SAPIENZA
UNIVERSITÀ DI ROMA

Autonomous and Mobile Robotics - group project presentation

QUADRUPED GAIT GENERATION BASED ON IS-MPC

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STATE OF THE ART

- Due to non linearity of quadruped simplified model as inverted pendulum is used.

- Commonly the classic approach aims to let the ZMP lie inside the support polygon with vertex in the contact feet, this constraints are generally nonlinear in the decision variables.

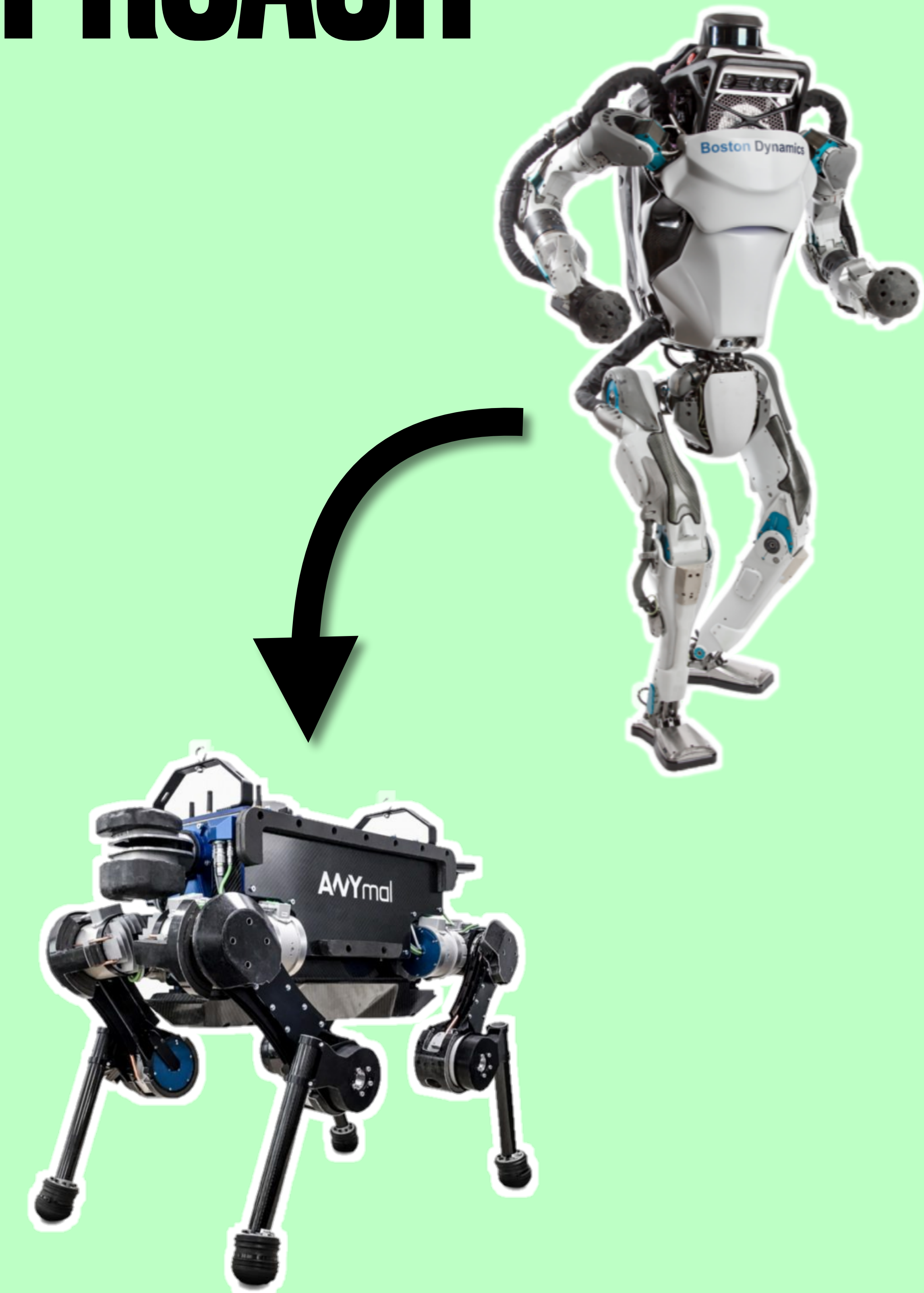
- In a MPC framework this will lead to an increase of decision variables defined as the footsteps.

OUR PROBLEM APPROACH

- Approach the quadruped problem, considering an ideal humanoid:
  - Computationally less intensive.
  - Linear ZMP constraints.
  - Restrictive approach case.

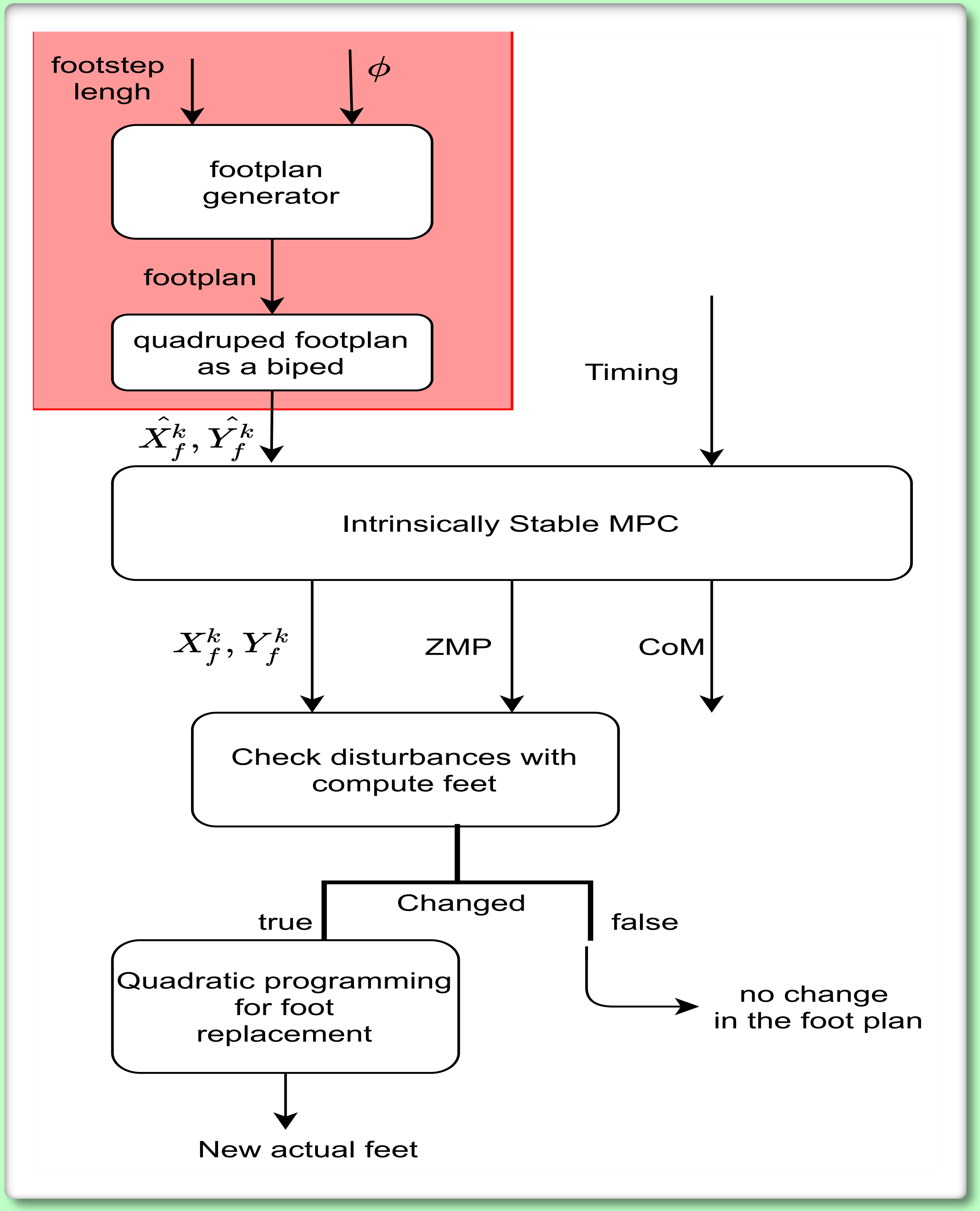
- Consider then a second QP for the foot replacement.

- Trotting/Walking scenarios.

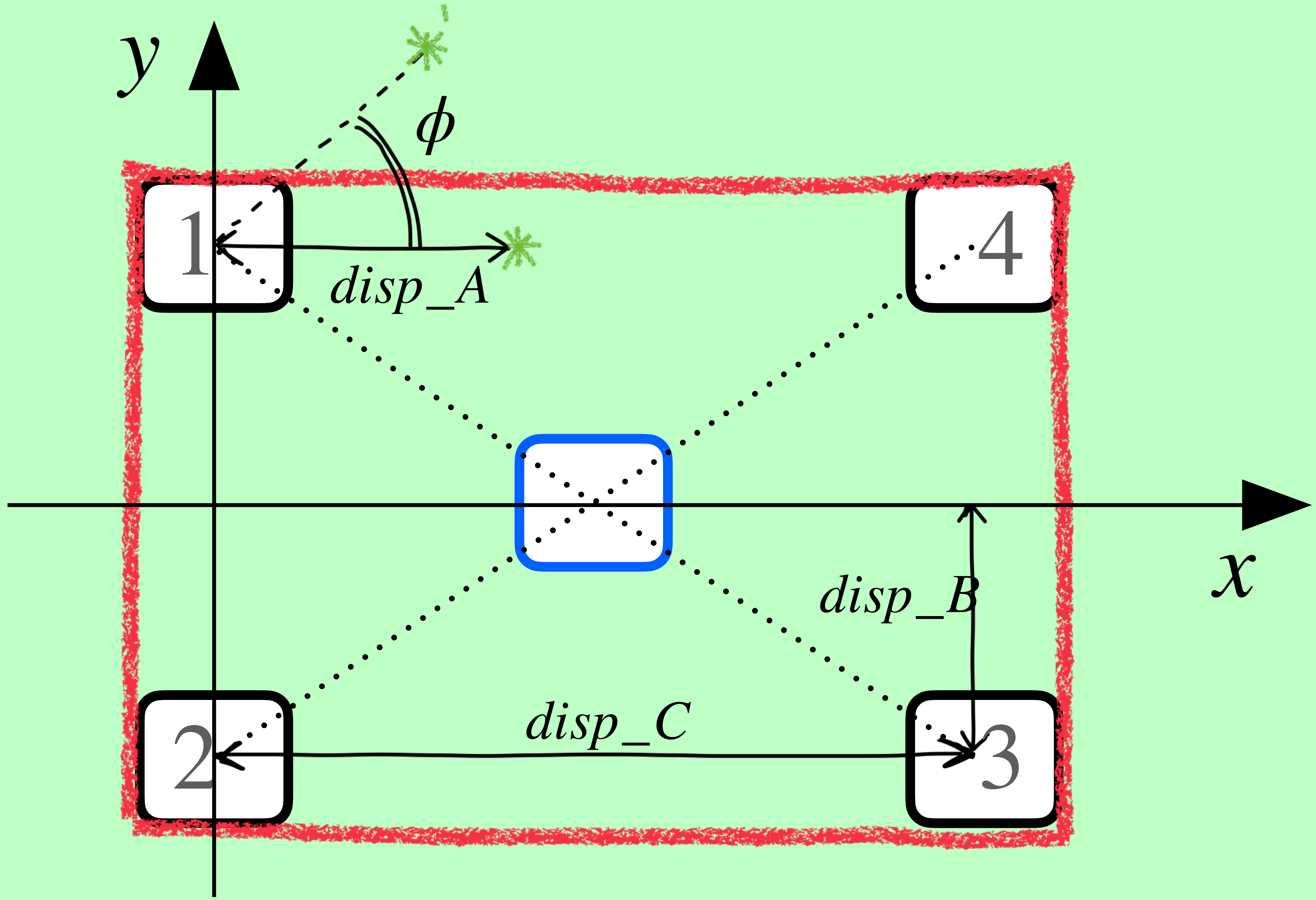


GENERAL BLOCK DIAGRAM

- Initialization of the foot plan for the quadruped and the ideal biped.
- IS-MPC on the biped.
- Check for eventual disturbances.
- Second QP for the foot replacement.



