Code ▾

Appendix for: Measuring the response diversity of ecological communities experiencing multifarious environmental change.

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1 Introduction

Researchers have previous suggested that the response diversity of a community be measured by the diversity of responses to environmental change. For example, one can measure the response of each of the species' intrinsic growth rate to temperature, quantify the strength and direction of these responses (e.g., as the first derivative of the response curve), and calculate the diversity of responses (e.g., by calculating variation in the first derivatives among the species in a community). When responses are nonlinear, the response diversity will be a function of the environmental state (i.e. the first derivative is a function of the value of the environmental state). So far we demonstrated this approach for quantifying response diversity in the context of a single environmental factor, but given that multiple environmental factors can change simultaneously, we need an approach that works in that context.

2 The principle

To learn about the mathematical principles watch these youtube videos:

- Surfaces and Partial Derivatives (https://www.youtube.com/watch?v=k4wNIZr8GU4)
- 54. Slope of the Surface in Any Direction - Directional Derivative, and Properties of the Gradient (https://www.youtube.com/watch?v=wfjipWmyRYg)

Imagine that the growth rate of a population depends on two environmental factors, e.g. temperature and salinity. We can represent the dependency as $G = f(T, S)$, where G is growth rate, T is temperature, and S is salinity. It may be that the dependencies are linear, nonlinear, and with an interaction between temperature and salinity, hence our approach needs to be able to accommodate this phenomena.

The response of growth rate to change in temperature and salinity is the gradient / slope of this surface, with units of growth rate [per time] per temperature [degrees C] per salinity [parts per thousand]. Because the slope (first derivative) of the surface can (when dependencies are nonlinear) vary across the surface (location on the surface), and can vary in different directions on the surface, to calculate a slope we must specify the current environment (location on the surface) and the trajectory of change in the environment. The location on the curve is the current environmental condition, (T_0, S_0), and the trajectory of environmental change is the unit vector $\hat{u} = \langle U_T, U_S \rangle$.

Put another way, we calculate a directional derivative at a point on the response surface. We can write this as $D_u f(T_0, S_0)$ and can calculate it as $f_T(T_0, S_0)U_T + f_S(T_0, S_0)U_S$, where f_T is the partial derivative of $f(T, S)$ with respect to T and f_S is the partial derivative of $f(T, S)$ with respect to S.

Efficient evaluating in n dimensions can be done by taking the dot product of the partial derivatives at the location and the direction unit vector: $D_u f(T_0, S_0) = \triangledown f \cdot \hat{u}$ where, $\triangledown f = \langle f_T, f_S \rangle$. (In R, the dot product of `a` and `b` is `sum(a*b)`)

Figure 2.1 is an illustration of the principle of directional derivatives on a surface.

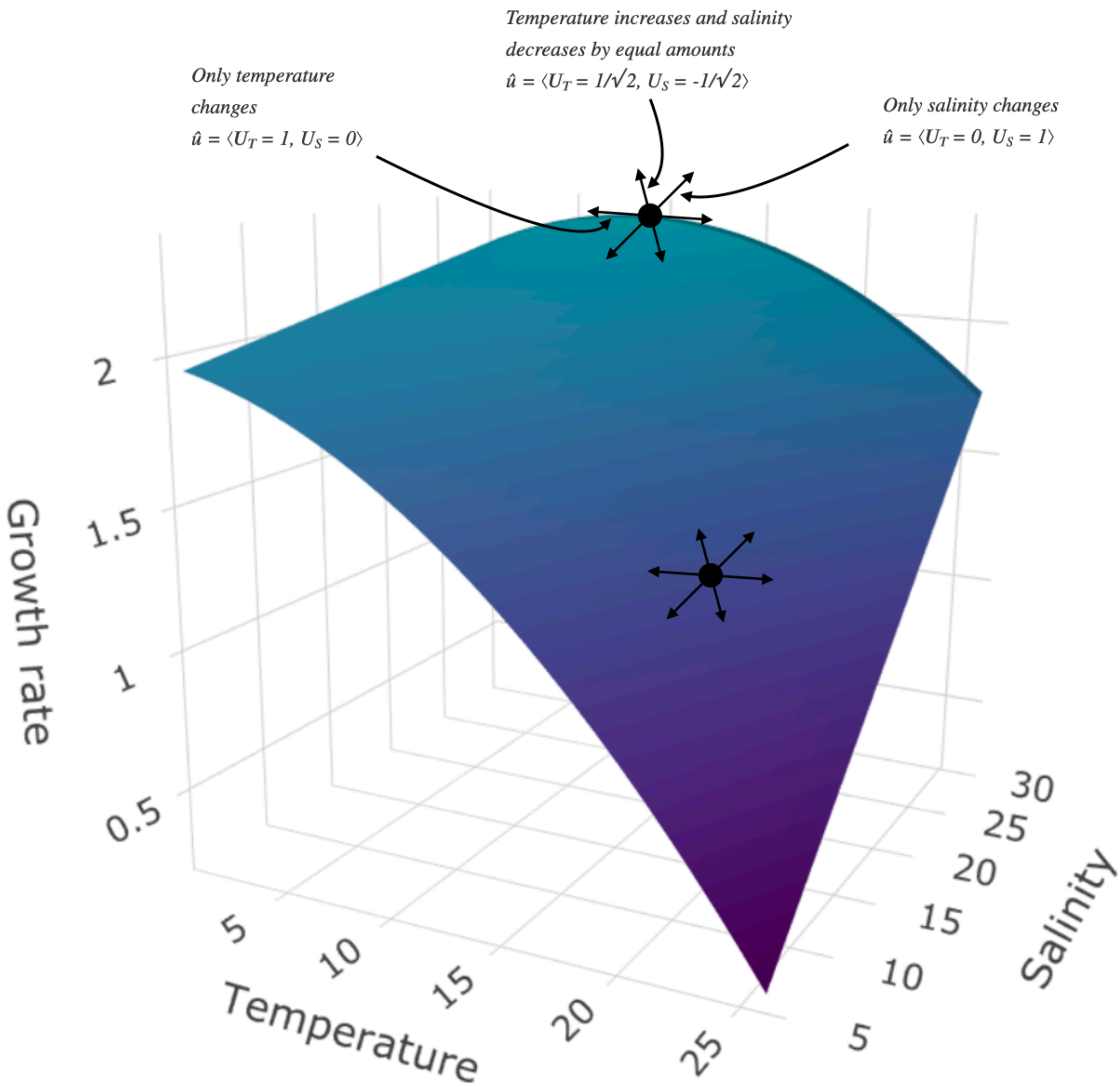

Figure 2.1: Response surface and principle illustration

3 A simulated empirical example

Numerous mathematical functions have been used to represent how organismal performance changes with an environmental driver. Moreover, multiple mathematical functions have been used to represent an interactive effect of two or more environmental drivers on species performance e.g. Thomas et al 2017 (https://onlinelibrary.wiley.com/doi/full/10.1111/gcb.13641).

3.1 Simulating performance curves

Let us use the Eppley performance curve, which was used, for example, in this paper Bernhardt et al. 2018 (https://royalsocietypublishing.org/doi/10.1098/rspb.2018.1076).

With one environmental variable, the performance (i.e., rate) is given by:

- $rate(E) = ae^{bE}(1 - (\frac{E-z}{w/2})^2)$
- E is the values of the environmental factor.
- z controls location of maximum.
- w controls range of E over which the rate is positive.
- a scaling constant.
- b controls rate of increase towards the maximum rate, as E increases.

Adding a second environmental variable gives:

$rate(E_1, E_2) = a_1 e^{b_1 E_1}(1 - (\frac{E_1 - z_1}{w_1/2})^2) + a_2 e^{b_2 E_2}(1 - (\frac{E_2 - z_2}{w_2/2})^2)$

In this case, it is clear the effect of E_1 and E_2 is defined as being additive. For example, the value of E_2 does not affect the value of E_1 at which the rate is maximised (z_1), and vice-versa.

Including an interaction. One way to do this is to make the value of E_1 at which the rate is maximised depend on the value of E_2:

$rate(E_1, E_2) = a_1 e^{b_1 E_1}(1 - (\frac{(E_1 + z_{int21} * E_2 - z_1)}{w_1/2})^2 + a_2 e^{b_2 E_2}(1 - (\frac{E_2 - z_2}{w_2/2})^2$

When $z_{int21} = 0$ then this equation becomes the previously mentioned additive one. When $z_{int} \neq 0$ then the value of E_1 at which the rate is maximised is a function of the value of E_2. We used this method for adding an interaction due to its simplicity. Other methods could be used, and if also or otherwise used could add confidence about the robustness of the method for calculating response diversity.

3.2 Simulating multiple species' performance curves

3.2.1 No interacting environmental effects

First we create (or import) a table of parameter values of each species, with species in the rows and parameters in the columns. In the following example, only values of the z parameters differ among the species (which determine the location of the maximum rate).

Show 10 entries

Search:

	a1	b1	z1	w1	a2	b2	z2	w2	z_int21	sd_rate
			All				All		A	A
1	1e-9	0.063	270.0840140227228	60	0.001	0.02	22.43456760421395	10	0	0
2	1e-9	0.063	277.0911623351276	60	0.001	0.02	16.46992172580212	10	0	0
3	1e-9	0.063	290.8805276826024	60	0.001	0.02	10.01292706932873	10	0	0
4	1e-9	0.063	286.0165206925012	60	0.001	0.02	24.24311844166368	10	0	0
5	1e-9	0.063	294.4974396913312	60	0.001	0.02	16.17267773486674	10	0	0
6	1e-9	0.063	291.0067399148829	60	0.001	0.02	25.73636558372527	10	0	0
7	1e-9	0.063	281.938274691347	60	0.001	0.02	26.77791999652982	10	0	0
8	1e-9	0.063	294.50001725927	60	0.001	0.02	15.46716617885977	10	0	0
9	1e-9	0.063	296.8717755982652	60	0.001	0.02	23.17856966517866	10	0	0
10	1e-9	0.063	272.5634251185693	60	0.001	0.02	20.36314472556114	10	0	0

Showing 1 to 10 of 10 entries

Previous 1 Next

For convenience we then convert the table of parameters into a list-column (https://dcl-prog.stanford.edu/list-columns.html). We can then easily make performance curves of each of the species, and put those into a list-column in the same table.

Show

Here are some examples of the species' performance curves (only with additive effects of E_1 and E_2).

