Omega}^{b}_{eb}(l)\,\delta T)}$$

$$\hat{\mathbf{v}}^{e}(l) = \hat{\mathbf{R}}^{e}_{b}(l)\hat{\mathbf{f}}^{b}(l) - 2\Omega^{e}_{ie}\hat{\mathbf{v}}^{e}(l) + \mathbf{g}^{e}(\hat{\mathbf{r}}^{e}(l)).$$

$$\overline{\exp(\Omega^{b}_{eb}(l)\,\delta T)} = (2\mathbf{I} + \Omega^{b}_{eb}(l)\,\delta T)(2\mathbf{I} - \Omega^{b}_{eb}(l)\,\delta T)^{-1}$$

   - Transform the accelerometer vector form the body frame to the NED frame
   - Predict the state of the receiver (position and velocity) forward in time using the Kalman filter state transition model.
   - Update the covariance matrix to account for process noise.
3. **Measurement Update**
   - Uses the computed GPS position, velocity to update the prior estimate of the state.
4. **Iterative Refinement**
   - Repeat steps 2 and 3 iteratively for each time step or measurement update.
   - Use the updated state and covariance estimates as the new initial conditions for the next iteration.
5. **Output**
   - Output the reconstructed path.