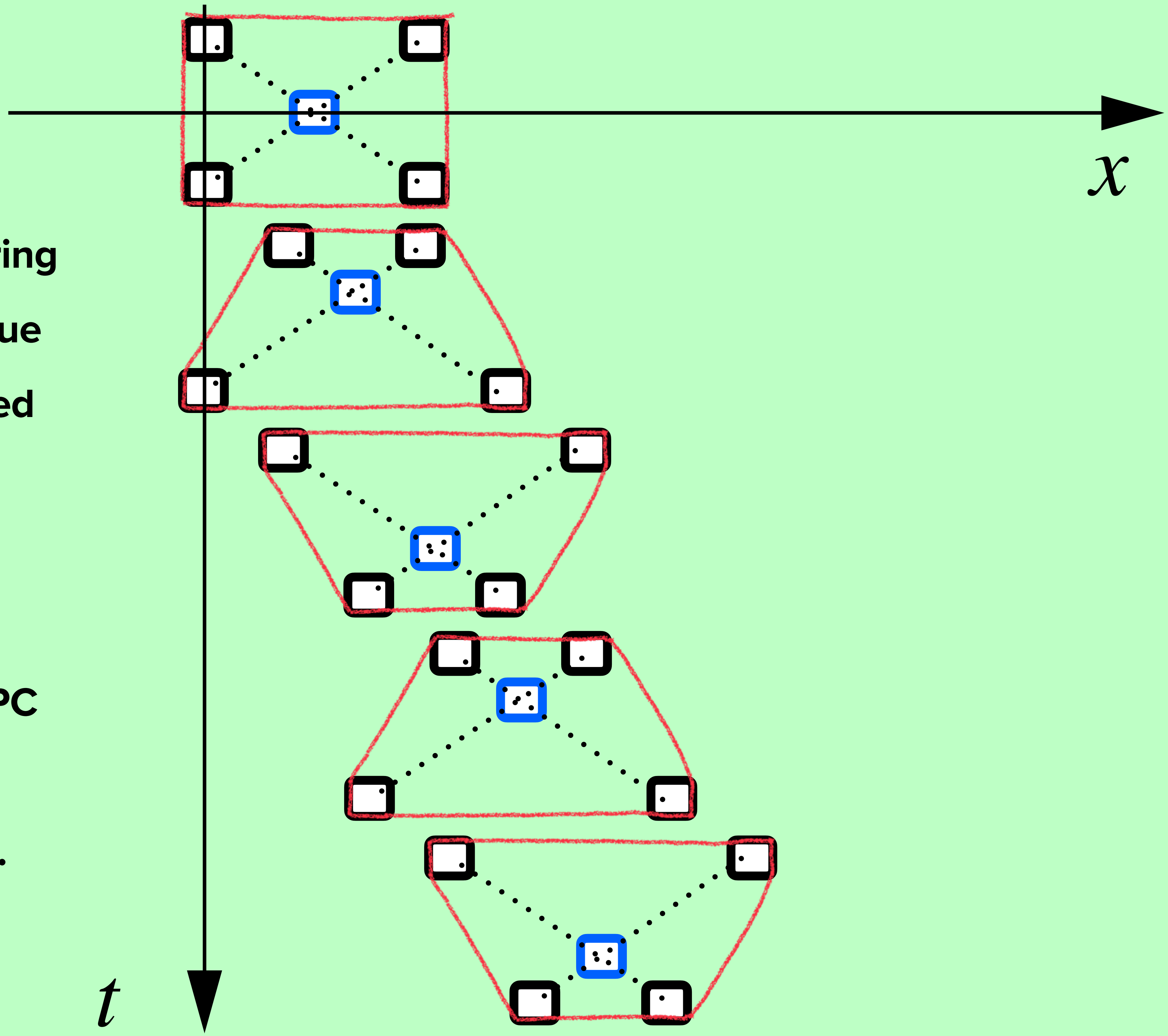
QUADRUPED PARAMETRIZATION



MAIN IDEA

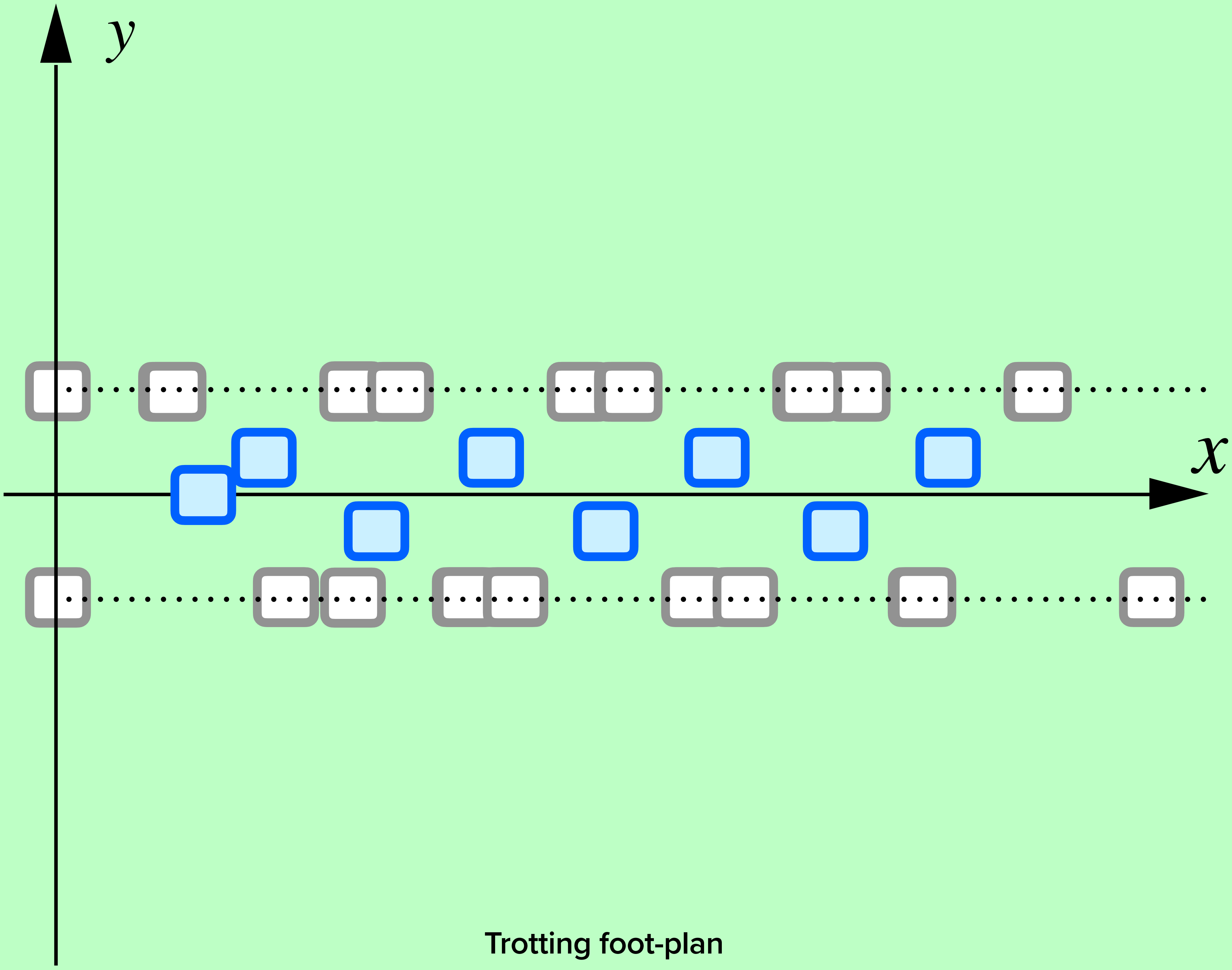
- During time and considering a trotting scenario, the blue squares are the ideal biped steps.

- We exploits these blue squares to feed the IS-MPC QP with a ZMP trajectory that interpolates all them.



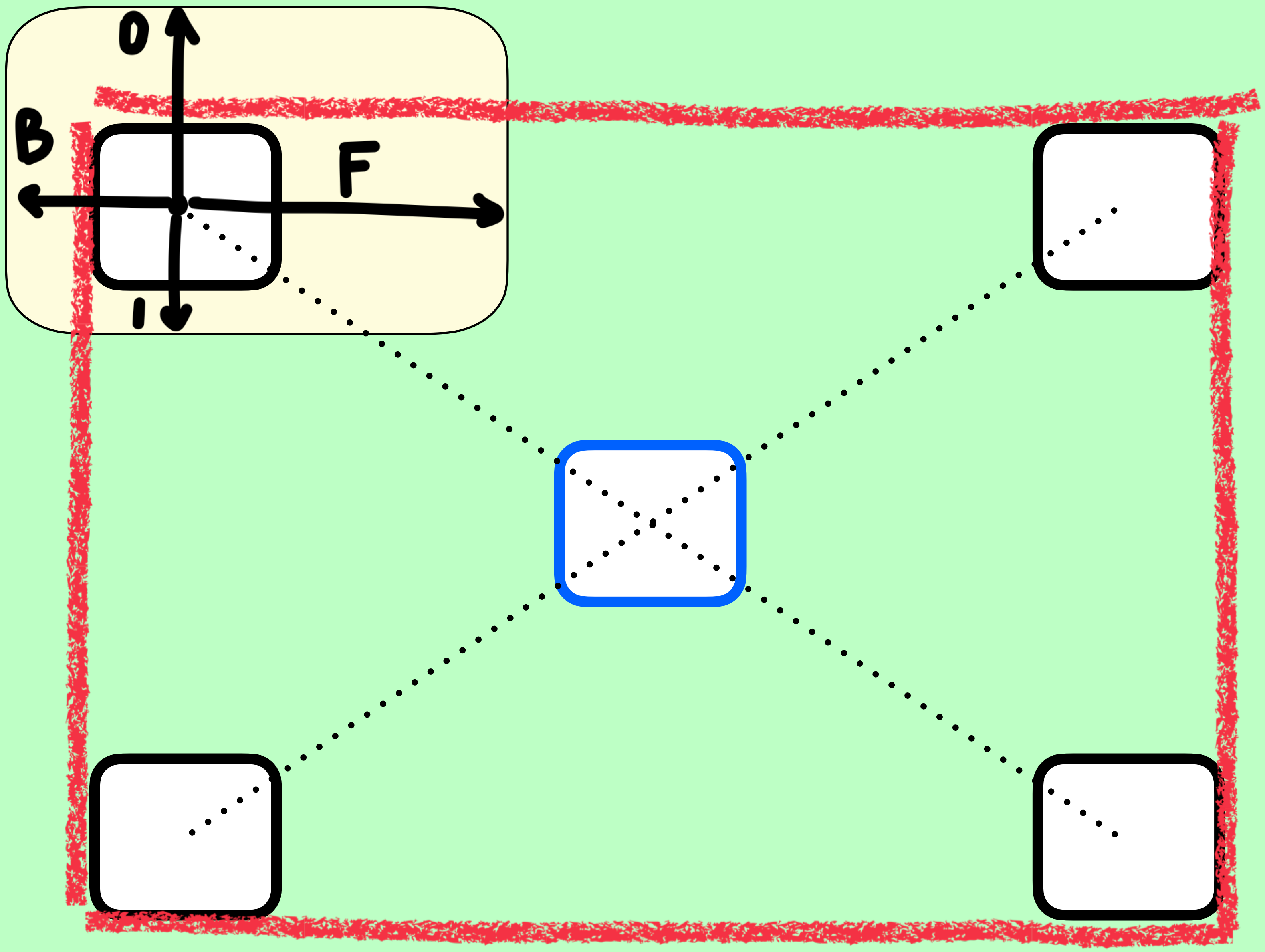
Gait phases for trotting maneuver

MAIN IDEA

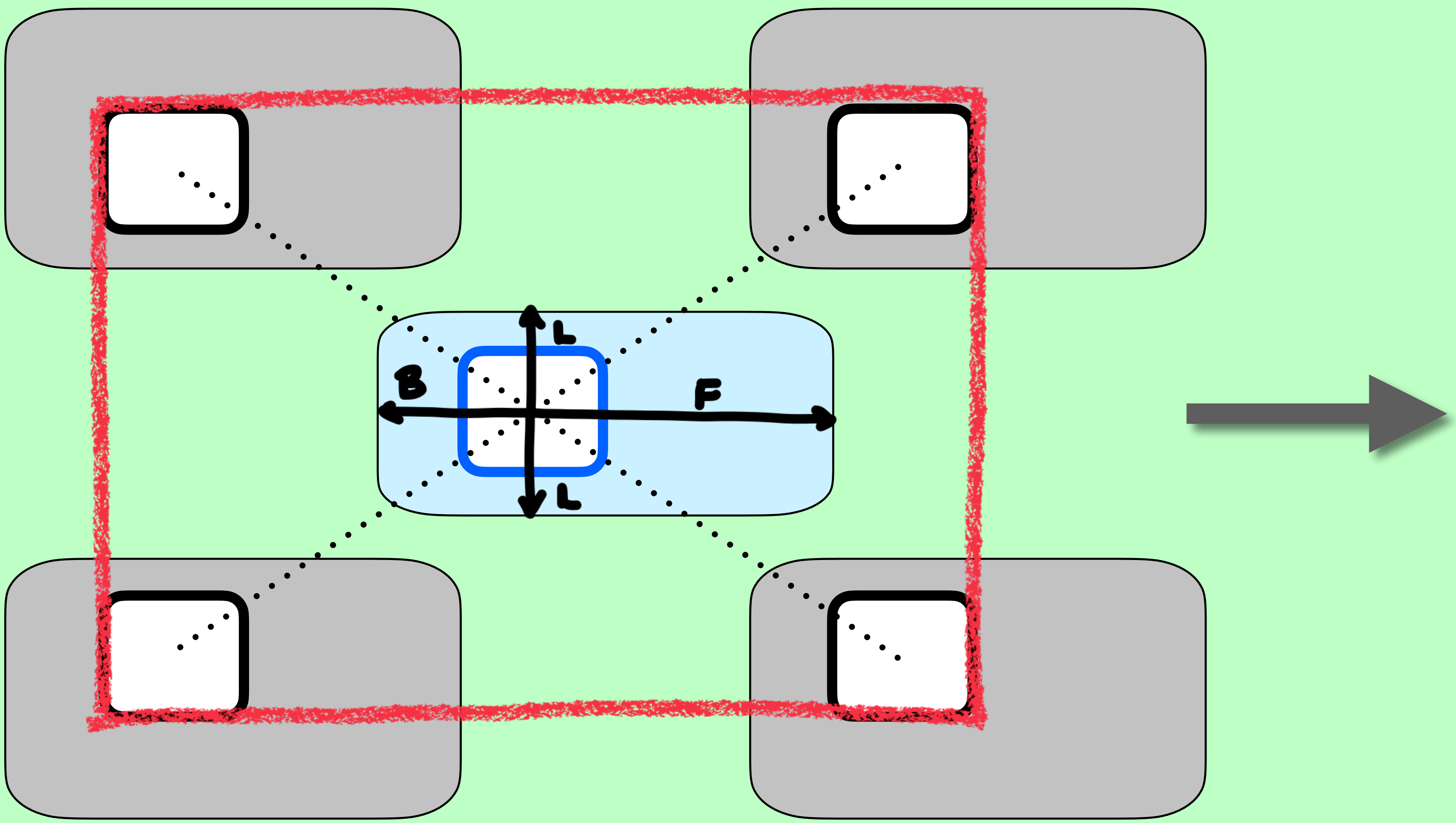


Trotting foot-plan

FOOT KINEMATIC LIMITATION



FICTITIOUS FOOT KINEMATIC LIMITATION



$$L = \frac{I + O}{2}$$

IS-MPC FORMULATION

$$\min_{\substack{\dot{X}_z^k,\, \dot{Y}_z^k \\ X_f^k,\, Y_f^k}} \|\dot{X}_z^k\|^2 + \|\dot{Y}_z^k\|^2 + \beta(\|X_f - \hat{X}_f\|^2 + \|Y_f - \hat{Y}_f\|^2)$$

Subject to :

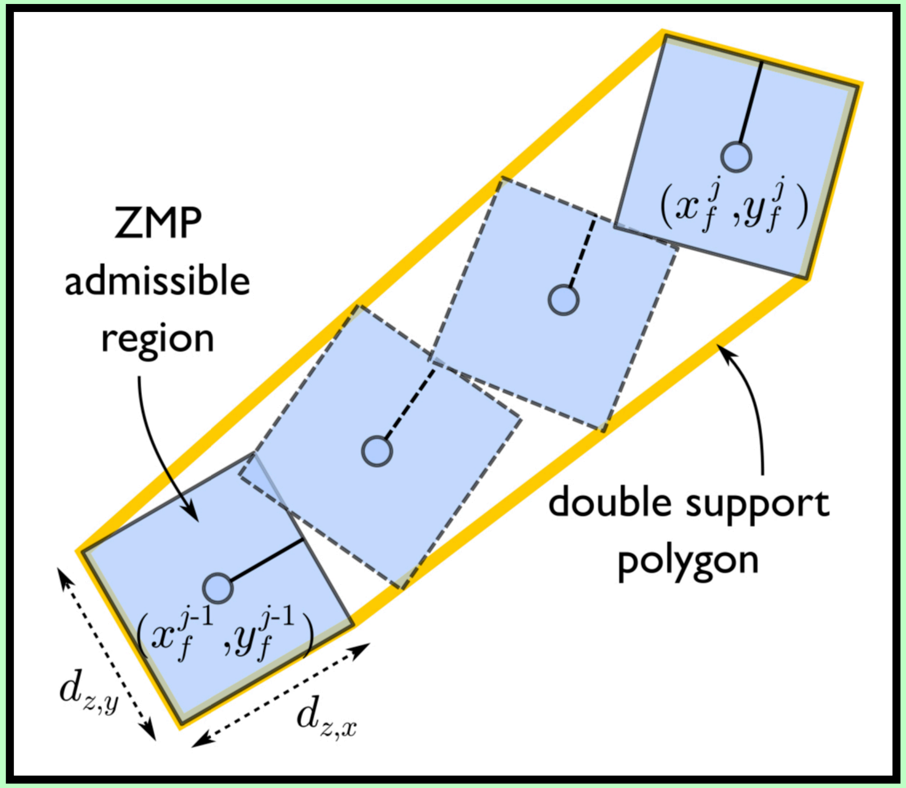
- **ZMP constraint**
- **Kinematic constraint**
- **Stability constraint for x and y**

All the decision variables are collected in vectors, like:

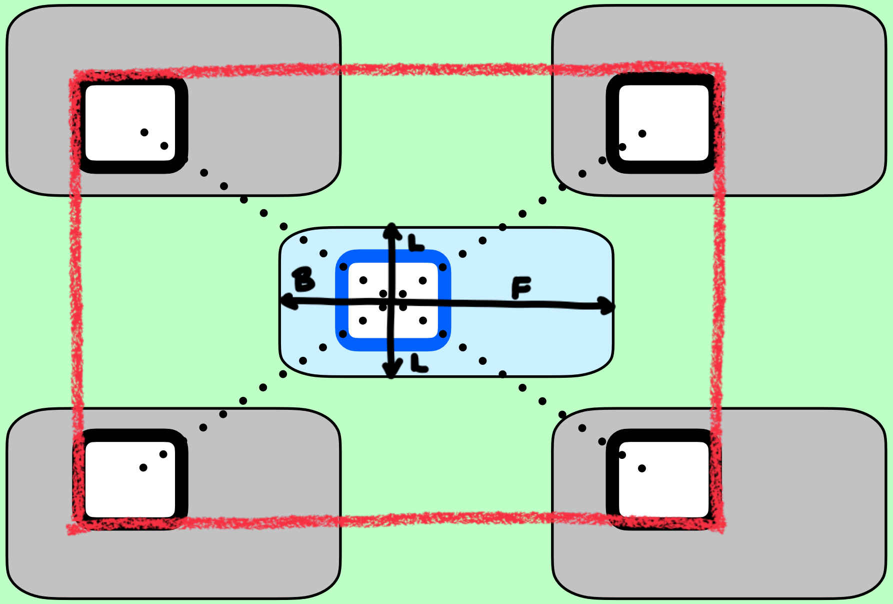
$$\begin{aligned} \dot{X}_z^k &= (\dot{x}_z^k, \ldots, \dot{x}_z^{k+C-1})^T \\ \dot{Y}_z^k &= (\dot{y}_z^k, \ldots, \dot{y}_z^{k+C-1})^T \\ X_f^k &= (x_f^1, \ldots, x_f^{F'})^T \\ Y_f^k &= (y_f^1, \ldots, y_f^{F'})^T \end{aligned}$$

ZMP & KINEMATIC CONSTRAINT

$$R_j^T \begin{pmatrix} \delta \sum_{l=0}^{i} \dot{x}_z^{k+l} - x_f^j \\ \delta \sum_{l=0}^{i} \dot{y}_z^{k+l} - y_f^j \end{pmatrix} \leq \frac{1}{2} \begin{pmatrix} d_{z,x} \\ d_{z,y} \end{pmatrix} - R_j^T \begin{pmatrix} x_z^k \\ y_z^k \end{pmatrix}$$



$$R_{j-1}^T \begin{pmatrix} x_f^j - x_f^{j-1} \\ y_f^j - y_f^{j-1} \end{pmatrix} \leq \pm \begin{pmatrix} 0 \\ disp\_L/2 \end{pmatrix} + \begin{pmatrix} disp\_F \\ disp\_L/2 \end{pmatrix}$$



