The start system of equation is:

$$\begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{pmatrix} = \begin{bmatrix} -sin\theta_1 & -sin\theta_2 & -sin\theta_3 \\ cos\theta_1 & cos\theta_2 & cos\theta_3 \\ l & l & l \end{bmatrix} \begin{pmatrix} f_1 \\ f_2 \\ f_3 \end{pmatrix}$$

Assuming that $l = 1$ we get that the matrix at the center, which call A, we are interested in solving the system of equations:

$$\begin{pmatrix} f_1 \\ f_2 \\ f_3 \end{pmatrix} = \begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{pmatrix} \begin{bmatrix} -sin\theta_1 & -sin\theta_2 & -sin\theta_3 \\ cos\theta_1 & cos\theta_2 & cos\theta_3 \\ l & l & l \end{bmatrix}^{-1} = \begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{pmatrix} A^{-1}$$

This is possible for Cramer's theorem.
Calculate the inverse of matrix A:

$$A = \begin{bmatrix} -sin\theta_1 & -sin\theta_2 & -sin\theta_3 \\ cos\theta_1 & cos\theta_2 & cos\theta_3 \\ 1 & 1 & 1 \end{bmatrix}$$

The inverse is:

$$A^{-1} = \begin{bmatrix} \frac{2\sin(\theta_1)(\cos(\theta_3) - \cos(\theta_2))}{2\sin(\theta_1)(\sin(\theta_1 - \theta_2) - \sin(\theta_1 - \theta_3) + \sin(\theta_2 - \theta_3))} & \frac{2\sin(\theta_1)(\sin(\theta_3) - \sin(\theta_2))}{2\sin(\theta_1)(\sin(\theta_1 - \theta_2) - \sin(\theta_1 - \theta_3) + \sin(\theta_2 - \theta_3))} & \frac{2\sin(\theta_1)\sin(\theta_2 - \theta_3)}{2\sin(\theta_1)(\sin(\theta_1 - \theta_2) - \sin(\theta_1 - \theta_3) + \sin(\theta_2 - \theta_3))} \\ \frac{2\sin(\theta_1)(\cos(\theta_1) - \cos(\theta_3)}{2\sin(\theta_1)(\sin(\theta_1 - \theta_2) - \sin(\theta_1 - \theta_3) + \sin(\theta_2 - \theta_3))} & \frac{-\cos(2\theta_1) + \cos(\theta_1 + \theta_3) - \cos(\theta_1 - \theta_3) + 1}{2\sin(\theta_1)(\sin(\theta_1 - \theta_2) - \sin(\theta_1 - \theta_3) + \sin(\theta_2 - \theta_3))} & \frac{-\cos(\theta_3) + \cos(2\theta_1 - \theta_3)}{2\sin(\theta_1)(\sin(\theta_1 - \theta_2) - \sin(\theta_1 - \theta_3) + \sin(\theta_2 - \theta_3))} \\ \frac{2\sin(\theta_1)(\cos(\theta_2 - \cos(\theta_1))}{2\sin(\theta_1)(\sin(\theta_1 - \theta_2) - \sin(\theta_1 - \theta_3) + \sin(\theta_2 - \theta_3))} & \frac{\cos(2\theta_1) - \cos(\theta_1 + \theta_2) + \cos(\theta_1 - \theta_2) - 1}{2\sin(\theta_1)(\sin(\theta_1 - \theta_2) - \sin(\theta_1 - \theta_3) + \sin(\theta_2 - \theta_3))} & \frac{\cos(\theta_2) - \cos(2\theta_1 - \theta_2)}{2\sin(\theta_1)(\sin(\theta_1 - \theta_2) - \sin(\theta_1 - \theta_3) + \sin(\theta_2 - \theta_3))} \end{bmatrix}$$

So some configurations isn't allowed.
The denominator of matrix vanishes for values of $\theta_1, \theta_2, \theta_3 = k\pi$ with $k \in [0,1]$.