**Mathematics for Financial Risks and Derivatives**
**School of Economics and Political Sciences**
**University of Padova**
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## Project A: Valuation of a Booster option

A Booster option is an exotic path-dependent derivative that is sometimes used in FX (foreign exchange) markets. The derivative delivers at payoff at maturity that depends on the amount of time that the underlying spends between two barriers $L$ (*low*) and $H$ (*high*), with $L < H$. More precisely, denoting by $S = (S_t)_{t\geq 0}$ the price process of the underlying, the payoff at maturity $T$ is given by the following quantity:

$$X := \int_0^T \mathbf{1}_{\{L\leq S_t\leq H\}}\,\mathrm{d}t, \tag{1}$$

where $\mathbf{1}_{\{L\leq S_t\leq H\}}$ denotes the indicator function, which takes the value 1 if $L \leq S_t \leq H$ and is 0 otherwise. Observe that $0 \leq X \leq T$, in particular the maximum payoff is $T$ (which occurs if the underlying stays always between the two barriers $L$ and $H$ until maturity $T$).

1. Assume that $S$ is described by the Black-Scholes model. Derive a semi-explicit valuation formula for the payoff (1).[1] Semi-explicit means that the valuation formula is explicit up to an integral that cannot be computed in closed form.

2. Compute a numerical approximation of the price of the Booster option by discretizing the integral obtained in step 1.

3. Always assuming that $S$ is described by the Black-Scholes model, compute the value of the Booster option by means of Monte Carlo simulations.

4. Compute the value of the Booster option by relying on a multi-period binomial model for the market index $S$.

5. Compare the numerical results obtained in parts 2, 3, 4, discussing their accuracy and computational efficiency.

6. By relying on your favorite computational method among 2, 3, 4, perform a sensitivity analysis of the value of the Booster option with respect to the volatility of the underlying.

[1] Hint. To obtain an explicit valuation formula for (1) you need to use Fubini-Tonelli's theorem: for any non-negative random process $(\phi_t)_{t\geq 0}$, it holds that $\mathbb{E}[\int_0^T \phi_t\,\mathrm{d}t] = \int_0^T \mathbb{E}[\phi_t]\,\mathrm{d}t$.