Esercitazioni per il corso di STRUMENTAZIONE SPAZIALE

prof. E. Lorenzini

USO DI MATLAB PER LA DETERMINAZIONE D'ASSETTO E L'ANALISI DI COVARIANZA

- Single-axis attitude determination
- Three-axis attitude determination

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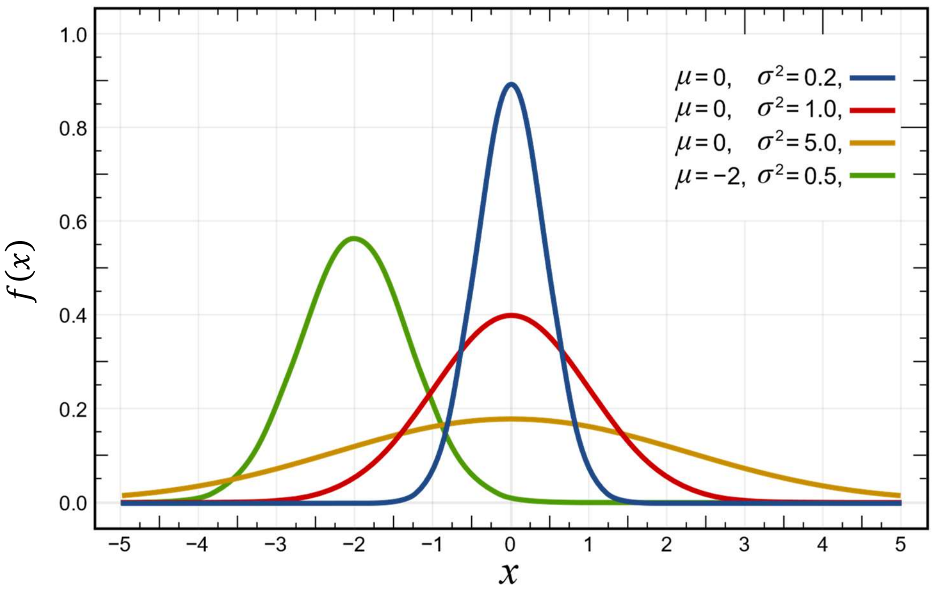
Univariate Normal or Gaussian distribution $\mathcal{N}(\mu, \sigma)$

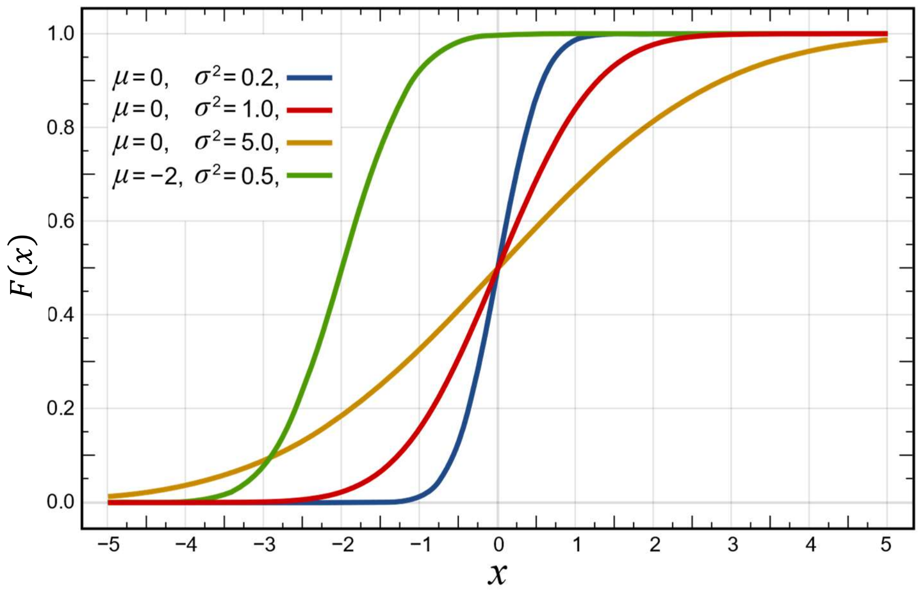
Probability Density Function (PDF)



$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

$\mu \Rightarrow$ mean value
$\sigma \Rightarrow$ standard deviation
$\sigma^2 \Rightarrow$ variance

Cumulative Distribution Function (CDF)



$$F(x) = \int_{-\infty}^{x} f(t)dt = \frac{1}{2}\left[1 + \operatorname{erf}\left(\frac{x-\mu}{\sigma\sqrt{2}}\right)\right],$$

