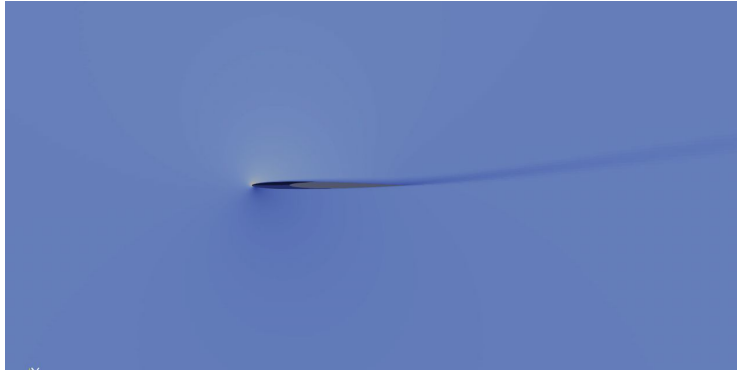


# Inferring Functional Properties from CFD

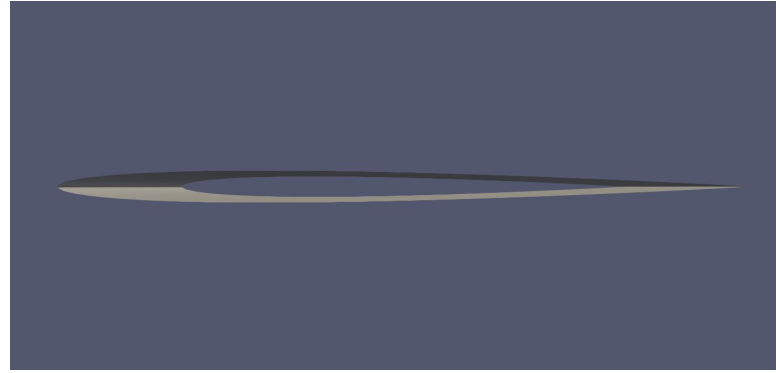


Francesco Montanaro

# Aim of the study



*CFD-computed flow fields around the airfoil*



*shape of the airfoil*

The aim of the study is to derive the geometrical features of the airfoils from the CFD outcomes by means of their **NACA** numbers:

- **Maximum Camber.**
- **Maximum Camber position.**
- **Thickness.**

Since an analytical link between the CFD-computed flow fields and the required information is not available, we refer to the use of **Machine Learning**.

# Dataset

2600 *.vtk* files containing the values of the CFD-computed flow fields within the space.

Target values: **NACA** numbers.

1. Maximum camber  $\rightarrow [0 - 9]$  in units of  $c/100$
2. Maximum camber position  $\rightarrow [0 - 9]$  in units of  $c/10$
3. Maximum thickness  $\rightarrow [05 - 50]$  in units of  $c/100$

# Features extraction

1. **Regional Averages** of the flow fields.
2. **Arrival times** of the streamlines.
3. **Regional arrival times** of the streamlines.



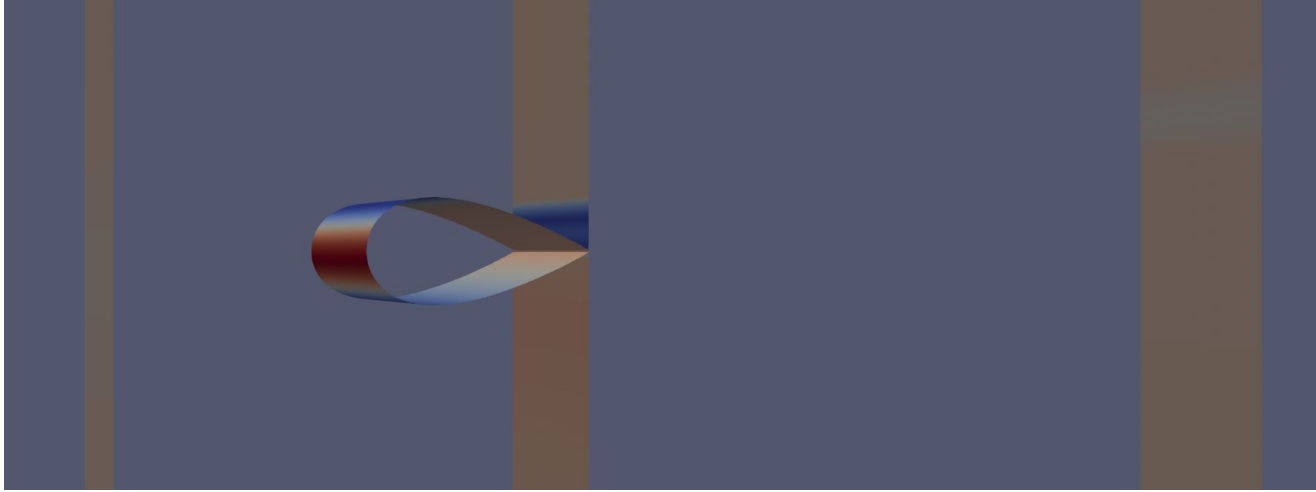
→ **Machine Learning**

4. **Signals** of the flow fields.
5. **Signals** of the streamlines.



→ **Deep Learning**

# 1. Regional Averages

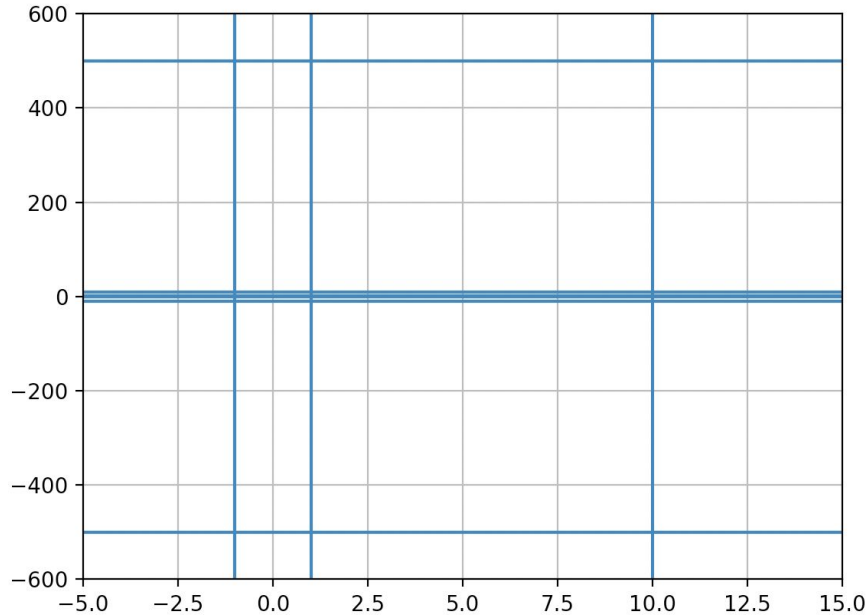


Sections extracted from the space

Goal:

- Extract the regions of interest.
- Compute the average of the flow fields for each region.

# 1. Regional Averages: cutting the space



- 3 sections considered.  
 $X = [-c, c, 10c]$
- 8 regions per section.  
 $Y = [500, 10, 1, 0.1, 0, -0.1, -1, -10, -500]$
- 24 regions in total.

*c: chord length*

# 1. Regional Averages: Features extraction

$$\overline{P}_k = \frac{\sum_i^n A_i p_i}{\sum_i^n A_i}$$

Regional average of the **pressure** field.

*n*: number of cells belonging to the *k*-th region.

*p<sub>i</sub>*: pressure of the *i*-th cell.

*A<sub>i</sub>*: surface of the *i*-th cell.

$$\overline{U}_k = \frac{\sum_i^n A_i U_i}{\sum_i^n A_i}$$

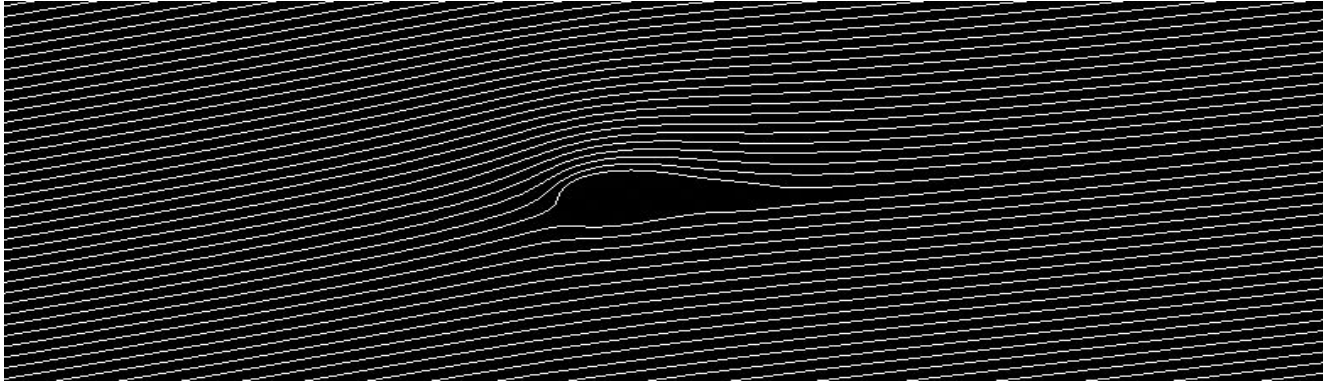
Regional average of the **velocity** field.

*n*: number of cells belonging to the *k*-th region.

*U<sub>i</sub>*: velocity of the *i*-th cell.

*A<sub>i</sub>*: surface of the *i*-th cell.

## 2. Arrival times



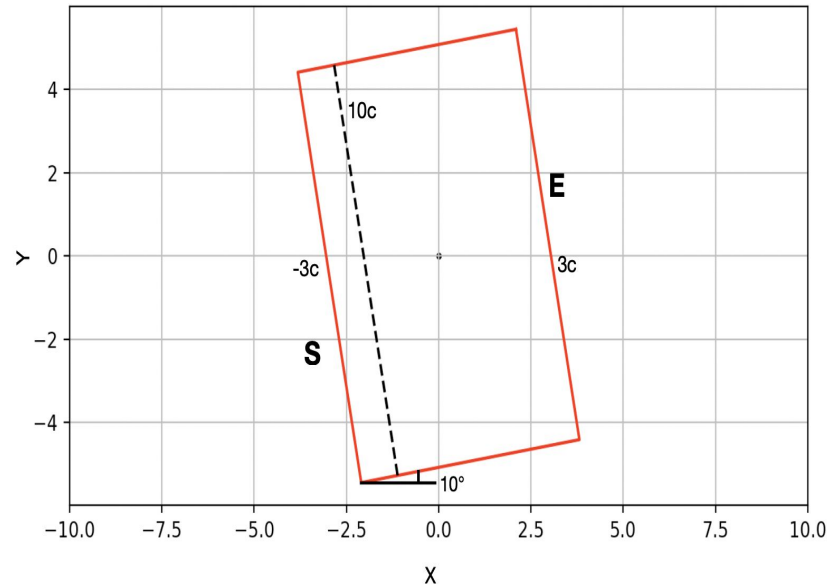
*Streamlines affecting an airfoil*

Goal:

- Extract streamlines.
- Compute the arrival time of each streamline from a starting section  $S$  to an arrival section  $E$ .



