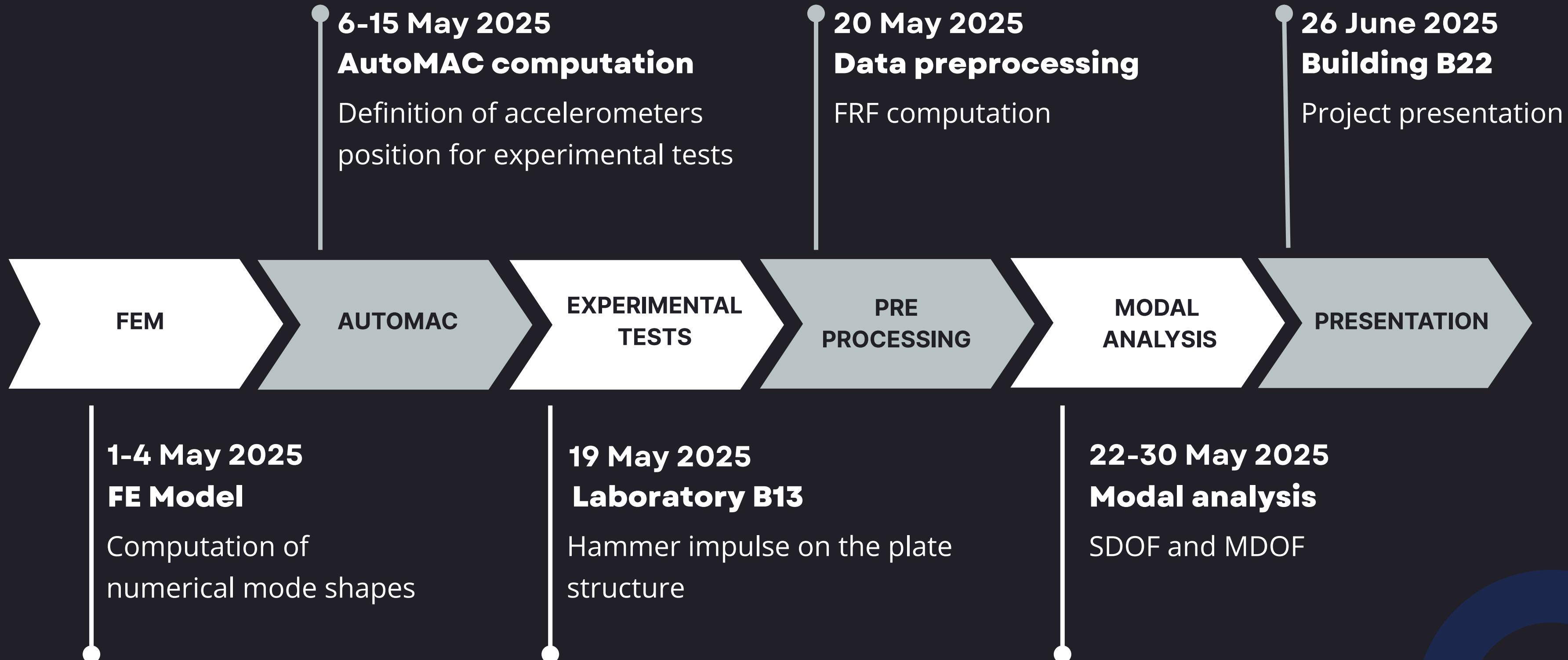


Description of the problem

Perform a modal analysis on a plate with defined geometry

PROJECT TIMELINE

Data Analysis Assignment
a.a.2024/2025





STEPS

- 01** FE Model
- 02** AUTOMAC Computation
- 03** Experimental Tests
- 04** Data preprocessing
- 05** Modal Extraction (SDOF, MDOF)
- 06** Comparison between numerical and experimental datas



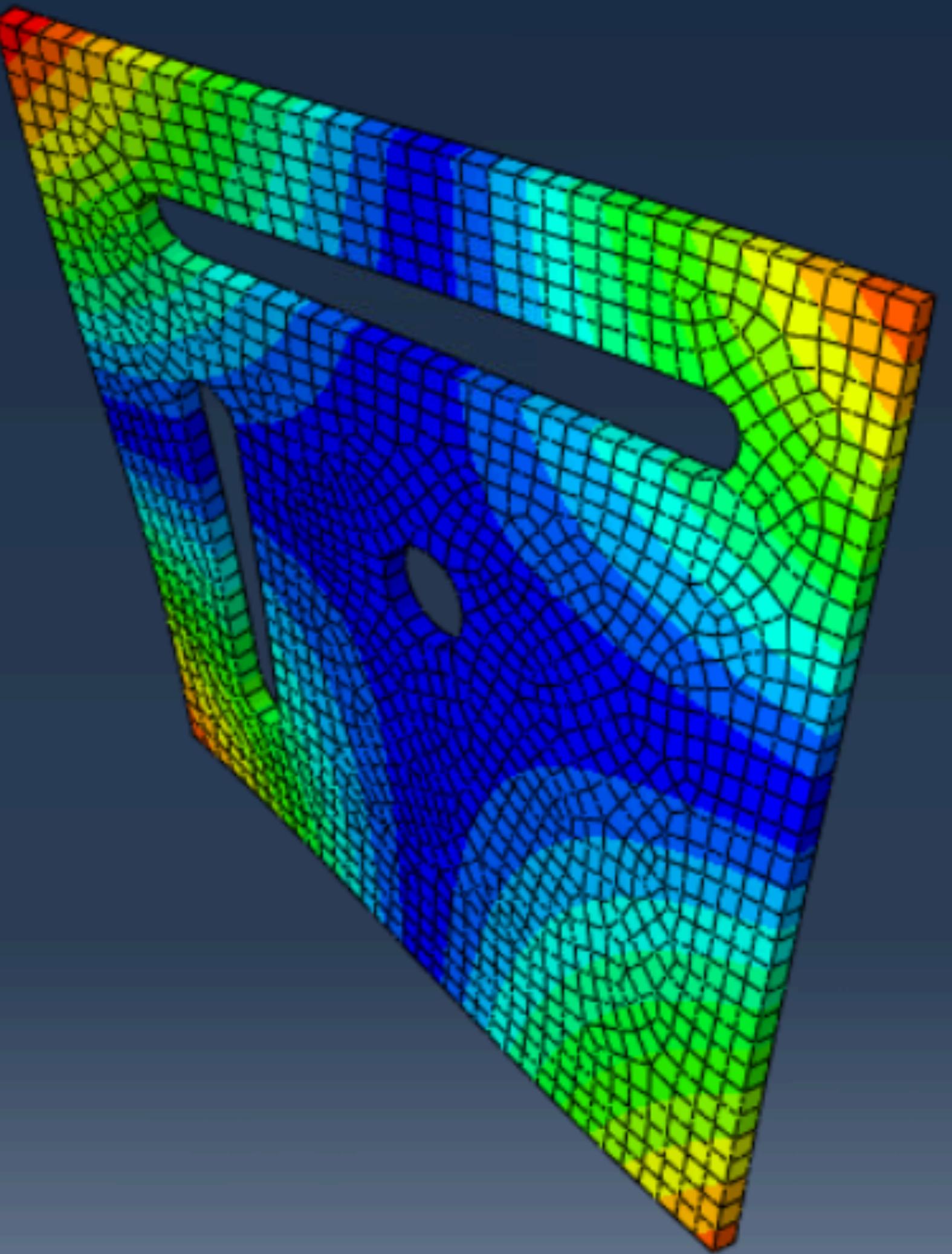
Data Analysis Assignment a.a. 2024/2025

FEM

Finite element model - pre experiment

FEM

**Finite Elements Model of a plate to
estimate natural frequencies**



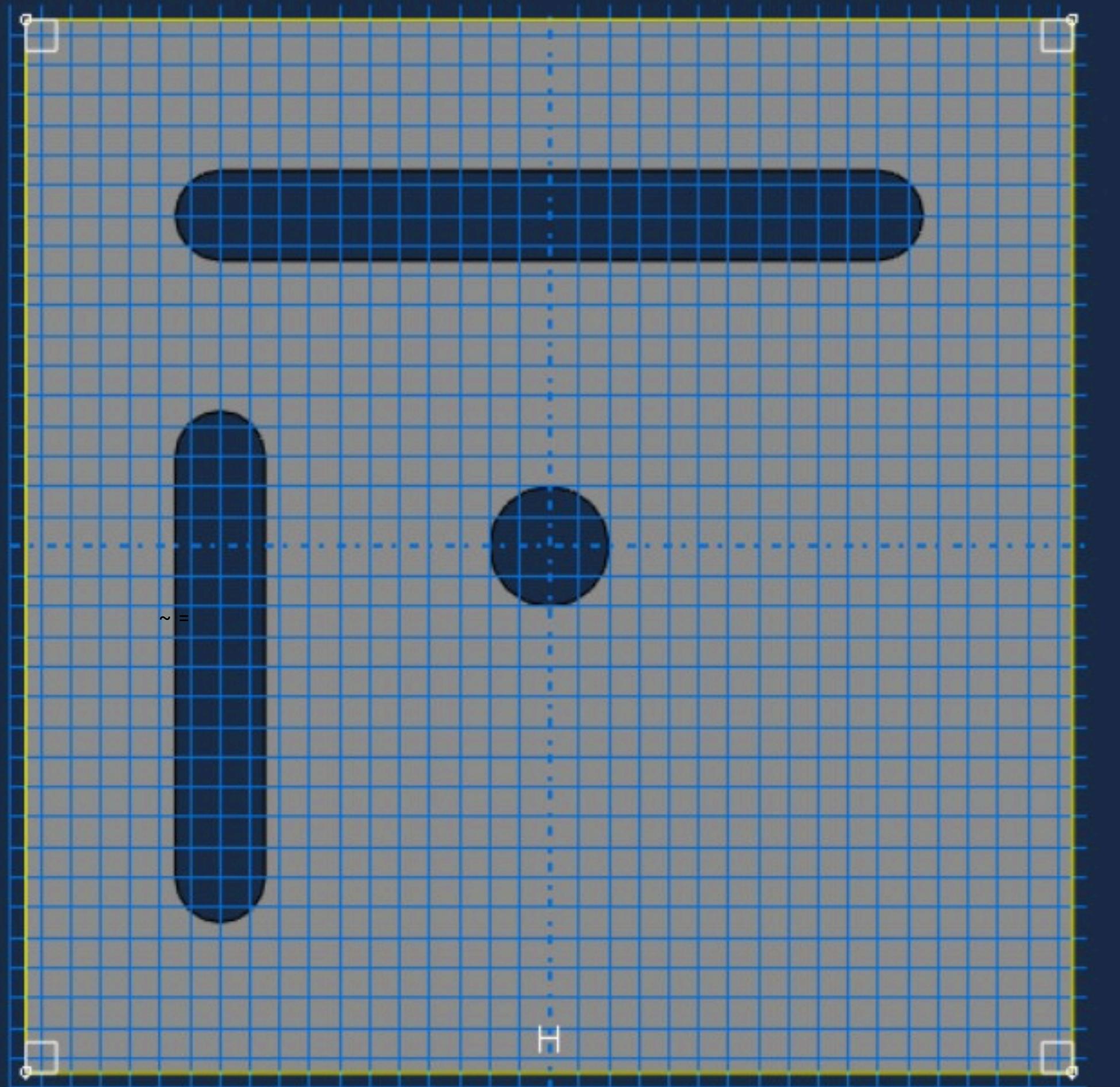
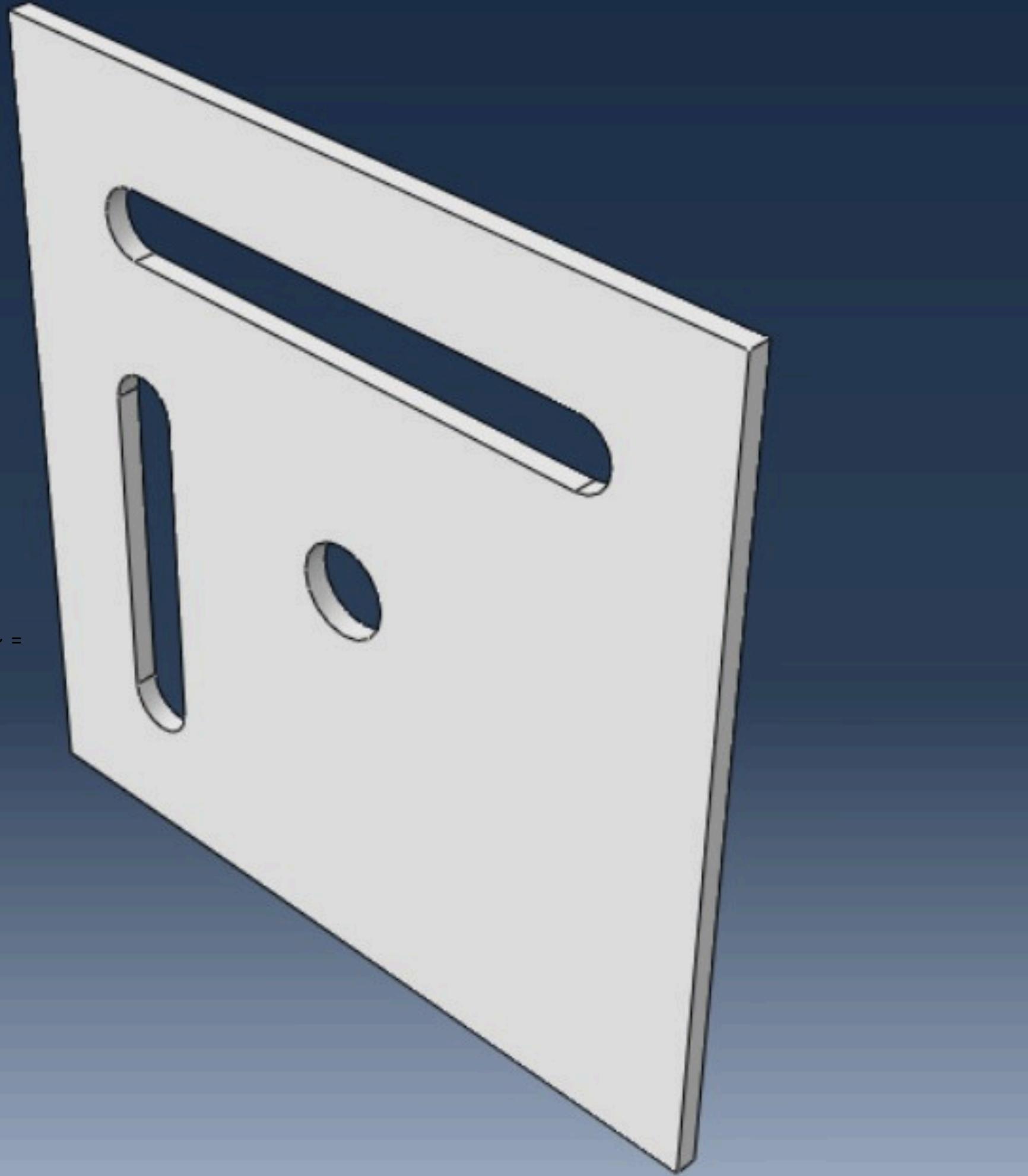


PLATE MODEL

Definition of perforated
plate geometry



PERFORATED PLATE 3D MODEL

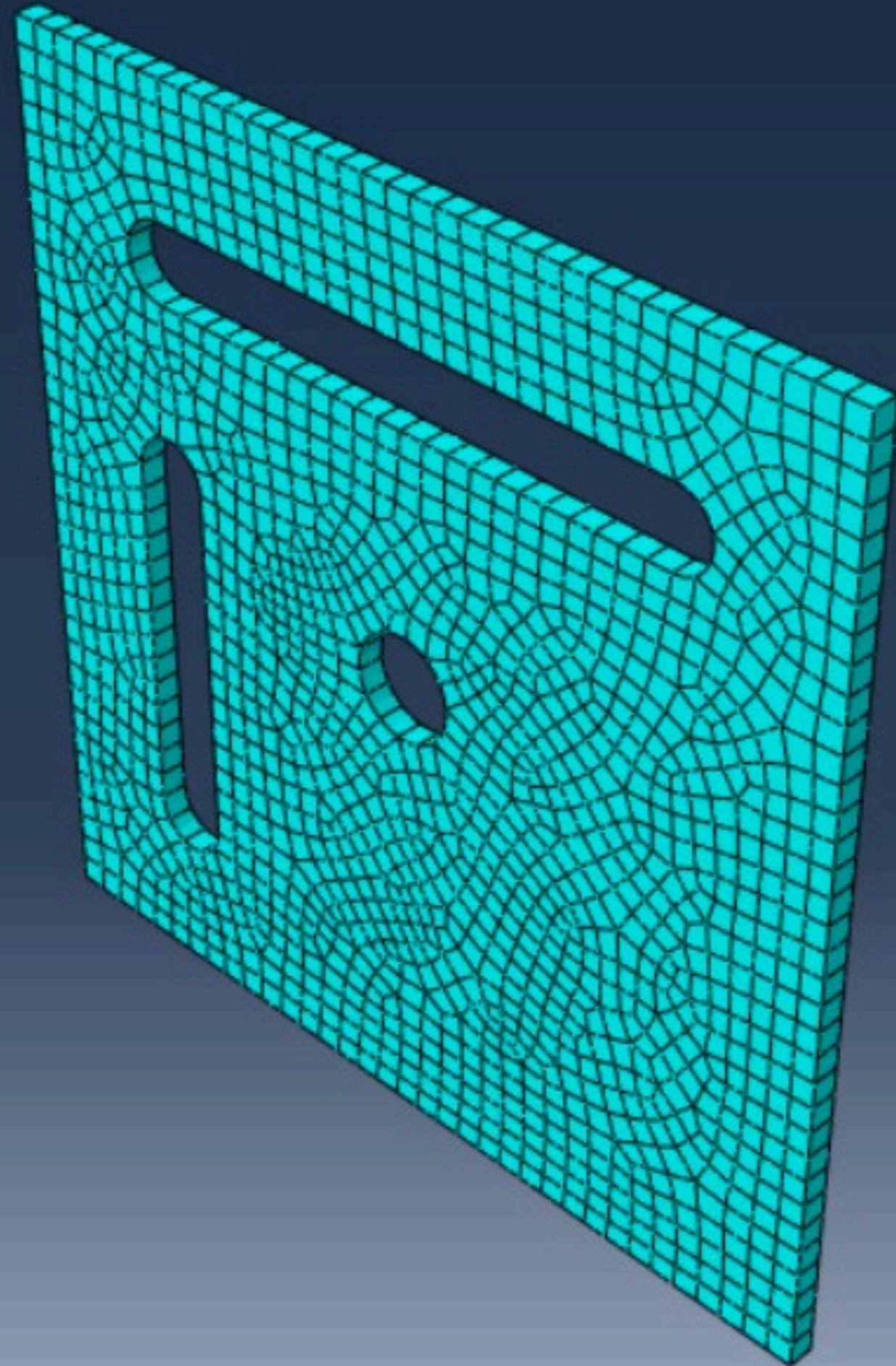
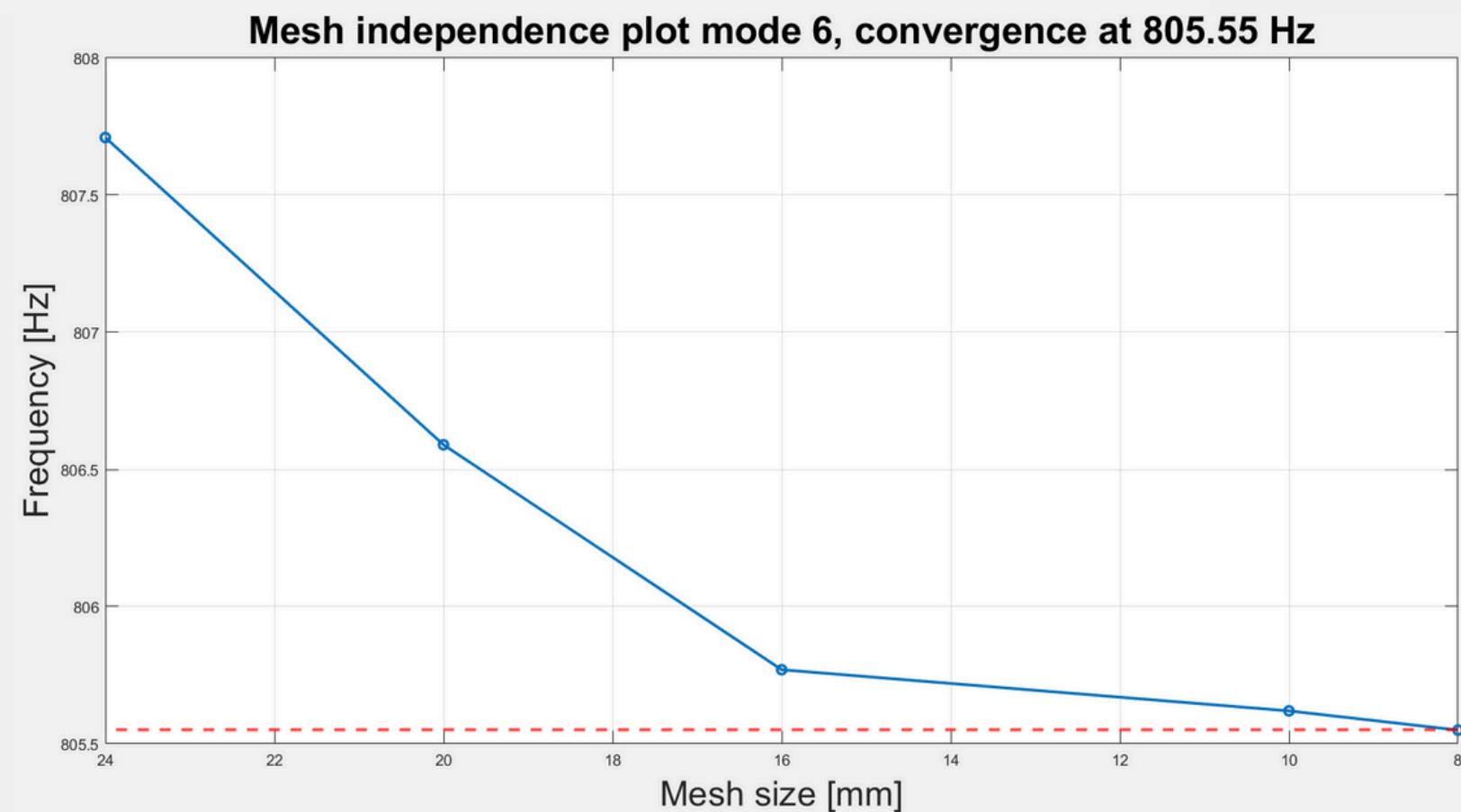


PLATE MODEL

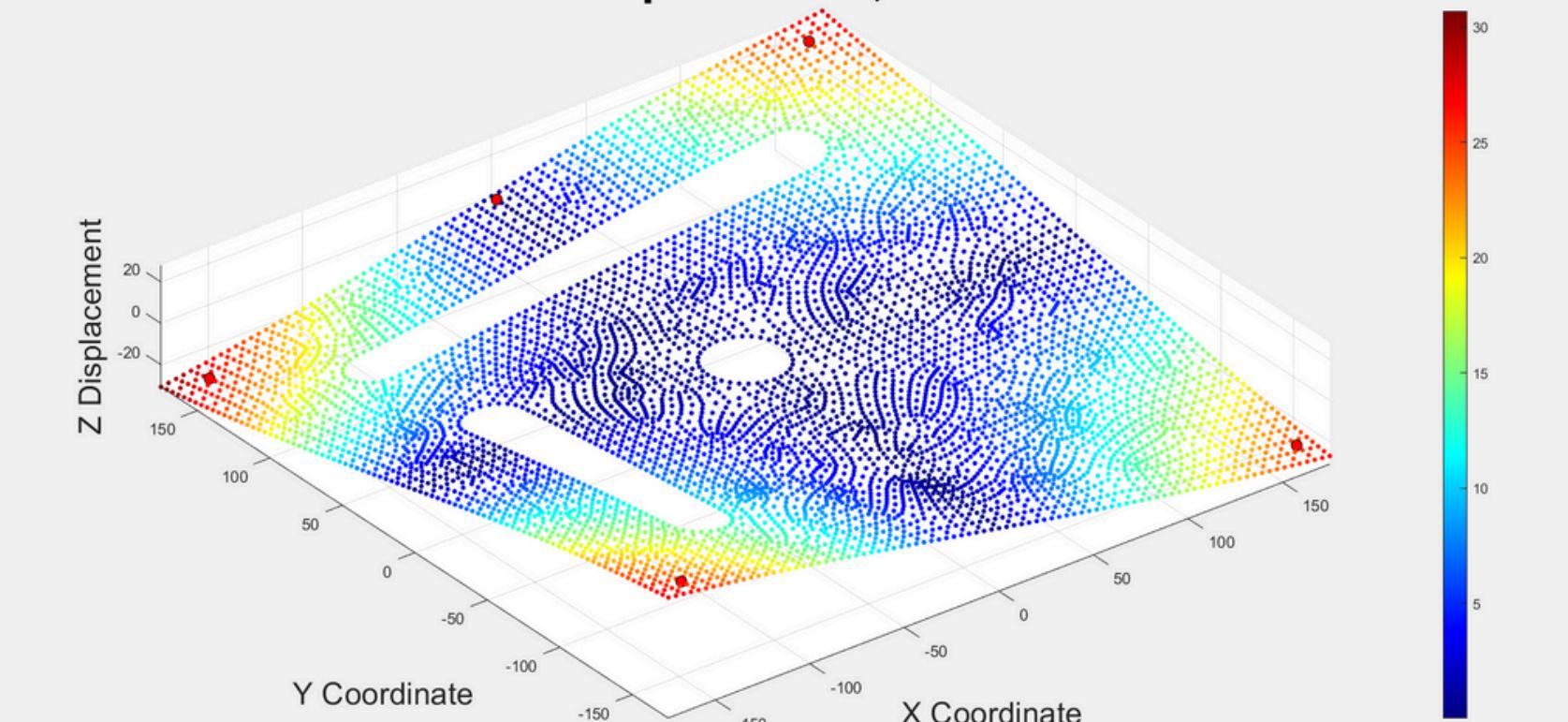
Is it the right mesh size?