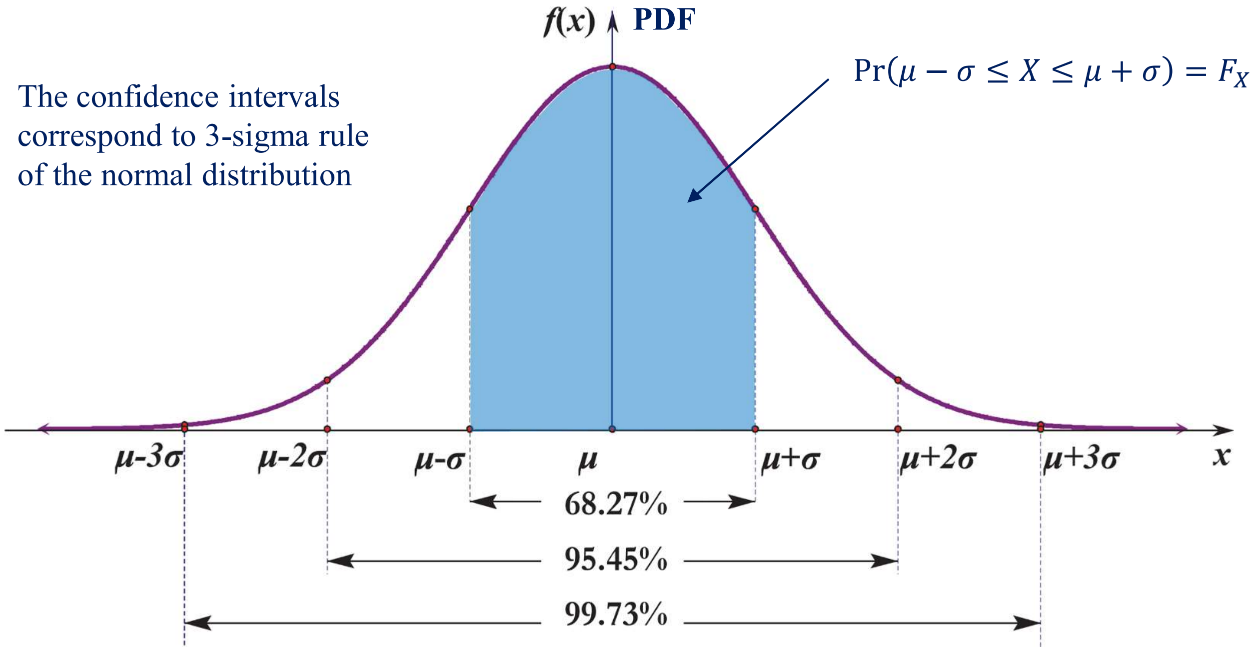
$$\operatorname{erf}(y) = \frac{2}{\sqrt{\pi}} \int_{-\infty}^{y} e^{-\frac{t^2}{2}}\, dt$$

$F_X(x) = \Pr(X \leq x) \Rightarrow$ probability that the random variable X takes on a value less than or equal to x.

https://en.wikipedia.org/wiki/Normal_distribution

The 3-sigma rule

The confidence intervals correspond to 3-sigma rule of the normal distribution



$\Pr(\mu - \sigma \le X \le \mu + \sigma) = F_X(\mu + \sigma) - F_X(\mu - \sigma)$

univariate normal distribution		
r	p	p [%]
1	0.6827	68.27
2	0.9545	95.45
3	0.9973	99.73

Matlab functions:
- `erf`
- `erfinv`

See also:
- `norminv`
- `normcdf`
- `tinv`
- `tcdf`
- `unifinv`
- `unifcdf`

p ⇒ confidence level (livello di confidenza)