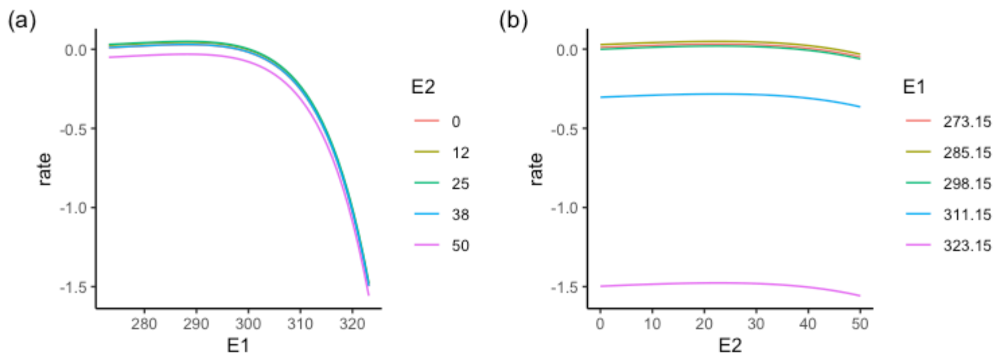

Figure 3.1: Performance curves for a species with maximum growth at **low** values of E_1. Without interacting environmental effects.

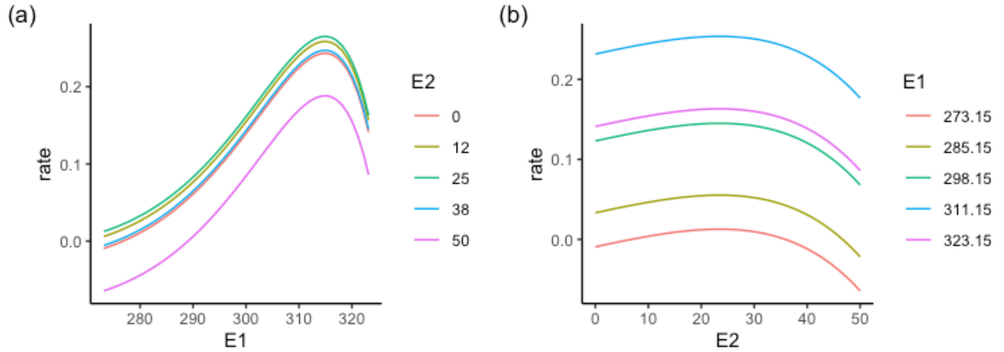

Figure 3.2: Performance curves for a species with maximum growth at **high** values of E_1. Without interacting environmental effects.

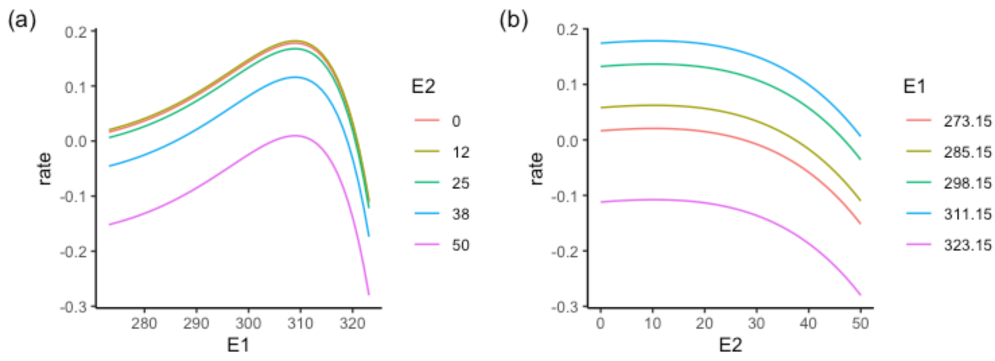

Figure 3.3: Performance curves for a species with maximum growth at **low** values of E_2. Without interacting environmental effects.

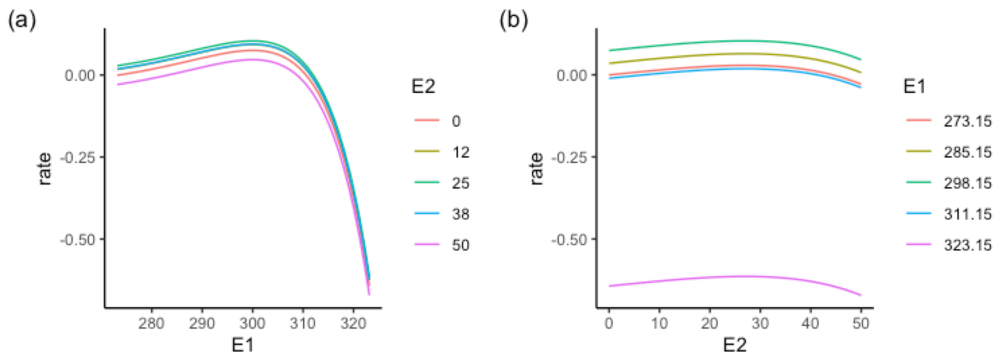

Figure 3.4: Performance curves for a species with maximum growth at **high** values of E_2. Without interacting environmental effects.

3.2.2 Interacting environmental effects

And now with interacting environmental effects…

Show 10 **entries** **Search:**

	a1	b1	z1	w1	a2	b2	z2	w2	z_int21	sd_rate
			All				All		All	A
1	1e-9	0.063	288.3548494032584	60	0.001	0.02	27.06988717895001	10	-0.2016422196385337	0
2	1e-9	0.063	271.3576483167708	60	0.001	0.02	12.60907350108027	10	-0.2285076291670147	0
3	1e-9	0.063	286.3001320743933	60	0.001	0.02	21.76479984074831	10	-0.2174869779064336	0
4	1e-9	0.063	272.1065990324132	60	0.001	0.02	21.33847609162331	10	-0.1396200451061001	0
5	1e-9	0.063	284.9350237450562	60	0.001	0.02	25.31842658761889	10	-0.1835694195781805	0
6	1e-9	0.063	279.786555566825	60	0.001	0.02	27.29842943139374	10	-0.1800884709868218	0
7	1e-9	0.063	295.2997326571494	60	0.001	0.02	11.09688625670969	10	-0.2108279609970924	0
8	1e-9	0.063	288.7897690641694	60	0.001	0.02	13.99729071650654	10	-0.2290171280622909	0
9	1e-9	0.063	274.4790320307948	60	0.001	0.02	27.27455640211701	10	-0.2421644933359179	0
10	1e-9	0.063	280.4067178932019	60	0.001	0.02	18.33161390386522	10	-0.1803280442100247	0

Showing 1 to 10 of 10 entries Previous 1 Next

For convenience we then convert the table of parameters into a list-column (https://dcl-prog.stanford.edu/list-columns.html). We can then easily make performance curves of each of the species, and put those into a list-column in the same table.

Show

Here are some examples of the species' performance curves (with interacting effects of E_1 and E_2).

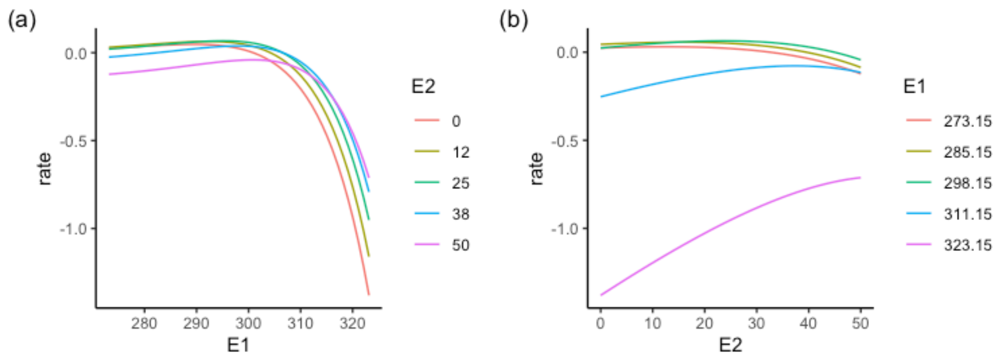

Figure 3.5: Performance curves for a species with maximum growth at **low** values of E_1. With interacting environmental effects.

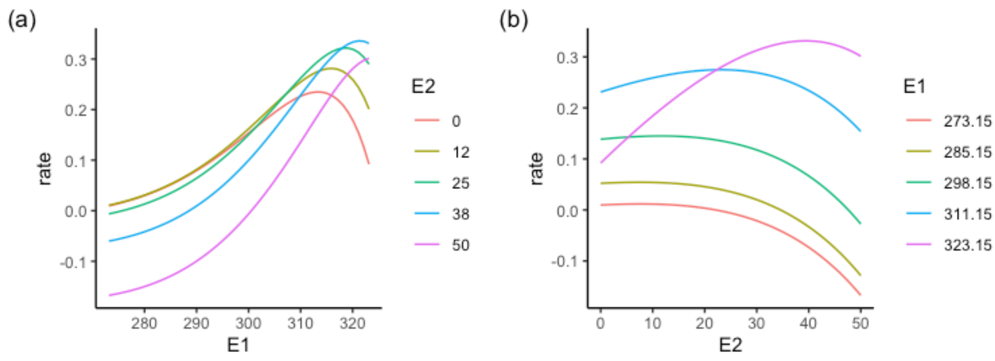

Figure 3.6: Performance curves for a species with maximum growth at **high** values of E_1. With interacting environmental effects.

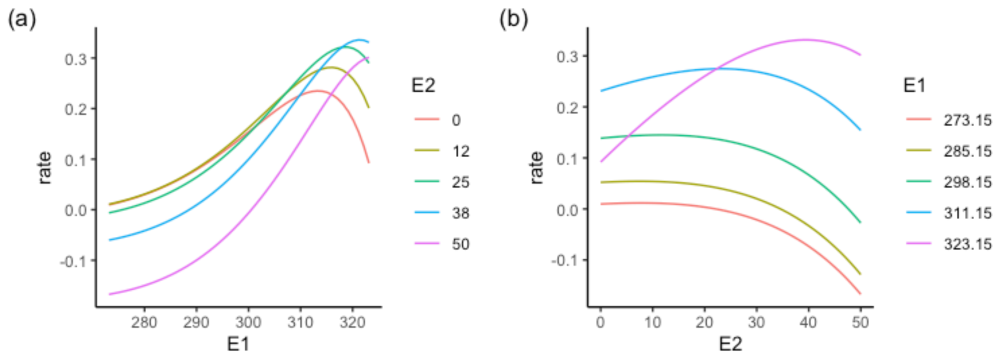

Figure 3.7: Performance curves for a species with maximum growth at **low** values of E_2. With interacting environmental effects.

