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Defeasible normative reasoning

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Abstract
The paper is motivated by the need of accounting for the practical syllogism as a piece of defeasible reasoning. To meet the need, the paper first refers to ranking theory as an account of defeasible descriptive reasoning. It then argues that two kinds of ought need to be distinguished, purely normative and fact-regarding obligations (in analogy to intrinsic and extrinsic utilities). It continues arguing that both kinds of ought can be iteratively revised and should hence be represented by ranking functions, too, just as iteratively revisable beliefs. Its central proposal will then be that the fact-regarding normative ranking function must be conceived as the sum of a purely normative ranking function and an epistemic ranking function (as suggested in qualitative decision theory). The distinctions defends this proposal with a comparative discussion of some critical examples and some other distinctions made in the literature. It gives a more rigorous justification of this proposal. Finally, it starts developing the logic of purely normative and of fact-regarding normative defeasible reasoning, points to the difficulties of completing the logic of the fact-regarding side, but reaches the initial aim of accounting for the defeasible nature of the practical syllogism.

Keywords Defeasible reasoning · Normative reasoning · Legal reasoning · Deontic logic · Ranking theory · Practical syllogism · Qualitative decision theory

1 Introduction

We all have been raised with the Naturalistic Fallacy: you must not and cannot validly infer any ought from any is, any normative, prescriptive, deontic assertion from any empirical, descriptive, ontic assertion. (There are many nearly equivalent labels. In this paper I will stick to the pair normative/descriptive.) Without doubt, David Hume has discovered a deep truth here, a landmark of ethical theorizing and a

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firewall against ideological sway. However, it's not so easy to say what precisely the deep truth is.

It's not easy because there are clear counter-examples to Hume's Thesis. There is the "ought implies can" principle implying that not-can implies not-ought; this seems to contradict Hume's Thesis. There is a problem with classifying mixed assertions—as normative? as descriptive?—and Prior's paradox showing that, however they are classified, they generate a violation of Hume's Thesis (see Schurz 1997, pp. 68f.). There is Aristotle's practical syllogism, which is not yet an exception to Hume's Thesis since it infers a normative conclusion from a normative and a descriptive premise.[1] However, it generates an exception, since the deduction theorem seems to allow shifting the normative premise to the conclusion and thus deriving a hypothetical normative sentence from a purely descriptive premise. So, there seem to be bridge principles relating the descriptive and the normative. If so, which status may they have? The list may be continued.

One may point out that the practical syllogism is clearly only a piece of defeasible or non-monotonic reasoning, while Hume's Thesis is certainly intended to apply only to monotonic, deductive inference.[2] However, this observation moves us out of the frying pan into the fire. Do we have any account of defeasible reasoning jointly for descriptive, normative, and mixed sentences? Yes and no.

The literature on descriptive defeasible reasoning has vastly amassed in the last 40 years, with many competing paradigms none of which is generally accepted. Compared to this, the literature on normative defeasible reasoning is small. It is, however, building up—alas, again with little consensus; even the logical formats proposed look quite different. The entire field still seems to be in a quite fragmented and tentative state. In particular, I don't find much about the practical syllogism, which will be of central concern in this paper.

To be a bit more specific: One may perhaps distinguish a modal logic, a default logic, and an argumentation theoretic approach. The boundaries are not clear-cut, though; one may emphasize the similarities or the differences. The oldest approach is the modal one. It originates from deontic logic as a branch of modal logic and unfolds in dyadic or conditional deontic logic in the way proposed by Hansson (1969) and axiomatized by Lewis (1973, Sects. 5.1 and 6.1) in response to the problems of somehow representing conditional obligations with the help of material implication. Hansson's and Lewis' point was that the conditional, unlike material implication, is of a non-monotonic or defeasible nature. This already raises the issue whether conditionality and defeasibility are the same kind of phenomenon. This will be an important issue below. There is, secondly, the default logic approach to defeasibility; it didn't take long to wed it with deontic logic. Both traditions are well

[1] Aristotle actually takes the conclusion of a practical syllogism to be the action itself and not a sentence saying that that action ought to be performed. Many interpreters take this to be significant. Here, however, I neglect this point in order to treat the practical syllogism as an ordinary inference between sentences.

[2] Pigden (2010) is an up-to-date collection of papers on Hume's Thesis, in which, however, defeasibility issues play no significant role.

represented in the collection of Nute (1997). The default logic tradition has been further developed, e.g., by Horty (2012).[3]

There is, thirdly, the argumentation theoretic approach, the formal theory of which was arguably initiated by Pollock (1987). It has the advantage that it is able to cope with all types of arguments across the board. For instance, the ASPIC and ASPIC+ framework developed by Prakken (2010) and Mogdil and Prakken (2013) is not specifically about normative reasoning, but applies there as well due to its generality.

Philosophers of law contributed to the topic from the beginning (see, e.g., Alchourrón and Bulygin 1971); Beltrán and Ratti (2012) is a detailed collection on defeasible legal reasoning. The work of Alchourrón may still be subsumed under the modal logic approach. However, the argumentation theoretic approach has gained strong influence in legal theorizing, as documented by Sartor (2018). This is just to mention a few references, but they already indicate how variegated the field is.

I mention them also in order to make clear that, although I will address the same field as they do, my approach will differ from all of them; it may even be called idiosyncratic. In any case, it looks as starting within modal logic, but it will end up in the field of qualitative decision theory, which rather belongs to microeconomics. So, it will widen the scope still more, but in a fruitful way, I hope. For this reason, the comparative remarks in this paper will be quite sparse; still they may sufficiently contrast my approach to the existing alternatives. Tightly connecting my paper with the variegated literature would vastly exceed the available space.

To be a bit more specific: Since 1983 I have been developing ranking theory, most fully in Spohn (2012). I always intended this theory to present a paradigm of defeasible reasoning, however only about empirical matters; it was pure epistemology dealing with our (empirical or a posteriori) beliefs and their change.[4] Recently, however, I became more and more convinced that ranking theory can also be used to account for our defeasible *normative* thinking. This is what this paper is going to explain.[5]

Therefore I will simply assume that ranking theory delivers a good account of defeasible reasoning. I am inclined to make stronger claims, but "good" is good

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[3] See, however, the criticism of Fuhrmann (2017).

[4] In the present context it is particularly noteworthy that Cohen's theory of Baconian probability, as he called it, is an important predecessor of ranking theory; cf. Spohn (2012, sect. 11.1). In Cohen (1977) he unfolds in book length that his theory is especially tailored to legal reasoning.

[5] A short note about the history of this paper: In my master thesis 1973 I presented a sound and complete calculus for Hansson's (1969) semantics of conditional deontic logic, unaware of the fact that Lewis (1973) had contemporaneously and more elegantly done the very same. My result is published in Spohn (1975). At that time I felt the strong urge that conditional deontic logic needs to be extended to an alethic-deontic logic, e.g., in order to represent the practical syllogism. I did not see, though, how this could be done; the only way to account for the interaction of norms and facts, or rather of beliefs and desires, seemed to be to turn to quantitative decision theory. That's what I did then in my dissertation. However, I never made whole peace with this move. Reading Schurz (1997), which is still the most thorough-going investigation into Hume's thesis and into alethic-deontic logic, inspired me to think again about this issue and to find a way to fill the old lacuna with ranking theoretic means (whence this paper is well placed in this collection in honor of Gerhard Schurz). The defeasible aspect is not well belabored in his book (as he would be the first to agree). That's where I hope to make progress.

enough a starting point of the present investigation, which I will not further defend. I have done so amply in Spohn (2012, 2015), but still incompletely in view of the vastness of the topic of defeasible reasoning. My aim will rather be to make progress on the normative side.

For this purpose, Sect. 2 will briefly review the results of Schurz (1997) concerning is-ought bridge principles and his account of the practical syllogism. This is to serve as a more substantial motivation of this paper. Section 3 will briefly recap ranking theory as a theory of (empirical) belief and explain how it entails an account of defeasible reasoning or conditional logic. Section 4 will argue that the very same structure applies to our normative convictions as well, with all the advantages ranking theory has in my view, and hence that this structure also accounts for purely normative defeasible reasoning or conditional deontic logic. The main task, then, will be to combine the descriptive and the normative side. For this purpose, Sect. 5 suggests that there is not only the purely normative ought dealt with in Sect. 4, but also a fact-regarding kind of ought, in analogy to intrinsic and extrinsic utility in utility theory. Thus an important claim of the paper will be that normative reasoning is systematically ambiguous; it divides into a pure and a fact-regarding version both of which are defeasible. Moreover, the section makes a rigorous proposal—which is central to this paper—for combining descriptive and purely normative ranking functions in order to account for this fact-regarding ought. Section 6 discusses the explicatory power of this proposal vis à vis various critical examples and various important distinctions raised in the literature; surely, this discussion could be extended indefinitely. Section 7 suggests three different ways of more rigorously justifying the proposal in Sect. 5 and precisely states one of them. Section 8 will finally, but only briefly discuss the logic of the fact-regarding ought and its relation to descriptive and purely normative conditionals. In particular, it will return to the practical syllogism, from which we departed, and explain how it is accounted for by the present proposal. This will also make clear why a complete axiomatization of the ensuing conditional (or defeasible) logic must remain an open issue.

2 Bridge principles between is and ought?

Let's start with asking for relations between descriptive and normative reasoning. How do they combine? Are there bridge principles between is and ought? Do they undermine Hume's Thesis? Let's take a brief look at the results of Schurz (1997). In chapter 3 he introduces the *Special Hume Thesis*: "no consistent set of purely descriptive premises deductively implies any purely normative sentence that is not logically true" (p. 70). And he distinguishes four versions of the thesis according to four notions of a purely normative sentence. He also introduces the *Generalized Hume Thesis*: "for every deductive inference … with a purely descriptive premise set D and a possibly mixed conclusion A, A is a completely O[ught]-irrelevant conclusion of D" (p. 77)—where everything depends on the precise definition of O-irrelevance. In chapter 5 he shows how the Generalized Thesis entails all four versions of the Special Thesis, given a further property (Halldén-completeness) of the logic considered. And in chapter 4 he proves the Generalized Thesis for any logic that does not contain any is-ought bridge principle—where X is a bridge principle iff “X contains at least one schematic letter which has in X at least one occurrence in the scope of an O and at least one occurrence out of the scope of any O” (p. 91).

This entails the crucial question: should we assume any such bridge principle? Schurz ponders this issue at length in his chapters 6, 11, and 12. He does so within the context of alethic-deontic logics building on first-order logic with identity and containing an alethic necessity operator $\Box$ (and the corresponding possibility operator $\Diamond$) and the deontic operator O (“ought” or “is obligatory”) in arbitrary mixtures and nestings. So, in a way, he considers a much more general framework than we will do here. Within this framework he studies three generally accepted bridge principles (whereas all other principles one might consider are argued to be tied to special ethical conceptions), namely:

(1) $O\varphi \rightarrow \Diamond\varphi$ (the OC principle: ought implies can),
(2) $\Box\varphi \rightarrow O\varphi$ (the MO principle: must implies ought), and
(3) $O\varphi \wedge \Box(\varphi \rightarrow \psi) \rightarrow O\psi$ (the ME (means-end) principle), or equivalently,
(4) $\Box(\varphi \rightarrow \psi) \rightarrow (O\varphi \rightarrow O\psi)$.

In the literature, (4) is often considered as a deontic version of a rule of necessitation. However, the equivalent (3) may also be taken as an explication of the practical syllogism.[6] Informally, it says that, if φ ought to be the case and ψ is a necessary condition of φ, then ψ ought to be the case, too. In its form (4) one sees immediately how an alethic formula entails a deontic one. As Schurz (1997, p. 128) explains, the OC principle is entailed by the MO principle, which in turn is equivalent to the ME principle. And he goes on to argue in chapter 6 that these bridge principles do not seriously affect Hume’s Thesis, since they allow only ‘practically trivial inferences’ in a sense precisely explicated.

All of this is highly recommended for close reading. At the same time this brief summary already displays why it does not provide what we are looking for. The means-end principle discussed by Schurz is the best he can state within his context, but it is not the ordinary one. An ordinary example of a practical inference is (apart from the stiltedness of the use of “ought”): “If I ought not to get wet and if I would get wet without an umbrella, then I ought to take an umbrella.” The point is that the second premise is stated as a subjunctive conditional, which we know to be of a defeasible nature. Normally, if I don’t have an umbrella, I will get wet (under the prevailing weather conditions); but this may be defeated. You could add: “Yes, it’s going to rain. Don’t worry, though; the path you want to take is completely roofed.” Thereby you undermine the subjunctive conditional and hence the normative conclusion that I should take an umbrella. I could object: “No, they have removed the roof due to danger of collapse”, and thereby save the conclusion. And so on.

[6] Well, perhaps not in the form originally found in Aristotle, but in a form as it is discussed today.

In more technical terms, the weakness of Schurz' ME principle is that it is stated in terms of *strict implication*, whereas we would like to have the practical syllogism or the means-end principle stated with something like a *variably strict implication*. That is, (4) should rather employ a subjunctive reading of "$\Box(\varphi \rightarrow \psi)$" and correspondingly of "$O\varphi \rightarrow O\psi$" as well and read as something like:

(5) $(\varphi \rhd \psi) \rightarrow (O\varphi \rhd O\psi)$,

where $\rhd$ is a subjunctive conditional not yet specified. This is what we would like to establish, at least as a defeasible inference.