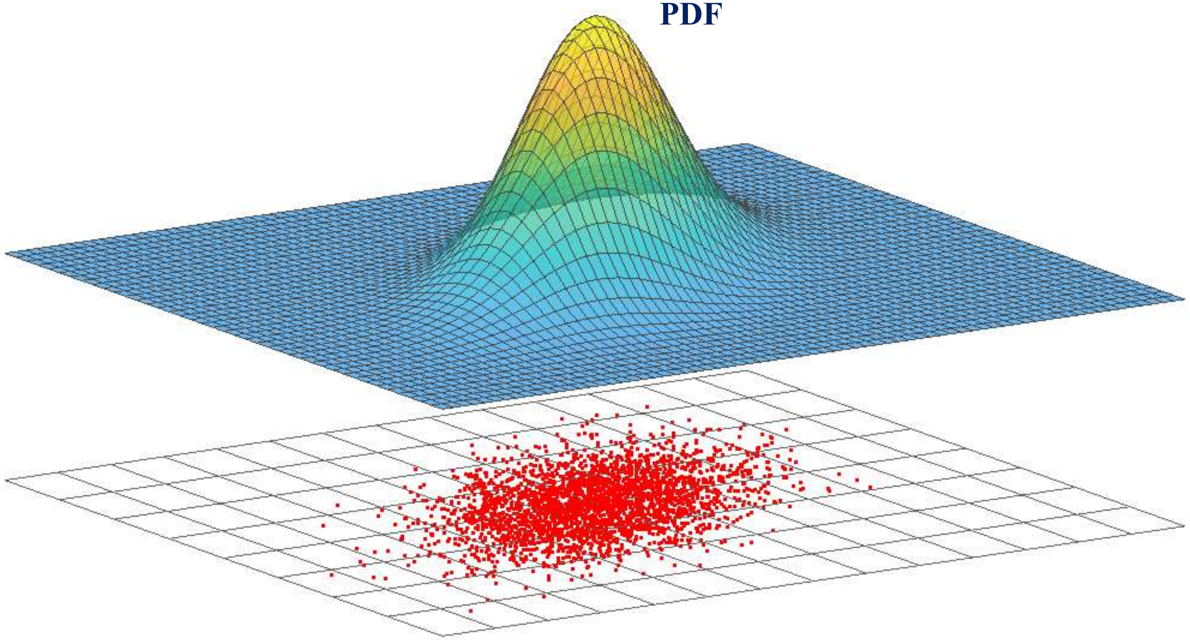
r ⇒ magnification or coverage factor (fattore di copertura)

Univariate Normal or Gaussian distribution

$$p = \mathrm{erf}\left(\frac{r}{\sqrt{2}}\right) \Rightarrow r = \sqrt{2}\ \mathrm{erf}^{-1}(p)$$

Multivariate Normal or Gaussian distribution $\mathcal{N}(\boldsymbol{\mu}, \boldsymbol{\Sigma})$

Probability Density Function (PDF) $f(\boldsymbol{x})$



$$f(\boldsymbol{x}) = \frac{1}{\sqrt{(2\,\pi)^n |\boldsymbol{\Sigma}|}} e^{-\frac{1}{2}(\boldsymbol{x}-\boldsymbol{\mu})^T \boldsymbol{\Sigma}^{-1}(\boldsymbol{x}-\boldsymbol{\mu})}$$

$n \Rightarrow$ space dimensionality

$\boldsymbol{\mu} \in \mathbb{R}^n \Rightarrow$ mean value

$\boldsymbol{\Sigma} \in \mathbb{R}^{n\times n} \Rightarrow$ covariance matrix

$$\Sigma_{i,j} = \begin{cases} var(x_i) & if\ \ i = j \\ cov(x_i, x_j) & if\ \ i \neq j \end{cases}$$

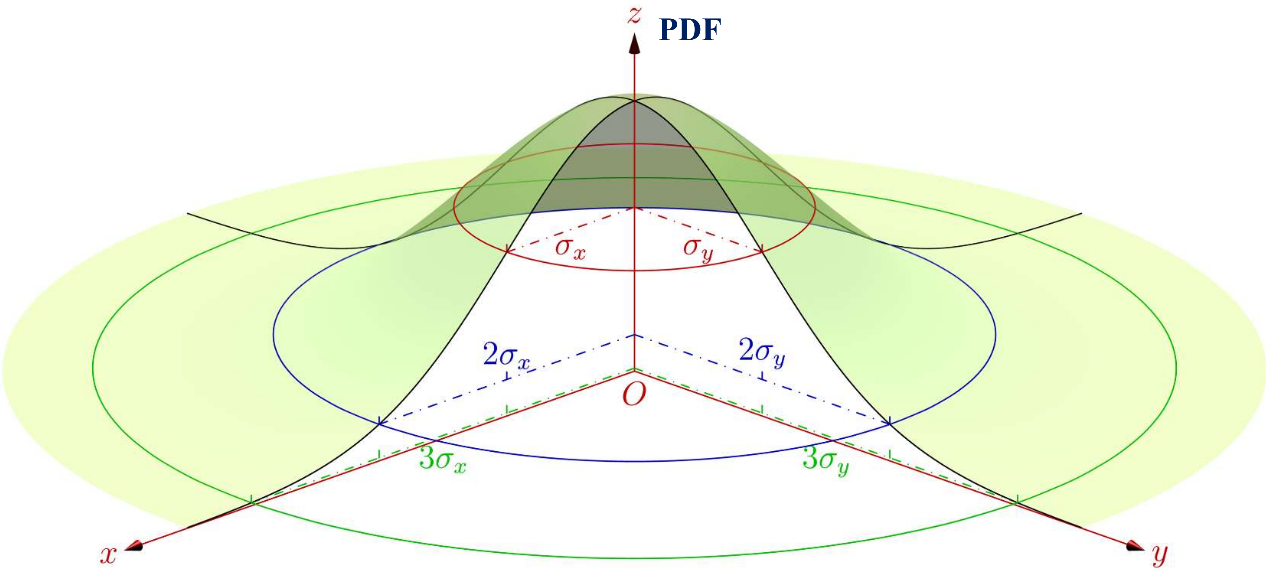
Cumulative Distribution Function (CDF) $F(\boldsymbol{x})$

