OPTIMAL CONTROL OF A MAGNETIC BEARING

Project for the course of Mechatronic Systems and Laboratory

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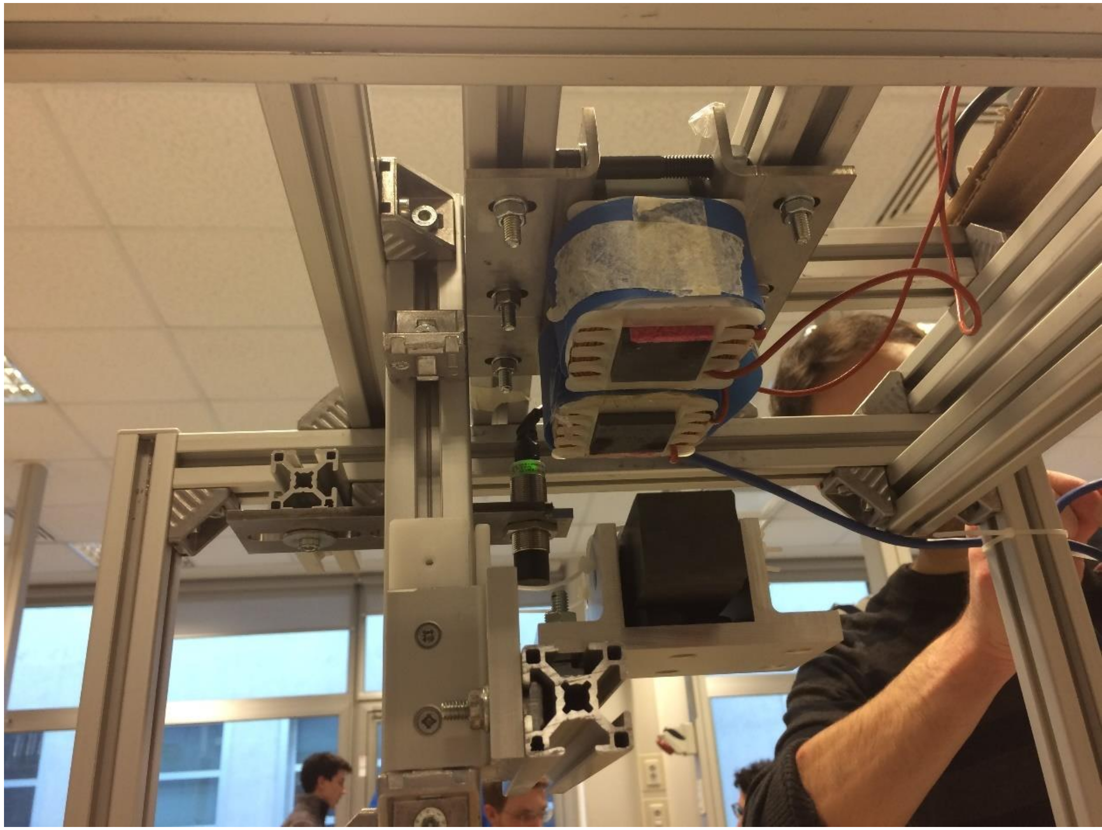
INTRODUCTION

INTRODUCTION: WORKBENCH DESCRIPTION

Aim of the laboratory experience: position control of a suspended mass through the generation of an electro-magnetic force.

Workbench's main components

- Frame;
- Lumped mass & roller;
- Electromagnet;
- Control board;
- Proximity sensor;
- PC.



INTRODUCTION: DYNAMIC & STATE EQUATIONS

System's equations:

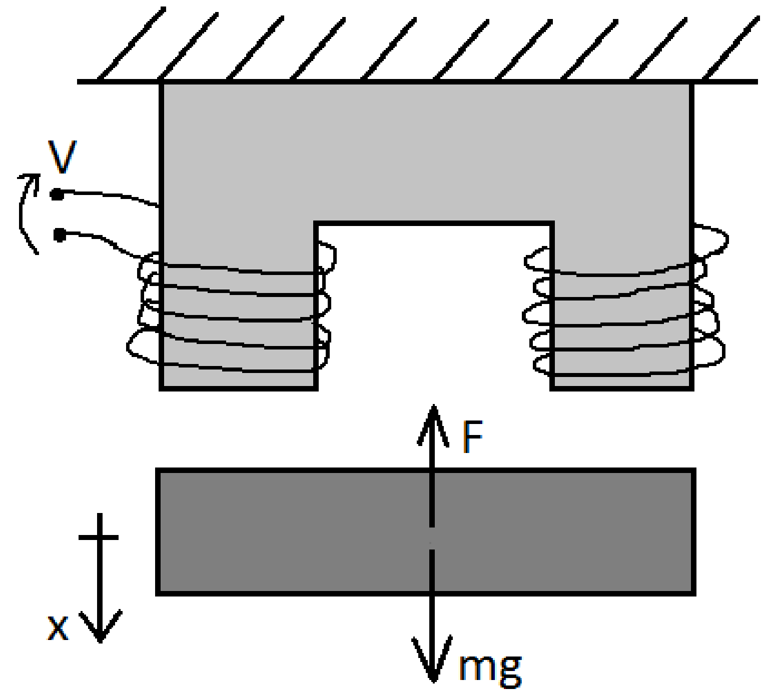
$$\begin{cases} m\ddot{x} = mg - F(x, I_1) \\ V_1 = RI_1 + \frac{d}{dt}(L(x)I_1) \end{cases}$$

Considering only the upper magnet, with x starting from the magnet and directed to the bottom:

$$\begin{cases} f(x, I_1) = \ddot{x} = +g - \frac{1}{m}F(x, I_1) \\ g(x, I_1, V_1) = \frac{dI_1}{dt} = \frac{1}{L(x)}[V_1 - RI_1 - \frac{dL(x)}{dx}\dot{x}I_1] \\ F(x, I_1) = \frac{\mu_0 A_s N^2 I_1^2}{(\frac{L_{fe}}{\mu_r}+2\cdot x)^2} \approx K_f \frac{i^2}{x^2} \\ L(x) = \frac{\mu_0 N^2 A_s}{\frac{L_{fe}}{\mu_r}+2\cdot x} = K_0 + \frac{K_1}{x} \end{cases}$$

$\longrightarrow$ Theoretical force equation

$\longrightarrow$ Theoretical inductance equation



INTRODUCTION: LINEARIZATION

System's nonlinearities are introduced by the **force** and the **inductance**. The linearization of the model is thus necessary in order to be able to develop our control logic.

By imposing an equilibrium condition: $(\dot{x} = \ddot{x} = 0,\ x = \hat{x})$ we find the parameters corresponding to an equilibrium position:

$$\hat{I}_1 = \sqrt{\frac{mg(\frac{L_{fe}}{\mu_r} + 2\hat{x})^2}{\mu_0 N^2 A_s}}$$

$$\hat{V}_1 = R \cdot \hat{I}_1$$

Assuming as state vector: $\underline{x} = \begin{Bmatrix} \dot{x} \\ x \\ I \end{Bmatrix}$

Linearizing the system around its equilibrium position, $\underline{x} = \underline{x}_o = \begin{Bmatrix} \dot{x}_o \\ x_o \\ I_o \end{Bmatrix}$, we obtain:

$$\begin{bmatrix} \delta\ddot{x} \\ \delta\dot{x} \\ \delta\frac{dI_1}{dt} \end{bmatrix} = \begin{bmatrix} 0 & \frac{df}{dx}\big|_{\hat{\underline{x}}} & \frac{df}{dI_1}\big|_{\hat{\underline{x}}} \\ 1 & 0 & 0 \\ 0 & \frac{dg}{dx}\big|_{\hat{\underline{x}}} & \frac{dg}{dI_1}\big|_{\hat{\underline{x}}} \end{bmatrix} \begin{bmatrix} \delta\dot{x} \\ \delta x \\ \delta I_1 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \frac{dg}{dV_1}\big|_{\hat{\underline{x}}} \end{bmatrix} \delta V_1$$

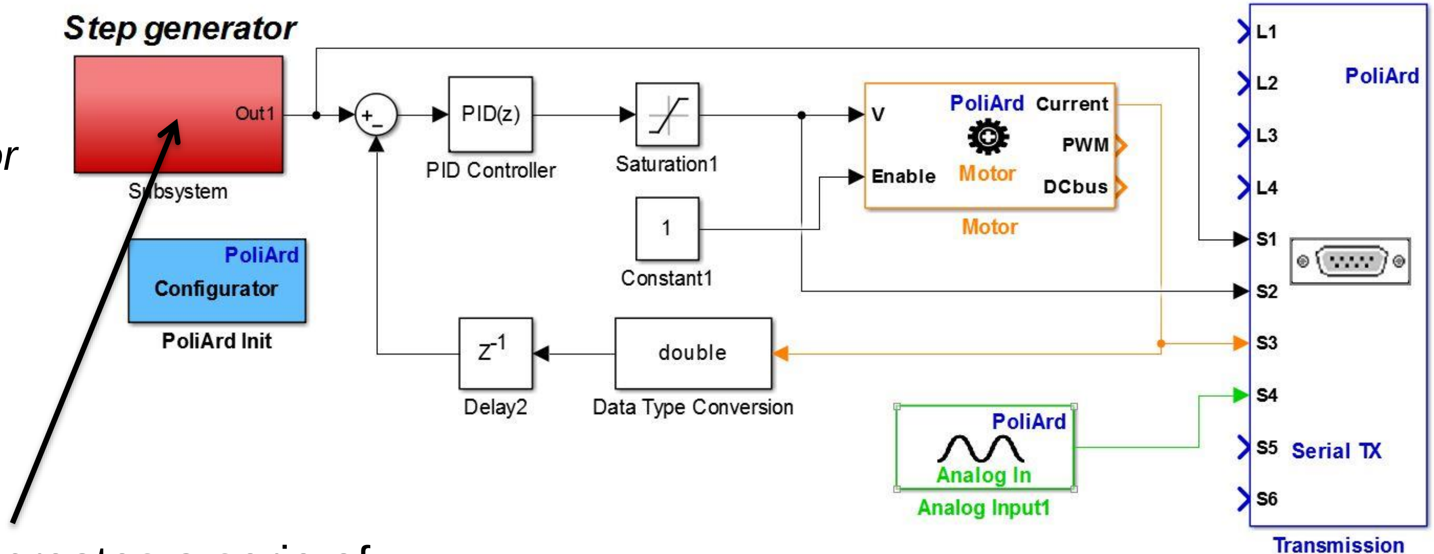
$$\delta\dot{\underline{x}} = [A]\ \delta\underline{x} + [B]\ \delta u$$

PARAMETERS ESTIMATION

PARAMETERS ESTIMATION : L & R ESTIMATION

Inductance depends on the relative distance of the mass from the electromagnet itself and the current.

Simulink model for L & R estimation.



Step generator: creates a serie of steps with increasing amplitude.

Inductance has been estimated analizing the **transient responce** of the current in front of an applied voltage.

PARAMETERS ESTIMATION: L & R ESTIMATION

Resistance is assumed to be constant with time, current and mass position.

The experiments have been repeated for different values of distance of the mass from the electromagnet.