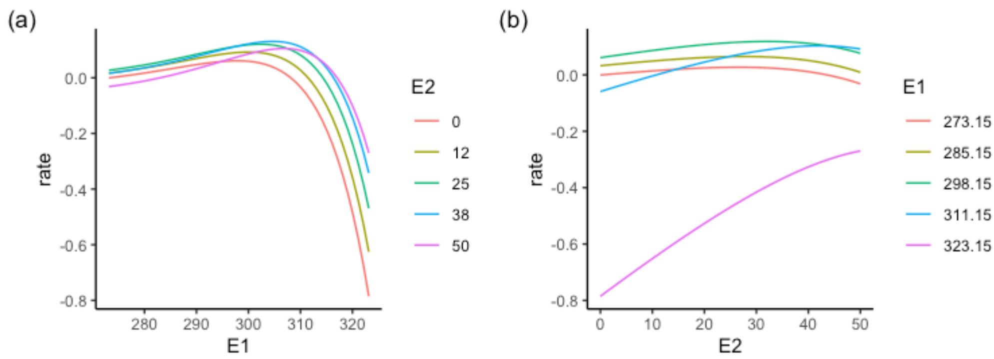

Figure 3.8: Performance curves for a species with maximum growth at **high** values of E_2. With interacting environmental effects.

3.3 Fitting GAMs to noisy rate observations

Try with and without an interaction. Therefore make two species, one with no interaction `z_int = 0` and the other with `z_int = 0.1`. All other parameters are the same. Note that noise is added to the rate observations.

Bottom line is that the gam picks up an interaction when we have included one in the parameters used to generate the rates, and does not pick one up when we have not. This confirms that our more mechanistic thinking and methods are matching our statistical thinking and methods, and confirms that each are promising, so far.

Show 10 **entries** **Search:**

	a1	b1	z1	w1	a2	b2	z2	w2	z_int21	sd_rate
	All	All	All	All	All	All	All	All	All	All
1	1e-9	0.063	285	60	0.001	0.02	20	10	0	0.02
2	1e-9	0.063	285	60	0.001	0.02	20	10	-0.2	0.02

Showing 1 to 2 of 2 entries

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3.3.1 Without interaction

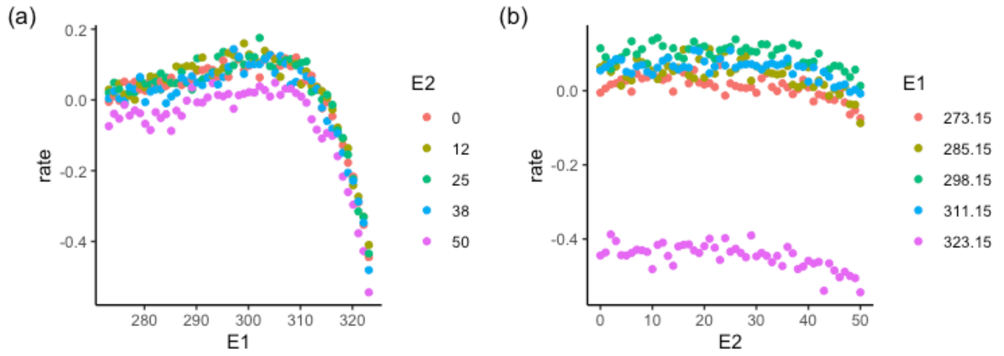

Figure 3.9: Performance curves for a species *without* interacting environmental effects and with some noise in the rate.

```
##
## Family: gaussian
## Link function: identity
##
## Formula:
## rate ~ ti(E1) + ti(E2) + te(E1, E2)
##
## Parametric coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept) 0.015428   0.000499   30.92   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Approximate significance of smooth terms:
##                edf Ref.df       F p-value
## ti(E1)    3.999310  4.000 14148.5  <2e-16 ***
## ti(E2)    3.936725  3.997   608.8  <2e-16 ***
## te(E1,E2) 0.001897 16.000     0.0   0.992
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## R-sq.(adj) =  0.958   Deviance explained = 95.8%
## -REML = -5820.5  Scale est. = 0.00064775  n = 2601
```

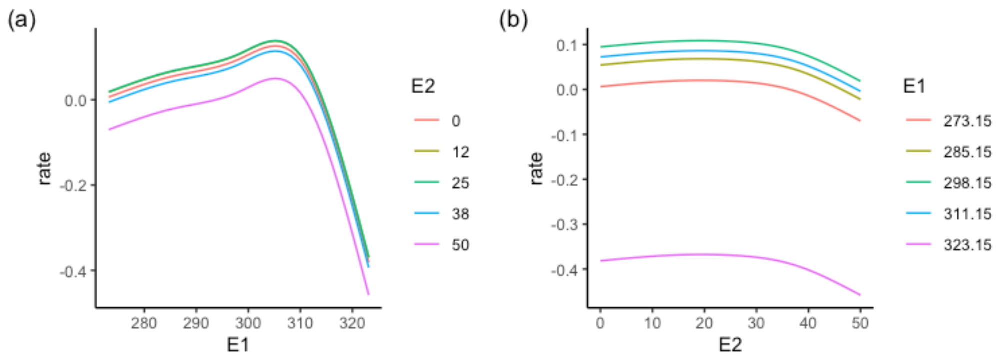

Figure 3.10: Performance curves for a species *without* interacting environmental effects and with some noise in the rate.

```
##
## Family: gaussian
## Link function: identity
##
## Formula:
## rate ~ s(E1) + s(E2)
##
## Parametric coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept) 0.0154285  0.0003952   39.04   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Approximate significance of smooth terms:
##         edf Ref.df       F p-value
## s(E1) 8.970  9.000 10199.8  <2e-16 ***
## s(E2) 7.146  8.187   475.4  <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## R-sq.(adj) =  0.974   Deviance explained = 97.4%
## -REML = -6407.3  Scale est. = 0.00040624  n = 2601
```

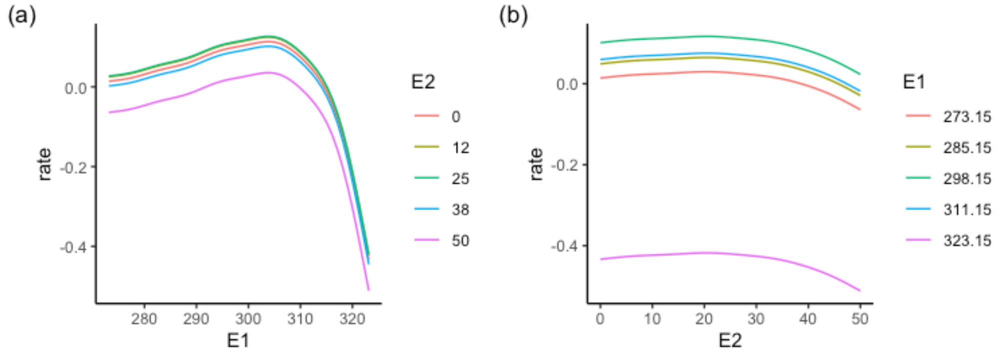

Figure 3.11: Performance curves for a species *without* interacting environmental effects and with some noise in the rate.

3.3.2 With interaction

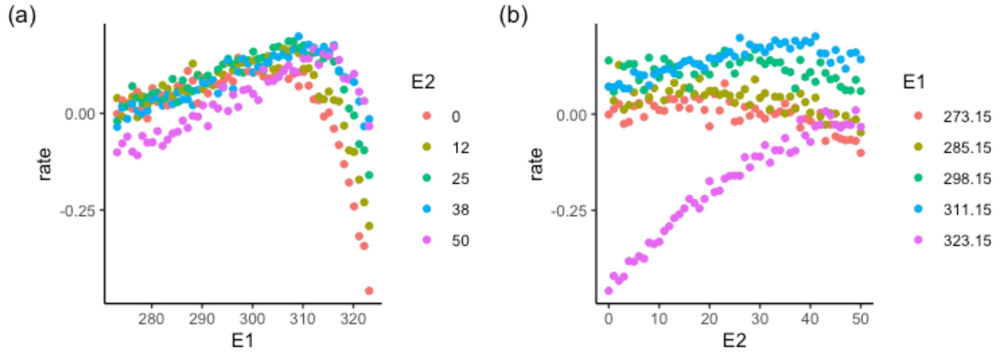

Figure 3.12: Performance curves for a species *with* interacting environmental effects and with some noise in the rate.

```
##
## Family: gaussian
## Link function: identity
##
## Formula:
## rate ~ ti(E1) + ti(E2) + te(E1, E2)
##
## Parametric coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept) 0.0656082  0.0004677   140.3   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Approximate significance of smooth terms:
##              edf Ref.df      F p-value
## ti(E1)     3.991  3.999 1899.8  <2e-16 ***
## ti(E2)     3.991  4.000  302.6  <2e-16 ***
## te(E1,E2) 15.614 16.000  489.1  <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## R-sq.(adj) =  0.924   Deviance explained = 92.5%
## -REML = -5943.3  Scale est. = 0.00056892  n = 2601
```

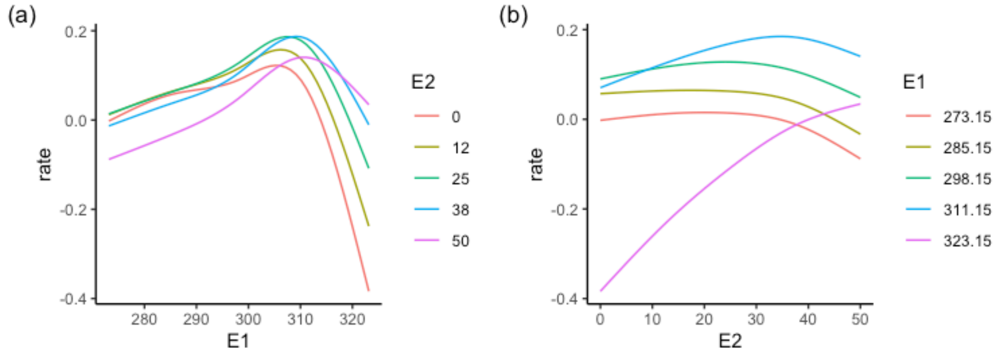

Figure 3.13: Performance curves for a species *with* interacting environmental effects and with some noise in the rate.

```
##
## Family: gaussian
## Link function: identity
##
## Formula:
## rate ~ s(E1) + s(E2)
##
## Parametric coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept) 0.0656082  0.0009042   72.56   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Approximate significance of smooth terms:
##         edf Ref.df     F p-value
## s(E1) 8.561  8.945 672.0  <2e-16 ***
## s(E2) 4.421  5.434 101.2  <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## R-sq.(adj) =  0.717   Deviance explained = 71.8%
## GCV = 0.0021379  Scale est. = 0.0021264  n = 2601
```

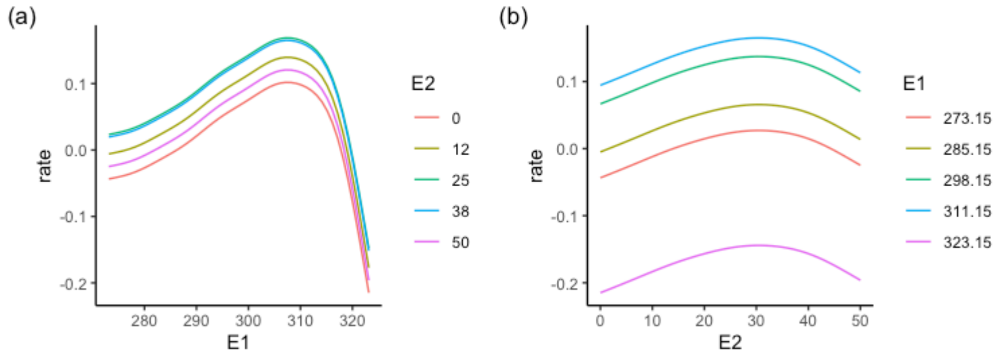

Figure 3.14: Performance curves for a species *with* interacting environmental effects and with some noise in the rate.