STABILITY CONSTRAINT

- Using a change of coordinates, the LIP can be decoupled in stable and unstable dynamics.

$$x_u = x_c + \frac{\dot{x}_c}{\eta}$$

$$x_s = x_c - \frac{\dot{x}_c}{\eta}$$

- The decoupled dynamics are:

$$\dot{x}_s = \eta(-x_s + x_z) \qquad \dot{x}_u = \eta(x_u - x_z)$$

- The CoM is bounded if and only if :

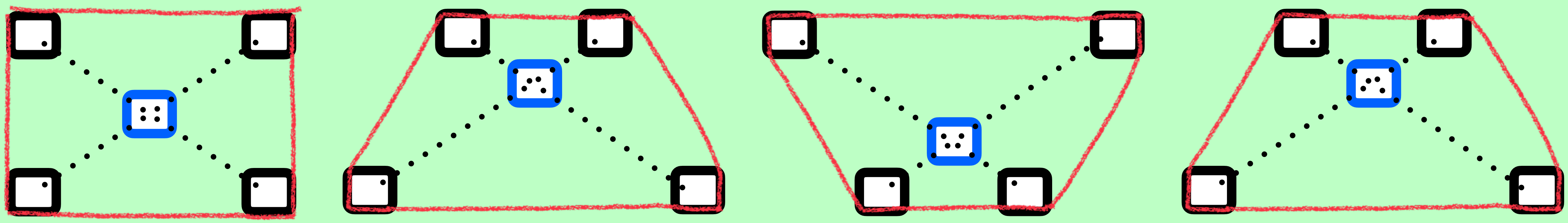
$$x_u^k = \eta \int_{t_k}^{\infty} e^{-\eta(\tau - t_k)} x_z(\tau) d\tau$$

$$\sum_{i=0}^{C-1} e^{-i\eta\delta} \dot{x}_z^{k+1} = -\sum_{i=C}^{\infty} e^{-i\eta\delta} \dot{\tilde{x}}_z^{k+i} + \frac{\eta}{1 - e^{-\eta\delta}} (x_u^k - x_z^k)$$

$$\sum_{i=0}^{C-1} e^{-i\eta\delta} \dot{x}_z^{k+1} = -\sum_{i=C}^{P-1} e^{-i\eta\delta} \dot{x}_{z,ant}^{k+i} - \sum_{i=P}^{\infty} e^{-i\eta\delta} \dot{\tilde{x}}_{z,ant}^{k+i} + \frac{\eta}{1 - e^{-\eta\delta}} (x_u^k - x_z^k)$$

Anticipative tail

TROTTING



- The idea is to move two opposite leg simultaneously, e.g. Back_Left with Front_Right and Back_Right with Front_Left.

Back Left		Back Right		Front Right		Front Left	
x	y	x	y	x	y	x	y
0	disp_B	0	-disp_B	disp_C	-disp_B	disp_C	disp_B
+x_pd	+y_pd	•	•	+x_pd	+y_pd	•	•
•	•	+x_p	+y_p	•	•	+x_p	+y_p
+x_p	+y_p	•	•	+x_p	+y_p	•	•

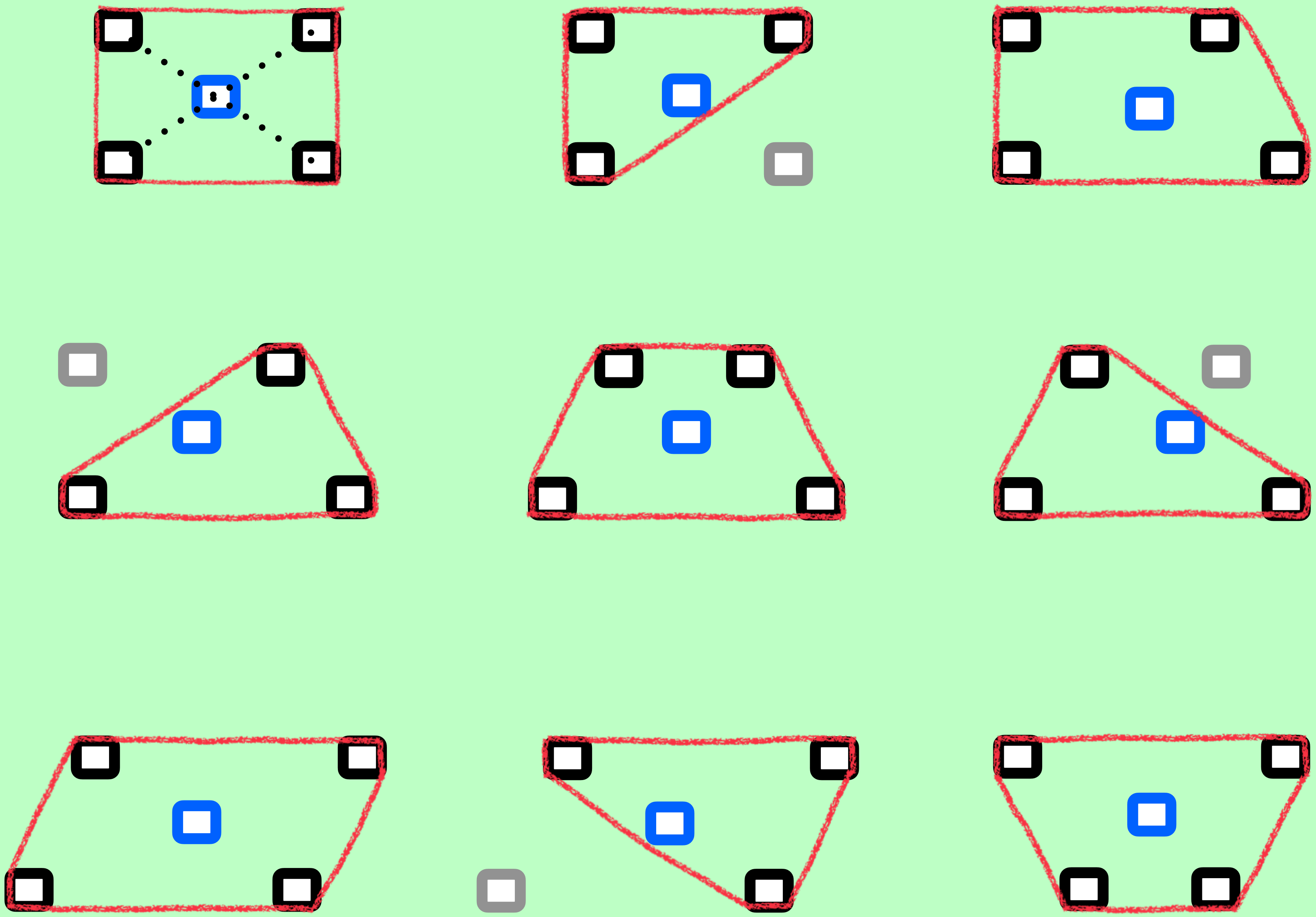
Recursive procedure to generate a trotting gait, starting from an initial feet configuration.

WALKING

- In the walking scenario instead, the robot is performing the gait just moving one leg at each step.

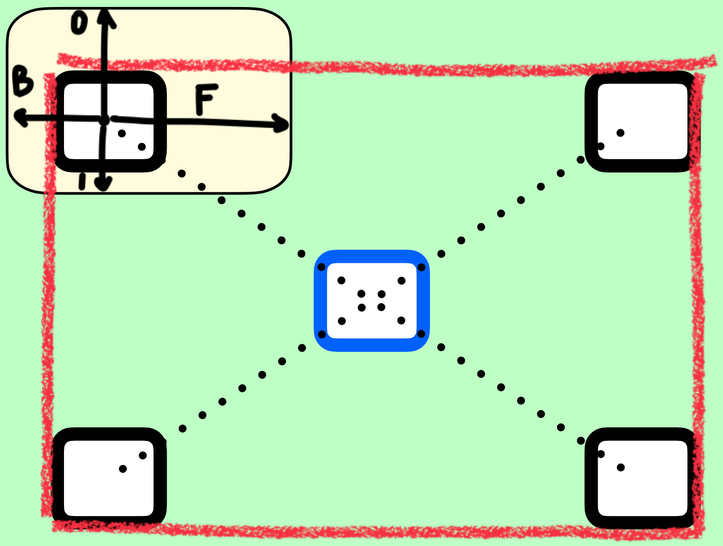
- It’s a static gait.

- During the triangular shape support polygon, we have constrained the ZMP to stay inside the blue square relative to the previous support polygon.



QP FOR FOOT REPLACEMENT

$$\min_{\substack{x_{fl}, y_{fl} \\ x_{fr}, y_{fr}}} (X_{fl} - x_{fl})^2 + (Y_{fl} - y_{fl})^2 + (X_{fr} - x_{fr})^2 + (Y_{fr} - y_{fr})^2$$



Subject to :

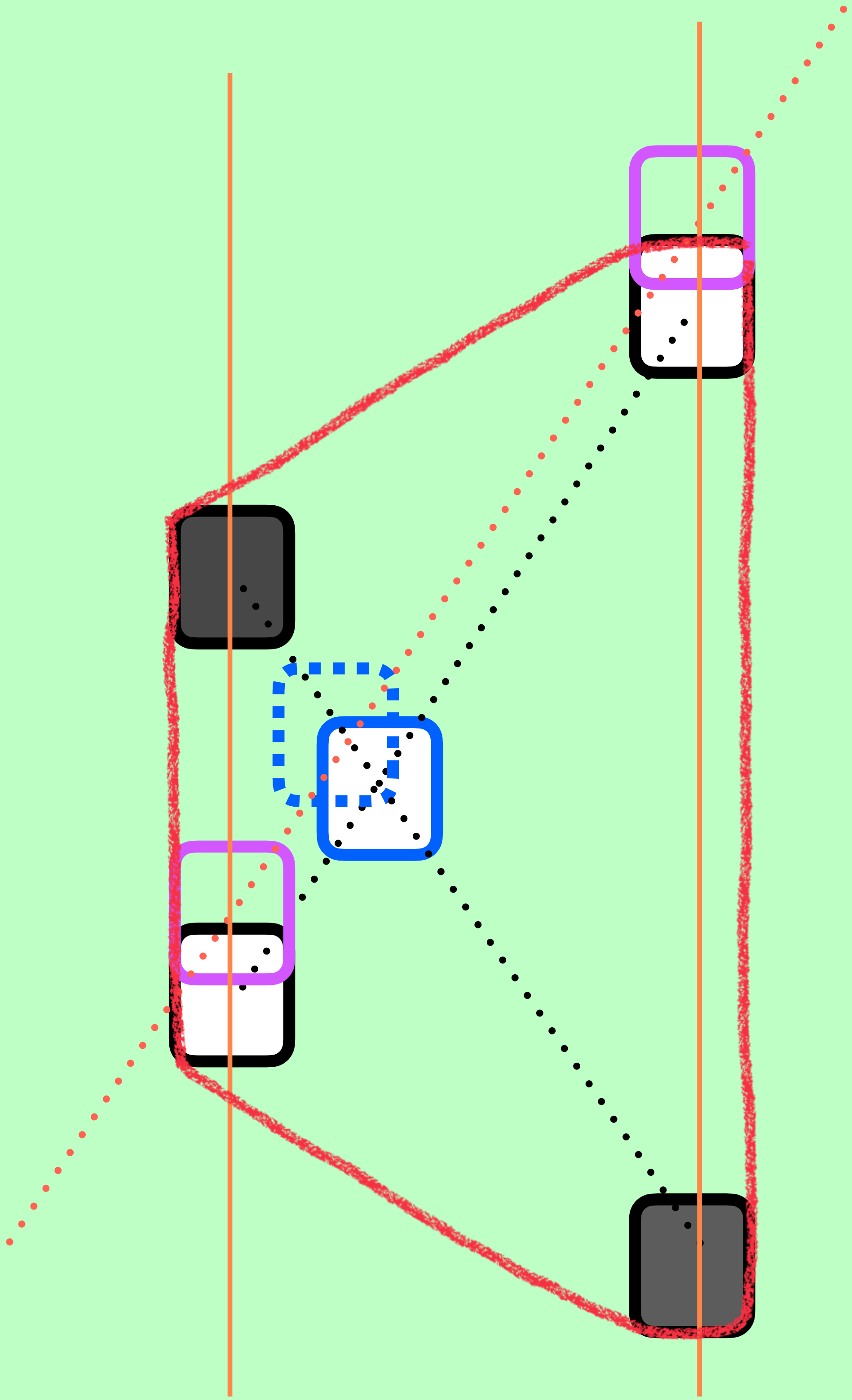
- **Kinematic foot constraints**

Where the decision variables are the coordinates for left and right foot

COMPUTE FEET: TROTTING

- When a disturbance is acting on the robot we recompute the feet prioritizing the gait orientation (ϕ angle).

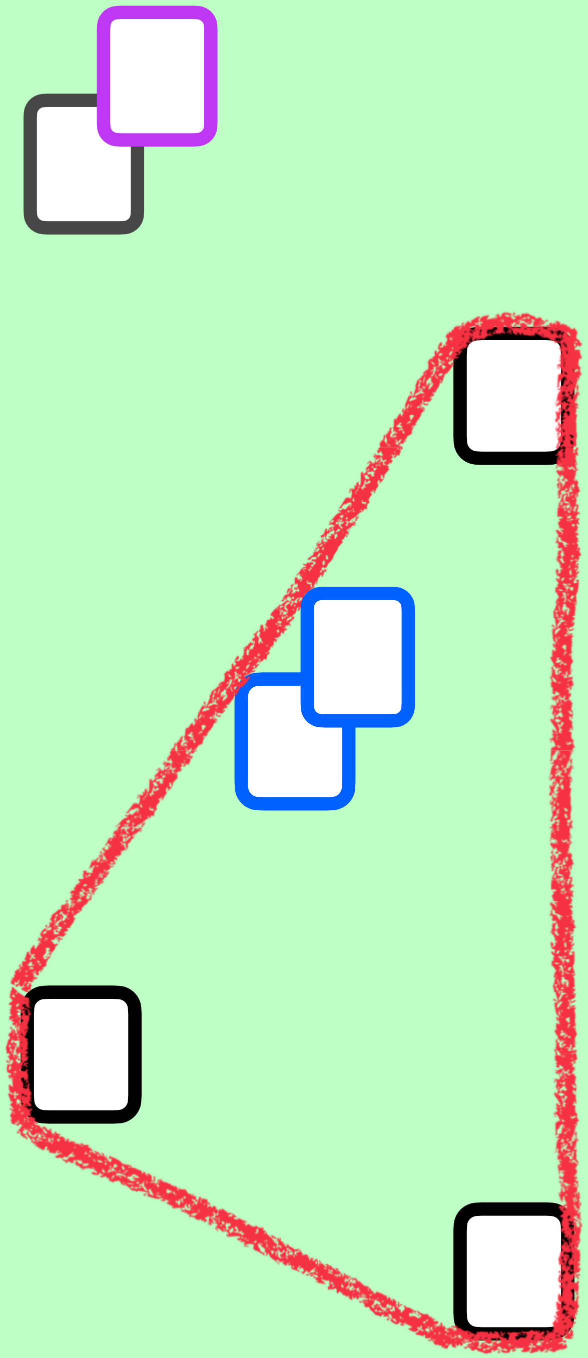
- The aim of this procedure is to impose that the ZMP at the next step lies inside the double support polygon.



COMPUTE FEET: WALKING

- In the walking case we try to compensate the error between the actual ZMP coordinates and the predicted ones.

- This is done by adding the aforementioned difference to the free-foot coordinate.