Note that this aim sets this paper apart from the papers at least in the modal logic tradition in an important respect. Schurz' procedure is very common. Whenever the alethic-deontic relations are considered, the alethic side is represented by one or several necessity operators. For instance, Prakken and Sergot (1997) add just a necessity operator for which (4) holds for conditional obligations (cf. p. 232), and Carmo and Jones (2002) operate with two modal notions, potential possibility and actual possibility, and correspondingly with two kinds of ought, which obey restricted versions of (2) (cf. p. 297). I would not know how the literature referenced could construe something like (5).

There is another issue we might clarify right away. What do φ, ψ, etc. stand for? Throughout this paper I take them to represent descriptive sentences having a truth condition and thus expressing a proposition. A *proposition* in turn is a subset of a given set W of *possibilities* or possible worlds. So $\mathcal{P}(W)$, the power set of W, is at the same time the set of propositions considered. I assume W to be finite, so that there are only finitely many propositions. This avoids various problems that are irrelevant in our context.

Hence, the simple normative sentences we will study here will always be of the form $O\varphi$, i.e., I take a normative statement as saying that the proposition expressed by φ *ought* to be the case. I will not consider any other basic form. This might seem wrong from the start. Deontic logic is beset by many problems. For some of them it has turned out fruitful to refer ought statements especially to actions; in the end, it seems, obligations are always about what ought to be *done*. Other problems require attending to the temporal structure; this suggests time-indexing the objects of obligations as well as the obligations themselves.

Here, I want to avoid all these complications. We will come across examples where it might be plausible to resort to those measures. However, I think they do not hit the heart of the matter. We are about to comprehensively treat problems posed by *conditionality*, not by agency in time. Prakken and Sergot (1996, sect. 3–4) argue convincingly that those problems emerge already on an atemporal propositional level and cannot be solved by resorting to more finely structured objects of obligations. Hence, they recommend the same starting point as I have chosen here. It is a different, though surely worthwhile task to extend this paper to those other structures. But those other approaches do not really serve as a standard of comparison for our present purposes.

So let's tackle (5). This will be long way to go. The plan is first to analyze the antecedent of (5) in terms of ranking theory (in Sect. 3), then to argue that ranking theory can also be used for analyzing the consequent (in Sect. 4), and finally to study how antecedent and consequent may be related (in the remaining sections).

3 Ranking theory as an account of descriptive defeasible reasoning

Let's recap ranking theory and the account of defeasible reasoning it delivers. Basically, ranking theory is a theory of belief, which, in order to account for the dynamics of belief, also assumes degrees of belief. It builds on a representation of the objects of belief. We just assumed the simplest possible representation according to which those objects form a finite algebra of *propositions*, which are subsets of a finite set W of *possibilities*. When dealing with propositional belief, as we do in this section, this is good enough. For the moment, we may defer the explicit introduction of a language expressing those propositions.

Given this simple starting point, the basic representation of a belief state just consists in the set of propositions believed or taken to be true in that state, its *belief set*. Traditionally, i.e., since Hintikka (1962), a belief set $\mathcal{B} \subseteq \mathcal{P}(W)$ has to satisfy two rationality postulates: $\mathcal{B}$ must be *consistent*, i.e., $\mathcal{B} \neq \emptyset$, and $\mathcal{B}$ must be *deductively closed*, i.e., for any propositions A, B, C, if $A, B \in \mathcal{B}$ and $C \supseteq A \cap B$, then $C \in \mathcal{B}$. On the one hand, these postulates seem still to be maintained by the majority. On the other hand, there have always been doubts about logical omniscience apparently presupposed by them. These doubts may be mitigated by our finite context. Still, even in the finite context the postulates have come under pressure by the lottery and the preface paradox. However, here is no place for discussing this issue. We will simply go with the majority and proceed from these postulates, without which ranking theory could not be developed.

The notion of a belief set is static. However, we must also account for the dynamics of belief sets, how they rationally change. The well-founded claim of ranking theory is that we can most adequately do so by introducing also degrees of belief in the following way[7]:

Definition 1 κ is a *negative ranking function* for W iff κ is a function from $\mathcal{P}(W)$ into $\mathbf{N} \cup \{\infty\}$ (the set of natural numbers plus infinity) such that for all $A, B \subseteq W$:

(6) $\kappa(W) = 0$ and $\kappa(\emptyset) = \infty$,

(7) $\kappa(A \cup B) = \min\{\kappa(A), \kappa(B)\}$ (*the law of disjunction*)

The standard interpretation is that such a function expresses degrees of *disbelief* (whence the qualification 'negative'). If $\kappa(A) = 0$, A is not disbelieved at all. This allows that $\kappa(\bar{A}) = 0$ holds as well (where $\bar{A}$ = non-A); this expresses indifference or suspense of judgment regarding A. If $\kappa(A) > 0$, A is disbelieved or taken to be false, and the more so, the larger $\kappa(A)$. So, positive *belief* in A, the fact that A is taken to be true, is expressed by disbelief in $\bar{A}$ i.e., by $\kappa(\bar{A}) > 0$.

[7] The following material is much more extensively explained in Spohn (2009, 2012, ch. 5).

This interpretation explains axioms (6) and (7). (6) says that the tautology W is not disbelieved, i.e., that the contradiction $\emptyset$ is not believed. (6) also says that the contradiction is maximally disbelieved. (7) states that you cannot disbelieve a disjunction less strongly than its disjuncts. This entails in particular that if you believe two conjuncts, i.e. disbelieve their negations, you also believe their conjunction, i.e., disbelieve the disjunction of these negations. (7) also entails $\kappa(A) \geq \kappa(B)$ for $A \subseteq B$; i.e., believing $\bar{B}$ entails believing any logically weaker $\bar{A}$. Thus, beliefs are deductively closed according to a ranking function. Together with the non-belief of $\emptyset$ this entails the consistency of beliefs. Hence, the belief set $\mathcal{B} = \{A \mid \kappa(\bar{A}) > 0\}$ associated with a ranking function κ satisfies the two basic rationality postulates. Note, moreover, that (6) and (7) entail:

(8) either $\kappa(A) = 0$ or $\kappa(\bar{A}) = 0$ (or both) (*the law of negation*),

which expresses the consistency of belief directly; you cannot (dis)believe A and $\bar{A}$ at the same time.

We may also introduce positive ranking functions expressing positive belief directly and not via double negation as does the negative version, and so-called two-sided ranking functions expressing belief and disbelief at once. All three notions are interdefinable. However, the negative version is the only one we need; it will be particularly germane when we get to normative theorizing.

Beliefs can be more or less firm. This fact is naturally represented by ranking functions. However, I said that the degrees of (dis-)belief were needed to account for the dynamics of belief. How? The crucial notion is that of conditional ranks:

Definition 2 Let κ be a negative ranking function for W and $\kappa(A) < \infty$. Then the *conditional rank* of *B given A* is defined as $\kappa(B \mid A) = \kappa(A \cap B) - \kappa(A)$.

This amounts to an explication of conditional belief. That is, continuing the above explanations, we can say that B is *believed conditional on* A (relative to κ) iff $\kappa(\bar{B} \mid A) > 0$.

We might rewrite this definition as:

(9) $\kappa(A \cap B) = \kappa(A) + \kappa(B \mid A)$ (*the law of conjunction*).

This is highly intuitive. For, what is your degree of disbelief in $A \cap B$? One way for $A \cap B$ to be false is that A is false; this contributes $\kappa(A)$ to that degree. However, given A is true, B must be false; this adds $\kappa(B \mid A)$.

It immediately follows for all propositions A and B with $\kappa(A) < \infty$:

(10) $\kappa(B \mid A) = 0$ or $\kappa(\bar{B} \mid A) = 0$ (*the conditional law of negation*).

This law says that even conditional belief must be consistent. If both, $\kappa(B \mid A)$ and $\kappa(\bar{B} \mid A)$, were >0, both, B and $\bar{B}$, would be (dis-)believed given A. This ought to be excluded, as long as the condition A itself is considered possible, i.e., $\kappa(A) < \infty$. Indeed, given Definition 2 and axiom (6), we could axiomatize ranking theory by (10) instead of (7). Hence, the only substantial assumption written into ranking functions is conditional consistency. This provides strong normative foundations of ranking theory.[8]

Axioms (6) and (7) did not refer to any cardinal properties of ranking functions. However, the definition of conditional ranks involves arithmetical operations and thus presupposes a cardinal understanding of ranks. We will see below how this may be justified.

With the help of this central notion we can state the dynamics of beliefs and ranks. As in probability theory, we might say that we should simply move to the degrees of belief conditional on the total evidence E received. Thereby, though, the evidence E acquires maximal certainty, either probability 1 or positive rank ∞. This seems too restrictive. In general, evidence may be uncertain, and our rules for epistemic change through evidence should take account of this. In probability theory this is done by two principles realized in Jeffrey conditionalization (Jeffrey 1965, ch. 11). In ranking theory, it is achieved by the analogous principles: first, the evidence E itself cannot change conditional ranks given the evidence E and given its negation $\bar{E}$—how could it?—and second, the evidence E does not become maximally certain, but only to degree n, where n is a free parameter varying according to the effects of the specific information process at hand. These two assumptions uniquely determine ranking-theoretic change:

Definition 3 Let κ be a negative ranking function for W, $E \subseteq W$ with $\kappa(E)$, $\kappa(\bar{E}) < \infty$, and $0 \leq n \leq \infty$. Then the *$E \rightarrow n$-conditionalization* κ' of κ is defined by $\kappa'(A) = \min\{\kappa(A \mid E), \kappa(A \mid \bar{E}) + n\}$.

It is easily checked that $\kappa'(A \mid E) = \kappa(A \mid E)$ and $\kappa'(A \mid \bar{E}) = \kappa(A \mid \bar{E})$; i.e., those conditional ranks are preserved. Moreover, we obviously have $\kappa'(\bar{E}) = n$, saying that E believed in κ' with firmness n. Thus our two principles are satisfied by Definition 3.

The associated rule of epistemic change then is that you rationally move from your prior κ to your posterior κ', whenever you receive total evidence E in between with firmness n. This is perhaps the simplest and most natural rule of learning. But note that you can easily apply it iteratively. Thereby it provides a complete dynamic law of epistemic change.

In belief revision theory three kinds of epistemic change are studied: expansions (adding beliefs), contractions (giving up beliefs), and revisions (replacing some old beliefs by their negation) (see, e.g., Gärdenfors 1988). In any case you must arrive at a new (consistent and deductively closed) belief set. If you have to delete beliefs (in contractions and revisions), this is not so easy. If you delete one belief, you have to delete many others as well. This is governed by a so-called *entrenchment order*,

[8] This point is fully elaborated in Huber (2007). There are various other justifications of ranking theory.

and the rule is that you first delete least entrenched beliefs and then second least, and so on, till you have reached a belief set in which the intended beliefs are deleted. It is not clear, however, how to iterate these kinds of epistemic change, since it is not clear how to also change the entrenchment order.

Now, $E \rightarrow n$-conditionalization comprises these three kinds of belief change (for details see Spohn 2012, sect. 5.5). Indeed, a negative ranking function may be interpreted as delivering not just an entrenchment order, but an *entrenchment grading*. And Definition 3 says what the posterior entrenchment grading is, so that the conditionalization rule can be applied again. This is important because we thereby attain a measurement procedure for ranks that justifies the cardinality of ranks. This procedure will turn out relevant below. Still, a rough sketch will suffice for our purposes. It works as follows:

The procedure—for all details see Hild and Spohn (2008)—refers to belief contraction ($= E \rightarrow 0$-conditionalization) and presupposes that you are able to say what your beliefs were after various iterated contractions. In fact, it suffices to look at two non-vacuous contractions. Definition 3 implies a certain rational behavior of those iterated contractions (characterized by an axiom of 'restricted commutativity' and an axiom of 'path independence', which add to the standard contraction axioms in belief revision theory and the contraction counterparts of the familiar Darwiche-Pearl postulates for revision; see Darwiche and Pearl 1997). The point now is this: if your contraction behavior conforms to these axioms, then that behavior uniquely determines your ranking function up to a multiplicative constant. That is, your ranks can thereby be *measured on a ratio scale*. This result may be taken to justify the specific cardinal structure of ranks.

Let me add that there are other ways to measure ranks as well. So, doubts vis à vis numeric ranks are as little justified as, say, doubts vis à vis numeric utilities (which had been effectively dispelled by the utility theory of von Neumann and Morgenstern more than 70 years ago). However, the method just sketched will be most useful below.