4 Partial derivatives

First step in calculating directional derivatives is estimating the two partial derivatives $f_{E1}(E1_0, E2_0)$ and $f_{E2}(E1_0, E2_0)$ (please review the section The principle if necessary).

4.1 Getting the partial derivatives

Partial derivatives. Draw response surface for sp 1 and calculate partial derivatives at a specific location (E1 = 300, E2 = 20). To calculate the partial derivative with respect to E1, E2 must be held constant.

Visualising the partial effect of E1 at a fixed level of E2.

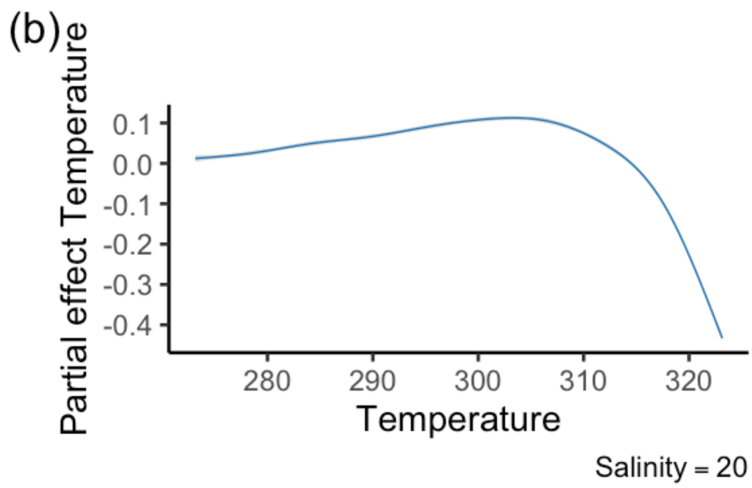

Figure 4.1: Partial effect of E1 on the growth rate of sp 1 when E2 is held constant at E2 = 20.

Partial derivative with respect to E1 when E2 is constant at 20.

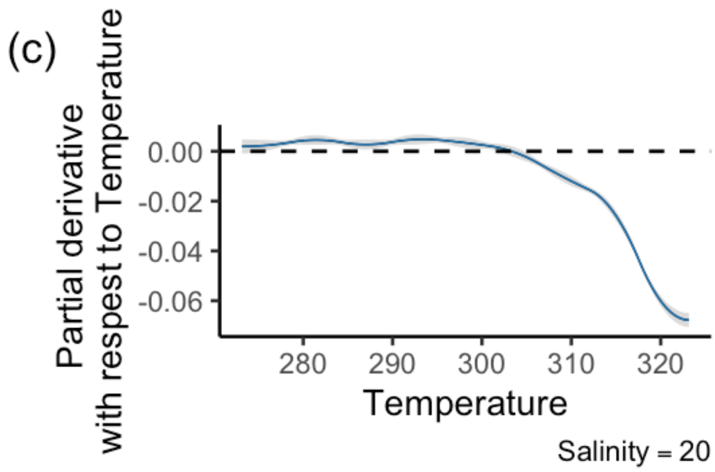

Figure 4.2: Partial derivative with respect to E1 when E2 is constant at 20.

Partial derivatives with respect to E2 (E1 held constant)

Partial effect of E2 on the growth rate of sp 1 when E1 is held constat at E2 = 300

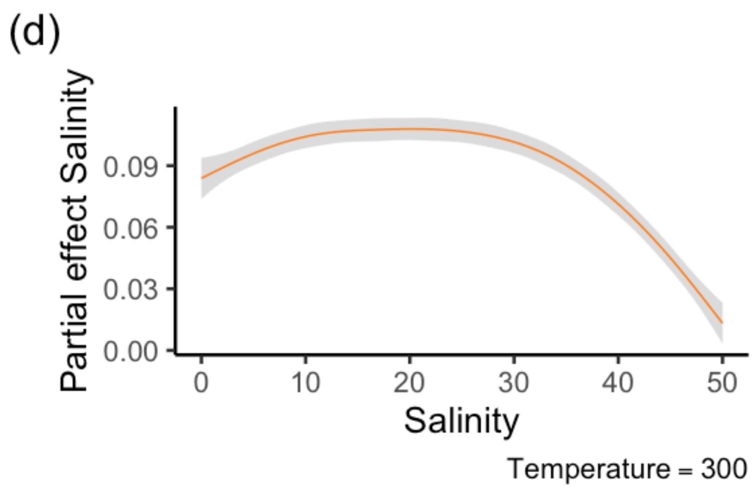

Figure 4.3: Partial effect of E2 on the growth rate of sp 1 when E1 is held constant at E1 = 300.

Partial derivative with respect to E2 when E1 is constant at 300

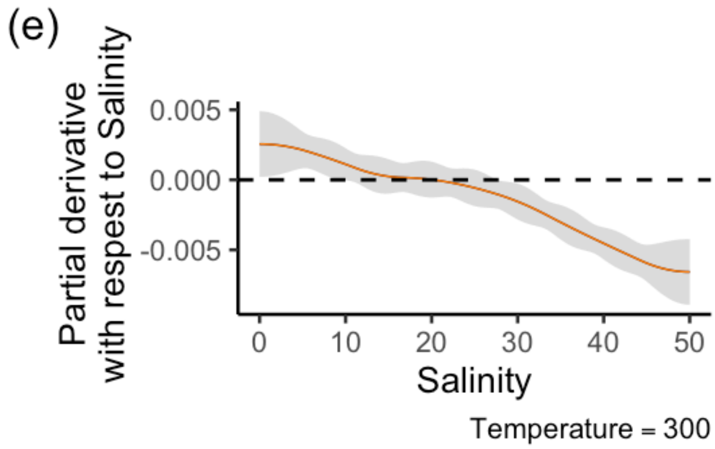

Figure 4.4: Partial derivative with respect to E1 when E2 is constant at 20.

Plot the two partial derivatives and relative effects

5 Directional derivatives

5.1 No trajectory of environmental change specified

5.1.1 One point

We start showing how directional derivatives can be calculated even when the trajectory of the environmental change is unknown. This may be the case when we want to calculate response diversity for future scenarios, and the future trajectory of environmental change is thus not known. Or we may have data for a species or a community at only one environmental location (E1 = x, E2 = y). It is therefore important to be able to measure directional derivatives when the trajectory of the environmental change is unknown, as this can provide useful information on response diversity nonetheless, for instance, by taking the mean of the slopes calculates in all directions. Measuring response diversity when the trajectory of environmental change is unknown may represent a way to systematically measuring response diversity to all possible environmental changes. This represents, in our view, an absolute measure of overall response diversity, since it captures the complete insurance capacity of a system under all possible environmental conditions. We thus put some emphasis on this approach here, and we call this new way of measuring response diversity *Response Capacity*

Here, we calculate, for a specific point (E1 = 300, E2 = 20), directional derivatives in all directions.

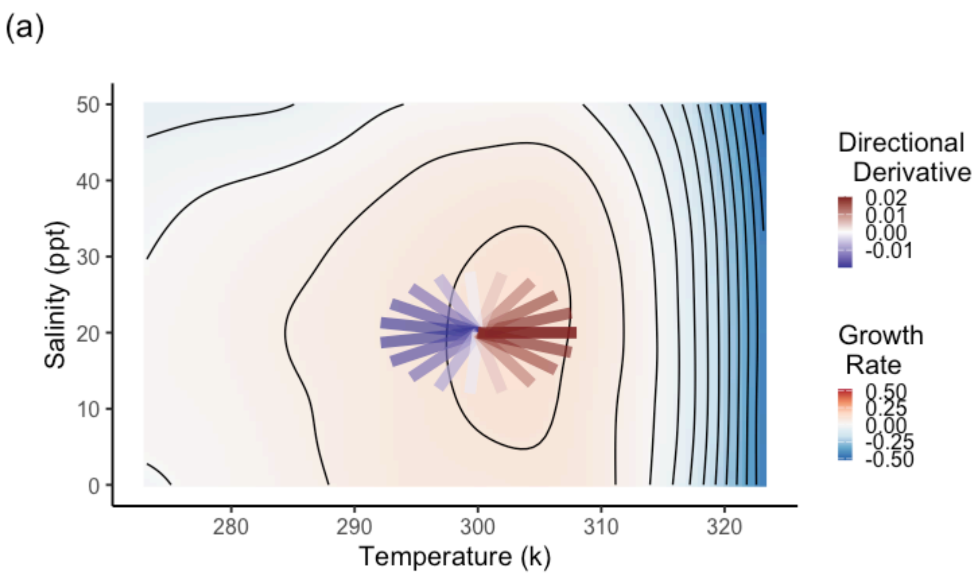

Figure 5.1: Directional derivatives calculated in all possible direction for a specific point on the response surface of sp1. Clearly, the slope of the directional derivative depends on the direction (red positive, blue negative). Note: the size of the radius was only chosen for representation purposes, and does not have any implication. The slope of the segments departing from the point have each their fixed slopes independently of the size of the radius.

5.1.2 Several points

We can measure all possible directional derivatives also for several points on the surface. This might be the case when we know that a species or a community occurs at multiple locations on the surface (multiple environmental conditions), but we do not know the trajectory of change.