Data gathering

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GREYBOX MODEL

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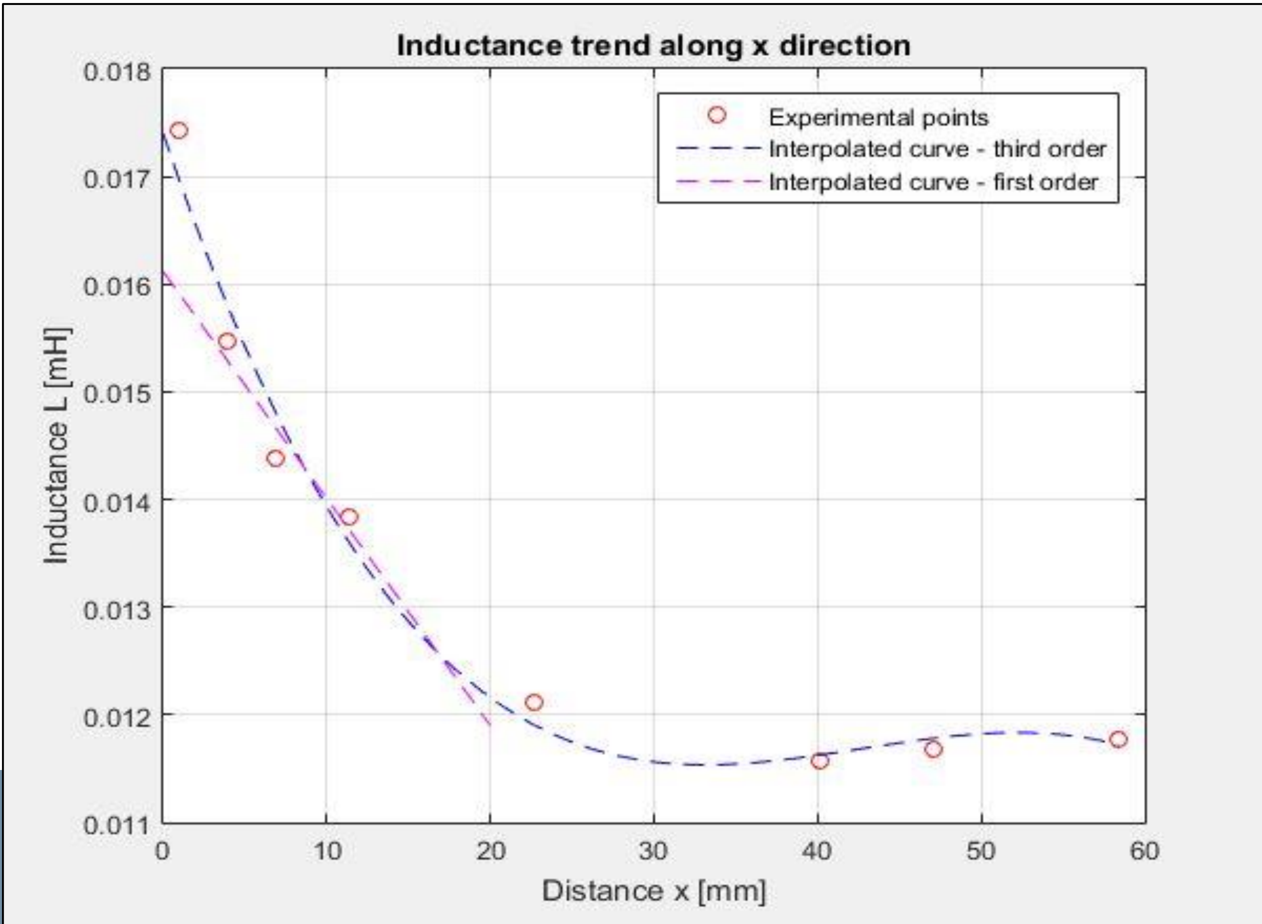
Parameters estimation:

- R = 3,695 ohm;
- L = see figure →

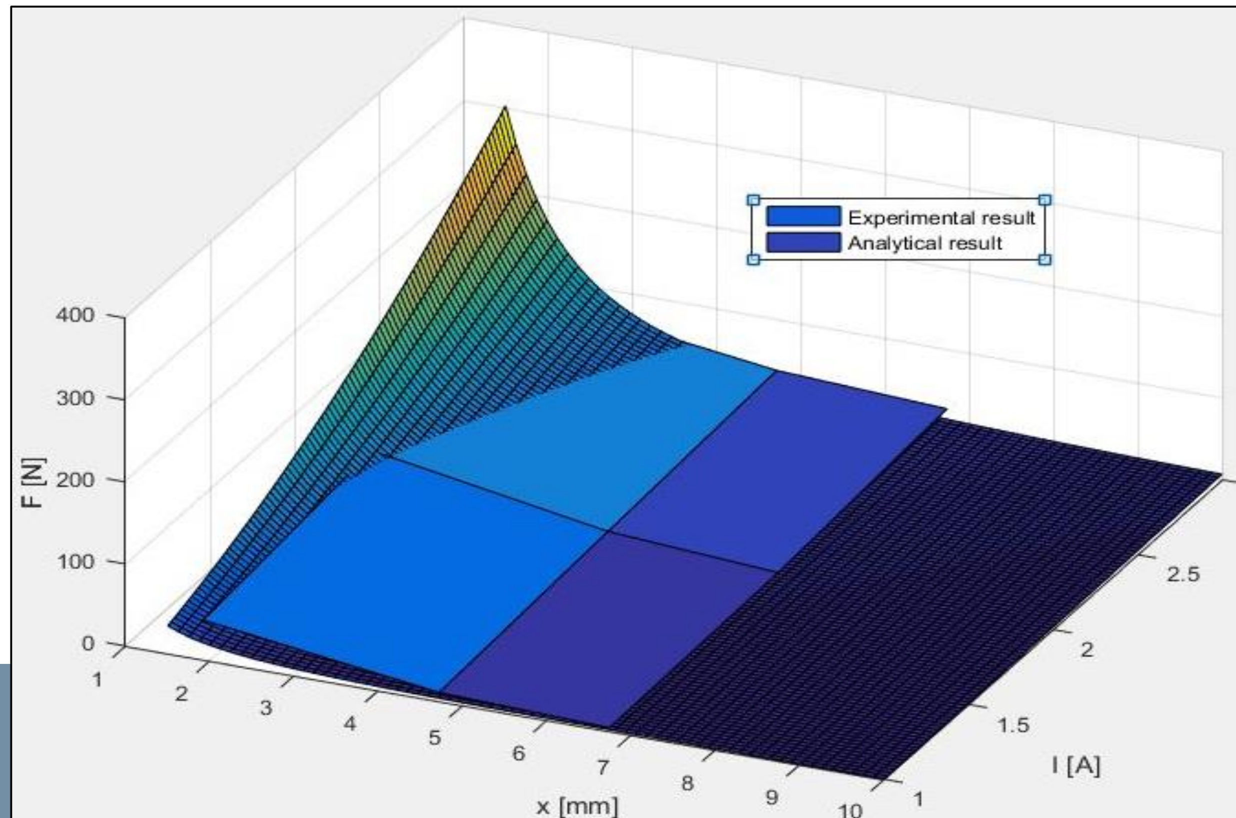


PARAMETERS ESTIMATION: F ESTIMATION

Attractive force generated by the electromagnet depends on the relative distance between the electromagnet itself and the mass and the current.

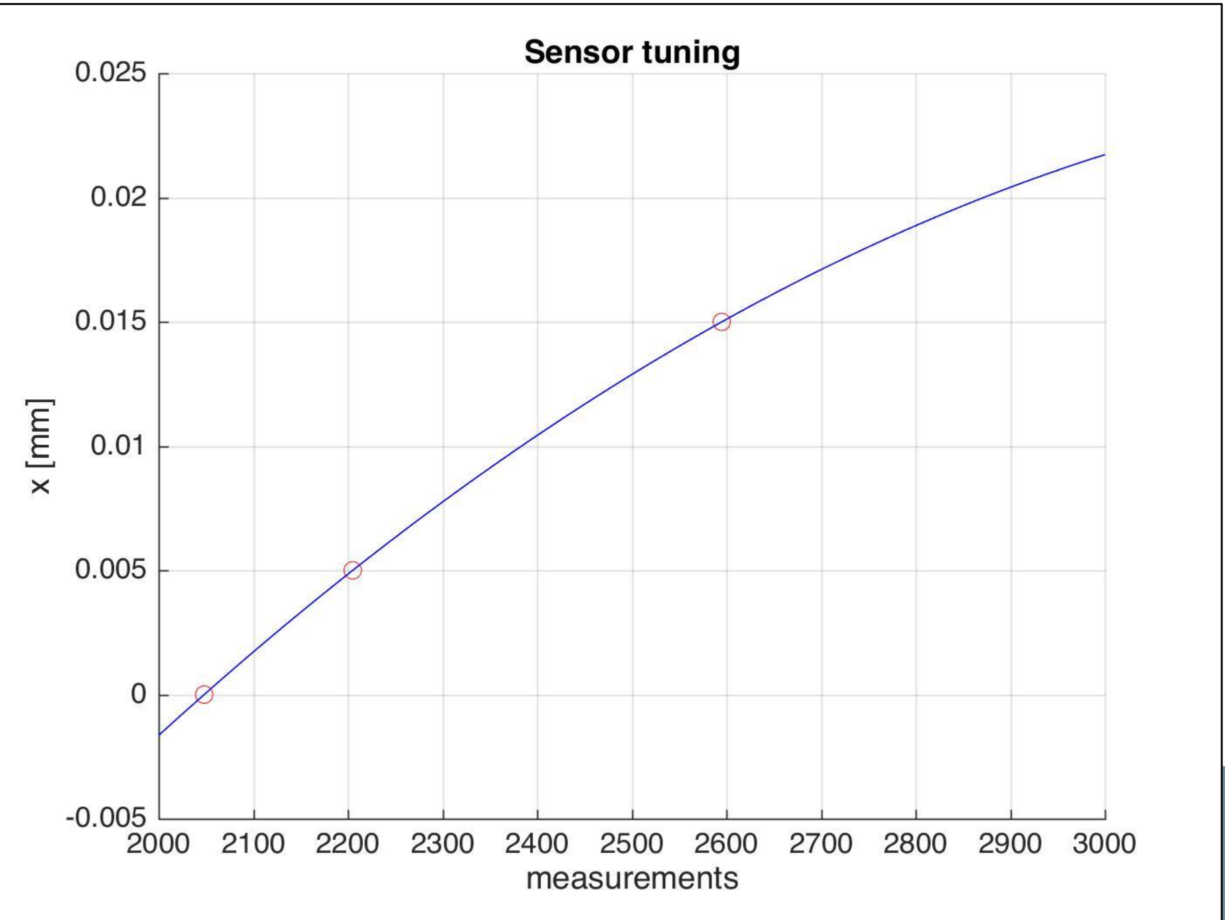
<u>*Analitical expression:*</u> $F(x, I_1) = \frac{\mu_0 A_s N^2 I_1^2}{(\frac{L_{fe}}{\mu_r} + 2 \cdot x)^2} \approx K_f \frac{i^2}{x^2}$ $\rightarrow K_f = 1{,}95e - 4$

<u>*Experimental results*</u>: experiments performed with the available load cell keeping constant the distance and the current once per time. Data gathered have been corrected removing the gravitational effect.



PARAMETERS ESTIMATION: SENSOR TUNING

Position sensor calibration has been performed through linear interpolation of a sequence of data taken at known distances.



The calibration was carried out before each use of the sensor.