What has ranking theory to do with defeasible reasoning or arguments? Well, arguments (about empirical matters) are to change our (empirical) beliefs; so, if you understand belief change, you have a basic understanding of arguments. Or you may say: Reasoning is about unfolding conclusions from given premises, it answers the question what you should believe on the basis of those premises; so, you might just as well say that reasoning is about conditional beliefs. This is at least the basic connection. Of course, belief change is treated in pure epistemology, whereas argumentation is a linguistic activity involving other things as well. Still, the connection is close. Let me give some more substantial hints:

As for the connection between belief revision theory and non-monotonic logic, Rott (2001, ch. 4–5) provides the most thoroughgoing information, by establishing a comprehensive one–one correspondence between all the belief revision axioms and all the non-monotonic inference rules discussed in the literature. So, the two fields fully converge. Quite a different connection between ranking theory and default logic is established by the so-called system **Z** proposed by Pearl (1990), an algorithm that constructs a minimal ranking function jointly satisfying a given set of default rules. Thus system **Z** at the same time provides a consistency test for sets of default rules.[9] Thereby, ranking theory serves as semantics of default logic. This point is, in turn, related to my basic criticism of formal argumentation theory à la Pollock (1995): namely that it is not furnished with any semantics and hence assumes various ad hoc rules like what he calls the weakest link principle or the no-accrual-of-reasons principle without systematic justification. Ranking theory could justify them, but only in a restricted and modified form (see Spohn 2002). I am not aware that the situation in formal argumentation theory has principally changed.

However, let me treat the issue not only by way of reference, but also by specifically stating how a ranking-theoretic semantics induces a basic conditional logic. This is to serve, though, no more than expository interests. First, let L_0 be the language of a finite part of propositional logic containing only finitely many sentence letters. Let V be the set of valuations (of those sentence letters) of L_0. For $\varphi \in L_0$ and $v \in V$ $v \vDash_0 \varphi$ says that φ is true in v. Finally define $V(\varphi) = \{v \mid v \vDash_0 \varphi\}$ to be the set of valuations in which φ is true. We may connect this with the algebra introduced above by simply identifying W and V. Thus, for each $\varphi \in L_0$ $V(\varphi)$ is the proposition expressed by φ.

As indicated, defeasible entailment and conditional logic may be seen to be exchangeable. So, instead of considering a defeasible entailment relation among sentences of L_0 we expand L_0 by a conditional $\rhd$, but only in a non-nested way. That is, we consider the following language L_{1d} ("*d*" stands for "descriptive"): if φ and ψ are sentences of L_0, $\varphi \rhd \psi$ is a sentence of L_{1d}, and if φ and ψ are sentences of L_0 or L_{1d}, propositional combinations of φ and ψ are sentences of L_{1d}, too. Thus, L_{1d} does not contain iterations of $\rhd$.

One may find iterations desirable simply for syntactic reasons. However, not everything syntactically constructible is semantically construable. One may point out that ordinary language allows at least some elementary iterations. But it does no more. So this hint is at best ambiguous. In any case, it is well known that any semantics of the conditional guided by the Ramsey test has great difficulties with iterating the conditional.[10] Since ranking theoretic semantics belongs to this category, too, I avoid struggling with this issue by appropriately restricting L_{1d}. In our context, this issue is simply irrelevant.

Ranking theory indeed offers various interpretations of the conditional, which may be needed for coping with the wildly variegated uses of the conditional in ordinary language (see Spohn 2015). Here, however, we may be content with the most common interpretation inspired by the Ramsey test. Then the semantics for L_{1d} runs thus:

First, for any ranking function κ for V, let $\mathcal{B}(\kappa) = \{\varphi \mid \kappa(V(\neg\varphi)) > 0\}$ be the set of sentences expressing beliefs held in κ, and let $\mathcal{CB}(\kappa) = \{\langle\varphi, \psi\rangle \mid \kappa(V(\neg\psi) \mid V(\varphi)) > 0\}$ be the set of sentence pairs $\langle\varphi, \psi\rangle$ corresponding to the conditional beliefs in κ.

[9] Goldszmidt and Pearl (1991) develop system **Z** into system $\mathbf{Z}^+$, which allows default rules to have varying strength. This is similar to endowing arguments with varying strength, as done by Mogdil and Prakken (2013). Kern-Isberner (2004) has proposed an alternative algorithm applying a different notion of minimality.

[10] See Lewis (1976) and Gärdenfors (1988, ch. 7). I take Rott (2011) to be the relatively best that can be done.

Then we may recursively define truth for all sentences in L_{1d} relative to a valuation $v \in V$ and a ranking function κ for V by specifying the following recursive base: $\langle v, \kappa\rangle \vDash p$ iff $v \vDash_0 p$ for any sentence letter p of L_0, and $\langle v, \kappa\rangle \vDash \varphi \rhd \psi$ iff $\langle \varphi, \psi\rangle \in \mathcal{CB}(\kappa)$ or $V(\varphi) = \varnothing$.[11] This embodies the Ramsey test. This recursive base may then be truth-functionally expanded to all sentences of L_{1d}.

Concerning logical truth we then have a choice: We may call $\chi \in L_{1d}$ *semi-epistemically logically true*, $\vDash^{se} \chi$, iff $\langle v, \kappa\rangle \vDash \chi$ for all valuations $v \in V$ and all ranking functions κ for V. Or we may epistemically restrict that notion by requiring that all (unconditional) beliefs in κ must be true in the valuation v. That is, we may define a sentence χ of L_{1d} to be *epistemically logically true*, $\vDash^{e} \chi$, iff $\langle v, \kappa\rangle \vDash \chi$ for all ranking functions κ for V and all valuations $v \in V$ such that $v \vDash_0 \varphi$ for all $\varphi \in \mathcal{B}(\kappa)$.

It is easily checked, then, that the restriction of Lewis' logic **V** (cf. Lewis 1973, pp. 132ff.) to the fragment L_{1d} is sound and complete with respect to $\vDash^{se}$. In particular, neither

(11) $\varphi \wedge \psi \rightarrow (\varphi \rhd \psi)$ (Centering) nor

(12) $(\varphi \rhd \psi) \rightarrow (\varphi \rightarrow \psi)$ (Weak Centering)

hold with respect to $\vDash^{se}$, simply because there is no relation between the facts according to v and the conditional beliefs according to a ranking function κ for V. By contrast, it is Lewis' logic **VC** restricted to the fragment L_{1d} that is sound and complete with respect to $\vDash^{e}$, since $\vDash^{e}$ specifies such a relation. This includes Weak Centering and Centering. However, according to $\vDash^{e}$ these axioms only indicate a relation between conditional and unconditional beliefs. This agrees with how things are set up by Gärdenfors (1988, pp. 148ff.) who accepts these axioms as well.

Non-monotonic logic is all about weakening monotony or restricting strengthening the antecedent. Many ways to do so are discussed in the literature. The crucial axiom of **V** and **VC** is:

(13) $\neg (\varphi \rhd \neg \psi) \rightarrow ((\varphi \wedge \psi \rhd \chi) \leftrightarrow (\varphi \rhd (\psi \rightarrow \chi)))$ (Rational Monotony).

It is one salient weakening of monotony and characterizes ranking theory, too. That is, it is the ranking way of moving up and down, in the terminology of Prakken and Sergot (1997), i.e., of relating what holds under less and what holds under more specific conditions or in less and more specific contexts. In other words: ranking theory so far just duplicates traditional and well-investigated paths. Anything else would have been disconcerting.

Indeed, as far as conditional logic is concerned, there is no advantage in moving from entrenchment orders to entrenchment gradings (= ranking functions). On the contrary, we are thereby nailed down to considering only complete entrenchment

[11] The latter condition makes conditionals with impossible antecedents vacuously true. For a full treatment of those exceptional antecedents see Raidl (2018).

orders, while it may be of logical interest to consider less demanding order types, as, e.g., considered by Prakken and Sergot (1997). However, as indicated, moving to entrenchment gradings is required when it comes to iterated belief change. This is not our present concern. What will be our concern, though, is combining the above with conditional deontic logic of Sect. 5; and there gradings will be indispensible.

4 Purely normative reasoning: a ranking-theoretic conception

Why do I repeat all this familiar stuff? In one sentence: because all of this applies directly and equally well to normative reasoning! That's a claim I have not made before. However, descriptive and normative reasoning are widely conceived to work more or less the same; and I concur. This is already suggested by the structural identity of standard doxastic and deontic logic, as displayed, e.g., in Hintikka (1962) and Hansson (1969). The similarity was first extended to conditional versions of those logics by Lewis (1973). Interestingly, AGM belief revision theory has an equally important normative origin in Alchourrón and Makinson (1981), although the main bulk of subsequent theorizing was devoted to formal epistemology. As mentioned, the argumentation theoretic approach treats all arguments on a par, anyway. So, basically the claim is not surprising. Merin (2006, pp. 339ff.) and Huber (2014, p. 2180) have already suggested that it extends to ranking theory. I would like to argue in this section that it indeed extends to the *full* resources of that theory.

To begin with, who may be the subject or the instance issuing the norms? Anyone: a religious or moral code, any state's legal code, a legislative, a judge applying a legal code or creating precedents, or, of course, any person's normative convictions. What we intend to grasp is such a subject's normative conception. In this section I will focus only on her *purely* normative conception *as such*, which only looks at how things ought to be, unaffected by the 'reality principle', i.e., which does not yet take any kind of facts or empirical conditions into account. What "pure" or "as such" is to mean will get clearer below and in Sect. 5, where we will contrast the pure case and the joint case of interacting of norms and facts.

As explained in Sect. 2, such a purely normative conception is considered here to be just about propositions (= subsets of W) and not about more specific or structured entities. So same propositions, but different attitudes. The question now is not whether $A \subseteq W$ should be believed or taken to be true; it is whether A ought to be the case or should be taken as a norm. Standard deontic logic (cf. Hansson 1969) assumes that a *norm set*, i.e., the set $\mathcal{N}$ of norms held by a subject, the set of propositions that ought to be the case, should be consistent and deductively closed—as it is with belief sets in doxastic logic. Again, this standard has been contested from the beginning, not only because "ought" was suspected to be just as hyperintensional as "believe". Deductive closure of obligations became doubtful due to Ross' paradox and the non-sensical air of asserting that the tautology ought to be the case. More importantly, while inconsistency might be neglected as an irrationality in the epistemic case (which deals only with rational belief), this negligence seems inappropriate in the deontic case, namely vis à vis moral dilemmas, which are not irrational, but real and painful and apparently confront us with incompatible obligations. There are many proposals for doing justice to such criticism. However, pursuing them would divert us from our main concerns. Hence, I will accept the standard assumptions here as well.[12]

The parallel extends. As mentioned in the introduction, there also exist dyadic or conditional deontic logics, dealing with assertions of the form that *B* ought to be the case on the condition *A*. Given I haven't kept my promise, I should at least apologize. Given a murderer kills somebody, he should at least do so gently. Hansson (1969) has proposed that these conditional norms are governed by a preference order. One of the best worlds, i.e., full compliance with the norms ought to be case; this is the normative ideal, and the other worlds are forbidden. Given though that this ideal is unattainable, one of the second best possibilities should realize. And so on. Lewis (1973, sect. 5.1) has taken over this idea, and it is still one of the main paradigms of conditional deontic logic. As just expressed, this preference order is rather an order of prohibition. Given something prohibited is the case, the least prohibited that is left open thereby ought to be the case. This makes clear that this prohibition order works very much like the entrenchment order in belief revision theory.

This is how conditional deontic logic is usually explained. However, it is *not* exactly how it is to be understood in this section. I sense a pervasive confusion in all of those conditional normative assertions. They hide the ambiguity whether the condition *A* is *given as a fact* or *given as a norm*; prima facie, these are two different things. The previous paragraph suggested that conditions are given as a fact. The unattainability of, say, the best worlds is taken to have factual reasons. Given that, against the norms, I didn't keep the promise, I should at least apologize. However, as long as we are considering only a purely normative conception, conditions must be given not as facts, but in a normative way. Given that, against our norms, I would be *obliged* to break my promise, I should at least apologize. A different, but perhaps more natural way might be this: Given I would be *permitted* to break my promise, what should I do then? Well, I should still avoid breaking the promise without apologies. In both formulations, the condition is stated as a normative supposition. This is the only case we can consider in the present context. The other case where the condition is given as a fact presupposes treating the interaction of norms and facts, and we turn to this only in Sect. 5. We will then discuss some prominent examples that crucially embody this ambiguity in my view and make it more vivid at the same time.

Let me explain this point a bit more carefully. One should think that the ambiguity has been familiar since we think about contrary-to-duty obligations. We will

[12] Carmo and Jones (2002), e.g., try to avoid deductive closure, although they accept substitutivity of logical equivalents. Concerning moral dilemmas, see the excellent overview of Goble (2013), who discusses in Sects. 5 and 6 how revisions of standard deontic logic might encompass normative conflicts. In Sect. 4, however, he explains how normative conflicts may be treated without such revisions—a line of thought which I definitely prefer and will take up again in Sect. 6.

more fully discuss this in Sect. 6, but let us just look at how Chisholm (1963) first raised the problem. He introduced the following set of sentences (which I state here also for future reference):

(14) Jones ought to go to the aid of his neighbors.
(15) If Jones goes to the aid of his neighbors, then he ought to tell them that he is coming.
(16) If Jones does not go to the aid of his neighbors, then he ought not to tell them that he is coming.
(17) Jones does not go to the aid of his neighbors.