## 2. Arrival times: cutting the space - 1/2

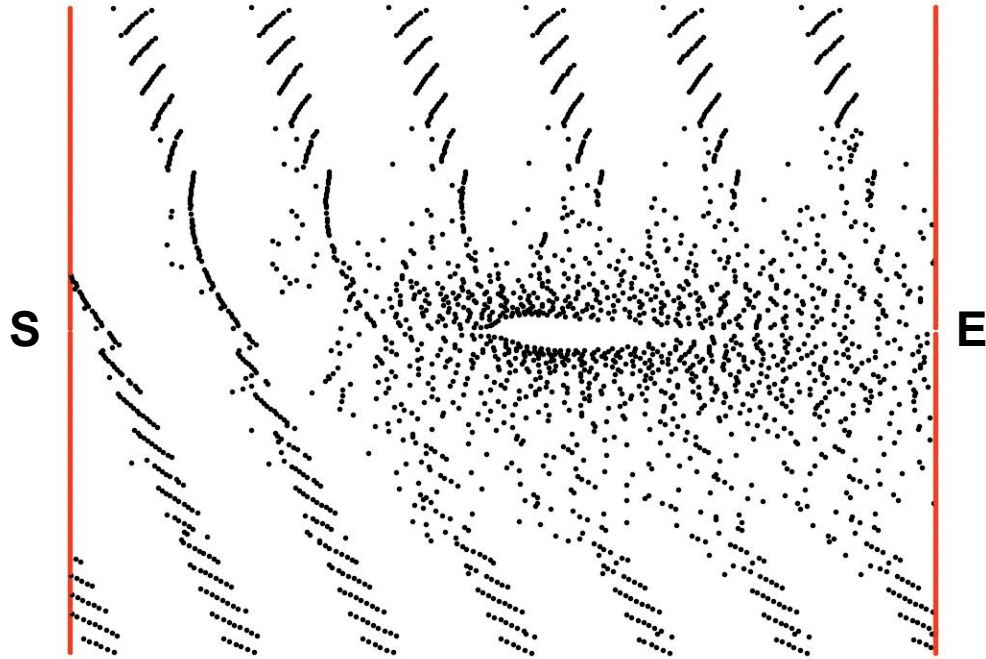


Starting section **S** and arrival section **E** both **orthogonal** to the free stream direction ( **$10^\circ$** ):

- Sections length:  **$10c$**
- Sections distances from the origin:  **$3c$**

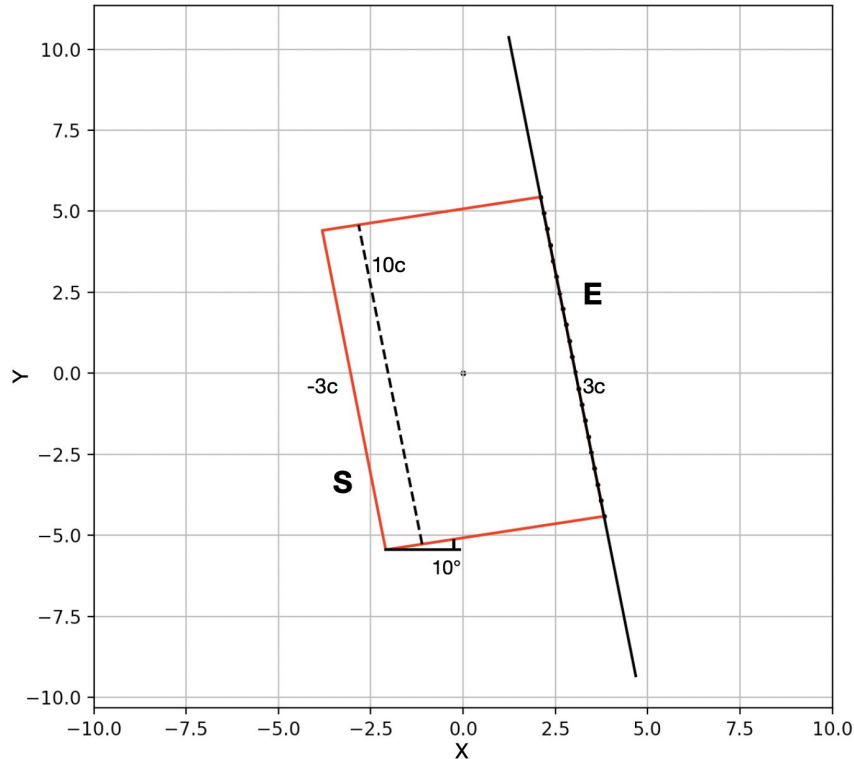
*c: chord length*

## 2. Arrival times: cutting the space - 2/2



- Extracting streamlines belonging to the cutting plane.
- Streamlines computed by using an adaptive integrator (**RungeKutta45**) with a variable step length.

## 2. Arrival times: Binning operation



- 1 segment considered at the arrival section  $E$ .  
*section length =  $10c$*
- Segment divided into 64 bins of equal length.

*$c$ : chord length*

## 2. Arrival times: features extraction

$$\Delta t_{i,j} = \frac{2\sqrt{(x_j - x_i)^2 + (y_j - y_i)^2}}{|U_j| + |U_i|}$$

$$T_k = \sum_{i=1}^{n-1} \Delta t_{i,i+1}$$

$$T_{b_i} = \frac{1}{n_i} \sum_{h=1}^{n_i} T_h$$

**Time distance** of two consecutive points.

***U<sub>i</sub>**: velocity field of the i-th point.*

***x<sub>i</sub>, y<sub>i</sub>**: spatial coordinates of the i-th point*

**Arrival time** of the k-th streamline.

***n**: number of points, of the k-th streamline, belonging to the cutting plane.*

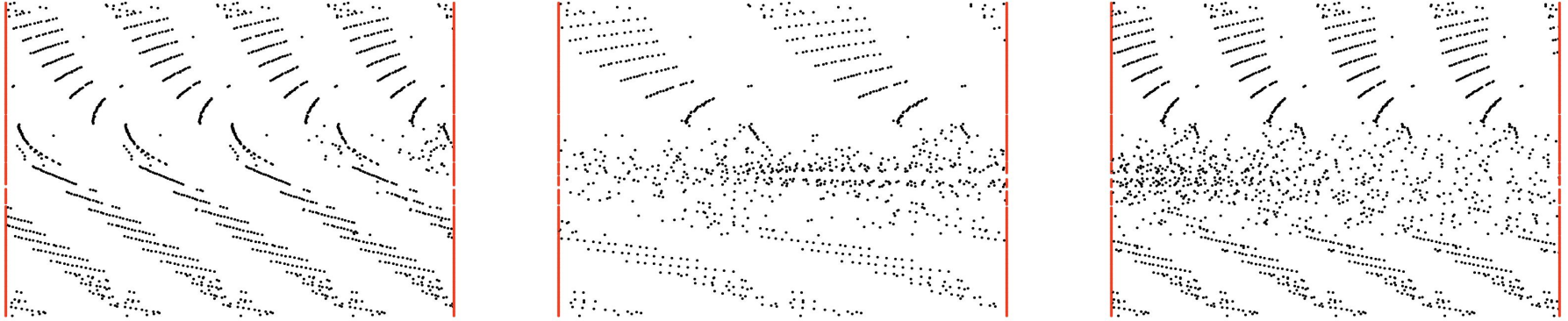
**Arrival time** of the i-th bin.

***b<sub>i</sub>**: i-th bin.*

***n<sub>i</sub>**: number of streamlines belonging to the i-th bin.*

***T<sub>h</sub>**: arrival time of the h-th streamline.*

### 3. Regional arrival times

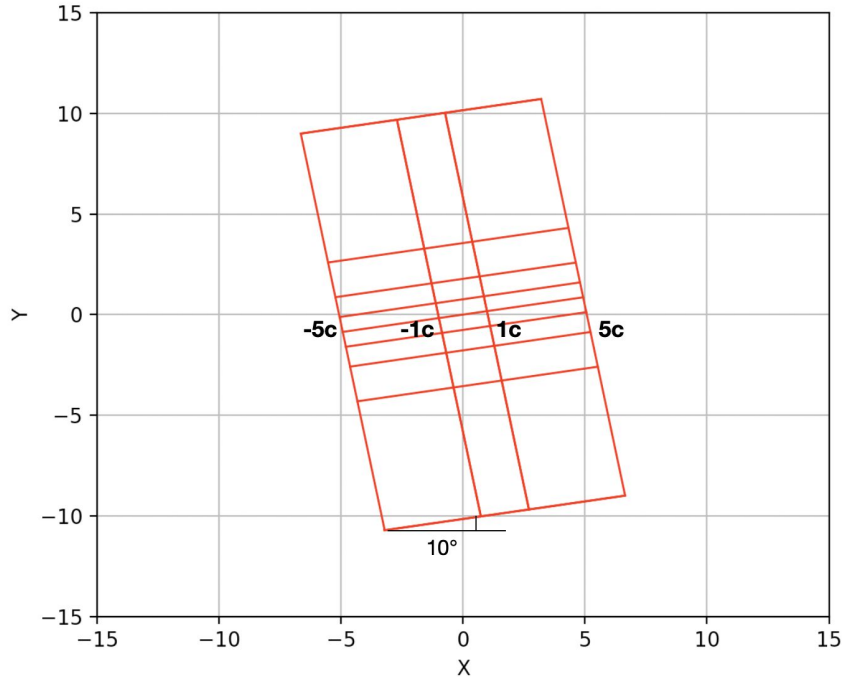


*Streamlines extracted from different sections*

Goal:

- Extract streamlines.
- Compute the arrival time of each streamline for multiple cutting sections.