probability that all components of $\boldsymbol{X}$ are less than or equal to the corresponding values in the vector $\boldsymbol{x}$

$$F_{\boldsymbol{X}}(\boldsymbol{x}) = \Pr(\boldsymbol{X} \leq \boldsymbol{x})$$

There is no closed form for $F(\boldsymbol{x})$; however, there are diverse algorithms to estimate it numerically.

Bivariate Normal or Gaussian distribution $\mathcal{N}(\boldsymbol{\mu}, \boldsymbol{\Sigma})$



Bivariate normal distribution		
r	p	p [%]
1	0.3935	39.35
2	0.8647	86.47
3	0.9889	98.89

Bivariate Normal or Gaussian distribution

$$p = 1 - e^{-\frac{r^2}{2}} \;\Rightarrow\; r = \sqrt{-2\ln(1-p)}$$

p ⇒ confidence level (livello di confidenza)

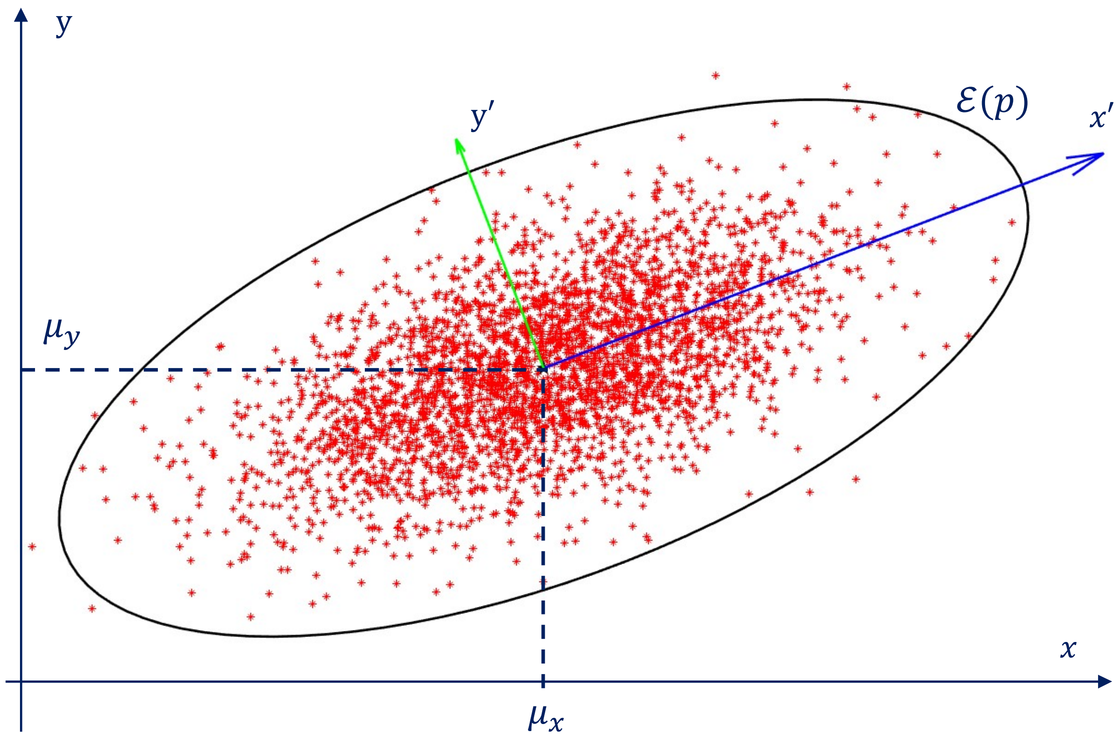
r ⇒ magnification or coverage factor

(fattore di copertura)