MESH INDEPENDENCE ANALYSIS

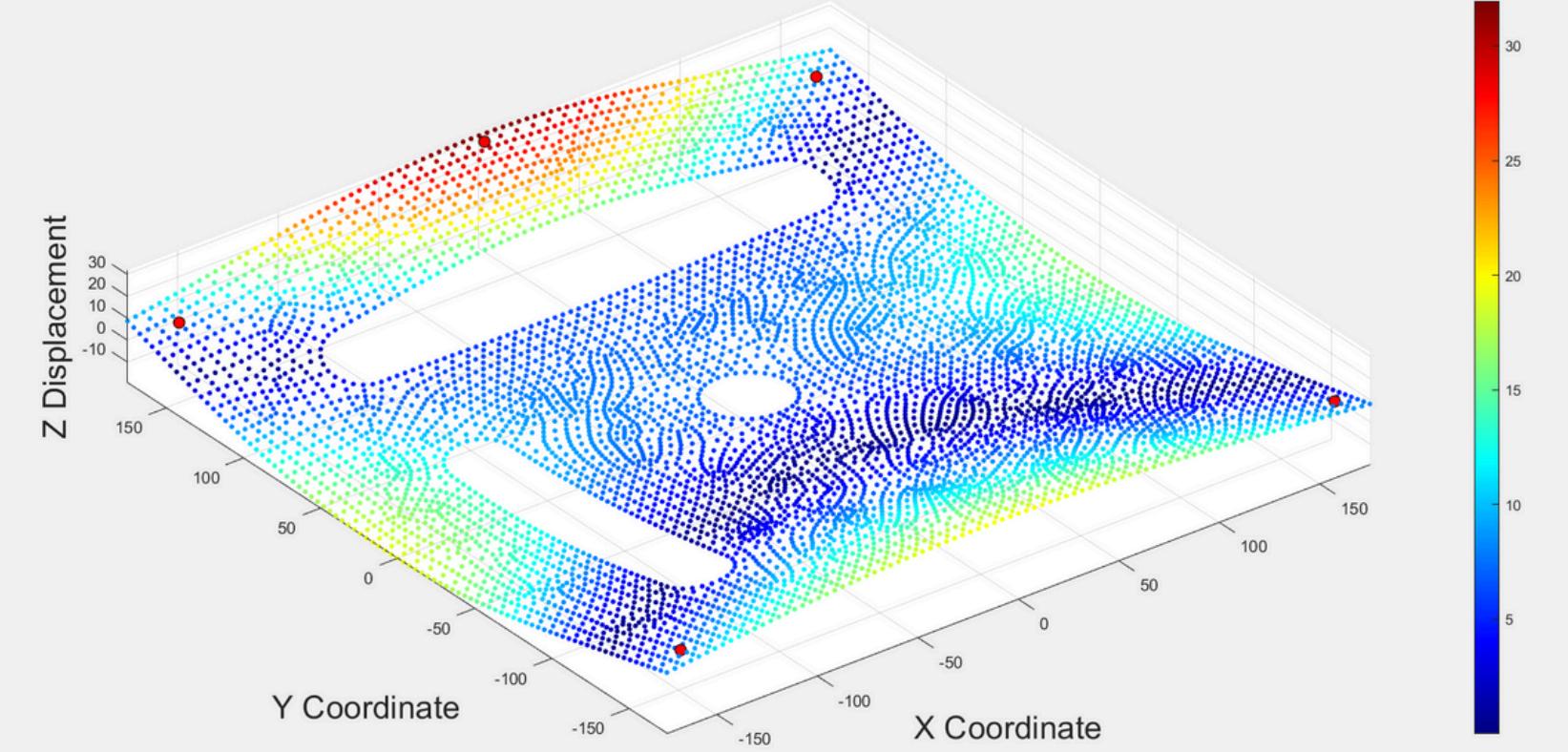


RESULTS

3D Plot of Plate Displacement, mode: 1 245.6 Hz

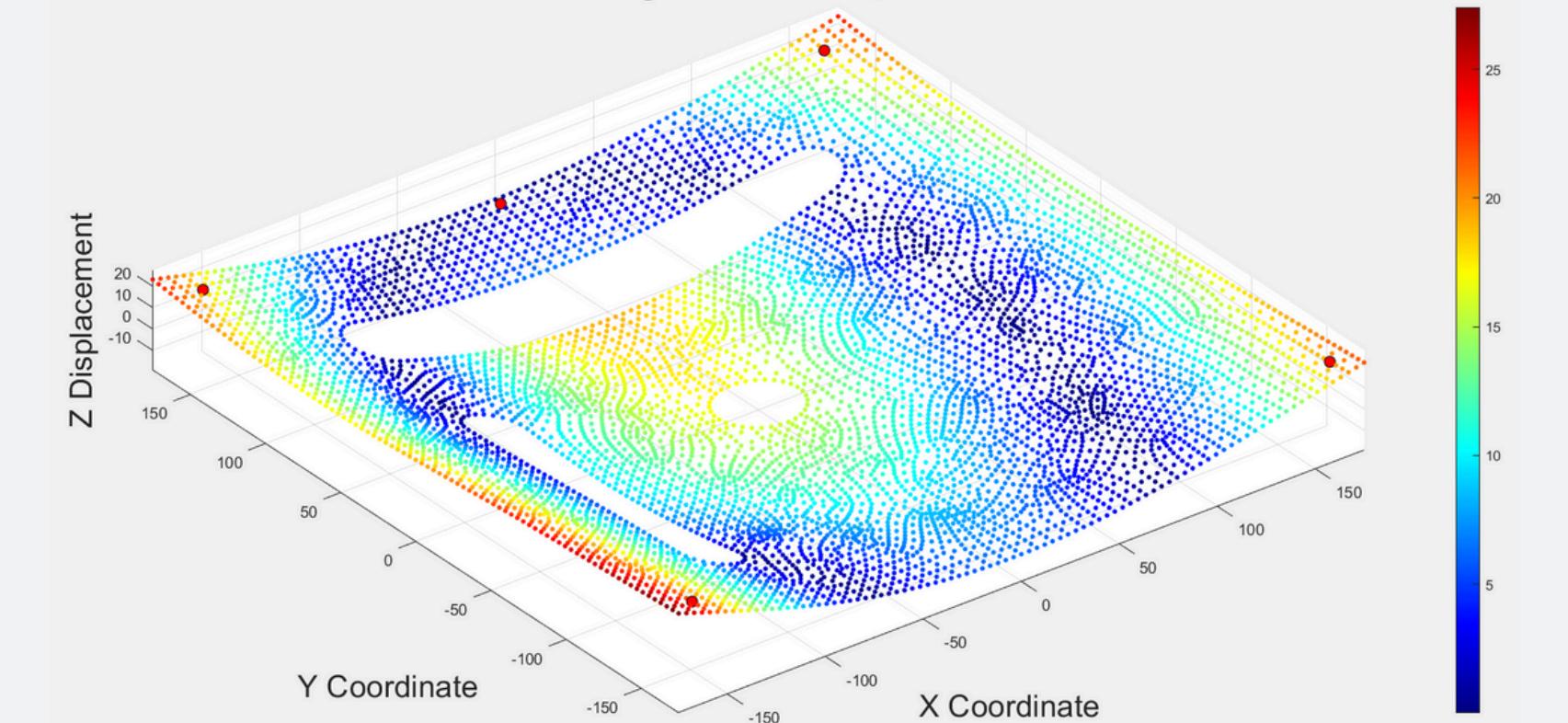


3D Plot of Plate Displacement, mode: 2 327.2 Hz

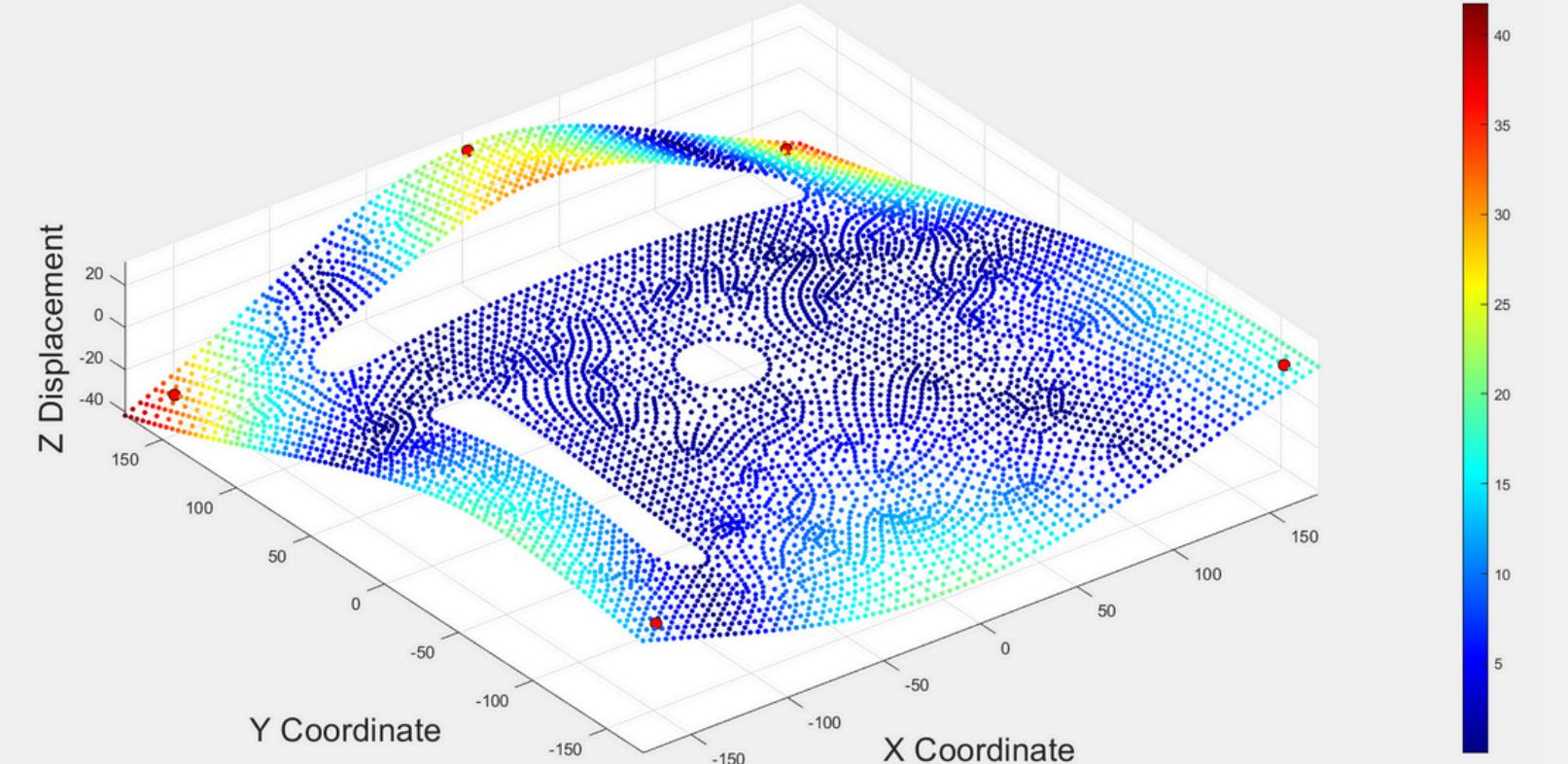


RESULTS

3D Plot of Plate Displacement, mode: 3 389.55 Hz

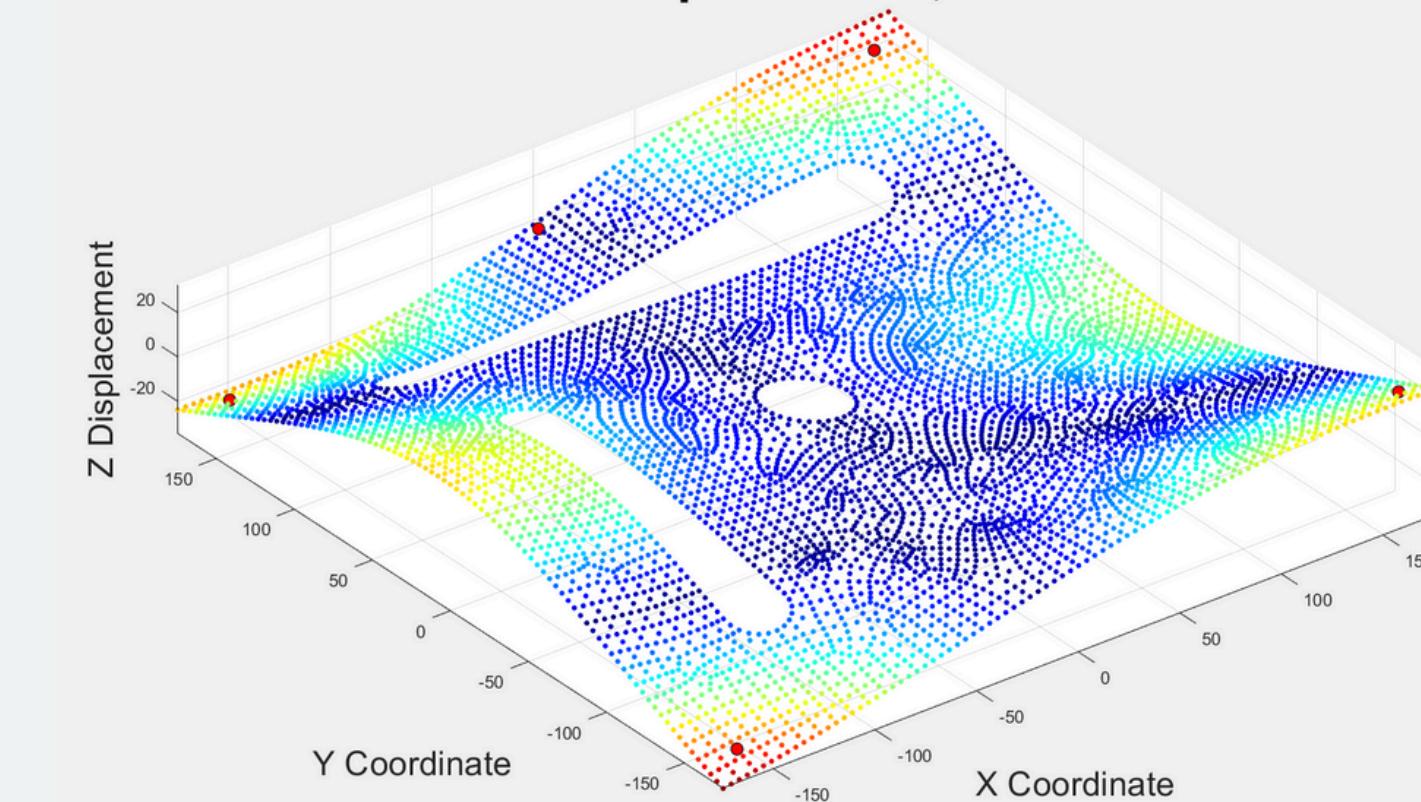


3D Plot of Plate Displacement, mode: 4 594.36 Hz

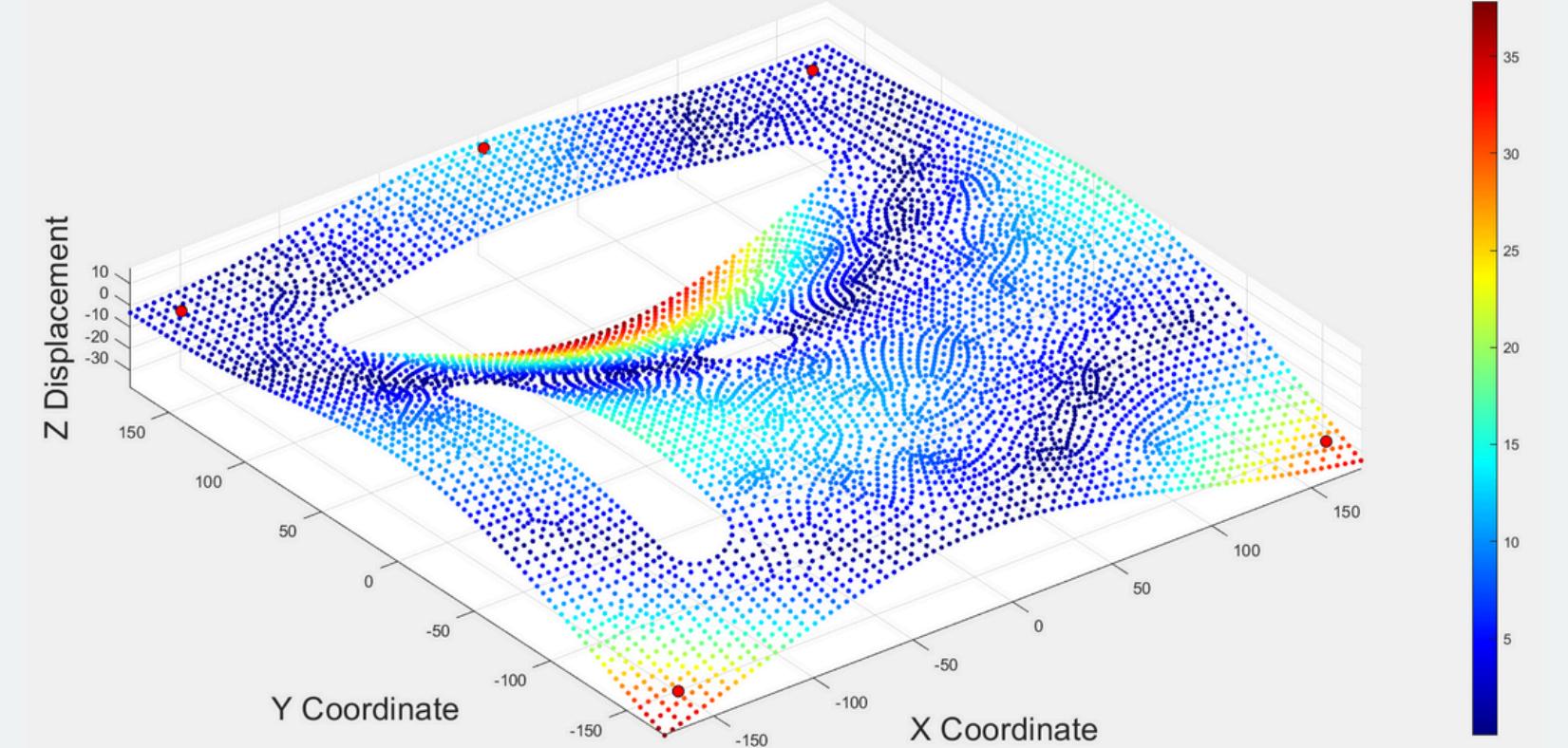


RESULTS

3D Plot of Plate Displacement, mode: 5 618.99 Hz



3D Plot of Plate Displacement, mode: 6 805.55 Hz





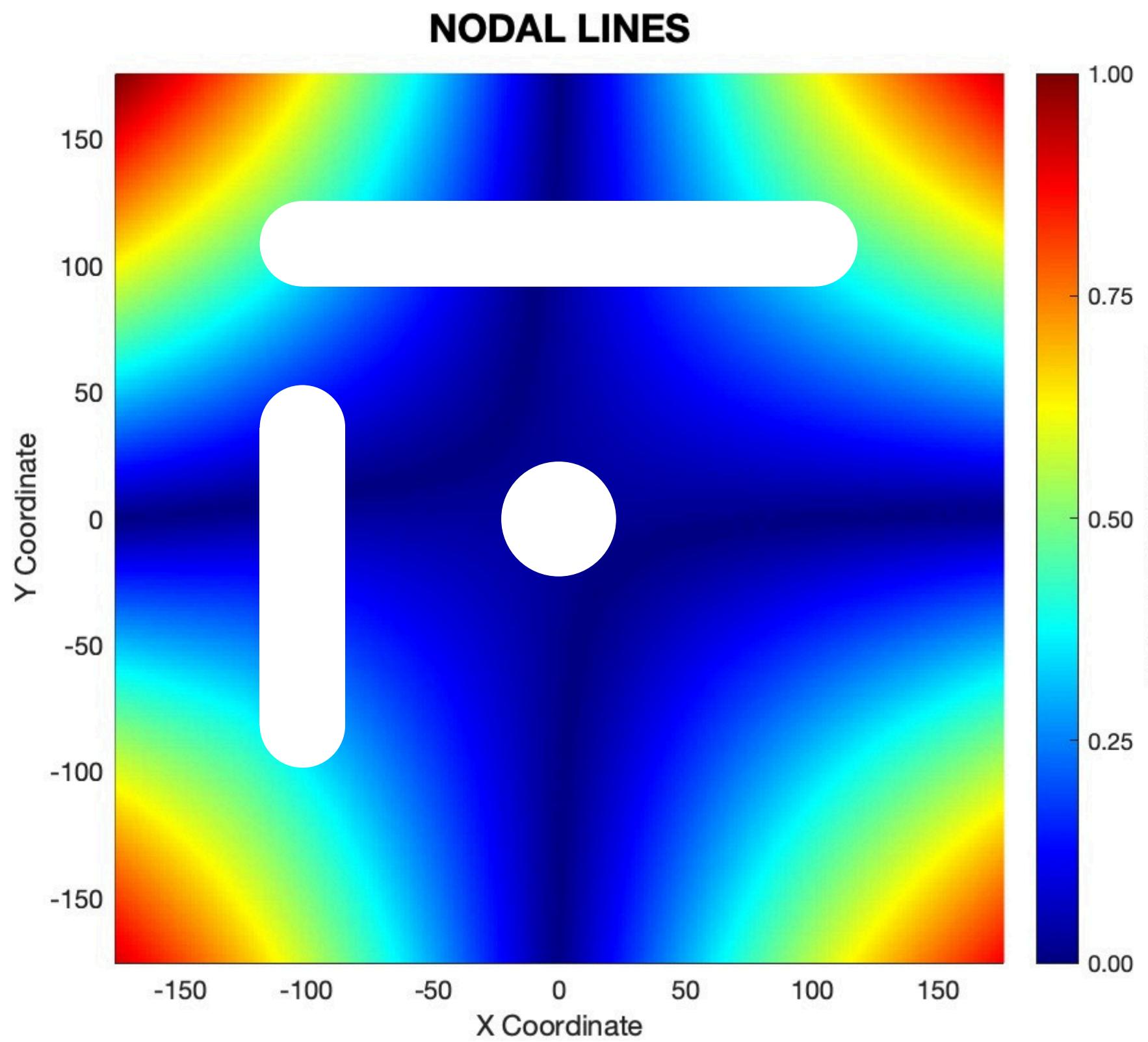
Data Analysis Assignment a.a. 2024/2025

AUTO-MAC

Auto Modal Assurance Criterion - pre experiment

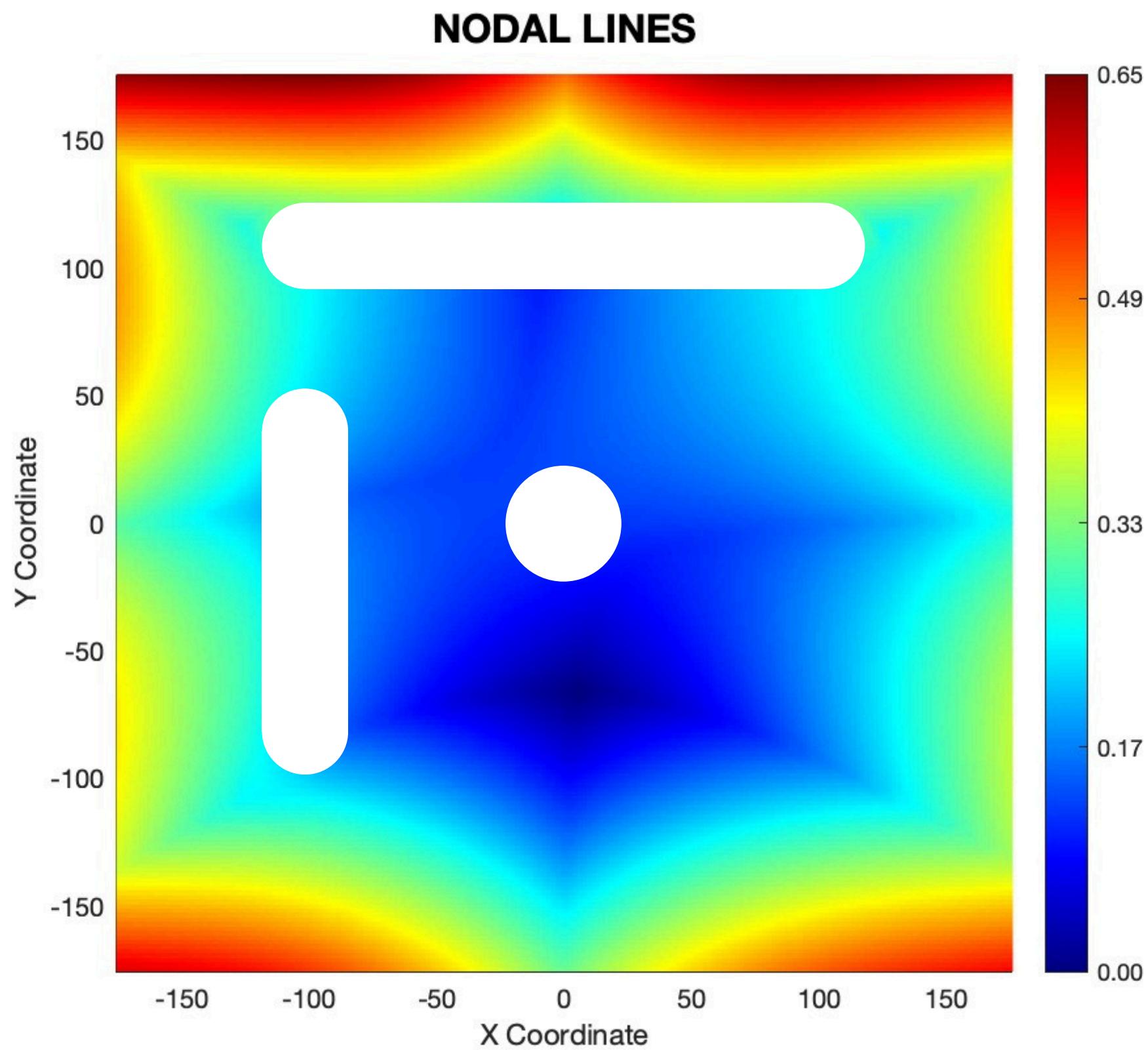
NODAL SURFACES DISTRIBUTION

**SUPERIMPOSITION OF THE
NORMALIZED DISPLACEMENT OF
THE DIFFERENT MODES**

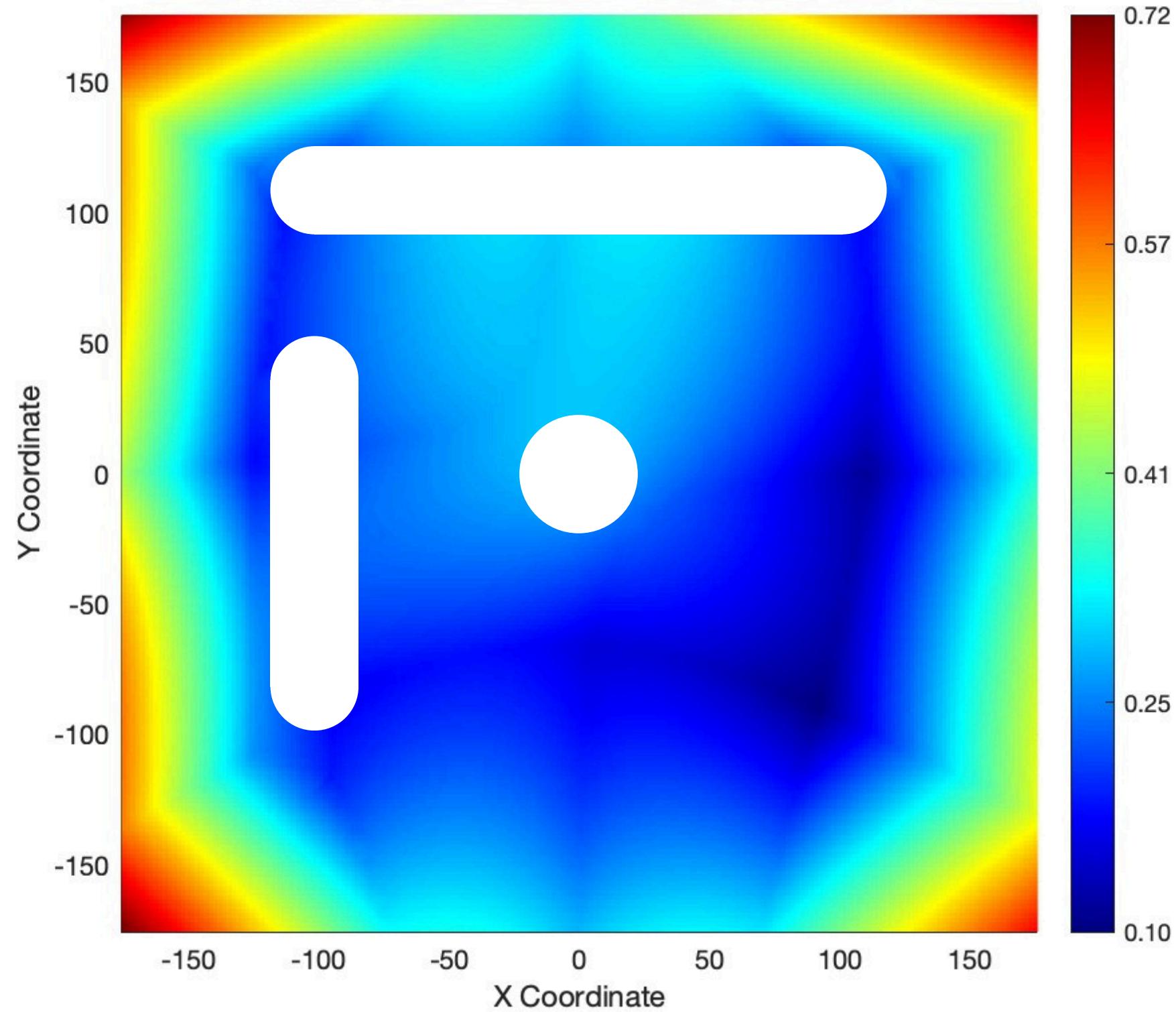


NODAL SURFACES DISTRIBUTION

SUPERIMPOSITION OF THE
NORMALIZED DISPLACEMENT OF
THE DIFFERENT MODES



NODAL LINES



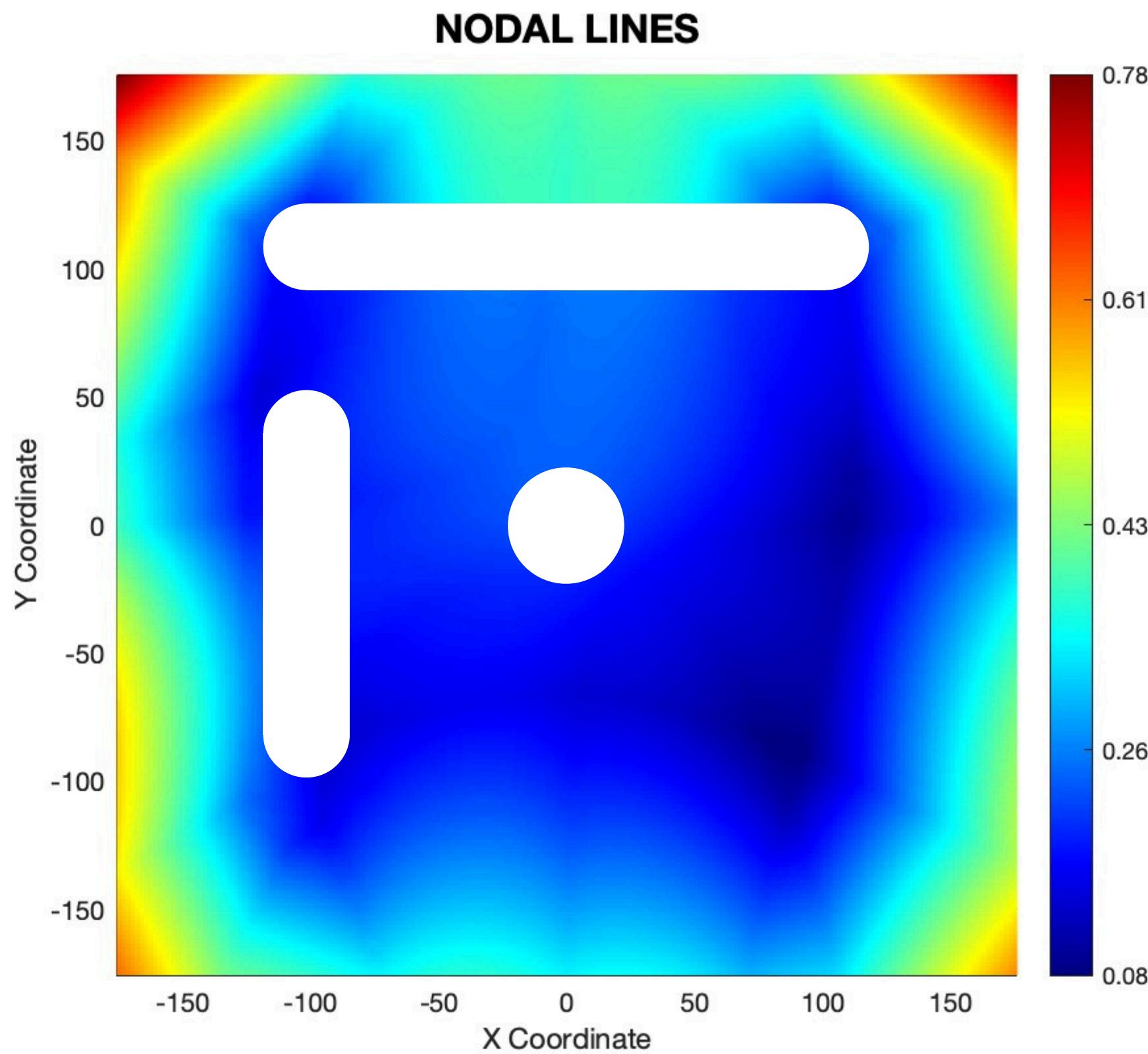
MODE 1+2+3

NODAL SURFACES DISTRIBUTION

**SUPERIMPOSITION OF THE
NORMALIZED DISPLACEMENT OF
THE DIFFERENT MODES**

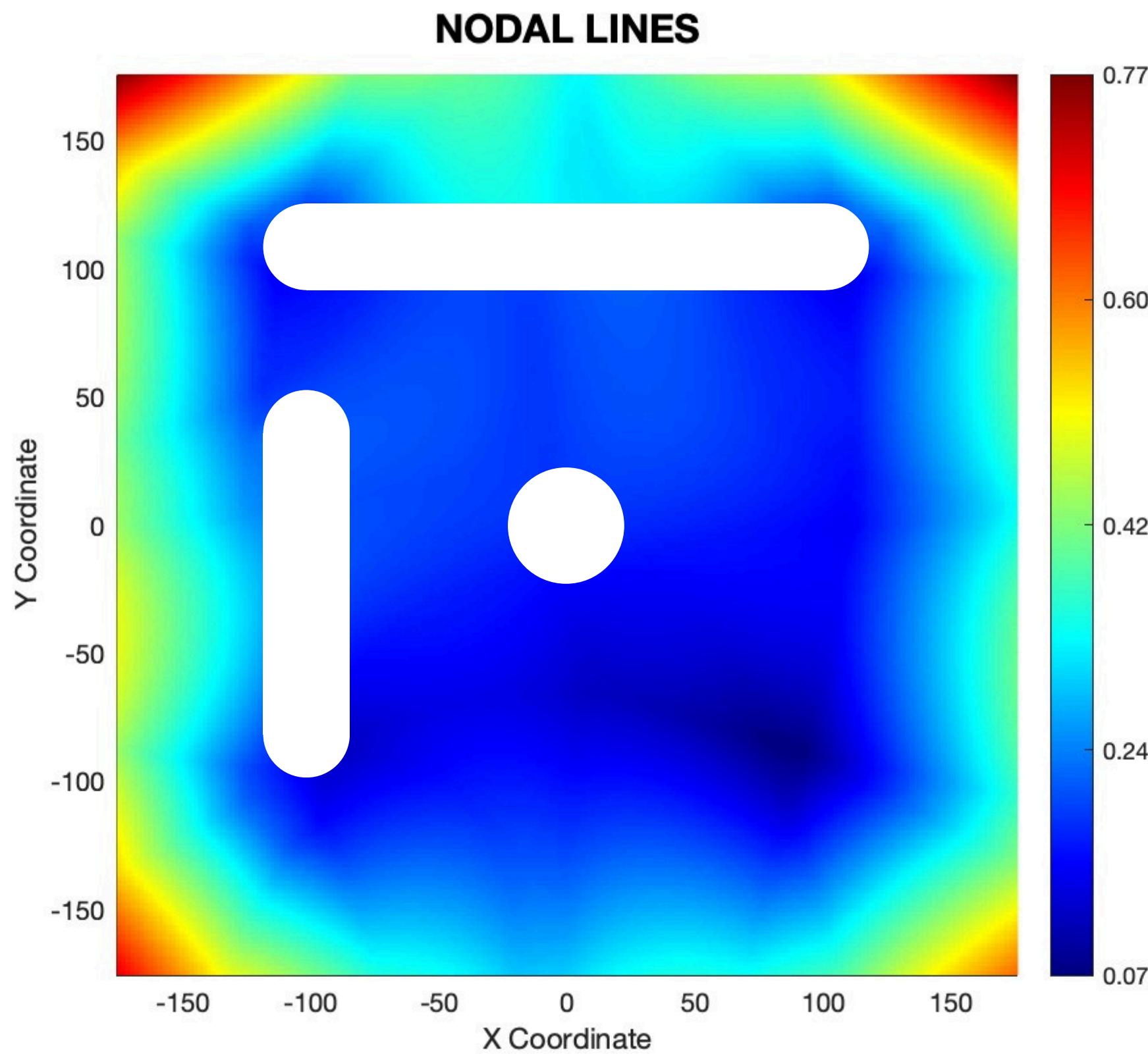
NODAL SURFACES DISTRIBUTION

SUPERIMPOSITION OF THE
NORMALIZED DISPLACEMENT OF
THE DIFFERENT MODES



NODAL SURFACES DISTRIBUTION

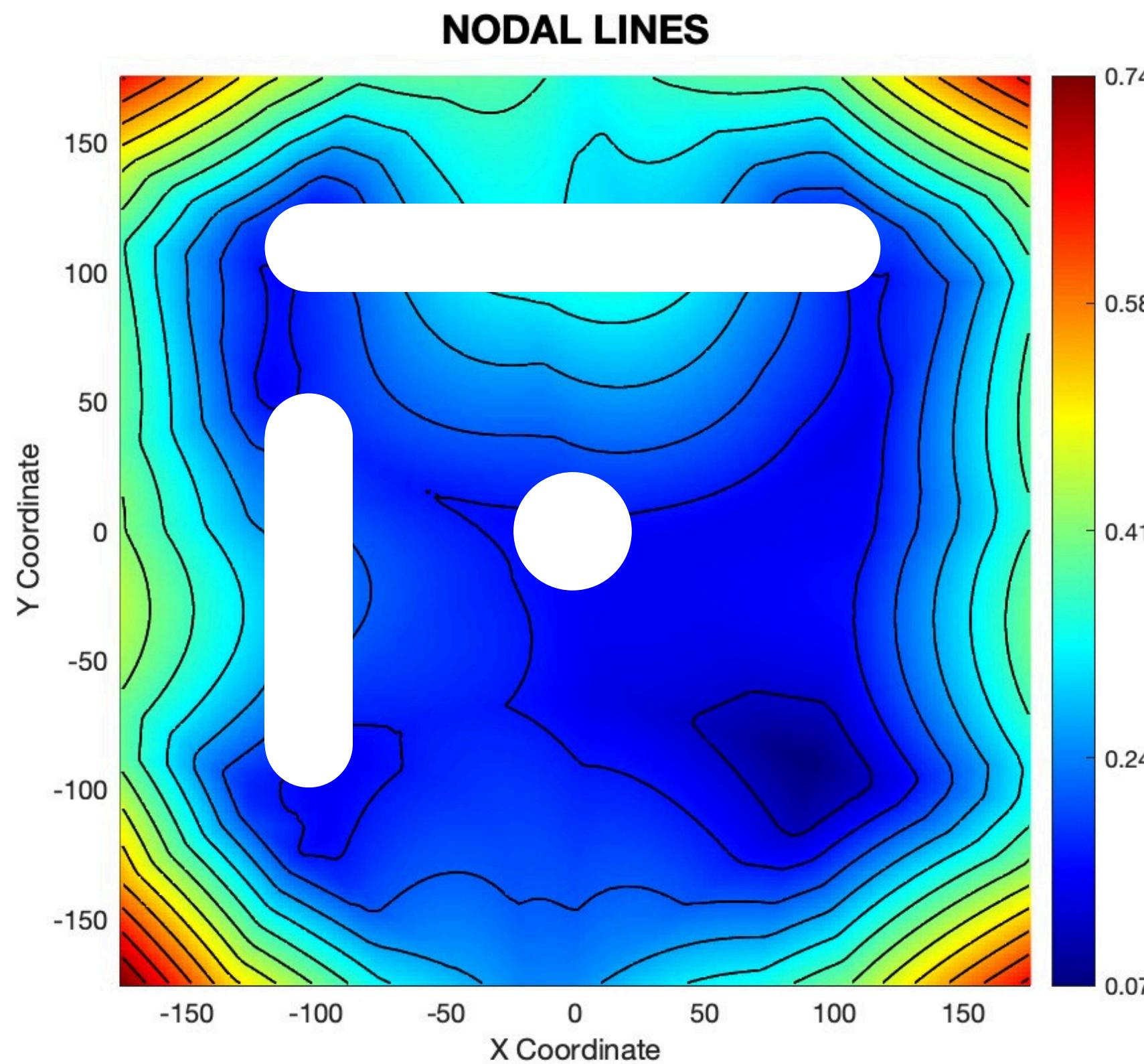
**SUPERIMPOSITION OF THE
NORMALIZED DISPLACEMENT OF
THE DIFFERENT MODES**



MODE 1+2+3+4+5

NODAL SURFACES DISTRIBUTION

SUPERIMPOSITION OF THE
NORMALIZED DISPLACEMENT OF
THE DIFFERENT MODES



ALL 6 MODES

THEORETICAL DEFINITION

Used to find whether a certain measurement mesh is correct or produce spatial aliasing

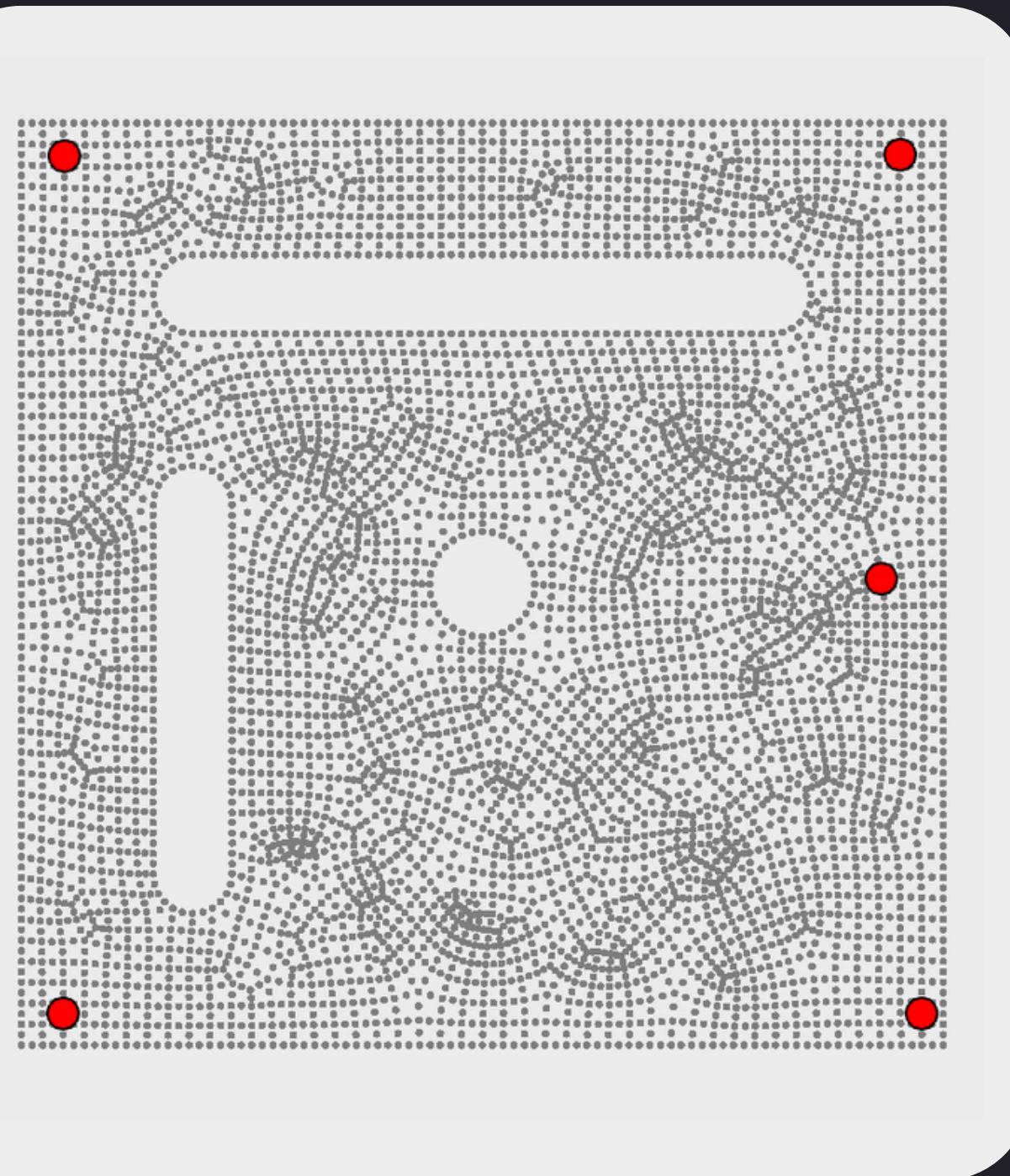
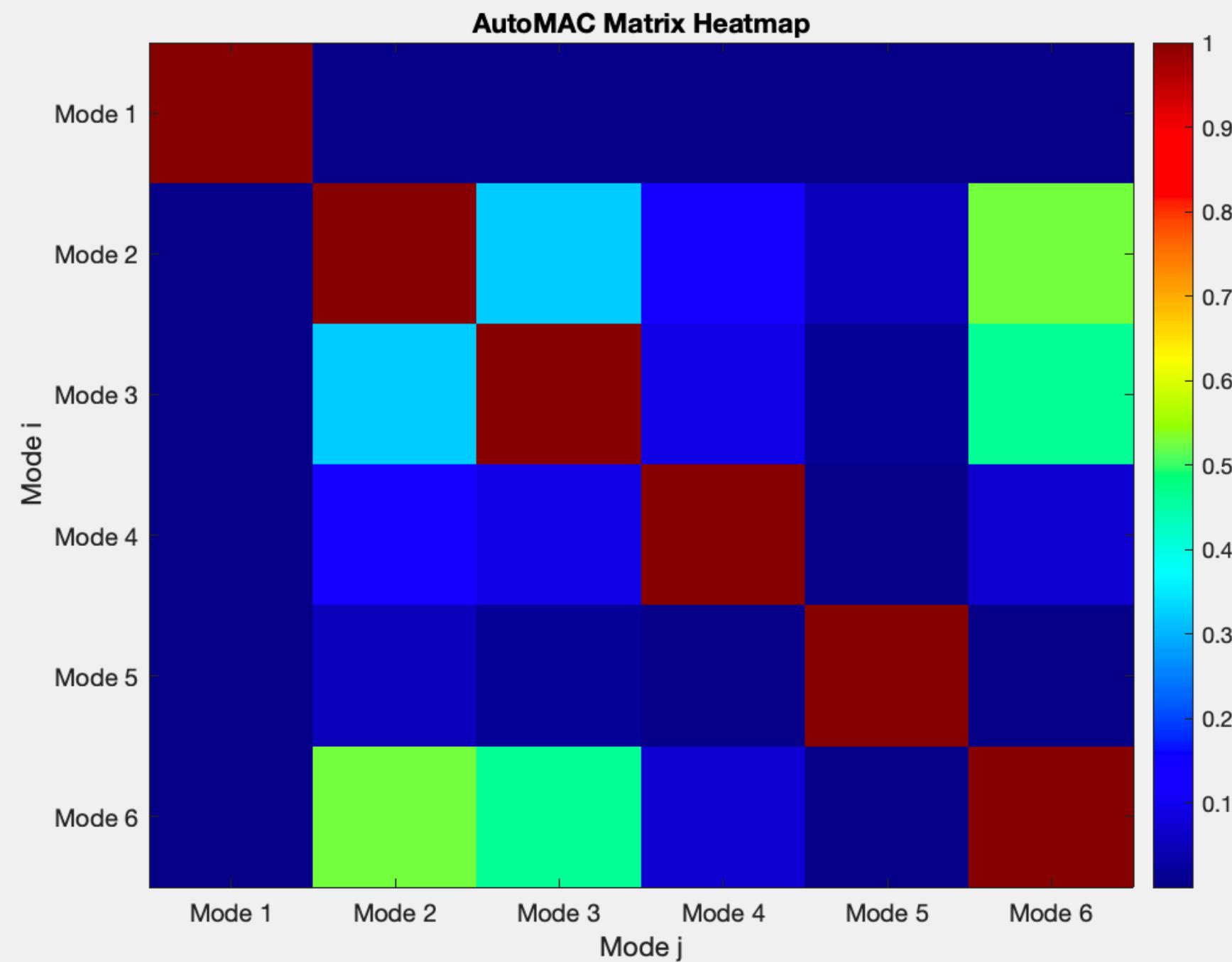
AutoMac

$$AutoMAC(A_i, A_j) = \frac{\|\{\psi_{A,i}\}^T \{\psi_{A,j}\}\|^2}{(\{\psi_{A,i}\}^T \{\psi_{A,i}\}) \cdot (\{\psi_{A,j}\}^T \{\psi_{A,j}\})}$$

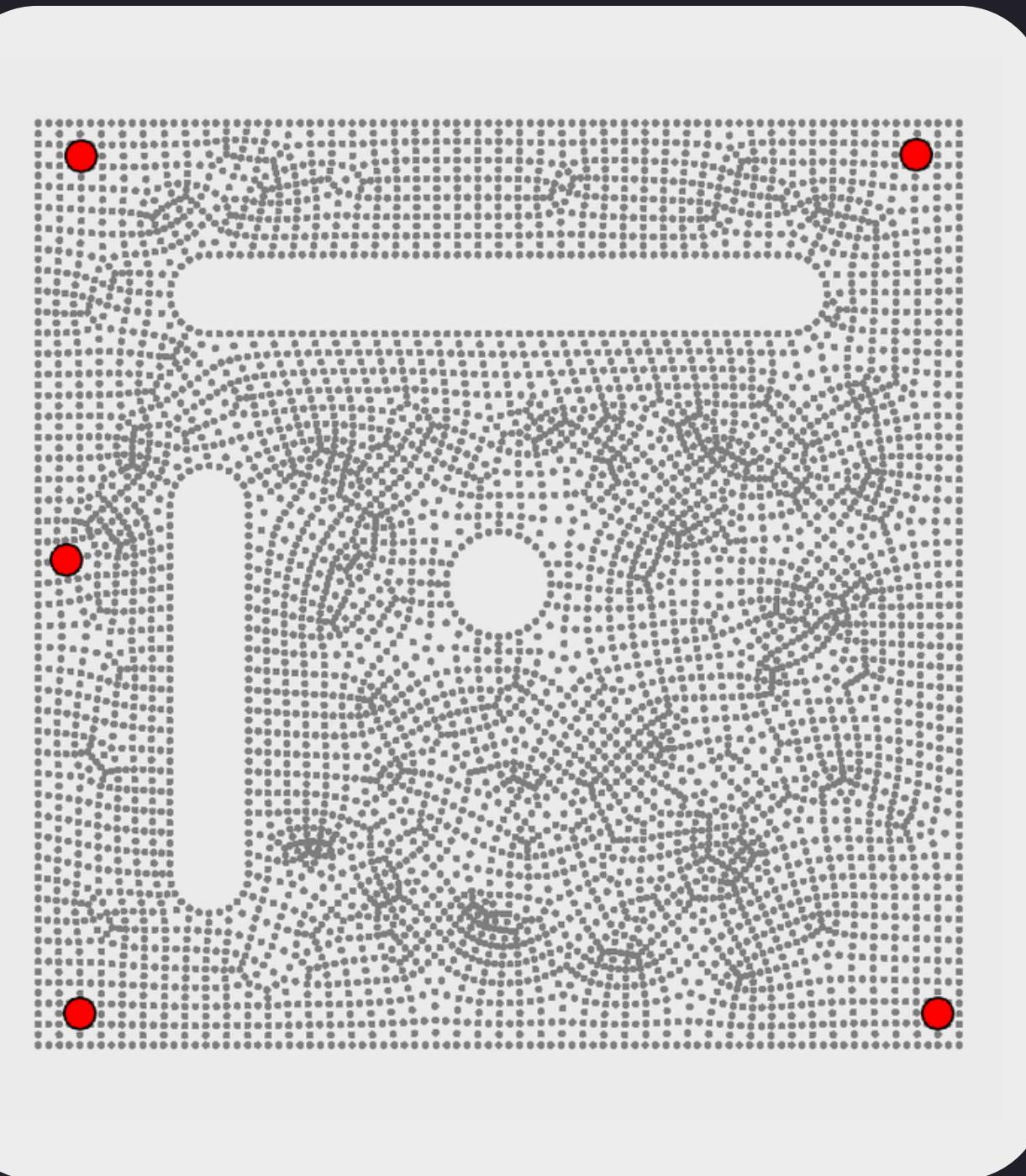
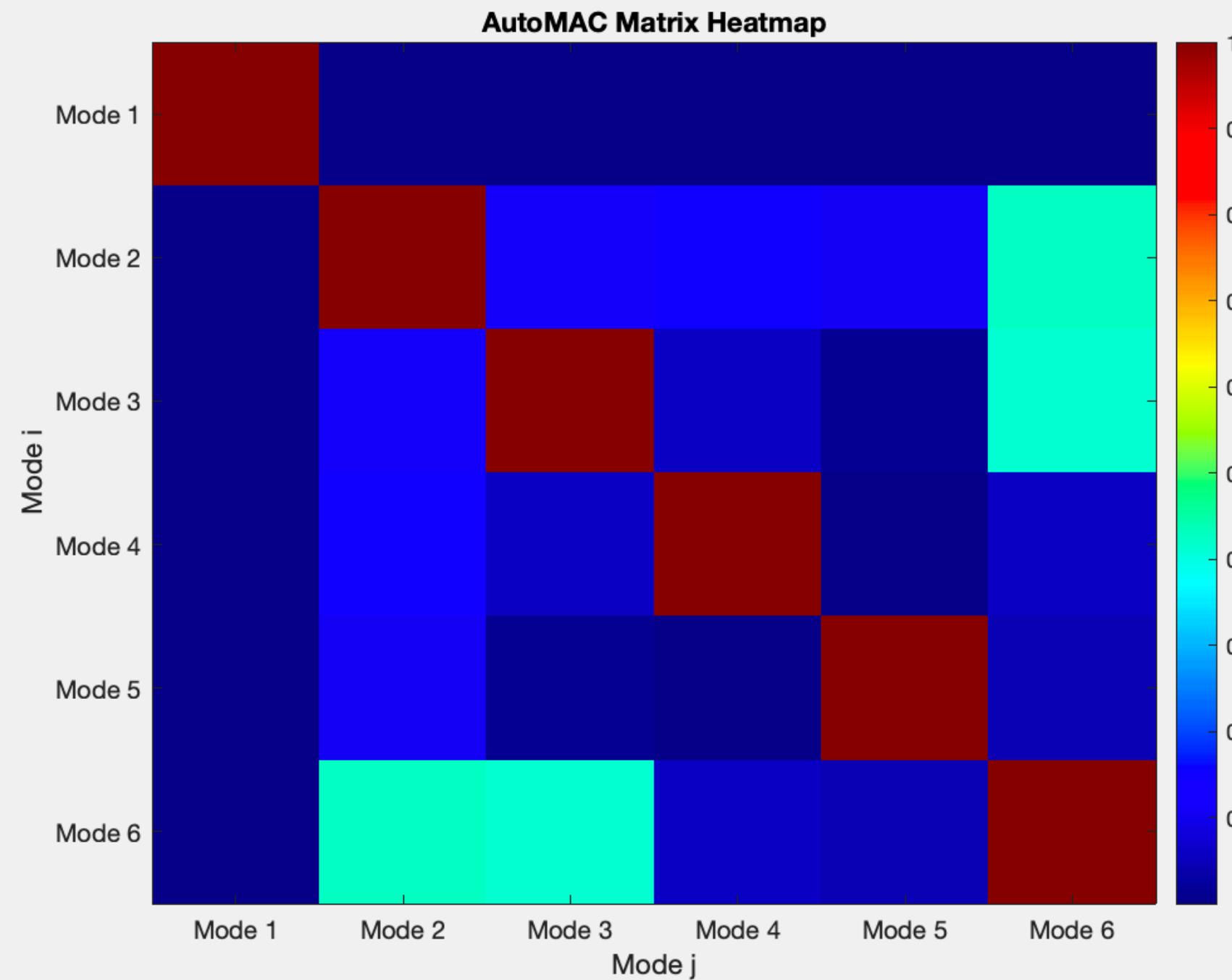