(14) and (15) seem to imply that Jones should tell his neighbors that he is coming, whereas (16) and (17) seem to imply that he should not do so. Thus, the Chisholm set seems to generate a deontic contradiction. Intuitively, though, there is no such contradiction. The inference from (14) and (15) applies deontic detachment, as it was called, and the inference from (16) and (17) applies factual detachment. So, not both kinds of detachment can be used without restriction. The debate between the factual and the deontic detachers is nicely displayed in Loewer and Belzer (1983) and Nute (1997). We will return to it.

The present point is only: Is my above ambiguity of conditions given as a fact and given as a norm not precisely this familiar ambiguity between the two kinds of detachment? Superficially yes. But perhaps no. My suspicion is raised by the fact that the antecedents of both, (15) and (16), are stated descriptively; they don’t say “if Jones ought to go” or “if Jones were permitted not to go”, as I wanted to have it above. Maybe it is intended this way. But this would make the ordinary language conditional even more ambiguous.

My suspicion is strengthened by the following observation: A plausible diagnosis of the Chisholm set is that there are two kinds of ought involved, a primary or ideal ought and a secondary ought generated by contrary-to-duty obligations.[13] I, too, will argue below that we find two kinds of ought here. This diagnosis is, however, associated with the idea that the secondary ought is already compromising, as it were, with the bad facts violating ideal obligations, whereas the primary ought only captures the ideal normative point of view.[14]

That’s not my idea at all. That ideal normative point of view is not the purely normative point of view I have in mind. Rather, there are also two kinds of sub-ideals and two kinds of fallback positions related to them. Adverse reality or the mere supposition of it refers us to the one kind (which we will analyze in the next sections). However, there are also normative suppositions. We are able to imagine that the ideal is different from what it actually is. This carries us to the second kind of sub-ideal and its fallback positions. Surely, this second kind is also part of a purely normative perspective. Hence, it is not merely about a normative ideal. It is this

[13] See Jones and Pörn (1985) for an early implementation of this diagnosis, which can be found in several variants, e.g., in Carmo and Jones (2002, pp. 299ff.).

[14] See the references in the previous footnote. We find this idea also in Schurz (1997, p. 41, footnote 31).

ambiguity which I would like to uncover and which I do not find well-articulated in discussions of the Chisholm set (14)–(17).

So, we consider the purely normative question: Given something prohibited ought to be the case (for whatever or no reason), what else ought to be the case? By definition, the least prohibited that is left open thereby. Equivalently, given that the present ideal is forbidden, what is ideal then? Well, by definition, the second best possibilities. And so on. That's how Hansson's preference or prohibition order is to be interpreted within the present purely normative perspective. It is in this sense that conditional obligations are understood in this section. Or else we might ask: Given something forbidden were permitted. What would still be forbidden? Well, by definition, all the possibilities that are worse than the previously and now admissible ones. In any case, both kinds of questions are rightly called questions about contrary-to-duty obligations, not in the sense what the obligations would be if some duty were actually violated, but in the sense what the obligations would be if duties were different from what they actually are. So, we can and do address questions of contrary-to-duty obligations within the present purely normative perspective.

The next point to observe is that there are not only conditional norms in the sense just explained. There are in fact all three kinds of hypothetical changes in our purely normative conception described by belief revision theory. There are expansions; we may consider strengthening our normative ideal. There are hypothetical revisions; that was what I was asking for above. And we have also been considering hypothetical contractions, which may be even more natural than hypothetical revisions: What if something forbidden were permitted? (What if I were permitted to break my promise? Then I should at least add apologies.) Indeed, I think that such contractions are more salient in the normative case than in the epistemic case.

Of course, there are not only hypothetical changes. There are also actual changes. This is very clear in the case of our legal codes. We expand them, we partially retract them, and we revise them. If it comes to subjective normative conceptions, such changes are less explicit, but they exist nevertheless. How they come about may be obscure: by thinking and insight, by information or experience, by a mere change of mind, or whatever. But their effects are clear. My wife utters a wish, and that turns normative for me. I was liberal concerning the internet. But now I think that the power and the misuse in the internet should be strictly subject to enforceable law. Both cases are expansions of my normative conception. When young I thought that everybody should excommunicate, because there is no God. While I am still convinced that there is no God, I have become more liberal and think everybody may belong to whichever non-dictatorial religious community he or she likes. That's a case of norm contraction. And so on.

These changes are governed by a prohibition order as understood now, again in analogy to the entrenchment order in belief revision theory. Even if that order is largely implicit, it operates in the background and controls the consistency of all such changes. This has indeed been the original intention of Alchourrón and Makinson (1981).

I guess that jurists feel uncomfortable with this description. Our subjective normative conceptions are indeterminate and unfathomable, and it is hard speculating which dispositions tacitly guide their changes. By contrast, our explicit legal codes are shining paradigms for normative systems. But their changes do not seem to work as just sketched. Rather, legal codes are conceived in analogy of so-called belief *bases* instead of belief sets. A belief base is just a collection of sentences, the basic beliefs, from which we obtain a belief set by (consolidation, if necessary and) deductive closure, i.e., by adding all derived beliefs. Similarly, our statute books are the norm base of our legal code or norm set. Legal change, then, seems to be a matter of adding and deleting sentences in our statute books.

Belief revision theory has examined the three kinds of changes also with respect to belief bases. It is instructive to see that this simple model of deleting and adding sentences to the belief base does not generally work. In general, one has to appeal to an entrenchment order of beliefs as well, and the rules of change take even a more complicated form than in the case of belief sets (see, e.g., Hansson 1999). By analogy, one may expect that the change of legal codes cannot be so simply described, either. Rather, one must again refer to something like a prohibition order, at least implicitly.

Indeed, when one looks more closely, one finds that our legal codes are full of vague overarching principles and priority rules. There is the general, but vague principle of proportionality governing the relation of rules and provisions. Whenever laws get into conflict in a given case, judges try to appeal to, or create, rules telling which law takes precedence. Whatever, e.g., the legal provisions for the infamous cum-ex transactions (where banks helped investors—legally, as they claim—to get multiply refunded a tax they paid only once), it remains a superior rule that no tax paid may be refunded twice or more times. Often, one wants to remove undesirable, but only implicit implications of laws. Hence, one explicitly states an exception, but this goes along with assumptions over which laws the exception takes precedence. The exception is usually not without exception in turn. And so on. Clearly, no legal code gets along without priority rules and similar provisions.

Of course, in practice these rules are fragmented and far from being integrated into one comprehensive and coherent prohibition order. But assuming such an order is a legitimate idealization. That's the ideal jurists need to strive for in order to make their codes coherent. And then the theoretician may and must spell out in the abstract how the ideal works.

Now, the next step in my argument is predictable. There are not only single-step, but also iterated expansions, revisions, and contractions of norm sets and legal codes, and the prohibition orders may change as well. If we accept this, then we should take the very same move as in the epistemic case. A normative conception in fact consists in a *purely normative ranking function* ν over the set W of possibilities to be purely normatively assessed, i.e., ν conforms to Definition 1. I will henceforth call ν a *pure norm function*, for short.

The interpretation, however, is different. $\nu(A)=n$ now says that, from the purely normative point of view, A is *prohibited* to degree n according to ν. So, $\nu(A)>0$ says that A is *prohibited* at all, $\nu(A)=0$ says that A is *permitted*, and hence $\nu(\bar{A})>0$ says that A is *obligatory* or *ought to be the case*. In other words, those worlds w for which $\nu(\{w\})=0$ are the best of all possible worlds or the ideal ones, and all the other worlds are less and lesser ideal, as measured by ν. In the epistemic case it was a bit convoluted to speak in negative terms of disbelief and its degrees. However, in the normative case, the negative way sounds quite natural, and more natural than the positive way.

Above, I said that jurists probably feel uncomfortable with talking of prohibition orders and the like. I tried to dispel those doubts with reference to legal practice, which is full of such ordering devices. But now the discomfort will reappear even more strongly. What are *degrees* of prohibition supposed to be? We definitely find them nowhere in legal practice. They seem entirely fictitious.

This is why I took pains in the previous section to at least informally explain the measurement of ranks. The theorem was that all possible twofold non-vacuous contractions already fix the numeric values of ranks on a ratio scale. This theorem applies to the normative case as well. Twofold normative contractions are not an abstract phantasm, but just as ordinary as single-step contractions; you only have to answer twice the question: what if A (or B) were permitted? That's easier than in the epistemic case. So, if we are prepared to say how our norm set or our legal code looks like after all kinds of hypothetical twofold contractions (even of norms we would ordinarily never envisage to contract), then we have thereby specified our degrees of prohibition (on a ratio scale), provided those contractions satisfy the standard AGM axioms for single-step contractions, the Darwiche-Pearl postulates, and the axioms of restricted commutativity and path independence mentioned above. Those degrees are implicit in our contraction behavior. This is certainly an argument far away from legal practice. However, in the abstract I see no reason at all why the argument should not apply to our normative conceptions just as well as to our epistemic states.[15]

If we accept this normative interpretation of ranking theory, this opens up a whole cosmos of new investigations. In Spohn (2012), ranking theory is developed in detail as a tool in epistemology and philosophy of science. All the notions explored there now acquire also a normative interpretation. The notion of (positive or negative) epistemic relevance or dependence (= (dis-)confirmation) lies at the heart of many epistemological applications. Likewise, we can now explicate a notion of normative relevance with similarly wide applications. My ranking-theoretic explication of empirical laws and their nomicity is novel and subversive (see Spohn 2012, ch. 12), but it seems to me to apply to normative laws all the more. For instance, this explication immediately endorses our attitude that, if a normative law is violated once, it must still not be violated a second time; I know of no account of natural law the normative analogue of which would endorse this attitude. In the empirical sciences

[15] At this point I may add a remark on my own behalf. In formal epistemology, Bayesianism is the dominating conception. Bayesians think that other conceptions of epistemic states must be reducible to the basic conception of subjective probabilities. I have always argued that ranking functions are not so reducible; prima facie, at least, we should accept ranking theory as an independent epistemological theory. I think the above ranking-theoretic construal of normative conceptions strengthens my case. In the case of epistemology, Bayesianism is quite plausible, no doubt. However, its normative counterpart has no plausibility at all. Probabilities should then be interpreted as degrees of obligation or prohibition or value. But how? I don't see how a normative reinterpretation of Bayesianism could make sense. For an opposing view see, however, Merin (1996, ch. 4), who gives Aristotle's endoxastic conception of probability a normative twist by interpreting endoxastic probability as 'approvability'. Bayesians must judge whether they welcome this support.

we find the widespread phenomenon of so-called ceteris paribus laws, which turned out to be most puzzling for philosophers of science (and are so also for deontic logicians; cf. Goble 2013, sect. 4.3). One account of them is in terms of ranking theory (see Spohn 2014). Certainly, we find that phenomenon in legal practice just as well; many laws are implicitly qualified by a ceteris paribus clause. For instance, the principle of utmost good faith is a general clause to that effect. So, maybe this also finds a plausible ranking-theoretic treatment. Indeed, the notion of an exception is central to empirical as well as to normative laws, and it may be fruitfully analyzed in ranking-theoretic terms. All of this is only to suggest that the normative interpretation of ranking theory is full of fascinating perspectives. Alas, I cannot pursue any of them here.

Let me only explain, in perfect analogy to the end of the previous section, which kind of defeasible logic we can derive from pure norm functions. There is a problem, though, that did not emerge in the descriptive context. Syntactically, we can combine the normative operator O ($O\varphi =$ "φ is obligatory or ought to be the case") with a conditional $\rhd$ in at least three ways: $O(\varphi \rhd \psi)$, $\varphi \rhd O\psi$, and $O\varphi \rhd O\psi$. The formulae seem to say three different things, and they seem to be accompanied by three different ways of conditionality. However, they are not easily sorted out intuitively. We will turn to this issue in the next sections.

For the moment, however, we should focus only on the third type of formula. For, presently the formulae are to be interpreted through the purely normative conception ν of some subject or instance, which does not yet take any facts into account. That is, when we express conditional obligations, the conditions are of a normative and not of a descriptive kind. Such normative conditions seem best expressed by the third form $O(\varphi) \rhd O(\psi)$, while descriptive conditions, which we will study in the next sections, seem well expressed by the second form $\varphi \rhd O(\psi)$. Moreover, I suggest to entirely stay away from the first form $O(\varphi \rhd \psi)$, since it requires identifying $\varphi \rhd \psi$ with a proposition that can in turn be the object of obligations or pure norm functions. That's a dangerous step, making no immediate intuitive sense and raising the threat of trivialization theorems well known in conditional logic (cf. Lewis (1976) and Gärdenfors (1988, ch. 7)).

Is $\rhd$ here the same conditional as the one introduced in Sect. 3? One might say no, because it is applied to different antecedents and consequents. Or one might say yes; it is simply extended to new antecedents and consequents. I don't know how to decide the question. I will simply use the same symbol $\rhd$ in order not to burden my notation with too many distinctions. Still observe that in the present context $\rhd$ will have different syntactic restrictions and thus has at least a different use than in the previous section.