Uncertainty ellipse

The covariance matrix $\mathbf{\Sigma} \in \mathbb{R}^{2\times2}$ defines the **uncertainty ellipse $\boldsymbol{\mathcal{E}(p)}$**:

- ✓ the eigenvectors of $\mathbf{\Sigma}$ defines the principal axes x' and y$'$ of the uncertainty ellipse $\mathcal{E}$
- ✓ the eigenvalues of $\mathbf{\Sigma}$ defines the standard deviations $\sigma_{x'}$ and $\sigma_{y'}$
- ✓ the semi-axes of the uncertainty ellipse for a given confidence level p are:
  - semi-major axis: $a = r(p)\ \sigma_{x'}$
  - semi-minor axis: $b = r(p)\ \sigma_{y'}$



➢ The principal axes x' and y$'$ of the uncertainty ellipse $\mathcal{E}$ can be found by computing eigenvectors and eigenvalues of the covariance matrix $\mathbf{\Sigma}$

Trivariate Normal or Gaussian distribution $\mathcal{N}(\boldsymbol{\mu}, \boldsymbol{\Sigma})$

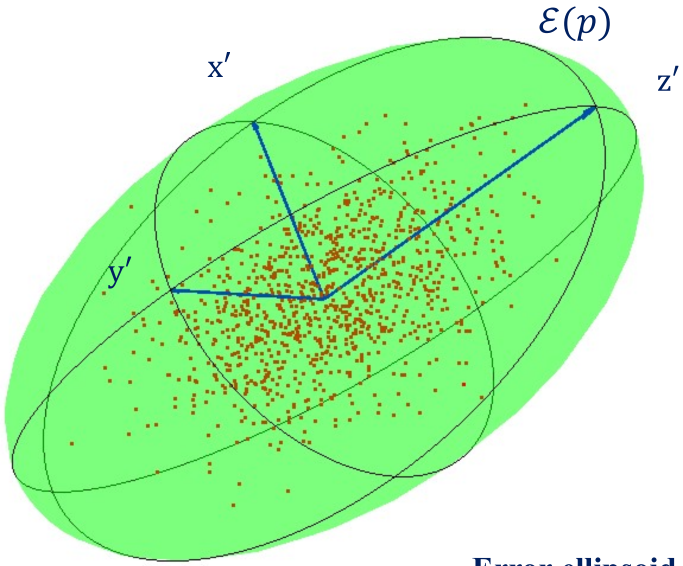
Trivariate Normal or Gaussian distribution

- ✓ There is not an explicit relation between the confidence level p and the magnification factor r, as for the univariate or bivariate Normal Distribution.
- ✓ There is instead a numeric algorithm, see for instance:

  https://journals.plos.org/plosone/article?id=10.1371/journal.pone.0118537
- ✓ Note: use interpolation data p-r to obtain the magnification factor r that corresponds to a given confidence level p.

Trivariate normal distribution		
r	p	p [%]
1	0.1987	19.87
2	0.7384	73.85
3	0.9707	97.07

- ➢ the covariance matrix $\boldsymbol{\Sigma} \in \mathbb{R}^{3\times3}$ defines the error ellipsoid $\mathcal{E}(p)$:
  - ✓ the eigenvectors of $\boldsymbol{\Sigma}$ defines the principal axes x', y′ and z′ of the error ellipsoid $\mathcal{E}(p)$;
  - ✓ the eigenvalues of $\boldsymbol{\Sigma}$ defines the standard deviations $\sigma_{x'}$, $\sigma_{y'}$ and $\sigma_{z'}$;
  - ✓ the semi-axes of the error ellipse for a given confidence level p are:
    - semi axes: $a_i = r(p)\, \sigma_{x_i'}$



Error ellipsoid

Magnification factor *r* vs. confidence level *p* vs. dimensionality

Confidence levels of scaled Standard Deviational Hyper-Ellipsoid (SDHE) vary with different magnification factors in spaces with the dimensionality ≤ 10.