### 3. Regional arrival times: cutting the space



- 3 regions on the x axis.  
 $X = [ [-5c, -1c], [-1c, 1c], [1c, 5c] ]$
- 8 regions on the y axis.  
 $Y = [-10, -3.5, -1.75, -0.75, 0, 0.75, 1.75, 3.5, 10]$
- 24 regions in total.

*c: chord length*

### 3. Regional arrival times: features extraction

$$\Delta t_{i,j} = \frac{2\sqrt{(x_j - x_i)^2 + (y_j - y_i)^2}}{|U_j| + |U_i|}$$

$$T_k = \sum_{i=1}^{n-1} \Delta t_{i,i+1}$$

$$R_{T_i} = \frac{1}{n_j} \sum_{j=1}^{n_j} T_j$$

**Time distance** of two consecutive points.

*$U_i$ : velocity field of the  $i$ -th point.*

*$x_i, y_i$ : spatial coordinates of the  $i$ -th point*

**Arrival time** of the  $k$ -th streamline.

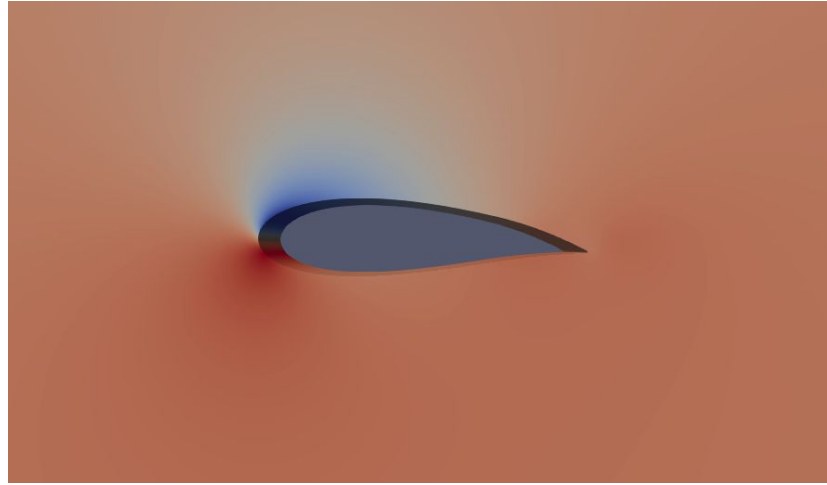
*$n$ : number of points, of the  $k$ -th streamline, belonging to the cutting plane.*

**Regional average** of the arrival  $i$ -th time.

*$n_j$ : number of streamlines belonging to the  $i$ -th region.*

*$T_j$ : arrival time of the  $j$ -th streamline.*

## 4. Signals of the flow fields: features extraction



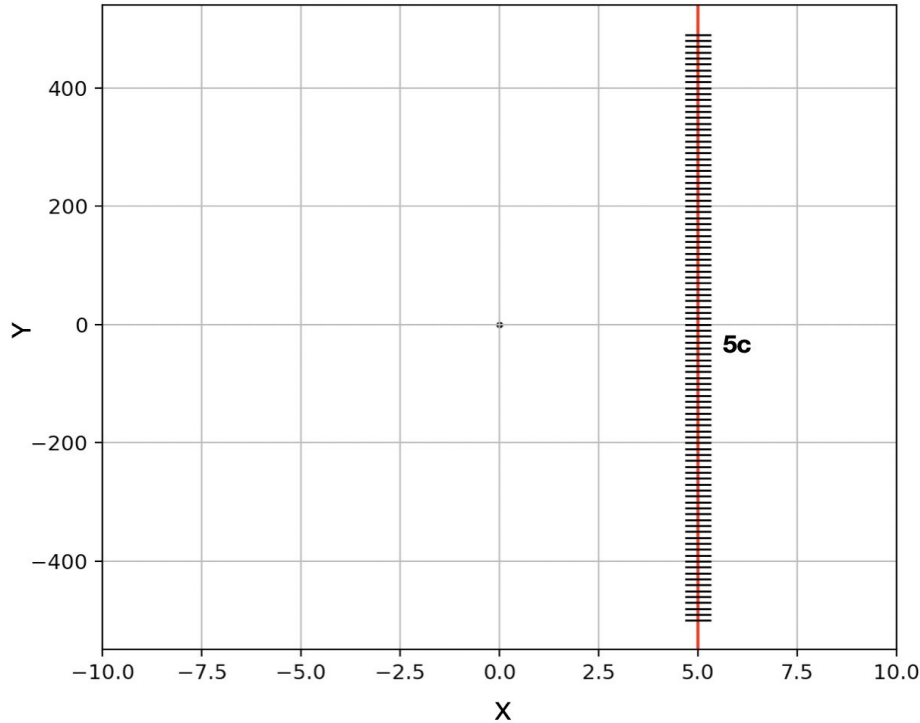
*CFD-computed flow fields around the airfoil*

Goal:

- Extract the values of the CFD-computed flow fields for a specific segment of the space.
- Generate spatial signals from the obtained values.



## 4. Signals of the flow fields: Binning operation



*Binning operation*

- 1 segment considered.  
 $x = 5c$   
 $y = [-500, 500]$   
 $z = 0.5$
- Section divided into 1024 bins of equal length.

*c: chord length*

## 4. Signals of the flow fields: features extraction - 1/2

$$P_{b_i} = \frac{1}{n_i} \sum_{h=1}^{n_i} p_h$$

**Pressure** of the  $i$ -th bin.

***$b_i$*** :  $i$ -th bin.

***$n_i$*** : number of points belonging to the  $i$ -th bin.

***$p_h$*** : pressure of the  $h$ -th point of the  $i$ -th bin.

$$|U|_{b_i} = \frac{1}{n_i} \sum_{h=1}^{n_i} |u_h|$$

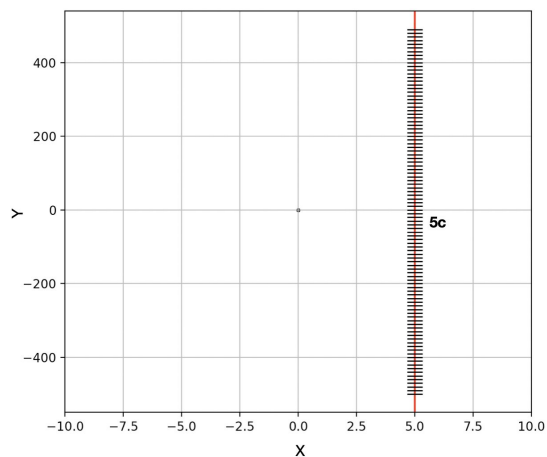
Magnitude of the **Velocity** of the  $i$ -th bin.

***$b_i$*** :  $i$ -th bin.

***$n_i$*** : number of points belonging to the  $i$ -th bin.

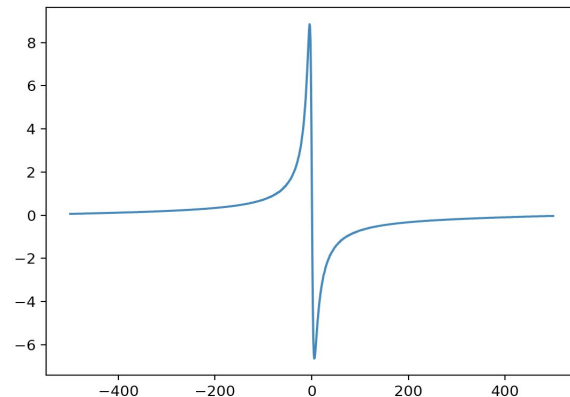
***$u_h$*** : magnitude of the velocity of the  $h$ -th point of the  $i$ -th bin.