We are obviously about to interpret the construction $O\varphi \rhd O\psi$ as a revision of norms. So let's briefly develop its logic. Above I claimed that our (conditional) normative talk is ambiguous between a pure and a fact-regarding version. So far, this seemed to concern only the interpretation of the conditions as descriptive or normative. And we focused on the latter interpretation. However, the ambiguity actually concerns "ought" itself. This will be argued in the next section. In anticipation, though, this should be distinguished in our notation. So $\hat{O}$ is to represent the *purely normative* ought we deal with in this section.

Then the above explanations suggest the following syntax: We keep the basic finite language L_0 with its finite set V of valuations that may be identified with the set W of possible worlds. And we expand it to a purely normative language L_{1p} in the following way: If φ and ψ are sentences of L_0, then $\hat{O}\varphi$ and $\hat{O}\varphi \rhd \hat{O}\psi$ (= "if φ ought to be the case, then ψ ought to be the case") are sentences of L_{1p}. Moreover, if φ and ψ are sentences of L_0 or L_{1p}, propositional combinations of φ and ψ are sentences of L_{1p}, too. So, note the syntactic restrictions of $\rhd$ within L_{1p}; antecedents and consequents of $\rhd$ can only be $\hat{O}$-sentences. Moreover, L_{1p} does not contain iterations of $\hat{O}$ and $\rhd$; they could not be explained in the semantics specified below. Concerning $\rhd$ iterations would be problematic, anyway, as mentioned in the previous section. Concerning $\hat{O}$ I admit that iterations of monadic modal operators are quite common. I am not sure how meaningful they are. But we need not discuss this. In the present context, it's certainly no loss to omit them from the language L_{1p}.

Now, for any pure norm function ν for V, let $\mathcal{PN}(\nu) = \{\varphi \mid \nu(V(\neg\varphi)) > 0\}$ be the set of sentences expressing pure norms obtaining according to ν, and let $\mathcal{CPN}(\nu) = \{\langle\varphi, \psi\rangle \mid \nu(V(\neg\psi) \mid V(\varphi)) > 0\}$ be the set of sentence pairs $\langle\varphi, \psi\rangle$ corresponding to the conditional pure norms in ν; here, conditional ranks relative to ν are defined as in Definition 2. Then we may recursively define truth for all sentences in L_{1p} relative to a valuation $v \in V$ and a ranking function ν for V by specifying the following recursive base: $\langle v, \nu\rangle \vDash p$ iff $v \vDash_0 p$ for any sentence letter p of L_0, $\langle v, \nu\rangle \vDash \hat{O}\varphi$ iff $\varphi \in \mathcal{PN}(\nu)$, and $\langle v, \nu\rangle \vDash \hat{O}\varphi \rhd \hat{O}\psi$ iff $\langle\varphi, \psi\rangle \in \mathcal{CPN}(\nu)$ or $V(\varphi) = \emptyset$. Thus, $\rhd$ embodies a normative version of the Ramsey test. This recursive base may then be truth-functionally expanded to all sentences of L_{1p}.

Finally, logical truth: We may call $\varphi \in L_{1p}$ *purely normatively logically true*, $\vDash^p \varphi$, iff $\langle v, \nu\rangle \vDash \varphi$ for all valuations $v \in V$ and ranking functions ν for V. Then $\hat{O}$ obeys the standard non-nested deontic logic characterized by axioms D (consistency) and K (deductive closure), and $\rhd$ obeys a non-nested fragment of Lewis' standard conditional logic **V** without Centering and Weak Centering; i.e., the pertinent soundness and completeness theorems apply. This is the logic of contrary-to-duty conditionals within the present interpretation. That's not exciting, and not intended to be. It just seems worth stating that we preserve the orthodoxy of conditional deontic logic in this respect, despite the non-standard interpretation applied here.

Still, I would like to note that Rational Monotony (13) entails

(18) $\hat{o}\varphi \wedge (\hat{o}\varphi \rhd \hat{o}\psi) \rightarrow \hat{o}\psi$ (Cautious Monotony),

which is tantamount to deontic detachment. That is, (14) and (15) from the Chisholm set indeed imply that Jones ought to tell his neighbors that he is coming—provided one interprets "ought" in (14) and (15) in the purely normative sense.

Note again that there is no logical advantage in moving from prohibition orders to prohibition gradings (= ranking functions). The remarks at the end of Sect. 3 apply here as well. However, as said there, we could not launch our proposal for combining descriptive and normative matters without this move.

A final remark: Above I have suggested that contrary-to-duty conditionals may be more plausibly conceived as answers to the question: what if something forbidden were permitted? This question imagines a contraction of pure norms. Hence the construction follows the logic of contraction, which is easily stated: Define $\hat{P}\varphi$ as $\neg\hat{O}\neg\varphi$, saying that φ is permitted. Then we extend the use of $\rhd$ to sentences of the form $\hat{P}\varphi \rhd \hat{O}\psi$. Their semantics is given by: $\langle v, \nu\rangle \vDash \hat{P}\varphi \rhd \hat{O}\psi$ iff $\langle v, \nu\rangle \vDash \hat{O}\psi$ and $\langle v, \nu\rangle \vDash \hat{O}\varphi \rhd \hat{O}\psi$.[16] And their logic is characterized by the substitutivity of logical equivalents, by the axioms K (closure) and D (consistency) for the consequents given a fixed antecedent and by the following axioms for variable antecedents:

(19) $(\hat{P}\varphi \rhd \hat{O}\chi) \wedge (\hat{P}\psi \rhd \hat{O}\chi) \rightarrow (\hat{P}(\varphi \vee \psi) \rhd \hat{O}\chi)$,

(20) $\neg\,(\hat{P}(\varphi \vee \psi) \rhd \hat{O}\neg\varphi) \rightarrow ((\hat{P}(\varphi \vee \psi) \rhd \hat{O}\chi) \rightarrow (\hat{P}(\varphi \rhd \hat{O}\chi))$.[17]

So, this may also be called a logic of contrary-to-duty conditionals.

5 Fact-regarding normative reasoning: a new approach

After these preparations let us finally turn to the interactions of norms and facts, as, e.g., displayed in the practical syllogism (5). It may seem that we have already provided both components, descriptive conditionals in Sect. 3 and normative conditionals in Sect. 4. Can't we simply fuse the two languages L_{1d} and L_{1p} and their logics into one? Yes, we can, but this would be a mere composition without any additional insight and not providing any bridge principle. There is so far no connection whatsoever between the epistemic ranking function κ interpreting descriptive conditionals and the purely normative ranking function ν interpreting normative conditionals. Indeed, as long as we take only the purely normative perspective, we can't find a connection, it seems, just because of its purity.

What to do? Let me give a clarification right away.[18] I often used the catchy formulation of an interaction between norms and facts. What I really mean thereby within my subjectiveistic picture is the interaction of the normative conception of some subject or instance and the facts, *as they present themselves* to the subject or instance, i.e., its beliefs. Note moreover that, according to Sect. 3, descriptive conditionals do not state facts, but express the subject's conditional beliefs. So, the practical syllogism (5) does not literally state a relation between facts and norms, either. What we are really after, then, is how the epistemic and the purely normative attitudes of a subject or instance may combine.

As already indicated in Sect. 4, I think that we can make progress only when we become aware that there are at least two subtly different kinds of conditional and unconditional "ought". Look again at the umbrella example. Let's accept that I ought not to get wet (and abstract from the fact that this may only be a means for some other end in turn). Carrying an umbrella, though, is as such normatively entirely neutral;

[16] That's the so-called Harper identity translated into our present context. Cf. Gärdenfors (1988, p. 70).

[17] These are the familiar contraction postulates (K-7) and (K-(8) translated into our context. Cf. Gärdenfors (1988, p. 64).

[18] I am grateful to two reviewers demanding that clarification.

from the purely normative point of view, it is neither obligatory nor inadmissible for me to do so. Still, I have a conditional obligation: if it rains, I ought to carry an umbrella. However, this obligation is generated only by my conditional belief, that if it rains and I don't have an umbrella, I will get wet, and the norm that I ought not to get wet. And it turns into an unconditional obligation by my belief that it actually rains. So, the conditional as well as the unconditional obligation do not obtain as such; but they obtain when I take the facts into account, i.e., my beliefs—that should be understood for the rest of the paper. I will henceforth call them *fact-regarding* obligations.[19]

It seems difficult to tell apart the pure and the fact-regarding norms. The highly ambiguous conditional idiom does not assist in clarifying the senses, either. What helps, I hope, is explicitly appending such phrases as "from a purely normative point of view", "considering only how things ought to be as such", etc., versus "within a fact-regarding perspective", "taking the facts into account", etc.

What helps more is an analogy from utility and decision theory. There we have the familiar distinction between intrinsic and extrinsic utilities or values. The intrinsic utilities stand for the subjective value of things (or propositions) as such, while the extrinsic utilities measure the subjective value things have regarding other intrinsically valuable things, i.e., due to their empirical relation to those things. This distinction is formally represented in decision models, which contain an intrinsic utility function for the consequences and derive extrinsic, i.e., expected utilities for the possible actions. (Note that actions, for instance, can have both, intrinsic and extrinsic utility. Dancing is just fun and it may serve a purpose. Formally, this could be achieved by taking actions to be part of their consequences.)

I prefer to construe the intrinsic/extrinsic distinction as a model-relative one. Within each decision model there must be an intrinsic utility function to start with. And this combines then with the probability function of the model in order to determine extrinsic or expected utilities. We can leave it open, then, whether these intrinsic utilities are intrinsic in an absolute sense. Of course, one might ponder about absolute values or intrinsic utilities: pains and pleasures, or dignity, or equality? Thereby, though, we enter deep philosophical questions, which are not our present concern. The relative understanding is good enough.

Now, how exactly should we draw this analogy to utility theory? Well, the intrinsic utilities correspond to our pure norm function ν, while the expected utilities are to correspond to our fact-regarding norms or obligations, which are somehow determined by the pure norm function ν and by our beliefs or epistemic state captured by the ranking function κ. Let me use ν_κ to denote our *fact-regarding norm function* representing our norms as given by our pure norms ν and in consideration of the epistemic state κ. Recall that we do not move in contexts of agency and decision. We do not specifically consider obligations to do something; we just consider norms how things ought to be. Therefore I assume that the fact-regarding norm function applies to all propositions across the board (which may describe actions as

19 Instead of conditional and unconditional obligations (whether in the pure or the fact-regarding sense) I am tempted to speak of hypothetical and categorical imperatives. These terms should be released from their Kantian grip.

well). Taking an umbrella, e.g., is analogous to dancing and subject to both kinds of norms. In a pure perspective it is neither obligatory nor forbidden, in a fact-regarding perspective it is obligatory.

How, then, is ν_κ to be defined? I shall propose the simplest possible definition. Let me first introduce it and explain a little bit how it works. In the next section I shall discuss its explicatory potential vis à vis various kinds of obligations we find in the literature and apply it to a few examples. Only in Sect. 7 I shall offer a more rigorous justification by arguing that it must look as simple as it does.

Very roughly, the idea is that a world w may be normatively deviant as measured by $\nu(w)$ and epistemically deviant as measured by $\kappa(w)$, and that the two kinds of deviancy just add and thus result in the fact-regarding normative deviancy $\nu_\kappa(w)$. This is suggested not only by mathematical simplicity, but also by our decision-theoretic analogy. Consider Jeffrey's (1965) decision theory, where worlds get a probability and an intrinsic utility, the product of which is their expected utility. As should have been clear already from the introduction of ranking theory in Sect. 2, ranking theory may be generated from probability theory by translating the sum of probabilities into the minimum of ranks and the multiplication and the division of probabilities into the addition and the subtraction of ranks.[20] So, Jeffrey's product translates into the ranking-theoretic sum just suggested. Moreover, the expected utility Jeffrey assigns to a proposition is just the sum of the expected utilities of the worlds in that proposition (apart from a normalizing factor). Analogously, the 'expected' or fact-regarding degree of prohibition or norm violation of a proposition is the minimum of those fact-regarding normative deviancies of the worlds in that proposition. This is how 'expectation' works in ranking theory. I admit that this analogy so far sounds quite vague and unreliable. I will tighten it in Sect. 7. The analogy is made precise by:

Definition 4 The *fact-regarding norm function* ν_κ on the basis of the ranking functions ν and κ is defined as

(21) $\nu_\kappa(A) = \min_{w \in A}[\nu(w) + \kappa(w)] - \min_{w \in W}[\nu(w) + \kappa(w)]$
where $\nu(w) = \nu(\{w\})$, etc., and where the first minimum is ∞, if $A = \emptyset$.

Here, the second term is just a normalizing term entailing that $\nu_\kappa(W) = 0$. Without this term this would not be guaranteed because all admissible possibilities w with $\nu(w) = 0$ may be disbelieved due to $\kappa(w) > 0$.[21]

It's apt to say that Definition 4 specifies *fact-regarding degrees of prohibition* or *norm violation*. So, if $\nu_\kappa(A) > 0$, A is *prohibited* in the fact-regarding sense; if $\nu_\kappa(A) = 0$, A is *permitted*; and if $\nu_\kappa(\bar{A}) > 0$, A is *obligatory* in that sense. The basic idea of (21) is

[20] This formal translation works almost perfectly. However, I should emphasize that it does not help interpreting ranking theory. Its interpretation as an account of *belief* stands by its own and is orthogonal to that of probability theory.