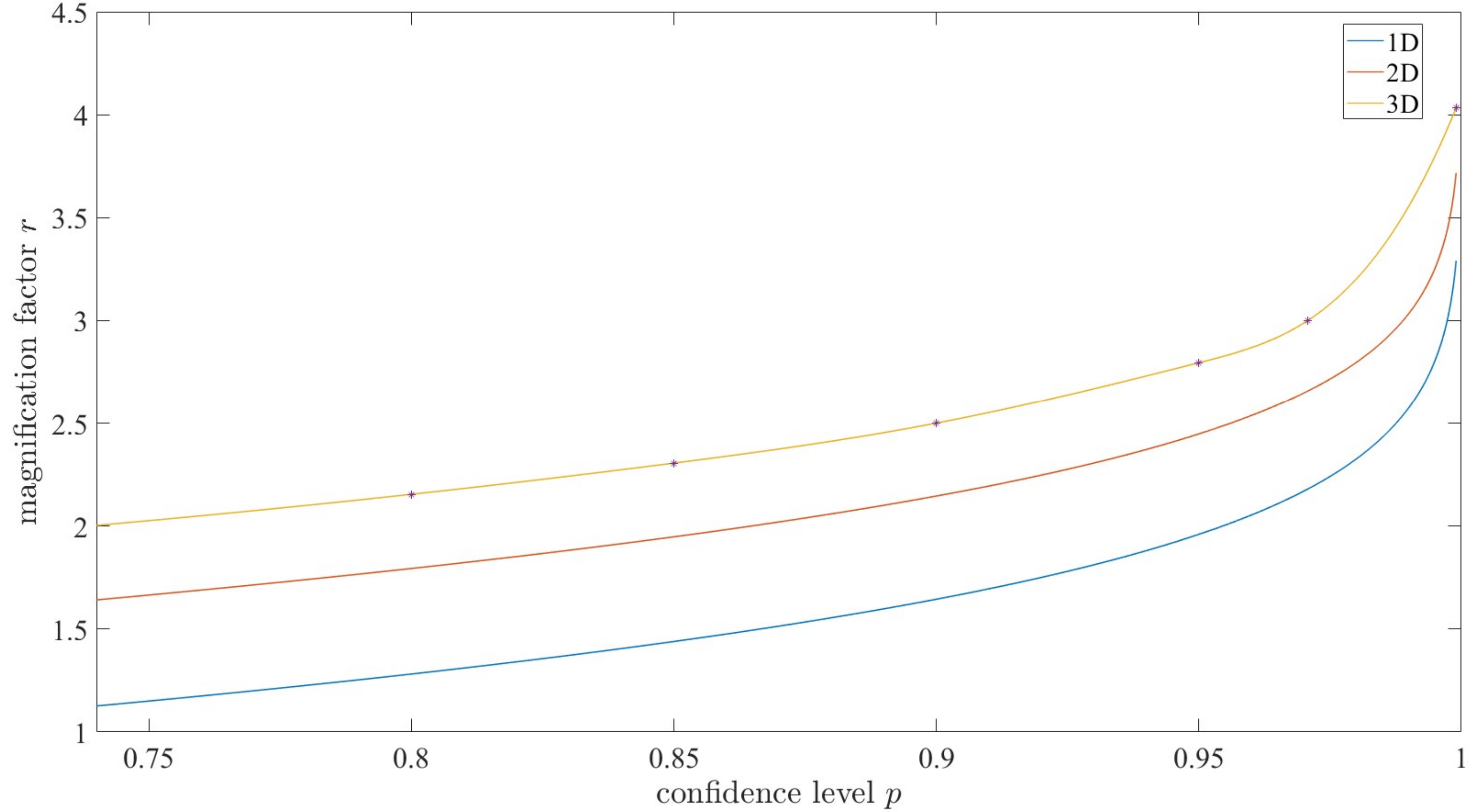
Dimensionality	Magnification factor						
	1	2	3	4	5	6	7
1	0.6827	0.9545	0.9973	0.9999	1.0000	1.0000	1.0000
2	0.3935	0.8647	0.9889	0.9997	1.0000	1.0000	1.0000
3	0.1987	0.7385	0.9707	0.9989	1.0000	1.0000	1.0000
4	0.0902	0.5940	0.9389	0.9970	0.9999	1.0000	1.0000
5	0.0374	0.4506	0.8909	0.9932	0.9999	1.0000	1.0000
6	0.0144	0.3233	0.8264	0.9862	0.9997	1.0000	1.0000
7	0.0052	0.2202	0.7473	0.9749	0.9992	1.0000	1.0000
8	0.0018	0.1429	0.6577	0.9576	0.9984	1.0000	1.0000
9	0.0006	0.0886	0.5627	0.9331	0.9970	1.0000	1.0000
10	0.0002	0.0527	0.4679	0.9004	0.9947	0.9999	1.0000

Magnification ratios of scaled SDHE corresponding to different specified confidence levels with space dimensionality ≤ 10.