modes computed numerically

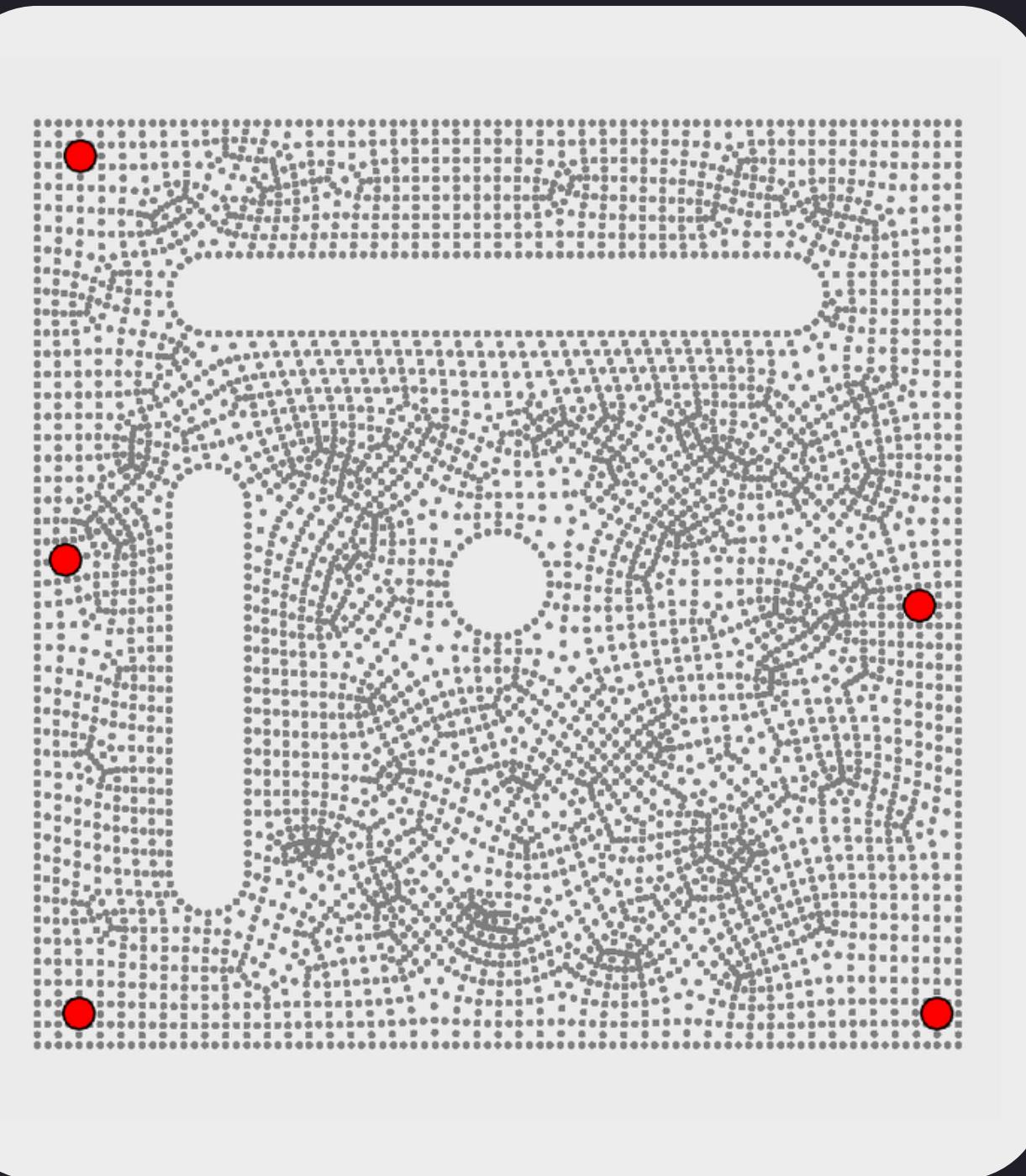
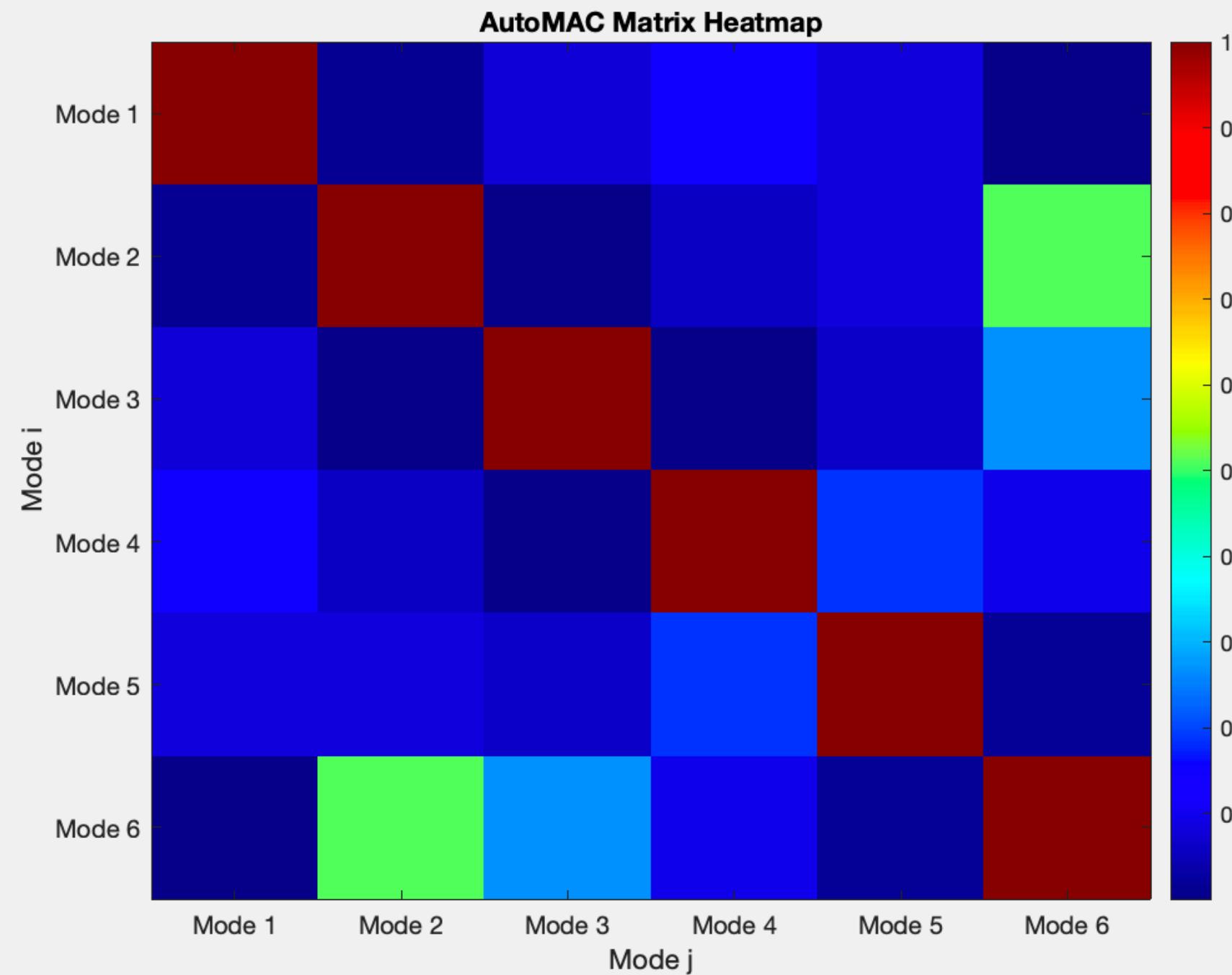
CONFIGURATION 1



CONFIGURATION 2



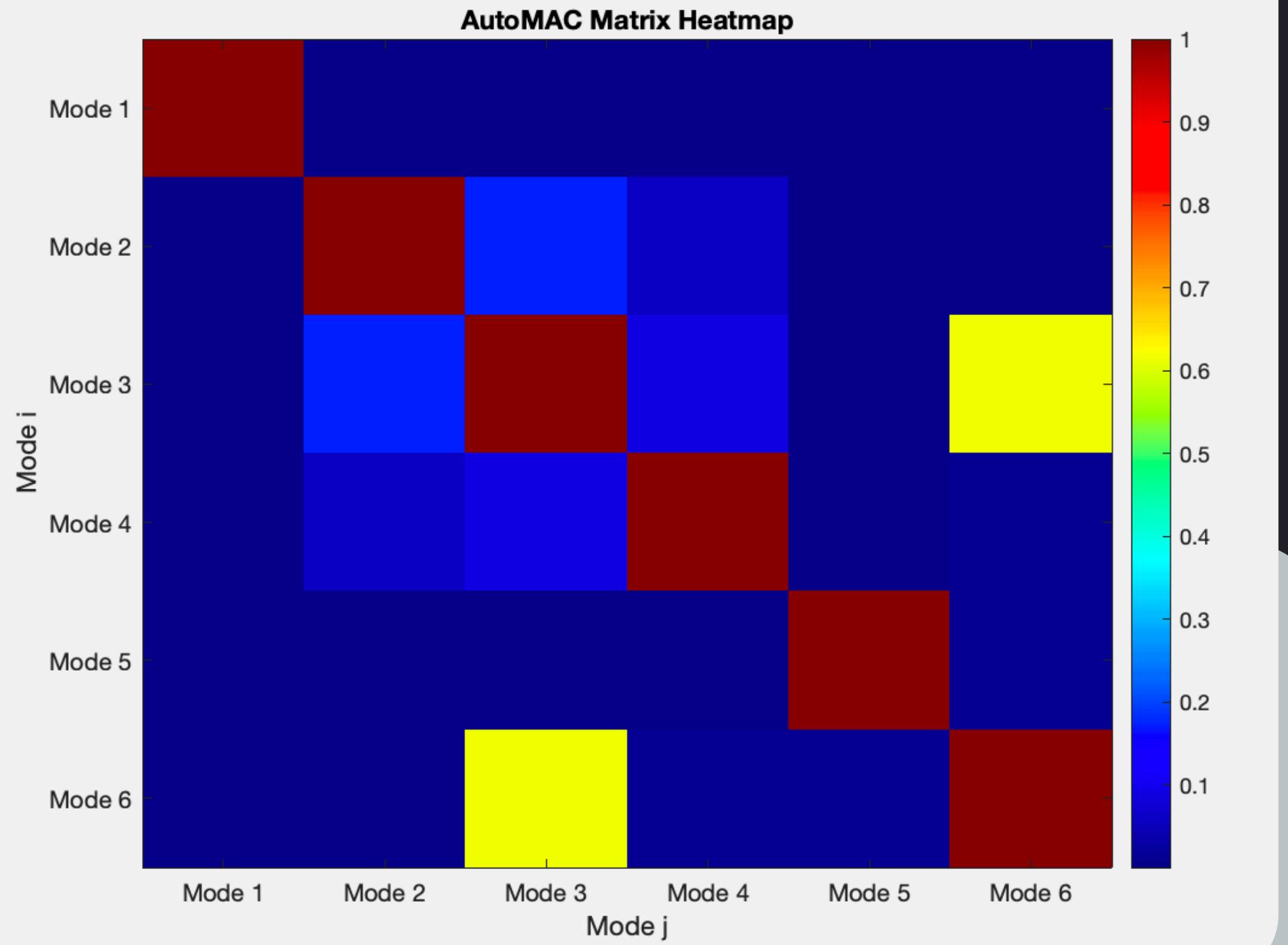
CONFIGURATION 3



CONFIGURATION 4



The chosen configuration



AUTOMAC

Configuration 4

Node 12110

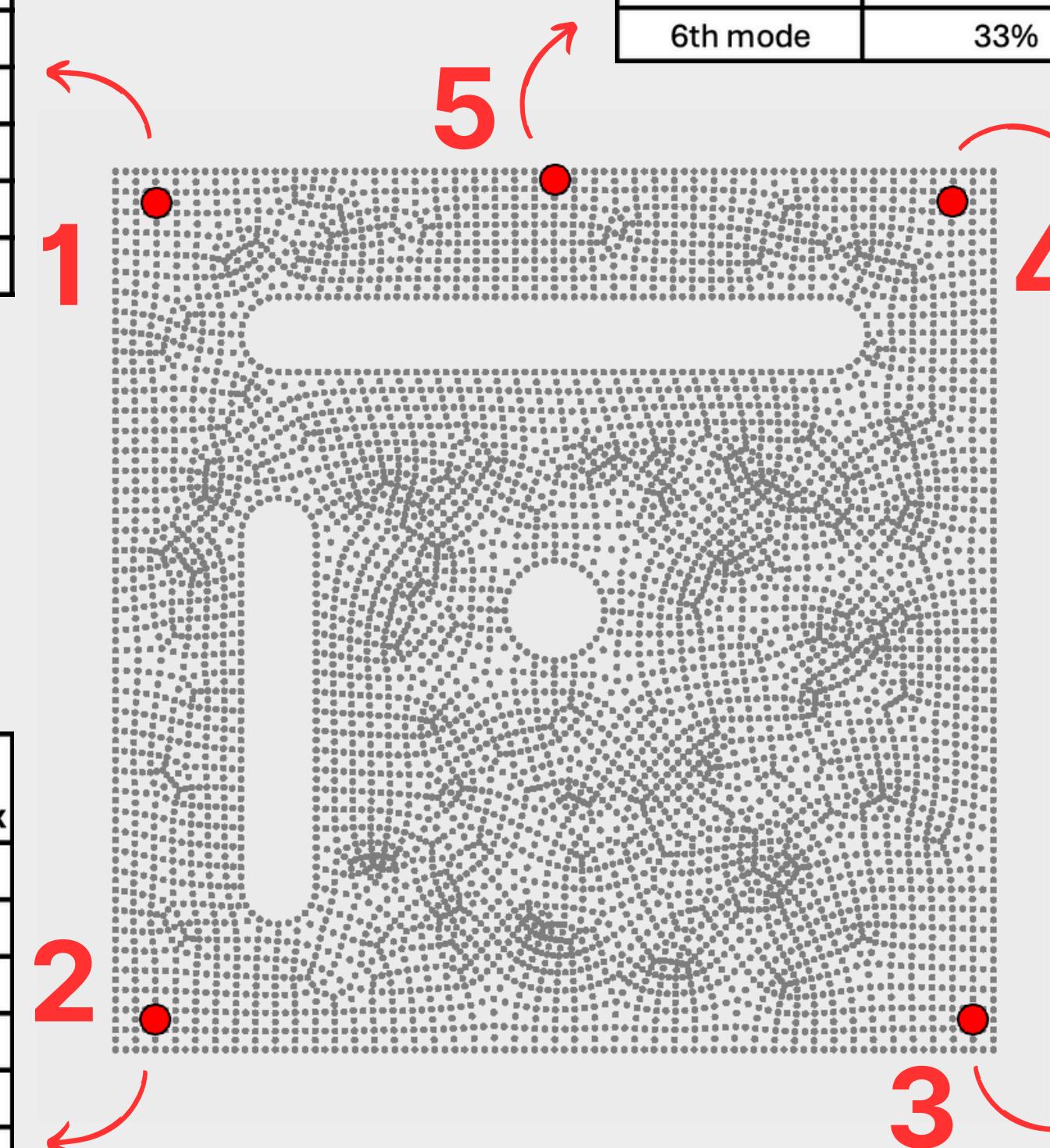
Mode	Successful acquisition index
1st mode	87%
2nd mode	25%
3rd mode	66%
4th mode	77%
5th mode	57%
6th mode	7%

Node 11299

Mode	Successful acquisition index
1st mode	0%
2nd mode	96%
3rd mode	5%
4th mode	58%
5th mode	6%
6th mode	33%

Node 11075

Mode	Successful acquisition index
1st mode	80%
2nd mode	24%
3rd mode	80%
4th mode	21%
5th mode	80%
6th mode	75%

**Node 10814**

Mode	Successful acquisition index
1st mode	80%
2nd mode	27%
3rd mode	67%
4th mode	65%
5th mode	77%
6th mode	9%

Node 7884

Mode	Successful acquisition index
1st mode	79%
2nd mode	21%
3rd mode	70%
4th mode	40%
5th mode	55%
6th mode	74%

Node 12110

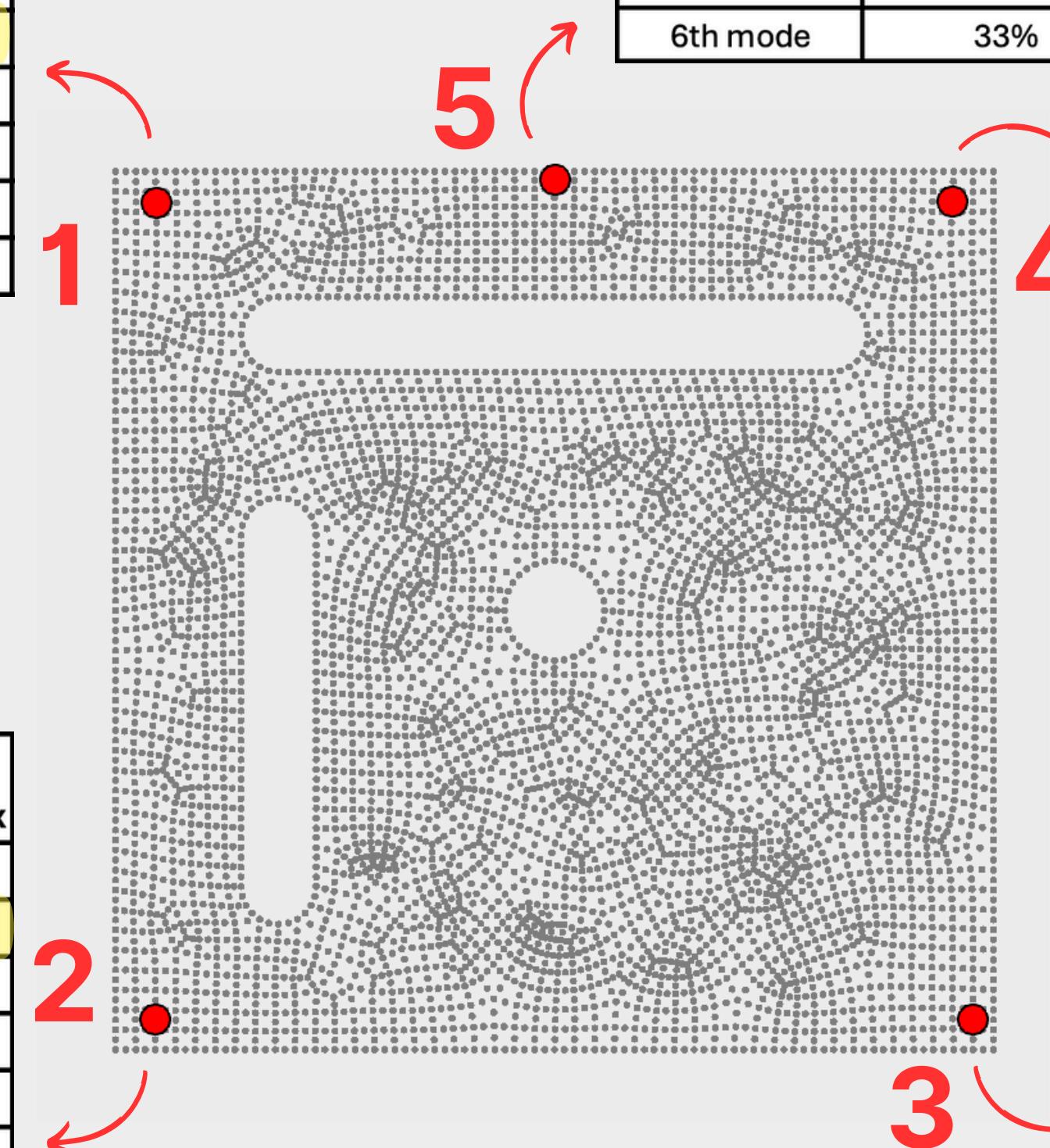
Mode	Successful acquisition index
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5th mode	57%
6th mode	7%

Node 11299

Mode	Successful acquisition index
1st mode	0%
2nd mode	96%
3rd mode	5%
4th mode	58%
5th mode	6%
6th mode	33%

Node 11075

Mode	Successful acquisition index
1st mode	80%
2nd mode	24%
3rd mode	80%
4th mode	21%
5th mode	80%
6th mode	75%

**Node 10814**

Mode	Successful acquisition index
1st mode	80%
2nd mode	27%
3rd mode	67%
4th mode	65%
5th mode	77%
6th mode	9%

Node 7884

Mode	Successful acquisition index
1st mode	79%
2nd mode	21%
3rd mode	70%
4th mode	40%
5th mode	55%
6th mode	74%

Node 12110

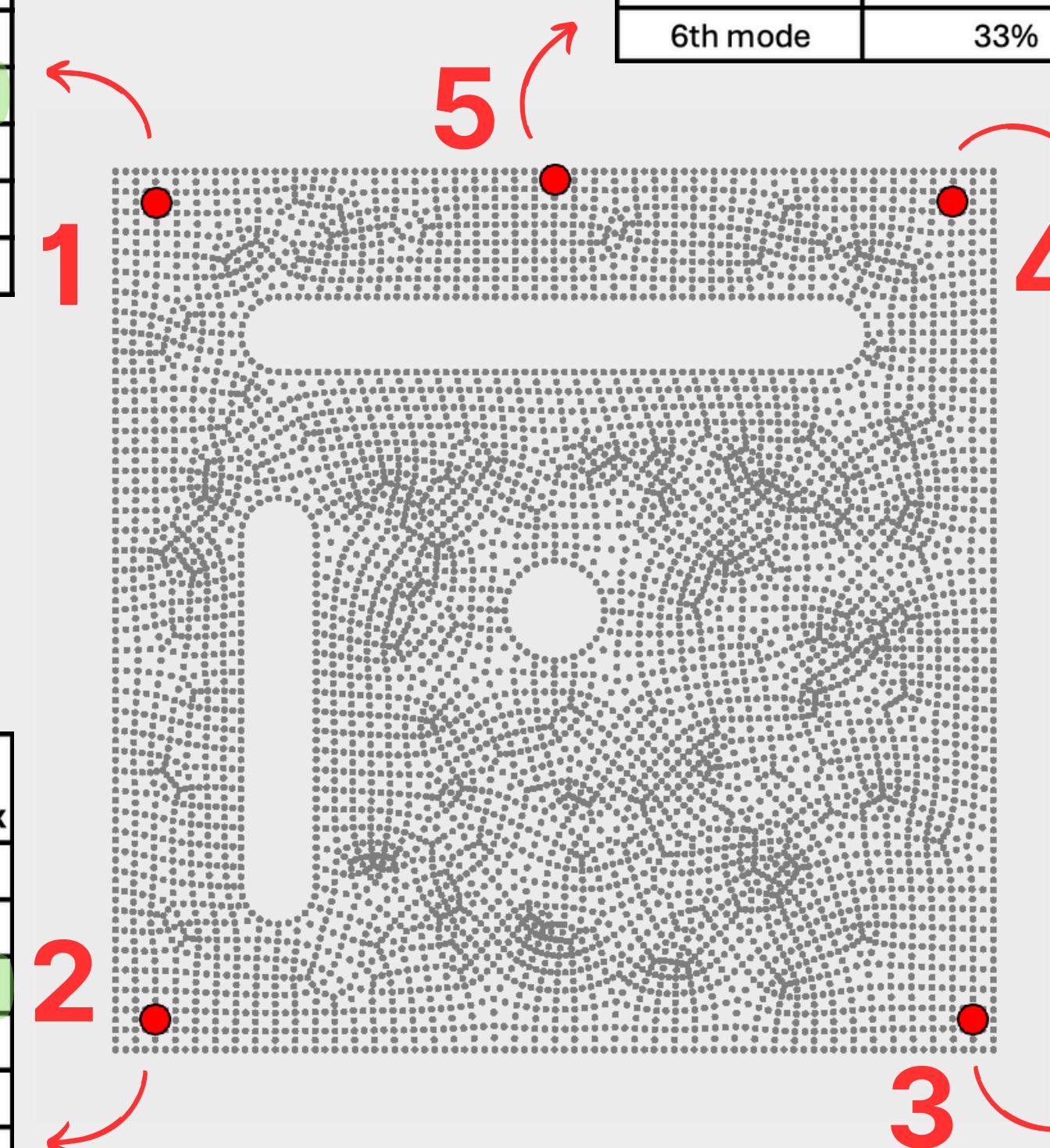
Mode	Successful acquisition index
1st mode	87%
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3rd mode	66%
4th mode	77%
5th mode	57%
6th mode	7%

Node 11299

Mode	Successful acquisition index
1st mode	0%
2nd mode	96%
3rd mode	5%
4th mode	58%
5th mode	6%
6th mode	33%

Node 11075

Mode	Successful acquisition index
1st mode	80%
2nd mode	24%
3rd mode	80%
4th mode	21%
5th mode	80%
6th mode	75%

**Node 10814**

Mode	Successful acquisition index
1st mode	80%
2nd mode	27%
3rd mode	67%
4th mode	65%
5th mode	77%
6th mode	9%

Node 7884

Mode	Successful acquisition index
1st mode	79%
2nd mode	21%
3rd mode	70%
4th mode	40%
5th mode	55%
6th mode	74%



Data Analysis Assignment a.a. 2024/2025

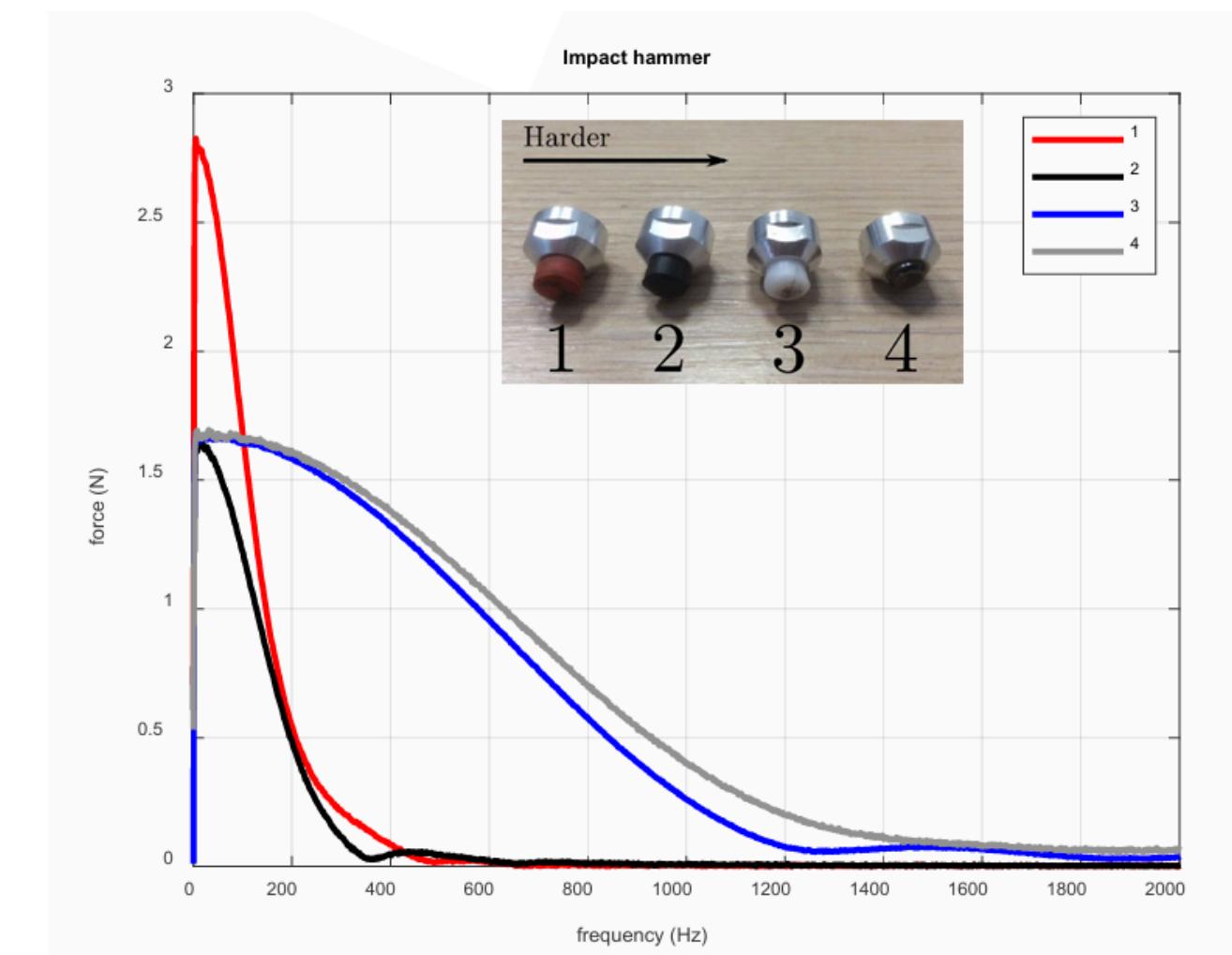
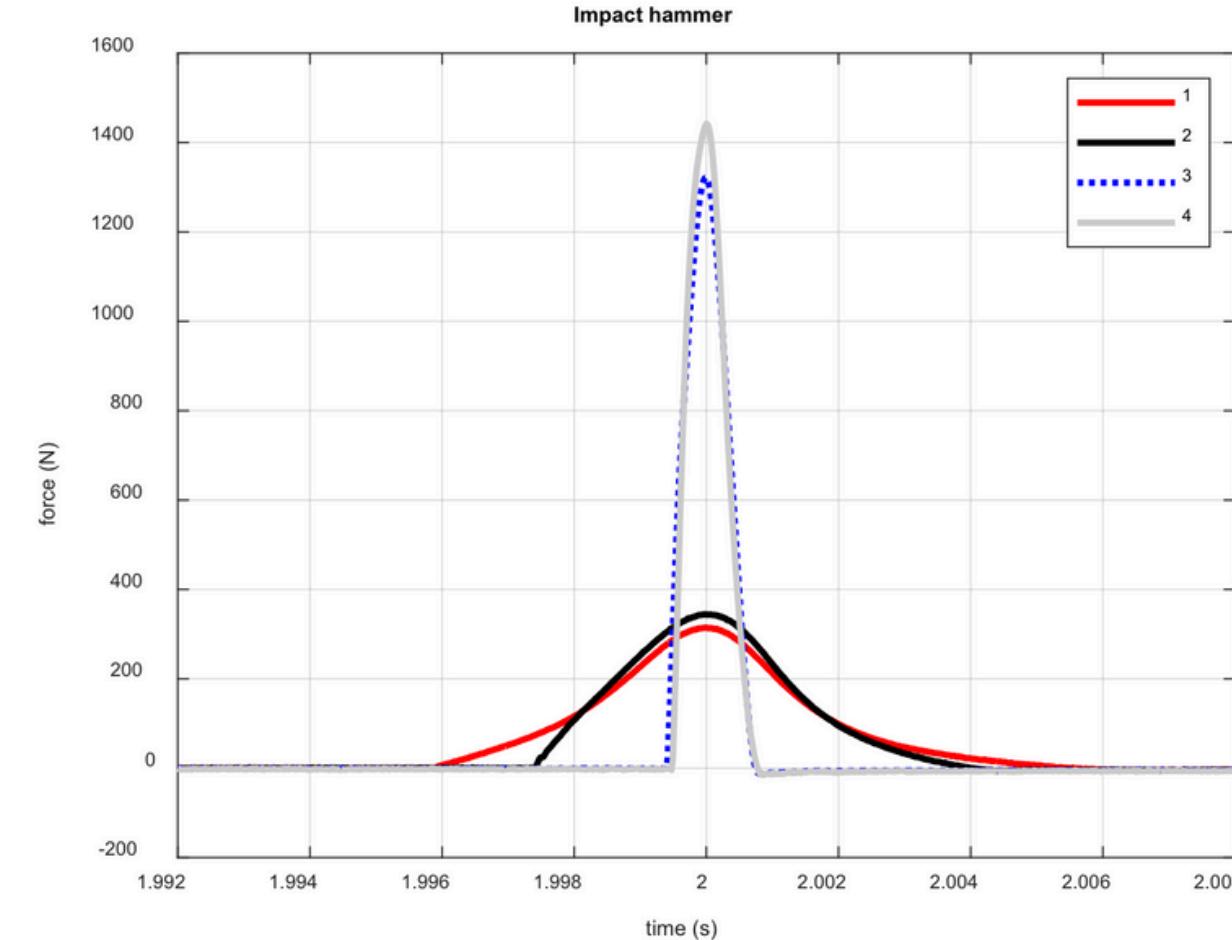
EXPERIMENTAL TESTS

INPUT FORCE, EXCITER

Type:

**Hammer (non attached
exciter)**

Sensitivity: 2.488 mV/N



Advantages:

- fastest approach
 - no load effect
 - cheap
-

Drawbacks

- input force control
- spectrum control

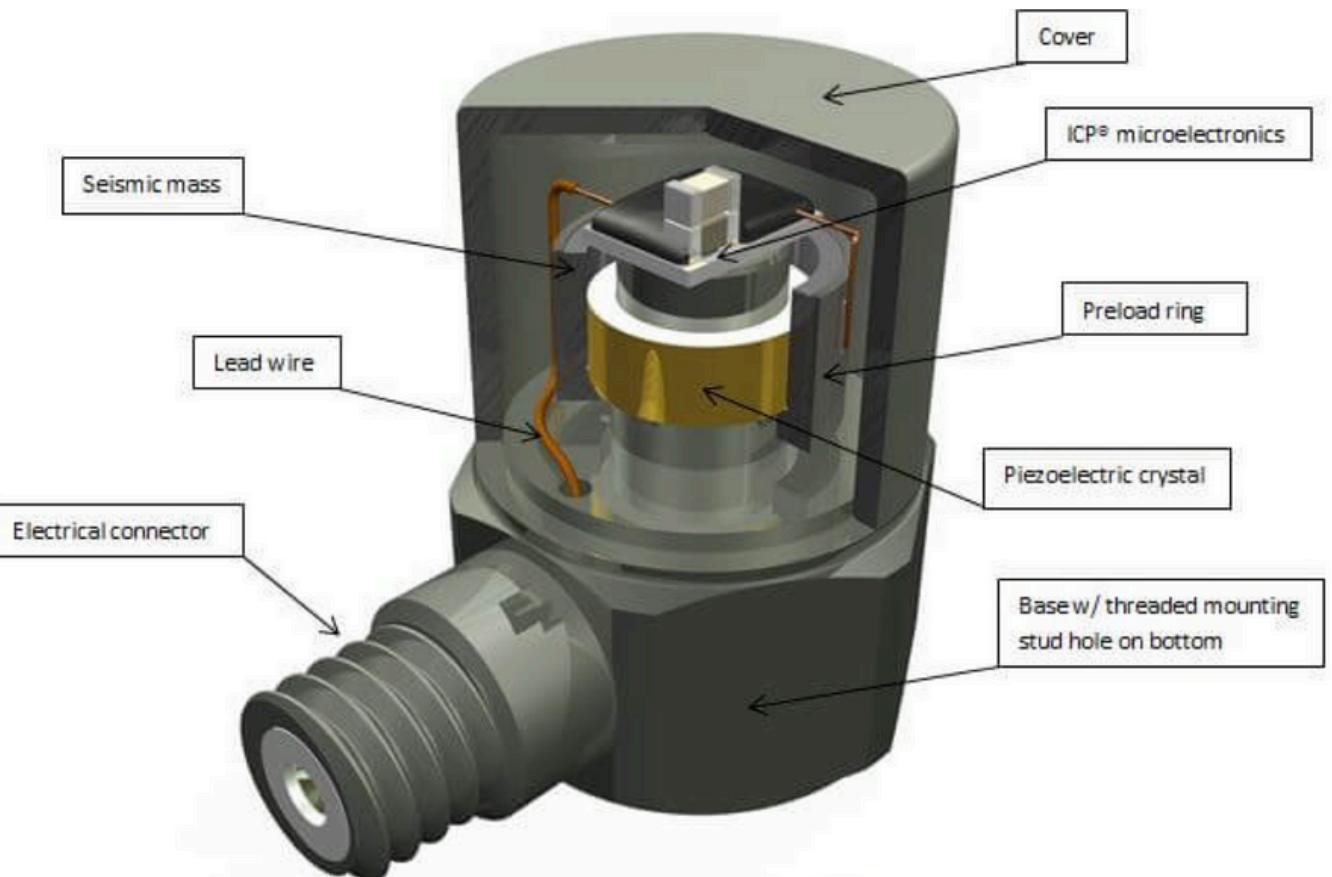
RESPONSE MEASUREMENT

Type:

Piezoelectric accelerometer

Sensitivity: $10.2 \text{ mV}/(\text{m/s}^2)$

Frequency range: 0.5 - 3000 Hz

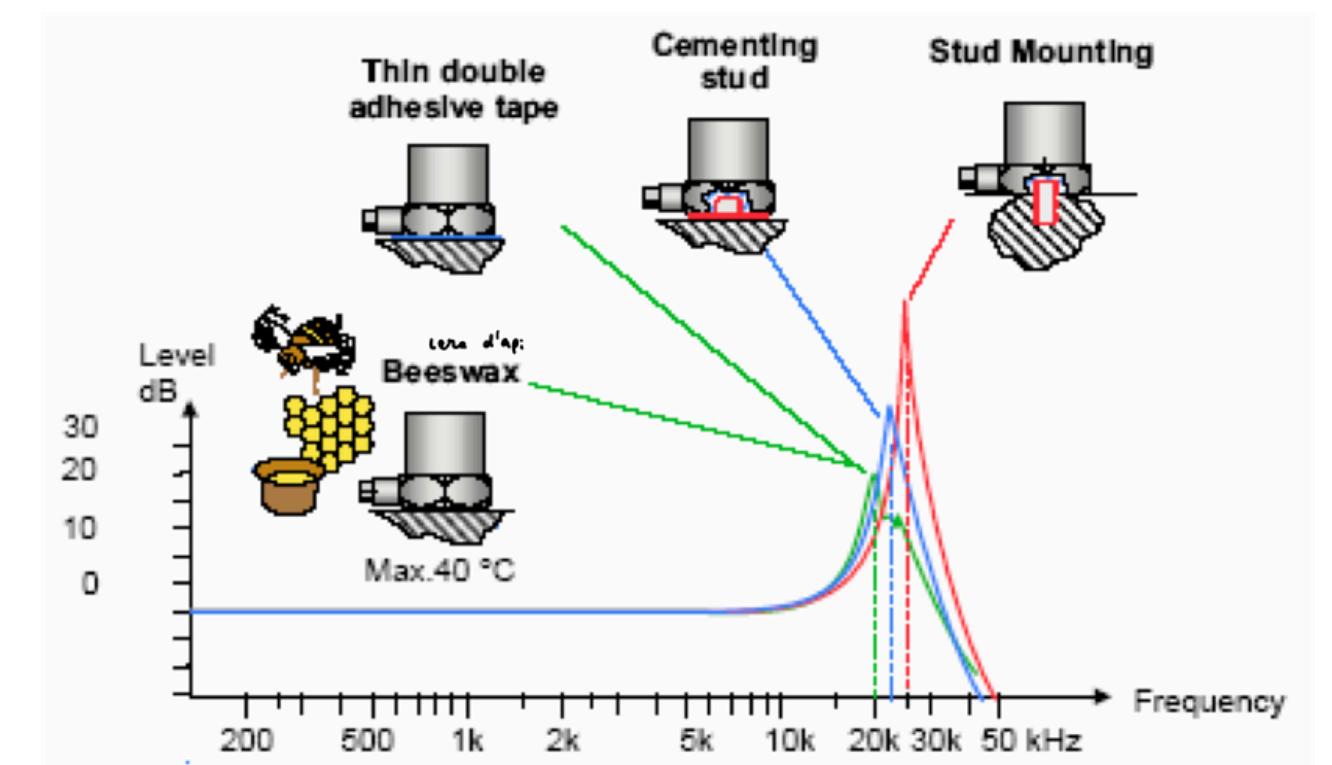


Mounting:

