Tutorial 2: Lorenz-Reservoir Computing

The well-known toy model for atmospheric variability is the Lorenz 1963 model given by the set of ODEs,

$$\frac{dx}{dt} = s(y-x) \tag{1a}$$
$$\frac{dy}{dt} = rx - y - xz \tag{1b}$$
$$\frac{dz}{dt} = xy - bz \tag{1c}$$

This system is, for standard values of parameters $s = 10, r = 28$ and $b = 2.667$, a chaotic system with sensitivity to initial conditions. In this project, we will look at the skill of Reservoir Computer (RC) based predictions, using the Python Notebook provided.

(i) Generate data by using a time step $\Delta t = 0.02$ and integrating over the time interval $[-100, 25]$. Use the interval $[-100, 0]$ as training data and the interval $[0, 25]$ as 'truth'.

(ii) Use initial hyperparameter values $d = 300$ (reservoir size), $< k >= 6$ (mean degree), $\rho = 1.2$ (spectral radius), $\sigma = 0.1$ (interval in uniform distribution defining W_{in}), and $\beta = 0$ (regularization), see B5_notes. Train the RC using the training data set and generate a RC prediction $\mathbf{x}_R$ of the 'truth' $\mathbf{x}$ over the test interval $[0, 25]$. Set a tolerance error

$$\epsilon = ||\mathbf{x} - \mathbf{x}_R||^2$$

and determine the time t_e when $\epsilon > 0.1$.

(iii) How could one determine the Lyapunov exponents of the Lorenz system, using the RC approach (you do not have to do this, but you may)?

(iv) Study the effect of the hyperparameter ρ on t_e and try to explain the behavior.

(v) To which of the hyperparameters is t_e most sensitive? How could one optimize the values of these hyperparameters to maximize t_e?