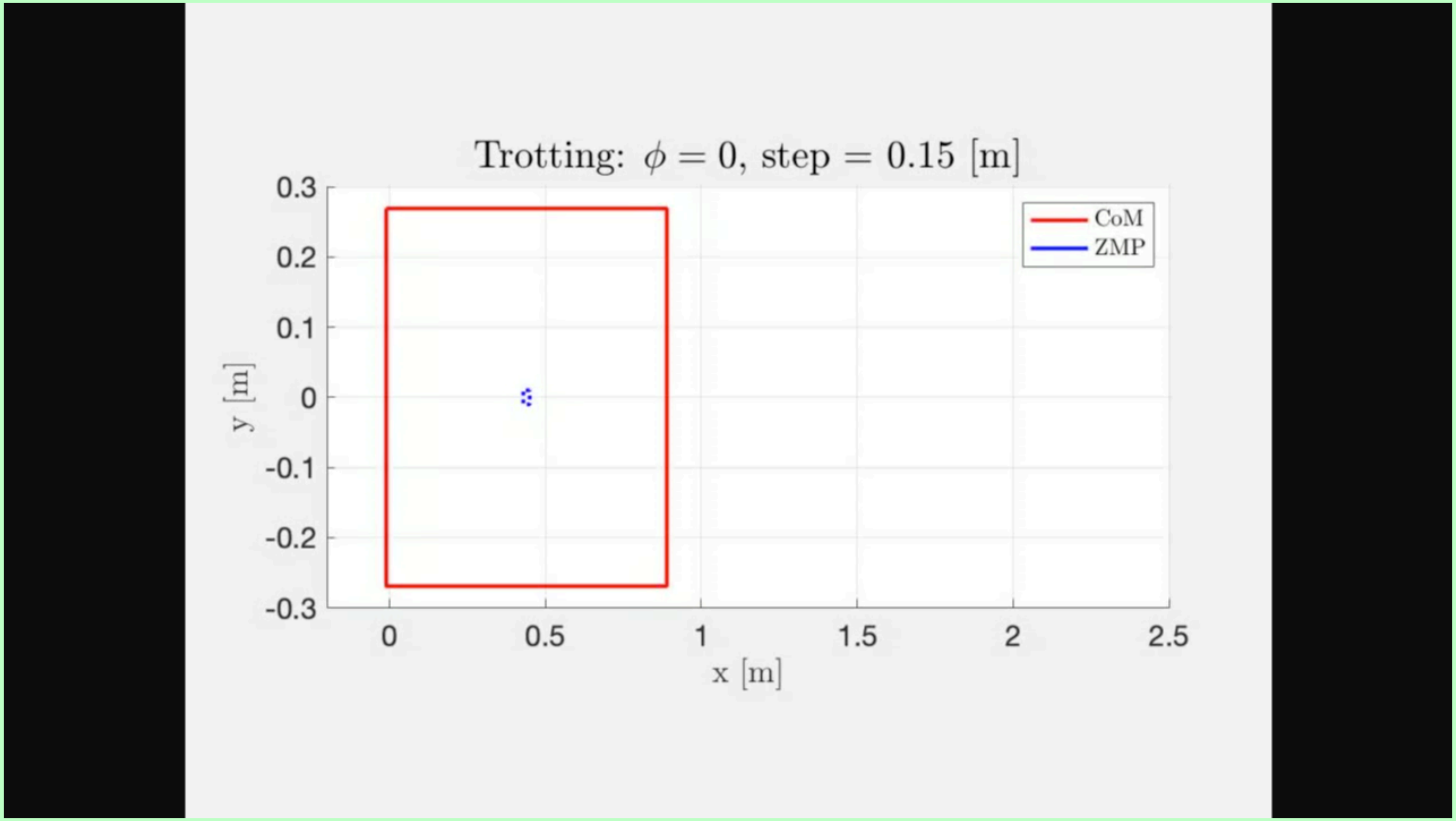


MATLAB - SIMULATION

Parameter	Symbol	Value	Unit
Robot Mass	m	30.5	kg
Gravity Acceleration	g	9.81	ms^{-2}
Foot size	f_s	0.02	m
Height	h	0.56	m
Length	$disp_C$	0.9303	m
Half body width	$disp_B$	0.259394	m
Lenght of footstep	$disp_A$	0.15	m
Sampling Time	δ	0.01	s
Trotting Control horizon	$C\delta$	160δ	s
Trotting Prediction horizon	$P\delta$	320δ	s
Walking Control horizon	$C\delta$	100δ	s
Walking Prediction horizon	$P\delta$	200δ	s
Footstep in the control horizon	F'	3	-

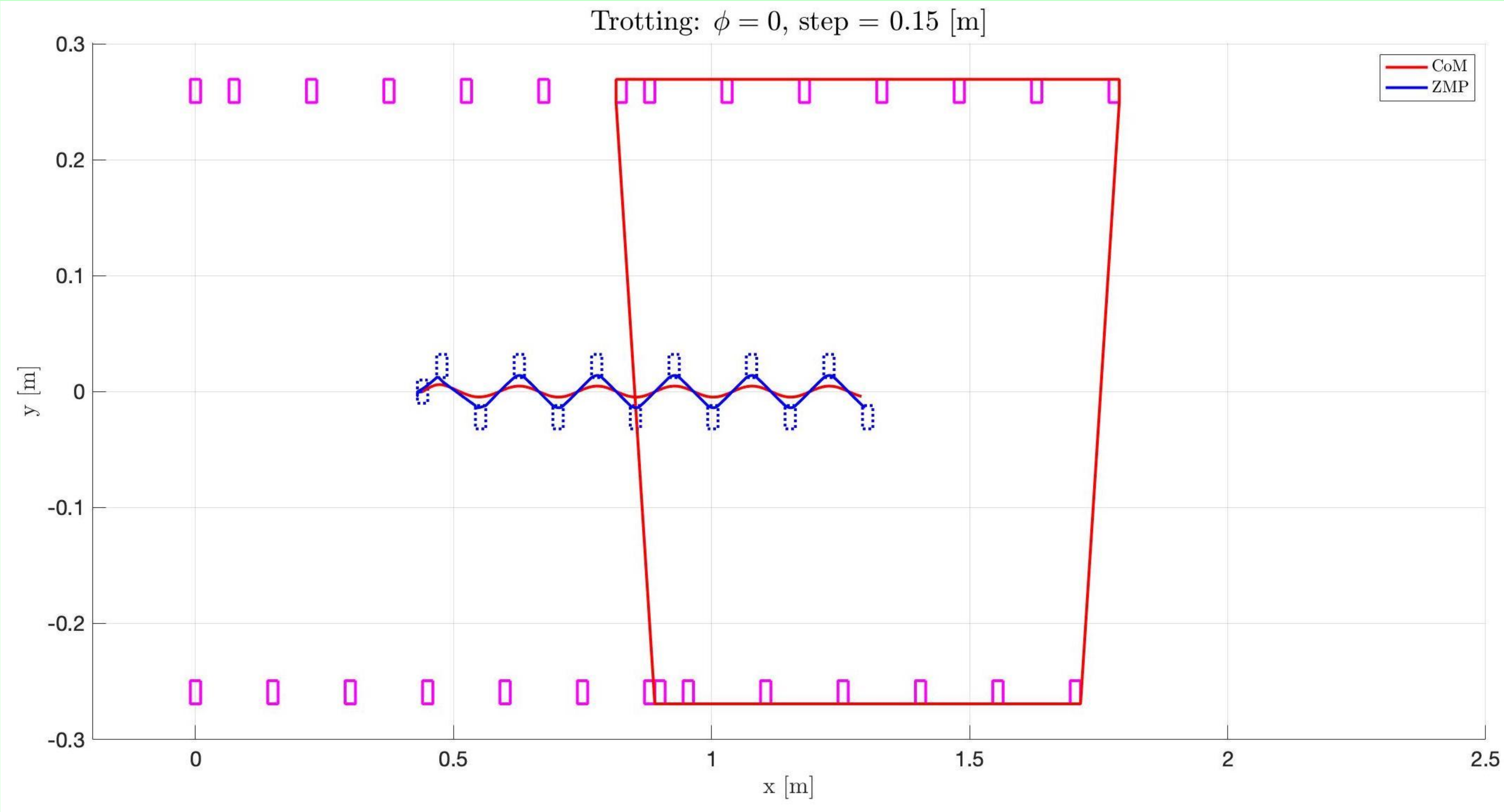
Table 1: Parameters used for the following simulations.

MATLAB



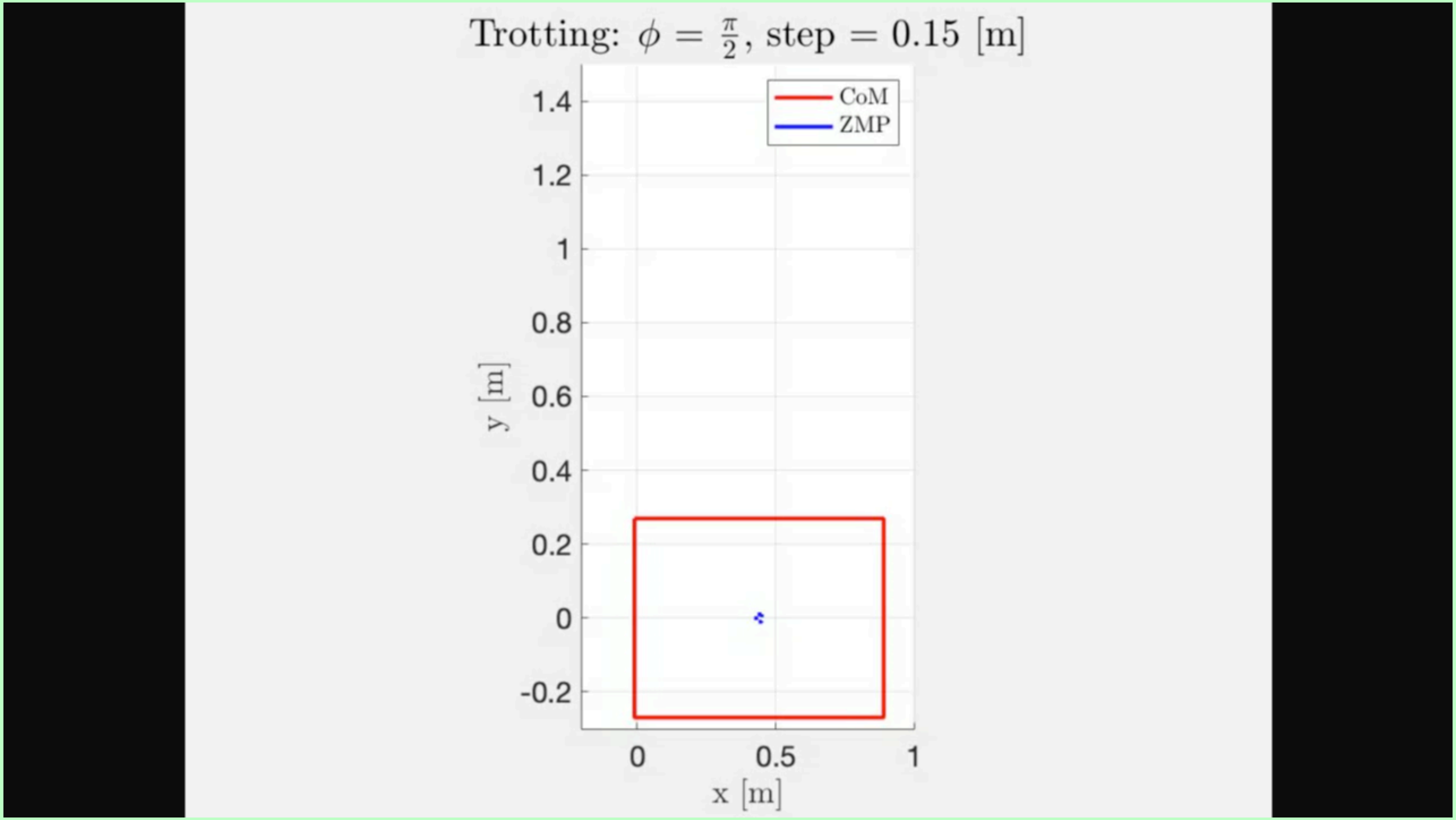
Trotting: $\phi = 0$, step = 0.15 [m]
CoM
ZMP
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
y [m]
0
0.5
1
1.5
2
2.5
x [m]

MATLAB



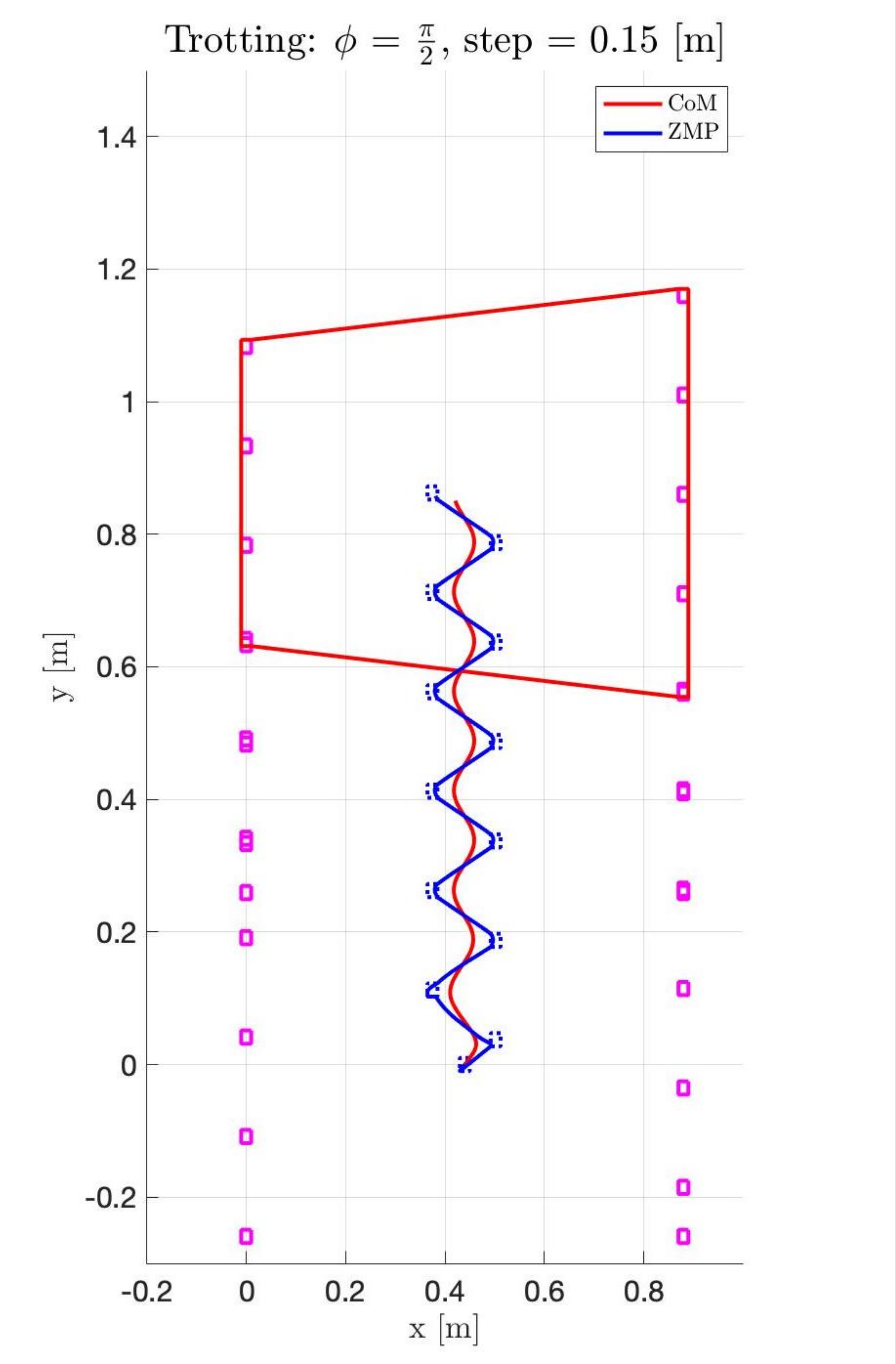
Trotting: $\phi = 0$, step = 0.15 [m]
CoM
ZMP
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
y [m]
0
0.5
1
1.5
2
2.5
x [m]