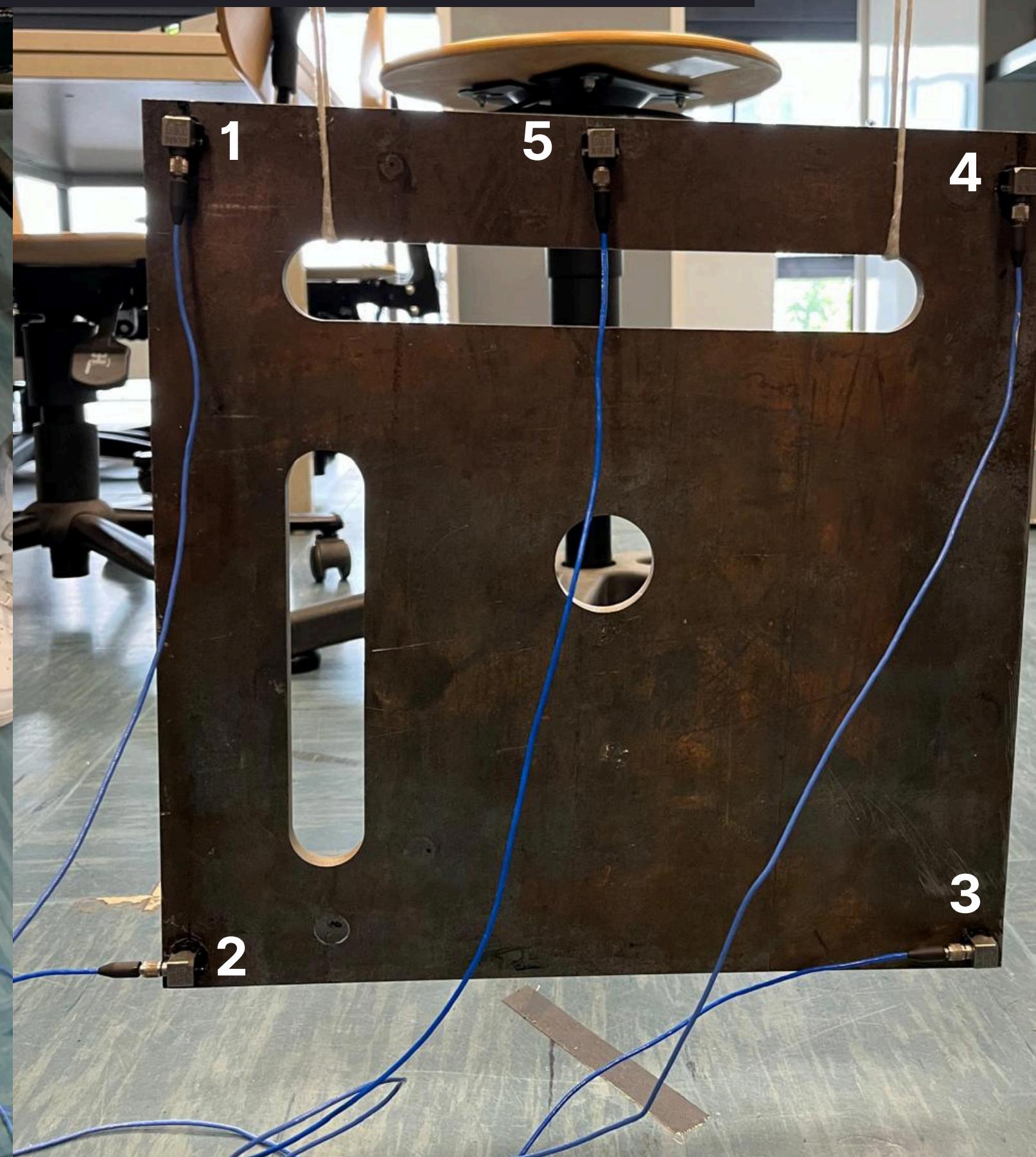
Glue

Advantages:

- not expensive
- absolute measurements



EXPERIMENTAL SETUP



MAIN CHALLENGES DURING ACQUISITION

PROBLEM

SOLUTION

ALIASING



SAMPLING FREQUENCY: 2500 Hz

ECCITATION FREQUENCY
RANGE



PROPER CHOICE OF THE
HAMMER TIP: PLASTIC,
NUMBER 3

HAMMER DOUBLE HITS
CO-LOCATED HITS
SATURATION OF ACCELEROMETERS



EXPERIENCE AND
PATIENCE



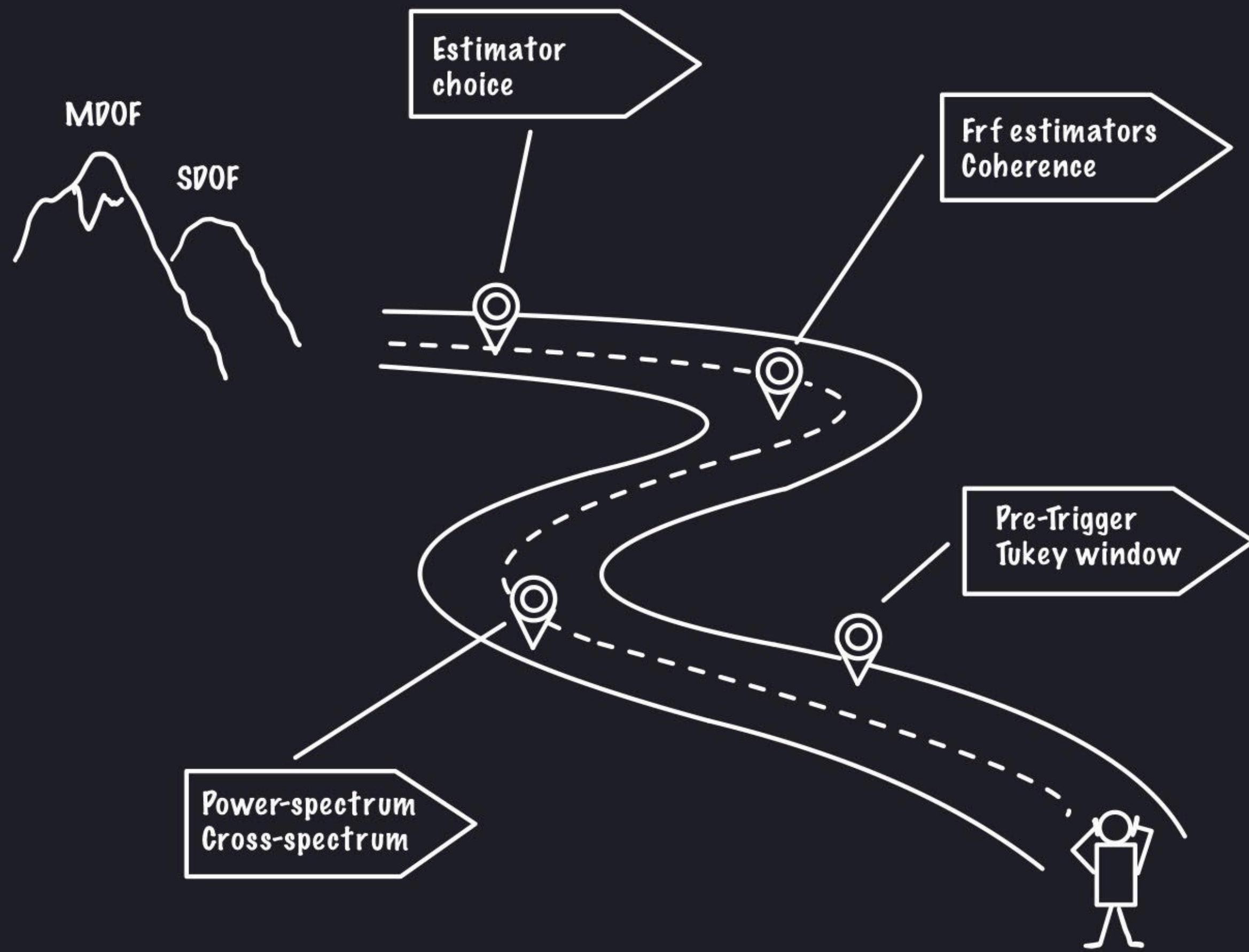
EXPERIMENTAL CAMPAIGN

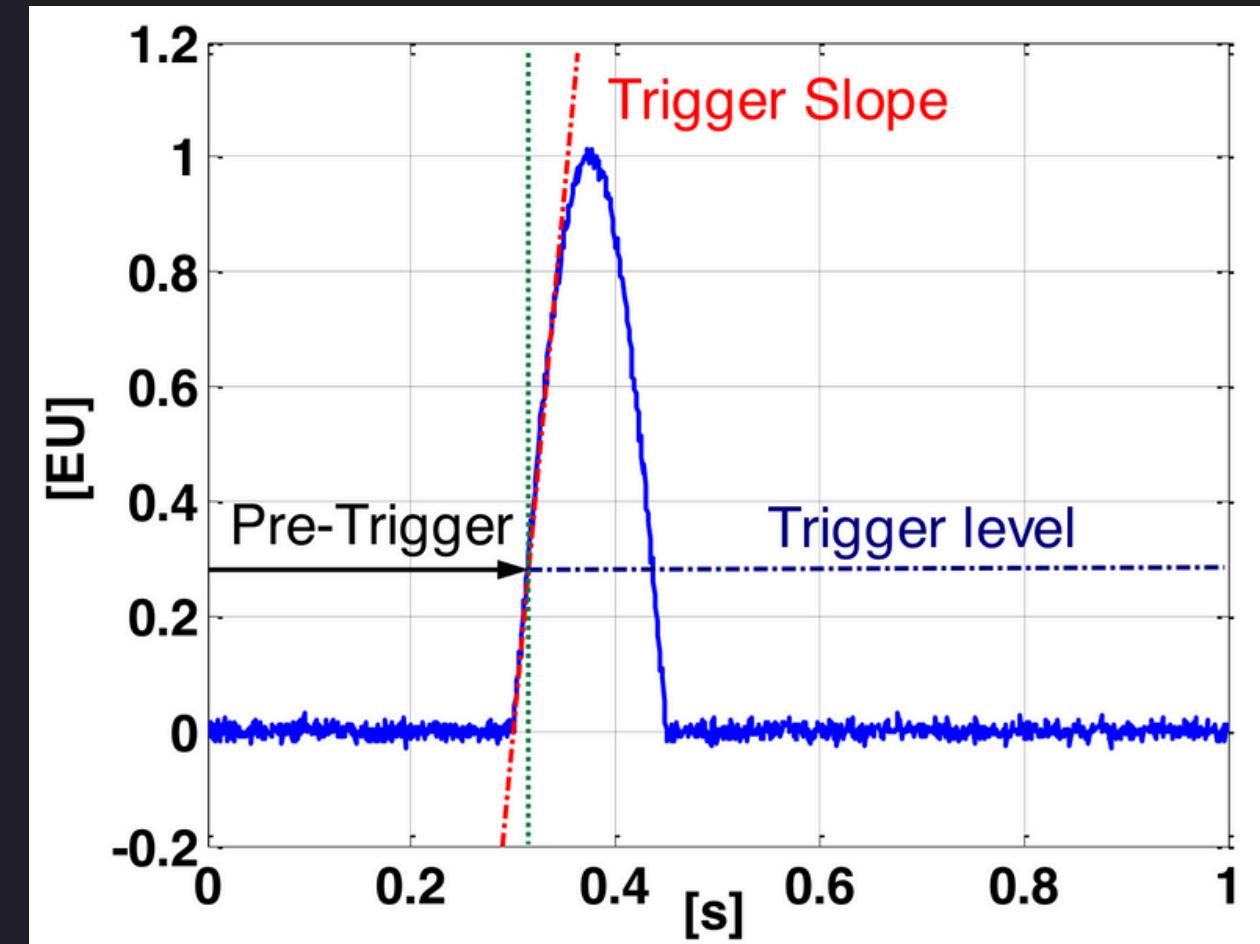
- Looked for the decay time, to decide about exponential window and acquisition time →
 $T = 25\text{ s}$
- 10 impacts for each sensor position, needed for the averages



Data Analysis Assignment a.a. 2024/2025

DATA PREPROCESSING

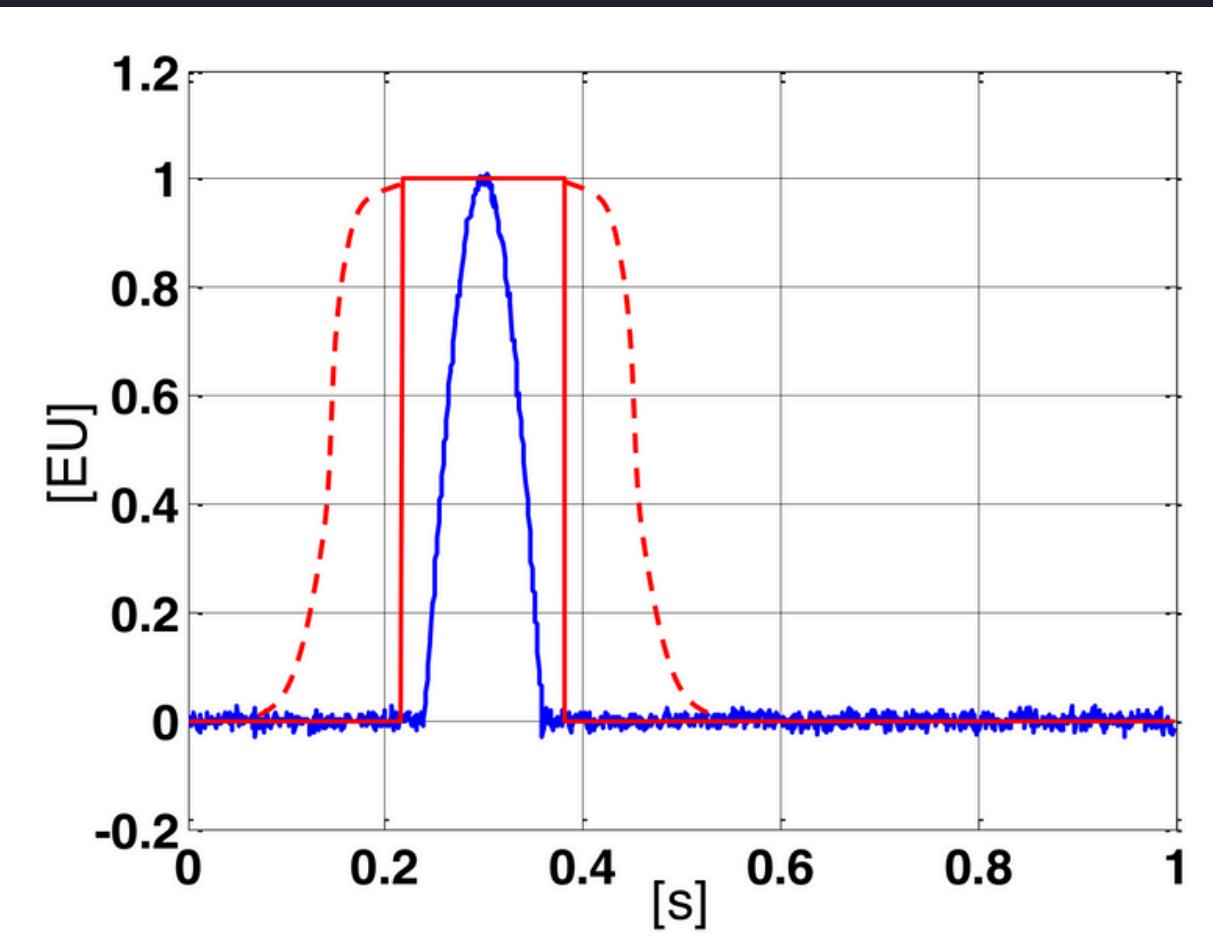


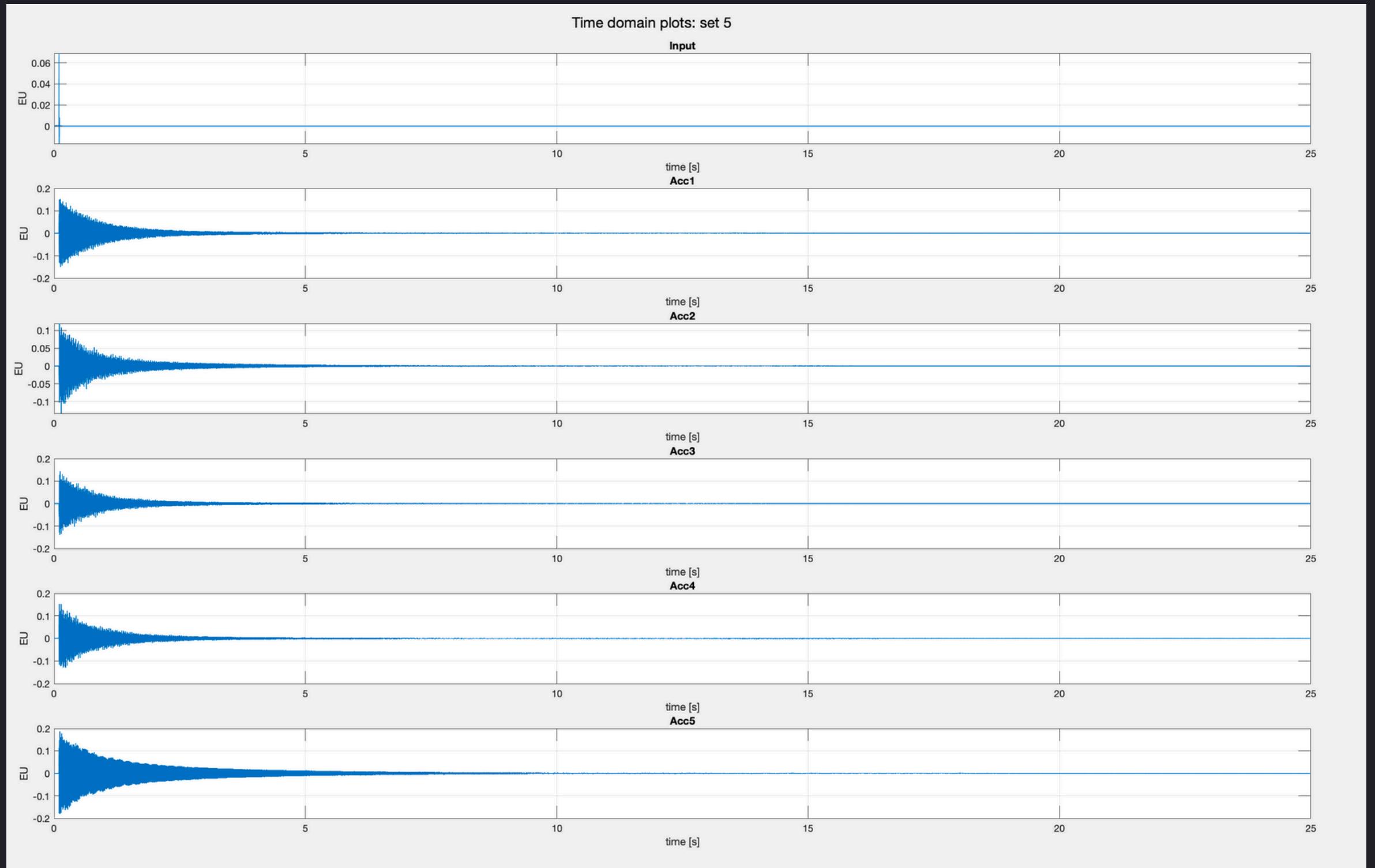


Pre Trigger: 0.1 s

STEP 01

SENSITIVITY - PRE TRIGGER -
TUKEY WINDOW



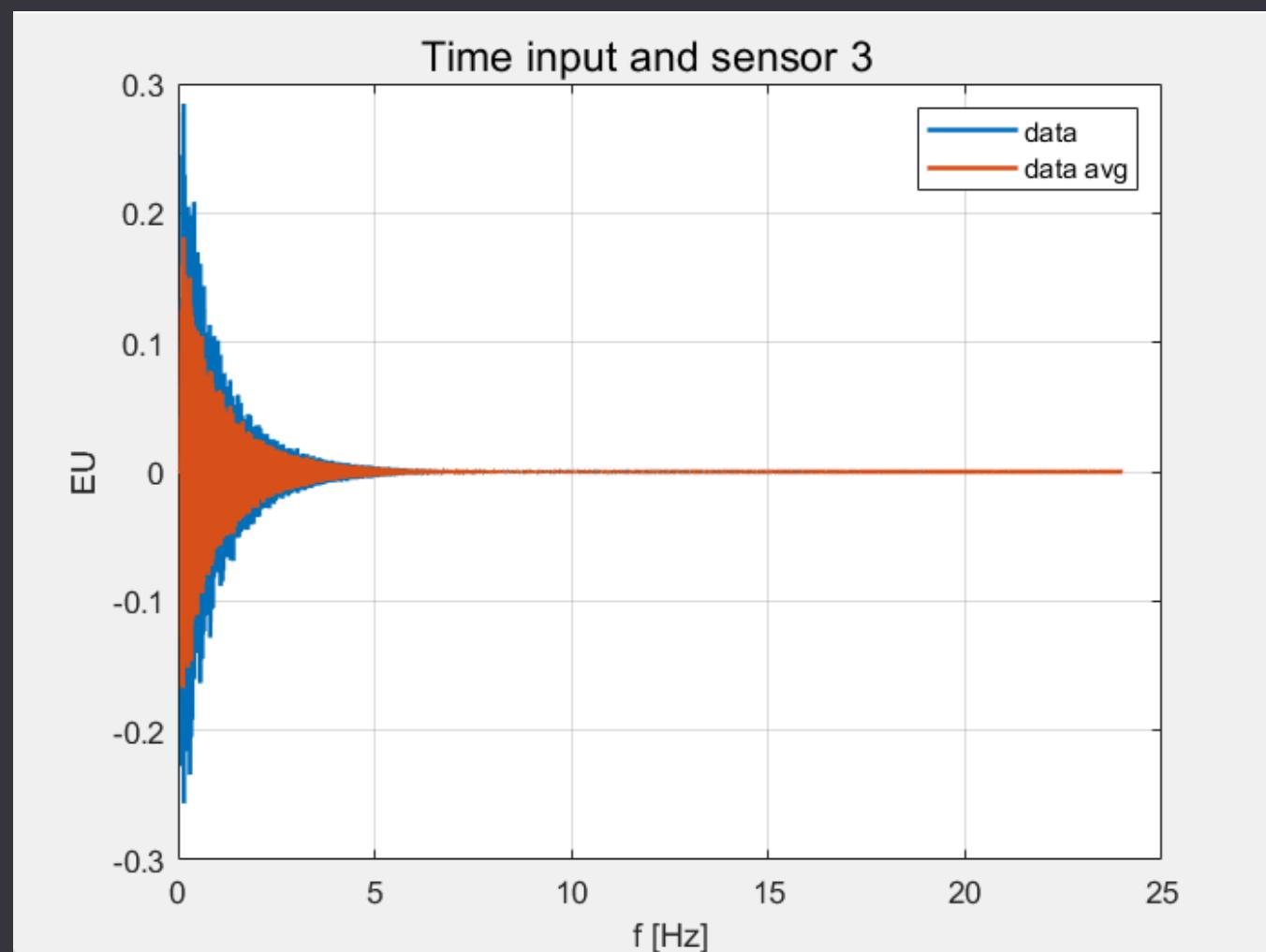
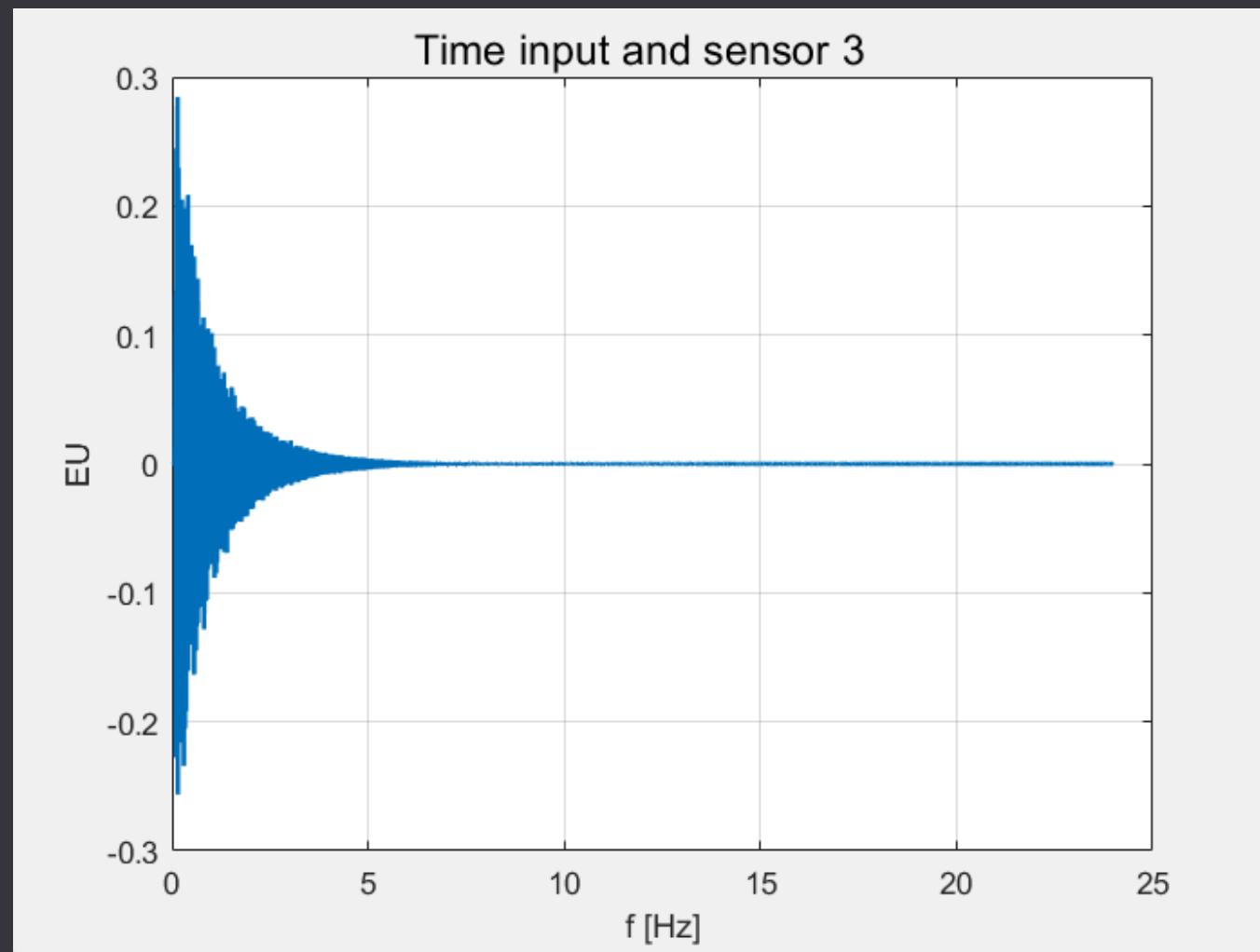


STEP 01

- Outputs from the sensors
 - 25 sec time acquisition

TIME AVERAGING

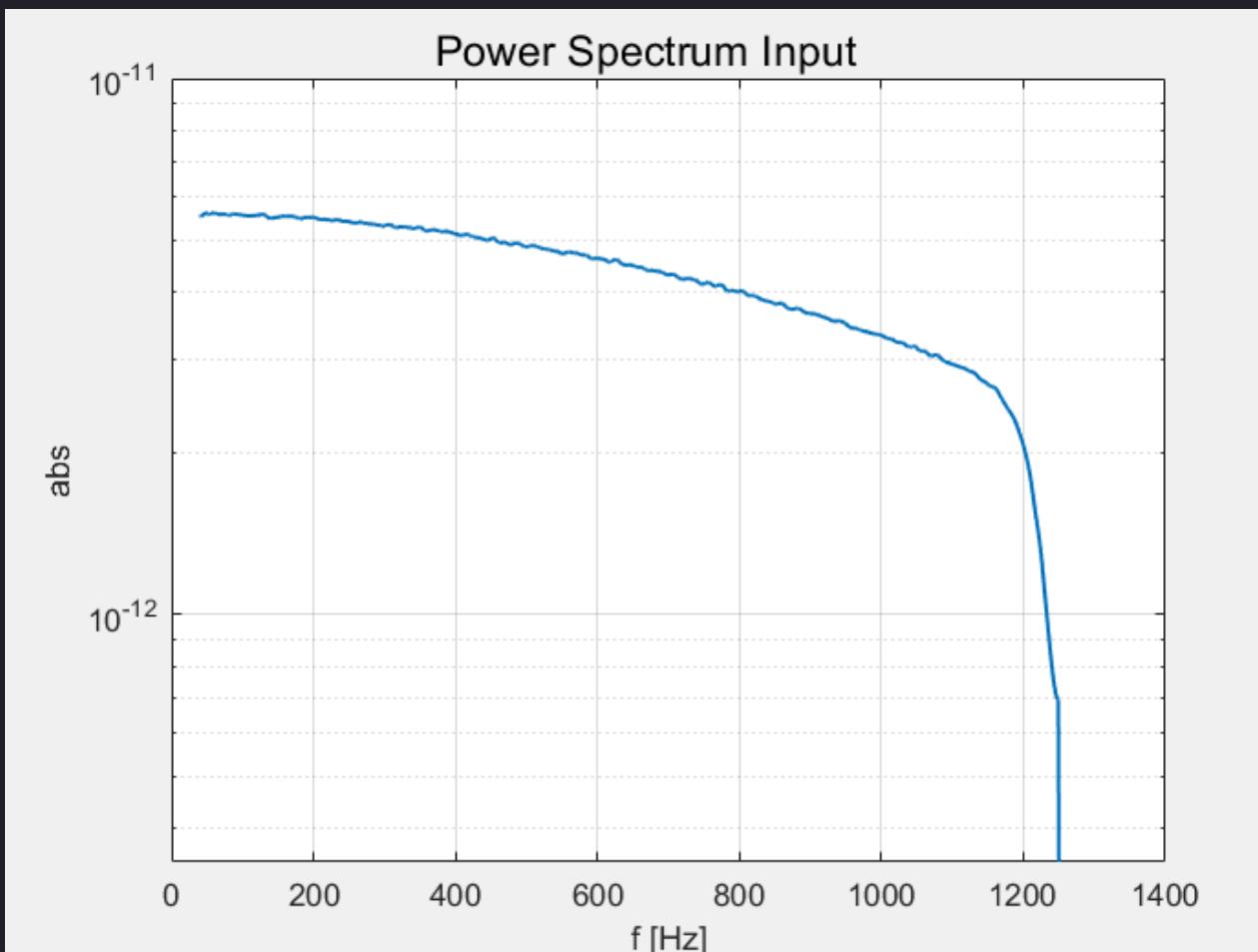
10 averages for each test to
reduce noise



STEP 02

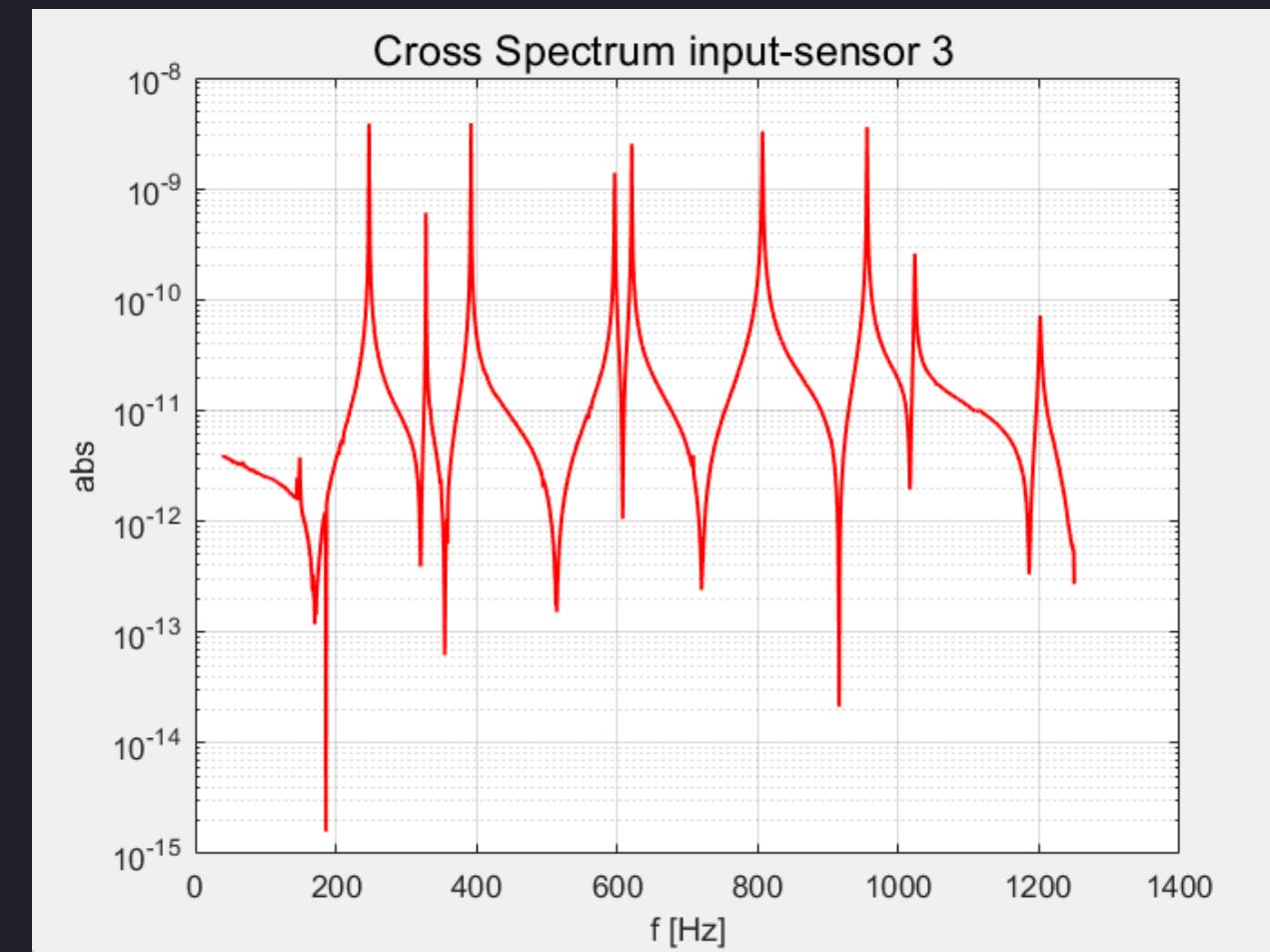
Power-spectrum

$$S_{AA}(f) = A^*(f) A(f)$$



Cross-spectrum

$$S_{AB}(f) = A^*(f) B(f)$$



STEP 03

FRF Estimators and Coherence

H1

$$H(f) = \frac{Y(f)}{X(f)} \equiv \frac{G_{xy}(f)}{G_{xx}(f)} = H_1(f)$$

H2

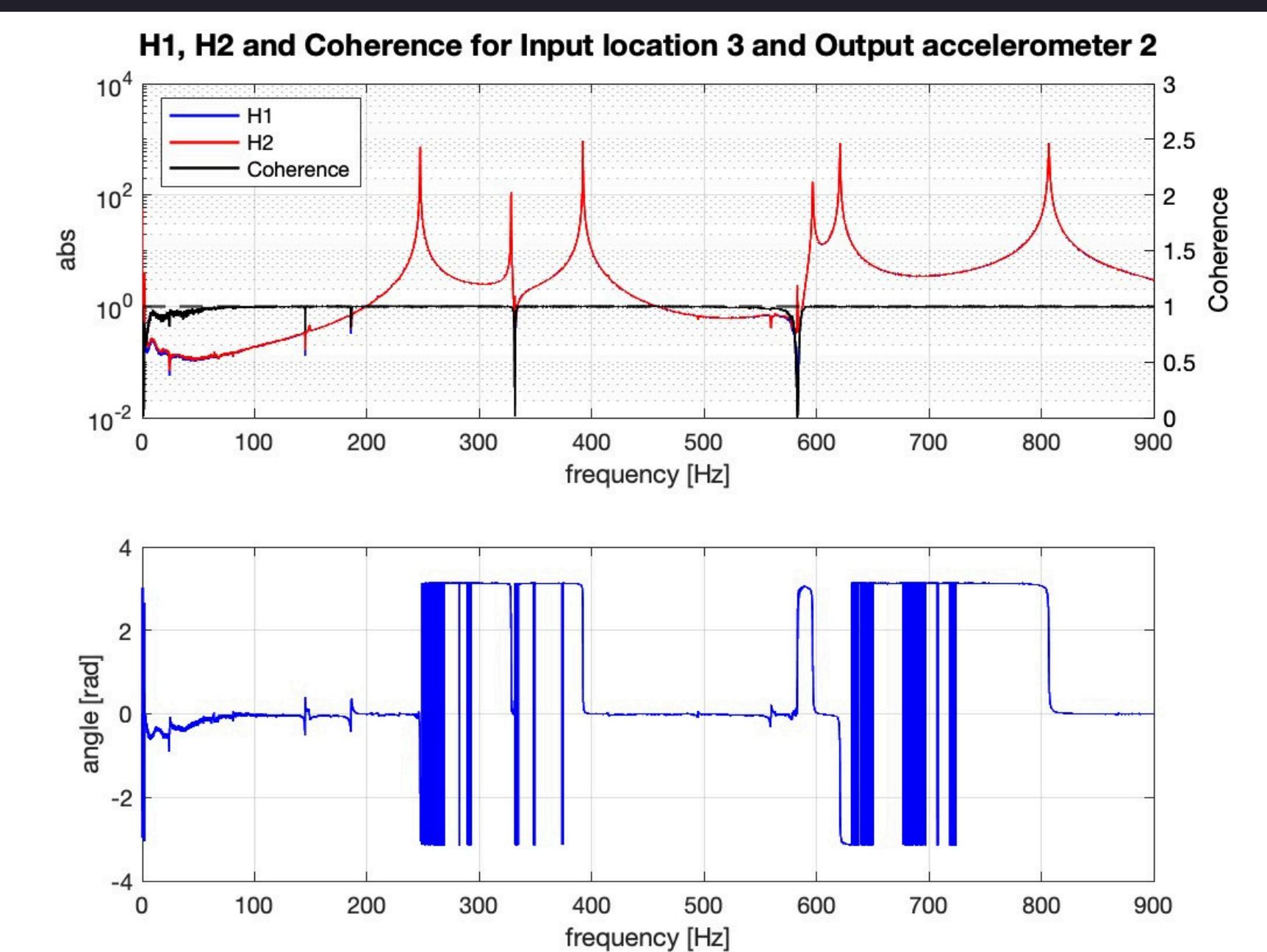
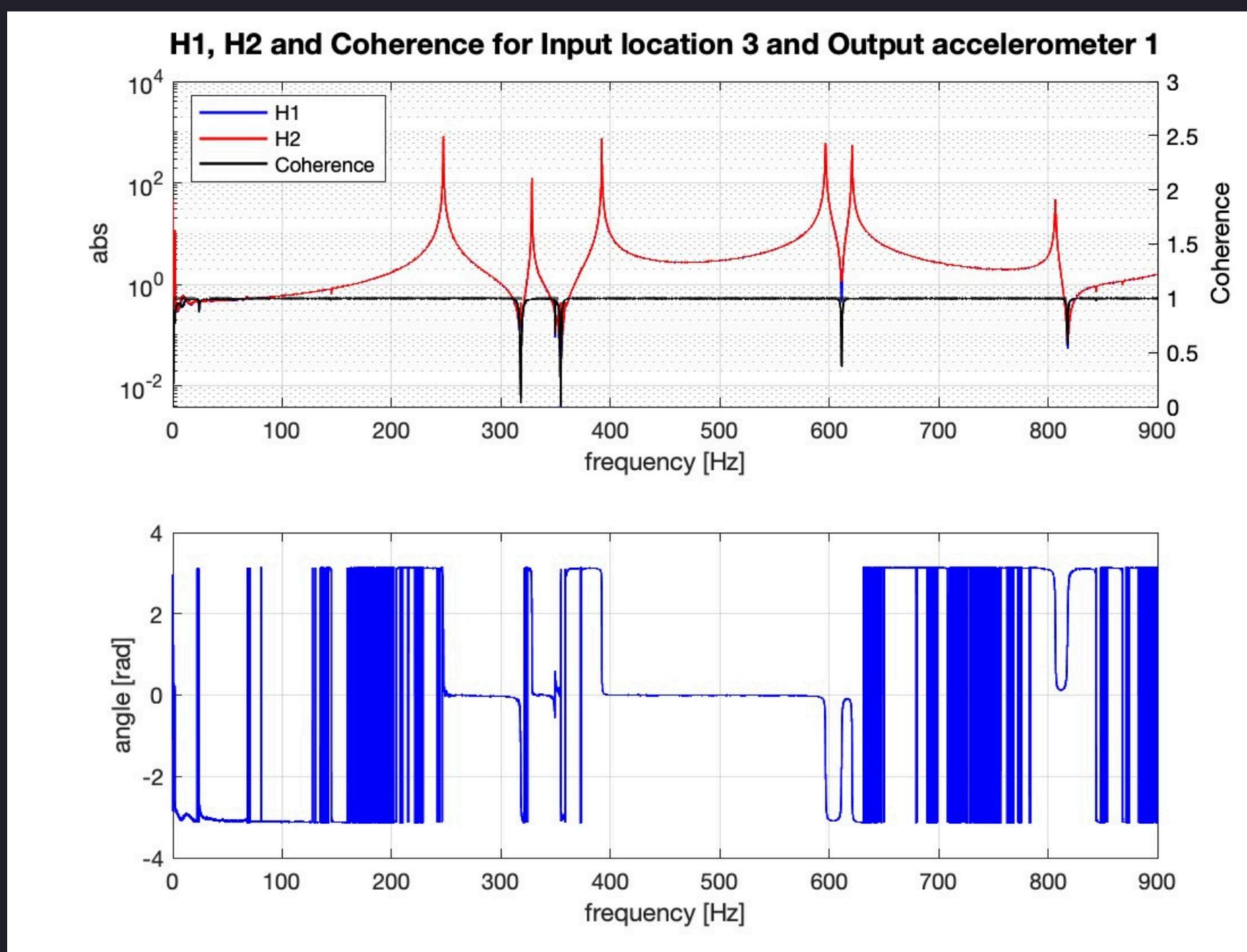
$$H(f) = \frac{Y(f)}{X(f)} \equiv \frac{G_{yy}(f)}{G_{yx}(f)} = H_2(f)$$

Coherence

$$\gamma_{AB}^2(f_k) = \frac{|G_{AB}(f_k)|^2}{G_{AA}(f_k) G_{BB}(f_k)} = \frac{|S_{AB}(f_k)|^2}{S_{AA}(f_k) S_{BB}(f_k)}$$