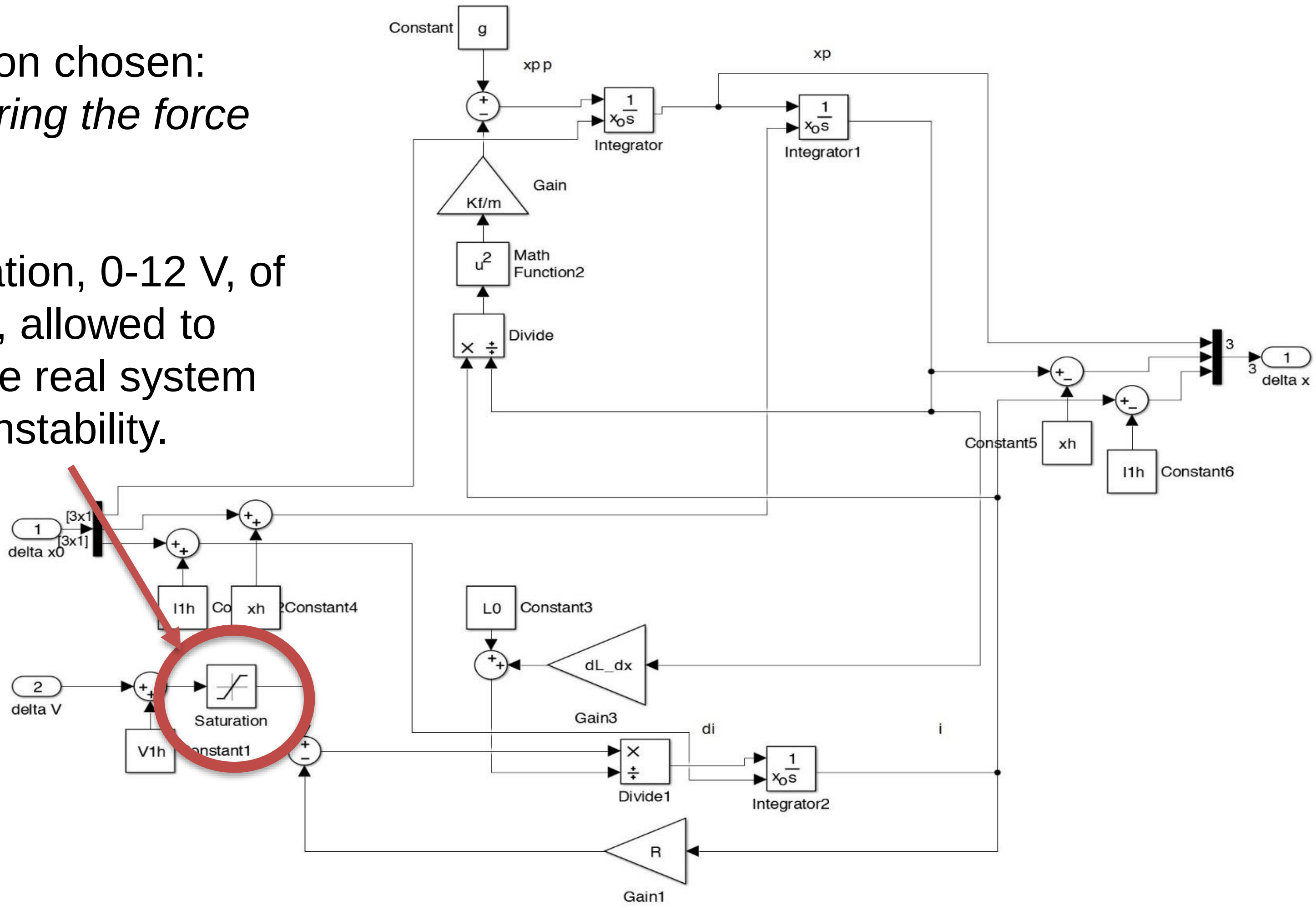
To improve signal quality, a small steel plate was used for the measurement.

SINGLE-INPUT LINEAR CONTROL

VIRTUAL SYSTEM

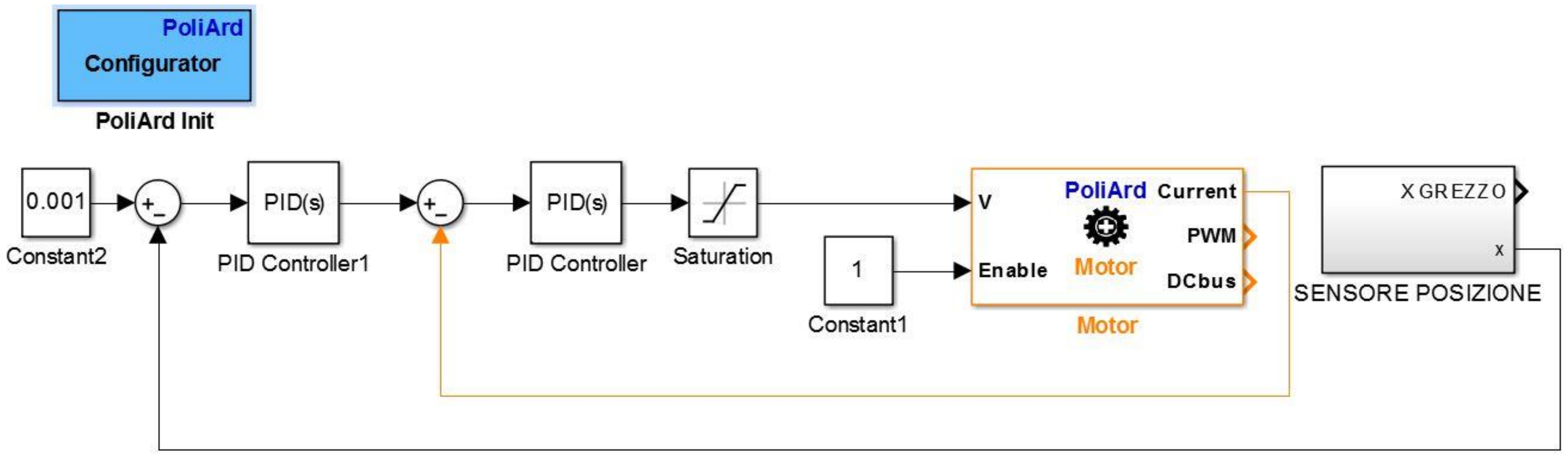
Equilibrium position chosen: 10mm (*remembering the force trend…*).

Adding the saturation, 0-12 V, of the control action, allowed to better simulate the real system and its eventual instability.

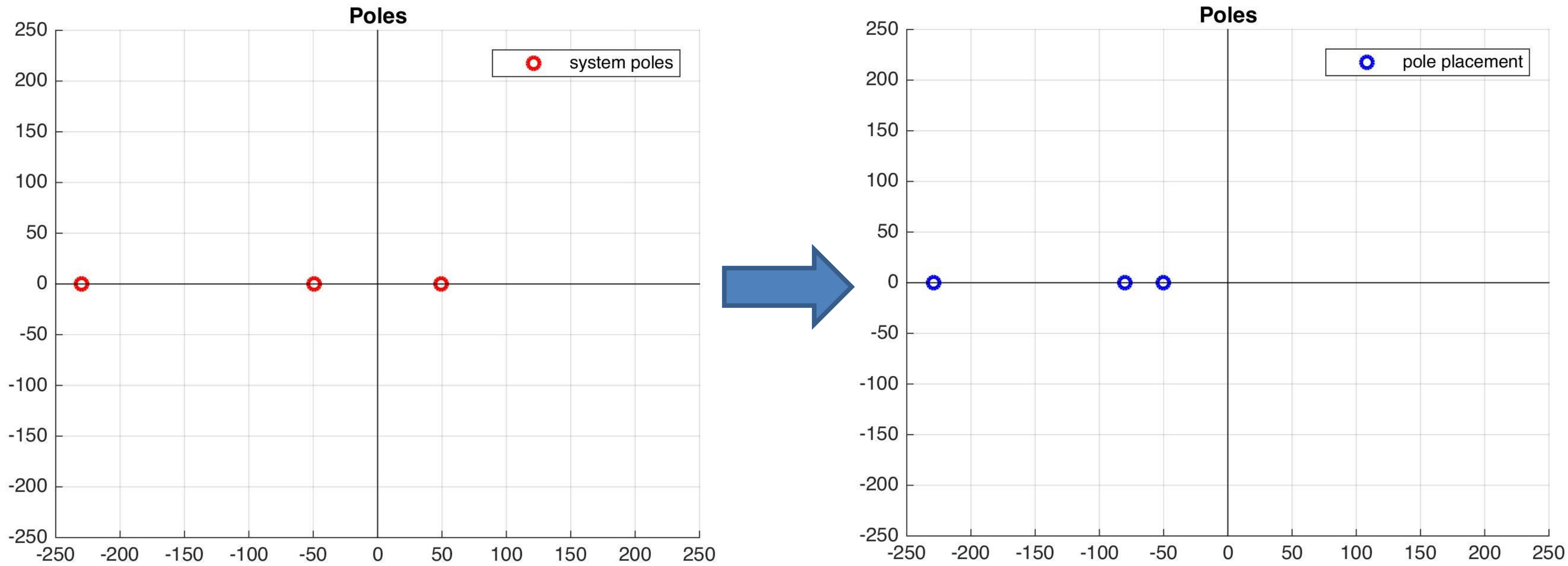


LINEAR CONTROL: CURRENT LOOP CONTROL

An external feedback loop for current control has not been implemented since the electrical and magnetic domain of the system are inherently coupled.