MATLAB



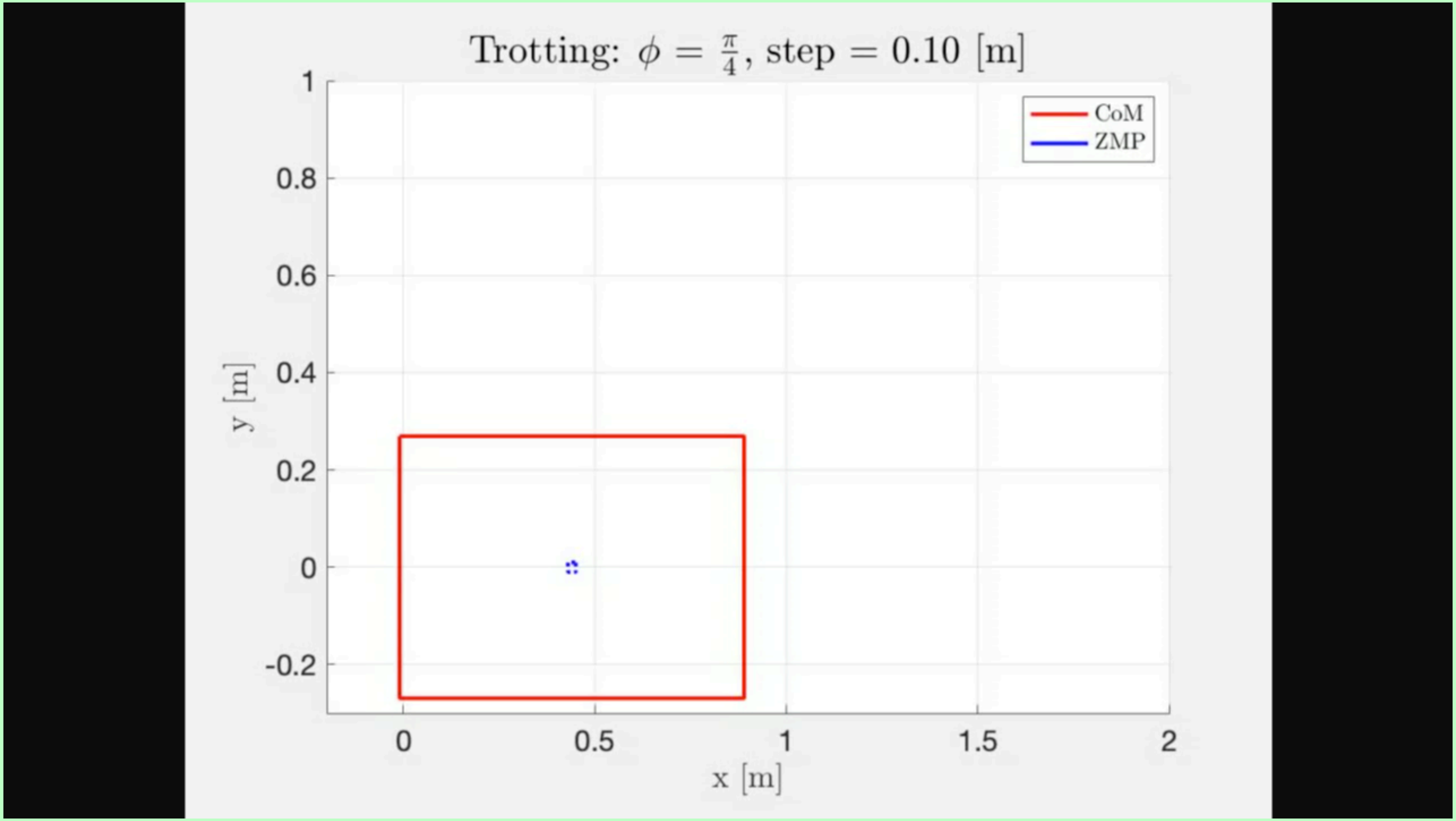
Trotting: $\phi = \frac{\pi}{2}$, step = 0.15 [m]
CoM
ZMP
1.4
1.2
1
0.8
0.6
0.4
0.2
0
-0.2
y [m]
0
0.5
1
x [m]

MATLAB



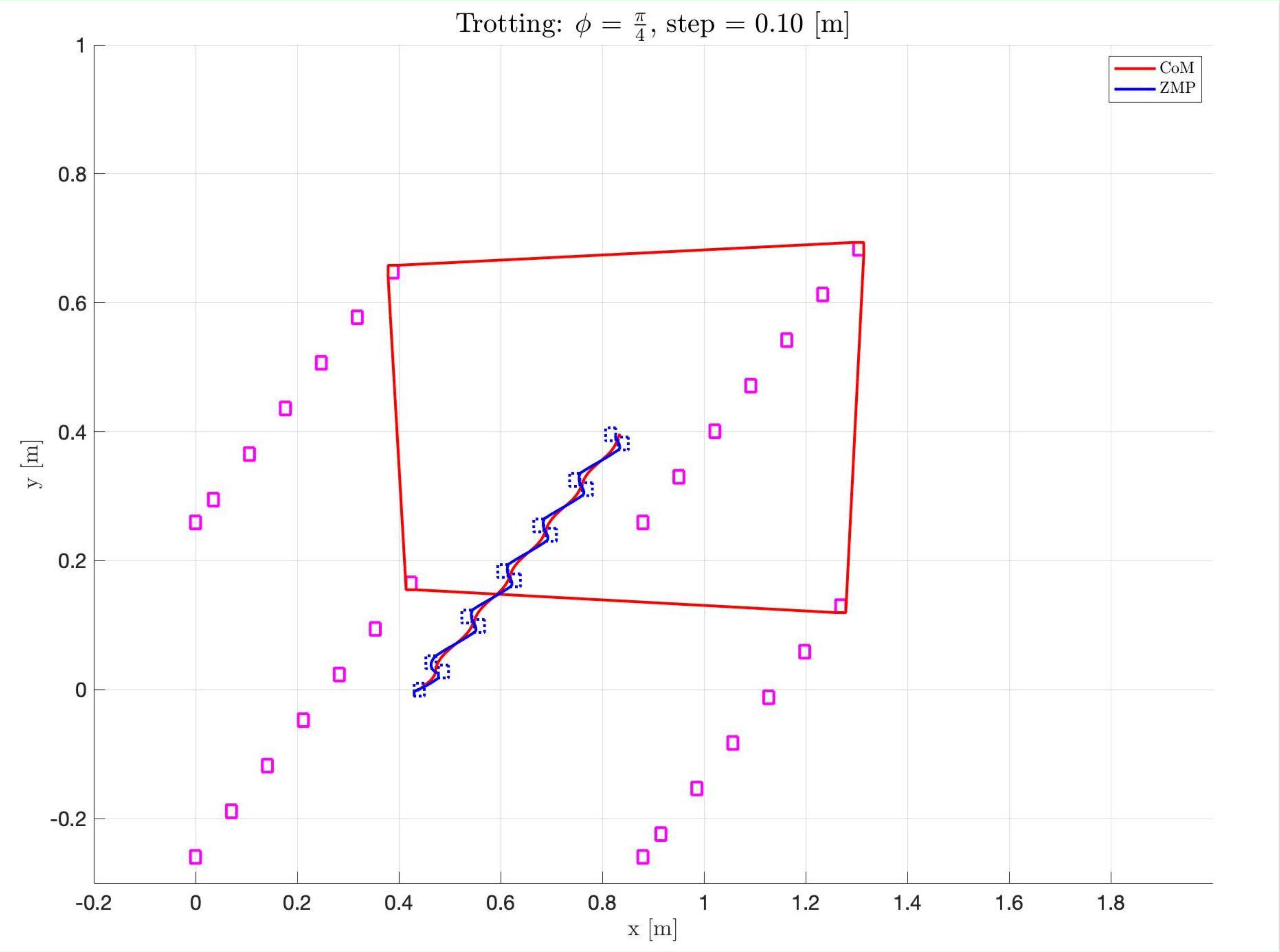
Trotting: $\phi = \frac{\pi}{2}$, step = 0.15 [m]
CoM
ZMP
y [m]
x [m]
1.4
1.2
1
0.8
0.6
0.4
0.2
0
-0.2
-0.2
0
0.2
0.4
0.6
0.8

MATLAB



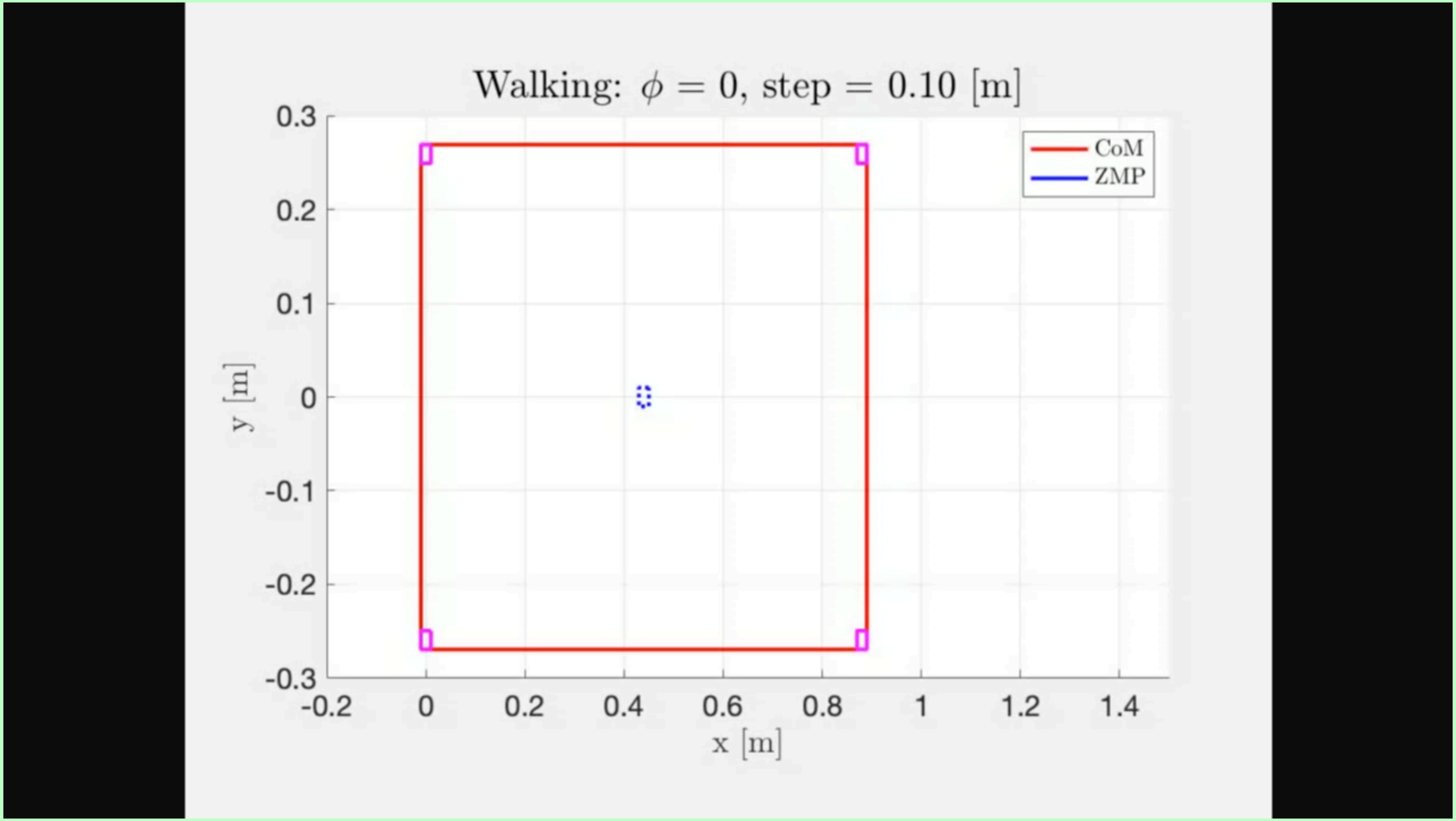
Trotting: $\phi = \frac{\pi}{4}$, step = 0.10 [m]
CoM
ZMP
y [m]
x [m]
1
0.8
0.6
0.4
0.2
0
-0.2
0
0.5
1
1.5
2

MATLAB



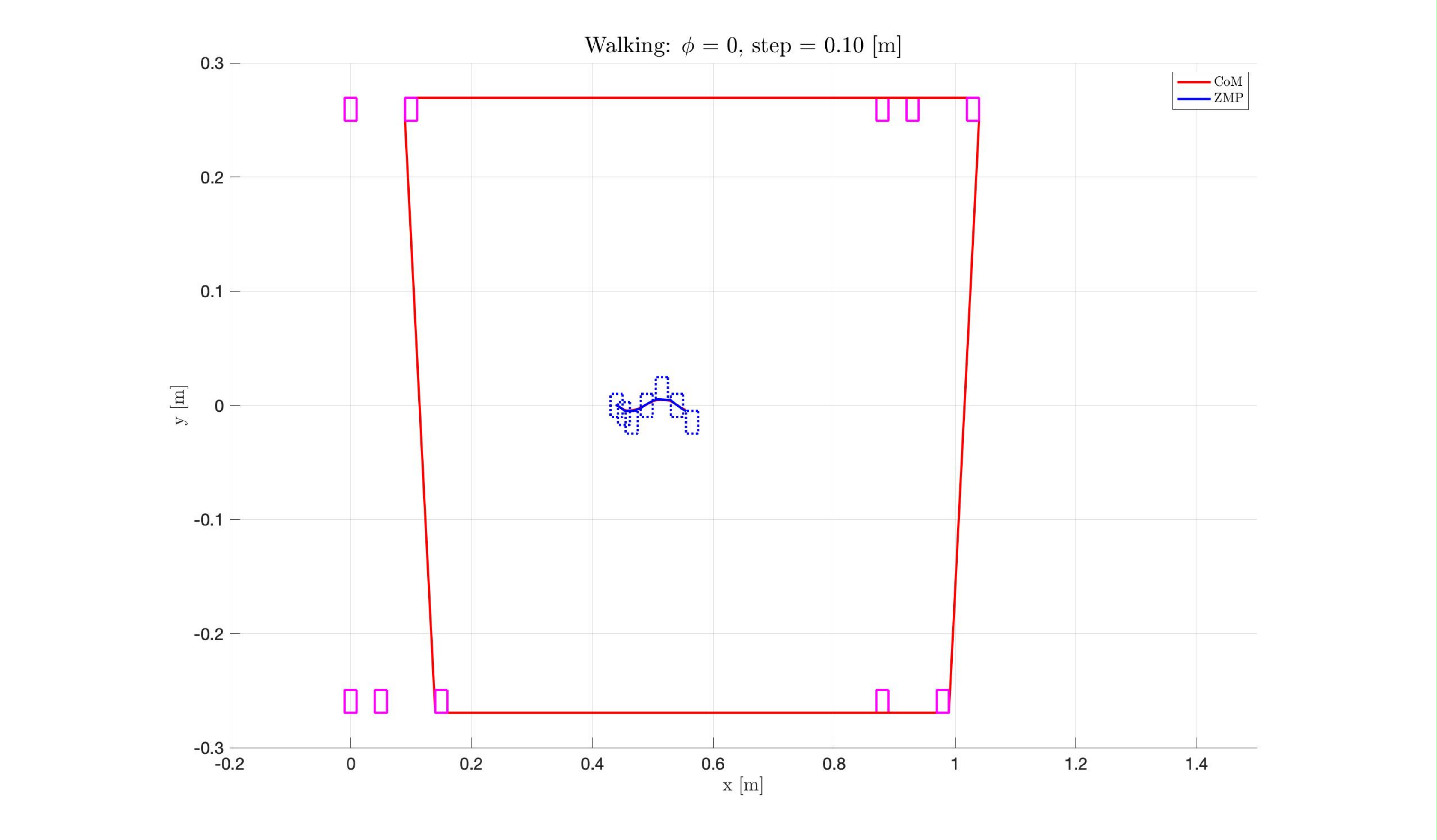
Trotting: $\phi = \frac{\pi}{4}$, step = 0.10 [m]
CoM
ZMP
y [m]
x [m]
1
0.8
0.6
0.4
0.2
0
-0.2
-0.2
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8

MATLAB

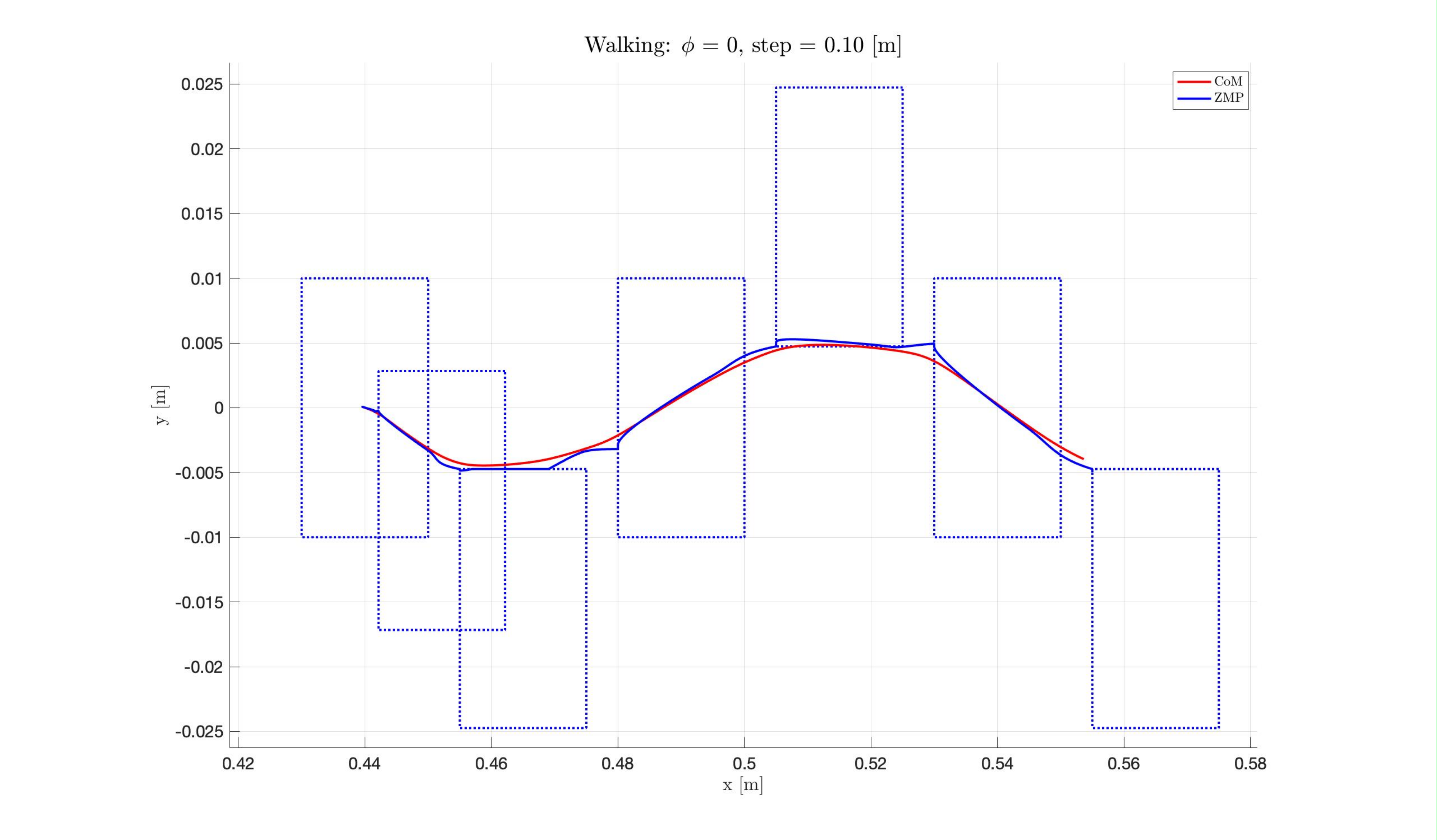


Walking: $\phi = 0$, step = 0.10 [m]
CoM
ZMP
y [m]
x [m]
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
-0.2
0
0.2
0.4
0.6
0.8
1
1.2
1.4