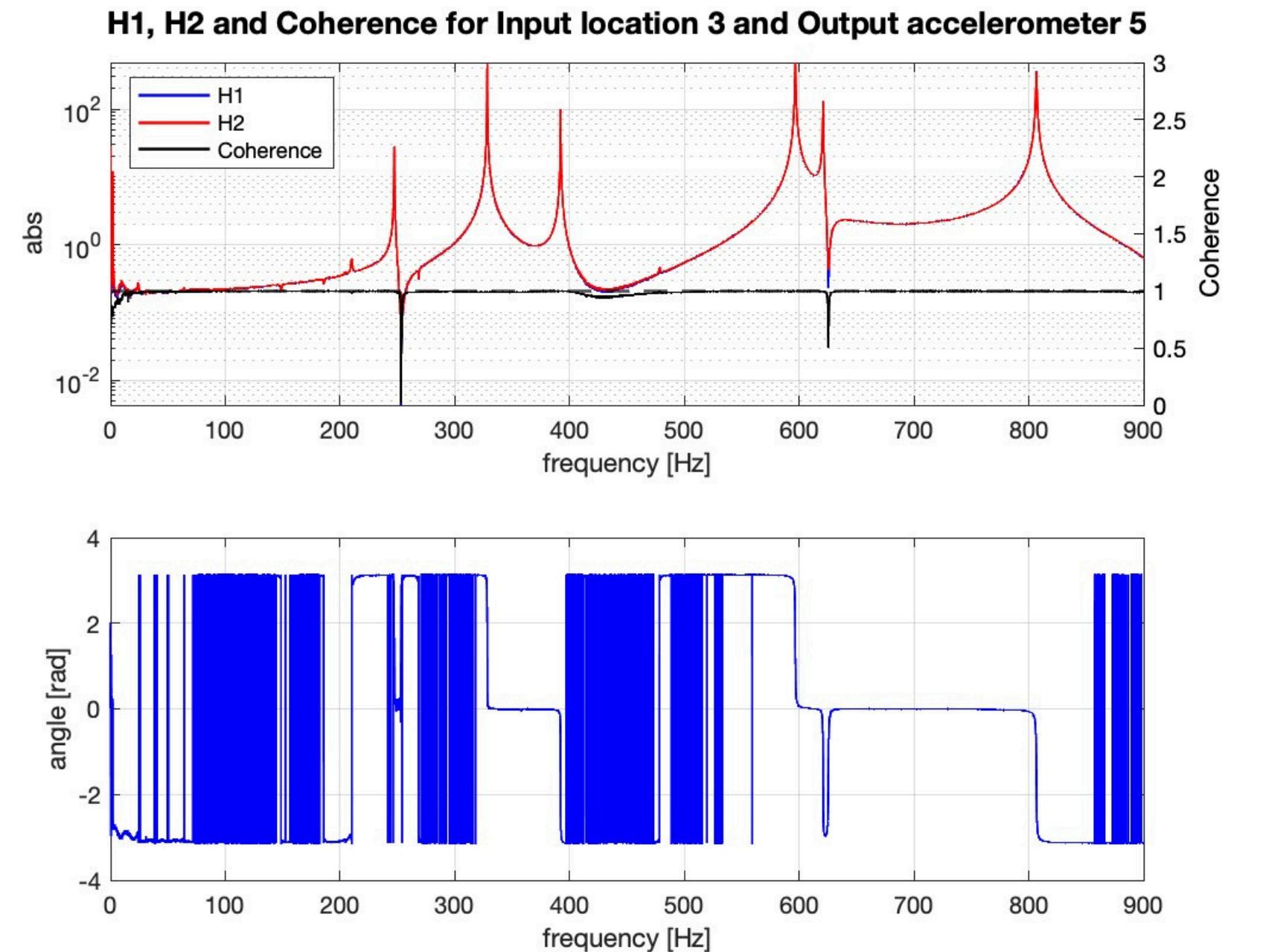
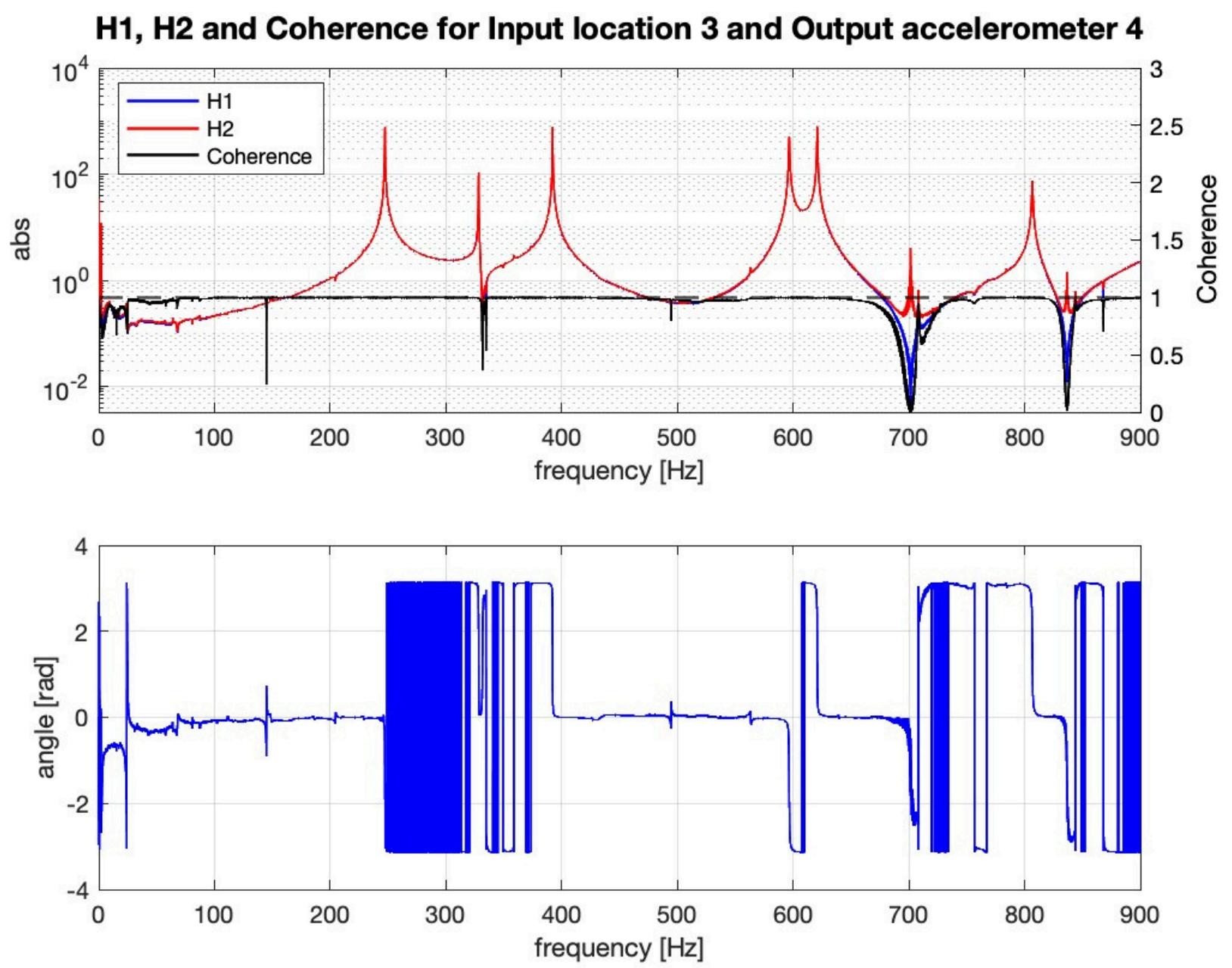
Non co-located FRFs (Input Location 3)

STEP 03

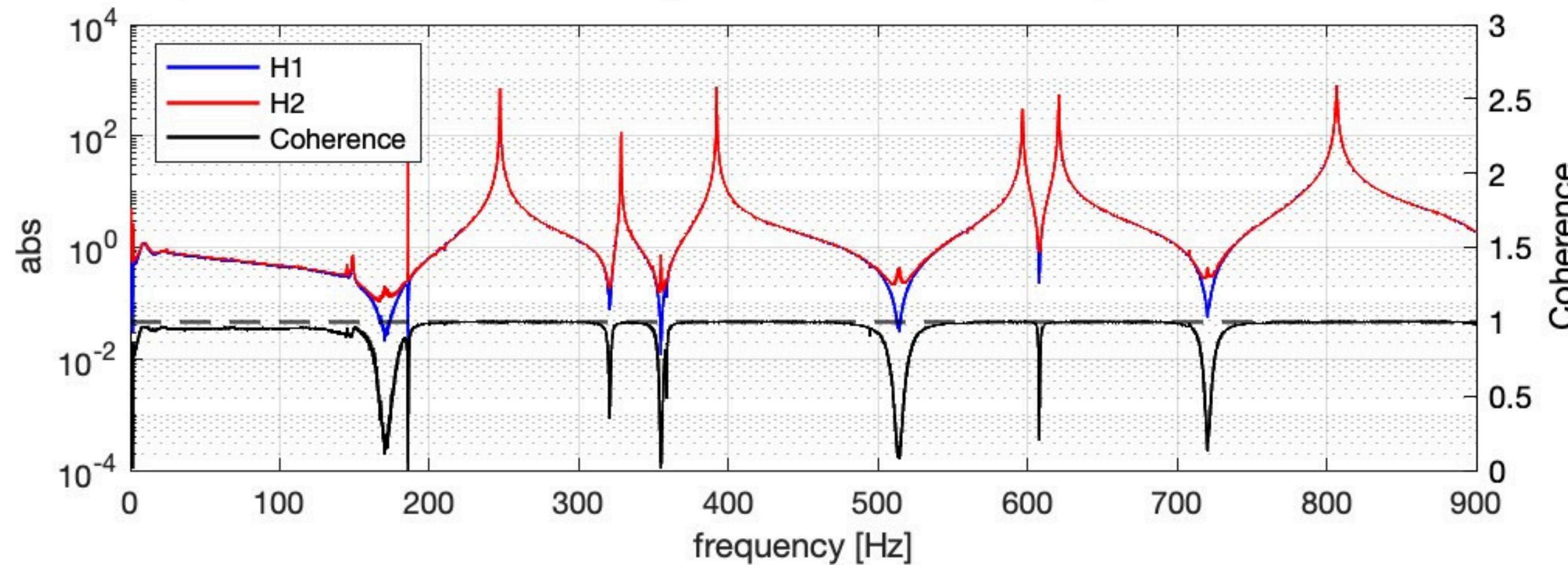


Non co-located FRFs (Input Location 3)

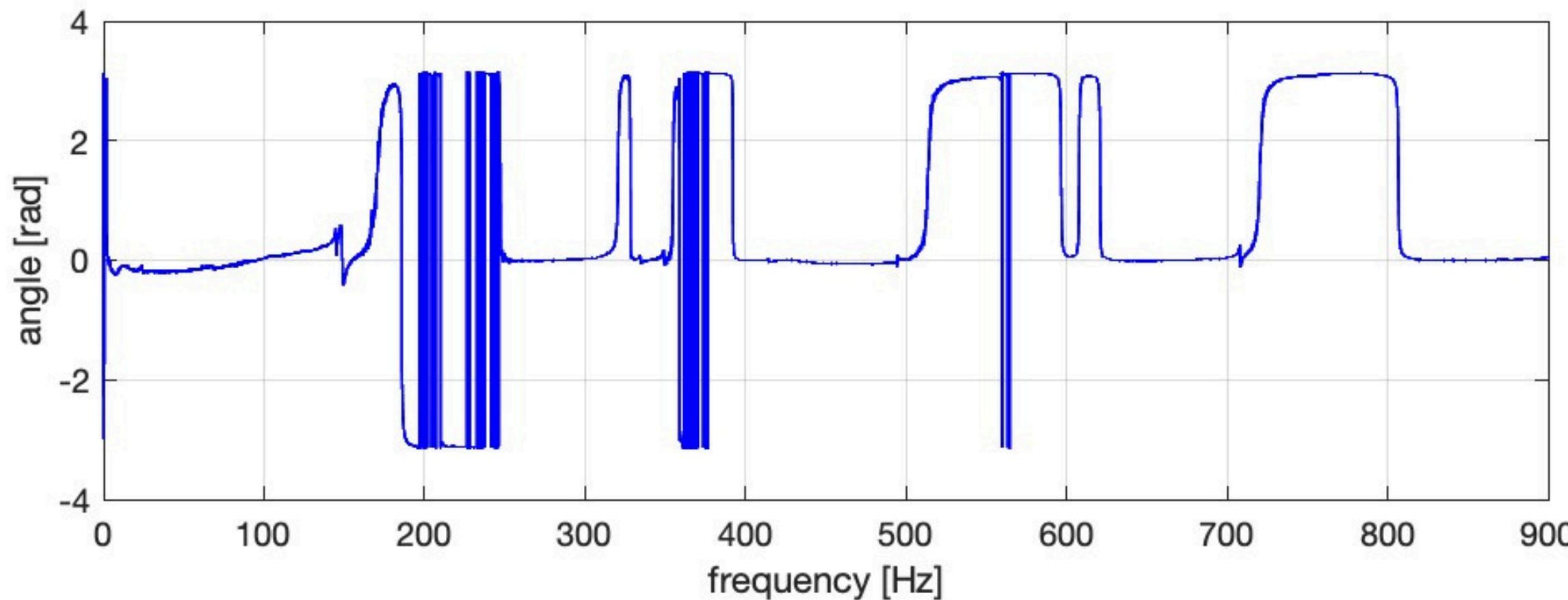
STEP 03



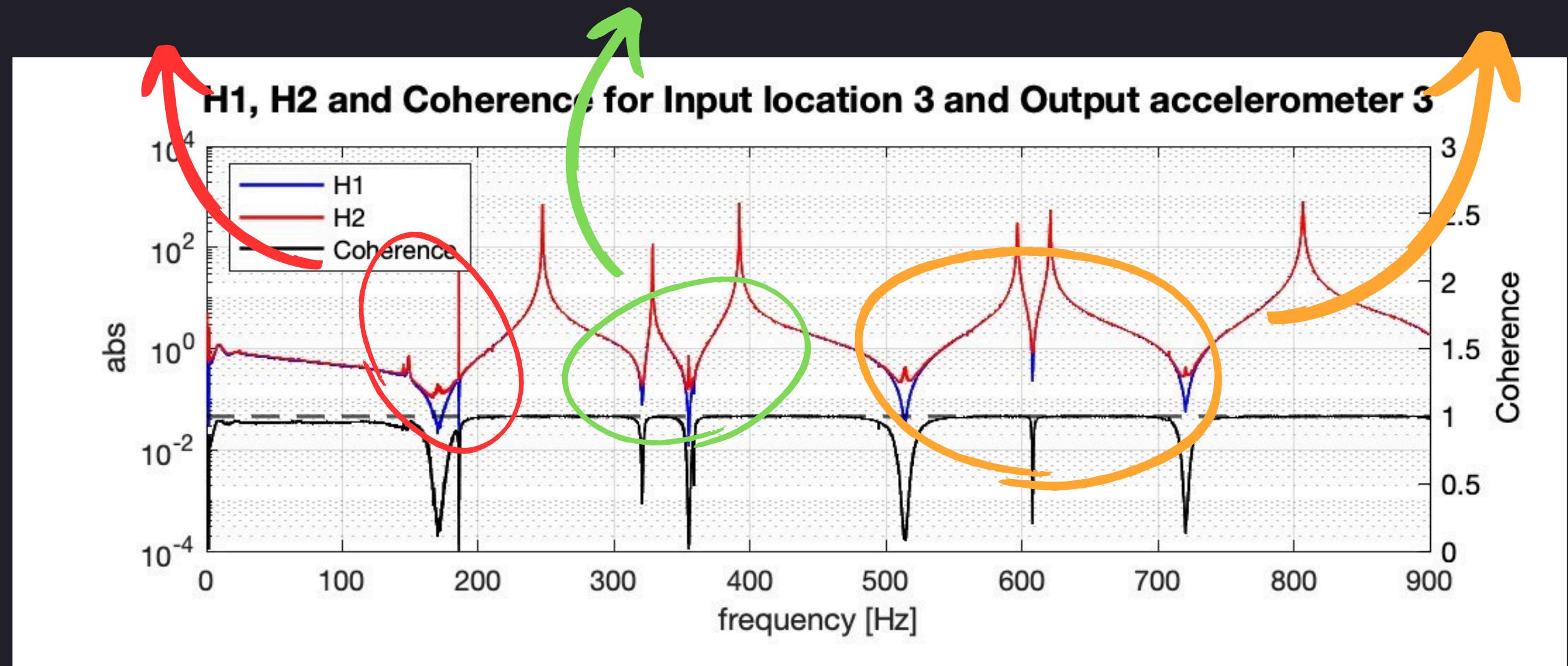
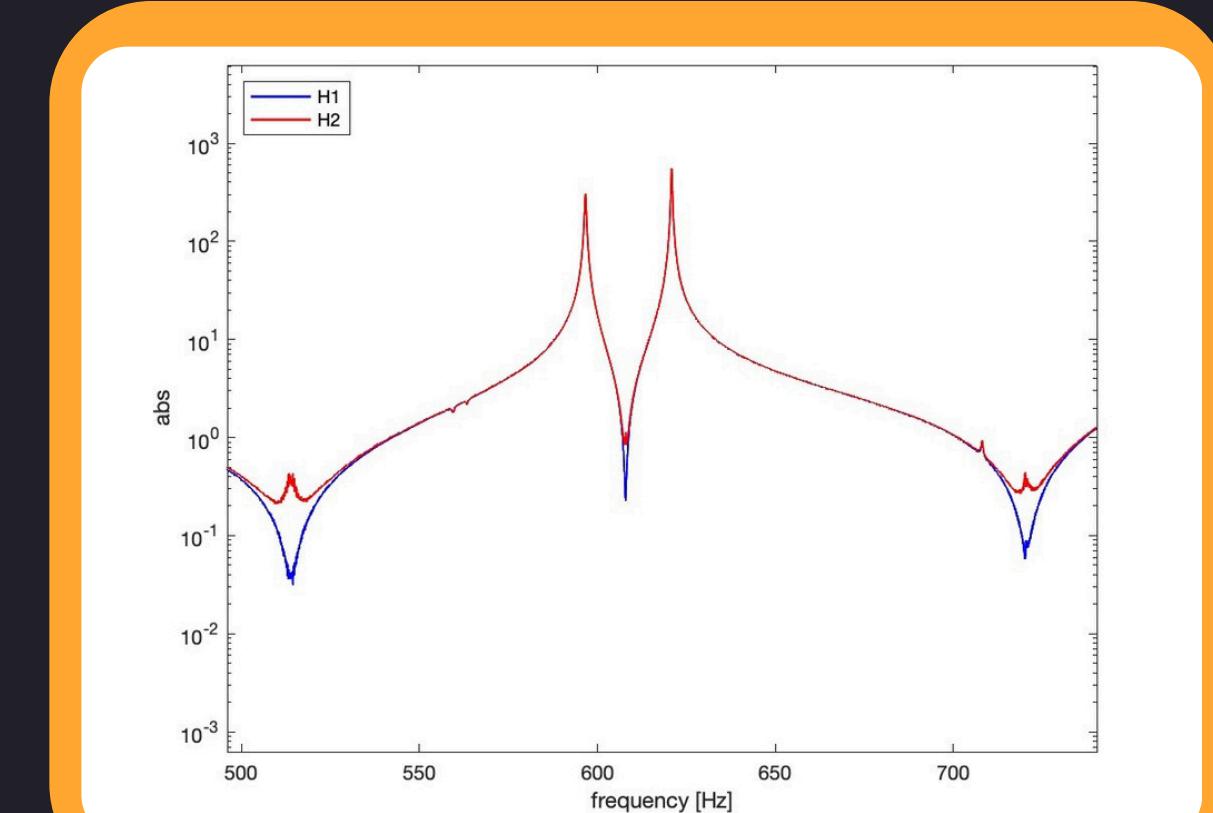
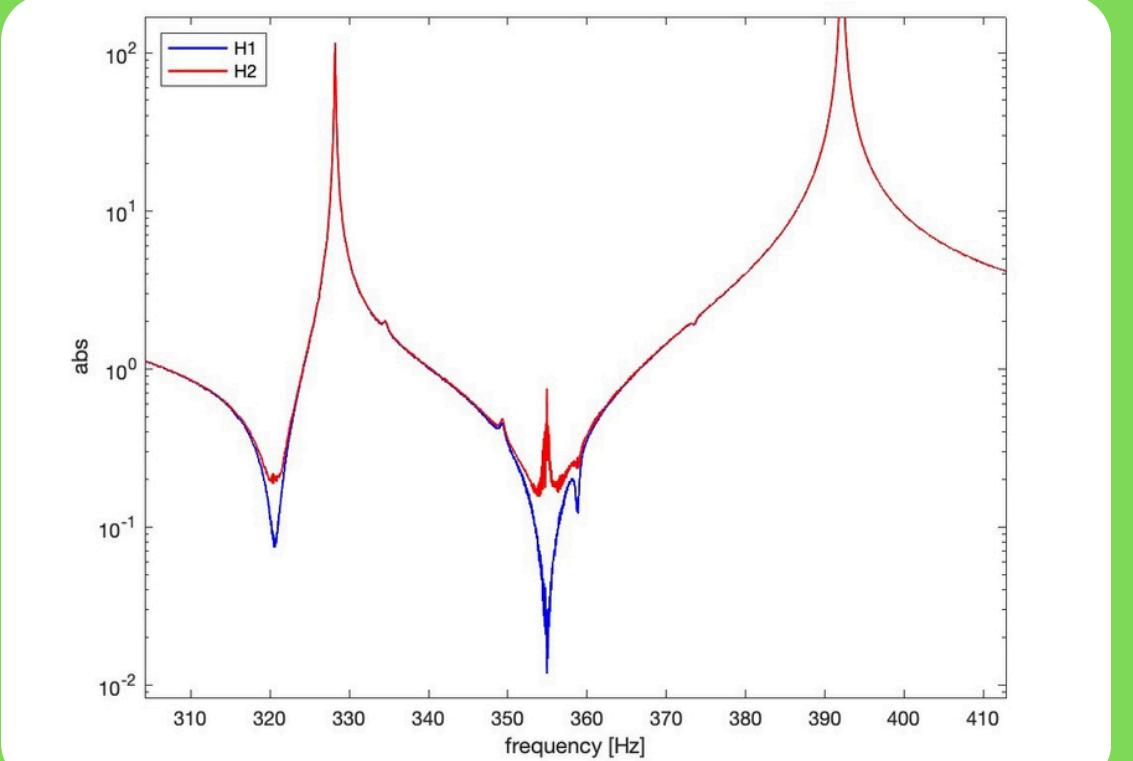
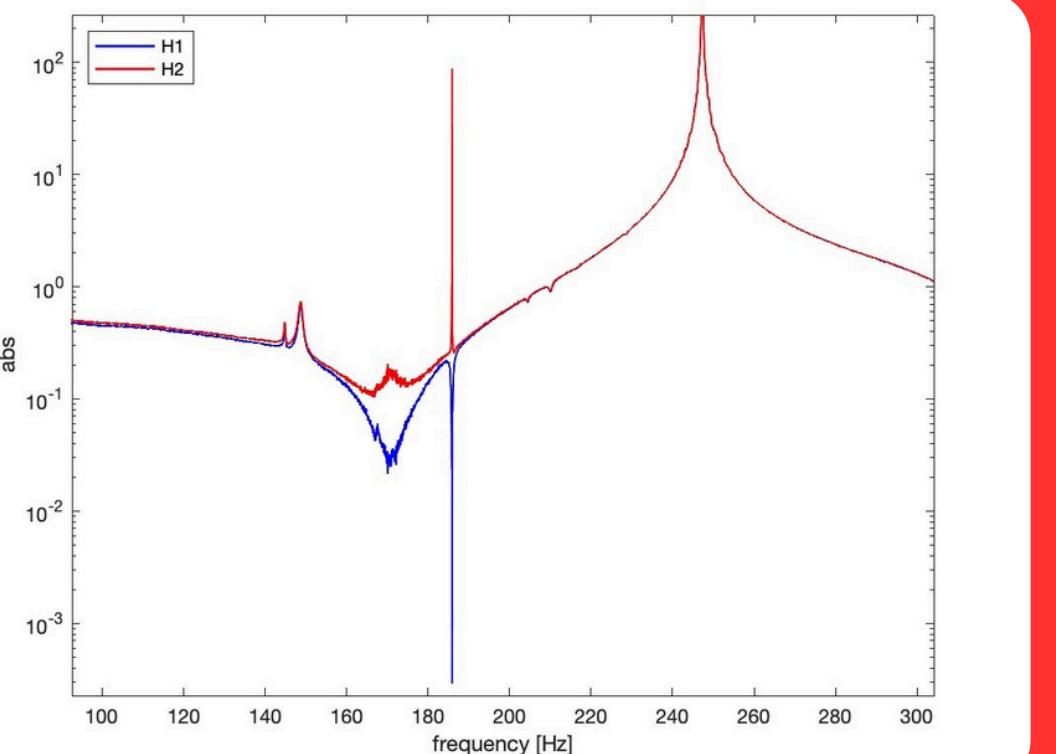
H1, H2 and Coherence for Input location 3 and Output accelerometer 3



STEP 03



Co-located FRF (Input Location 3)

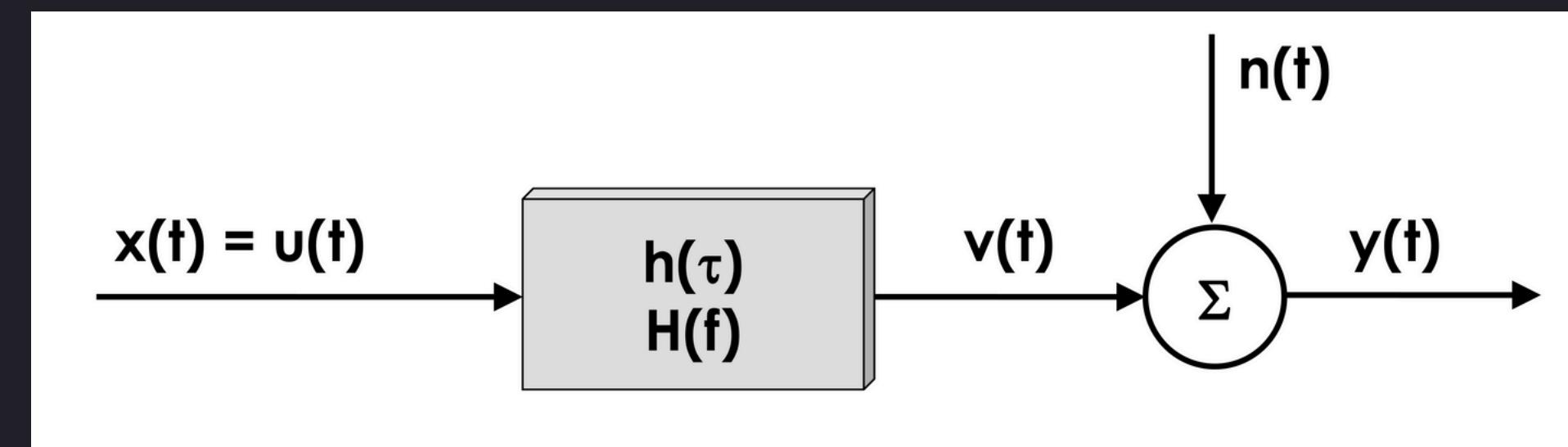


STEP 04

Choice of the estimator

Which one is better?

Noise at the output:



$H_1(f)$ allows to get a correct estimate of the FRFs

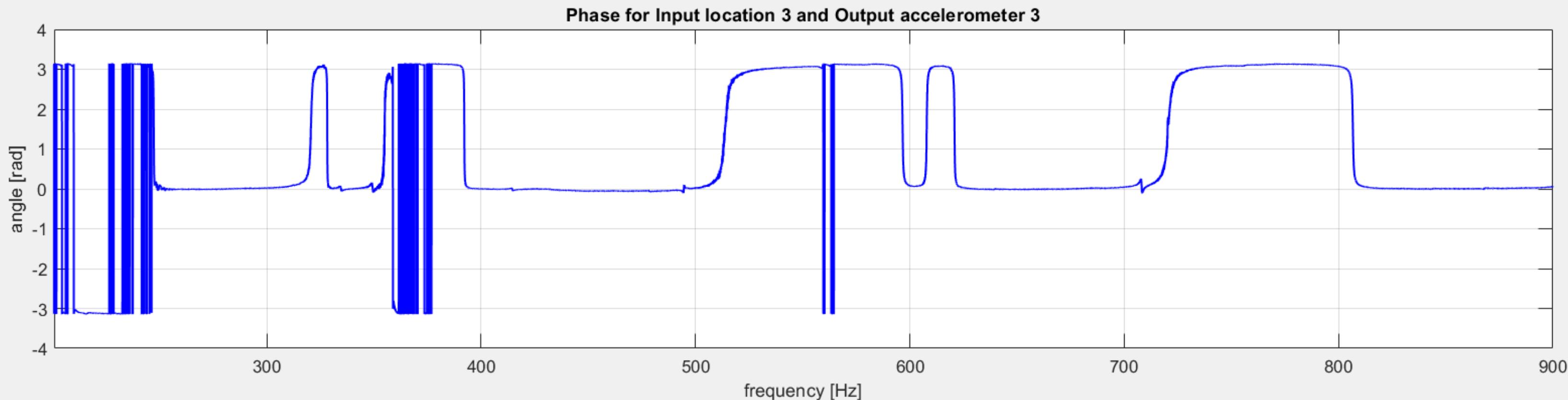
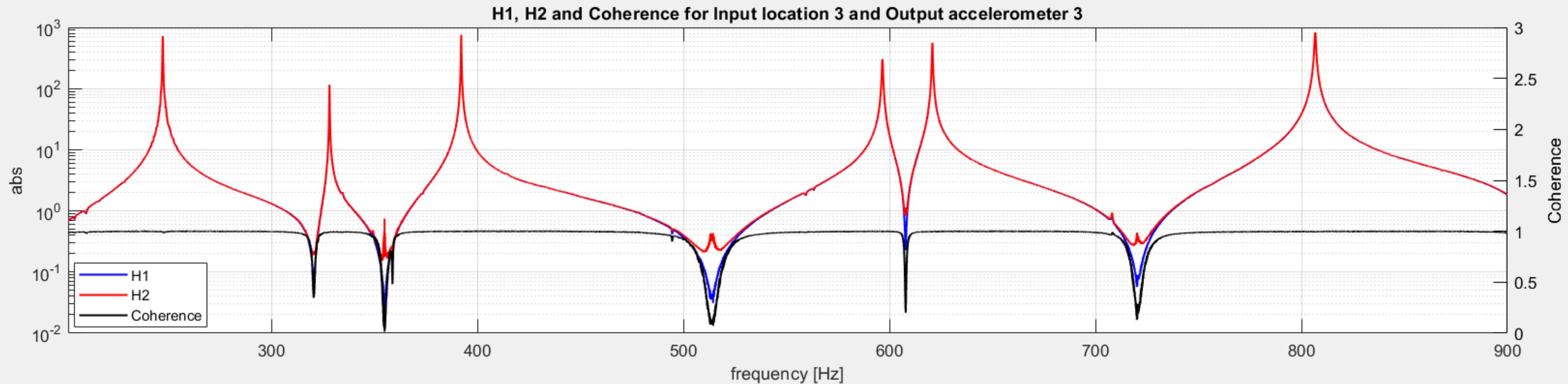


Data Analysis Assignment a.a. 2024/2025

SDOF

Single degree of freedom method for modal analysis

FRF of the 3rd accelerometer

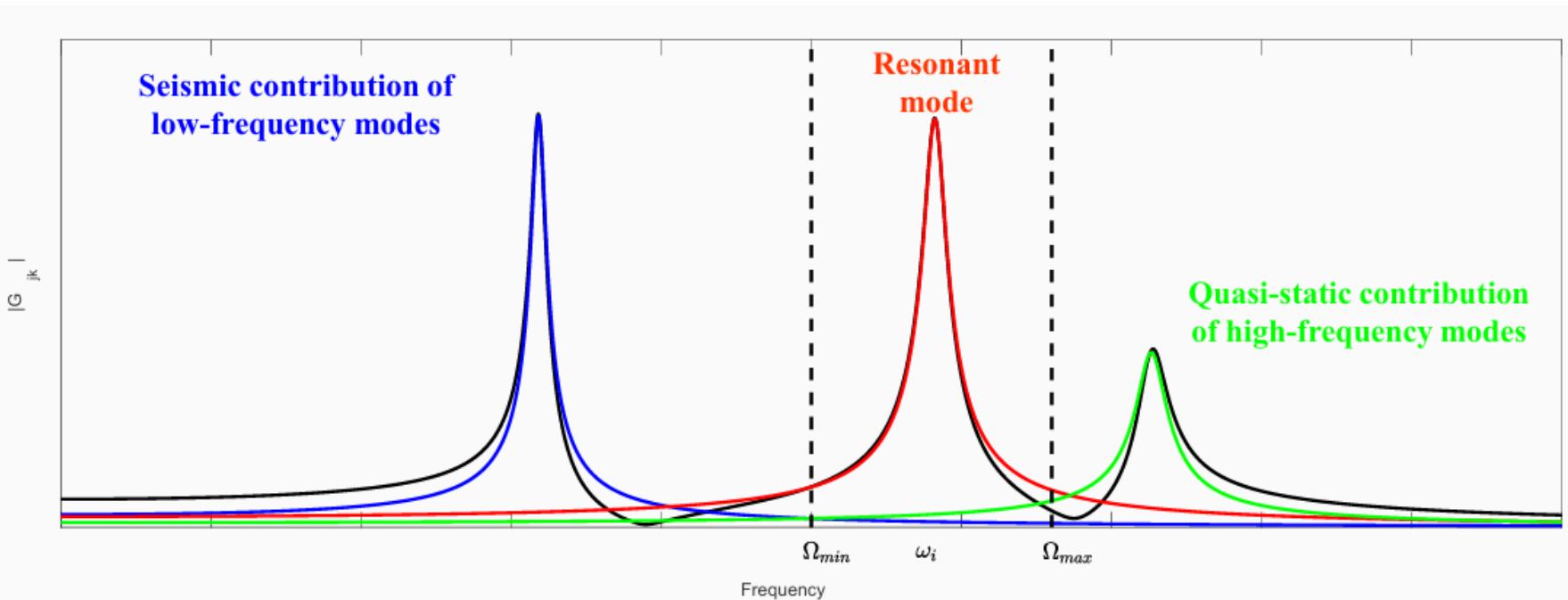


$$G_{jk}(\Omega) = \sum_{i=1}^N \frac{-\Omega^2 \phi_i(x_j)\phi_i(x_k) / m_i}{-\Omega^2 + j2\xi_i\omega_i\Omega + \omega_i^2}$$

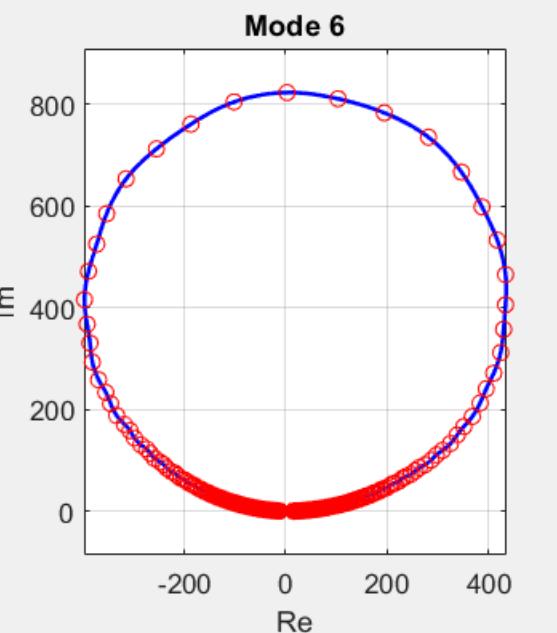
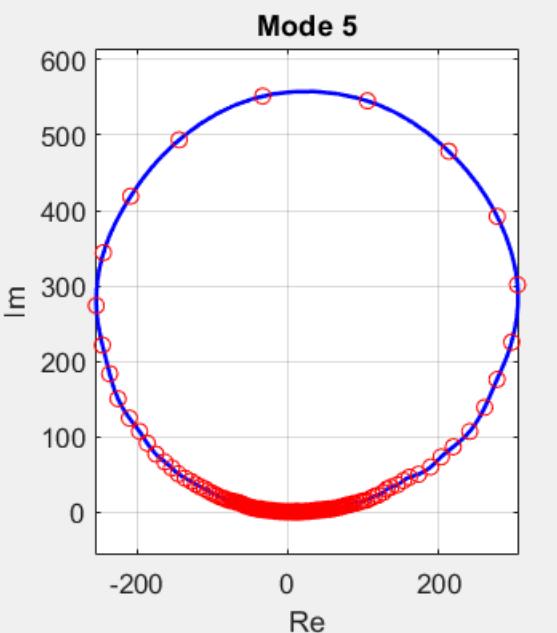
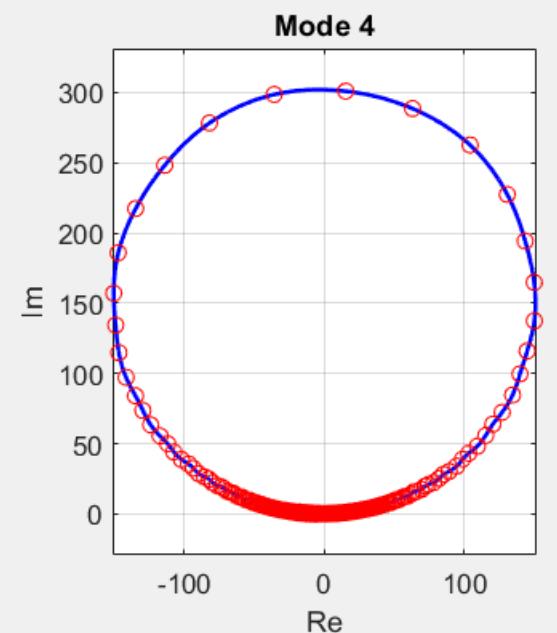
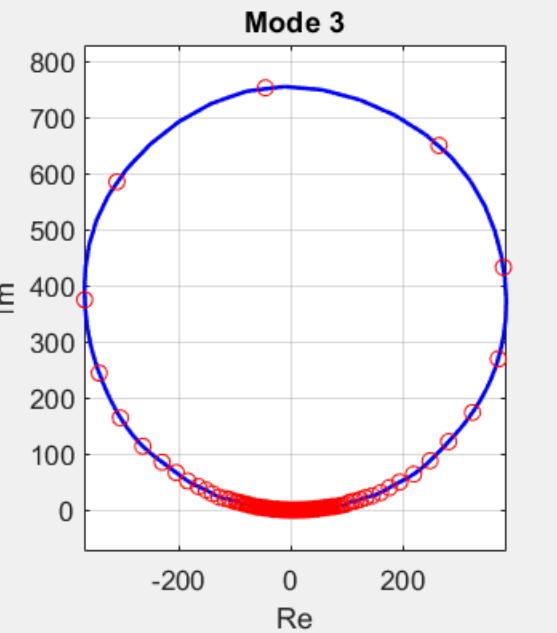
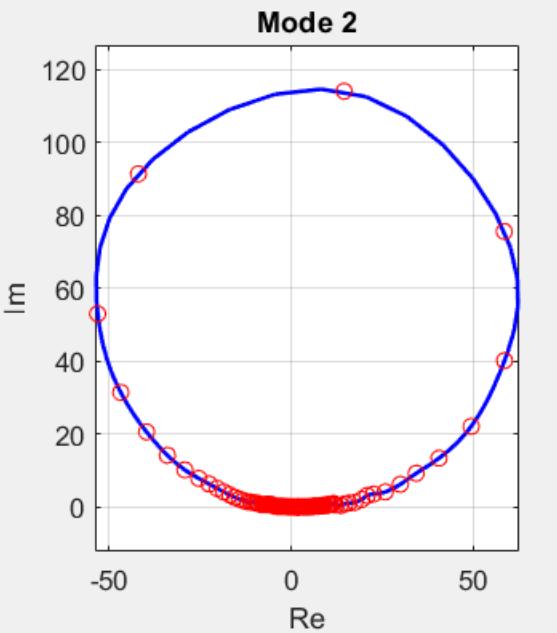
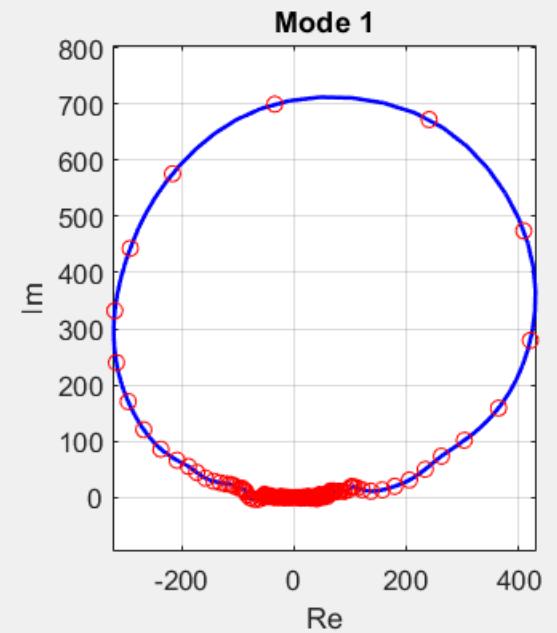
$$G_{jk}^{NUM}(\Omega) = \frac{A_{jk}^{(i)}}{-\Omega^2 + j2\xi_i\omega_i\Omega + \omega_i^2} + \frac{R_{jk}^L}{\Omega^2} + R_{jk}^H$$

APPROXIMATION FORMULA

- Reconstruction of FRF considering for each peak one resonant mode at time
- Extraction of modal parameters from optimization function



REQUIREMENTS TO USE SDOF MODAL ANALYSIS



- FRF peaks far from each other in frequency**
- Low damping structure**

INITIAL PARAMETERS

- Natural Frequencies

Computed as:

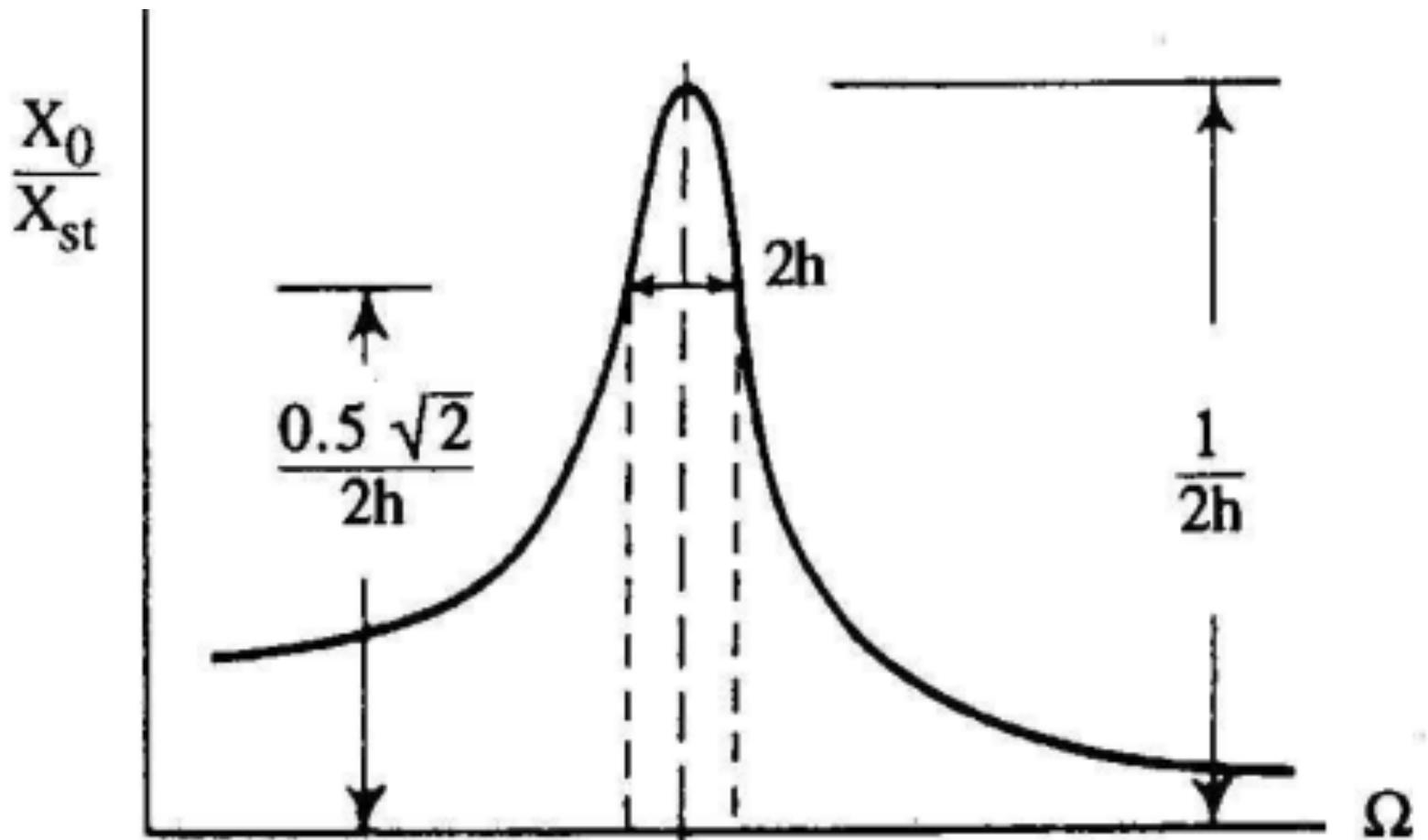
- Maximum value of the FRF magnitude
- Zero of Real part of the FRF
- Maximum of the amplitude of the imaginary part of the FRF

	frequency [Hz]
1st mode	247.32
2nd mode	328.20
3rd mode	392.10
4th mode	596.60
5th mode	620.84
6th mode	806.64

- Damping Ratios

- Half power method
- Phase derivative technique

	Csi
1st mode	3.80e-4
2nd mode	1.22e-4
3rd mode	1.53e-4
4th mode	3.53e-4
5th mode	2.40e-4
6th mode	7.15e-4

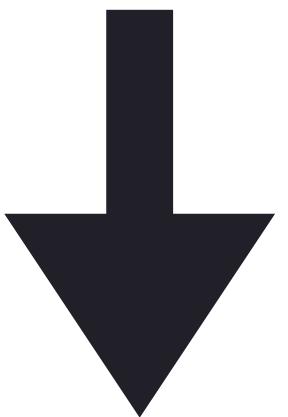


- **A constants and residuals**

Computation of A constants different from co-located FRF to non co-located FRF:

$$G_{jk}(\Omega) = \sum_{i=1}^N \frac{-\Omega^2 \phi_i(x_j)\phi_i(x_k) / m_i}{-\Omega^2 + j2\xi_i\omega_i\Omega + \omega_i^2}$$

in resonances:



$$G_{jk}(\omega_i) = \frac{\varphi_i(x_j)\varphi_i(x_k)}{j2\zeta_i}$$

- Co-located accelerometer:

$$A_{ik} = \varphi_i^2(x_k)$$

- Non co-located accelerometers:

$$A_{ij} = \varphi_i(x_j)\varphi_i(x_k)$$

Residuals:

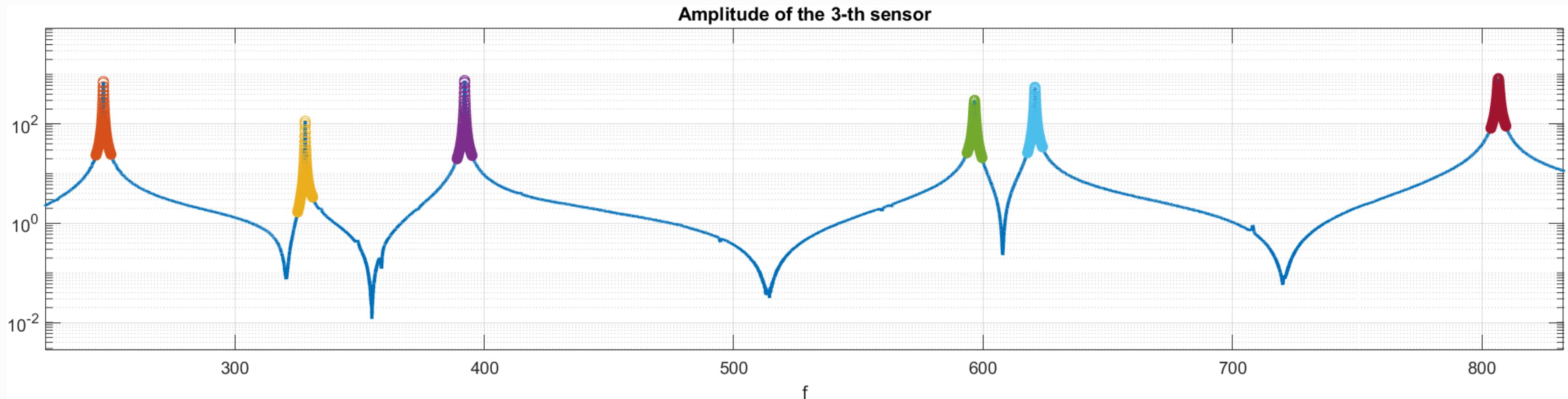
$$R_l = 0$$

$$R_h = 0$$

OPTIMIZATION METHOD

lsqnonlin MatLab function

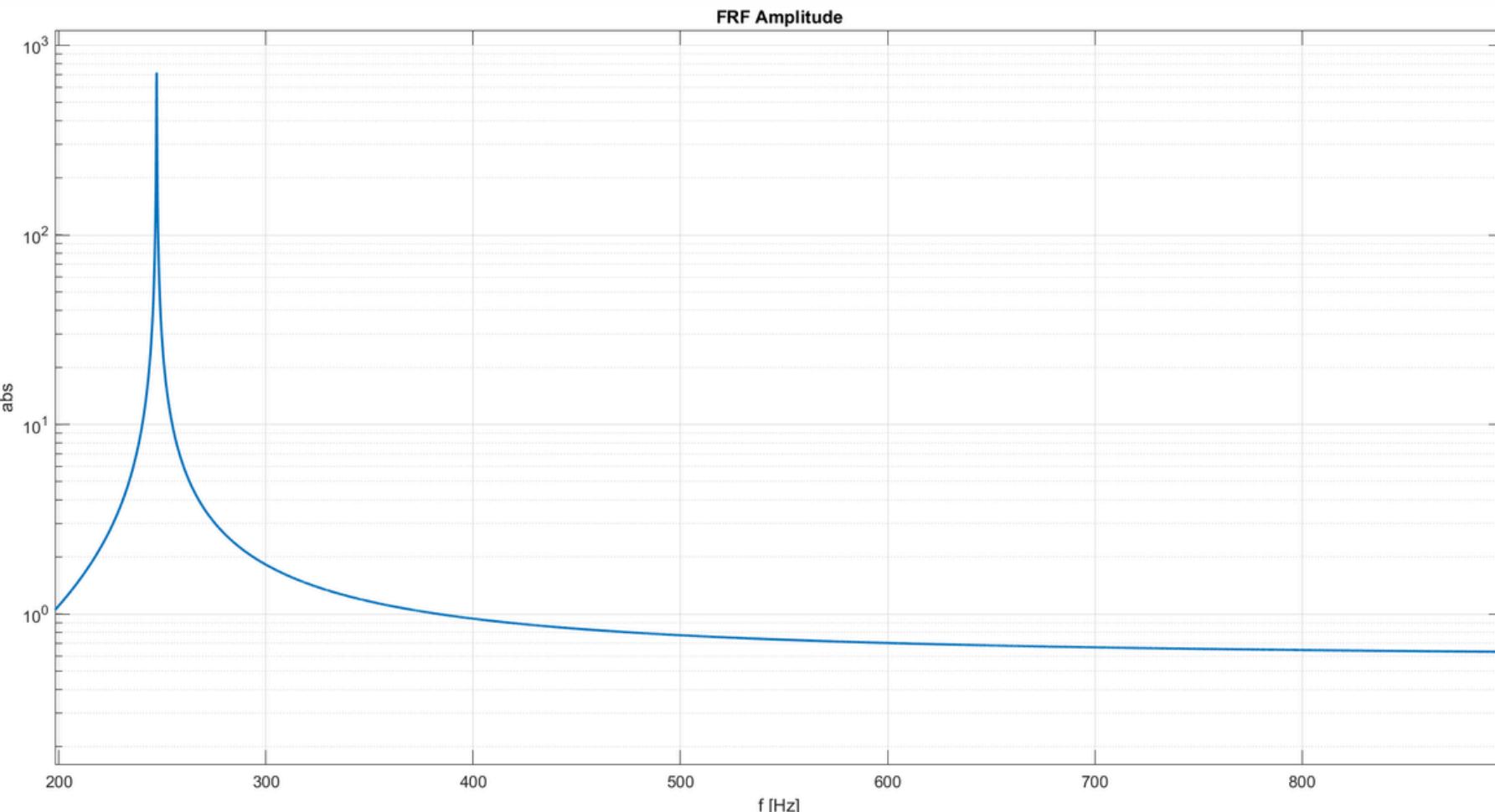
$$\epsilon = \sum_{r=1}^N \sum_{s=1}^M Re(G_r^{EXP}(\Omega_s) - G_r^{NUM}(\Omega_s))^2 + Im(G_r^{EXP}(\Omega_s) - G_r^{NUM}(\Omega_s))^2$$



OPTIMIZED PARAMETERS

1st mode:

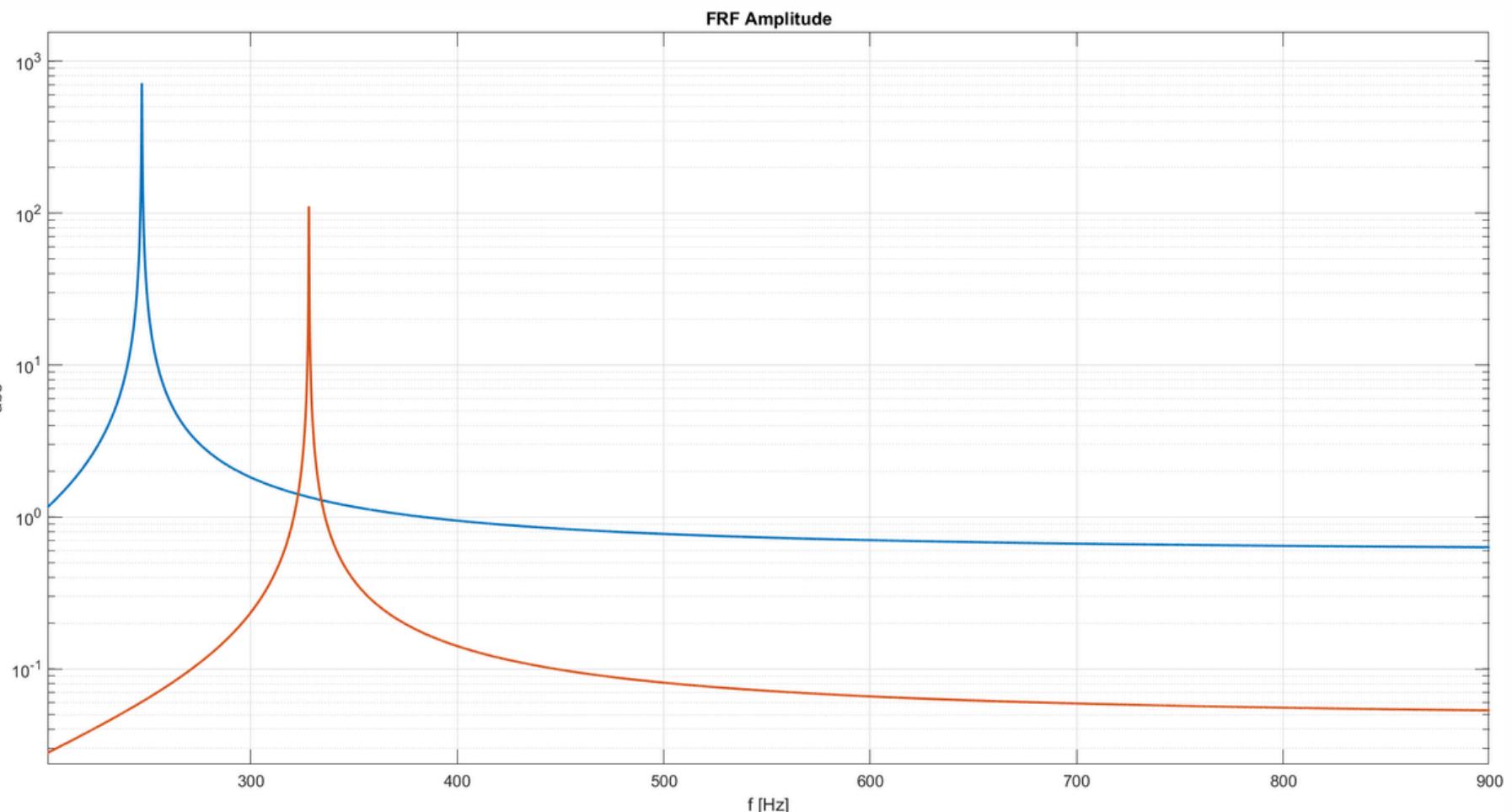
	VALUE
frequency	247.32 Hz
csi	4.047e-4
A	3.57e+4
RI	-6+2.41i
Rh	0.32-1.167i



OPTIMIZED PARAMETERS

2nd mode:

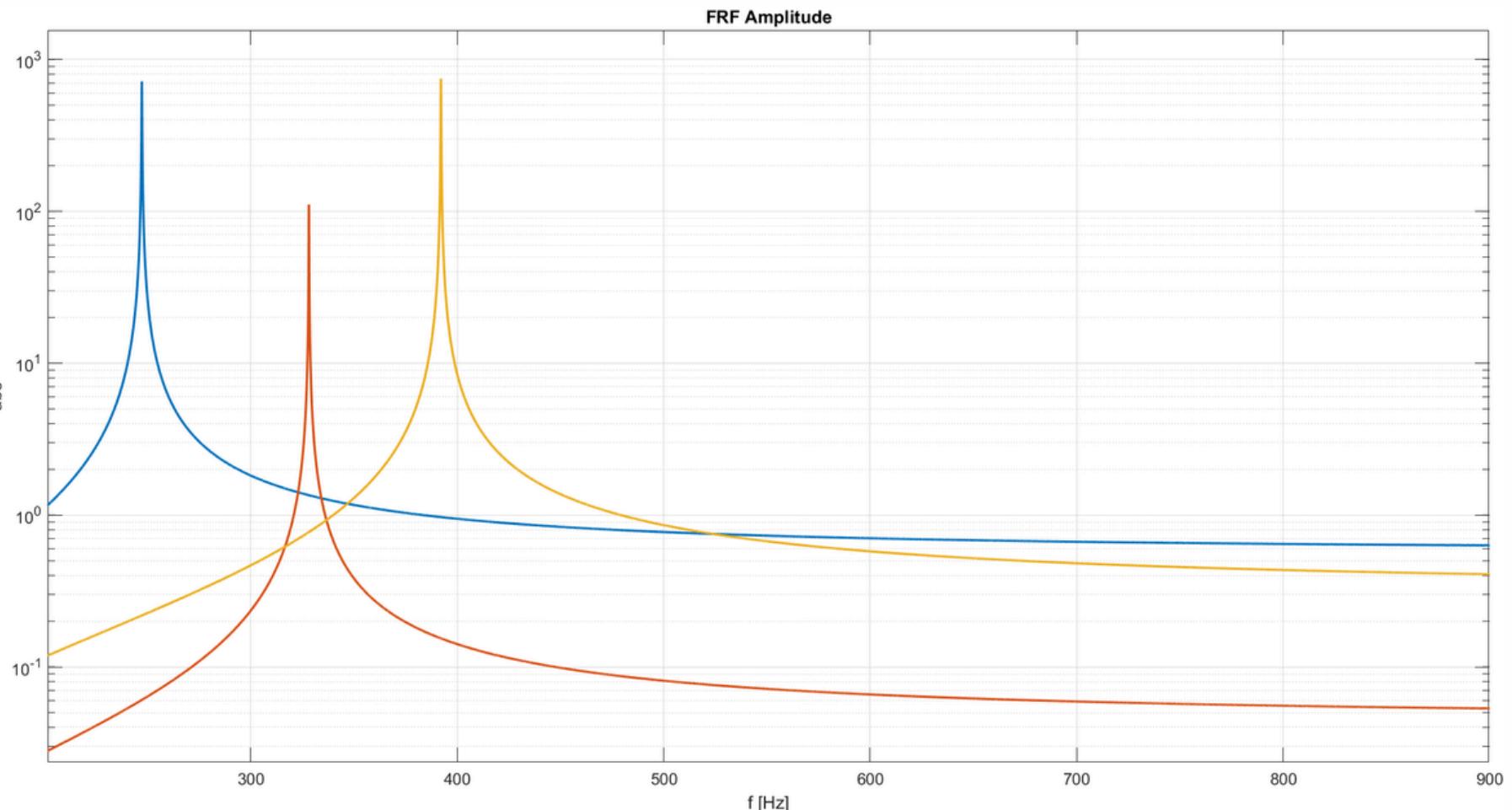
	VALUE
omega	328.20 Hz
csi	2.01e-4
A	4.98e+3
RI	0.0045+0.0024i
Rh	0.0081+0.0031i



OPTIMIZED PARAMETERS

3rd mode:

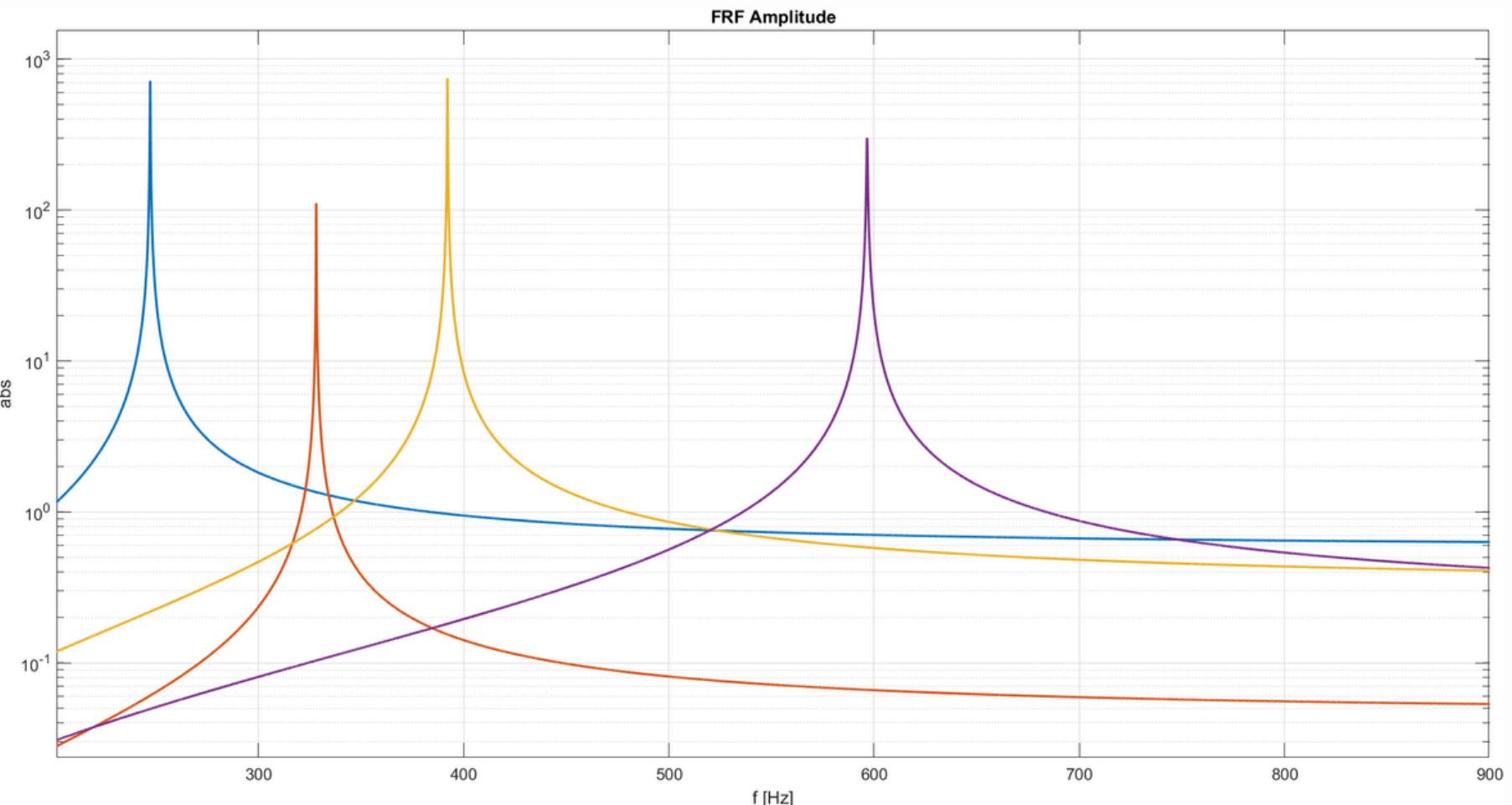
	VALUE
omega	392.12 Hz
csi	2.18e-4
A	5.07e+4
RI	-3.45+0.66i
Rh	1.35+0.009i



OPTIMIZED PARAMETERS

4th mode:

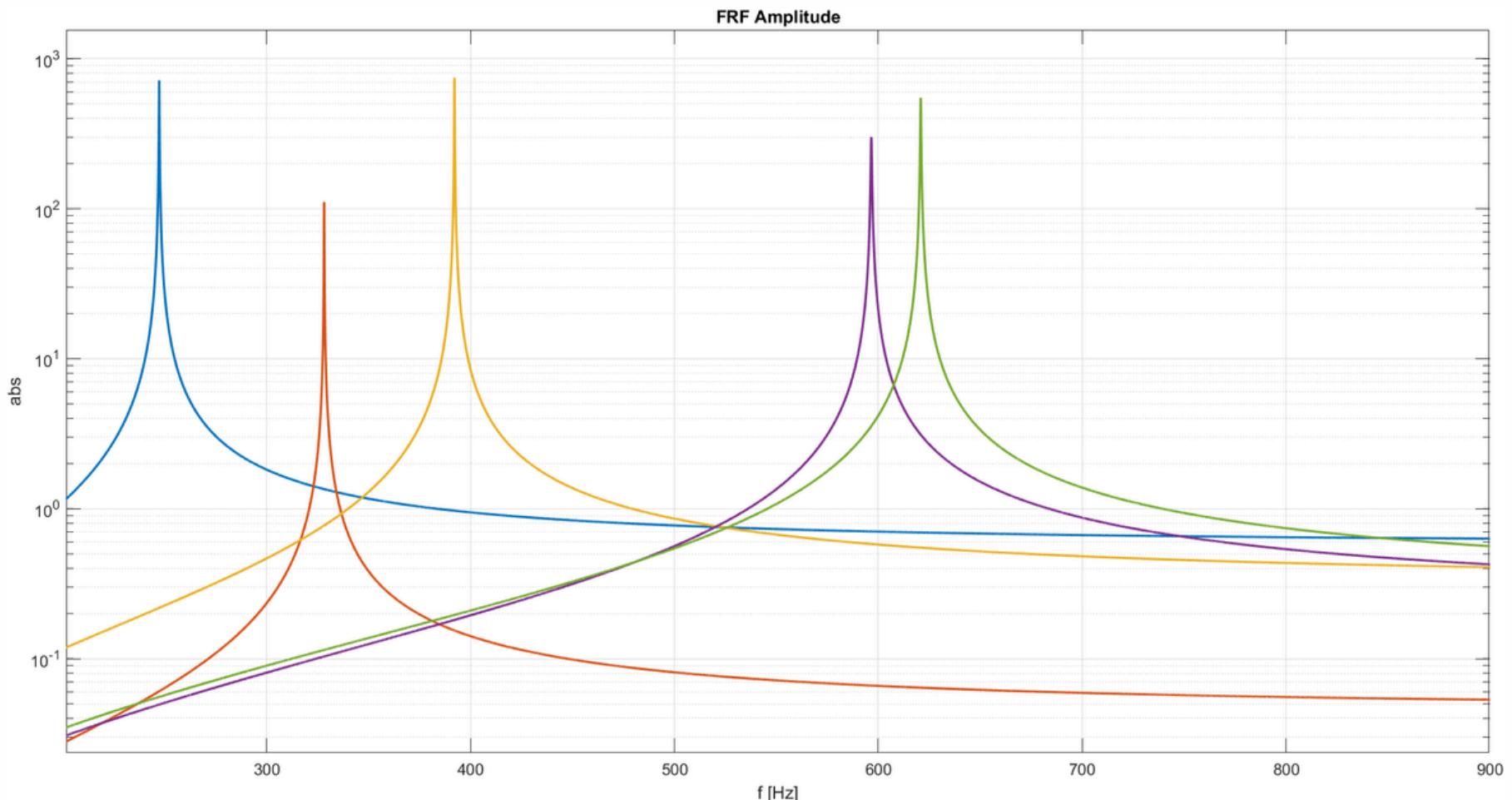
	VALUE
omega	596.60 Hz
csi	3.96e-4
A	8.49e+4
RI	0+1.8i
Rh	-2.76+0.28i



OPTIMIZED PARAMETERS

5th mode:

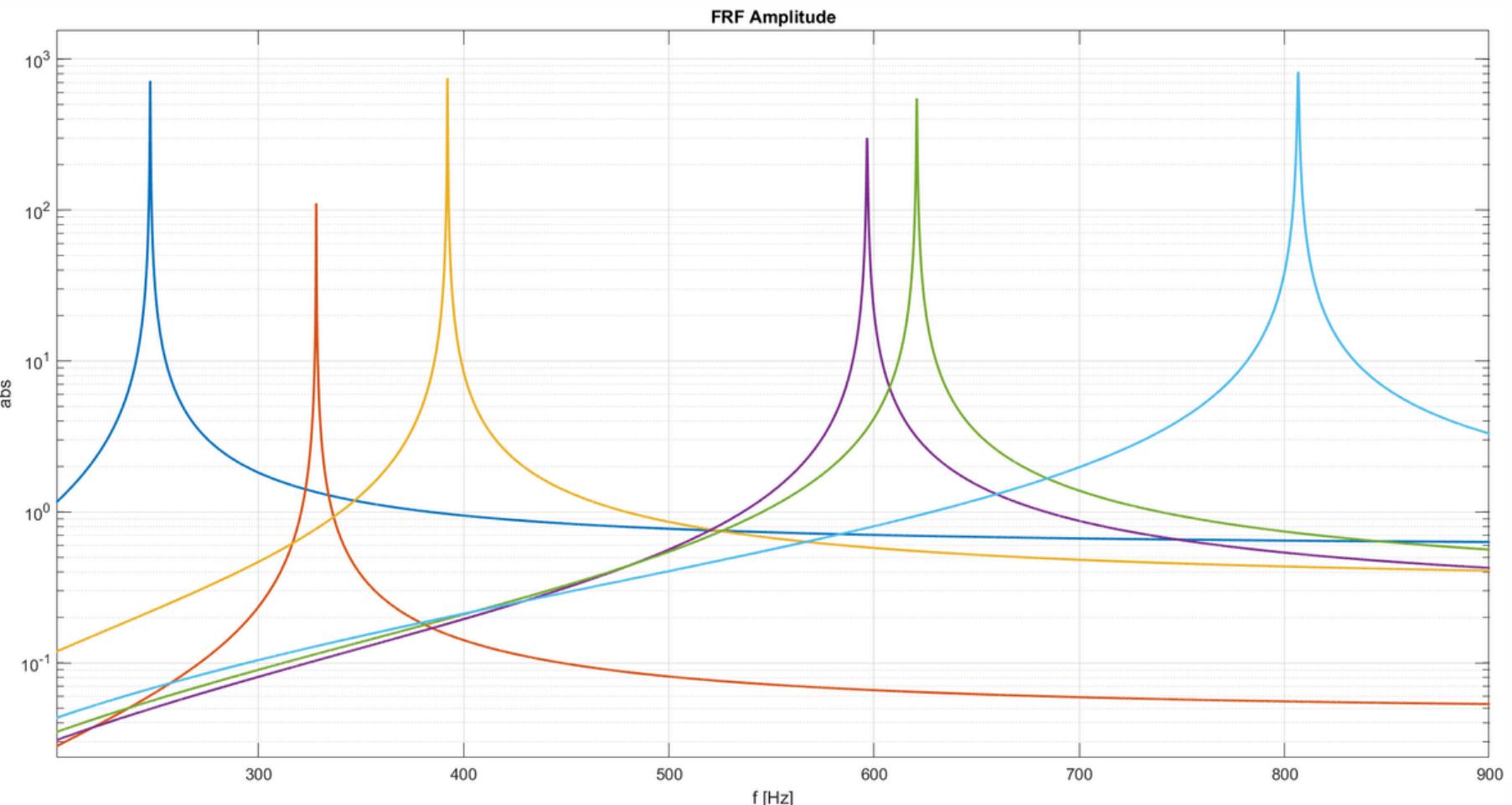
	VALUE
omega	620.81 Hz
csi	2.66e-4
A	1.11e+5
RI	0+17i
Rh	4.22+0.53i



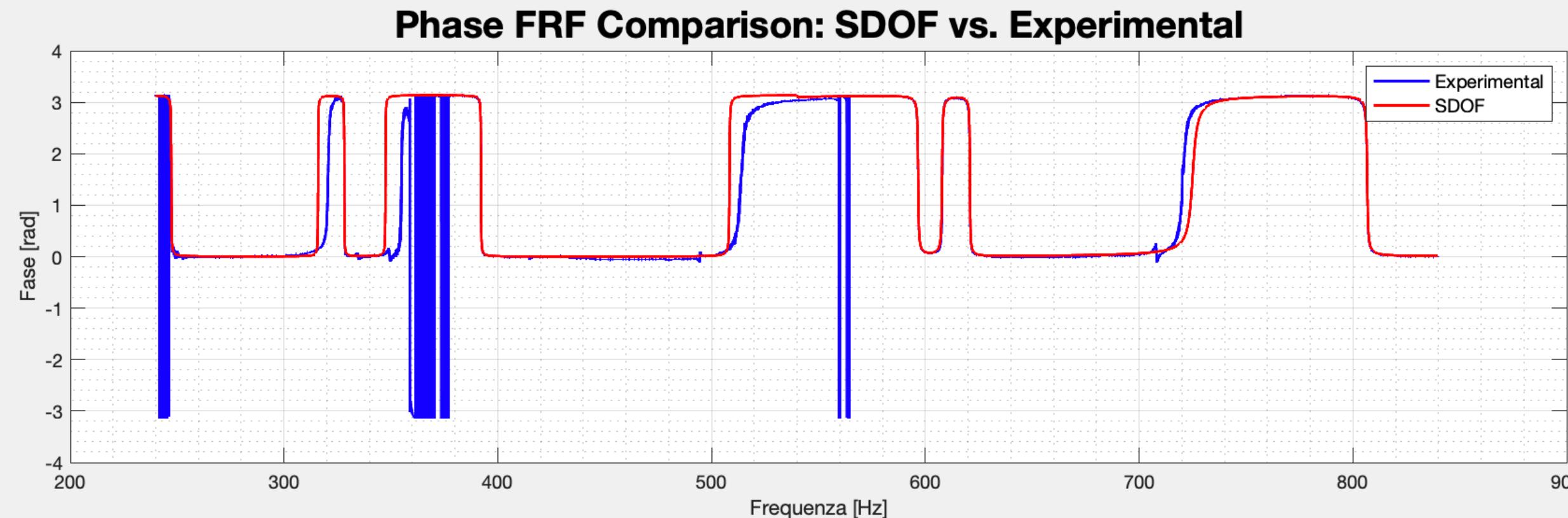
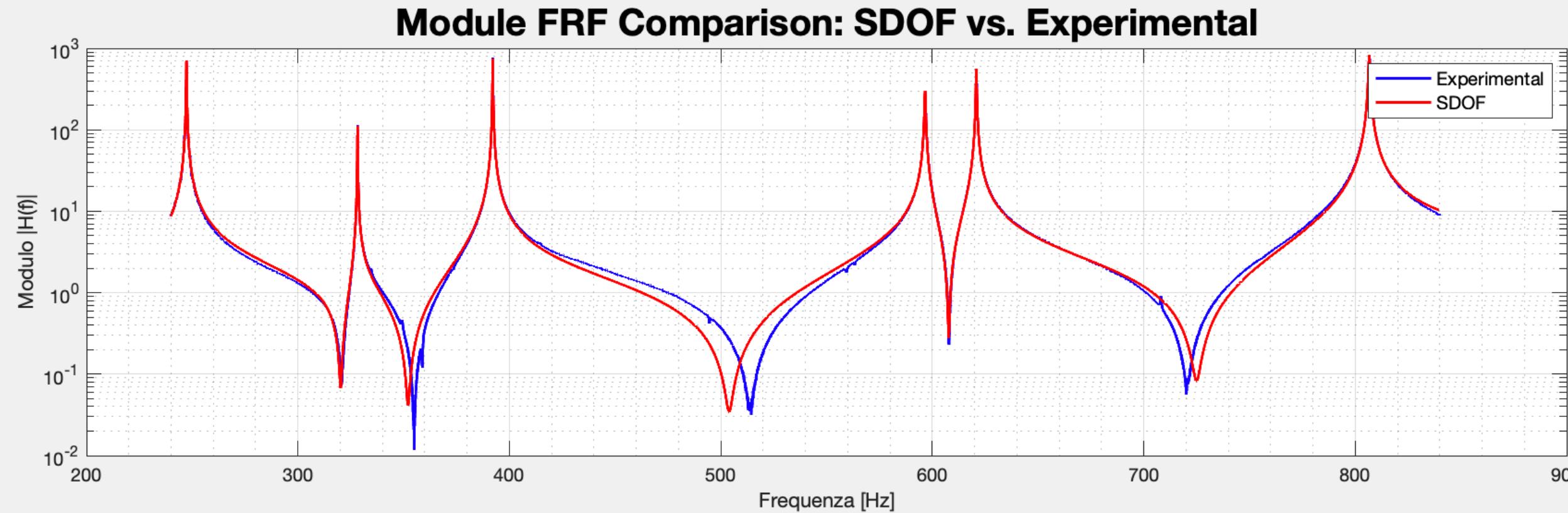
OPTIMIZED PARAMETERS

6th mode:

	VALUE
omega	806.68 Hz
csi	3.94e-4
A	4.22e+5
RI	-2.38+0.04i
Rh	3.89+1.1i



FRF RECONSTRUCTION - SDOF





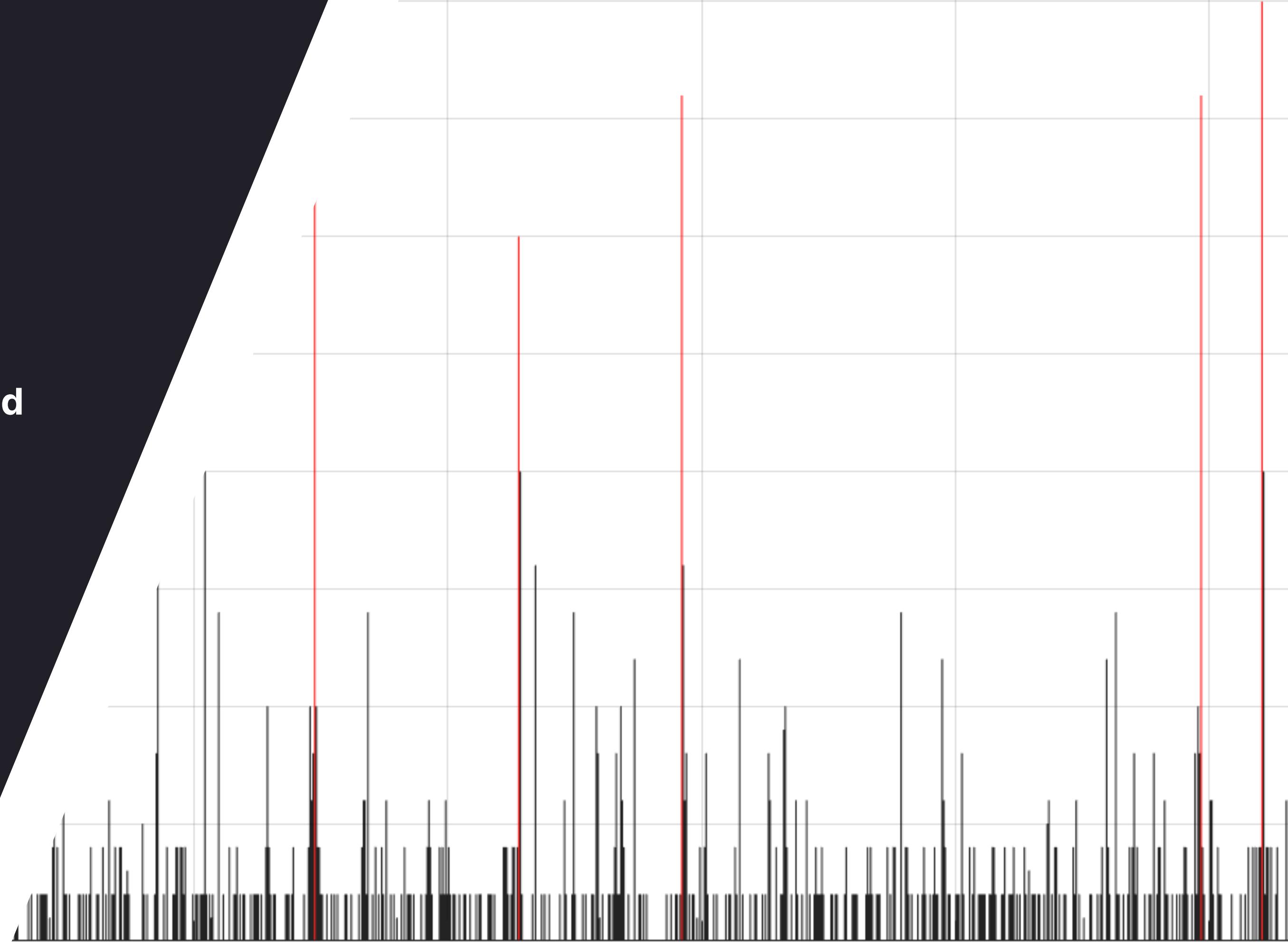
Data Analysis Assignment a.a. 2024/2025

MDOF

Multi degree of freedom method for modal analysis

MDOF PRONY

Complex exponential Method
(Prony Method)



STARTING POINT

Impulse Response Functions

$$h_{pq}(t) = \sum_{r=1}^N \frac{\psi_{p,r}\psi_{q,r}}{2jm_r\omega_r\sqrt{1-\xi_r^2}} e^{s_r t} + \left(\frac{\psi_{p,r}\psi_{q,r}}{2jm_r\omega_r\sqrt{1-\xi_r^2}} \right)^* e^{s_r^* t}$$

$$h_0, h_1, h_2, \dots, h_m = h(0), h(\Delta t), h(2\Delta t), \dots, h(m\Delta t)$$

SOME NOTATION

$$h(t) = \sum_{r=1}^{2N} A_r e^{s_r t}$$

FROM CONTINUOUS TO
SAMPLED TIME

$$e^{s_r \Delta t} \rightarrow V_r$$

$$h_\ell = \sum_{r=1}^{2N} A_r V_r^\ell$$

PRONY'S IDEA

INTRODUCE
COEFFICIENTS
BETAS

$$\begin{cases} h_0 = A_1 + A_2 + \dots + A_{2N} \\ h_1 = V_1 A_1 + V_2 A_2 + \dots + V_{2N} A_{2N} \\ h_2 = V_1^2 A_1 + V_2^2 A_2 + \dots + V_{2N}^2 A_{2N} \\ \vdots \\ h_m = V_1^m A_1 + V_2^m A_2 + \dots + V_{2N}^m A_{2N} \end{cases}$$