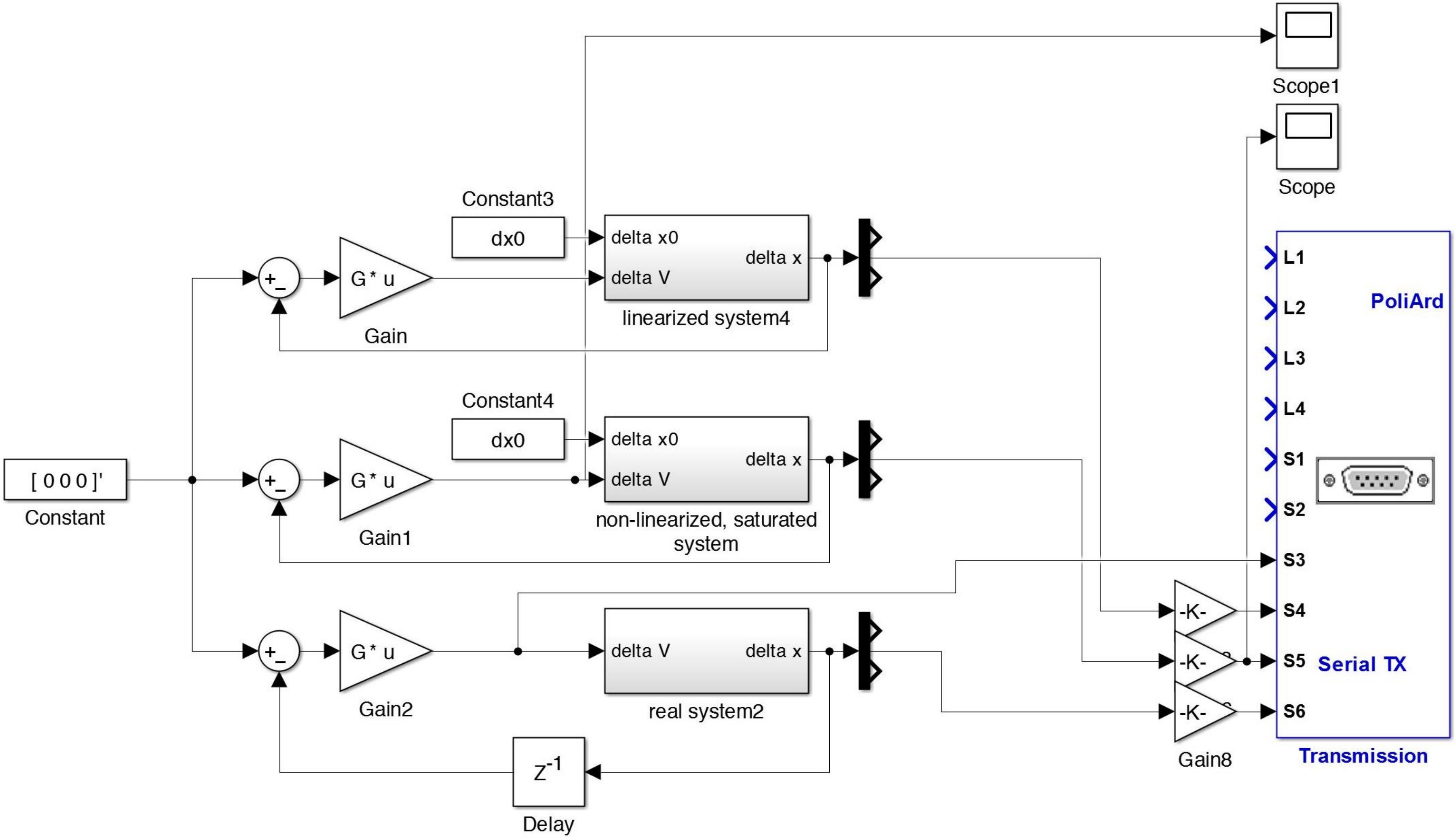


LINEAR CONTROL: DETERMINISTIC POLE PLACEMENT



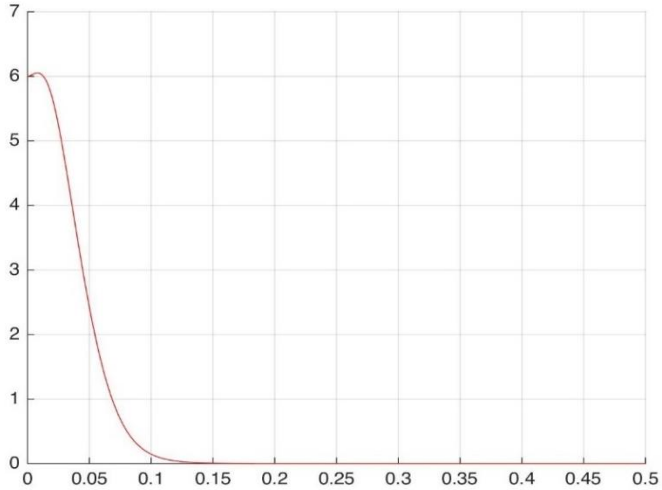
The unstable pole has been moved in the left half plane, while **the slower stable pole** of the mechanical system has been moved on the left to improve system's performances. **The fastest pole**, associated to the electrical dynamic, has not been moved. Moreover, the position of the poles has been chosen in order to minimize the control action (trial & error).

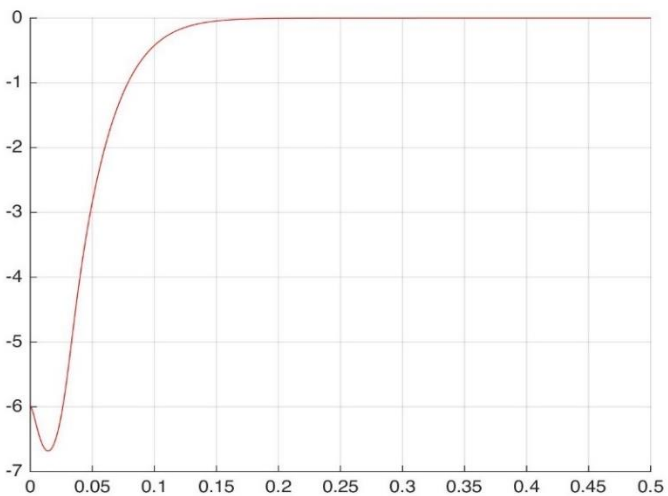
LINEAR CONTROL: DETERMINISTIC POLE PLACEMENT



LINEAR CONTROL: DETERMINISTIC POLE PLACEMENT

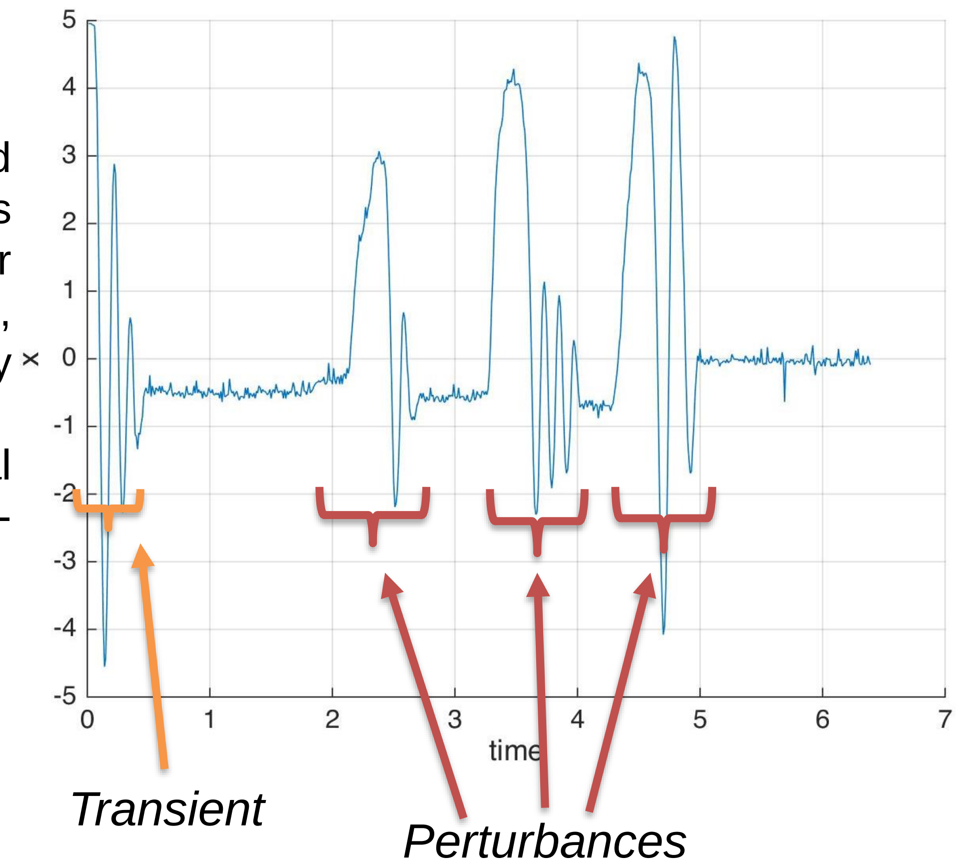
Virtual system





Max perturbed initial coditions possible for the system, achieved by simulations with the virtual system: +/- 6mm.

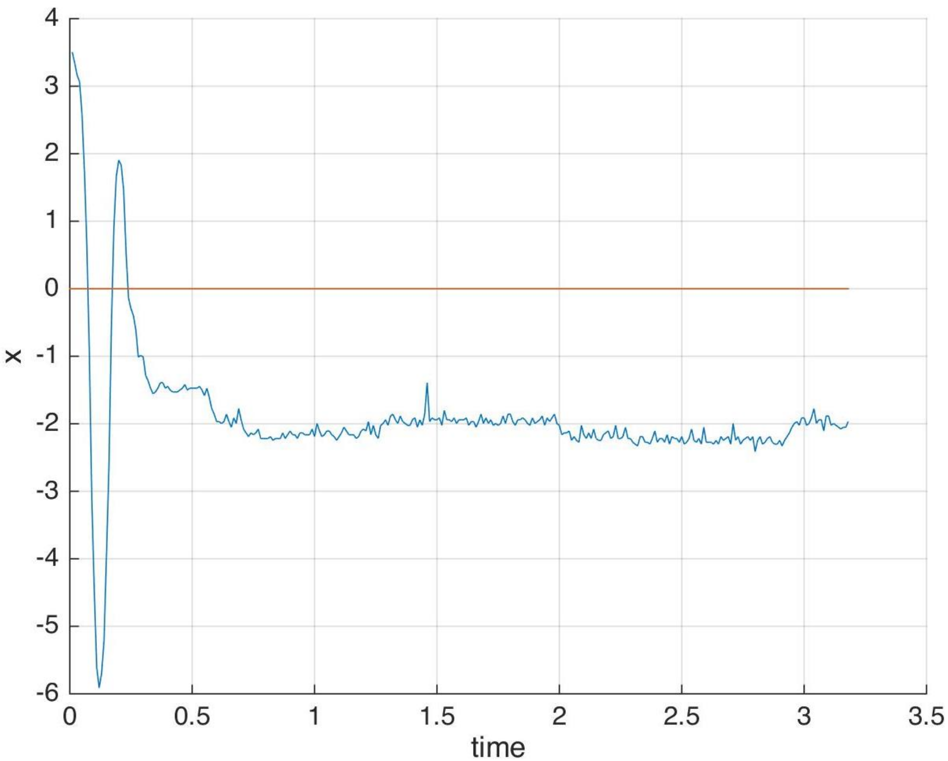
Real system

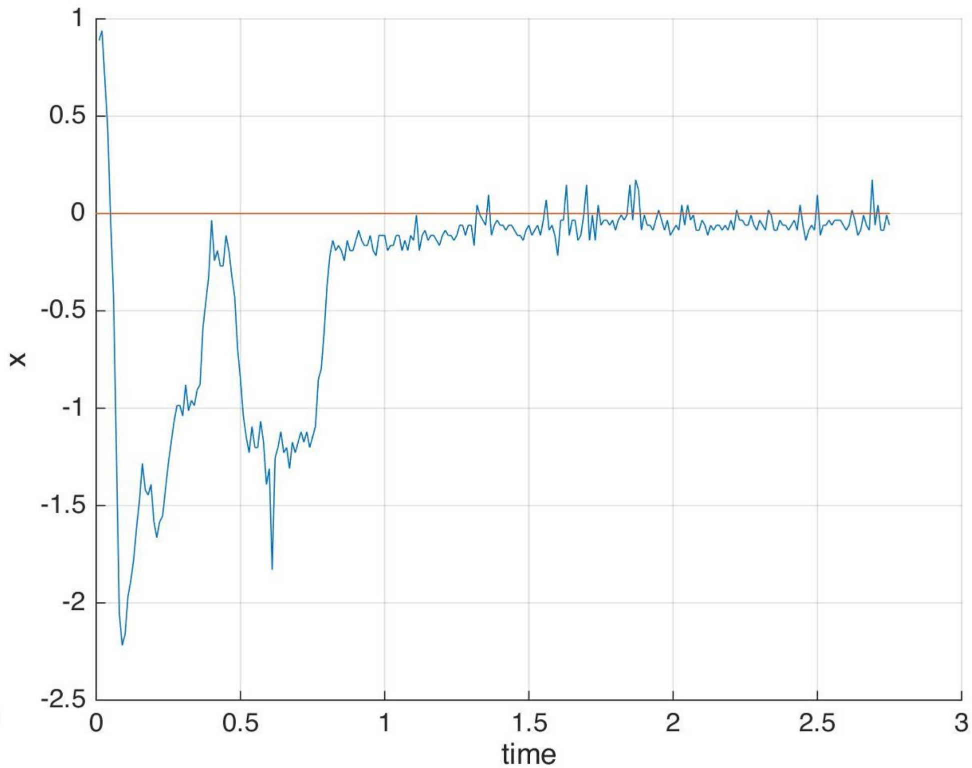


LINEAR CONTROL: KF ADJUSTATION

__Problem__: the reference position was not reached by the control action.
Kf value has been adjusted looking at steady state error.