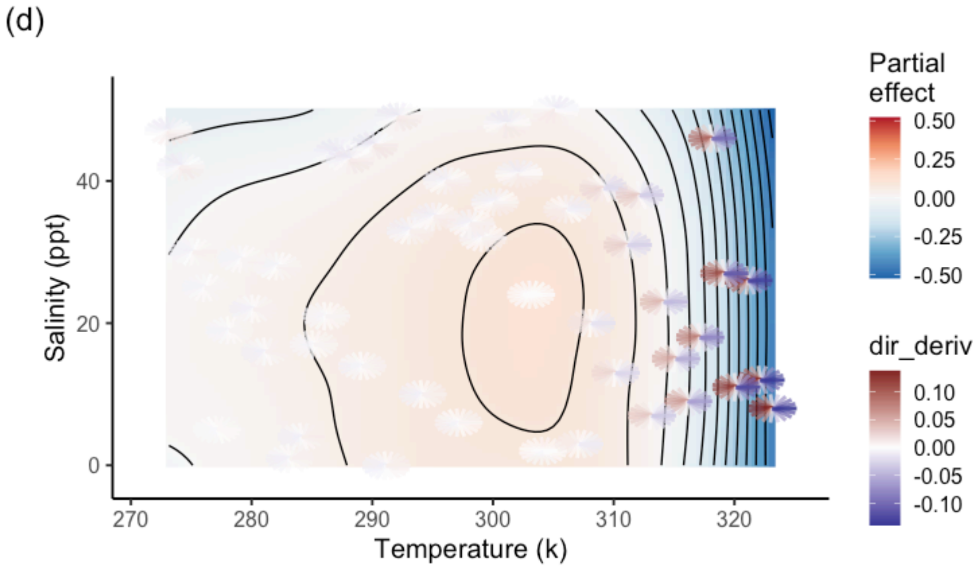

Figure 5.2: Directional derivatives calculated in all possible direction for several points on the response surface of sp1. Clearly, the slope of the directional derivative depends on the direction (red positive, blue negative).

5.1.3 Grid of points

Finally, we might do the same for a grid of points on the surface. We may want to do that when we do not have information on where a species or a community is living within the surface, but we know the range of values of E1 and E2.

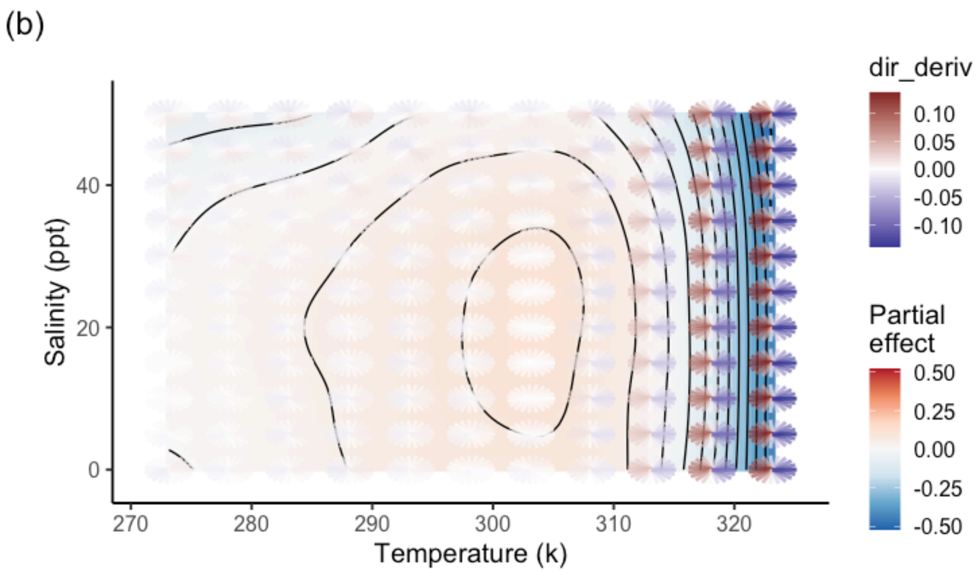

Figure 5.3: Directional derivatives calculated in all possible direction for a grid of points on the response surface of sp1. Clearly, the slope of the directional derivative depends on the direction (red positive, blue negative).

6 Response diversity calculation

We now move away from the example with temperature and salinity and use two general environmental variable E1 and E1.

Environmental variables may show different correlations between each other. The increase in one environmental variable may be directly correlated with the increase of another one (positive correlation), or vice versa, the increase in one driver may be correlated to a decrease in the other one (negative correlation). Yet, two environmental variables may change over time, or space, completely independently.
We may imagine that these different types of relationships between two environmental variables could determine specific trends in response diversity.

To explore this hypothesis, we calculate now response diversity for two communities (one with additive effect, and one including an interactive environmental effect) composed of 4 spp in 4 different cases: 1. Unknown trajectory of the environmental change 2. Trajectory of env change is given by the time series,and E1 and E2 change over time independently 3. Trajectory of env change is given by the time series,and E1 and E2 change over time with positive correlation 4. Trajectory of env change is given by the time series,and E1 and E2 change over time with negative correlation

We want to see if any consistent trend appears in the two communities when E1 and E2 have different correlations.

Steps:

i. Simulate spp performance curves with the modified Eppley function with and without interactive effect.

ii. Fit response surface for each sp (done with GAMs)

iii. Data wrangling and partials derivatives calculations

6.1 Community 1 - without interactive effect

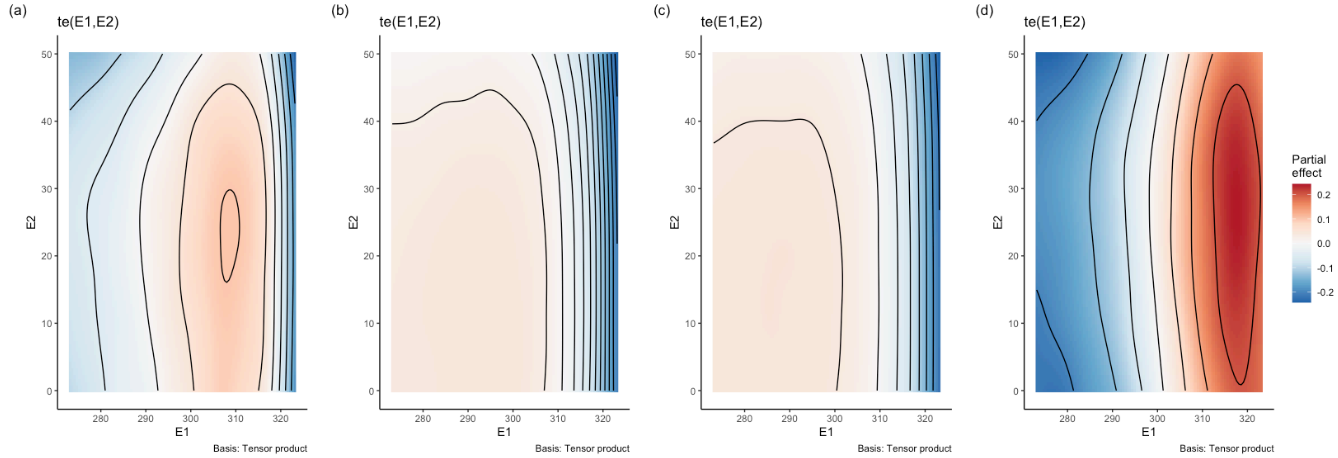

Figure 6.1: Response surface of the three species composing community 1. (a) Sp4. (b) Sp6. (c) Sp 11

6.2 Community 2 - interactive effect

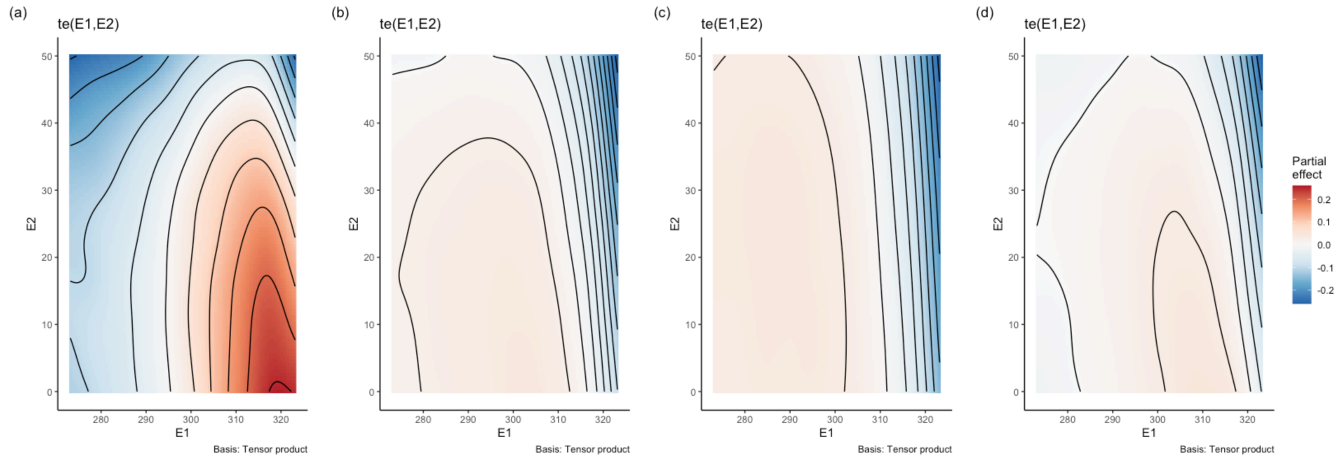

Figure 6.2: Response surface of the three species composing community 1. (a) Sp2. (b) Sp5. (c) Sp 13

6.3 Unknown trajectory of the environmental change

Table showing the calculated response diversity for one of the two communities when the trajectory of the environmental change is unknown. In this case, we calculated response diversity for a community in all possible directions across the surface, which represents in our opinion the most sensible way to measure the absolute response diversity of a specific community.

divergence	dissimilarity	community
0.2693827	1.011580	1
0.2473904	1.010359	2

6.3.1 E1 and E2 change independently over time

This example mimics a situation where the two environmental variables change over time completely independently. This is a common situation in field studies, where multiple drivers of environmental change are not correlated one another.

In this case the trajectory of the environmental change is given by the change of E1 and E2 over time.

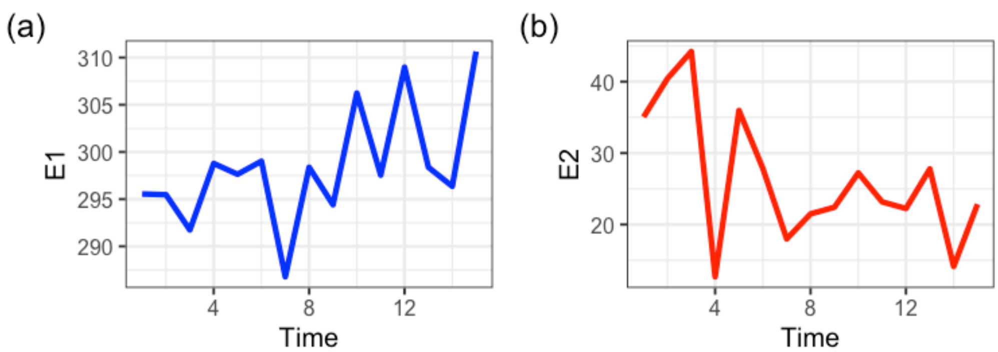

Figure 6.3: Time series of E1 and E2 changing independently over time.