[21] In July 2015, Franz Huber has suggested essentially the same formula to me in personal communication. His term for $\nu_\kappa(A)$ is 'anticipated regret'—'regret' because ν expresses only negative utilities, as it were, and 'anticipated' because he thinks that 'expected' is too firmly associated with probability.

given by the following abstract example: Assume that the minimum defining $\nu_\kappa(A) = 2$ is reached by just two worlds w and w' in the proposition A. Say, w reaches it because it is permissible as such, i.e., $\nu(w) = 0$ (that's good); alas, it is unexpected or epistemically deviant to some degree, i.e., $\kappa(w) = 2$. w' instead reaches it because it is not disbelieved, i.e., $\kappa(w') = 0$, but prohibited or normatively deviant to some degree, i.e., $\nu(w') = 2$. Then, in the fact-regarding way, w is no better than w' and w' no better than w; that's the crucial assumption contained in (21). Possible worlds can be deviant in an epistemic and in a normative way. However, both ways have the same effect on the overall deviancy expressed by the fact-regarding degree of prohibition or norm violation of A to. I will defend this assumption in the next section.

Is this to say that the two kinds of deviancy count the same? Why should they do so? This is the wrong way to ask. Recall that epistemic and normative ranking functions were measured on a ratio scale; they are uniquely determined only up to multiplicative constants. So, by basing our fact-regarding degrees of norm violation on the ranking functions κ and ν instead of $m\kappa$ and $n\nu$ we decide about the comparative weight of epistemic and normative deviancy. These weights are not discovered to be the same, but fixed to be the same by choosing this κ and this ν and no other ones. Still, it is inherent in the proposal that there is just this much to choose in the relation between epistemic and normative deviancy.

Let me emphasize right away that Definition 4 works only with cardinal gradings. For the purposes of Sects. 3 and 4 it may have sufficed to assume less demanding entrenchment and prohibition orders. However, orders cannot be added in a well-defined way. This was already the mathematical reason why belief revision theory has difficulties in explaining iterated belief revision.[22] And it is my present reason for applying here the cardinal resources of ranking theory on the epistemic as well as the normative side.

The mathematical behavior of ν_κ is fairly obvious. The crucial observation is that, due to the normalizing term, ν_κ is also a negative ranking function for W such that

(22) $\nu_\kappa(A) \geq \nu(A) + \kappa(A))$.

(Equality need not hold because min and plus do not commute.) Hence, the basic laws (6) and (7) apply to ν_κ as well. This entails in particular that we can also conditionalize ν_κ:

Definition 5 Given the ranking functions ν and κ, *conditional fact-regarding degrees* ν_κ $(B \mid A)$ *of norm violation* are defined for all propositions A and B with $\nu(A)$, $\kappa(A) < \infty$ by

(23) $\nu_\kappa(B \mid A) = \nu_\kappa(A \cap B) - \nu_\kappa(A)$.

Our conceptual machinery in fact allows distinguishing three kinds of conditionalization. We might base the fact-regarding degree of norm violation of B given A either on the epistemic ranks $\kappa(. \mid A)$ given A or on the purely normative ranks $\nu(. \mid A)$ given A

[22] Observe that Definition 3 of epistemic conditionalization essentially refers to the arithmetics of ranks.

(ought to be the case in the pure sense) or on the fact-regarding normative ranks $\nu_\kappa(.\mid A)$ given A (ought to be the case in the fact-regarding sense). Formally, this results in:

(24) $\nu_\kappa(B \mid_d A) = \min_{w \in A \cap B}[\nu(w) + \kappa(w|A)] - \min_{w \in A}[\nu(w) + \kappa(w|A)]$,

(25) $\nu_\kappa(B \mid_p A) = \min_{w \in A \cap B}[\nu(w|A) + \kappa(w)] - \min_{w \in A}[\nu(w|A) + \kappa(w)]$, and

(26) $\nu_\kappa(B \mid_f A) = \min_{w \in A \cap B}[\nu(w|A) + \kappa(w|A)] - \min_{w \in A}[\nu(w|A) + \kappa(w|A)]$,

where $\mid_d$ denotes *descriptive* or *epistemic conditionalization*, $\mid_p$ denotes *purely normative conditionalization*, and $\mid_f$ denotes *fact-regarding normative conditionalization*. The second terms, respectively, are again normalization terms ensuring that $\nu_\kappa(A \mid_d A) = \nu_\kappa(A \mid_p A) = \nu_\kappa(A \mid_f A) = 0$, as it should be.

On the one hand, this distinction will be helpful in the analysis of examples. On the other hand, it looks confusing. So many forms of conditionalization? Therefore it may be relieving to see that the distinction makes no difference and that all three agree with (23). For it is easily proved that:

(27) $\nu_\kappa(B \mid_d A) = \nu_\kappa(B \mid_p A) = \nu_\kappa(B \mid_f A) = \nu_\kappa(B \mid A) = \nu_\kappa(A \cap B) - \nu_\kappa(A)$.

So, Definition 5 is in fact all we need. Already here we may note, however, that (23) entails:

(28) if $\nu_\kappa(\bar{A}) > 0$ and $\nu_\kappa(\bar{B} \mid A) > 0$, then $\nu_\kappa(\bar{B}) > 0$,

i.e. if A is obligatory and B is so given A, then B is unconditionally obligatory—obligation always taken in the fact-regarding sense. That's a semantic version of detachment. Observe, though, that because of (27) the detachment can be taken in any sense, as factual detachment and as purely normative and as fact-regarding deontic detachment. You may suspect that this observation will become relevant in our discussion of examples. Indeed, (27) will acquire a crucial role in Sect. 7 where I try to more rigorously justify the basic Definition 4. In any case, we must not accept (27) simply as a consequence of our definitions, but have to argue for its intuitive acceptability. This is part of the next section.

6 A comparative discussion of the new approach

Let's go beyond the vague analogy of Definition 4 to decision theory and develop a better understanding of the function ν_κ by relating it to other distinctions found in the literature and applying it to examples. Perhaps it is apt to start with the practical syllogism, accounting for which is a main motive of this paper. I discuss its logic in Sect. 8. At present let's only look at our example: "I ought not to get wet. I would get wet without an umbrella. Therefore I ought to take an umbrella." Suppose I know it's raining, otherwise its subjunctive premise does not hold and the syllogism does not apply. Let U = "I take an umbrella" and W = "I get wet". Then my pure norm function ν is given by the table:

ν	W	$\bar{W}$
U	2	0
$\bar{U}$	2	0

This represents the purely normative point of view saying that I am prohibited (to some degree) to get wet (wherever this comes from), while I may or may not carry an umbrella. Now, my epistemic state is given by κ (since I know it's raining):

κ	W	$\bar{W}$
U	3	0
$\bar{U}$	0	3

This represents that it is open whether I carry an umbrella and whether I get wet, but that having an umbrella is a necessary and sufficient condition (of some strength) for avoiding getting wet. So, my fact-regarding norm function is given by

ν_κ	W	$\bar{W}$
U	5	0
$\bar{U}$	2	3

This obviously represents the fact-regarding obligation that I ought to avoid getting wet and also ought to carry an umbrella.

Intuitively, this practical syllogism should also run through, if having an umbrella is only a necessary condition for not getting wet. In this case, $\kappa(U \cap W)$ would have to be 0 instead of the upper left 3 in the table; this amounts to not believing that the umbrella saves me from getting wet. Even then the only 0 in the table of ν_κ is in the upper right field. Hence, this representation of the practical syllogism confirms this intuition. As mentioned, I don't know of any account in the literature that comes close to this representation.

It should be illuminating to relate my account to Prakken and Sergot (1997) who give a most differentiated discussion of various notions of obligation, in particular in Sect. 4. They distinguish ideal and actual, primary and secondary, prima facie and all-things-considered obligations, contextual and contrary-to-duty, conditional and unconditional, and defeasible and non-defeasible obligations. All these distinctions are somehow to shed light on this enigmatic notion of conditional obligation; and their sheer number shows that we are dealing with delicate matters. The discussion of Prakken and Sergot makes clear that one cannot expect these twelve terms to have consistent usages. This is one reason why I chose still other terms and speak of purely normative and fact-regarding obligations. How do they, and their conditional versions, relate to all the former notions?

We already discussed ideal versus actual obligations, where the latter come into force when the ideal is somehow unattainable. This agrees with how Hansson (1969, p. 393) speaks of primary and secondary obligations. But then, as Hansson was well aware, there are also tertiary obligations, etc. That's why, to repeat, a binary distinction of ideal and actual obligations won't do; a whole order of obligations is required. However, my criticism above was a different one, namely that the ideal obligations are contained in, but do not exhaust what I call the purely normative point of view, which embraces various sub-ideal levels already in itself. This suggests construing "actual" as referring not to sub-ideal obligations in the purely normative sense, but rather to the fact-regarding obligations.

However, "actual" may also mean "unconditional" as opposed to "conditional". The discussion of Prakken and Sergot is driven by the concern that conditional obligations somehow have no 'actuality' and are no obligations at all. The usual response is that conditional obligations turn unconditional either by deontic detachment or by factual detachment. However, how this works is precisely the issue.

Therefore Prakken and Sergot (1997, pp. 238f.) discuss whether conditional obligations might be understood as prima facie duties having more 'actuality', as it were, as opposed to all-things-considered obligations.[23] A standard example, the one used by them, is that I have a prima facie duty to keep my promise that I will visit my friend this afternoon. Given, however, that my mother fell ill, I rather ought to take care of my mother and am excused from keeping the promise. Thus, the prima facie duty is not necessarily an all-things-considered obligation; it entails the latter only defeasibly. That is, nothing withstanding it is an unconditional duty. This suggests that "prima facie/all-things-considered" and "defeasible/indefeasible" are the same distinction. However, this is precisely why Prakken and Sergot think that the interpretation of conditional obligations as prima facie ones does not fit, because they see no way how conditional deontic logic could account for this defeasible inference.

Therefore, their preferred interpretation of conditional obligations is as what they call contextual obligations. They write $O[B]A$ for the obligation A in the context B. Such a contextual obligation is to be undetachable; we cannot infer OA from B and $O[B]A$. Still, it is a genuine obligation that arises in the context. For instance, it is an absolute obligation not to kill, but in the context of killing somebody one has the contextual obligation to do it gently. This is to do justice to the intuition that in this context the absolute obligation not to kill still persists and also to the intuition that the case does not give rise to contradictory obligations. The absolute and the contextual obligation can coexist. In Prakken and Sergot (1994, 1996) they develop the theory of contextual obligations independently of Lewis' (1973) conditional deontic logic. In (1997) they argue first for the need of aligning their account with Lewis' and then for the need of going beyond. We need not go into those details.

[23] This distinction was introduced already by Ross (1930). It has become common to speak of pro tanto duties instead of prima facie duties (or moral reasons, etc.).

The notion of a contextual obligation is tailored to norm-violating or contrary-to-duty situations. Therefore they sharply distinguish it from prima facie obligations. They offer very plausible examples for this distinction. Look at the following triad (Prakken and Sergot 1997, p. 228) of regulations for holiday cottages somewhere near the sea (the example is to avoid considerations of time and agency):

(29) There must be no fence.
(30) If there is a fence, there must be a white fence.
(31) If the cottage is by the sea, there may be fence.

The point of the triad is obvious. Both, (30) and (31), are conditional statements, but their meaning is different. (30) is not an exception to (29); it states a contrary-to-duty conditional. By contrast, (31) does state an exception to (29) and thus reveals the defeasible nature of (29). (30) and (31) cannot be modeled by the same pattern of conditional obligation.

They also discuss another triad of regulations for those holiday cottages (p. 247):

(32) There must be no dog.
(33) If there is a dog, there must be a sign.
(34) There must be no sign.

Again, in the presence of (32), (33) is a contrary-to-duty obligation. How to understand (34)? One way discussed by Prakken and Sergot is to understand it non-defeasibly. Then they suggest that the triad is contradictory, because (34) forbids signs under all circumstances, i.e., even if there is a dog. Their point then is that this contradiction is not represented by conditional deontic logic à la Lewis. The other and more plausible, because non-contradictory way is to understand (34) defeasibly. Then, however, (32) and (34) are no longer analogous despite their analogous formulation. (33) is contrary to the duty unconditionally specified by (32); by contrast (33) states only an exception to the defeasible (34)—again a variance not explained within the Lewis paradigm.

How does all of this square with my terms and distinctions? I agree, and I disagree. Therefore, the subsequent dialectics is a bit complicated. One issue is about equating the "prima facie/all-things-considered" and the "defeasible/indefeasible" distinction. For Prakken and Sergot all-things-considered judgments are indefeasible precisely because no unconsidered thing can turn up that could change them. Thus understood, this seems to me to be a fictitious notion. We may speculate what all-things-considered judgments in this sense may be, but we never make them, because we never are as omniscient as required thereby. For me, an all-things-considered judgment considers everything that is presently accessible to me; presently

I can consider no more.[24] In this sense, the pure norm function ν takes into account all prima facie obligations relevant for a given case and integrates them into one all-obligations-considered scheme, while the fact-regarding norm function ν_κ can be said to consider all things, facts and norms. It is also this sense in which all-things-considered intentions or decisions are understood in action theory. However, this also means that those all-things-considered judgments are defeasible as well. Whenever I learn something new, be it about facts or about norms, I change my all-things-considered judgments.