Dimensionality	Confidence Level (%)					
	80.0	85.0	90.0	95.0	99.0	99.9
1	1.2816	1.4395	1.6449	1.9600	2.5758	3.2905
2	1.7941	1.9479	2.1460	2.4477	3.0349	3.7169
3	2.1544	2.3059	2.5003	2.7955	3.3682	4.0331
4	2.4472	2.5971	2.7892	3.0802	3.6437	4.2973
5	2.6999	2.8487	3.0391	3.3272	3.8841	4.5293
6	2.9254	3.0735	3.2626	3.5485	4.1002	4.7390
7	3.1310	3.2784	3.4666	3.7506	4.2983	4.9317
8	3.3212	3.4680	3.6553	3.9379	4.4822	5.1112
9	3.4989	3.6453	3.8319	4.1133	4.6547	5.2799
10	3.6663	3.8123	3.9984	4.2787	4.8176	5.4395

doi:10.1371/journal.pone.0118537.t002

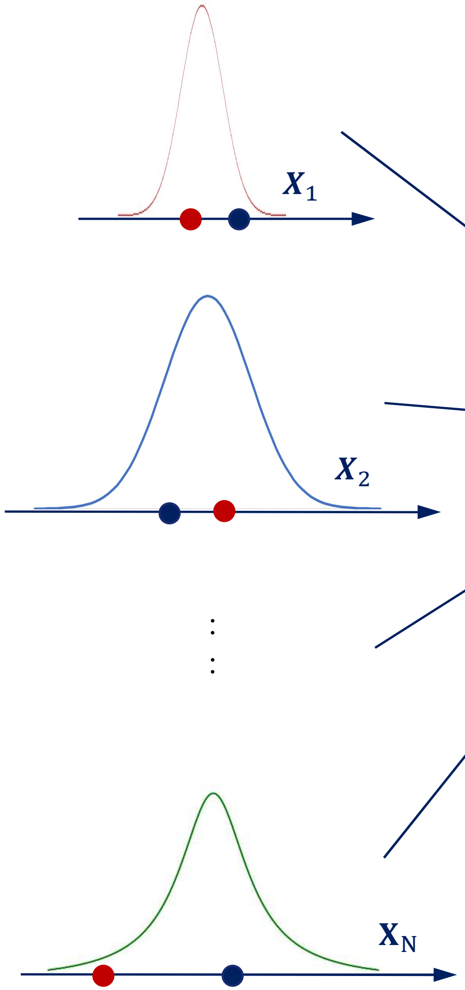
Magnification factor *r* vs. confidence level *p* vs. dimensionality



Covariance Analysis with Montecarlo Methods

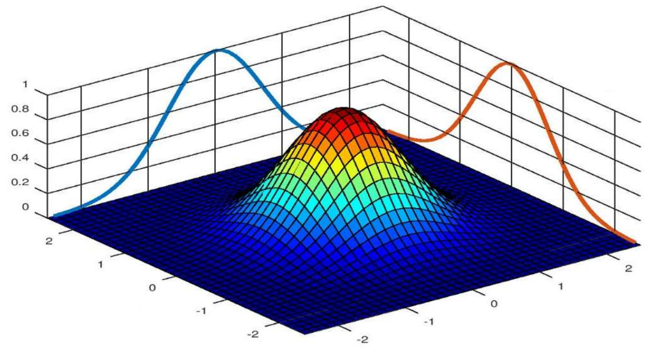
$\boldsymbol{X_i}$: influence variables, sources of uncertainty

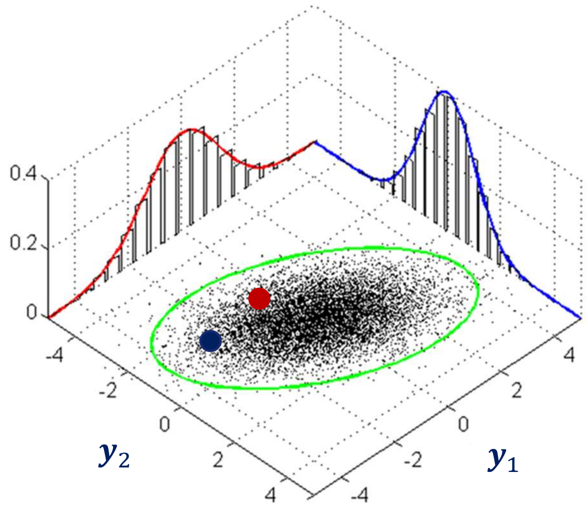
$\boldsymbol{Y}$: interest variable (measurand)