6.3.2 Response surfaces with change in environmental conditions

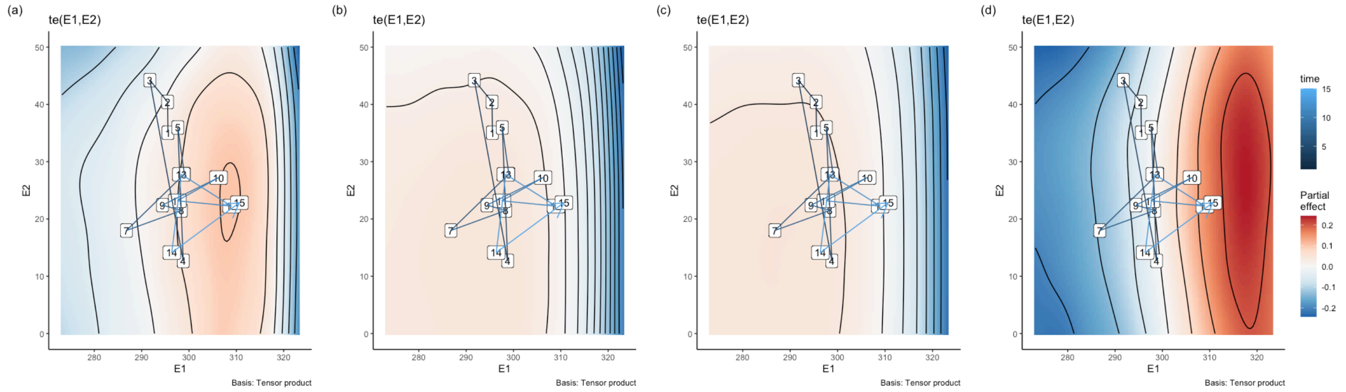

Figure 6.4: Response surface of the three species composing community 1. (a) Sp4. (b) Sp6. (c) Sp 11. The numbers on the response surfaces show the environmental location in the time steps of the time series and the arrows connect the time steps.

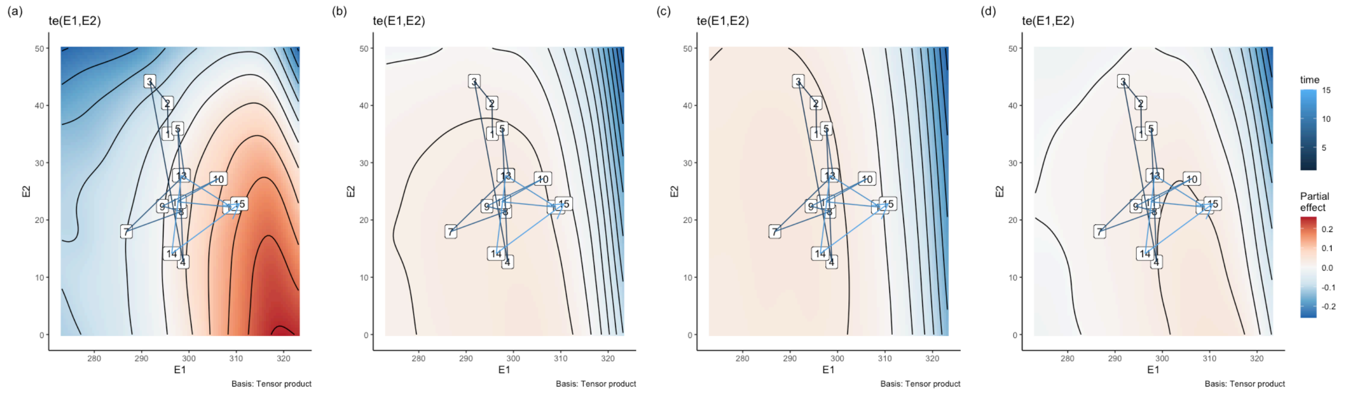

Figure 6.5: Response surface of the three species composing community 2. (a) Sp2. (b) Sp5. (c) Sp 13

Table showing the calculated response diversity for one of the two communities when the two environmental variables change independently over time (only first 6 rows shown).

time	E1_ref	E2_ref	s1	s2	s3	s4	rdiv	sign	Med
1	295.5499	35.06010	-0.0025569	-0.0043734	-0.0031682	-0.0019315	1.000992	0.0000000	1.004178
2	295.4694	40.43676	-0.0076105	-0.0038792	-0.0010685	-0.0079641	1.003054	0.0000000	1.004178
3	291.7350	44.23112	0.0059942	0.0082445	0.0069416	0.0049920	1.001339	0.0000000	1.004178
4	298.7794	12.68533	0.0010253	-0.0008450	0.0011959	0.0006765	1.000809	0.8280425	1.004178
5	297.6343	35.97065	0.0033619	0.0040078	0.0024411	0.0033568	1.000588	0.0000000	1.004178
6	299.0214	27.84749	-0.0045781	0.0049459	0.0077936	-0.0073761	1.006887	0.9724763	1.004178

Plot response diversity over time

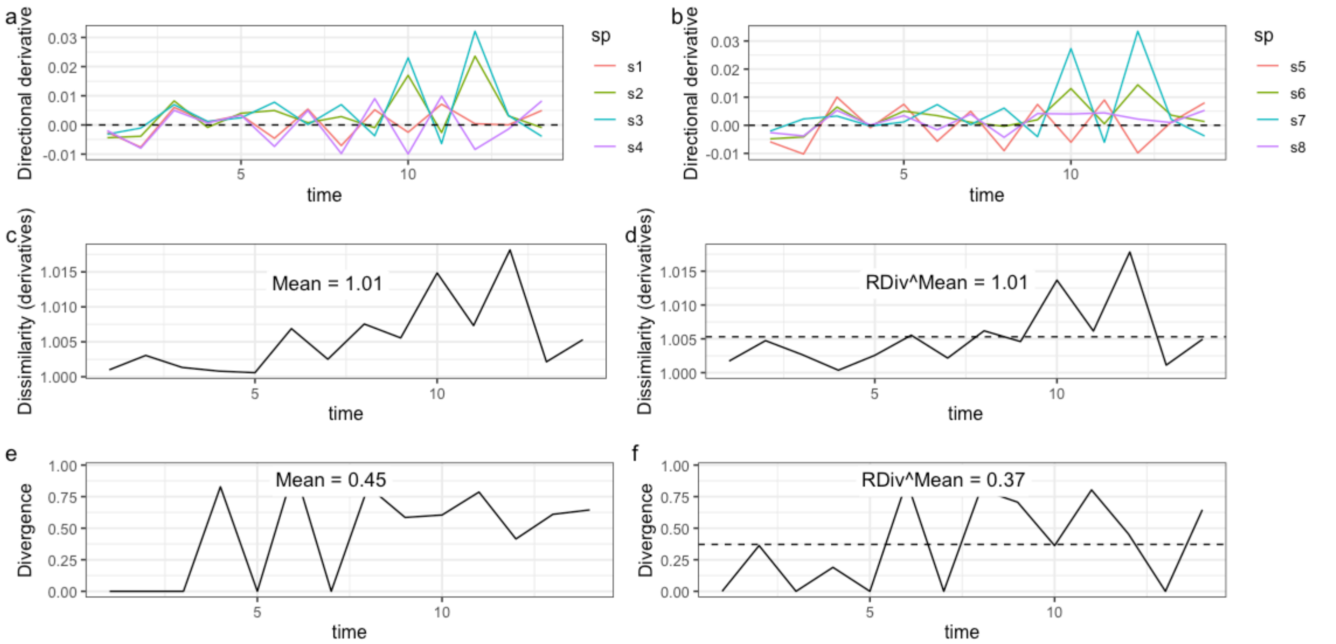

Figure 6.6: Directional derivatives and response diversity with known trajectory of env change. E1 and E2 change independently over time. a and b: Species directional derivatives over time. c and d: Response diversity measured as similarity-based diversity metric. e and : Response diversity measured as divergence (sign sensitive).

6.4 E1 and E2 change with negative correlation

This example mimics a situation where the two environmental variables change over time with negative correlation. This is common in field studies, where one environmental variable (e.g. CO2 concentration in oceans) increases, while another (e.g. pH) decreases e.g. Shirayama & Thornton (2005) (https://agupubs.onlinelibrary.wiley.com/doi/full/10.1029/2004JC002618).

Creating a time series with E1 and E2 changing over time with negative correlation.

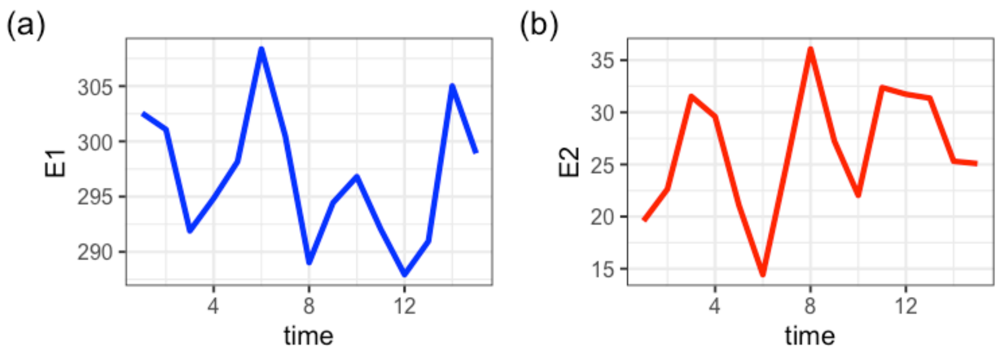

Figure 6.7: Time series of E1 and E2 changing with negative correlation over time.

6.4.1 Response surfaces with change in environmental conditions

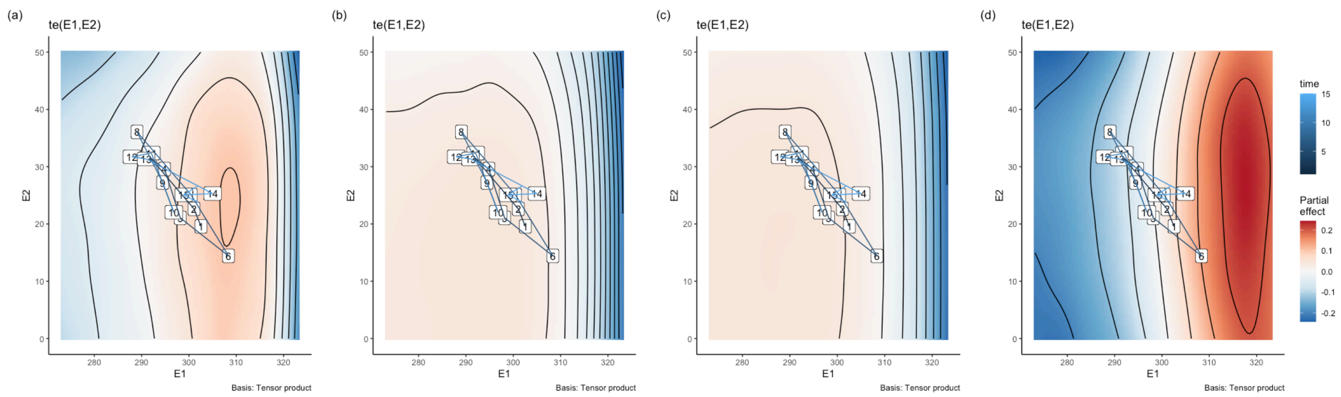

Figure 6.8: Response surface of the three species composing community 1. (a) Sp4. (b) Sp6. (c) Sp 11. The numbers on the response surfaces show the environmental location in the time steps of the time series and the arrows connect the time steps.