$\rightarrow K_f = 2 * 1{,}95e - 4$





LINEAR CONTROL: LQR

Trial & error procedure in order to find suitable weigths for state and control action.

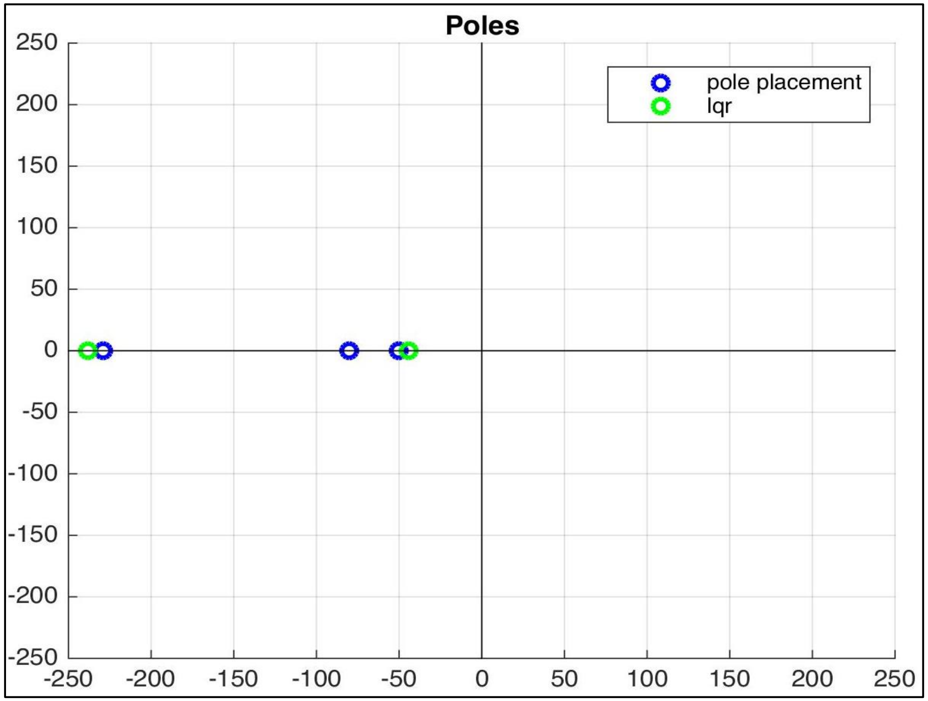
First attempt: higher weigth on the control action, but stability guaranteed only for small perturbations.

Final results:

$$R = \begin{bmatrix} 10^1 \end{bmatrix}$$

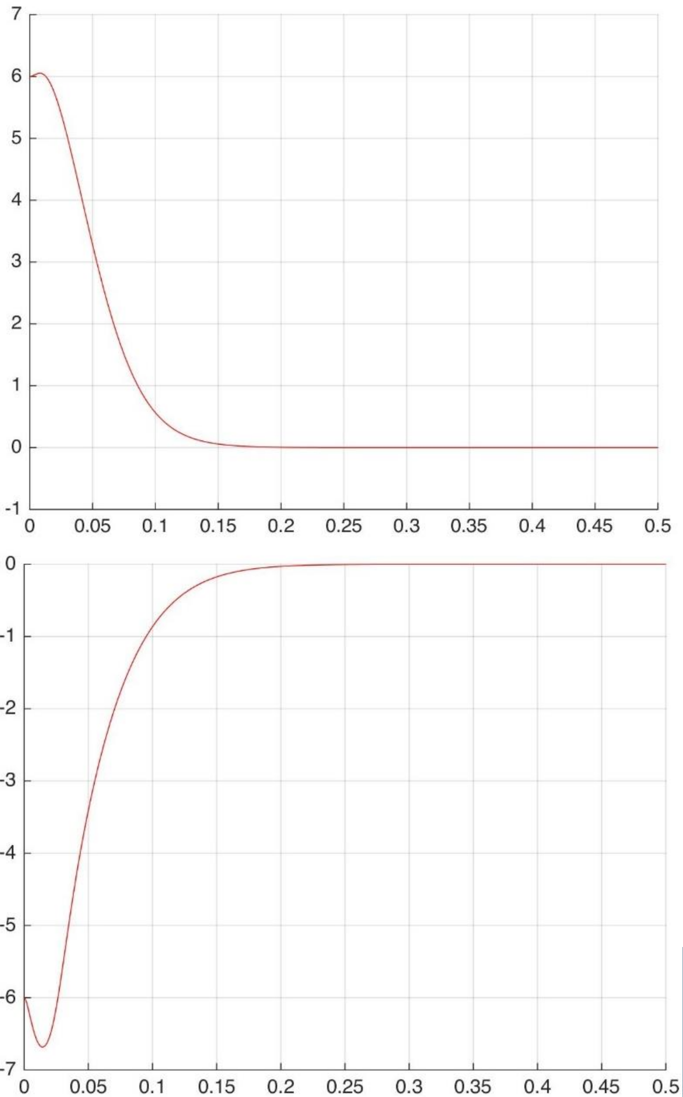
$$Q = \begin{bmatrix} 10^1 & 0 & 0 \\ 0 & 2 \cdot 10^4 & 0 \\ 0 & 0 & 10^1 \end{bmatrix}$$

$$\bar{x} = \begin{bmatrix} dx & x & i \end{bmatrix}$$

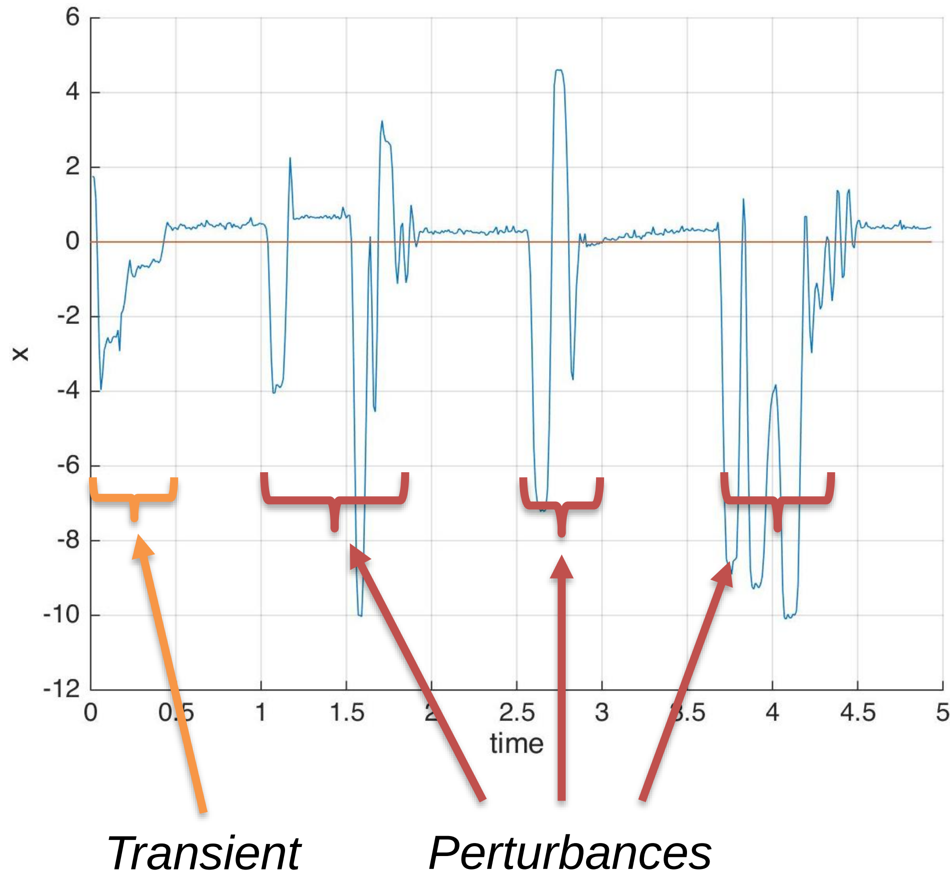


LINEAR CONTROL: LQR RESULTS

Virtual system



Real system

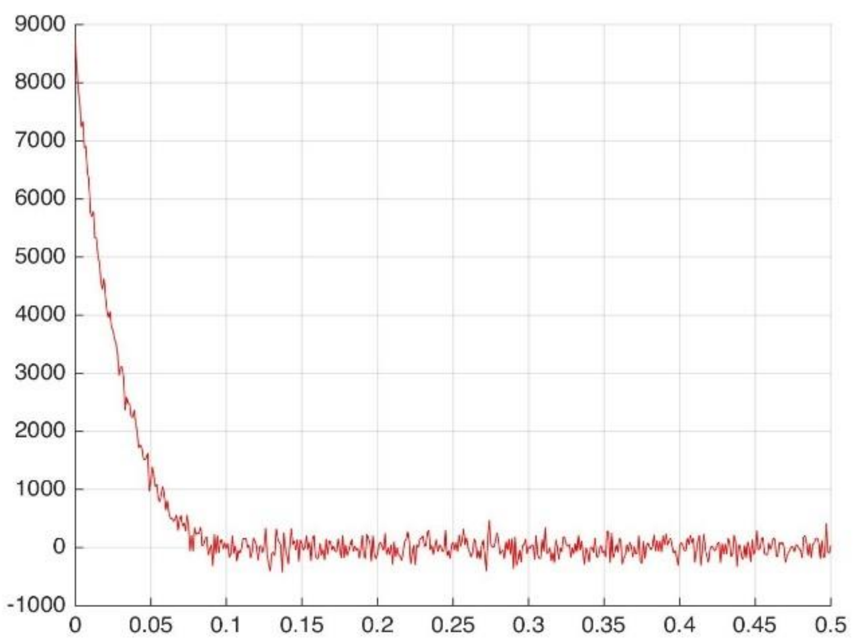


Max perturbed initial coditions possible for the system, achieved by simulations with the virtual system: +/- 6mm.

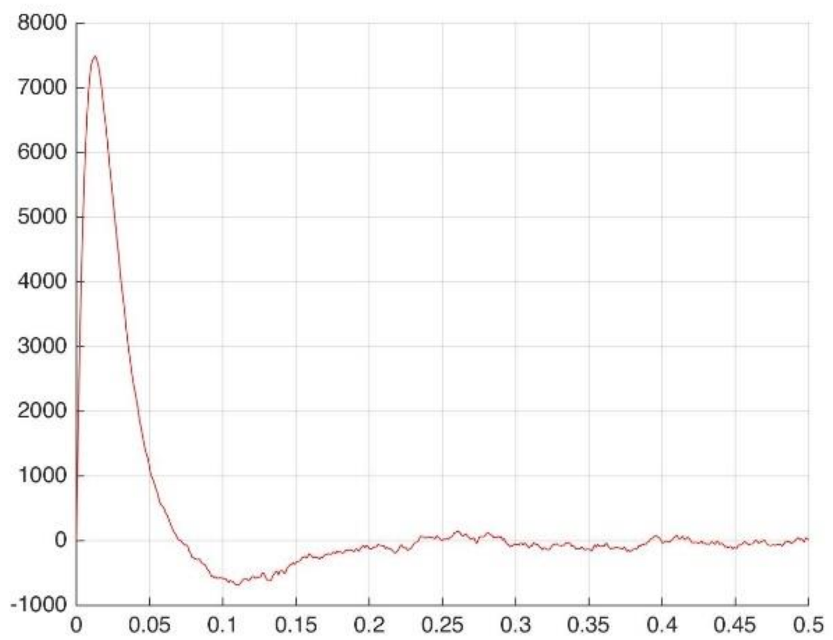
LINEAR CONTROL: EFFECTS OF KALMAN FILTER

The proximity sensor is affected by a **high frequency noise**.
Kalman filter showed good performances in the improvement of the virtual system only, while, for the real one, it showed a slower response with respect to the design for a traditional observer. Despite giving more weigth to the sensor's measurement, Kalman filter reduces the vibrations of the system but increases the instability region.