# features extraction - 2/2



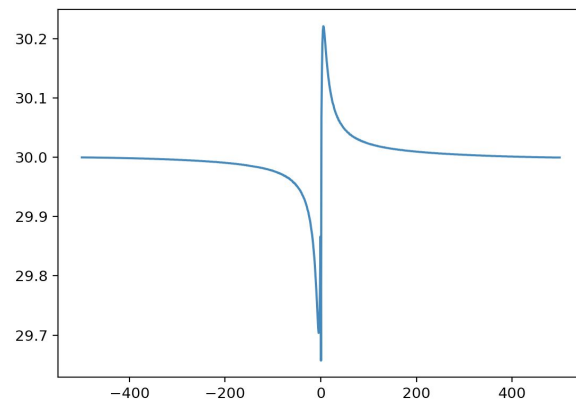
*Binning operation*

$$P_{b_i} = \frac{1}{n_i} \sum_{h=1}^{n_i} p_h$$



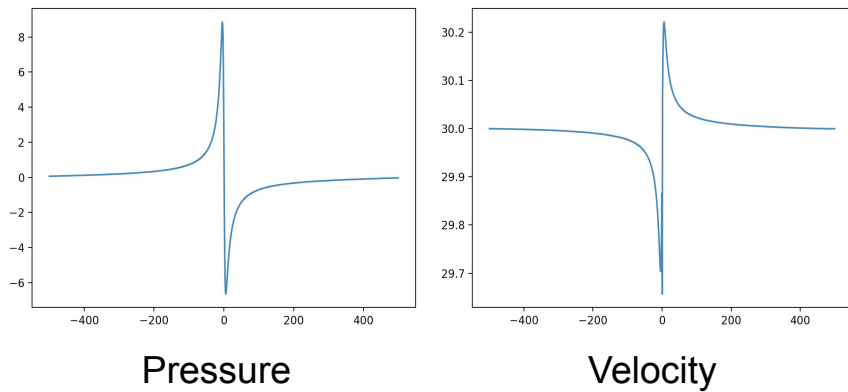
*Pressure signal*

$$|U|_{b_i} = \frac{1}{n_i} \sum_{h=1}^{n_i} |u_h|$$

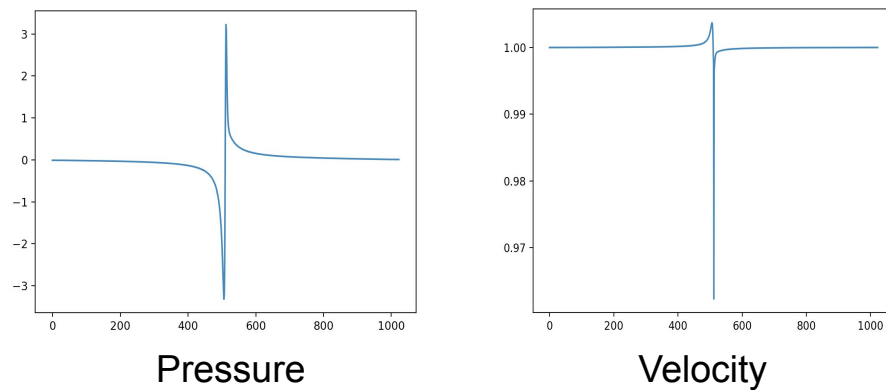


*Velocity signal*

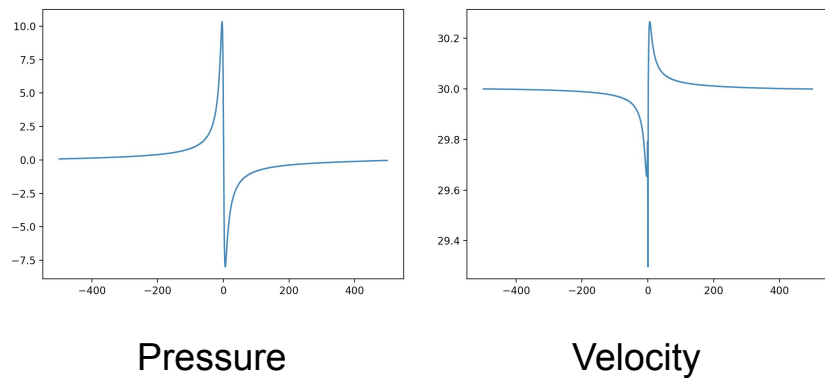
NACA: 0005



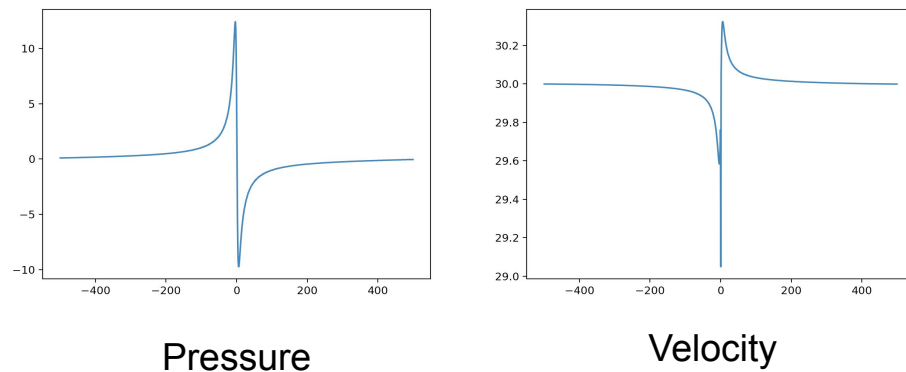
NACA: 0050



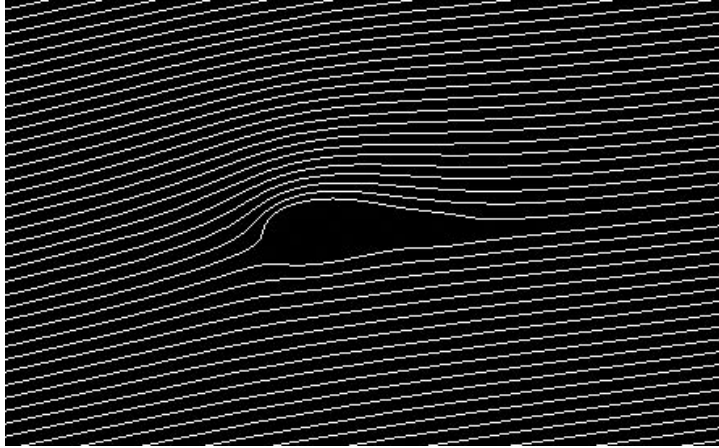
NACA: 5732



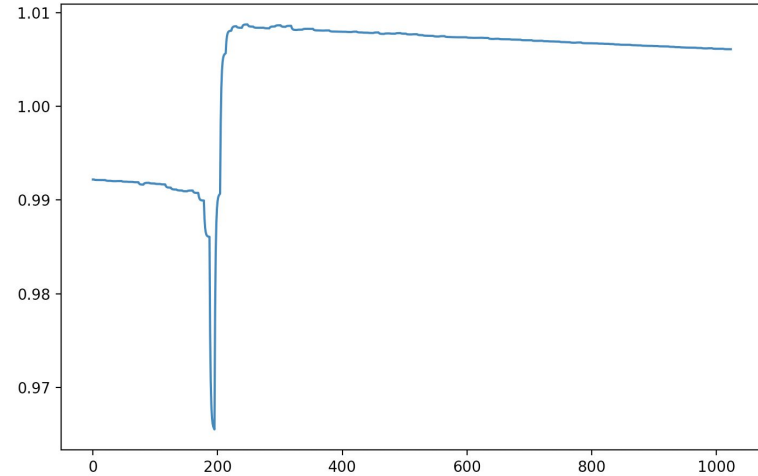
NACA: 8732



## 5. Signals of the streamlines



Streamlines affecting an airfoil

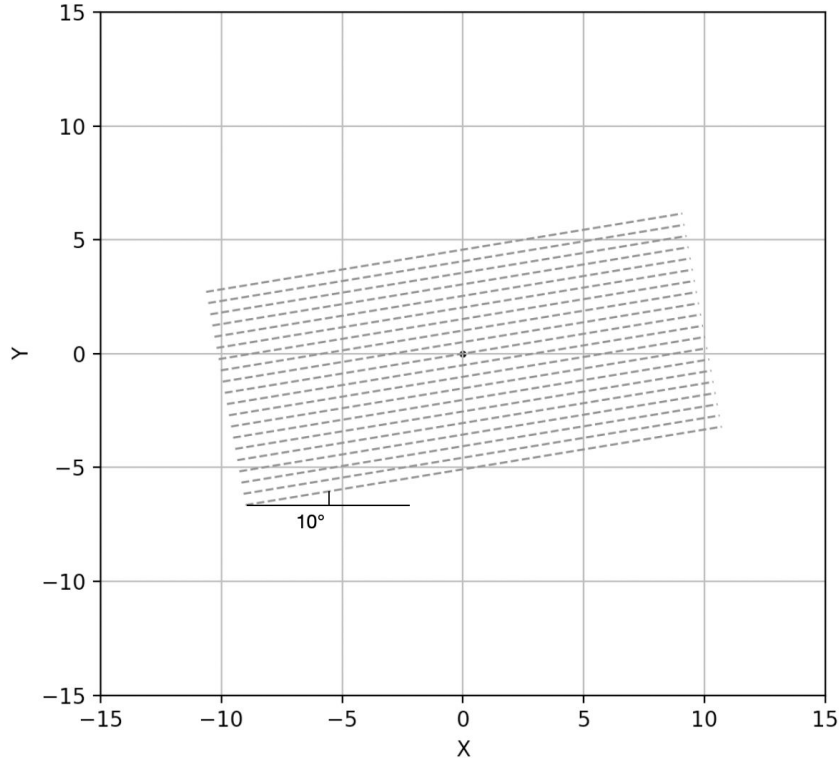


Signal of the streamline

Goal:

- Extract streamlines within the space.
- Generate spatial signals of the velocity field.

## 5. Signals of the streamlines: Binning operation



- Space divided into 1024 bins of equal length.
- Bins orthogonal to the free stream direction.

*c: chord length*

## 5. Signals of the streamlines: features extraction - 1/2

$$|u|_h = \frac{1}{n_h} \sum_{j=1}^{n_h} |u|_j$$

**Mean velocity** magnitude of the points belonging the h-th streamline.

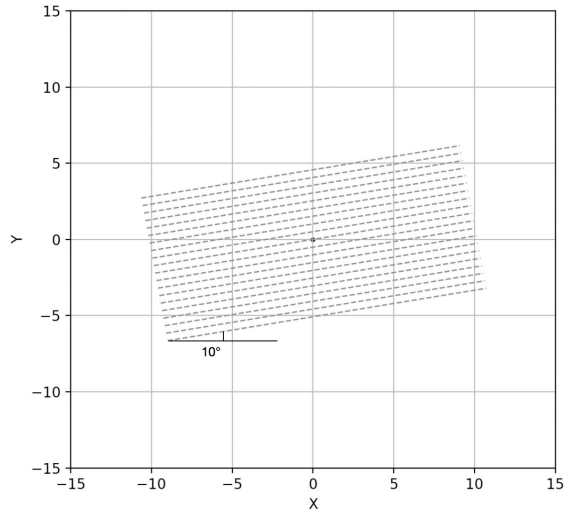
*ui*: velocity magnitude of the i-th point.  
*nh*: number of points of the i-th streamline.

$$|U|_{b_i} = \frac{1}{n_i} \sum_{j=1}^{n_i} |U|_j$$

**Velocity magnitude** of the i-th bin.

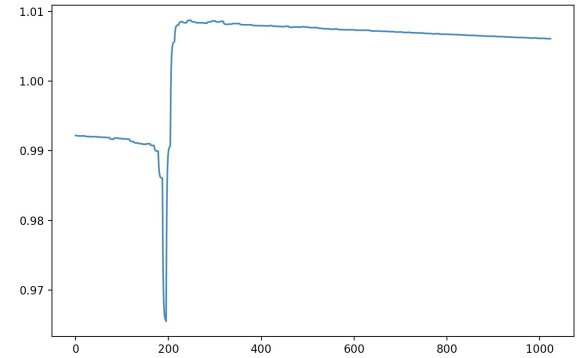
*bi*: i-th bin.  
*ni*: number of streamlines belonging to the i-th bin.  
*Ui*: mean velocity magnitude of the j-th streamline.