ILL CONDITIONED



$$\begin{cases} \beta_0 h_0 = \beta_0 A_1 + \beta_0 A_2 + \dots + \beta_0 A_{2N} \\ \beta_1 h_1 = \beta_1 V_1 A_1 + \beta_1 V_2 A_2 + \dots + \beta_1 V_{2N} A_{2N} \\ \beta_2 h_2 = \beta_2 V_1^2 A_1 + \beta_2 V_2^2 A_2 + \dots + \beta_2 V_{2N}^2 A_{2N} \\ \vdots \\ \beta_m h_m = \beta_m V_1^m A_1 + \beta_m V_2^m A_2 + \dots + \beta_m V_{2N}^m A_{2N} \end{cases}$$

$$\beta_0 + \beta_1 V + \beta_2 V^2 + \dots + \beta_m V^m = 0$$

CHARACTERISTIC POLYNOMIAL



$$\sum_{i=0}^m \beta_i h_i = \sum_{j=1}^{2N} \left(A_j \sum_{i=0}^m \beta_i V_j^i \right) \cancel{=} 0$$

DECOUPLING EIGEN VALUES AND VECTORS

SOLVING BETAS?

$$\sum_{i=0}^m \beta_i h_i = 0$$

FIX ORDER TO 2N

$$\begin{bmatrix} h_0 & \dots & \dots & h_{2N-1} \\ \vdots & \dots & \dots & \vdots \\ \vdots & \dots & \dots & \vdots \\ h_m & \dots & \dots & h_{m+2N-1} \end{bmatrix} \begin{Bmatrix} \beta_0 \\ \vdots \\ \beta_{2N-1} \end{Bmatrix} = - \begin{Bmatrix} h_{2N} \\ \vdots \\ h_{m+2N} \end{Bmatrix}$$

WITH $m > 2N$: LEAST SQUARE PROBLEM

$$\{\beta\} = - [h]^+ \{\tilde{h}\}$$

SISO
TO
SIMO

WE CHOSE THE 5 IRFs, WITH INPUT 3

$$\begin{bmatrix} h_0 & h_1 & h_2 & \cdots & h_{2N-1} \\ h_1 & h_2 & h_3 & \cdots & h_{2N} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ h_m & \cdots & \cdots & \cdots & h_{m+2N-1} \end{bmatrix} = R_{i3} \quad i=1,2,\dots,5$$

$$\begin{bmatrix} R_{13} \\ R_{23} \\ \vdots \\ R_{53} \end{bmatrix} = R \rightarrow R \begin{Bmatrix} \beta_0 \\ \beta_1 \\ \vdots \\ \vdots \\ \beta_{2N-1} \end{Bmatrix} = - \begin{Bmatrix} h_{2N} \\ h_{2N+1} \\ \vdots \\ \vdots \\ h_{5m} \end{Bmatrix}$$

WITH $5m \gg 2N$: LEAST SQUARE PROBLEM

$$\{\beta\} = - [R]^+ \{\tilde{h}\}$$

$$\beta_0 + \beta_1 V + \beta_2 V^2 + \dots + \beta_m V^m = 0$$

ROOTS OF THE POLYNOMIAL



$$V_r = e^{s_r \Delta t}$$

$$s_r = -\omega_r \xi_r \pm j\omega_r \sqrt{1 - \xi_r^2}$$

DETERMINING
POLES

$$\begin{bmatrix} 1 & 1 & \dots & \dots & 1 \\ V_1 & V_2 & \dots & \dots & V_{2N} \\ V_1^2 & V_2^2 & \dots & \dots & V_{2N}^2 \\ \vdots & \vdots & \dots & \dots & \vdots \\ V_1^{2N-1} & V_2^{2N-1} & \dots & \dots & V_{2N}^{2N-1} \end{bmatrix} \begin{Bmatrix} A_1 \\ A_2 \\ \vdots \\ A_{2N} \end{Bmatrix} = \begin{Bmatrix} h_0 \\ h_1 \\ h_2 \\ \vdots \\ h_{2N-1} \end{Bmatrix}$$

SOLVE THIS SYSTEM FOR EACH IRF



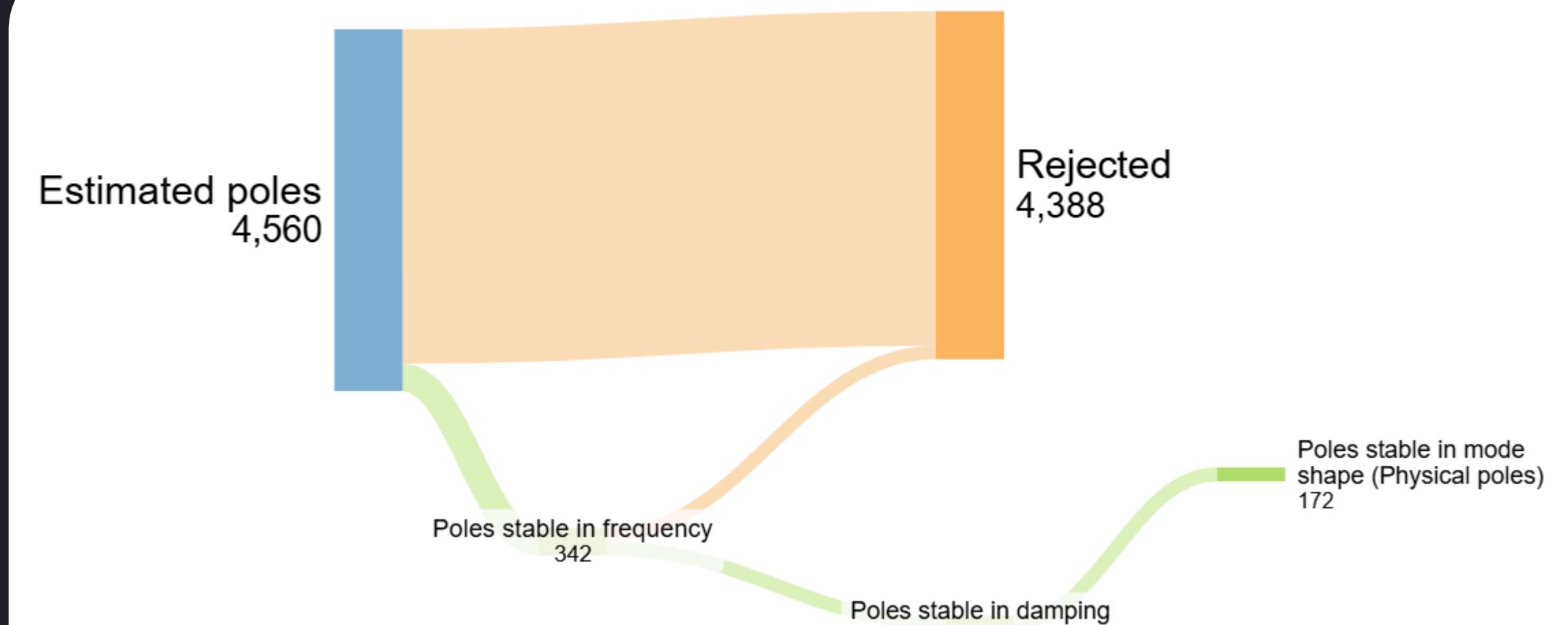
$$[A_{1,i3} \ A_{2,i3} \ \cdots \ A_{2N,i3}]^T \quad i=1,2,\dots,5$$

$$A_{r,i3} = \frac{\psi_{i,r} \psi_{3,r}}{2jm_r\omega_r\sqrt{1-\xi_r^2}}$$

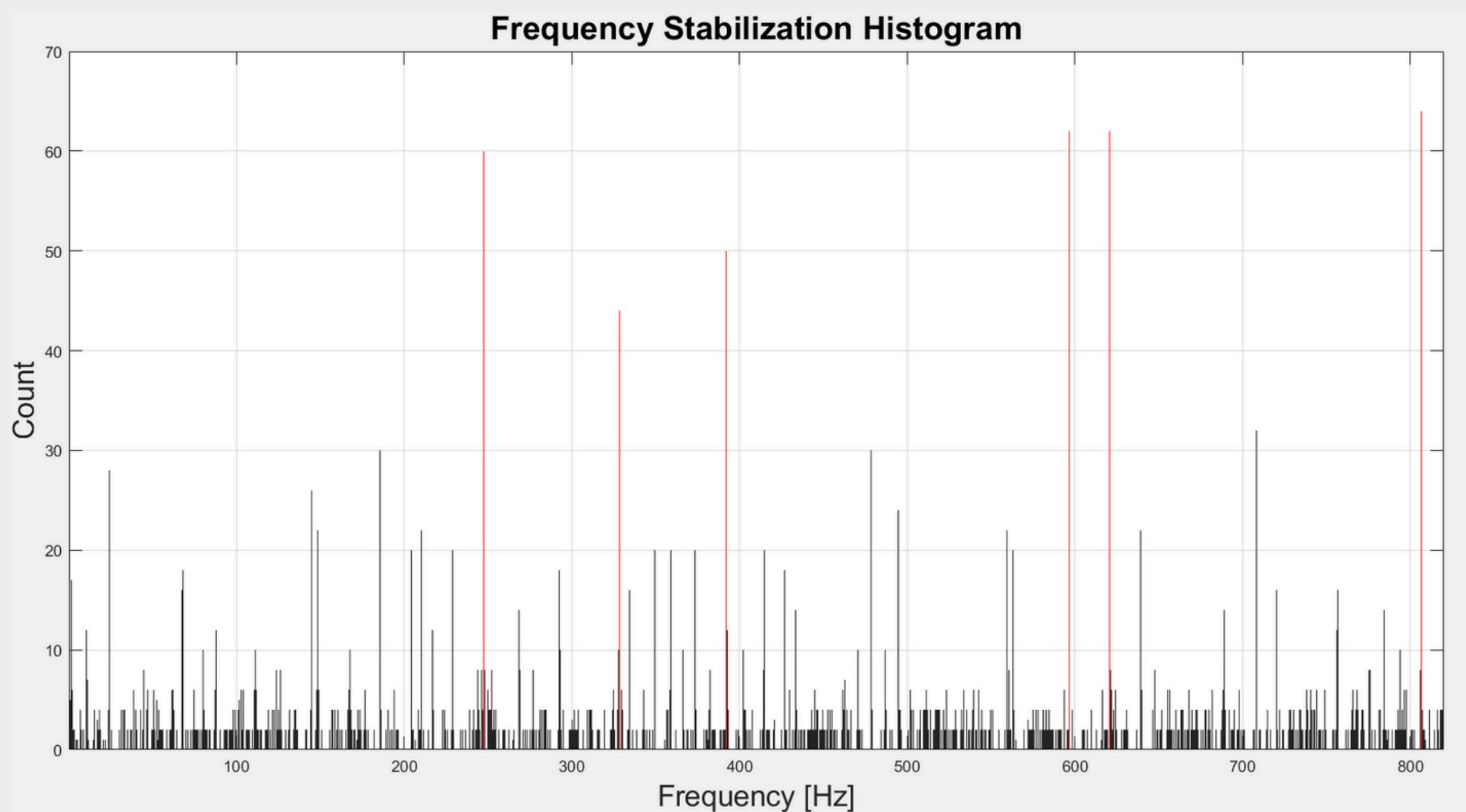
**DETERMINING
“MODES”**

FILTERING PHYSICAL POLES

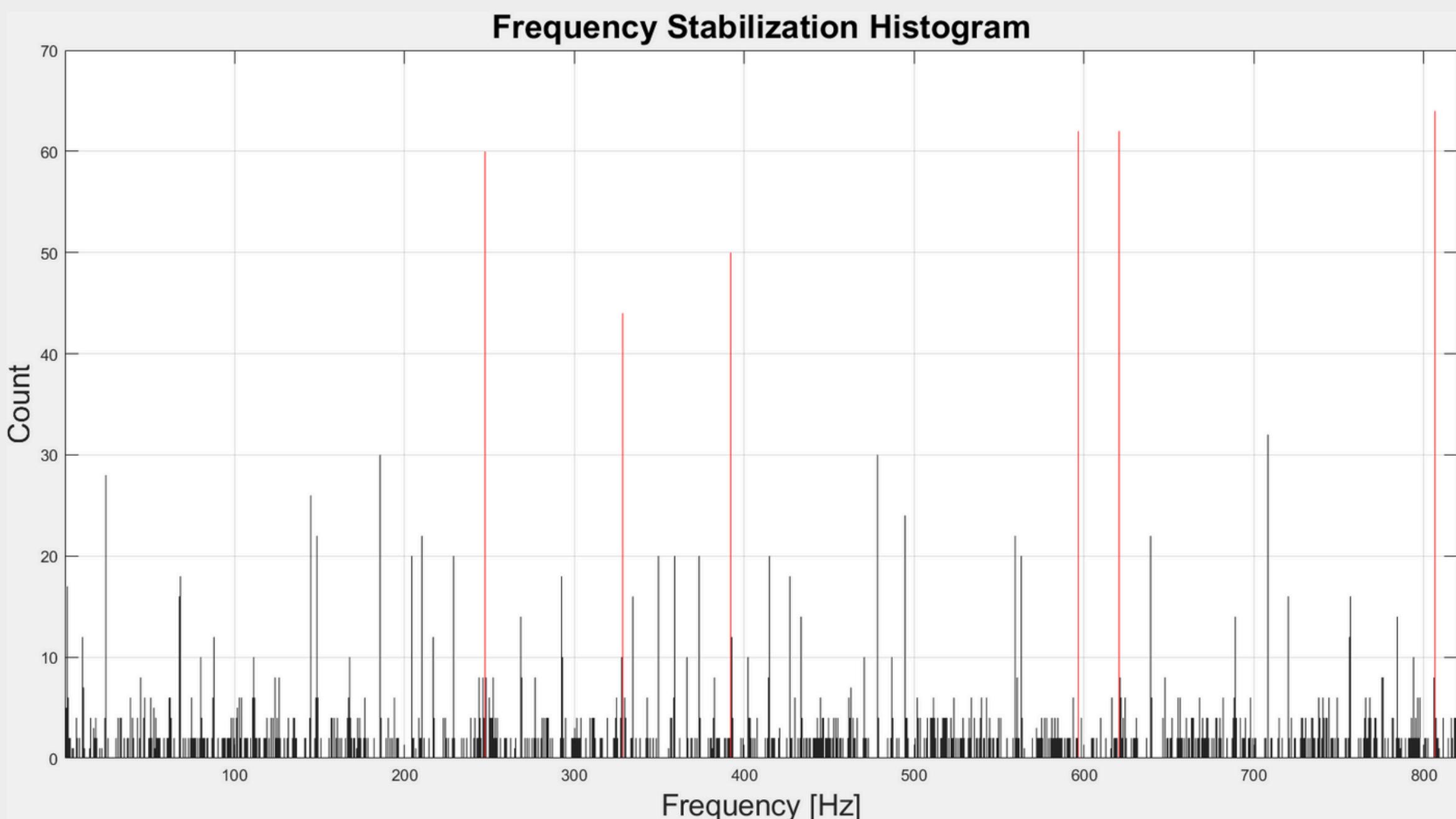
ONLY 3.8% SURVIVED



FREQUENCY STABLE POLES



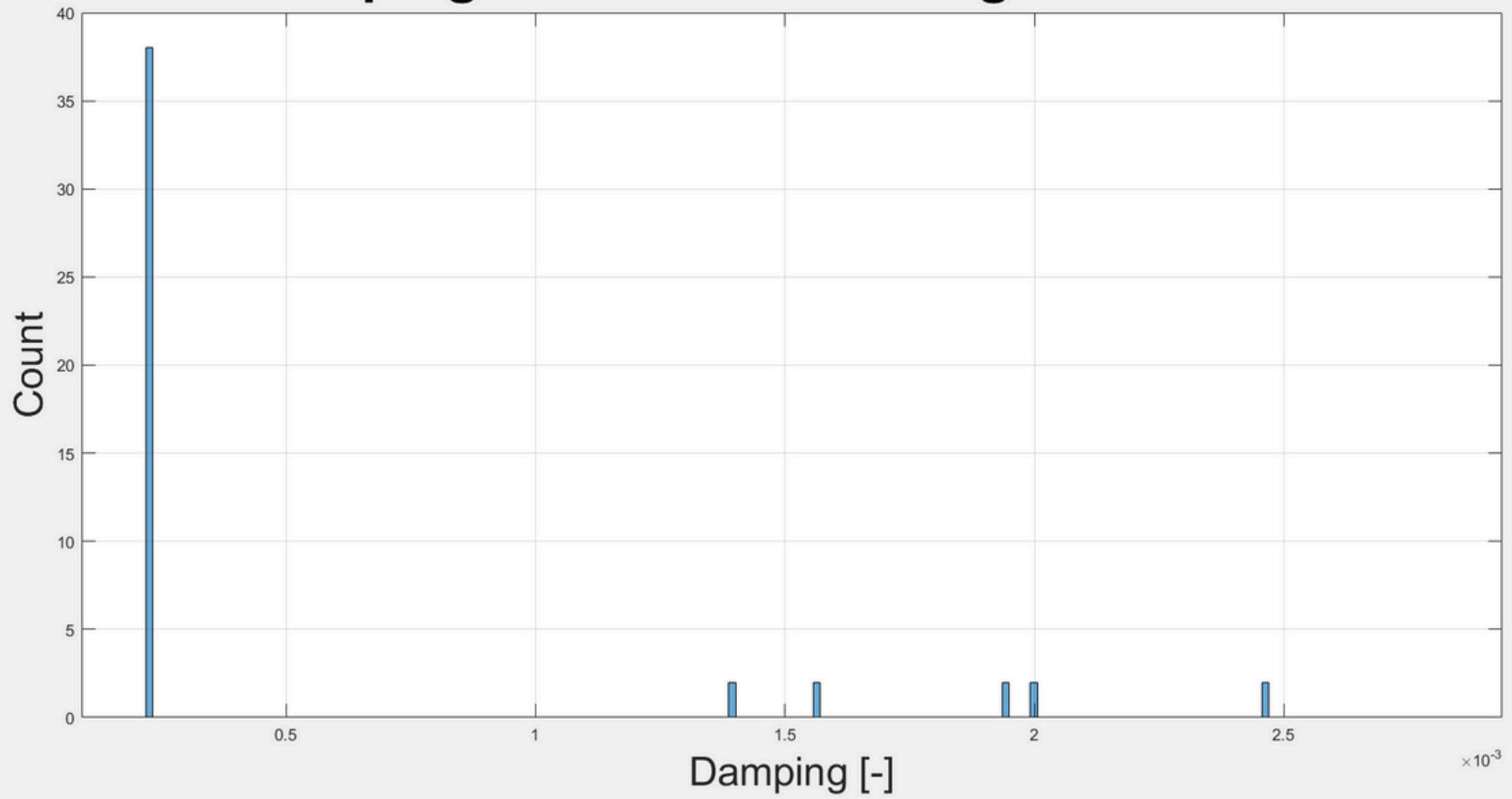
FREQUENCY STABLE POLES



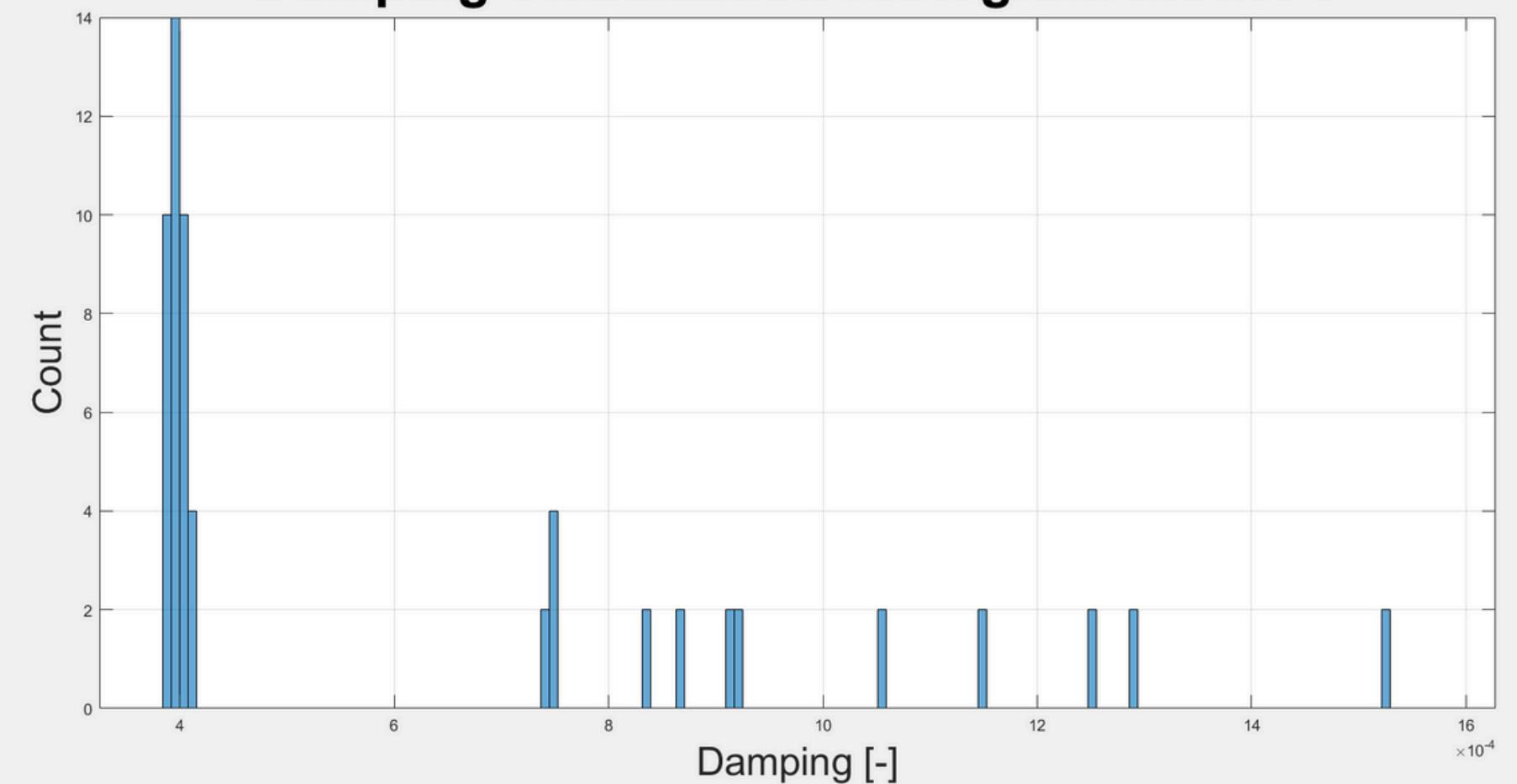
- $\approx 247 \text{ Hz}$
- $\approx 328 \text{ Hz}$
- $\approx 392 \text{ Hz}$
- $\approx 596 \text{ Hz}$
- $\approx 621 \text{ Hz}$
- $\approx 807 \text{ Hz}$

DAMPING STABLE POLES

Damping Stabilization Histogram mode: 3



Damping Stabilization Histogram mode: 6



MODE STABLE POLES

MAC VALUES

MODAL MASSES ESTIMATION

$$H_{pq}(\omega) = \sum_{r=1}^{2*6} \frac{\psi_{pr}\psi_{qr}}{j\omega - s_r} Q_r + R_U + \frac{R_L}{\omega^2}$$

SETUP A LEAST SQUARE PROBLEM

$$\left\{ \begin{array}{l} H_{pq}(\omega_1) \\ H_{pq}(\omega_2) \\ \vdots \\ H_{pq}(\omega_{N_f}) \end{array} \right\} = \left[\begin{array}{cccccc} \frac{\psi_{p1}\psi_{q1}}{j\omega_1 - s_1} & \frac{\psi_{p2}\psi_{q2}}{j\omega_1 - s_2} & \dots & \frac{\psi_{p2*6}\psi_{q2*6}}{j\omega_1 - s_{2*6}} & 1 & \frac{1}{\omega_1^2} \\ \frac{\psi_{p1}\psi_{q1}}{j\omega_2 - s_1} & \frac{\psi_{p2}\psi_{q2}}{j\omega_2 - s_2} & \dots & \frac{\psi_{p2*6}\psi_{q2*6}}{j\omega_2 - s_{2*6}} & 1 & \frac{1}{\omega_2^2} \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ \frac{\psi_{p1}\psi_{q1}}{j\omega_{N_f} - s_1} & \frac{\psi_{p2}\psi_{q2}}{j\omega_{N_f} - s_2} & \dots & \frac{\psi_{p2*6}\psi_{q2*6}}{j\omega_{N_f} - s_{2*6}} & 1 & \frac{1}{\omega_{N_f}^2} \end{array} \right] \left\{ \begin{array}{l} Q_1 \\ Q_2 \\ \vdots \\ Q_{2*6} \\ R_U \\ R_L \end{array} \right\}$$

MODAL MASS EXTRACTION

USING DEFINITION OF Q

$$Q_r = \frac{-\omega^2}{2j m_r \omega_r \sqrt{1-\xi_r^2}}$$



$$m_r = \frac{-\omega^2}{2j Q_r \omega_r \sqrt{1-\xi_r^2}}$$

LEADS TO MODAL MASSES

MODAL PARAMETERS SUMMARY

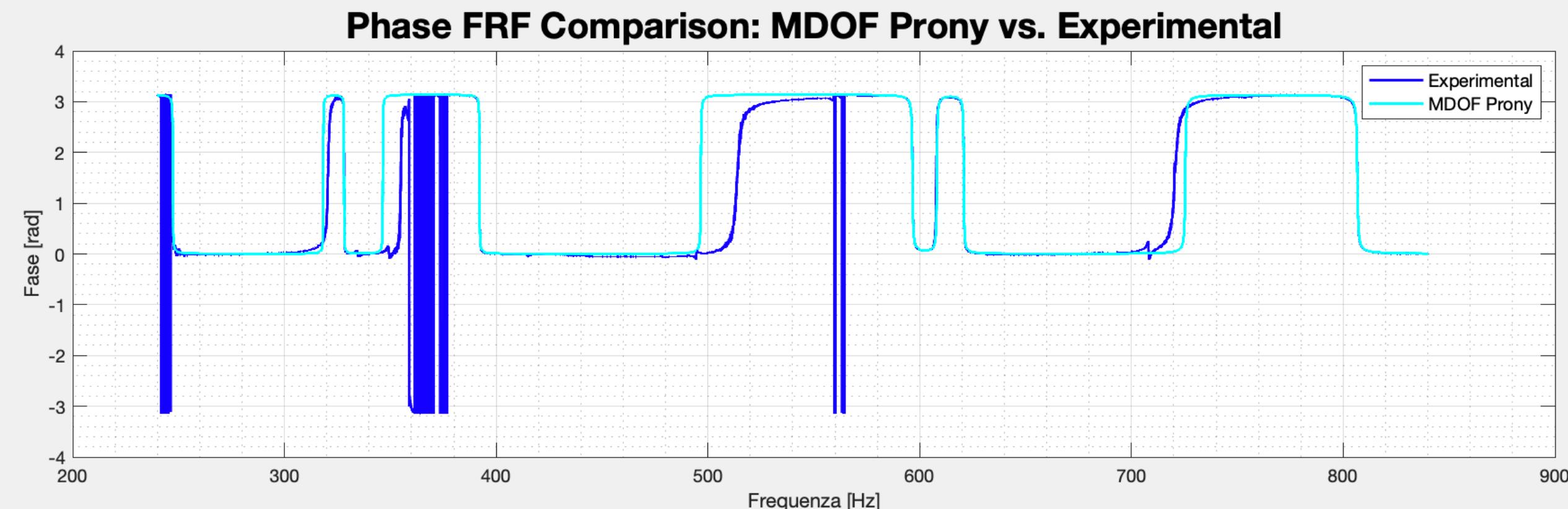
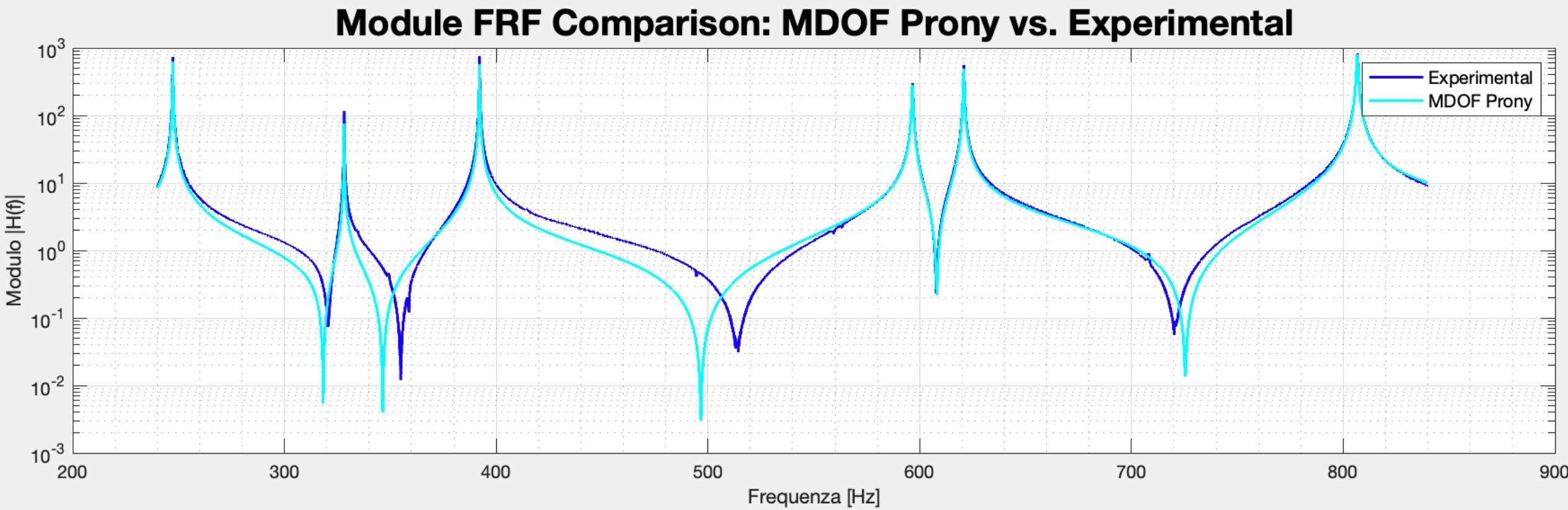
FREQUENCIES: CONCORDING WITH FEM

DAMPING: LOW -> THIN SOLID STEEL PLATE

MODE SHAPE: CONCORDING WITH FEM

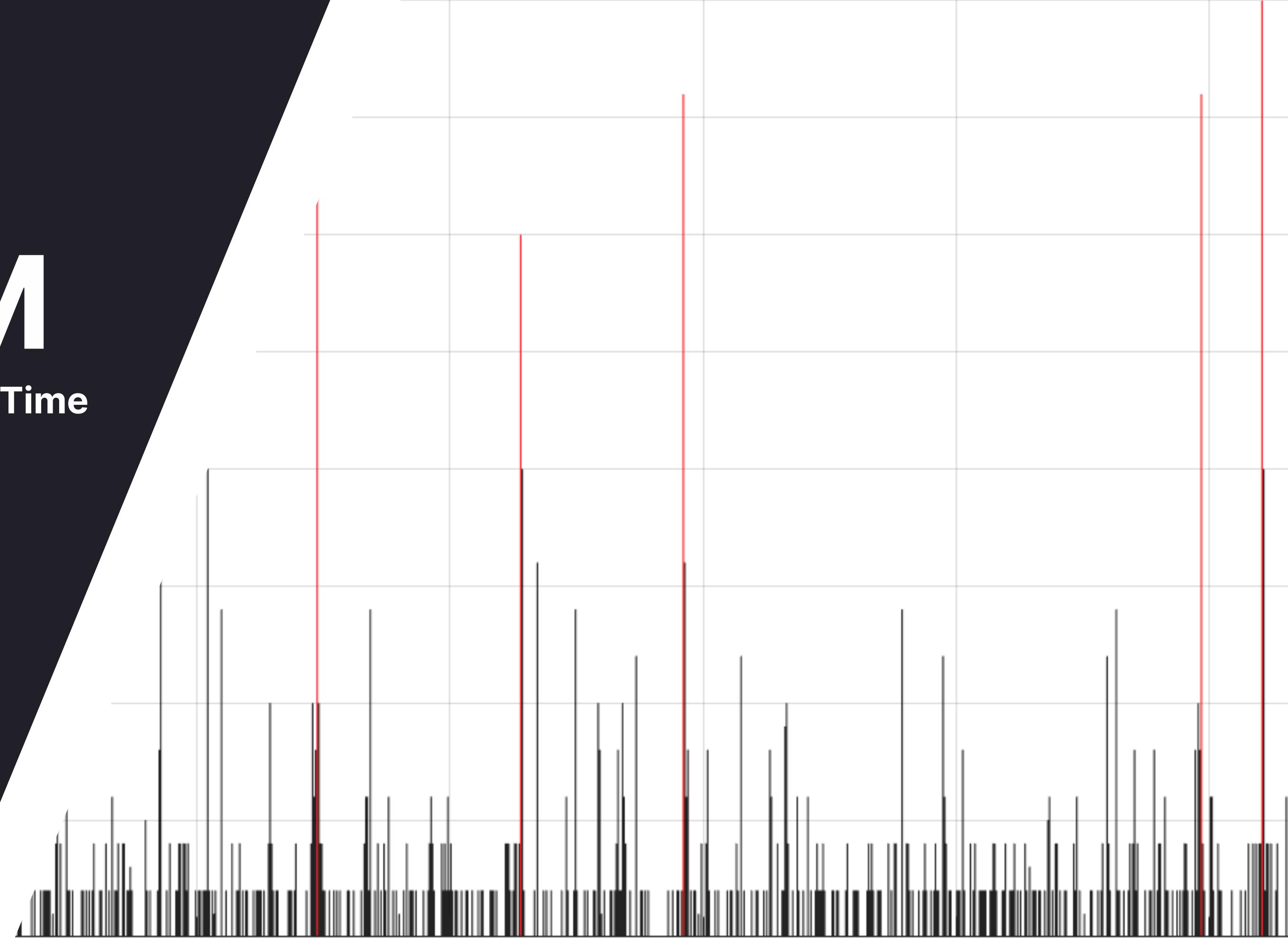
	Mode 1	Mode 2	Mode 3	Mode 4	Mode 5	Mode 6
Res.Frequencies [Hz]	247.29	328.17	392.12	596.57	620.81	806.65
Damping [-]	0.00040	0.00019	0.00022	0.00040	0.00027	0.00041
Norm. Mode shapes	-1	-0.2735	-0.8235	1	0.6604	-0.0584
	0.9313	-0.2929	-1	-0.2372	1	-1
	-0.8302	-0.2178	-0.7586	-0.4473	-0.5542	-0.8085
	0.9234	-0.2075	-0.8791	0.8707	-0.9111	-0.0870
	-0.0319	-1	0.1290	-0.8208	0.1669	0.4433

FRF RECONSTRUCTION - MDOF PRONY



MDOF IBRAHIM

Multiple-Reference Ibrahim Time
Domain Method (MITD)



FUNDAMENTAL IDEA

Impulse Response Functions

$$h_{pq}(t) = \sum_{r=1}^N \frac{\psi_{p,r}\psi_{q,r}}{2jm_r\omega_r\sqrt{1-\xi_r^2}} e^{s_r t} + \left(\frac{\psi_{p,r}\psi_{q,r}}{2jm_r\omega_r\sqrt{1-\xi_r^2}} \right)^* e^{s_r^* t}$$

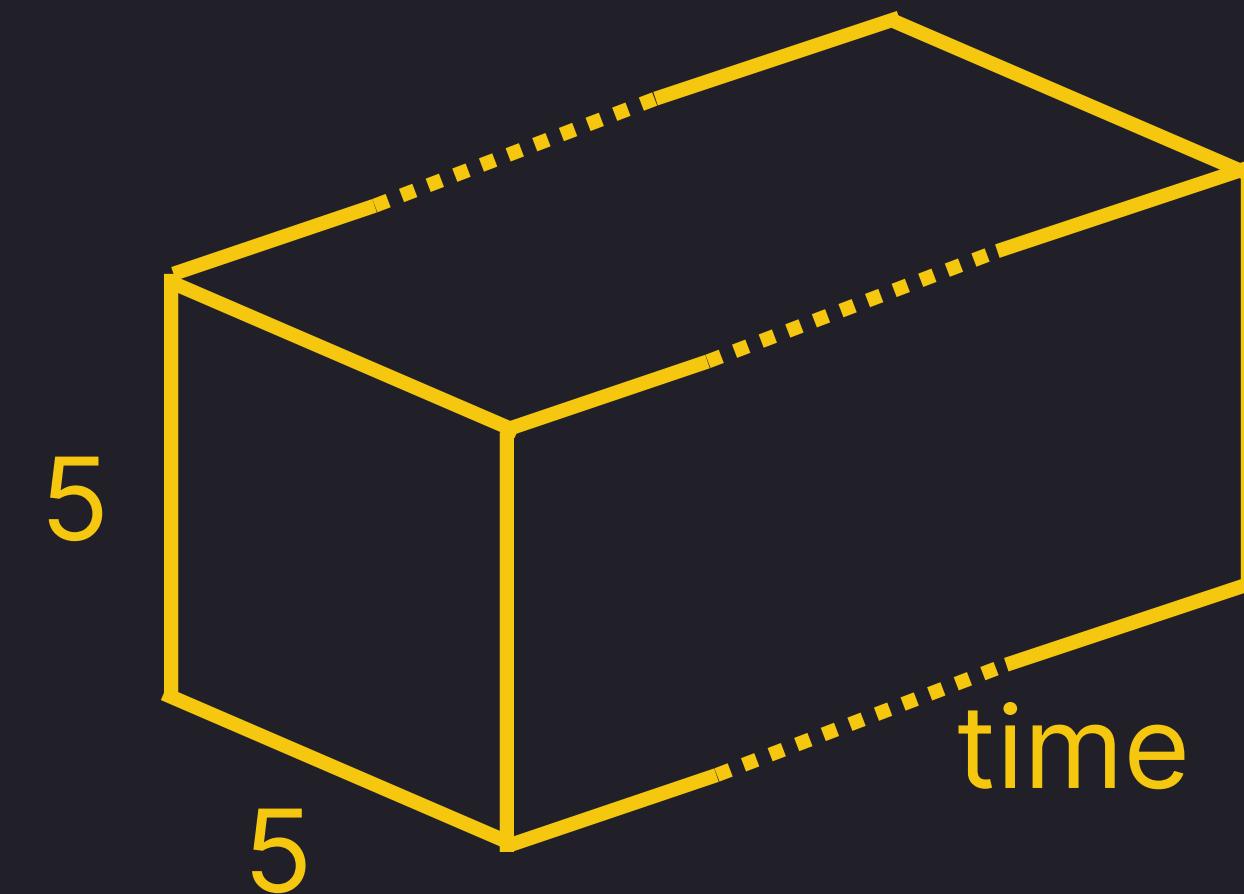
Matrix formulation:

$$[h(t)] = [\Psi] [e^{s_r t}] [L]^T$$

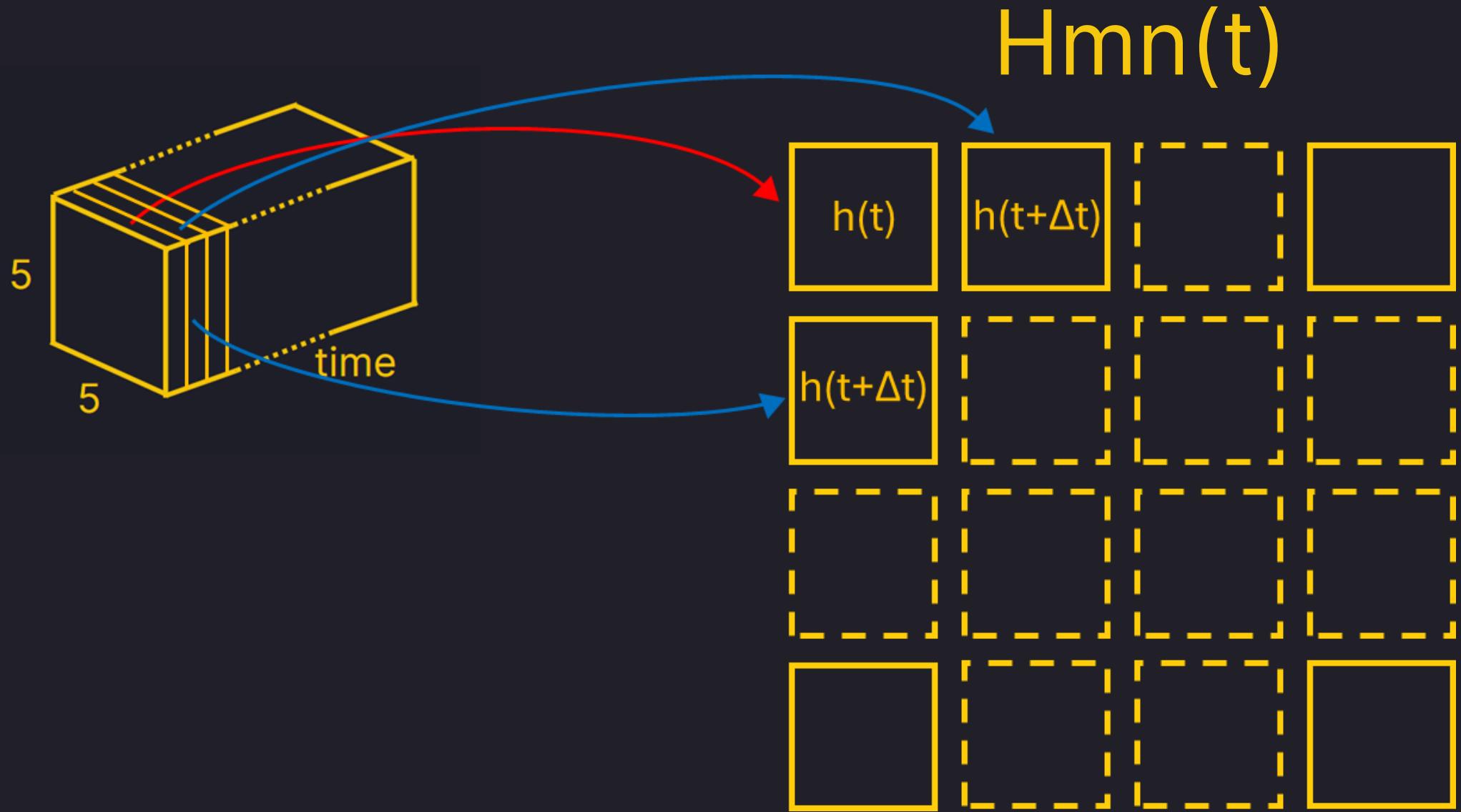
IRFs MATRICES

$\begin{cases} n_l = 5 & \text{number of outputs} \\ n_s = 5 & \text{number of inputs} \end{cases}$

$$[h(t)] = \begin{bmatrix} h_{11}(t) & h_{12}(t) & h_{13}(t) & h_{14}(t) & h_{15}(t) \\ h_{21}(t) & h_{22}(t) & h_{23}(t) & h_{24}(t) & h_{25}(t) \\ h_{31}(t) & h_{32}(t) & h_{33}(t) & h_{34}(t) & h_{35}(t) \\ h_{41}(t) & h_{42}(t) & h_{43}(t) & h_{44}(t) & h_{45}(t) \\ h_{51}(t) & h_{52}(t) & h_{53}(t) & h_{54}(t) & h_{55}(t) \end{bmatrix}$$

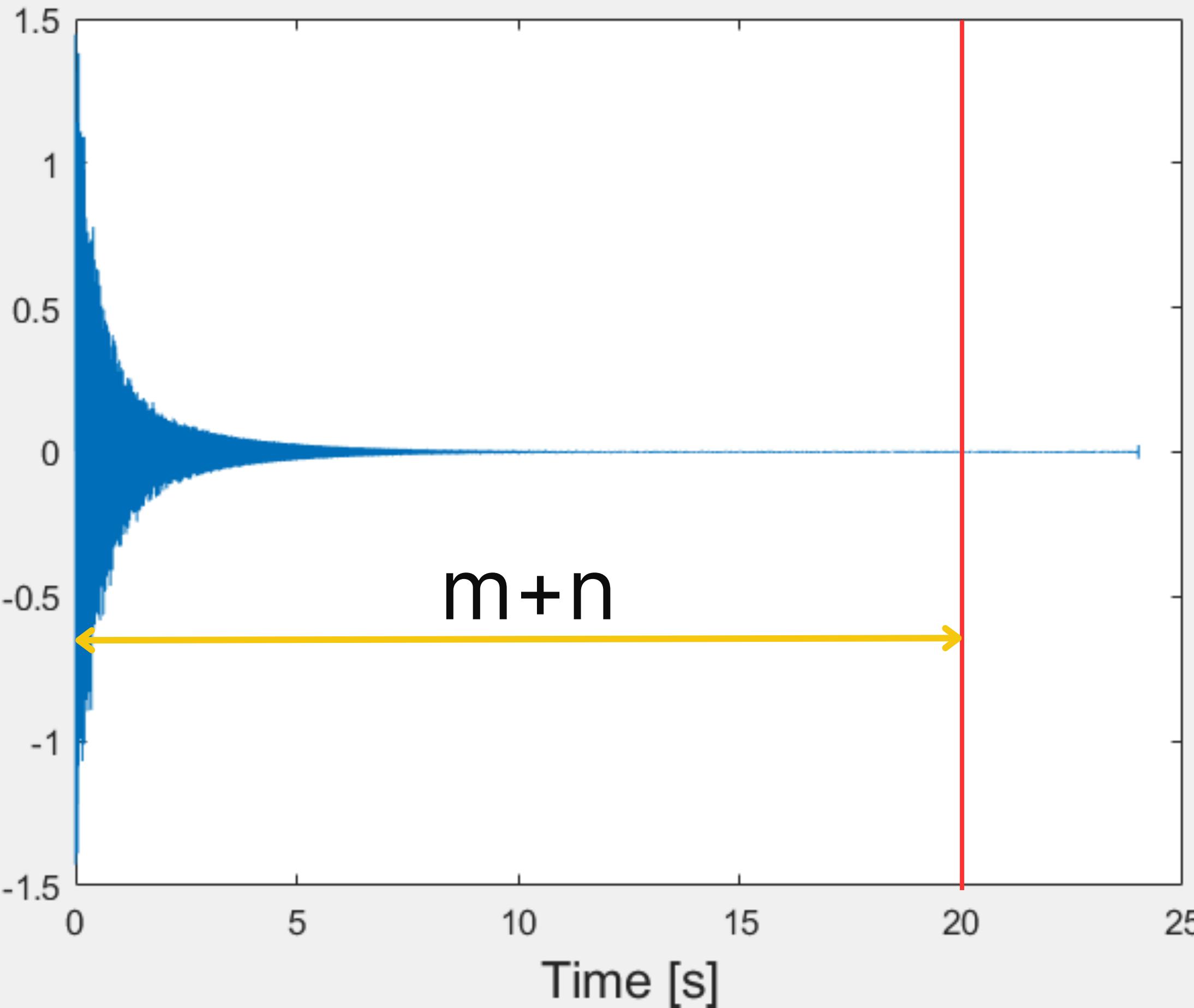


FROM 3D TO 2D



$$[H_{mn}(t)] = \begin{bmatrix} h(t) & h(t + \Delta t) & \dots & h(t + (n - 1)\Delta t) \\ h(t + \Delta t) & h(t + 2\Delta t) & \dots & h(t + n\Delta t) \\ \vdots & \vdots & \vdots & \vdots \\ h(t + (m - 1)\Delta t) & h(t + m\Delta t) & \dots & h(t + (m + n - 2)\Delta t) \end{bmatrix}$$

Fastest decaying Impulse Response Function



IS ALL THE DATA NECESSARY?