Virtual sys - Control action without KF

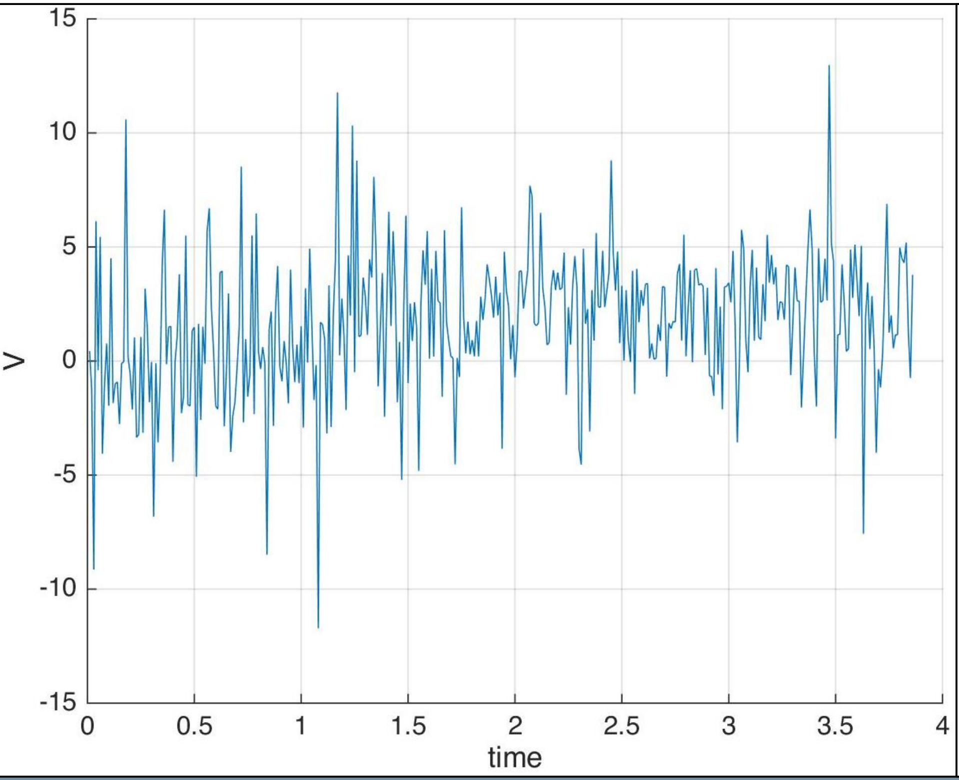


Virtual sys - Control action with KF

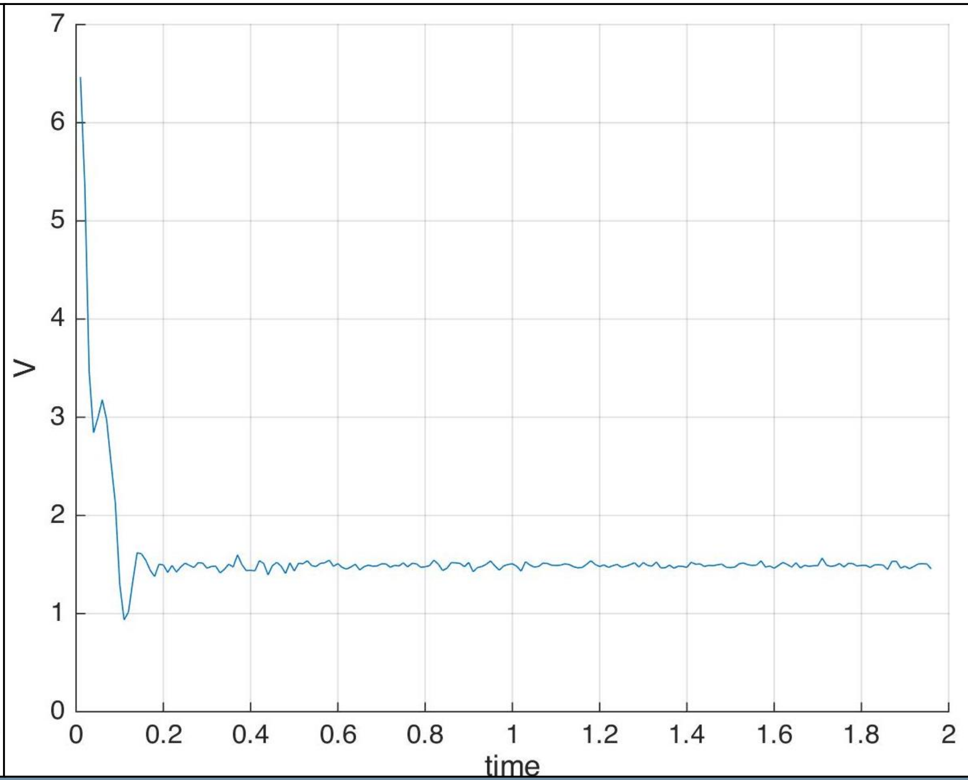


LINEAR CONTROL: EFFECTS OF KALMAN FILTER

Real sys - Control action without KF



Real sys - Control action with KF



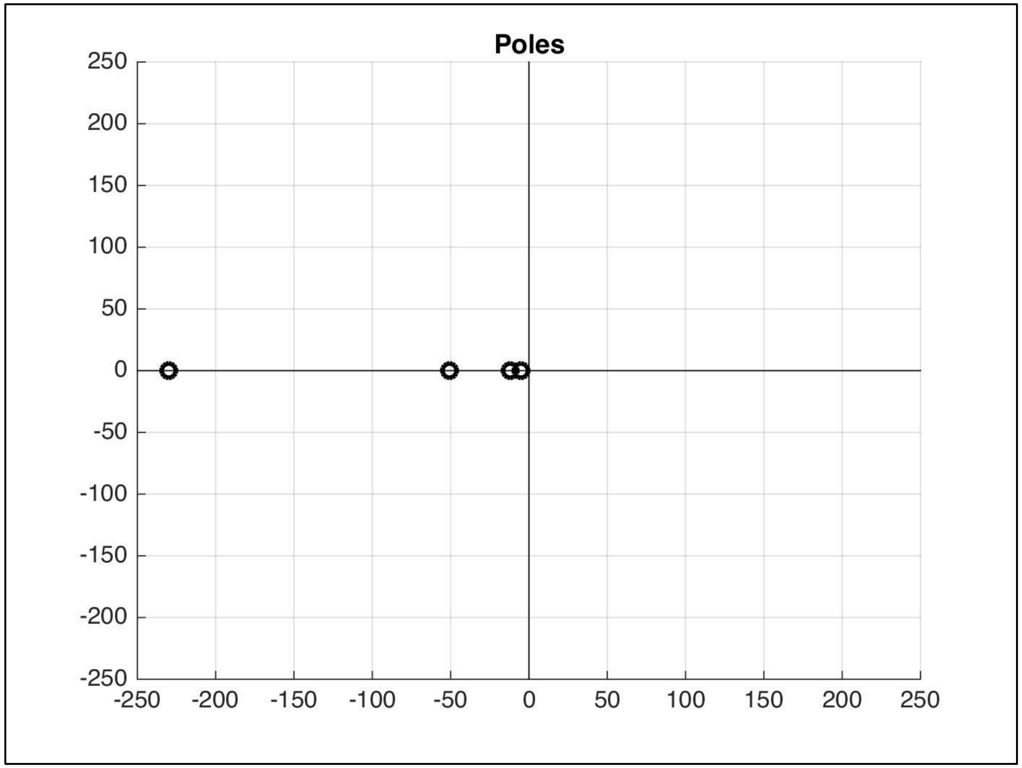
LINEAR CONTROL: LQR PERFORMANCE

LINEAR CONTROL:
POLE PLACEMENT + REFERENCE TRACKING

In order to implement the reference tracking, the state vector has been increased, resulting in an augmented state equations.

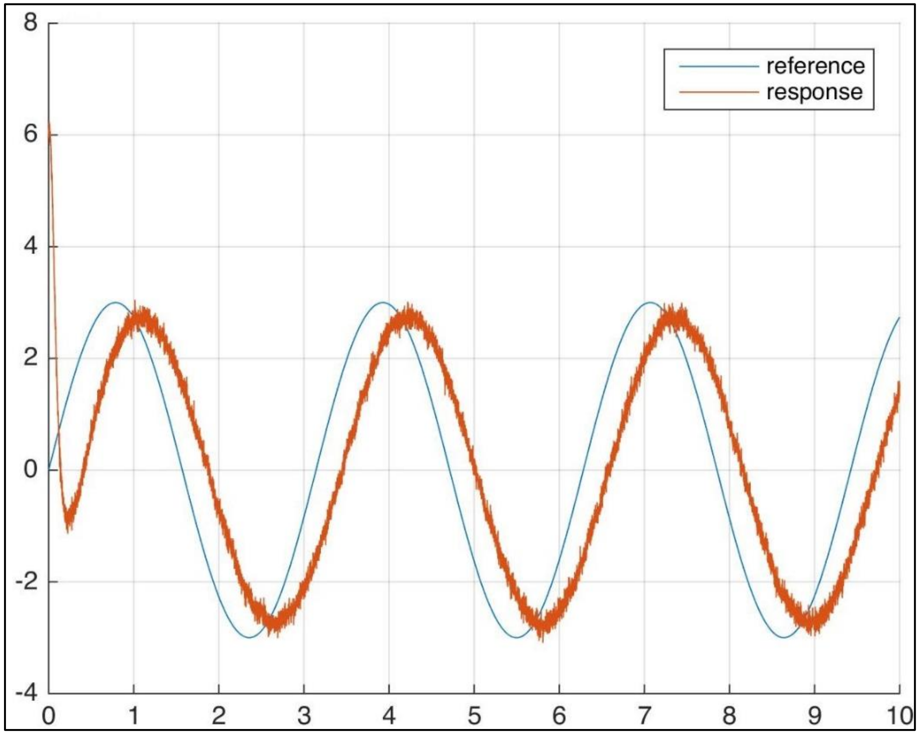
$$A_i = \begin{bmatrix} A & 0 \\ C & 0 \end{bmatrix}$$