## 5. Signals of the streamlines: Features extraction - 2/2



*Binning operation*

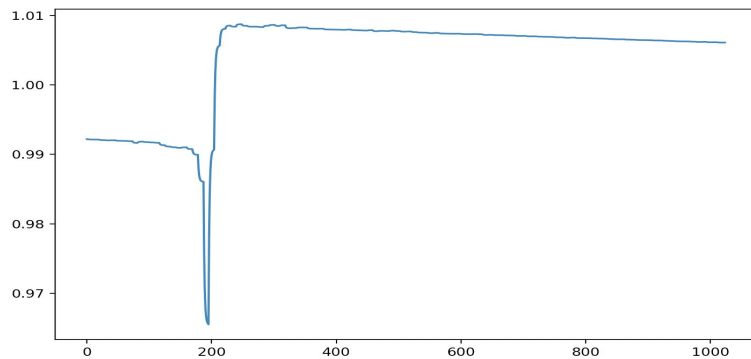
$$\Rightarrow |U|_{b_i} = \frac{1}{n_i} \sum_{j=1}^{n_i} |U|_j \Rightarrow$$



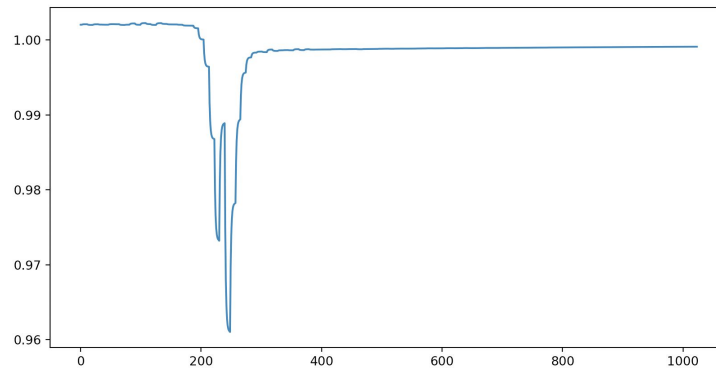
*Signal of the streamlines*



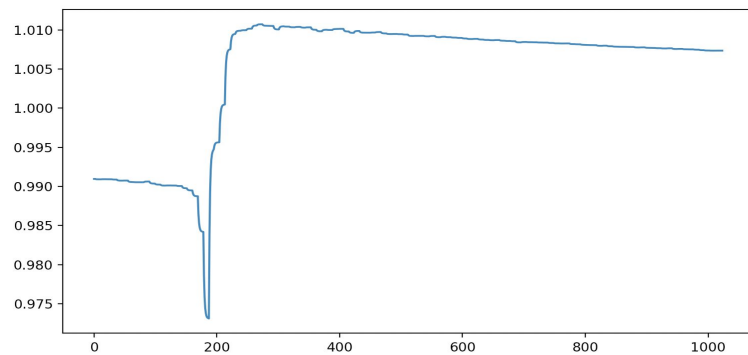
NACA: 0005



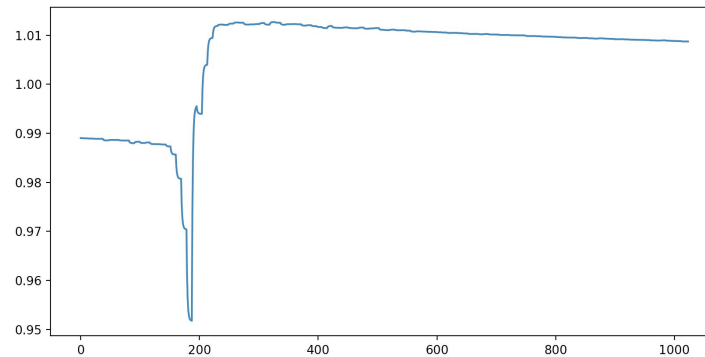
NACA: 0050



NACA: 5732



NACA: 8732



# Prediction of Geometrical Features



Building the models

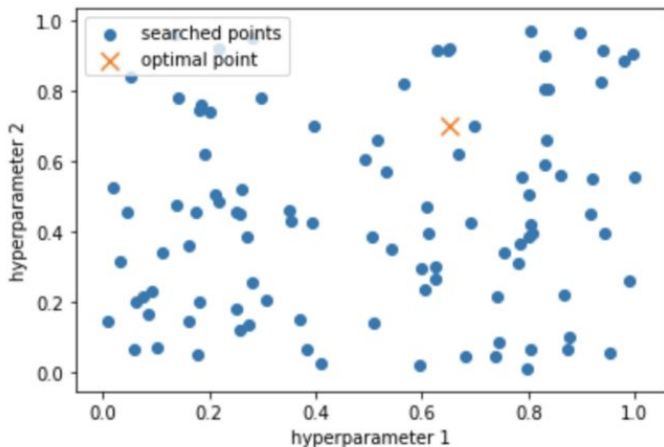


O P T U N A

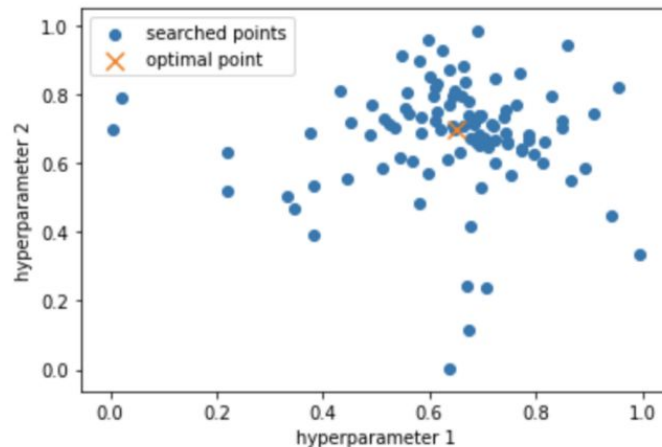


Optimizing the models

# Optuna for hyperparameters tuning - 1/2



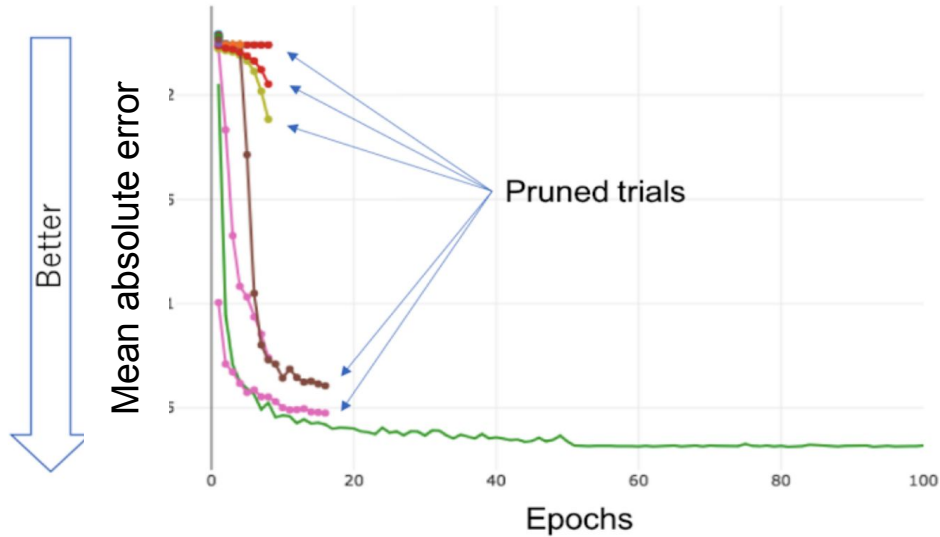
Random search



Sequential Model-based  
Optimization (TPE)

Optuna uses Tree-structured Parzen Estimator (TPE) to **search** more efficiently than a random search, by iteratively choosing points closer to previous good results.

# Optuna for hyperparameters tuning - 2/2



- More than **150 architectures tested** for each class of features.
- More than **750 architectures tested** in total.

Optuna uses a **Pruning** optimization technique to automatically stop unpromising trials at the early stages of the training, so that computing time can be used for trials that show more potential.

# Prediction of Geometrical Features

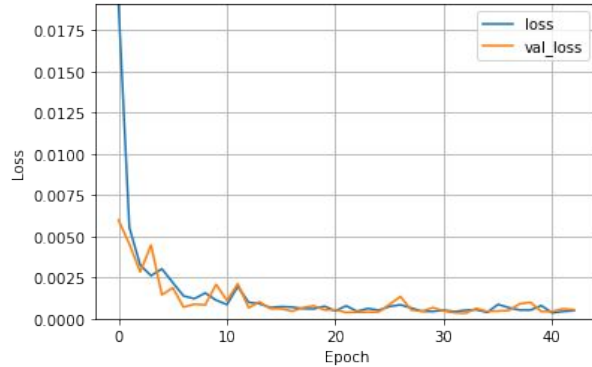
- Two classes of models considered:
  1. Fully Connected Neural Network (**FCNN**):
    - Regional Averages.
    - Arrival times.
    - Regional arrival times.
  2. Convolutional Neural Network 1D (**CNN-1D**):
    - Signals of the flow fields.
    - Signals of the streamlines.
- Loss function: Mean Square Error (**MSE**) computed on the NACA numbers.
- Performance evaluation metric: Mean Absolute Error (**MAE**) computed on the NACA numbers.

# Experimental setup

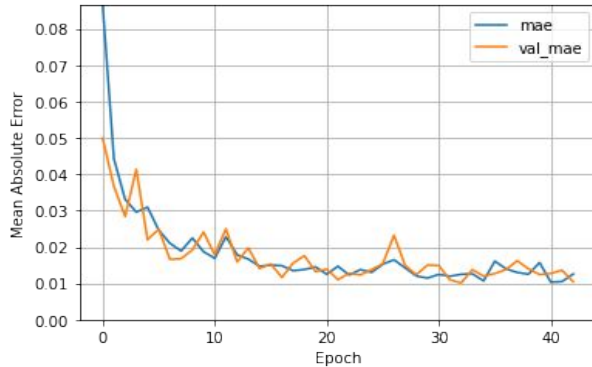
First Approach: all samples used in the training phase.

- Data **sampled randomly**.
- Data splitting:
  - **Training set**: 80% of the available samples.
  - **Validation set**: 20% of the training samples.
  - **Test set**: 20% of the available samples.