In another way, though, I also agree with equating the two distinctions. This is better explained in contrast with Goble (2013). In Sect. 4.3 he concurs with Prakken and Sergot insofar as he prefers to treat defeasible normative inference along the model of default logic and not with the help of conditional deontic logic. And I concur with him insofar as he would also count all-things-considered obligations among the defeasible ones.

However, Goble disagrees with both sides and insists on not equating the "prima facie/all-things-considered" and the "defeasible/indefeasible" distinction. Rather he presents them as two strategies towards normative conflict. The first distinction assumes two kinds of obligations. Prima facie obligations may be contradictory, as in the example above. However, all conflicts must somehow be resolved in the all-things-considered obligations, which have to be consistent.[25] By contrast, the second strategy does not make this distinction and assumes only one kind of defeasible obligation. In the example above, there simply are two defeasible obligations. There the second obligation to help the mother outweighs the first, and therefore the conflict is resolved in favor of the mother. There may also be irresolvable conflicts, where no defeasible obligation outweighs the other. In this case, logic can only tell that at least one of the obligations must be fulfilled; Goble (2013, p. 255) calls this the 'Disjunctive Response'. This situation resembles the famous Nixon diamond: Ceteris paribus, all Quakers are pacifist and all republicans are not pacifist. So, what is Nixon, who is both a Quaker and a republican? Default logic is silent on the issue and does not allow deriving that he is both, pacifist and not pacifist. So, in any case consistency is preserved.

Now, from a ranking-theoretic point of view I see no need to distinguish between "prima facie" and "ceteris paribus". My epistemic ranking function κ is such that I believe that Nixon is pacifist given that he is a Quaker and that he is not pacifist given he is a republican. It also somehow answers the question what Nixon is given he is both a Quaker and a republican. However, this answer cannot be derived from the first two conditional beliefs, but depends on their strength and how they mix; these details must be, and are, provided by my ranking function κ.[26] Similarly, my pure norm function ν or my fact-regarding norm function ν_κ—my point works for

[24] This point also reflects my above remarks about facts and beliefs. In our context, "all facts considered" can only mean "all beliefs considered".

[25] Hence, all-things-considered obligations may conform to standard deontic logic. Normative ranking functions do so as well, and hence they must be understood in this way, too.

[26] See also my extensive ranking-theoretic account of ceteris paribus conditions in Spohn (2014).

both—are such that I ought to do x given I have promised x and that I ought to care for my mother given that she is ill. The functions also tell what I ought to do given I have promised x *and* my mother is ill. But again the answer cannot be derived from the first two conditional obligations. However, these conditional obligations have weights in those norm functions, which determine whether the conflict is resolvable (in the sense of one obligation outweighing the other) or irresolvable; in the latter case, the 'Disjunctive Response' should be derivable from ranking theory.

So far my reasons why I concur with Prakken and Sergot in equating "prima facie" and defeasible", pace Goble. Surely, my remarks on the ranking-theoretic treatment of normative conflicts are very insufficient. They would need a lot of elaboration and defense. However, since this is not my central topic, I must leave it at those remarks.

My main disagreement with Prakken and Sergot, however, concerns their distinction between defeasibility and conditionality (or contextuality) as suggested by their examples about fences and dogs. My argument should also serve as a defense of the approach presented in Sect. 5. To begin with, a general point irritating me is the universal correspondence between belief revision theory, which is about conditional beliefs, and defeasible logic, which is about defeasible arguments (and, by the way, the theory of choice or revealed preference); see Rott (2001). Every axiom or rule proposed in one field has an exact duplicate in the other field; the discussions are in perfect parallel. Scholars seemed to work at the same issues, only in a different disguise. Or were they caught in collective error?

So, do I want to deny the intuitions about fences and dogs that appeared to be so convincing? No. I only want to account for them in a different way. Let's look at (29)–(31). There is no doubt that (30) rules a case contrary to the duty stated in (29). So, one way to understand (29) and (30) is from the purely normative point of view, even though the antecedent of (30) is stated descriptively; we have learned already that the grammatical form of conditional statements may be deceptive. Another way is to interpret all three statements (29)–(31) as being about fact-regarding obligations. Then the antecedent of (30) assumes a normatively deviant situation, just as in the purely normative interpretation, and its consequent states a fact-regarding obligation. Similarly, the antecedent of (31) describes an epistemically deviant situation. The case is perfectly analogous to Tweety. We learn that Tweety is a bird and conclude that it can fly, because the possibility that it is a penguin is epistemically far-fetched, exceptional, unexpected. Likewise, the norm (29) is for normal cottages. It is epistemically far-fetched, exceptional, unexpected that a cottage is by the sea. The point is that both, (30) and (31), move us into deviant conditions and tell then which fact-regarding obligations hold under those conditions. The one condition is contrary-to-duty, the other is not. Both are deviant in different ways as reflected in the different ways of conditionalizing fact-regarding obligations. The crucial point, though, is that the deviancies are processed in the same way and have the same effects. To emphasize, if (31) would be changed into "if the cottage ought to be by the sea, there may be a fence", then it would be a contrary-to-duty conditional as well, but the conditional permission would be the same. I see no need to assume a principled difference here. And this is why (27) seems right to me in equating the various forms of conditionalization.

The intuitions concerning the dogs can be similarly understood. If (34) is to be understood indefeasibly, this should be read as forbidding signs under all circumstances. Under the condition that there is a dog, (33) and (34) then state contradictory conditional obligations, in both, the purely normative and the fact-regarding sense. If, however, (34) is understood defeasibly, the two roles of (33) can be well explained, if we apply the fact-regarding interpretation to the triad. The condition of (33) that there is a dog is normatively deviant vis à vis (32); that's why it is a contrary-to-duty conditional. However, the condition is only epistemically deviant vis à vis (34). This is why (34) is defeasible. (Of course, (32) is presumably also defeasible, though not as far as the example goes. However, we might expand the story, e.g., by seeing-eyes dogs.) Again, as implied by (27), the different kinds of deviancy do not make a difference.

So, to return to the initial concern of Prakken and Sergot: what is the actual normative force of conditional obligations? The analogous question applies to conditional beliefs. My present conditional beliefs given A turn into unconditional ones, when A becomes my total evidence, when I actually learn A and nothing but A. Similarly, my conditional pure norms given A turn into unconditional ones, when I actually come to accept that A ought to be the case (and no more). Finally, my conditional fact-regarding norms given A turn into unconditional ones, when my empirical or normative total evidence is A, when I learn nothing but A. And it does not make a difference whether I accept A as a norm or learn A as a fact; the resulting unconditional fact-regarding obligations are the same.

This is perhaps a good place to address the objection that facts and norms do not seem so easily separable and then combinable as assumed by the fact-regarding interpretation. Aren't norms not usually infected by factual considerations? In the umbrella example, not getting wet is certainly not a pure norm. It is somehow the result of more basic norms and beliefs relating them to wetness. I agree. That's why I emphasized the model-relativity of the intrinsic/extrinsic distinction. The distinction between pure and fact-regarding norms is model-relative in the same way. So, in that example, not getting wet was simply assumed to be a pure norm, relatively speaking. And relative to this, carrying an umbrella is still a fact-regarding norm.[27]

[27] There may even be a direct relation between facts and norms blurring their distinction. (I am grateful to a reviewer for raising this issue.) We expect people to conform to the norms. That is, we expect that only permissible things happen; and the more strictly something is forbidden, the more firmly we disbelieve that it takes place. In the dogs example, e.g., the condition that there is a dog seems epistemically deviant only because it is normatively deviant and hence unexpected. The inference may be even reversed. When something occurs, it was presumably admissible, and when something doesn't occur, maybe that is so because it is inadmissible. Now, the latter is dangerously close to a rejection of Hume's Thesis and to Leibniz' claim that the actual world is the best of all worlds (according to God's proper standards). I admit that both inferences have some justification. However, they clearly are defeasible inferences, and the second is much weaker than the first. And they are relatively plausible only with respect to human actions, and the less plausible, the farther we move away from human influence. If it ought to rain, there is no way to infer that it will rain; the reverse would be even worse. There is a general simple explanation why the inference is sometimes plausible. We simply expect our fellows to respect the norms (at least if the norms are publicly known and not my idiosyncratic ones), and reversely, if we observe a behavioral regularity within our community without natural explanation, then it is presumably a normative rule. However, this does not reflect a general logical point, as we pursue it here. It is rather based on a special empirical claim, namely the human receptivity for norms. Therefore, the issue is besides our present interests, even though it would deserve getting developed.

In order to close the circle of our discussion, let us return to the Chisholm set (14)–(17). Our distinctions are made for coping with it. It is instructive to spell this out numerically, Let G = "Jones goes to the aid of his neighbors", and T = "Jones tells his neighbors that he is coming". The purely normative point of view is represented by the ranking function ν:

ν	T	$\bar{T}$
G	0	1
$\bar{G}$	3	2

This endorses (14), since $\nu(\bar{G}) = 2$; it endorses (15), since $\nu(\bar{T} \mid G) = 1$; and thus it verifies the conclusion $\nu(\bar{T}) = 1$ that T ought to be the case. It also represents, though this is secondary, that $\bar{T}$ is less of a sin than $\bar{G}$, that T is forbidden given $\bar{G}$, and that the worst for Jones is not to go, while pretending to do so. All this is intuitively plausible.

We may further assume that the initial epistemic point of view is completely neutral; all contingent propositions involved get rank 0. However, (17) tells us that in fact $\bar{G}$. So, the epistemic ranking function κ should be such that $\kappa(G) = n > 0$, and it should be indifferent vis à vis T and $\bar{T}$. This endorses (17). Thereby, the following fact-regarding norm function ν_κ results:

ν_κ	T	$\bar{T}$
G	$0 + n - 2$	$1 + n - 2$
$\bar{G}$	$3 - 2$	$2 - 2$

where –2 is added in each field for normalization. So, in any case we have $\nu(T \mid \bar{G}) = \nu_\kappa(T \mid \bar{G}) = 1$; this endorses (16) in any reading. If κ results from the completely neutral initial state by strict conditionalization with respect to $\bar{G}$, then $n = \infty$. But even if only $n > 2$, we have $\nu_\kappa(T) > 0$. And so ν_κ endorses the conclusion of (17) and (16), that Jones ought not to tell that he is coming, in the fact-regarding reading. In this way the paradox vanishes and intuition is restored. The two oughts in the apparently contradictory conclusions have two different readings, a pure and a fact-regarding one, and arise from different conditionalizations or detachments, as explicated.

7 How to justify the new approach

So far, I tried to make Definitions 4 and 5 plausible and to emphasize their explicatory power vis à vis contested examples. However, are they amenable to stricter kinds of justification? Yes, I can think of three very different ways. The first way is this: On the basis of (23) we may also develop a conditionalization rule for fact-regarding degrees of norm violation, and then we might run the same argument via iterated contractions that we used for measuring epistemic or purely normative ranks—provided, of course, those laws of iterated contraction are convincing also in the case of fact-regarding obligations. However, the idea is hard to realize because contractions relating to purely normative considerations and contractions relating to their fact-regarding counterparts are easily confused; the responses of the subjects might not be reliable. Moreover, it does not suffice to merely confirm the contractions axioms for fact-regarding contractions. We also need to establish some formal relation between the three kinds of contraction referring to the three kinds of ranking function, including some relation between the three units of their ratio scales which would determine the relative weights of epistemic and normative deviancy, as I expressed it in Sect. 5. Below I will argue that the logic of the fact-regarding conditional ought in relation to the other conditionals is a complicated matter. I suspect that the formal relation between the three kinds of contraction is just as difficult. Therefore I do not further pursue this line of thought.

A second, quite stringent idea is to render precise the decision-theoretic analogy with which I motivated Definition 4. In fact, this idea has already been carried out by Giang and Shenoy (2000). They establish a ranking-theoretic decision theory (see also Spohn 2012, sect. 10.4, and Spohn 2017) in strict formal analogy to von Neumann-Morgenstern utility theory (as paradigmatically presented in Luce and Raiffa 1957, ch. 2). That is, just as in the probabilistic case they proceed from a preference relation among simple and compound ‘ranking lotteries’ or prospects, assume the same kind of axioms for that preference relation, and end up with a utility function, such that the preference relation represents the ranking-theoretic ‘expected utilities’ of those prospects. The Giang-Shenoy utility function can be positive and negative. However, if one restricts it to negative values (‘degrees of prohibition’), then their expectation is precisely given by formula (21). These remarks strongly indicate that there is not just an analogy, but a strict justificatory argument on offer, which moreover has a surprising potential of relating qualitative decision theory and deontic logic or, more pompously, microeconomics and law. Still, I abstain from further elaborating this idea, since it is available in principle.