- Time series is cropped to limit noise introduction
- Length of time series fixes $m+n = \text{constant}$

$$[H_{mn}(t)] = [\tilde{\Psi}] [e^{s_r t}] [\tilde{L}]$$

Rectangular Matrix!

- $[H_{mn}(t)]$ is $m * 5 \times n * 5$
- $[\tilde{\Psi}]$ is $m * 5 \times 2N_{modes}$
- $[e^{s_r t}]$ is $2N_{modes} \times 2N_{modes}$
- $[\tilde{L}]$ is $2N_{modes} \times n * 5$

DIMENSIONS MATTER!

$$[H_{mn}(t)] = [\tilde{\Psi}] [e^{s_r t}] [\tilde{L}]$$

Rectangular Matrix!

- $[H_{mn}(t)]$ is $m * 5 \times n * 5$
- $[\tilde{\Psi}]$ is $m * 5 \times 2N_{modes}$
- $[e^{s_r t}]$ is $2N_{modes} \times 2N_{modes}$
- $[\tilde{L}]$ is $2N_{modes} \times n * 5$

TIME SHIFTED H_{mn}

$$[H_{mn}(t + \Delta t)] = [\tilde{\Psi}] [e^{s_r \Delta t}] [e^{s_r t}] [\tilde{L}]^T = [\tilde{\Psi}] [e^{s_r \Delta t}] [\tilde{\Psi}]^+ [H_{mn}(t)]$$

NOTICE:

$$[H_{mn}(t)] = [\tilde{\Psi}] [e^{s_r t}] [\tilde{L}]^T \Rightarrow [\tilde{\Psi}]^+ [H_{mn}(t)] = [e^{s_r t}] [L]^T$$



$$[H_{mn}(t + \Delta t)] [H_{mn}(t)]^+ = [\tilde{\Psi}] [e^{s_r \Delta t}] [\tilde{\Psi}]^+$$

SQUARE MATRIX $5m \times 5m$

**SQUARE
MATRIX**

EIGENVALUE-EIGENVECTOR APPROACH

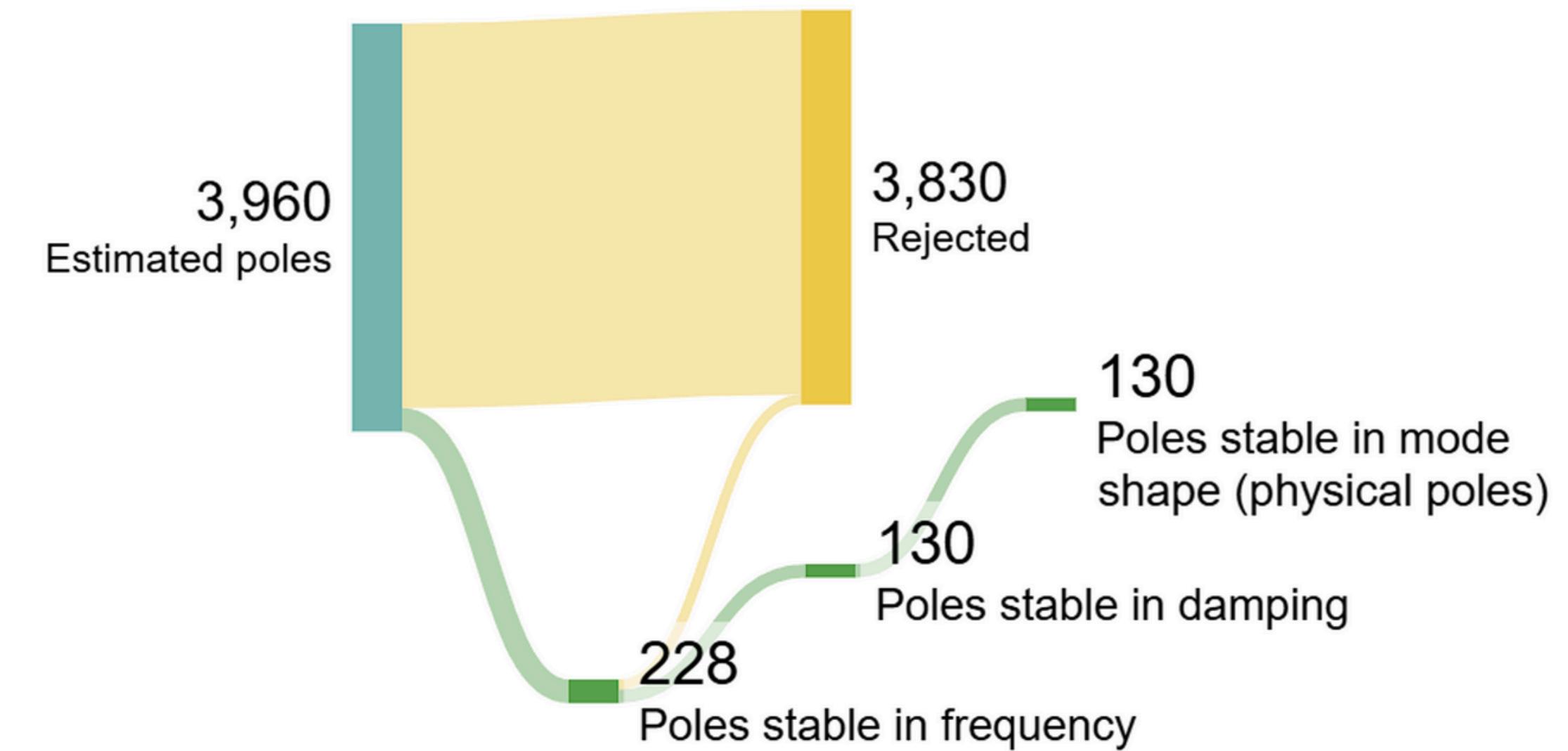
$$[H_{mn}(t + \Delta t)] [H_{mn}(t)]^+ = [\tilde{\Psi}] [e^{s_r \Delta t}] [\tilde{\Psi}]^+$$

- $[H_{mn}(t + \Delta t)][H_{mn}(t)]^+$ is $m * 5 \times m * 5$
- $[\tilde{\Psi}]$ is $m * 5 \times m * 5$
- $[e^{s_r \Delta t}]$ is $m * 5 \times 5 * m$
- $[\tilde{\Psi}]^+$ is $m * 5 \times m * 5$

SYSTEM'S
ORDER
 $2N = 5m$

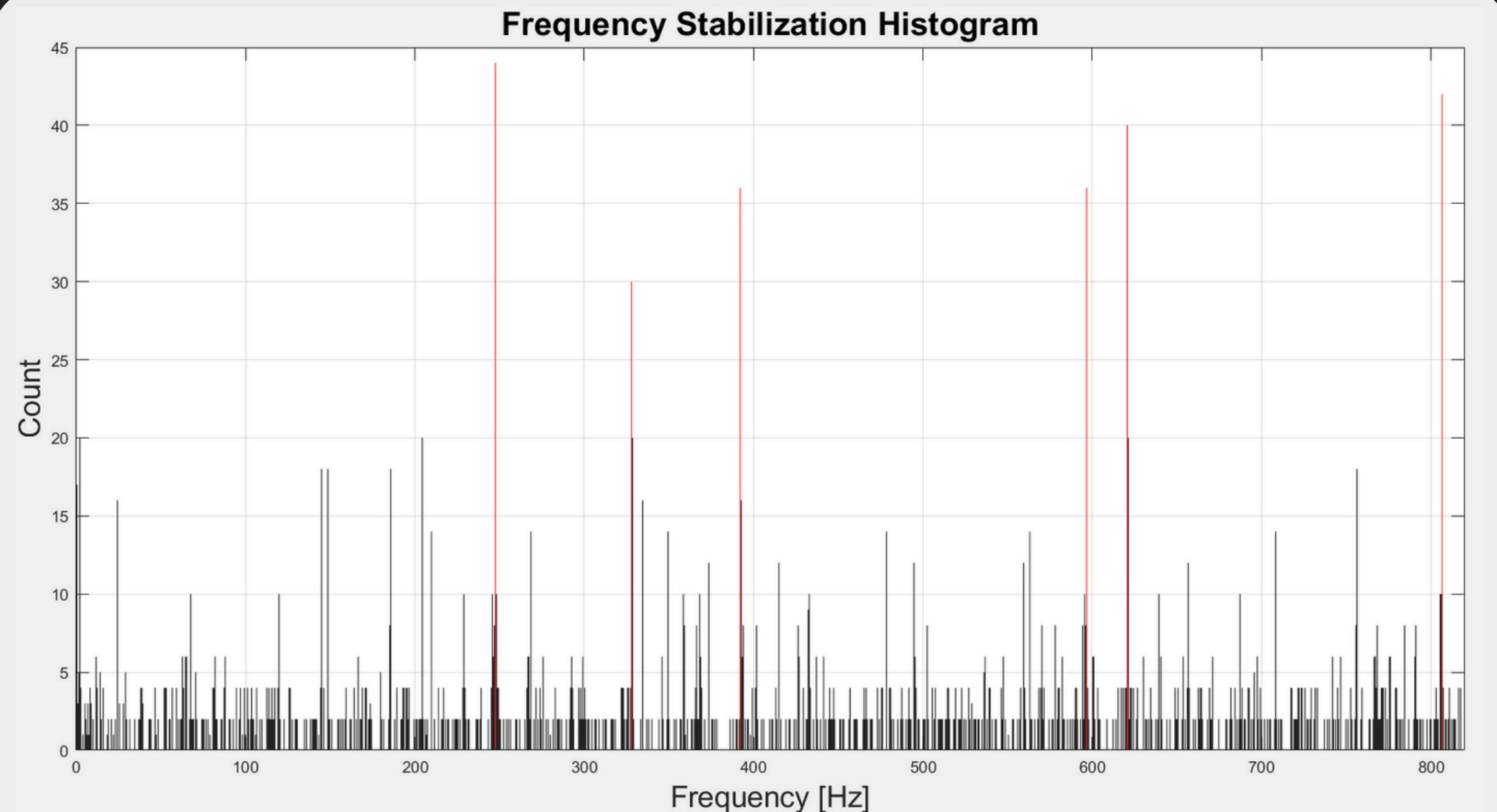
FILTERING PHYSICAL POLES

ONLY 3.3% SURVIVED



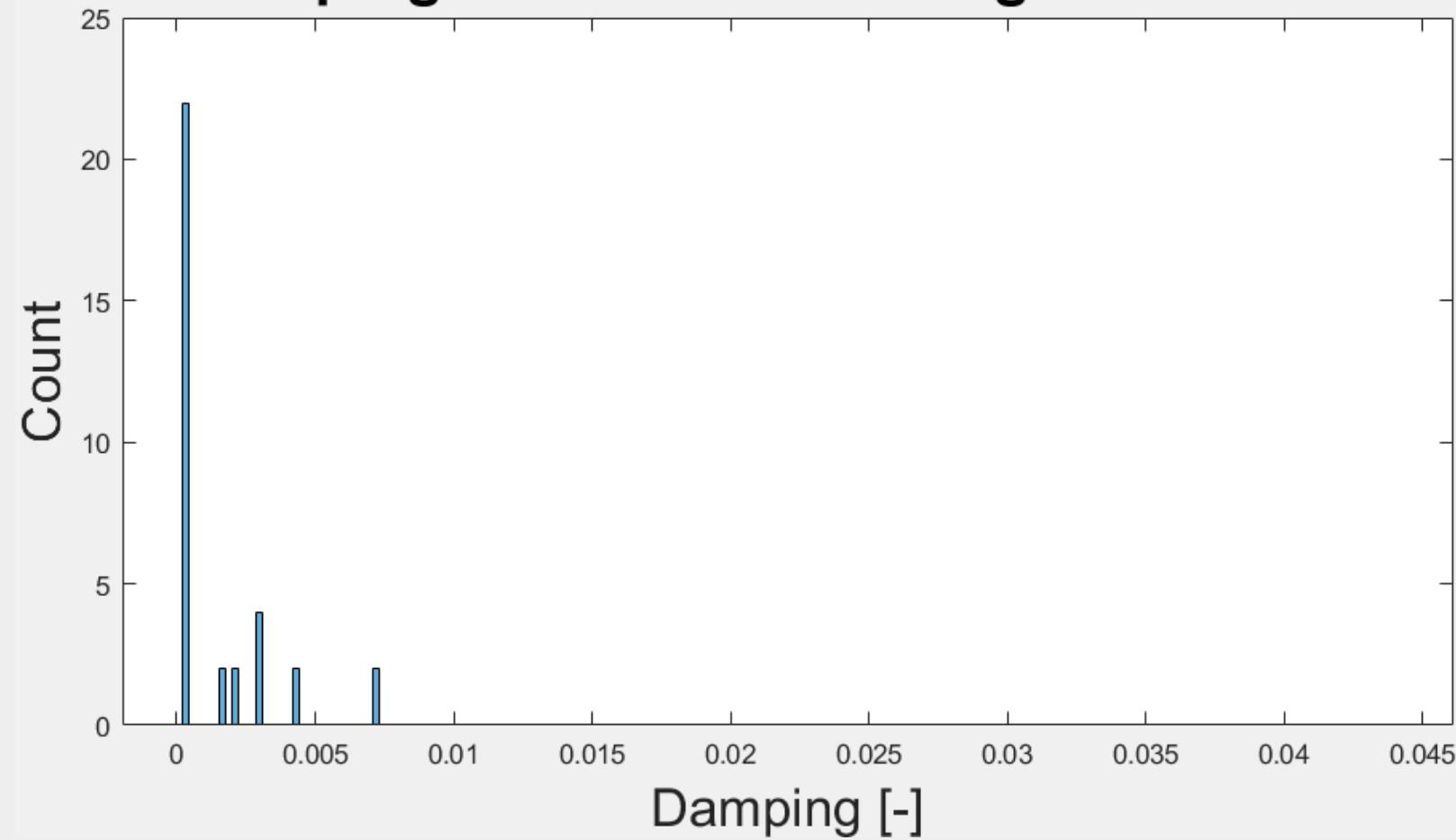
FREQUENCY STABLE POLES

- ≈ 247 Hz
- ≈ 328 Hz
- ≈ 392 Hz
- ≈ 596 Hz
- ≈ 621 Hz
- ≈ 807 Hz

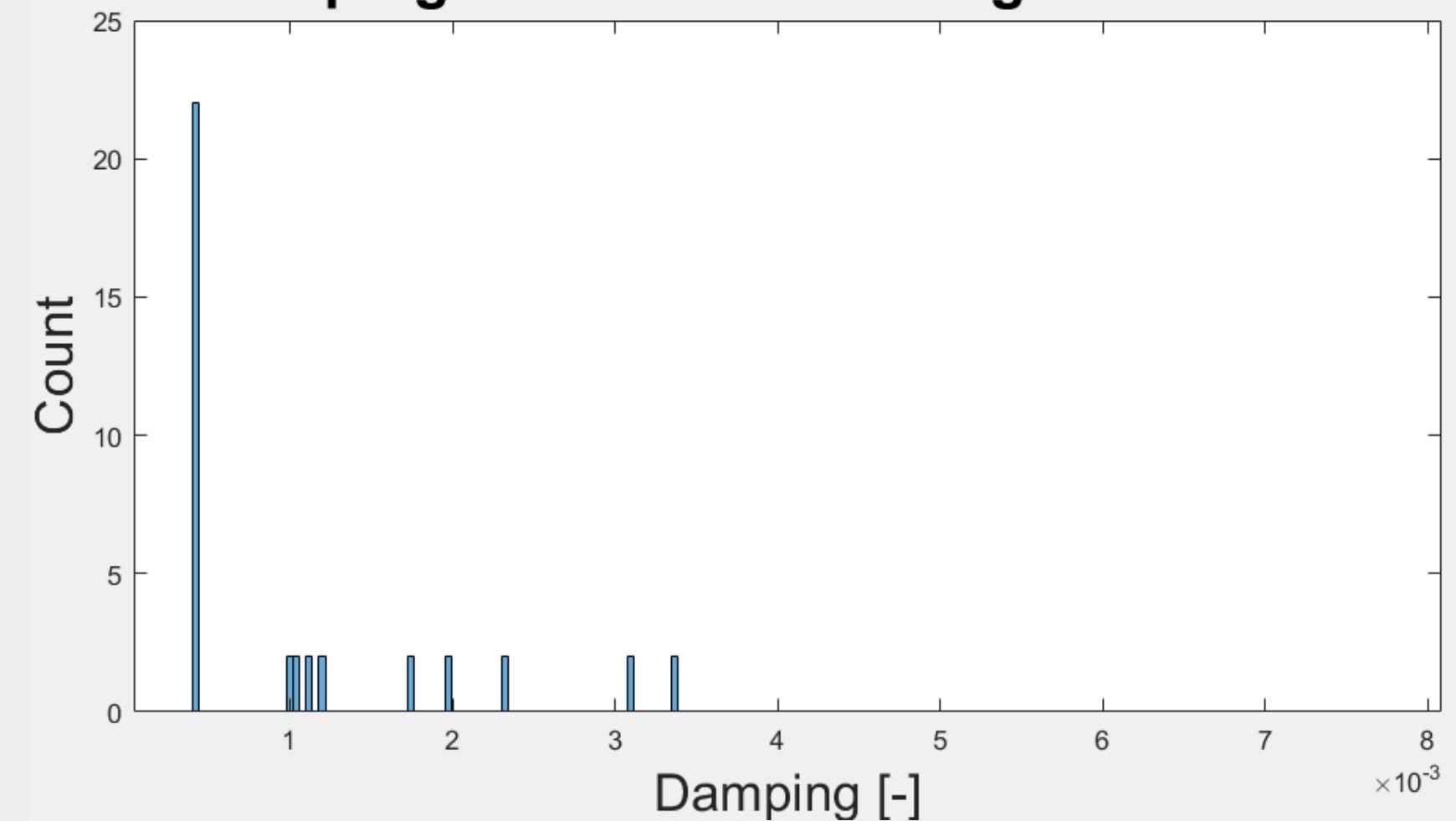


DAMPING STABLE POLES

Damping Stabilization Histogram mode: 3



Damping Stabilization Histogram mode: 6



MODE STABLE POLES

MAC Good enough...

	1	2	3	4	5	6	7	8	9	10	11
1	1	1	1	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
2		1	1	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
3		1	1	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
4		1	1	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
5		1	1	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
6		1	1	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	12	13	14	15	16	17	18	19	20	21	22
1	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0	0
2	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
3	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
4	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
5	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
6	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

ONLY 20
POLES
FOR MODE 1

MODAL MASSES ESTIMATION

$$H_{pq}(\omega) = \sum_{r=1}^{2*6} \frac{\psi_{pr}\psi_{qr}}{j\omega - s_r} Q_r + R_U + \frac{R_L}{\omega^2}$$

SETUP A LEAST SQUARE PROBLEM

$$\left\{ \begin{array}{l} H_{pq}(\omega_1) \\ H_{pq}(\omega_2) \\ \vdots \\ H_{pq}(\omega_{N_f}) \end{array} \right\} = \left[\begin{array}{cccccc} \frac{\psi_{p1}\psi_{q1}}{j\omega_1 - s_1} & \frac{\psi_{p2}\psi_{q2}}{j\omega_1 - s_2} & \dots & \frac{\psi_{p2*6}\psi_{q2*6}}{j\omega_1 - s_{2*6}} & 1 & \frac{1}{\omega_1^2} \\ \frac{\psi_{p1}\psi_{q1}}{j\omega_2 - s_1} & \frac{\psi_{p2}\psi_{q2}}{j\omega_2 - s_2} & \dots & \frac{\psi_{p2*6}\psi_{q2*6}}{j\omega_2 - s_{2*6}} & 1 & \frac{1}{\omega_2^2} \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ \frac{\psi_{p1}\psi_{q1}}{j\omega_{N_f} - s_1} & \frac{\psi_{p2}\psi_{q2}}{j\omega_{N_f} - s_2} & \dots & \frac{\psi_{p2*6}\psi_{q2*6}}{j\omega_{N_f} - s_{2*6}} & 1 & \frac{1}{\omega_{N_f}^2} \end{array} \right] \left\{ \begin{array}{l} Q_1 \\ Q_2 \\ \vdots \\ Q_{2*6} \\ R_U \\ R_L \end{array} \right\}$$

MODAL PARAMETERS SUMMARY

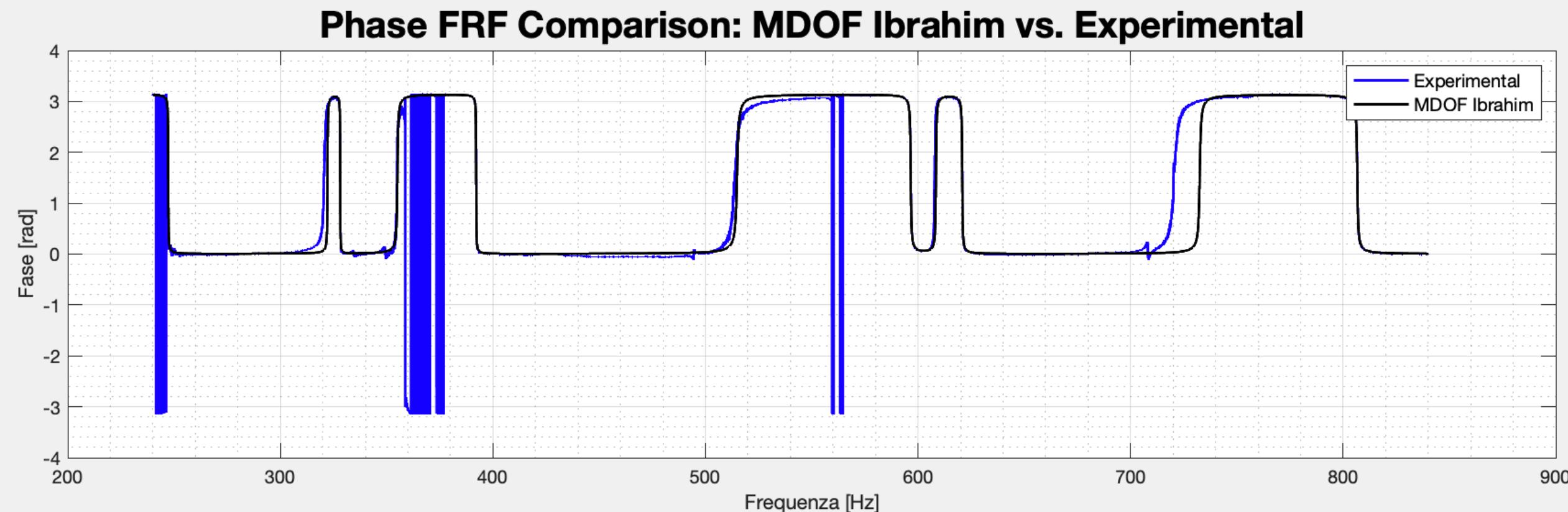
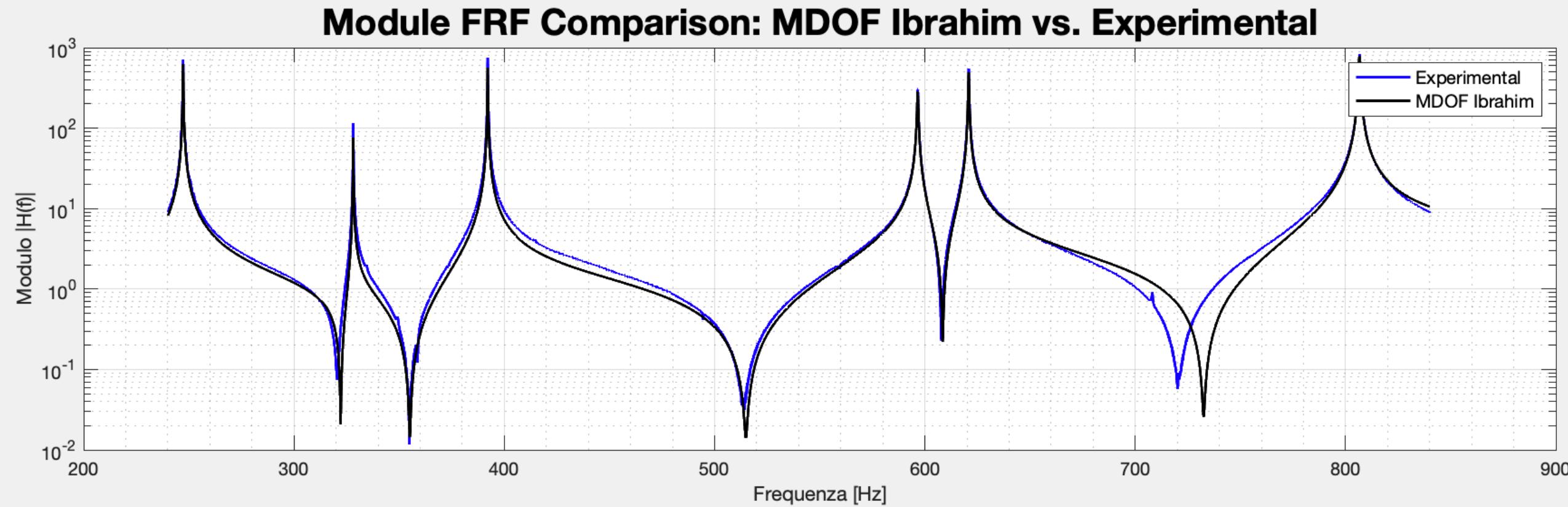
FREQUENCIES: CONCORDING WITH FEM

DAMPING: LOW -> THIN SOLID STEEL PLATE

MODE SHAPE: CONCORDING WITH FEM

	Mode 1	Mode 2	Mode 3	Mode 4	Mode 5	Mode 6
Res.Frequencies [Hz]	247.29	328.17	392.12	596.57	620.81	806.63
Damping [-]	0.00041	0.00019	0.00022	0.00040	0.00027	0.00041
Norm. Mode shapes	-1	0.26	0.8	1	0.66	-0.05
	0.91	0.24	1	-0.28	1	-1
	-0.89	0.25	0.80	-0.49	-0.66	-0.95
	0.92	0.23	0.81	0.82	-0.92	-0.09
	-0.03	1	-0.11	-0.79	0.16	0.43

FRF RECONSTRUCTION - MDOF IBRAHIM

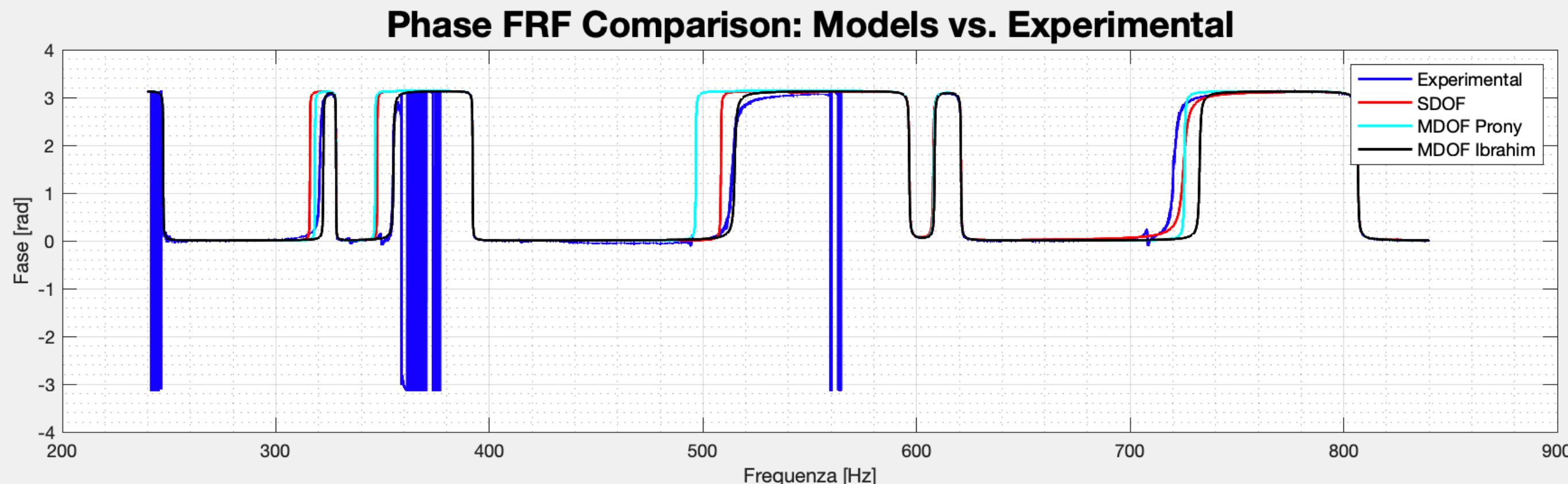
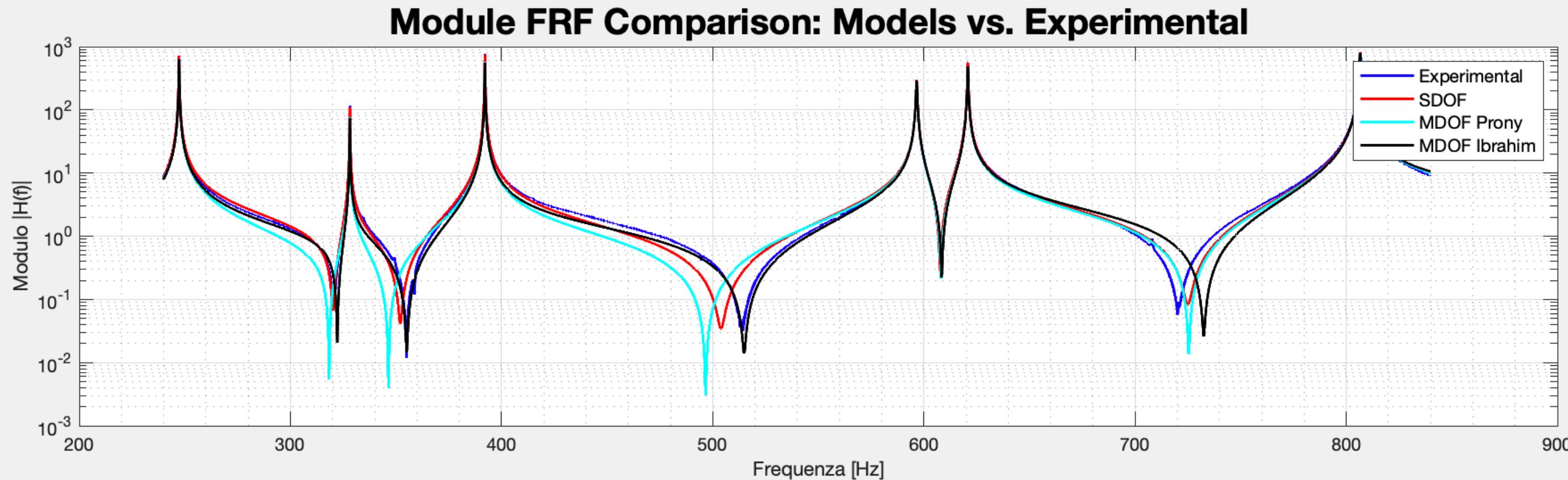




RESULTS

MAC computation between experimental and numerical mode shapes

FRF RECONSTRUCTION - FINAL COMPARISON



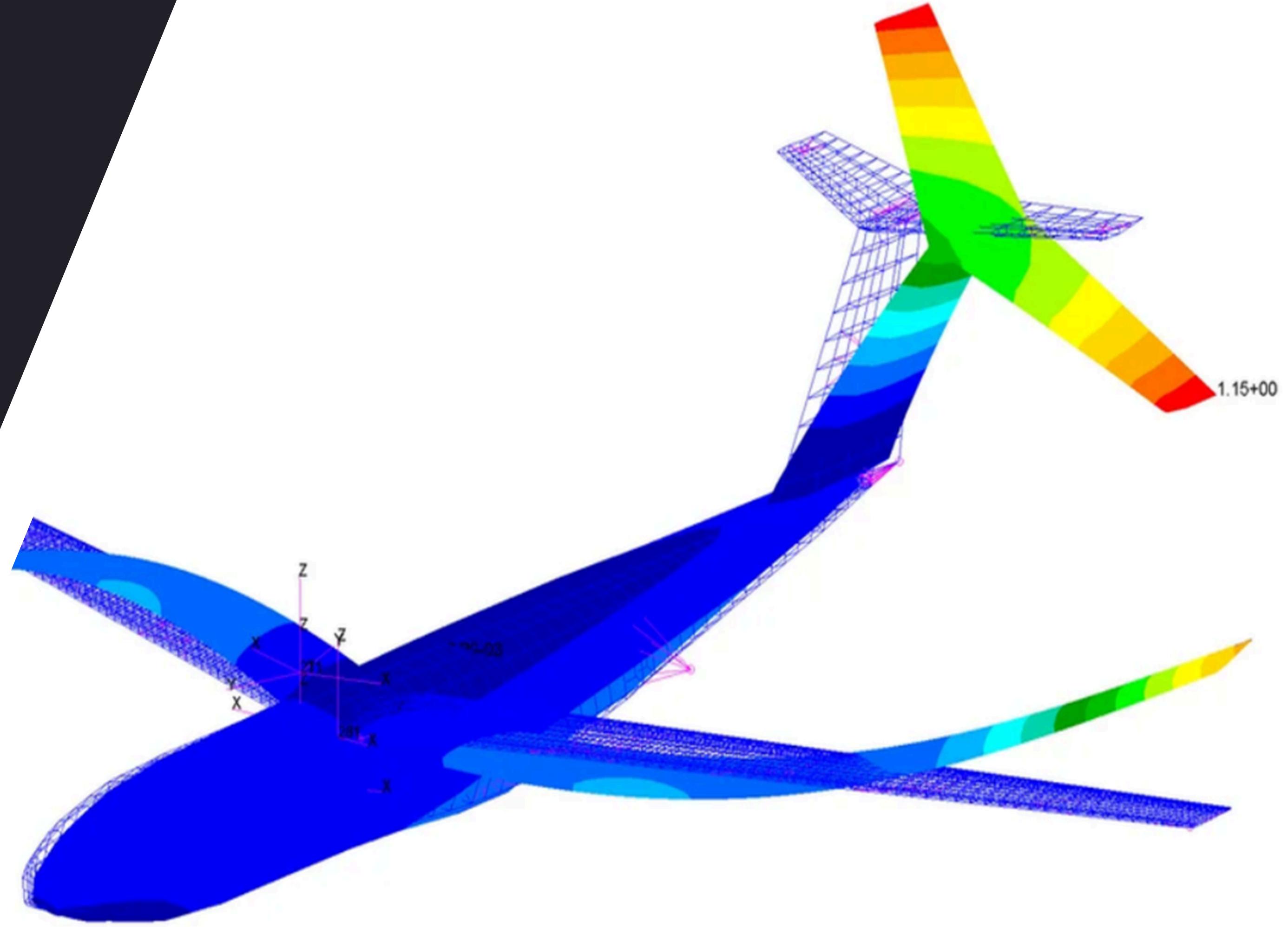
MAC COMPARISON

$$MAC(A, X) = \frac{\left| \sum_{j=1}^n (\psi_X)_j (\psi_A)_j^* \right|^2}{\left(\sum_{j=1}^n (\psi_X)_j (\psi_X)_j^* \right) \cdot \left(\sum_{j=1}^n (\psi_A)_j (\psi_A)_j^* \right)} = \frac{\left| \{ \psi_X \}^T \{ \psi_A \} \right|^2}{\left(\{ \psi_X \}^T \{ \psi_X \} \right) \cdot \left(\{ \psi_A \}^T \{ \psi_A \} \right)}$$

MODE	SDOF	PRONY	IBRAHIM
1st	0.9996	0.9979	0.9996
2nd	0.9973	0.9939	0.9972
3rd	0.9980	0.9938	0.9980
4th	0.9986	0.9954	0.9983
5th	0.9968	0.9922	0.9969
6th	0.9987	0.9902	0.9988



WHY ALL THIS?



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