$$\boxed{\boldsymbol{Y = g(X_1, X_2, \ldots X_N)}}$$

$\boldsymbol{PDF\ of\ Y = [y_1, y_2]}$

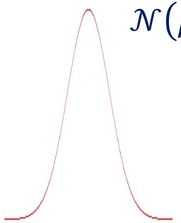




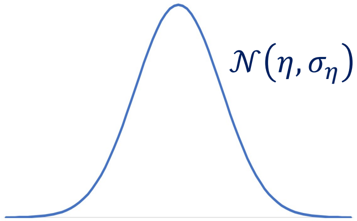
$\boldsymbol{\Sigma}$

covariance matrix of $\boldsymbol{Y}$

Covariance Analysis with Montecarlo Methods



$\mathcal{N}(\beta, \sigma_\beta)$



$\mathcal{N}(\eta, \sigma_\eta)$

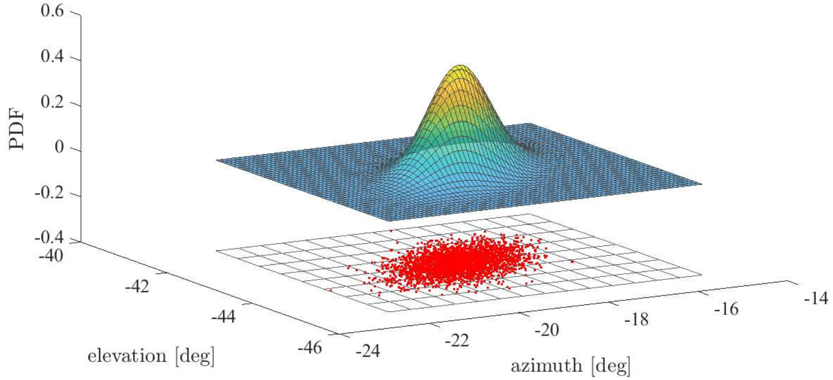
$\tilde{\beta} \sim \mathcal{N}(\beta, \sigma_\beta)$

$\tilde{\eta} \sim \mathcal{N}(\eta, \sigma_\eta)$

$\phi_S, \lambda_S, \phi_E, \lambda_E$

Single-axis Attitude Determination Algorithm

ϕ, λ



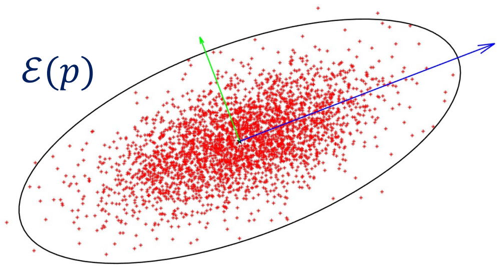
cov

$$\Sigma = \begin{bmatrix} \sigma_\phi^2 & \sigma_{\phi\lambda} \\ \sigma_{\phi\lambda} & \sigma_\lambda^2 \end{bmatrix}$$

eig

eigenvalues
eigenvectors

uncertainty
ellipse



$\mathcal{E}(p)$

Esercitazione 5/1 di Strumentazione Aerospaziale

- **Prima esperienza: Single-Axis Attitude Determination**
  - ✓ Metodo di Grubin per la determinazione ad asse singolo
  - ✓ Analisi di Covarianza con metodo Montecarlo per la stima dell'incertezza d'assetto
  - ✓ File Matlab:
    - ✓ main: «**Esercitazione_single_axis_determination_Montecarlo.m**»

      In questo script di Matlab, dovrete seguire le istruzioni ed implementare la chiamata alla funzione con il metodo di Grubin e la parte di analisi di covarianza. Nelloscript avrete il testo con la consegna sottoforma di testo, mentre nelle varie celle dovrete implementare il codice richiesto nei commenti.
    - ✓ subfunction: «SingleAxisAttDetAlgo.m» che implementa il metodo Grubin
    - ✓ funzioni aggiuntive (built-in) di Matlab: deg2rad, rad2deg, eig, cov

Esercitazione 5/2 di Strumentazione Aerospaziale

- **Seconda esperienza: Three-Axes Attitude Determination**
  - Metodo-q di Davenport => routine QUEST
  - File Matlab:
    - main: «**Esercitazione_three_axes_determination_quest.m**»

      Come nell'esercitazione precedente, sarete guidati nell'implementazione del metodo di Davenport e dell'analisi di covarianza.
    - subfunctions: «quest.m» (scaricata dal Matlab File Exchange)

      Chiamerete questa routine per la generazione delle matrici di rotazione per poi trovare gli angoli di Eulero
    - altre funzioni: libreria «matGeom» scaricabile da Matlab File Exchange, trasm, trasfg, pltassi, freccia, drawEllipsoidV2
    - funzioni aggiuntive (built-in) di Matlab: unifrnd, deg2rad, rad2deg, eul2rotm, rotm2eul, sph2cart, cart2sph, randn, rms, eig, cov