There is a third way of justification which is immediately related to our discussions in the previous section. In (24)–(26) we noticed that the fact-regarding norm function ν_κ can be conditionalized in three different ways, and in (27) we observed that the three forms of conditionalization come to the same. On the one hand, this follows from Definitions 4 and 5; on the other hand, I have argued in the previous section that this is also intuitively plausible. Interestingly we can reverse the argument and derive a justification of Definition 4 from the Eq. (27). Here is the reversal: Suppose we are already given some epistemic ranking function κ and some pure norm function ν, and let us grant that we are justified in assuming that epistemic states and purely normative conceptions already suffice to determine the fact-regarding norms, i.e., that the fact-regarding norm function ν_κ is just some function of ν and κ. So we assume that $\nu_\kappa(w) = F(\nu(w), \kappa(w))$ for some binary function F within $\mathbf{N} \cup \{\infty\}$ and all $w \in W$. That’s all what my argument assumes. Now we might define (24)–(26) more generally in terms of F replacing $+$. Denote the terms thus defined, respectively, by $\nu_\kappa(B \mid_d A)_F$, $\nu_\kappa(B \mid_p A)_F$, and $\nu_\kappa(B \mid_f A)_F$. Then the following theorem holds:

(35) $\nu_\kappa(B \mid_d A)_F = \nu_\kappa(B \mid_p A)_F = \nu_\kappa(B \mid_f A)_F$ for all A, B, ν, and κ if and only if $F(x, y) = n(x+y)$ for some $n>0$ and all $x, y \in \mathbf{N} \cup \{\infty\}$.

This theorem says, in other words, that, given we want to deny any difference between the three kinds of conditionalization (24)–(26) and to stick to the Eq. (27), we have no choice but assuming Definition 4 (where $n=1$ without loss of generality).

Here is a proof of (35): Only the "only if" direction needs proof. For $w \in A \cap B$ write the generalized version $(24)_F$ of (24) as $\nu_\kappa(w \mid_d A)_F = F(\nu(w), \kappa(w)-b)-(c-b)$, where $b=\kappa(A)$ and $c=\nu_\kappa(A)$. Similarly, $(25)_F$ turns into $\nu_\kappa(w \mid_p A)_F = F(\nu(w)-a, \kappa(w))-(c-a)$, where $a=\nu(A)$, and $(26)_F$ turns into $\nu_\kappa(w \mid_f A)_F = F(\nu(w)-a, \kappa(w)-b)-(c-a-b)$. Hence, $(24)_F=(26)_F$ translates into $F(x, y-b)-(c-b)=F(x-a, y-b)-(c-a-b)$, i.e., into $F(x, y-b)=F(x-a, y-b)+a$, for all $x, y \in \mathbf{N} \cup \{\infty\}$. This entails that F is linear in x. Similarly, $(25)_F=(26)_F$ entails that F is linear in y. Hence, $F(x, y)=mx+ny$ for some $m, n \in \mathbf{N}$. Finally, $(24)_F=(25)_F$ translates into $F(x, y-b)-(c-b)=F(x-a, y)-(c-a)$ for all $x, y \in \mathbf{N} \cup \{\infty\}$. Hence, F must be symmetric in x and y, i.e., $m=n$. QED.

This proof establishes Definitions 4 and 5 as firmly as its premise (the left side of (35) = the generalized (27)) is intuitively justified. This is why I took pains to provide this justification in Sect. 6.

8 Incomplete notes on the logic of the new approach

Having thus presented the third justificatory argument for Definition 4, we may finally proceed to stating the semantics and the logics of conditional fact-regarding obligations. This is essentially a repetition of the end of Sect. 3. Concerning the syntax, we stick to our simple framework. We first keep the basic finite language L_0 with its finite set V of valuations (= the set W of possible worlds). And we expand it to a fact-regarding normative language L_{1f} in the following way: Let us denote the fact-regarding ought by $\tilde{O}$. So, if φ is a sentence of L_0, then $\tilde{O}\varphi$ (= "in regard of the facts, φ ought to be the case") is a sentence of L_{1f}; and if ψ is also a sentence of L_0 and χ is either φ or $\hat{O}\varphi$ or $\tilde{O}\varphi$, then $\chi \rhd \tilde{O}\psi$ (= "if χ, then, in regard of the facts, ψ ought to be the case") is a sentence of L_{1f}. This reflects the fact that we have three kinds of conditionals according to the three kinds (24)–(26) of conditionalizations. Moreover, if φ and ψ are sentences of L_0 or L_{1f}, propositional combinations of φ and ψ are sentences of L_{1f}, too. Again, L_{1f} does not contain nestings of $\tilde{O}$ and $\rhd$ (see my pertinent remarks in Sect. 4), and $\rhd$ has a restricted syntax insofar as it takes only $\tilde{O}$-sentences as consequents and only purely descriptive sentences or $\hat{O}$- or $\tilde{O}$-sentences as antecedents. Again, it is futile to decide whether $\rhd$ represents one conditional with a mixed syntax or in fact three different kinds of conditional.

Now, for any fact-regarding norm function ν_κ for V based on the epistemic ranking function κ and the pure norm function ν, let $\mathcal{FN}(\nu_\kappa)=\{\varphi \mid \nu_\kappa(V(\neg\varphi))>0\}$ be the set of sentences expressing fact-regarding obligations obtaining according to ν_κ, and let $\mathcal{CFN}(\nu_\kappa)=\{\langle\varphi, \psi\rangle \mid \nu_\kappa(V(\neg\psi) \mid V(\varphi))>0\}$ be the set of sentence pairs $\langle\varphi, \psi\rangle$ corresponding to the conditional fact-regarding norms in ν_κ as defined in Definition 5. Then we may recursively define truth for all sentences in L_{1f} relative to a valuation $v \in V$, an epistemic ranking function κ and a pure norm function ν for V by specifying the following recursive base: $\langle v, \kappa, \nu\rangle \vDash p$ iff $v \vDash_0 p$ for any sentence letter p of $\boldsymbol{L}_0$, $\langle v, \kappa, \nu\rangle \vDash \tilde{O}\varphi$ iff $\varphi \in \mathcal{FN}(\nu_\kappa)$, and $\langle v, \kappa, \nu\rangle \vDash \varphi \rhd \tilde{O}\psi$, $\langle v, \kappa, \nu\rangle \vDash \hat{O}\varphi \rhd \tilde{O}\psi$, and $\langle v, \kappa, \nu\rangle \vDash \tilde{O}\varphi \rhd \tilde{O}\psi$ iff $\langle\varphi, \psi\rangle \in \mathcal{CFN}(\nu_\kappa)$. This means that $\rhd$ embodies a fact-regarding normative version of the Ramsey test. This recursive base may then be truth-functionally expanded to all sentences of L_{1f}.

Finally, logical truth: We may call $\varphi \in L_{1f}$ *normatively logically true* (in the fact-regarding sense), $\vDash^f \varphi$, iff $\langle v, \kappa, \nu\rangle \vDash \varphi$ for all valuations $v \in V$ and all ranking functions κ and ν for V. Then the same observations hold as in the purely normative case: $\tilde{O}$ follows the standard non-nested deontic logic. For $\rhd$ we have the axioms: $\vDash^f (\tilde{O}\varphi \rhd \tilde{O}\psi) \leftrightarrow (\varphi \rhd \tilde{O}\psi)$ and $\vDash (\tilde{O}\varphi \rhd \tilde{O}\psi) \leftrightarrow (\hat{O}\varphi \rhd \tilde{O}\psi)$, due to the equivalence (27). (Granted, the syntactic triplication of antecedents is therefore superfluous.) Otherwise, $\rhd$ follows a non-nested fragment of Lewis' standard conditional logic **V** without Centering and Weak Centering; i.e., the pertinent soundness and completeness theorems apply. All this holds because ν_κ is a ranking function in turn. As such, the fact-regarding ought behaves in the same way as the purely normative ought.

Now we are ready for the final step of the paper. In the beginning of Sect. 5 I envisaged fusing the languages L_{1d} and L_{1p}, but dismissed the proposal, because the logics of the two languages are entirely unconnected. Now, however, we can combine all three languages L_{1d}, L_{1p}, and L_{1f} to form a language L_1 which contains the two ought-operators and all syntactic constructions of the conditional we have considered so far, everything without nesting. Then we should be able to observe a lot of is-ought connections, due to the definition of the fact-regarding norm function ν_κ.

We can indeed. Alas, I have no idea how to completely axiomatize those connections. I better don't even start. Let me only indicate how formidable a task this is (also in order to excuse myself). For this purpose let us return to the practical syllogism in the intended version (5). One might conjecture that we can now represent it by the following inference:

(36) $\varphi \rhd \psi \vDash^f \tilde{O}\varphi \rhd \tilde{O}\psi.$

If the end φ is obligatory and the means ψ is a necessary condition of the end φ,[28] then the means ψ is obligatory in a fact-regarding perspective, too.

However, (36) is not a valid inference. If $V(\varphi)=A$ and $V(\psi)=B$, then the conclusion of (36) says, in semantic terms, that:

[28] Note that the fact that ψ is a necessary condition of φ could as well be formalized by $\neg\psi \rhd^{d} \neg\varphi$. As is well known, the two formalizations are not equivalent. Only our choice yields a promising inference.

(37) $\min_{w \in A \cap B} [\nu(w) + \kappa(w)] - \min_{w \in A} [\nu(w) + \kappa(w)] > 0$,

while the premise of (36) says

(38) $\min_{w \in A \cap B} \kappa(w) - \min_{w \in A} \kappa(w) > 0$.

Clearly, (38) does not entail (37). The superposition of κ and ν in (37) is generally unpredictable. So, in order to get (36), i.e., to move from (38) to (37), we need additional assumptions about the shape of ν.

Let me present only one fairly obvious sufficient condition for (36). Where might the minimum of $\nu(w)+\kappa(w)$ lie for $w \in A \cap B$? Well, a simple case is that it overlaps with the minimum of the $\kappa(w)$ within $A \cap B$. This is guaranteed when (a) the minimum of the $\nu(w)$ within $A \cap B$ overlaps with the minimum of the $\kappa(w)$ within $A \cap B$. Next, (37) requires that the minimum of $\nu(w)+\kappa(w)$ for $w \in A \cap \bar{B}$ is larger than the minimum of $\nu(w)+\kappa(w)$ for $w \in A \cap B$. Given (a), this is guaranteed when (b) the minimum of $\nu(w)$ for $w \in A \cap \bar{B}$ is not smaller than the minimum of $\nu(w)$ for $w \in A \cap B$, because (38) says already that the minimum of the $\kappa(w)$ within $A \cap \bar{B}$ is larger than the minimum of the $\kappa(w)$ within $A \cap B$. Now, given (b), we can express (a) as (c): the minimum of the $\nu(w)$ within the whole of A overlaps with the minimum of the $\kappa(w)$ within A. And (c) in turn implies (b). Thus we have shown that (c) is the only condition we need:

(39) Given there is a $w^* \in A$ with $\kappa(w^*) = \min_{w \in A} \kappa(w)$ and $\nu(w^*) = \min_{w \in A} \nu(w)$, (38) entails (37).

We can even express the additional condition of (39) in our language L_1. It simply translates into the condition that given A nothing forbidden according to ν obtains according to κ. Thereby, we have arrived at an inference that is deductively valid in $\boldsymbol{L}_1$:

(40) $\varphi \rhd \psi$, and for all χ in $\boldsymbol{L}_0$ $(\varphi \rhd \chi) \rightarrow \neg (\hat{O}\varphi \rhd \hat{O}\neg\chi) \vDash^f \tilde{O}\varphi \rhd \tilde{O}\psi$.

(Due to the finiteness of L_0 (40) has only finitely many premises.)

If you look again at the umbrella example as discussed in Sect. 6, you find the additional premise of (40) satisfied there. Hence, that example indeed instantiates a valid inference. Is (40) also intuitively plausible? I think so. (40) says, in other words, that we may defeasibly infer $\tilde{O}\varphi \rhd \tilde{O}\psi$ from $\varphi \rhd \psi$; but this inference is defeated by any χ which would be, but ought not to be the case (in the purely normative sense) given φ. This sounds correct. Indeed, it is obvious for $\chi = \psi$, i.e., if the means ψ would be inadmissible given the end φ in the purely normative sense. Interestingly, (40) points us to many more defeaters. This insight, though, is already helped by our explicit semantic model, which would also tell how those defeaters may be overturned again (although I did not pursue this question). In any case, we need that model in order to steer clear with defeasible is-ought bridge principles. So, this is how far I can advance here the topic of the practical syllogism.

A final noteworthy point about (40) is this. The derivation of (39) made clear that the additional premise of (40) is only sufficient for turning the practical syllogism (36) into a valid inference; it is not necessary. The derivation also displayed that the weighing of ν and κ may turn out favorably for the practical syllogism also under other conditions, which may, however, become quite complicated and hard to express in the language L_1. I don't see at all how to capture them completely. A fortiori, it seems to be a difficult task to completely specify the logic of L_1.

Still, it should have become clear that the proposed semantics is very plausible, that it well explicates the ambiguity between purely normative and fact-regarding oughts or obligations, and that it properly accounts for the defeasible character of normative reasoning in both interpretations. If so, this paper has fulfilled its purpose, and there is good reason to more fully study the is-ought relations implied by this semantics.

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