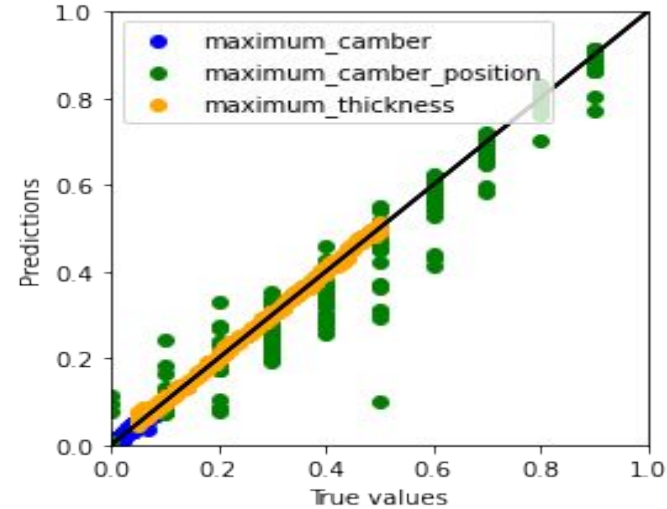
# Regional Averages



*Training and Validation loss trends across the epochs*



*Training and Validation MAE trends across the epochs*



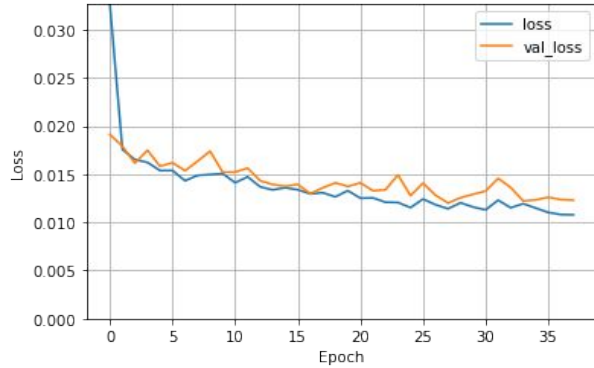
*Prediction of the geometrical features*

**Metrics on the test set:**

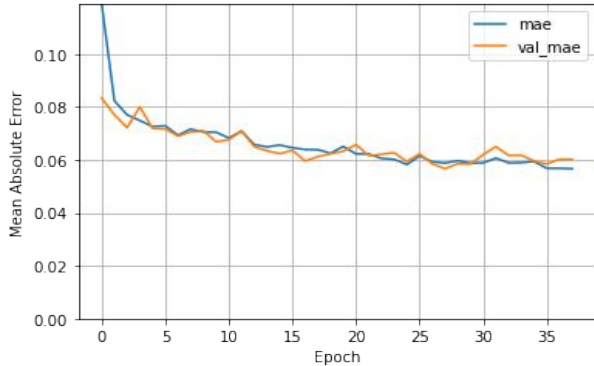
Loss (MSE): **0.00066293**

Mean Absolute Error (MAE): **0.011075435**

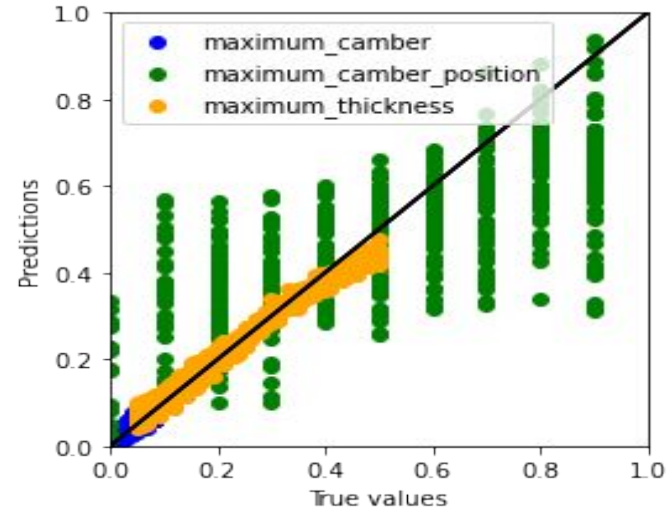
# Arrival times



*Training and Validation loss trends across the epochs*



*Training and Validation MAE trends across the epochs*



*Prediction of the geometrical features*

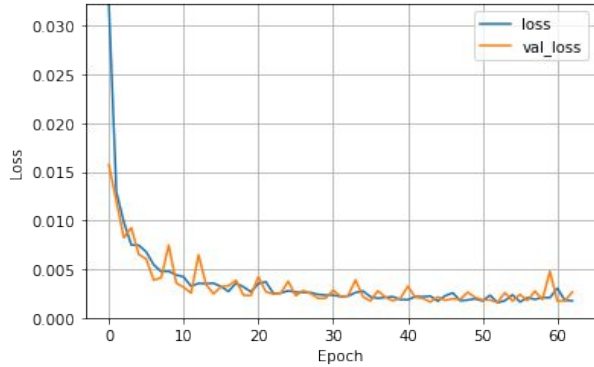
**Metrics on the test set:**

Loss (MSE): **0.012493741**

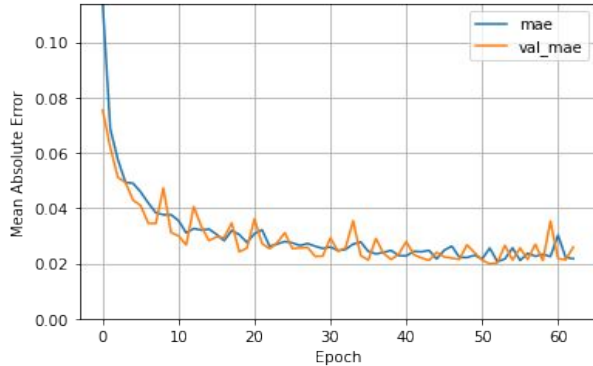
Mean Absolute Error (MAE): **0.060213528**



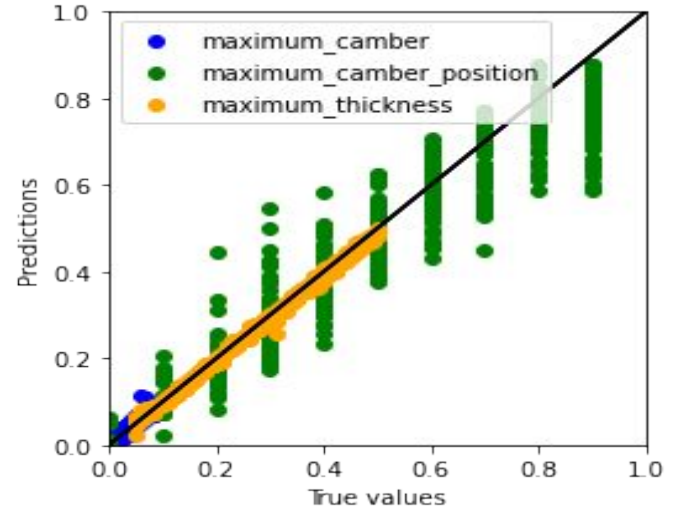
# Regional arrival times



*Training and Validation loss trends across the epochs*



*Training and Validation MAE trends across the epochs*



*Prediction of the geometrical features*

**Metrics on the test set:**

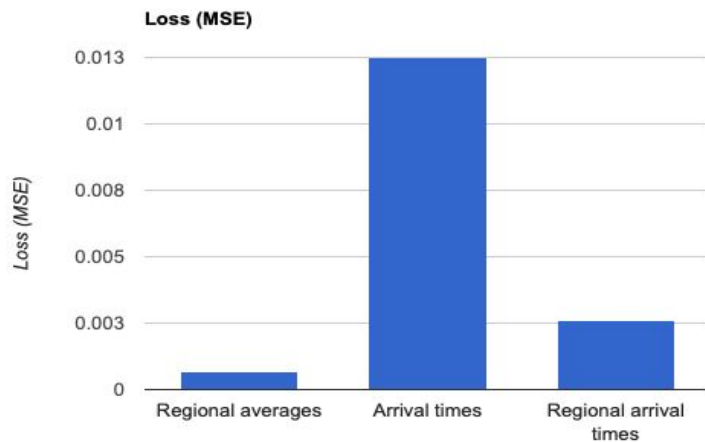
**Loss (MSE): 0.002624211**

**Mean Absolute Error (MAE): 0.025725699**

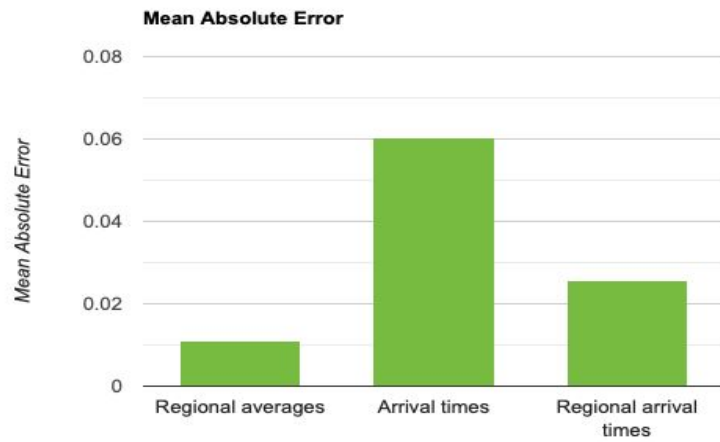
# Comparison of metrics - FCNN

	Regional Averages	Arrival times	Regional arrival times
Loss (MSE)	<b>0.00066293</b>	<b>0.012493741</b>	<b>0.002624211</b>
Mean Absolute Error (MAE)	<b>0.011075435</b>	<b>0.060213528</b>	<b>0.025725699</b>