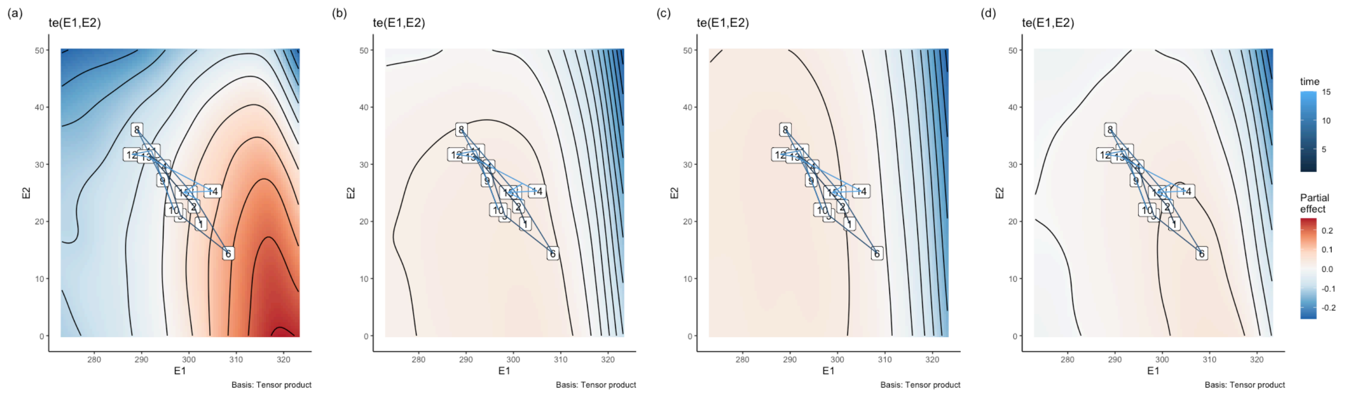

Figure 6.9: Response surface of the three species composing community 2. (a) Sp2. (b) Sp5. (c) Sp 13

Table showing the calculated response diversity for one of the two communities when the two environmental variables change with negative correlation over time (only first 6 rows shown).

time	E1_ref	E2_ref	s1	s2	s3	s4	rdiv	sign	Med
1	302.5483	19.58057	-0.0019757	0.0023263	0.0058283	-0.0041292	1.004276	0.8293662	1.003744
2	301.0725	22.63566	-0.0049556	0.0029915	0.0072241	-0.0076176	1.006567	0.9734861	1.003744
3	291.9013	31.52736	0.0050877	0.0022967	0.0006925	0.0073109	1.002833	0.0000000	1.003744
4	294.8456	29.58774	0.0027586	0.0026945	0.0006353	0.0029457	1.000874	0.0000000	1.003744
5	298.1674	21.06944	0.0060911	-0.0020108	-0.0052386	0.0080303	1.005995	0.7896076	1.003744
6	308.3573	14.42994	0.0010460	0.0155556	0.0197509	-0.0051795	1.011184	0.4155200	1.003744

Plot response diversity over time for the two communities

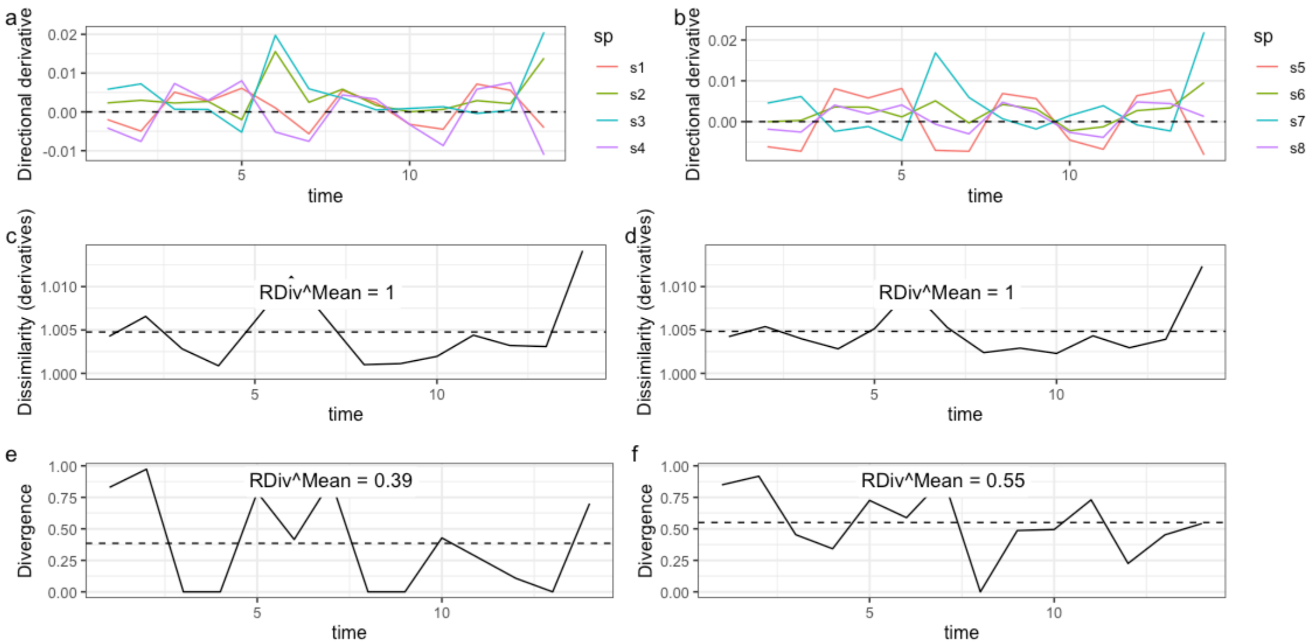

Figure 6.10: Directional derivatives and response diversity with known trajectory of env change. E1 and E2 change with negative correlation over time. a and b: Species directional derivatives over time. c and d: Response diversity measured as similarity-based diversity metric. e and : Response diversity measured as divergence (sign sensitive).

6.5 E1 and E2 change with positive correlation

Finally, two environmental variables can show positive correlation over time. A typical example is given by the positive correlation between air temperature and UV radiation e.g. Häder at al. 2015 (https://pubs.rsc.org/en/content/articlehtml/2015/pp/c4pp90035a).

Let us create a time series with E1 and E2 changing over time with positive correlation

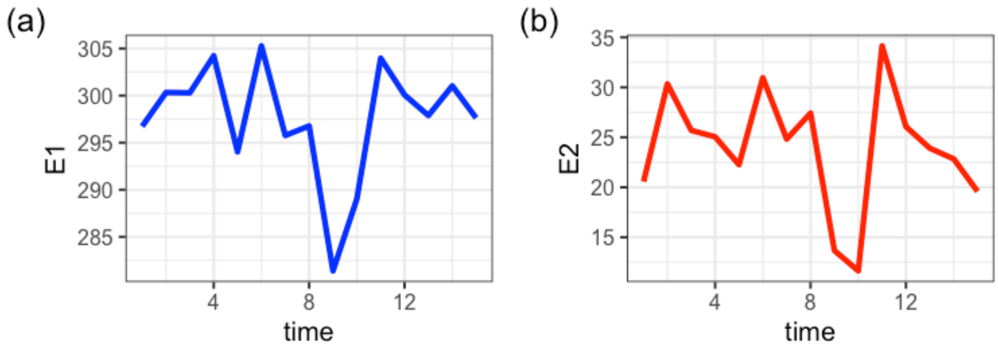

Figure 6.11: Time series of E1 and E2 changing with positive correlation over time.

6.5.1 Response surfaces with change in environmental conditions

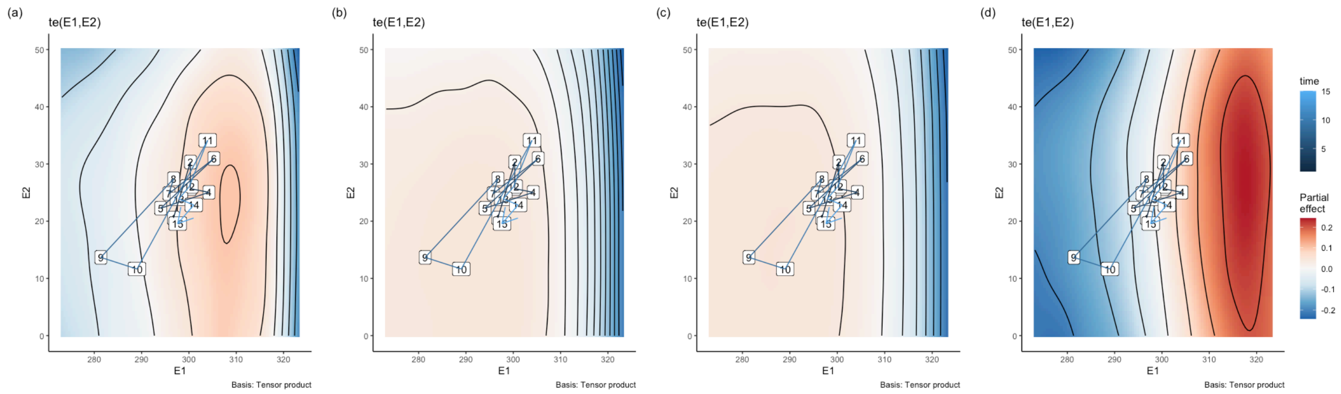

Figure 6.12: Response surface of the three species composing community 1. (a) Sp4. (b) Sp6. (c) Sp 11. The numbers on the response surfaces show the environmental location in the time steps of the time series and the arrows connect the time steps.