$$B_i = \begin{bmatrix} B \\ 0 \end{bmatrix}$$

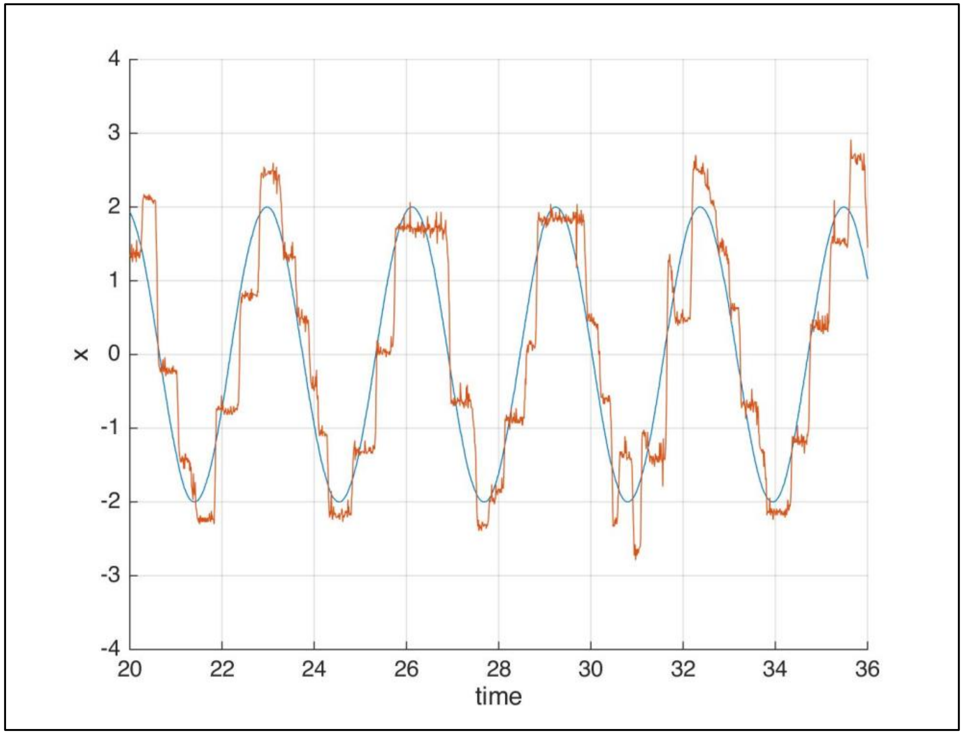


SIMO CONTROL: POLE PLACEMENT + REFERENCE TRACKING

Virtual system



Real system



Limitation of reference tracking: **signal frequency** must be not too high, **high friction** due to the cantilever configuration of the mass.

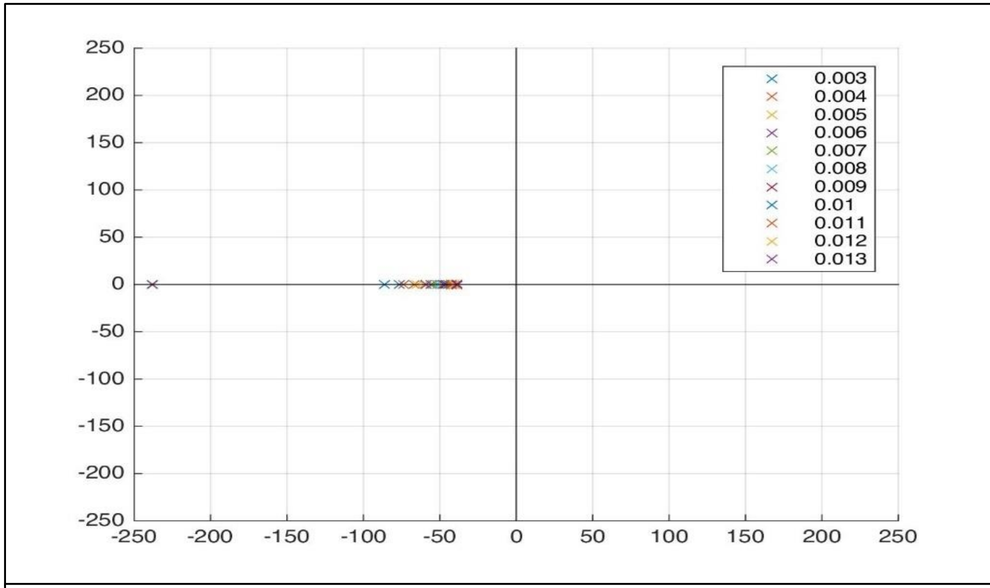
NON LINEAR CONTROL:
GAIN SCHEDULING

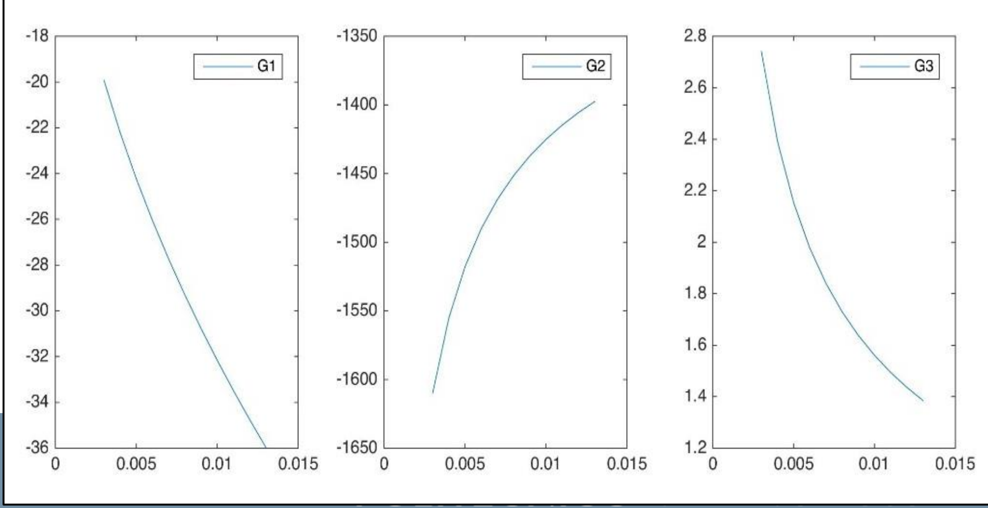
GAIN SCHEDULING

Since the system is **highly non linear**, it makes sense to try to use a non linear control logic like gain scheduling.

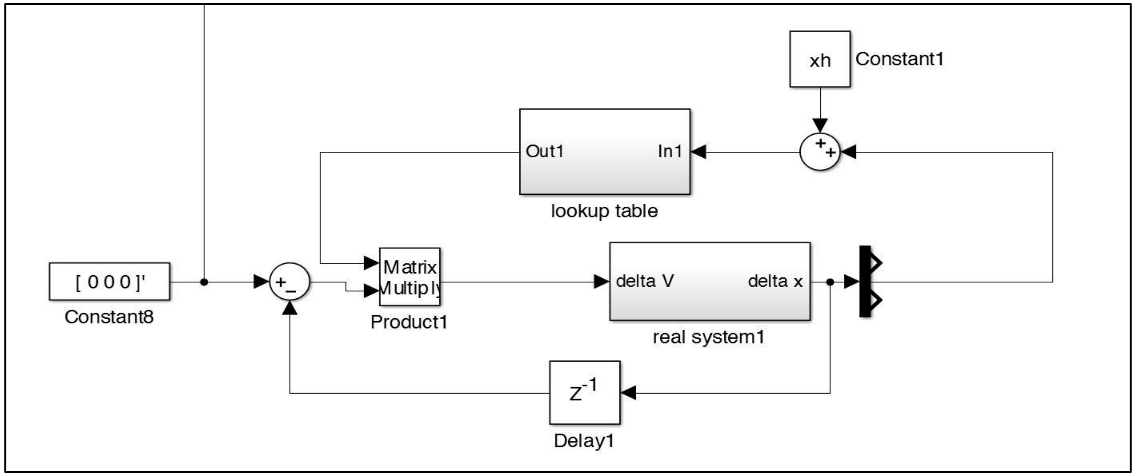
Procedure:

1. The system has been linearized around **a number of equilibrium positions**;
2. For each position the LQR control logic has been applied, resulting in **a gain matrix for each position**;
3. These gain matrices have been placed into a **look-up table** which allows the system to correctly select the gain regarding to current state;
4. The resultant model is thus a **linear time variant** (LTV) system.