*Metrics obtained from each class of features*

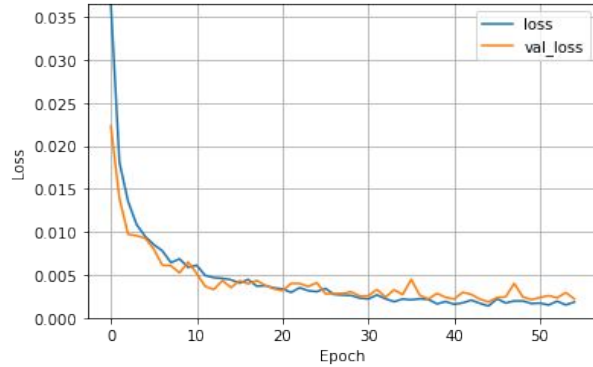


*Loss obtained from each class of features*

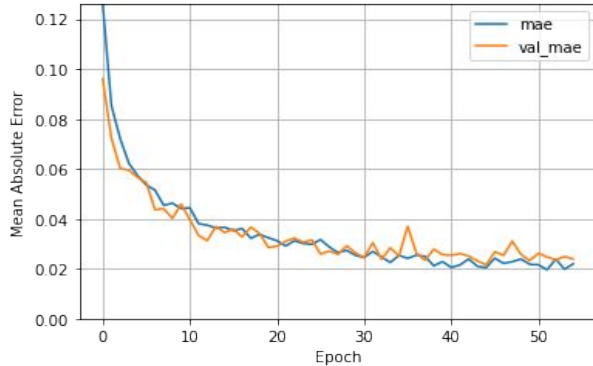


*MAE obtained from each class of features*

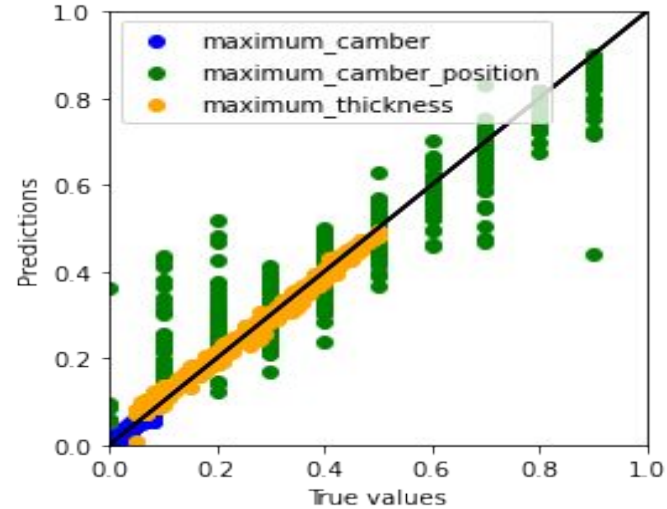
# Signals of the flow fields



*Training and Validation loss trends across the epochs*



*Training and Validation MAE trends across the epochs*



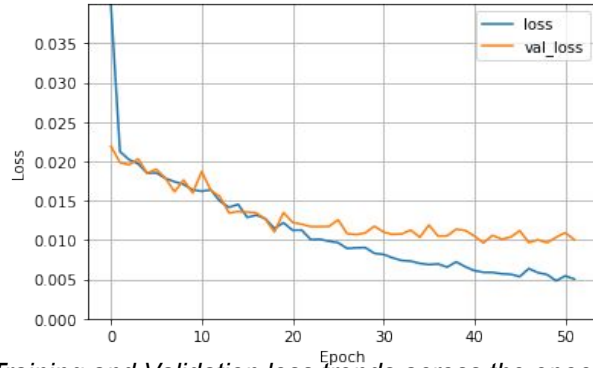
*Prediction of the geometrical features*

**Metrics on the test set:**

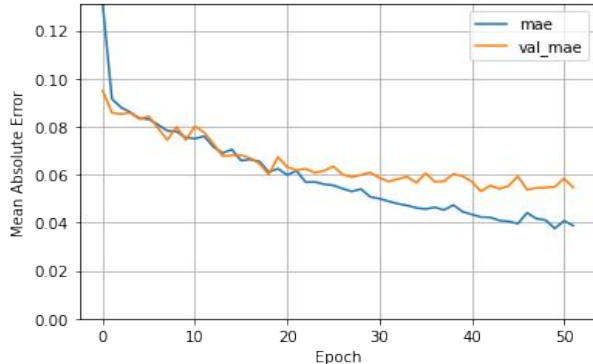
Loss (MSE): **0.002248327**

Mean Absolute Error (MAE): **0.024343444**

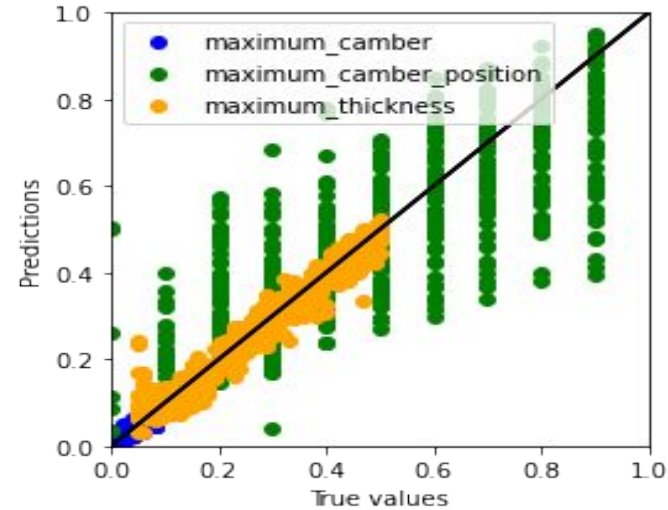
# Signals of the streamlines



*Training and Validation loss trends across the epochs*



*Training and Validation MAE trends across the epochs*



*Prediction of the geometrical features*

**Metrics on the test set:**

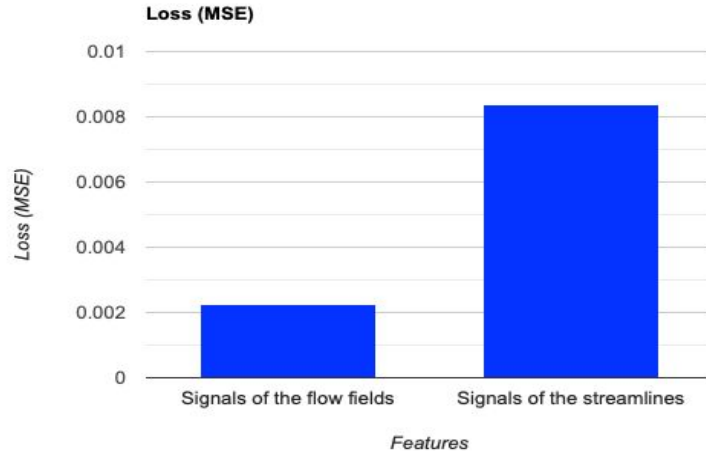
Loss (MSE): **0.008360184**

Mean Absolute Error (MAE): **0.049679368**

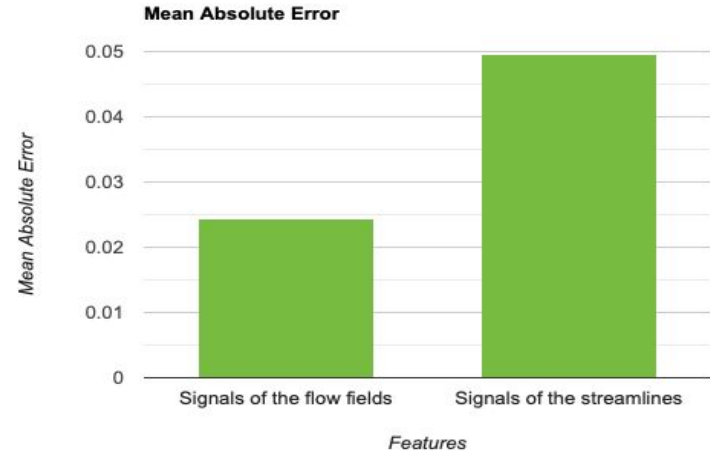
# Comparison of metrics - CNN

	Signals of the flow fields	Signals of the streamlines
Loss (MSE)	0.002248327	0.008360184
Mean Absolute Error (MAE)	0.008360184	0.049679368

*Metrics obtained from each class of features*

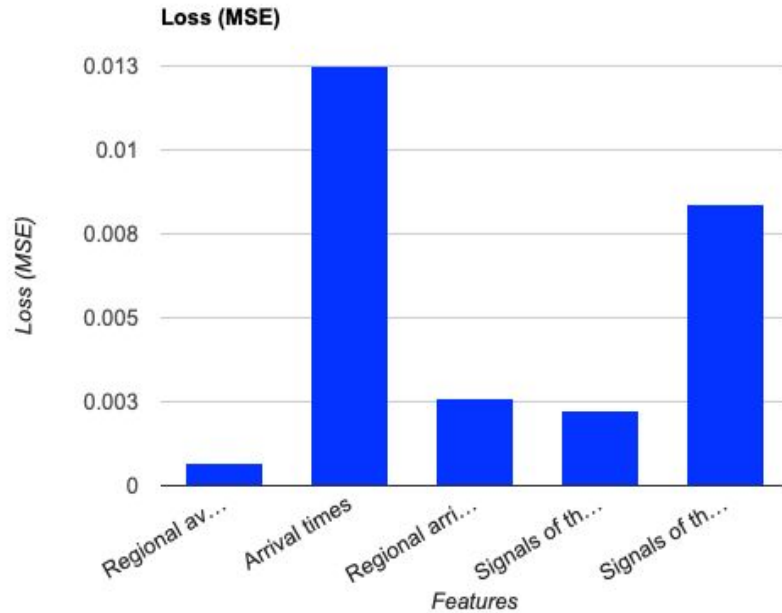


*Loss obtained from each class of features*

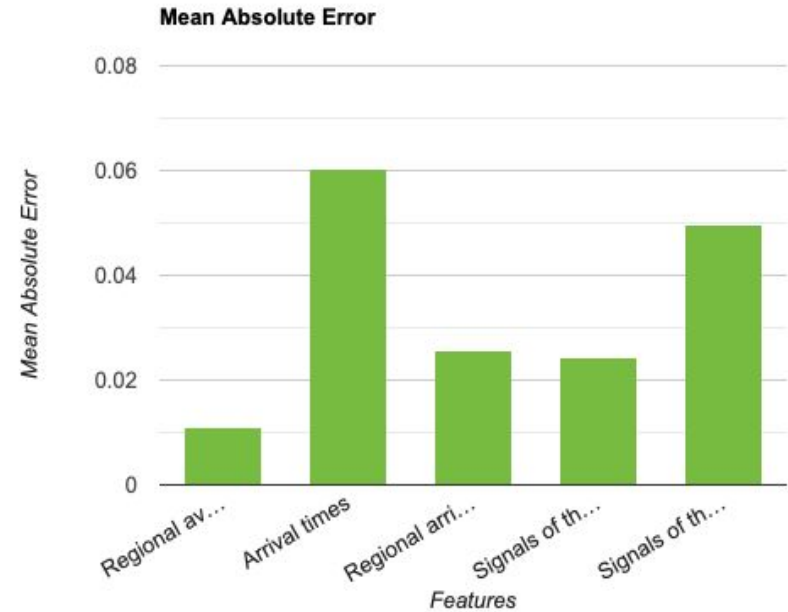


*MAE obtained from each class of features*

# Comparison of metrics



*Loss obtained from each class of features*



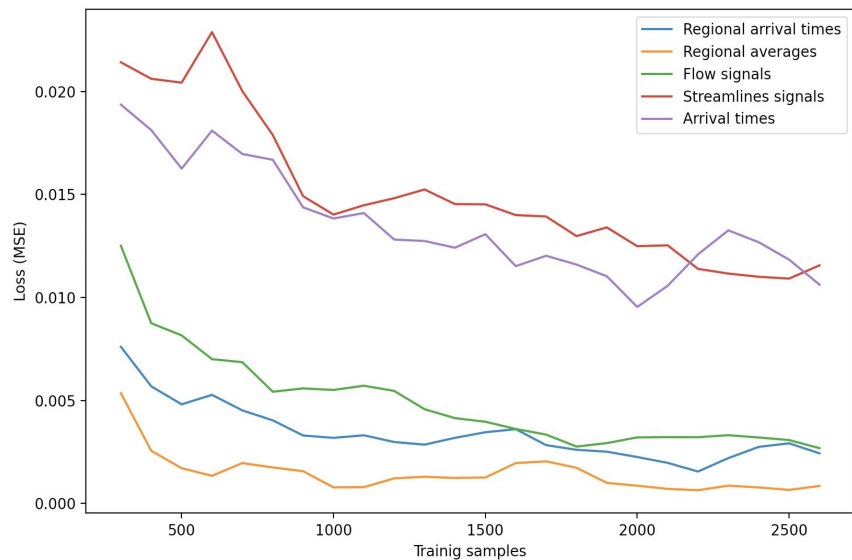
*MAE obtained from each class of features*