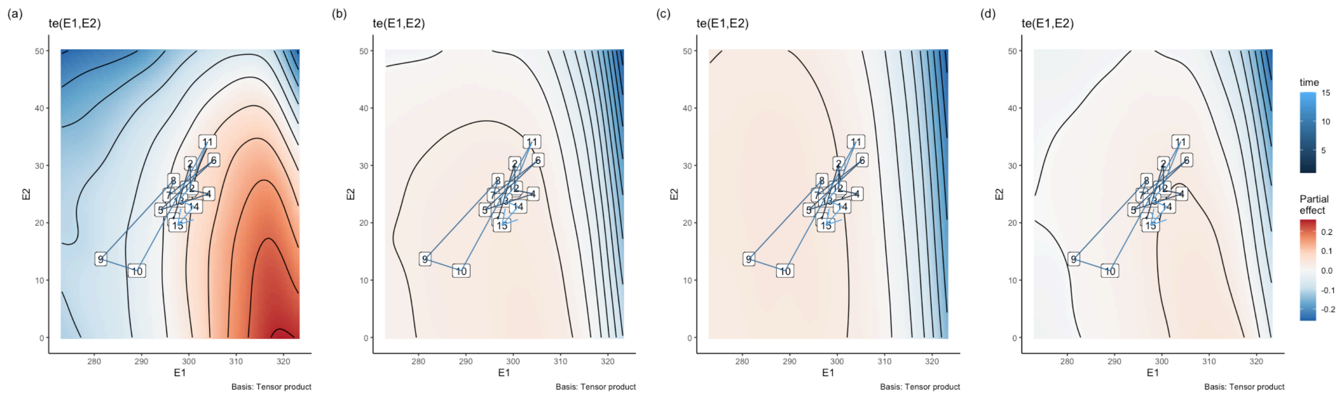

Figure 6.13: Response surface of the three species composing community 2. (a) Sp2. (b) Sp5. (c) Sp 13

Table showing the calculated response diversity for one of the two communities when the two environmental variables change with positive correlation over time (only first 6 rows shown).

time	E1_ref	E2_ref	s1	s2	s3	s4	rdiv	sign	Med
1	296.7312	20.56752	0.0021979	-0.0013016	-0.0030484	0.0044936	1.003268	0.8083766	1.006269
2	300.3276	30.33591	0.0010021	0.0029452	0.0025192	0.0003718	1.001155	0.0000000	1.006269
3	300.2941	25.69146	0.0069807	-0.0049171	-0.0093154	0.0101964	1.008817	0.9548461	1.006269
4	304.2220	25.04615	-0.0044516	0.0118764	0.0174888	-0.0106962	1.012639	0.7590010	1.006269
5	294.0289	22.26395	0.0043536	-0.0008374	-0.0032995	0.0078661	1.004841	0.5910078	1.006269
6	305.2743	30.95401	-0.0028616	0.0137421	0.0187414	-0.0097329	1.012782	0.6836275	1.006269

Plot response diversity over time

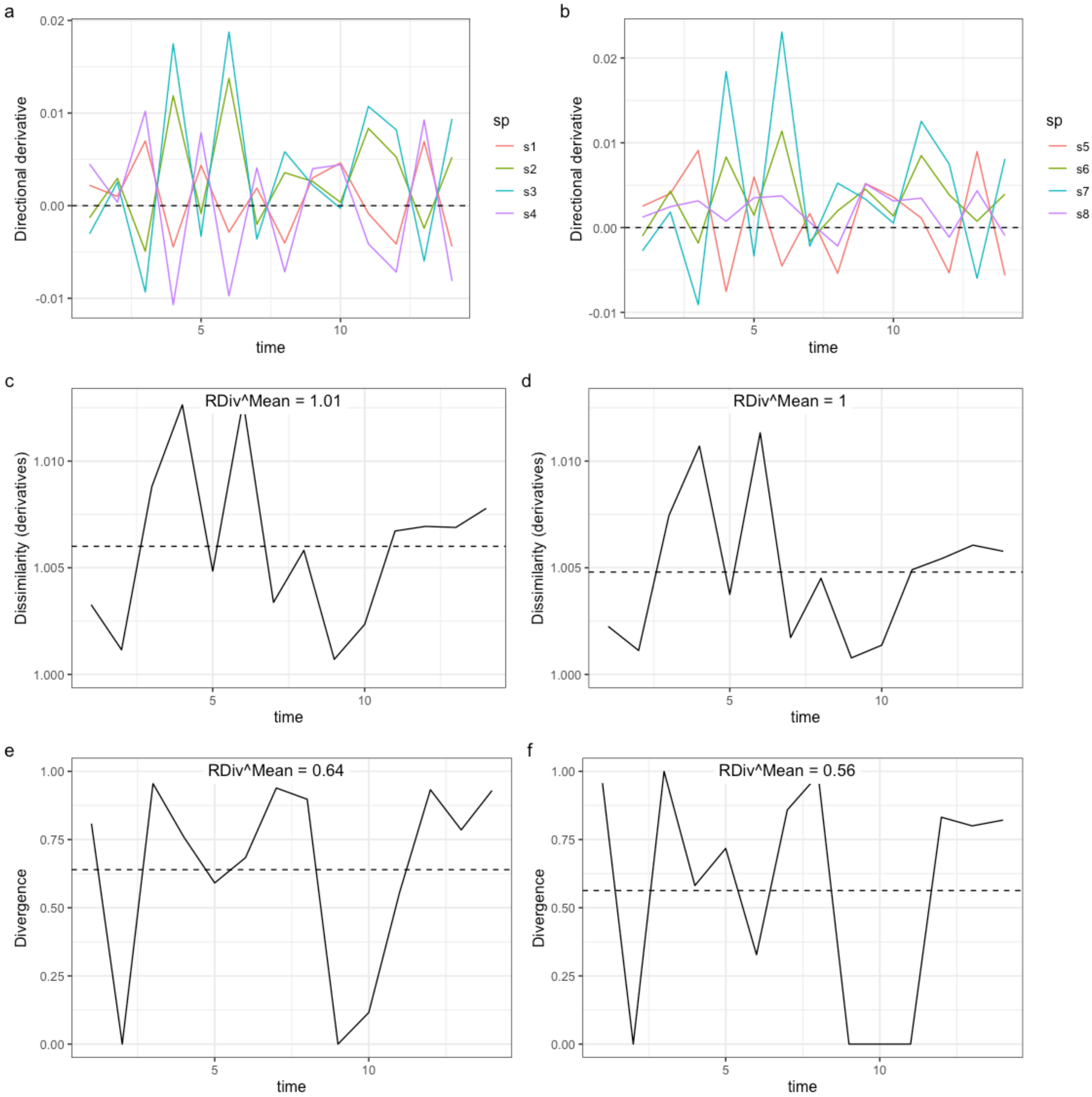

Figure 6.14: Directional derivatives and response diversity with known trajectory of env change for community 1 and 2. E1 and E2 change with negative correlation over time.a and b: Species directional derivatives over time. c and d: Response diversity measured as similarity-based diversity metric. e and : Response diversity measured as divergence (sign sensitive).

Now, we visualize the relationship between different correlations between the two environmental variables and response diversity.

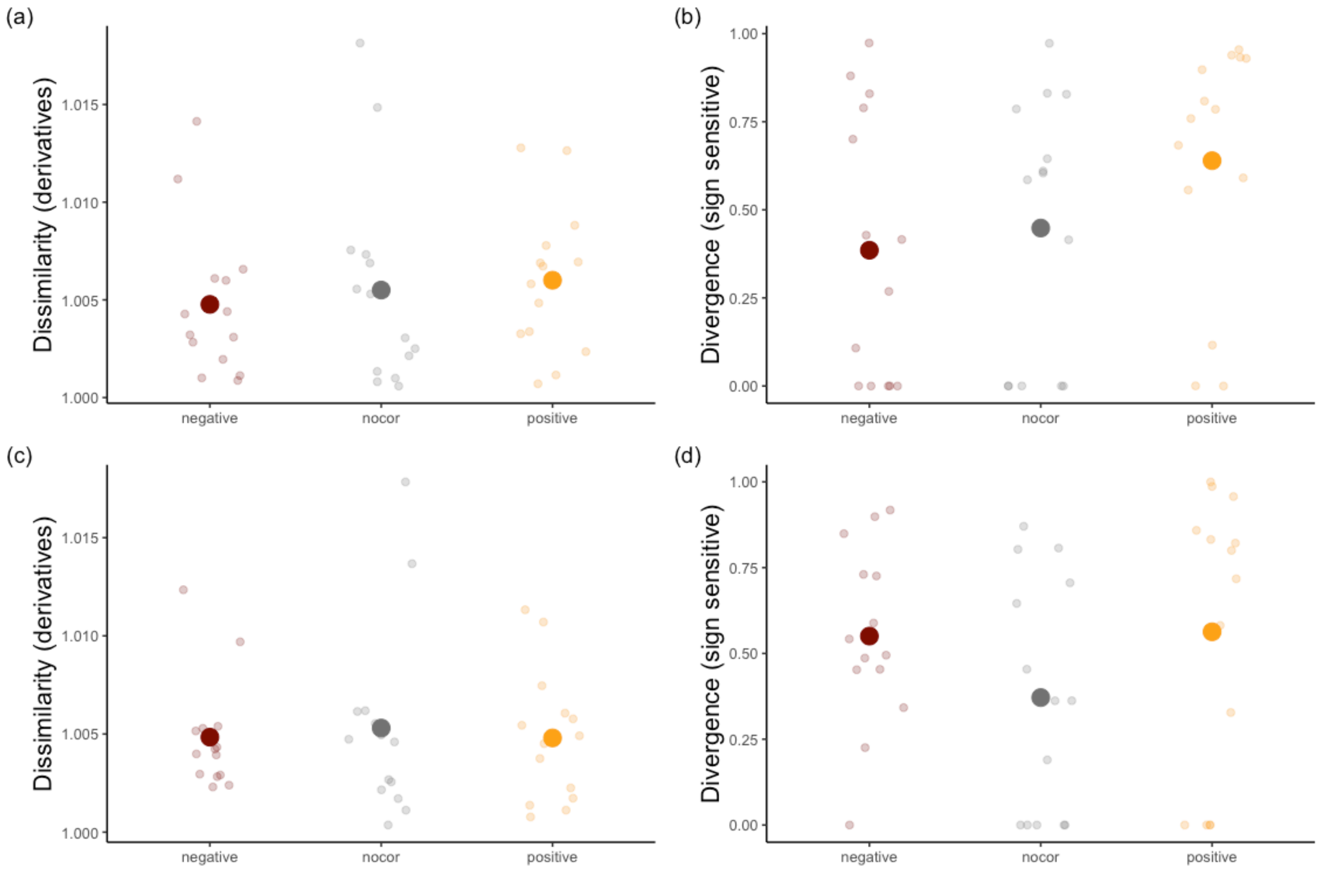

Figure 6.15: Correlation types and response diversity. a and c: correlation types and response diversity measured as dissimilarity in the first derivatives (sign insensitive) for community 1 and 2 respectively. c and d. correlation types and response diversity measured as divergence in the first derivatives (sign sensitive) for community 1 and 2 respectively

We can rule out the hypothesis that different types of relationships between two environmental variables could determine specific trends in response diversity.