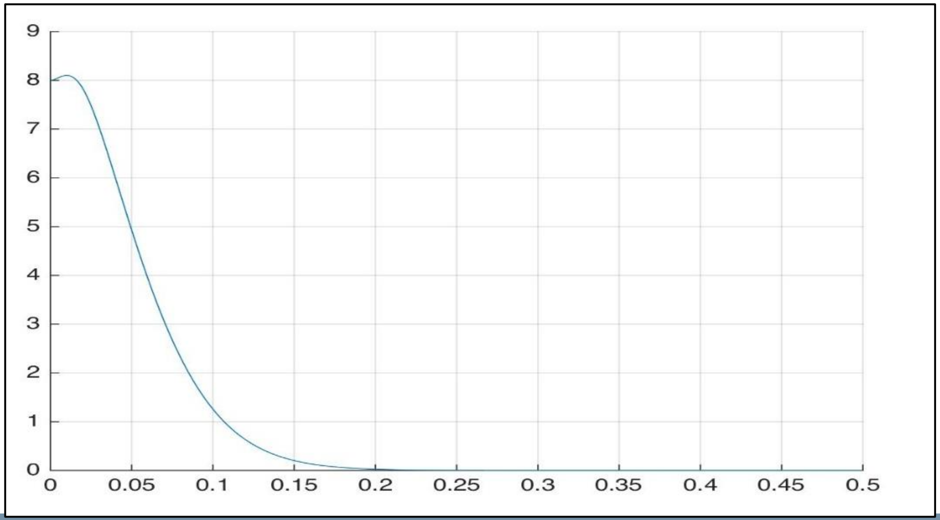


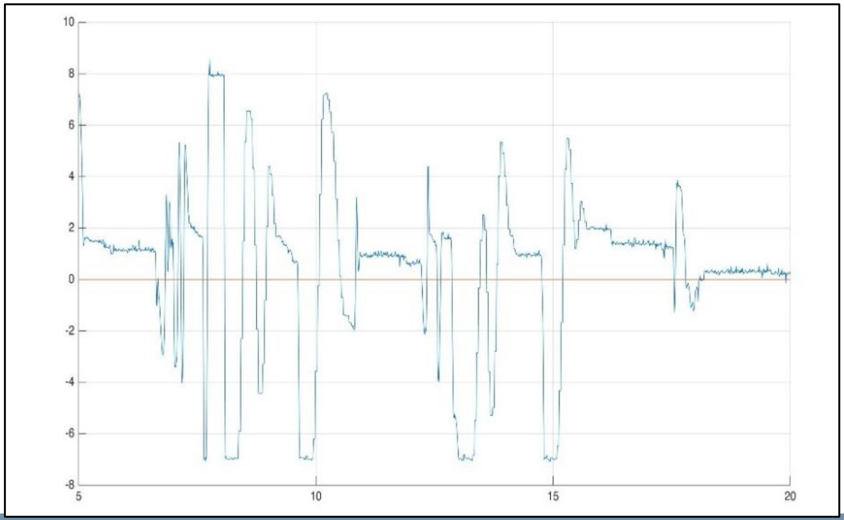


GAIN SCHEDULING

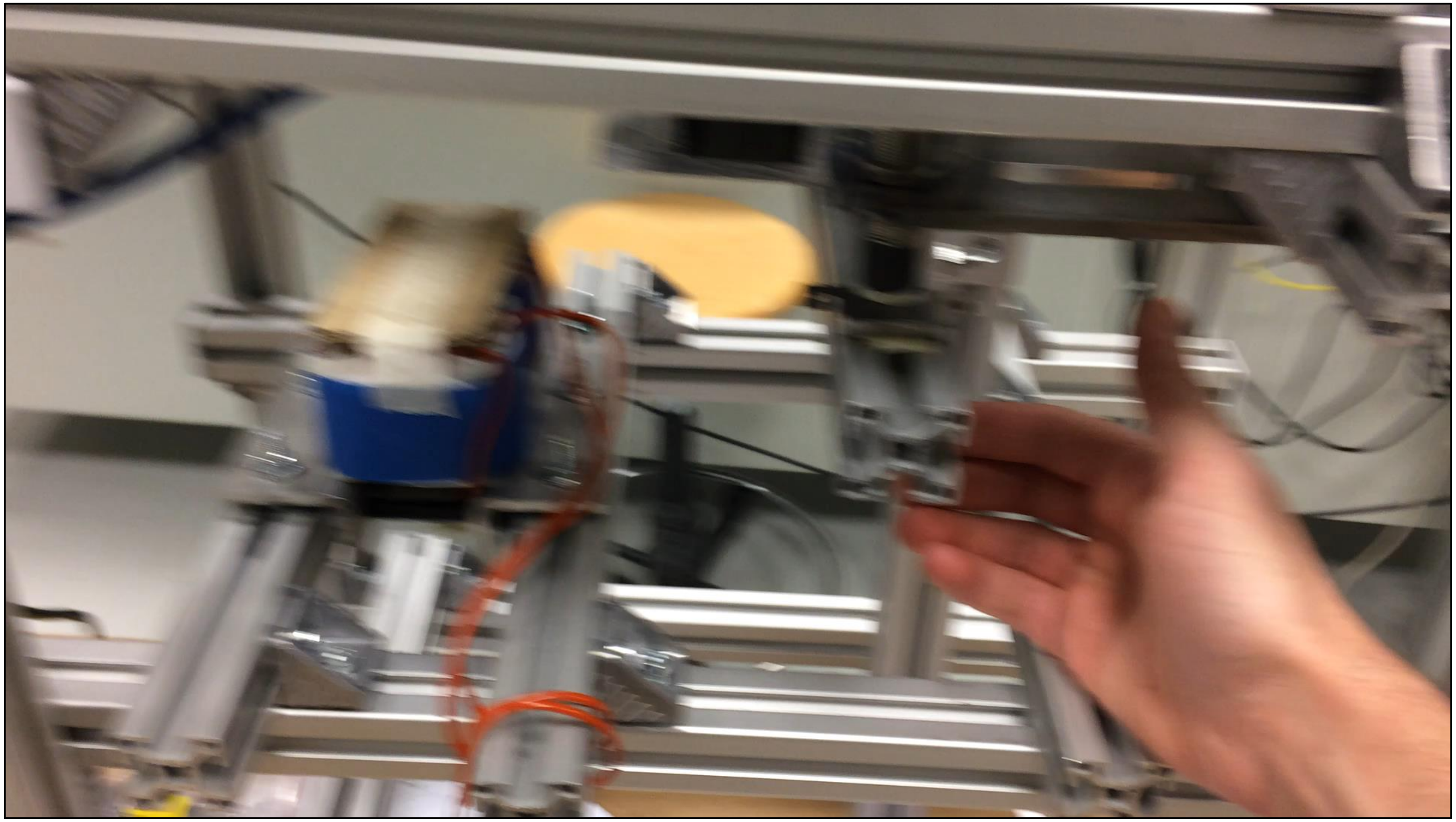


Here it is possible to see the scheme of the **control logic** and the **response of the virtual and the real system**.

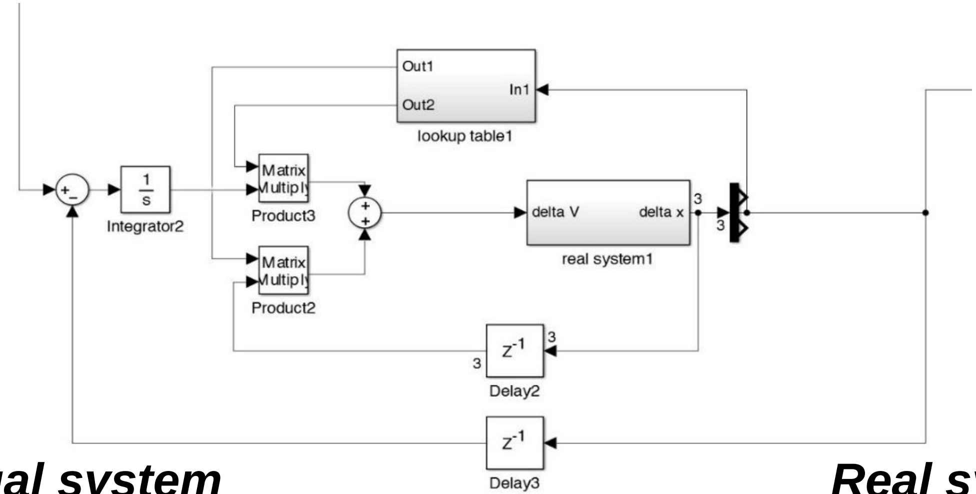




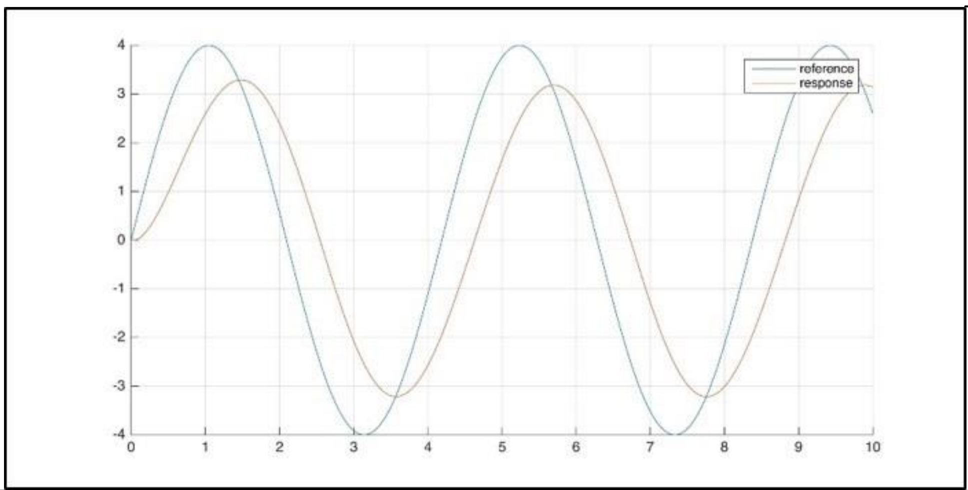
GAIN SCHEDULING - PERFORMANCE



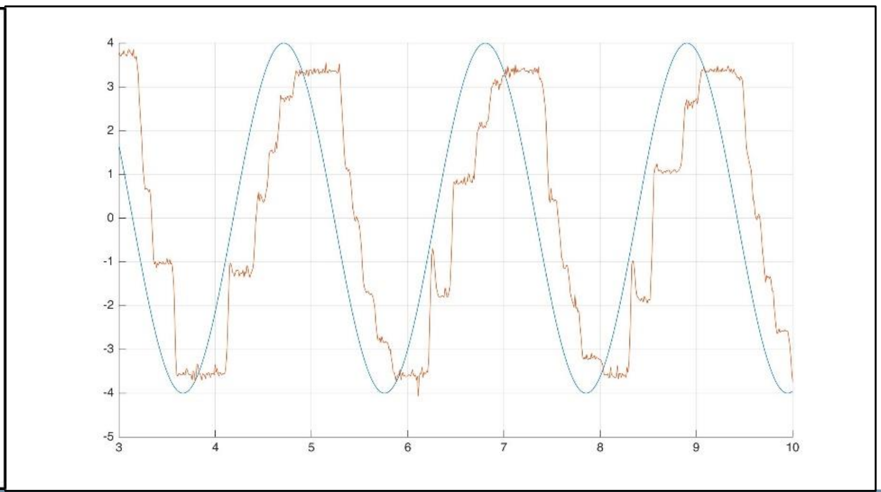
GAIN SCHEDULING FOR REFERENCE TRACKING



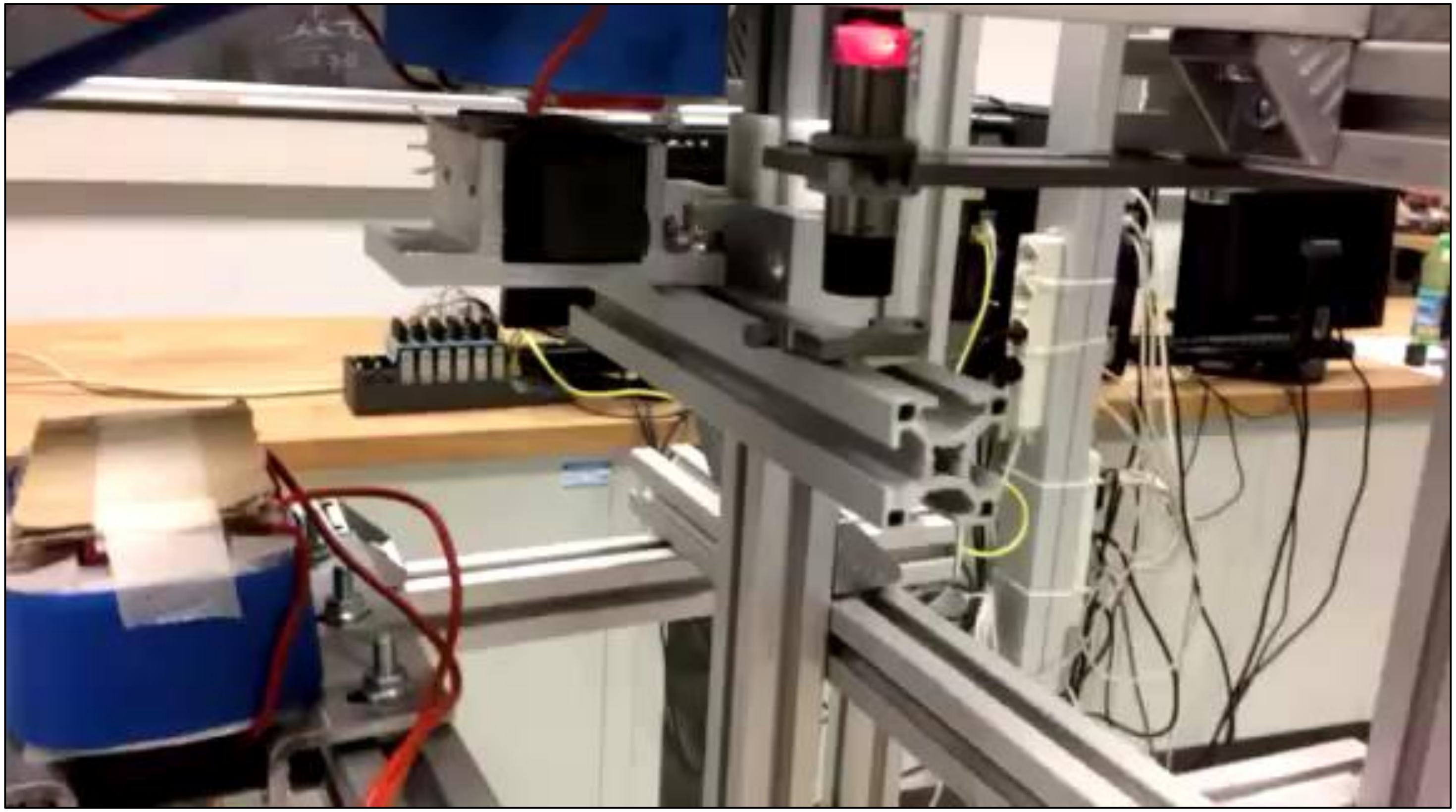
Virtual system



Real system



GAIN SCHEDULING FOR REFERENCE TRACKING – PERFORMANCE



Thanks for the attention!