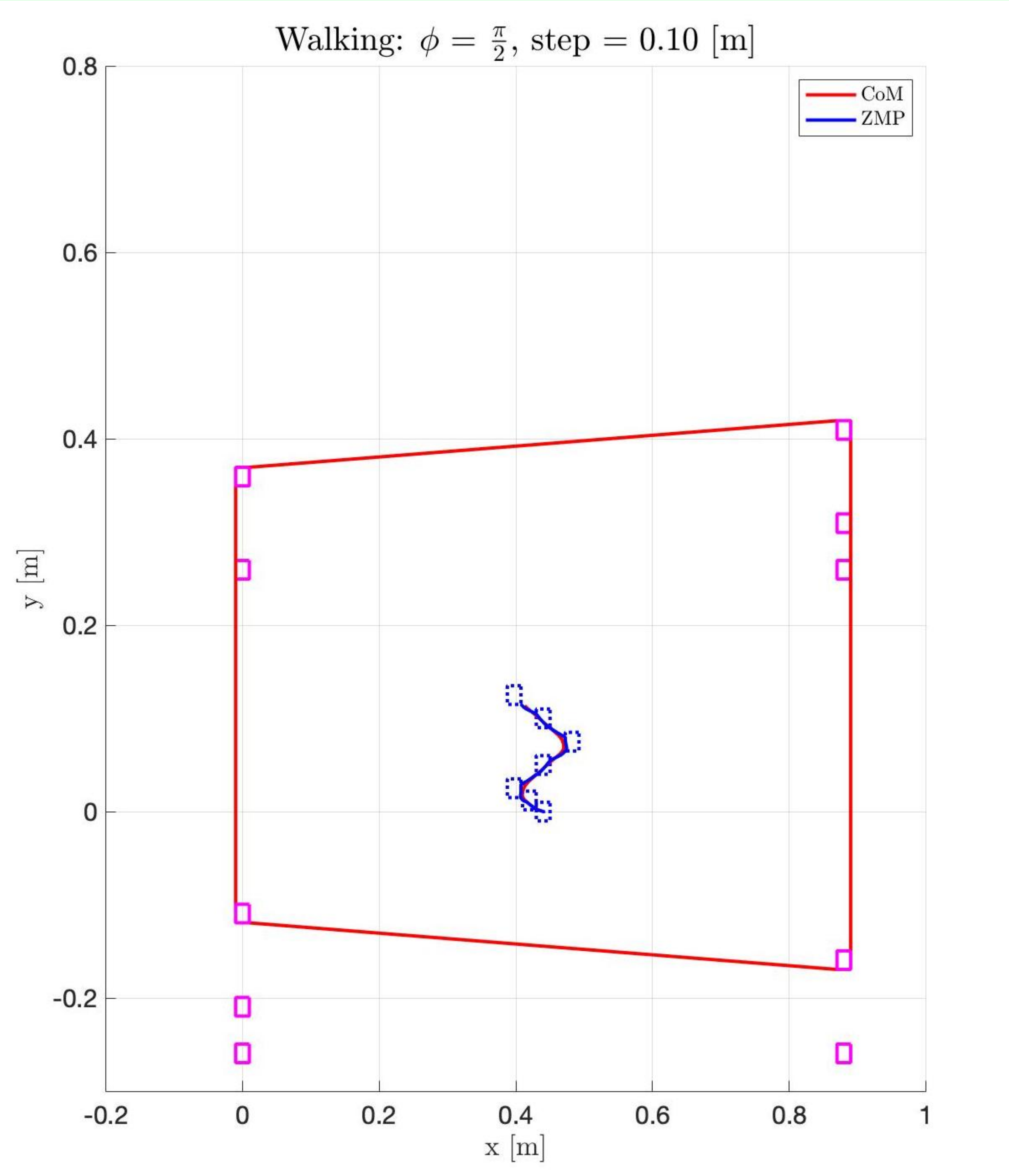
MATLAB


Walking: $\phi = 0$, step = 0.10 [m]
CoM
ZMP
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
y [m]
-0.2
0
0.2
0.4
0.6
0.8
1
1.2
1.4
x [m]

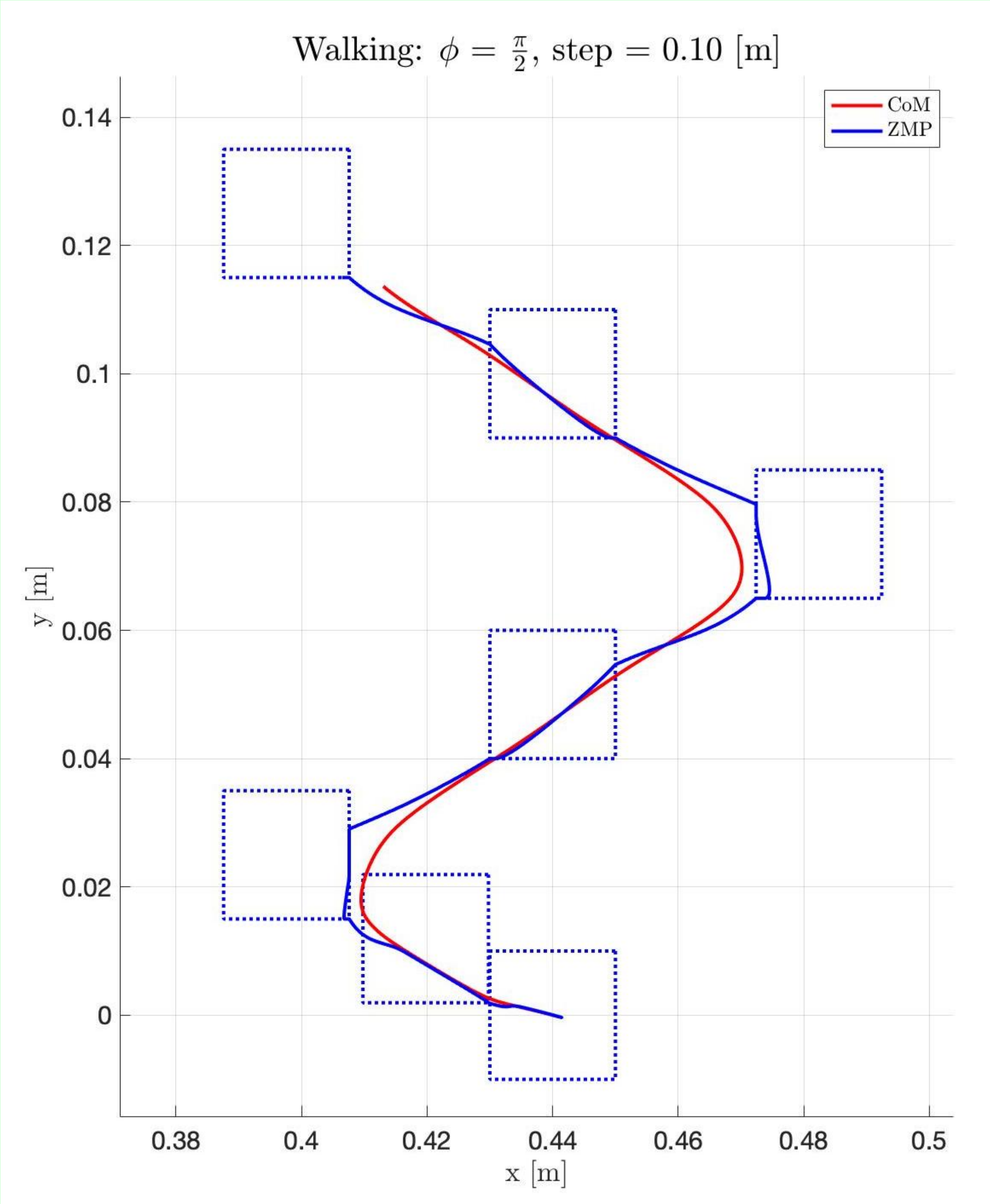
MATLAB


Walking: $\phi = 0$, step = 0.10 [m]
CoM
ZMP
y [m]
x [m]
0.025
0.02
0.015
0.01
0.005
0
-0.005
-0.01
-0.015
-0.02
-0.025
0.42
0.44
0.46
0.48
0.5
0.52
0.54
0.56
0.58

MATLAB

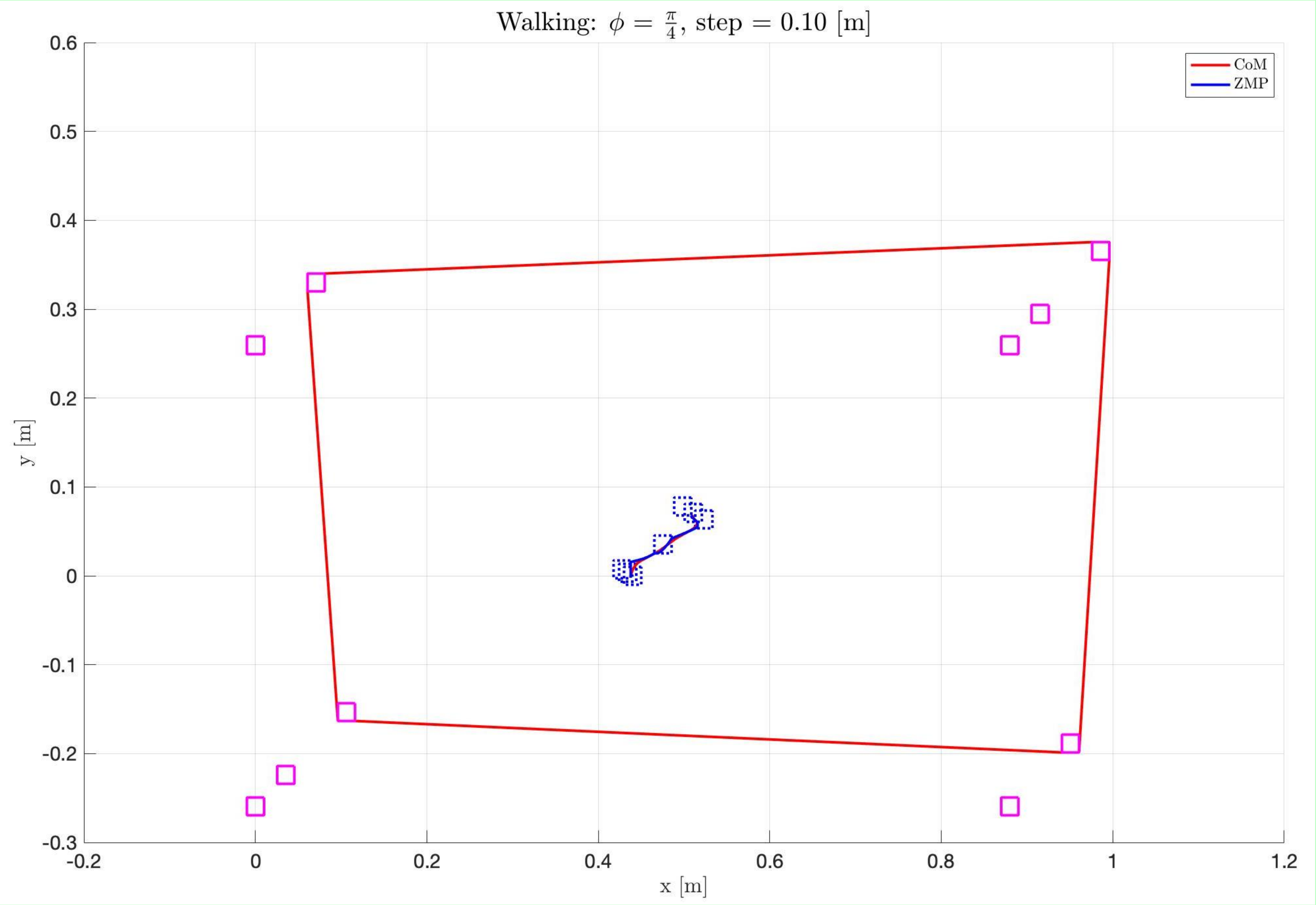


Walking: $\phi = \frac{\pi}{2}$, step = 0.10 [m]
CoM
ZMP
0.8
0.6
0.4
0.2
0
-0.2
y [m]
-0.2
0
0.2
0.4
0.6
0.8
1
x [m]

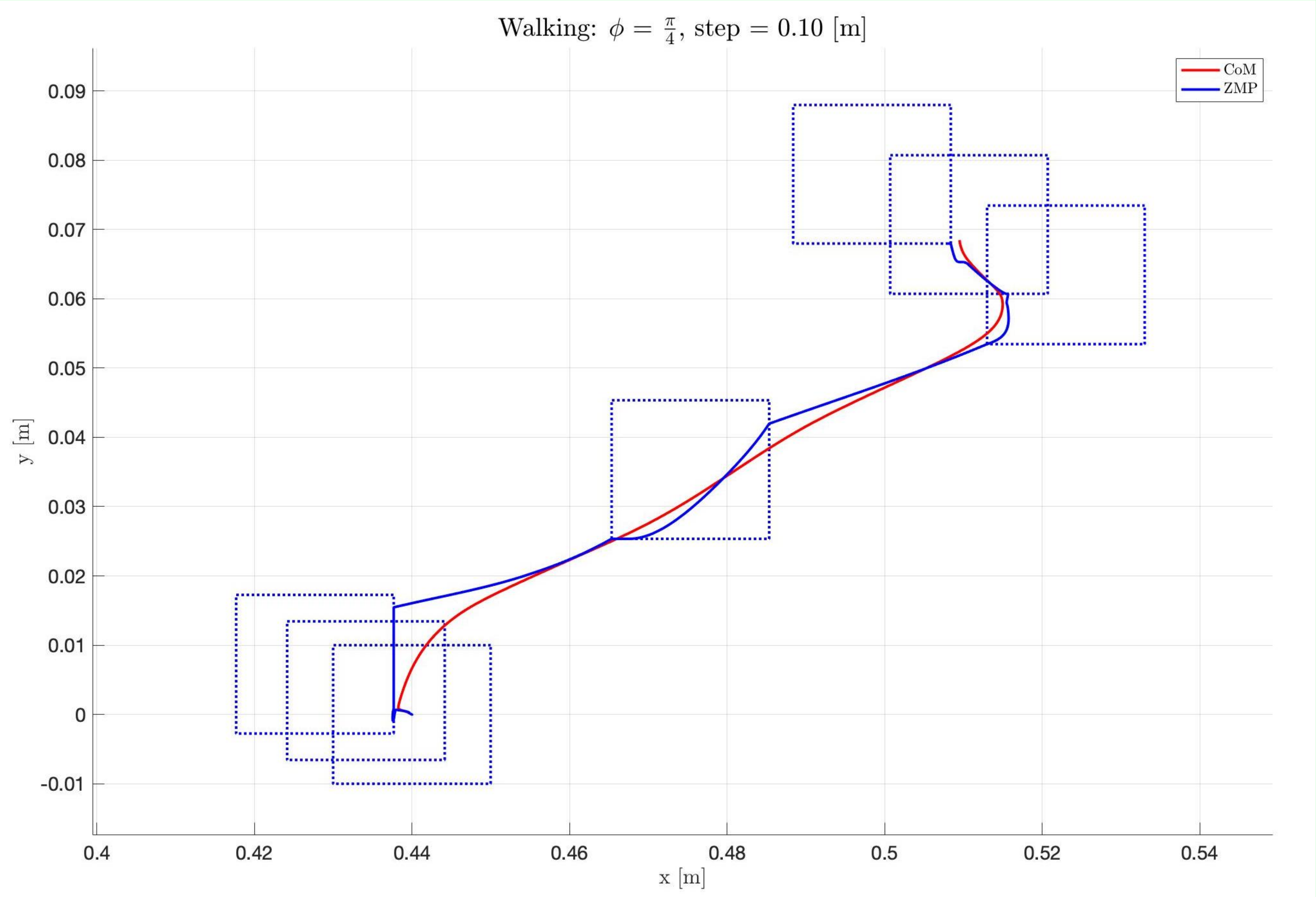


Walking: $\phi = \frac{\pi}{2}$, step = 0.10 [m]
CoM
ZMP
0.14
0.12
0.1
0.08
0.06
0.04
0.02
0
y [m]
0.38
0.4
0.42
0.44
0.46
0.48
0.5
x [m]