# Experimental setup

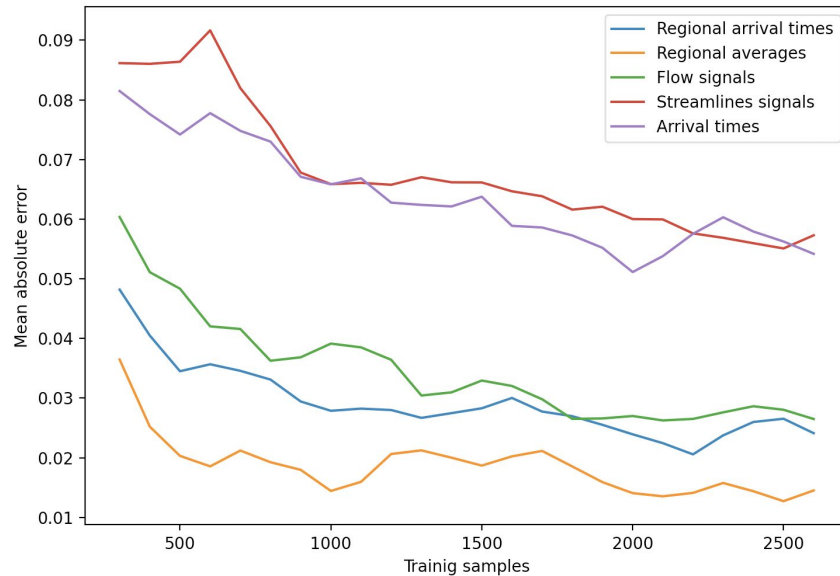
Second Approach: variation of the training set size.

- Incremental **step size** of *100* samples per experiment.
- Data **sampled randomly**.
- Data splitting:
  - **Training set**: 80% of the available samples for training.
  - **Validation set**: 20% of the training samples.
  - **Test set**: 20% of the available samples.

# Comparison of metrics



*Loss trend w.r.t. the training set size for each class of features*

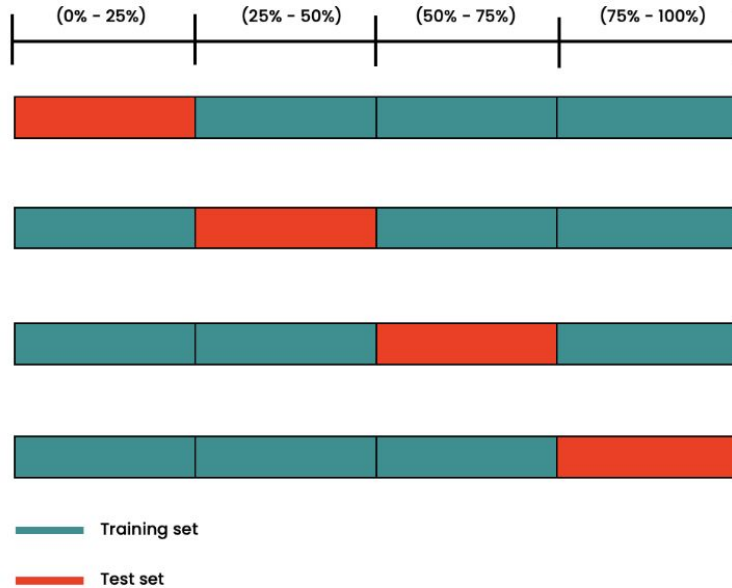


*MEA trend w.r.t. the training set size for each class of features*



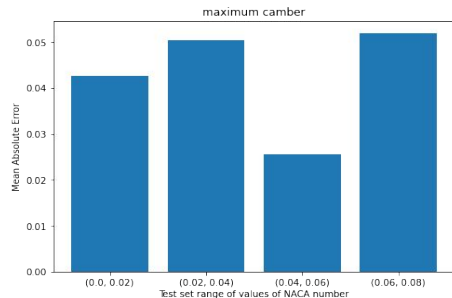
# Extrapolation

Studying how the composition of the training set can affect the performances.

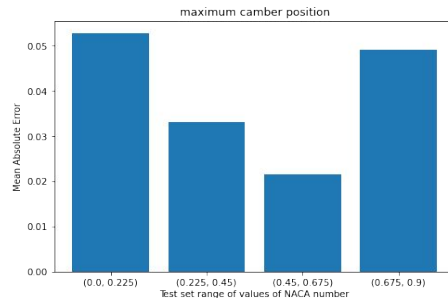


- **4 not overlapping** subsets extracted for each NACA number by excluding specific range of values from the training set.
- **12 subsets in total.**

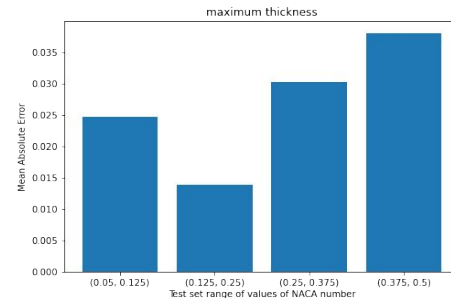
# Regional Averages



*Maximum camber*

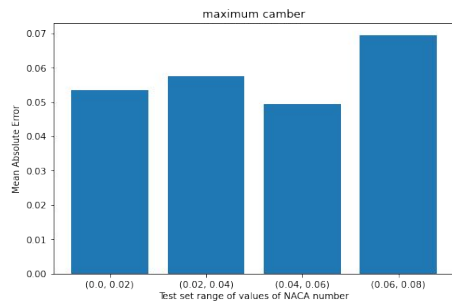


*Maximum camber position*

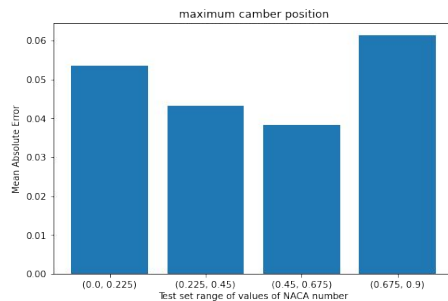


*Maximum thickness*

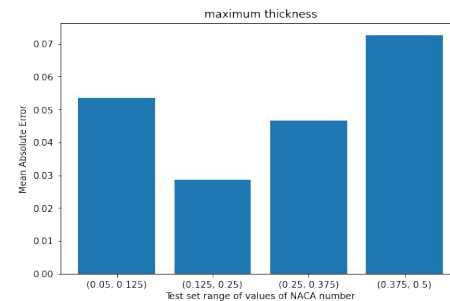
# Regional arrival times



*Maximum camber*

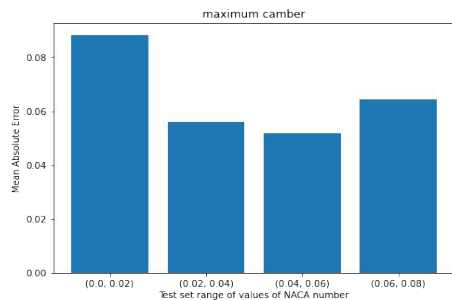


*Maximum camber position*

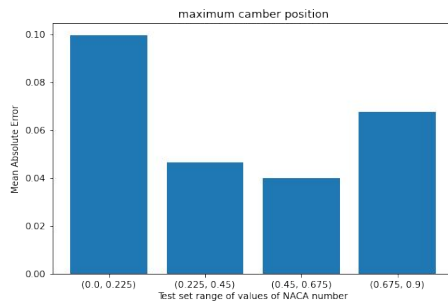


*Maximum thickness*

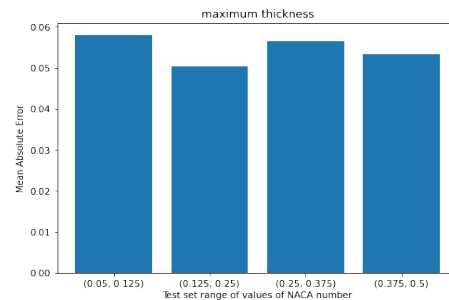
# Signals of the flow fields



*Maximum camber*

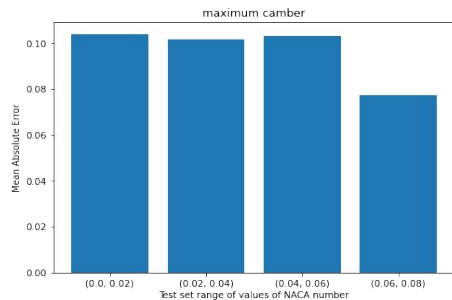


*Maximum camber position*

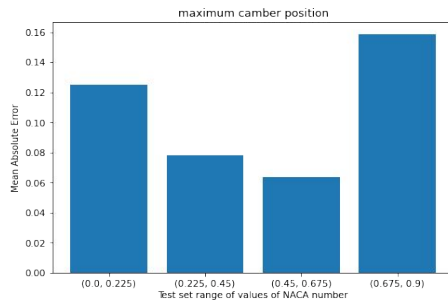


*Maximum thickness*

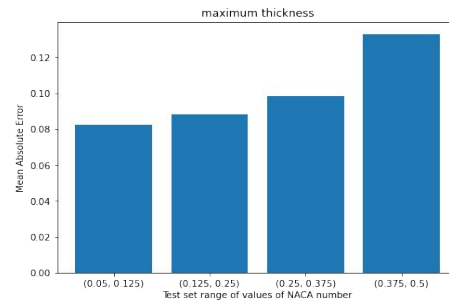
# Signals of the streamlines



*Maximum camber*



*Maximum camber position*



*Maximum thickness*