MATLAB


Walking: $\phi = \frac{\pi}{4}$, step = 0.10 [m]
CoM
ZMP
y [m]
x [m]
0.6
0.5
0.4
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
-0.2
0
0.2
0.4
0.6
0.8
1
1.2

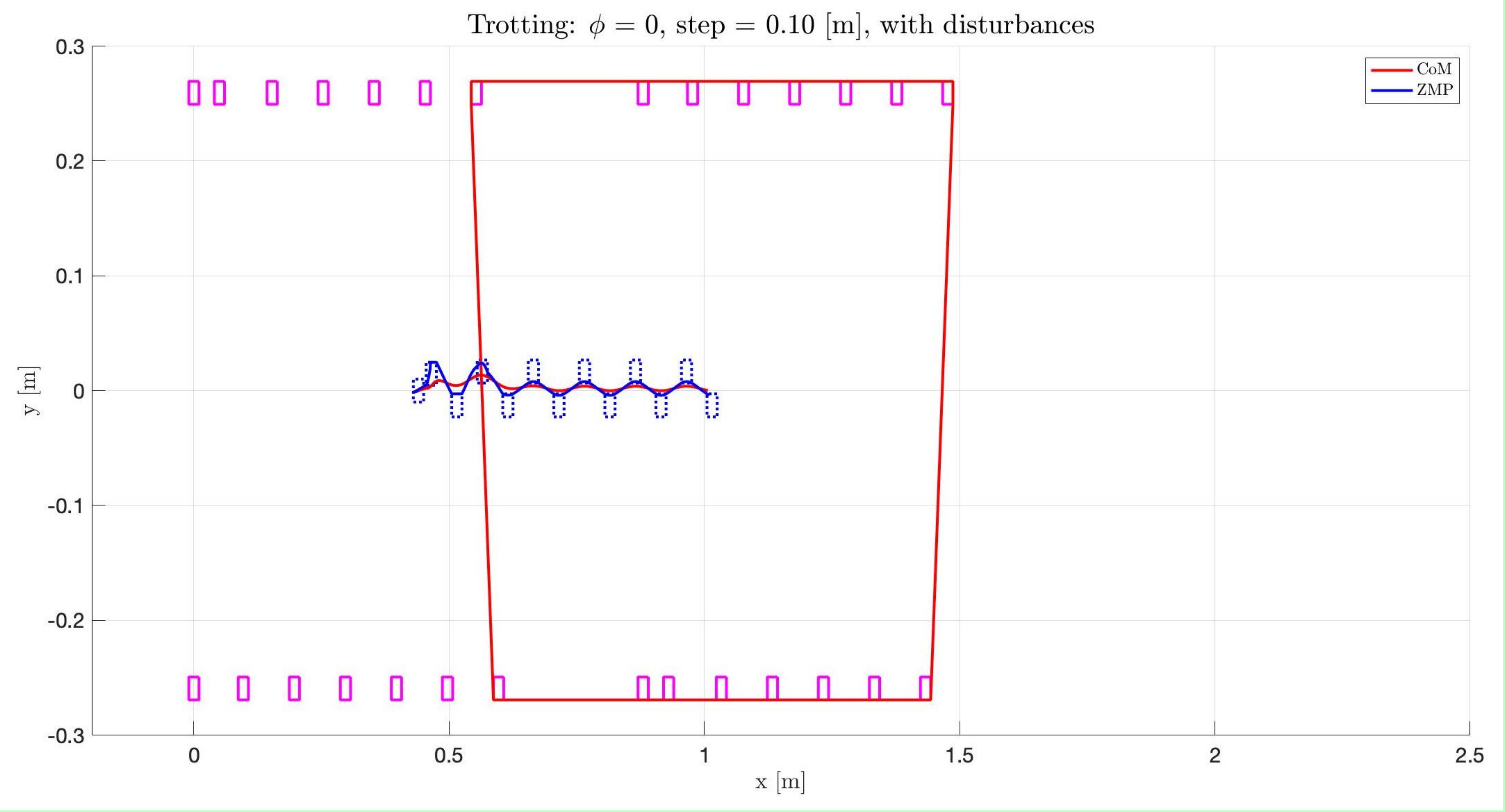
MATLAB



Walking: $\phi = \frac{\pi}{4}$, step = 0.10 [m]
CoM
ZMP
0.09
0.08
0.07
0.06
0.05
0.04
0.03
0.02
0.01
0
-0.01
y [m]
0.4
0.42
0.44
0.46
0.48
0.5
0.52
0.54
x [m]

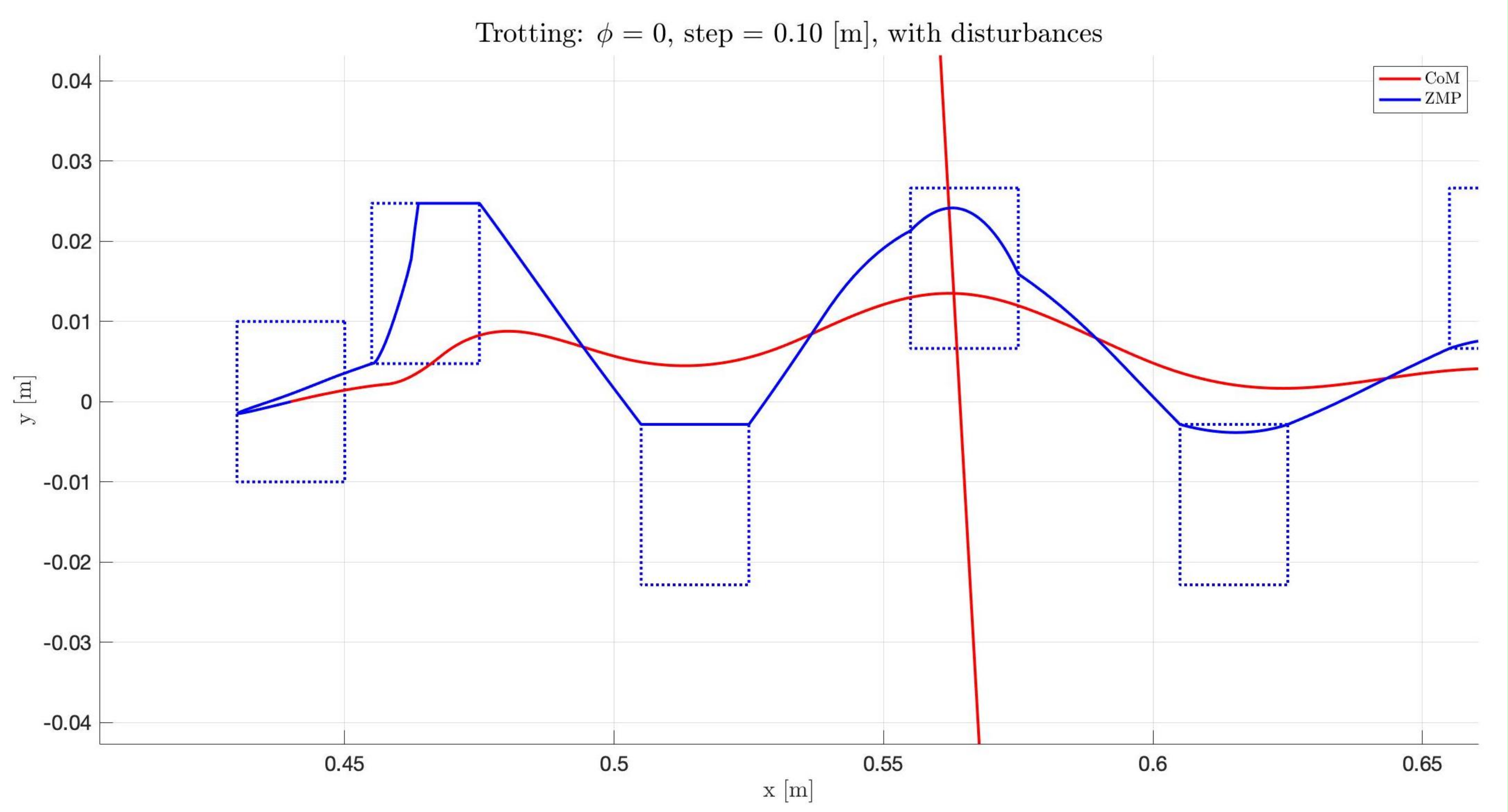
MATLAB

Disturbance of 0.4 m/s^2 along y-axis at the second step.



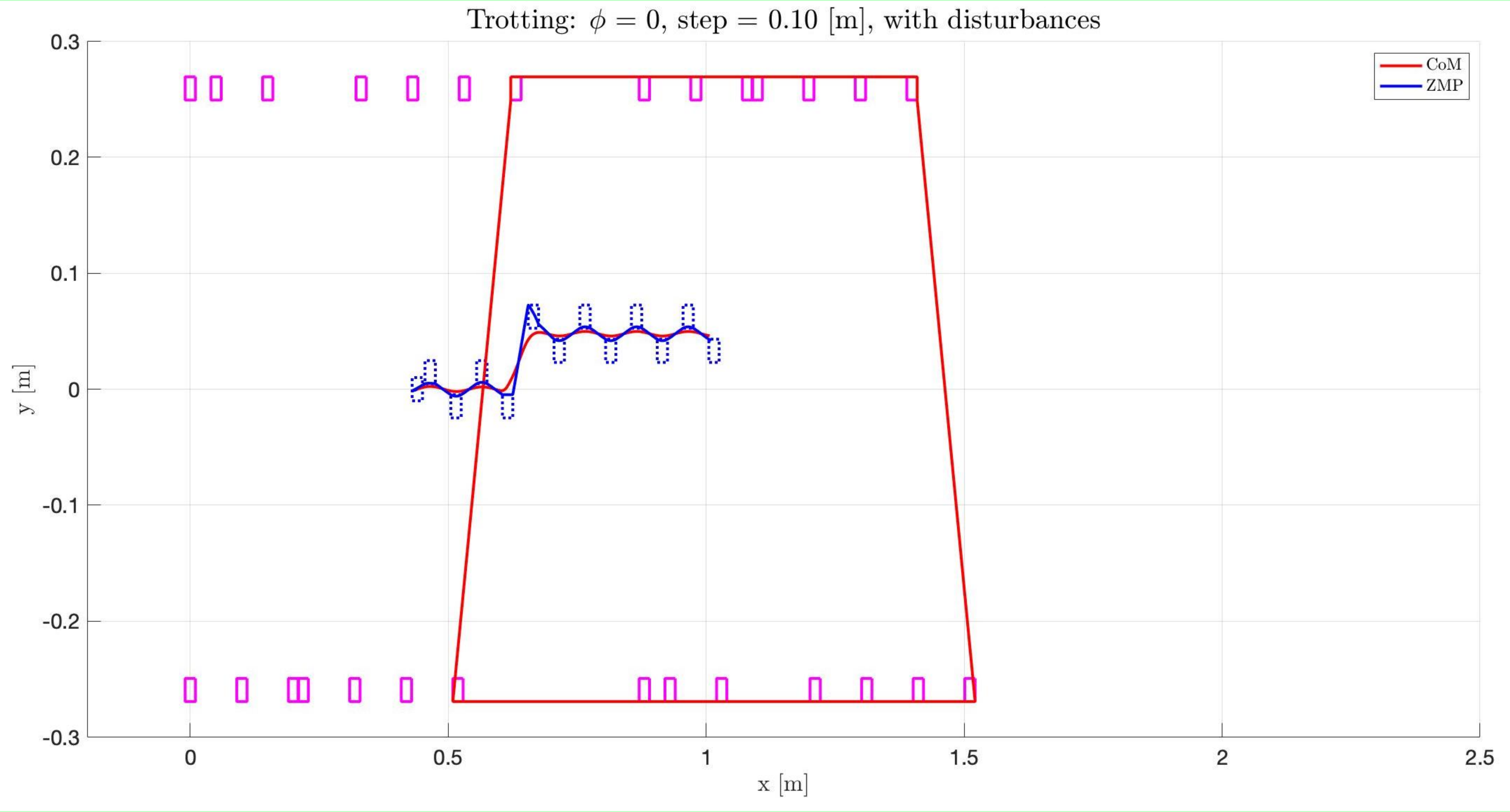
MATLAB

Disturbance of 0.4 m/s^2 along y-axis at the second step.



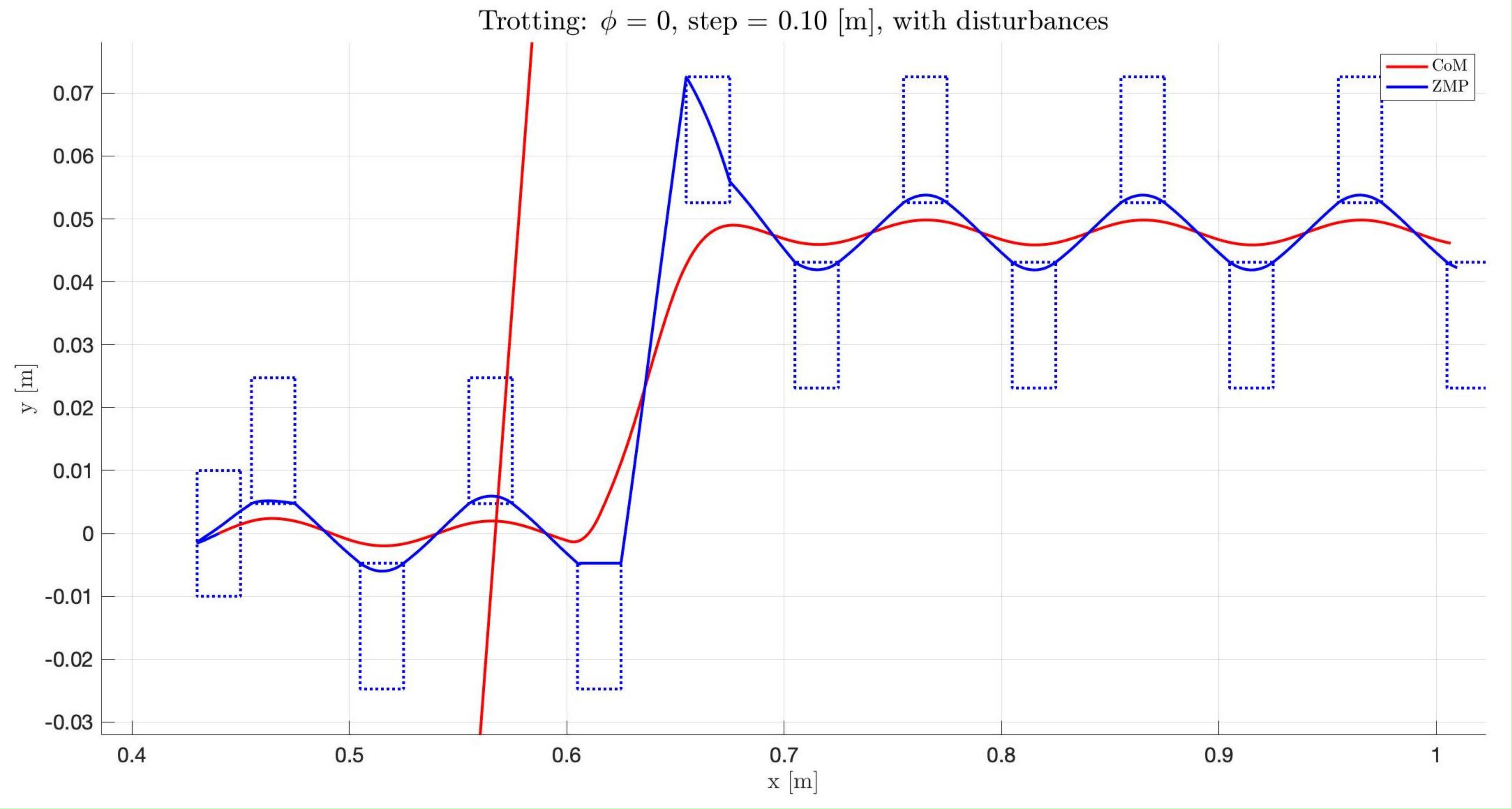
MATLAB

Disturbance of 0.5 m/s^2 along y-axis at the fifth step.



MATLAB

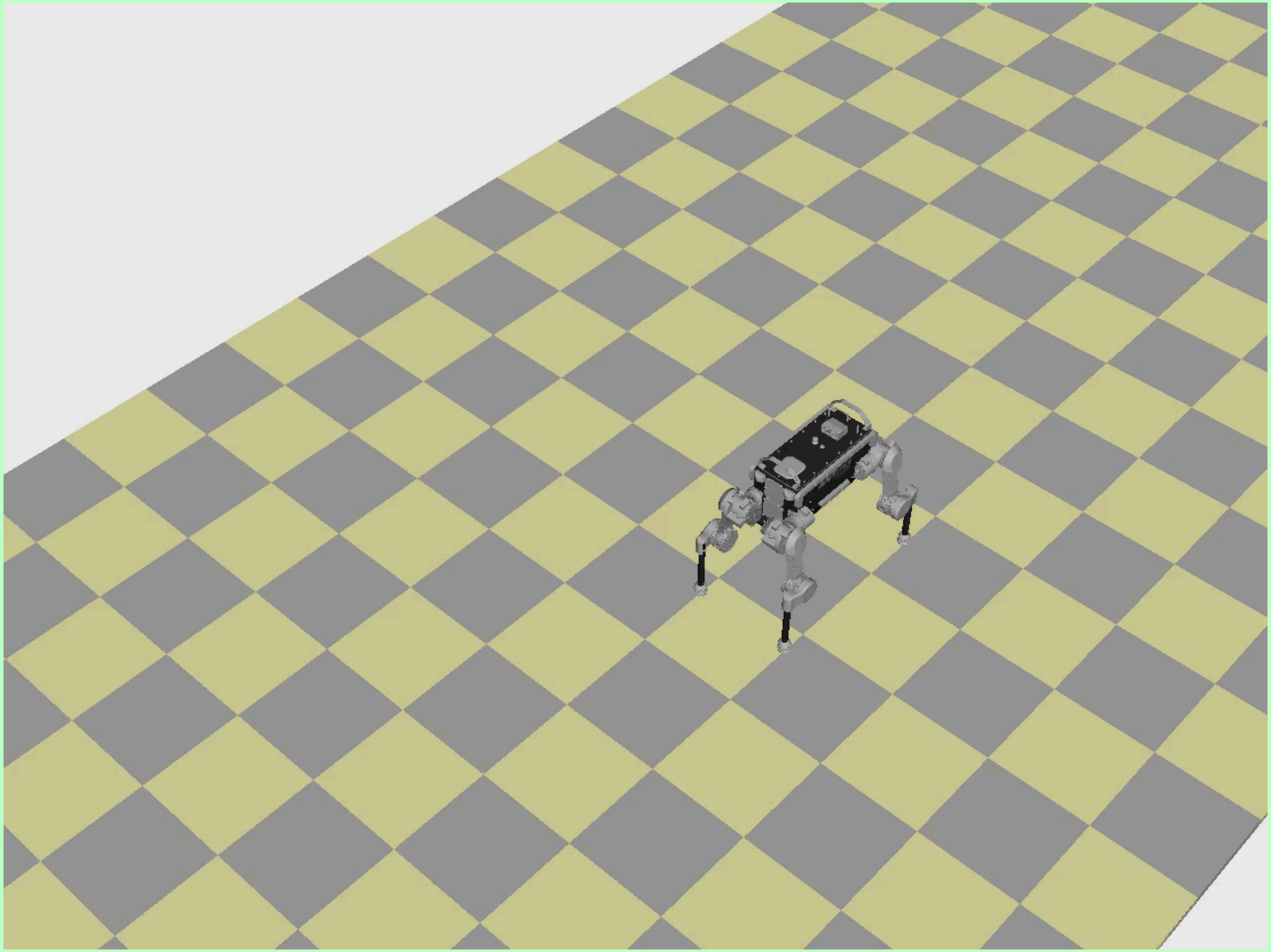
Disturbance of 0.5 m/s^2 along y-axis at the fifth step.



TROTTING: $\phi = 0$

Step size = 0.15 [m]

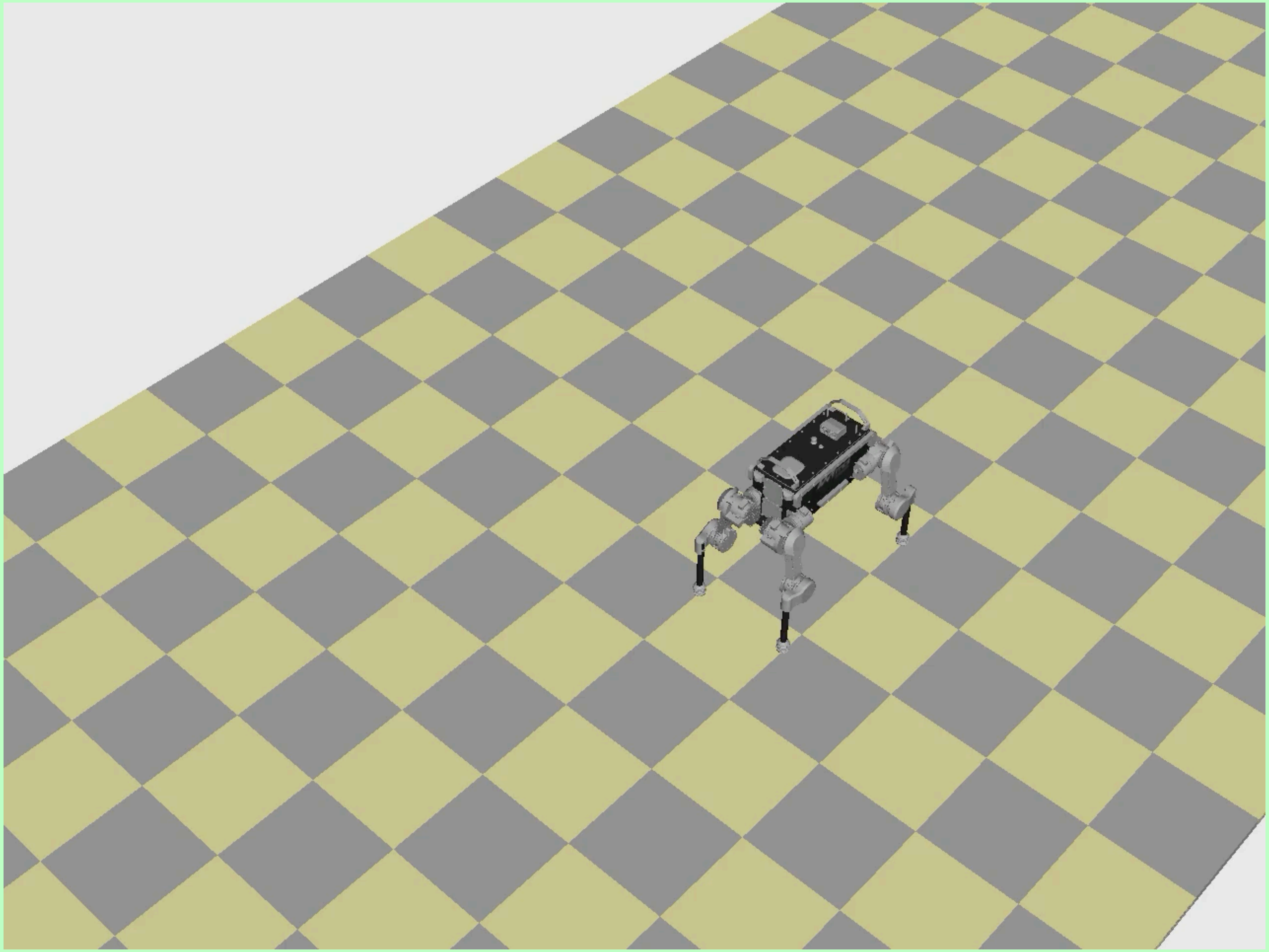
Step timing = 0.8 [s]



TROTTING: $\phi = \frac{\pi}{2}$

Step size = 0.15 [m]

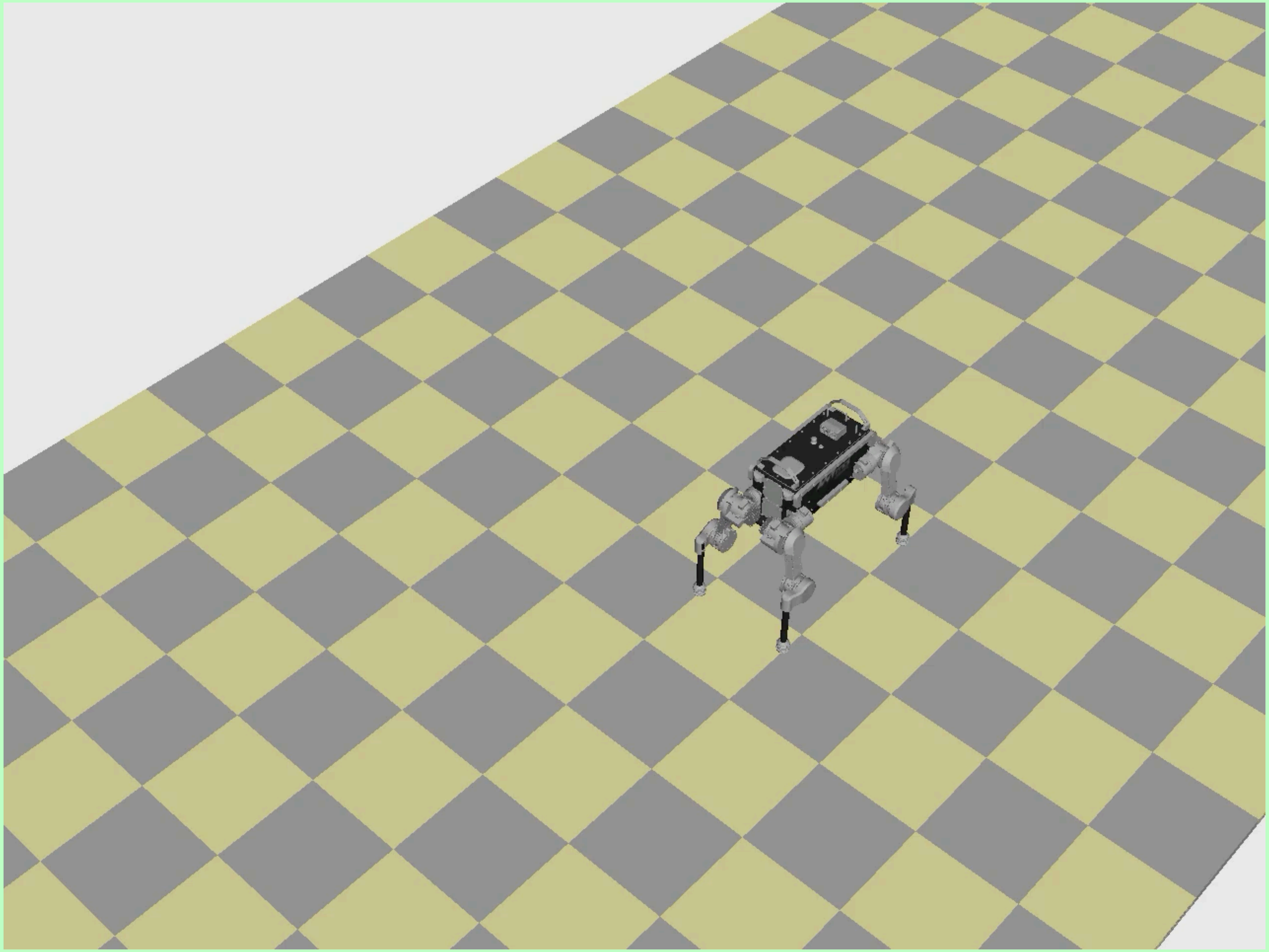
Step timing = 0.8 [s]



TROTTING: $\phi = \frac{\pi}{4}$

Step size = 0.10 [m]

Step timing = 0.8 [s]

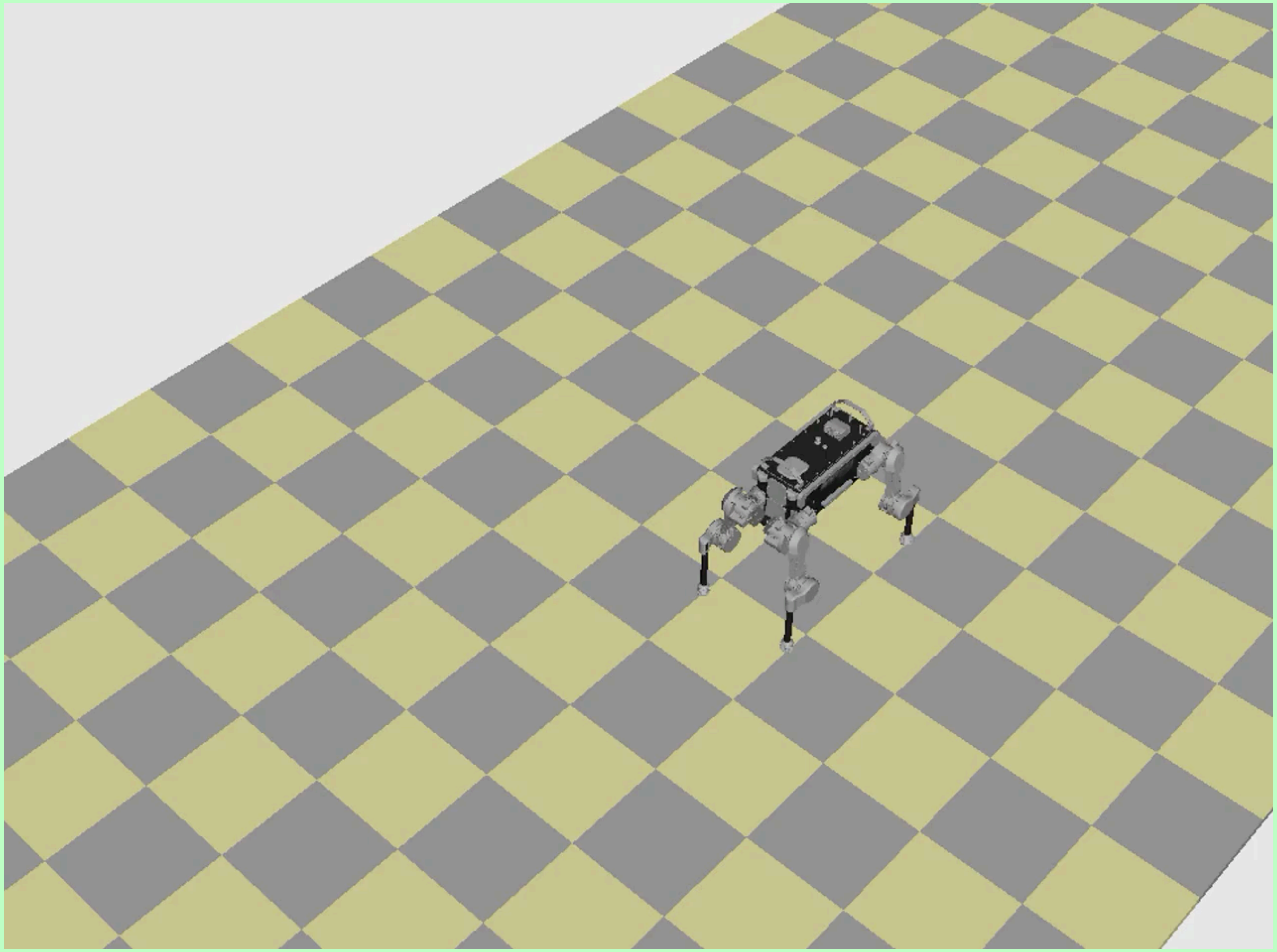


WALKNG:

$\phi = 0$

Step size = 0.10 [m]

Step timing = 0.5 [s]



CONCLUSIONS

- The approach is very conservative due to the limitation on the ZMP region.
- The approach that was developed for the trotting gait has shown to be suitable also for walking gait.
- We have performed dynamic simulation on DART and in absence of disturbances the nominal gait is well performed in any direction.
- Different foot replacement policies can be investigated in future works.
- Further developments point to the goal of exploring gaits different from trotting and walking with a deeper exploitation of ZMP admissible region and more complex disturbance recovery.

THANK YOU FOR THE ATTENTION

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Leonardo Salustri (M1762556)
Francesco Scotti (M1758391)