Lagrangian and Eulerian Multi-Scale Control of a Distributed Multibody Robotic System

M.Sc Thesis in Mechanical Engineering

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JET PROPULSION LABORATORY

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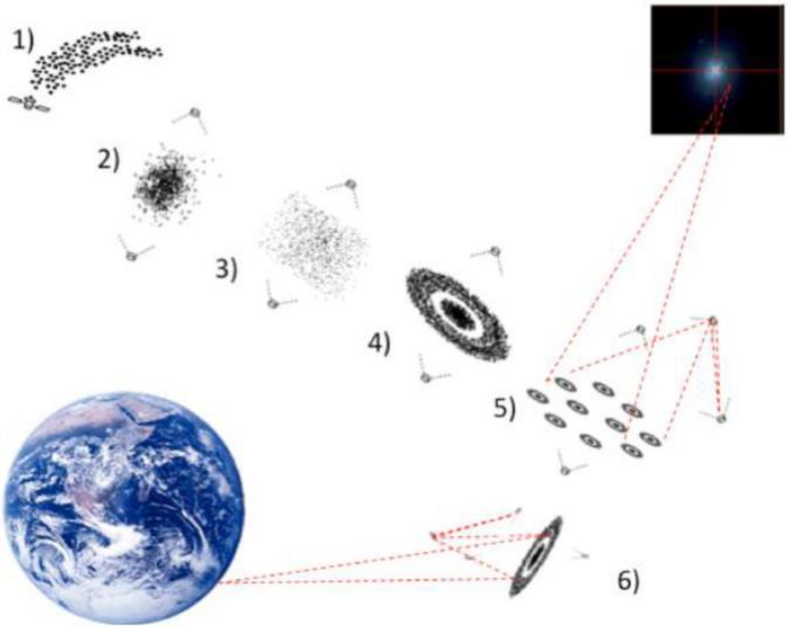
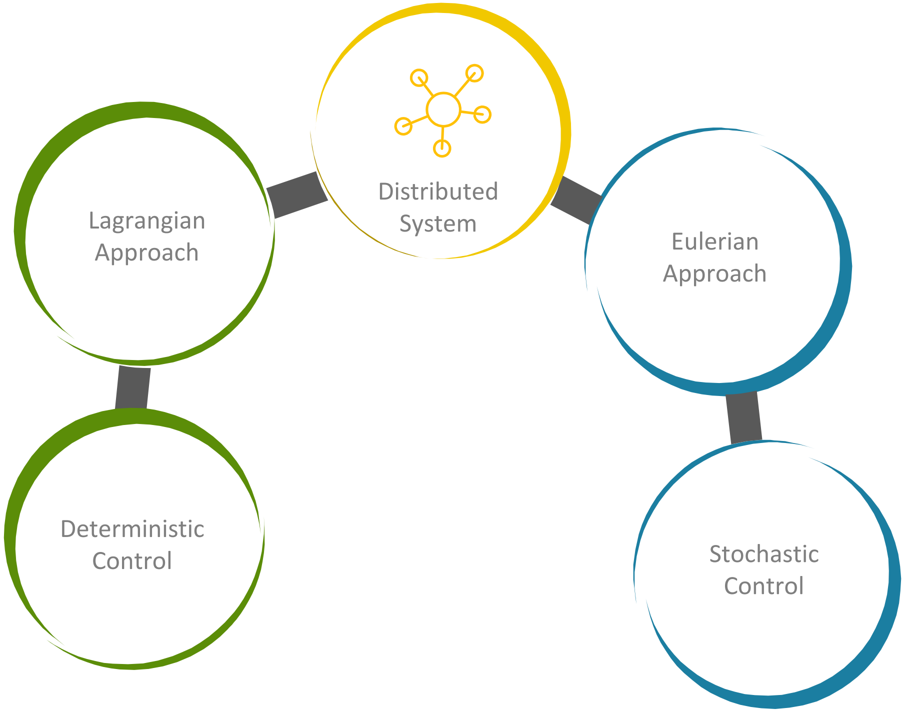
DISTRIBUTED MULTIBODY SYSTEM

- It is a **multibody** system composed of a **large number** of non-contacting **elements.**
- Its modeling is more challenging compared to conventional spacecrafts, since we are dealing with a **probabilistic vehicle**.
- Its advantages include: **lightness** and reduced **costs**, very large and **recofigurable** structure, ease of packaging and deployment, high **fault tolerance** with reduced vulnerability to impacts and losses.



HYBRID CONTROL OF A DISTRIBUTED SYSTEM

- A distributed system can be seen as a **fluid** in motion, widening the modeling possibilities beyond a single approach.
- A **Lagrangian** rather than **Eulerian** framework can be dictated by the utilisation circumnstances or by the specific requirements.
- An **Hybrid Control** formulation could grant higher **flexibility**, and aim at the concrete realization of a distributed system.



LAGRANGIAN APPROACH FOR THE CONTROL OF A DISTRIBUTED SYSTEM

VERY LARGE SCALE ROBOTIC (VLSR) SYSTEM

- Very Large Scale Robotic (**VLSR**) systems are composed of hundreds to possibly tens of thousands of small-scale autonomous robots.

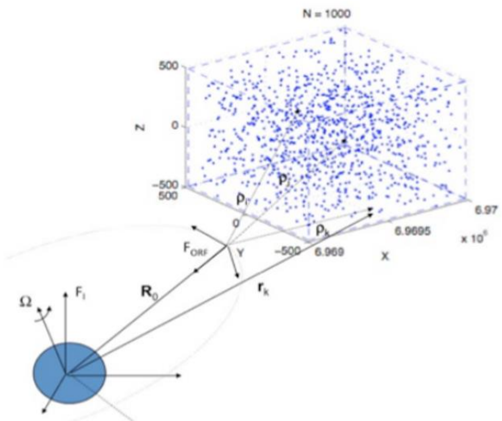
- Ground applications for industrial assembling, exploring or patrolling tasks, or in military field applications.

- Space applications for creating **large-aperture** and **reconfigurable** observatories from confined **swarms** of robots.

DISTRIBUTED SYSTEM MODELING

MAIN ASSUMPTIONS :

- System composed of **N rigid bodies**, orbiting around an Earth-centered inertial (**ECI**) frame.
- The formation dynamics is described (and numerically integrated) with respect to an Orbiting Reference Frame (**ORF**).
- The ORF follows a geostationary orbit.
- The rotational quantities are expressed in **quaternion** form and are referred to the ECI reference frame.



$$\mathbf{x} = \left\{ \begin{array}{c} \boldsymbol{\rho}_i \\ \boldsymbol{q}_i \\ \dot{\boldsymbol{\rho}}_i \\ \boldsymbol{\omega}_i \\ \cdots\cdots \\ \boldsymbol{R}_0 \\ \boldsymbol{V}_0 \\ \cdots\cdots \\ \boldsymbol{\omega}_{wi} \\ \boldsymbol{\omega}_{wi}^{Ref} \end{array} \right\}$$

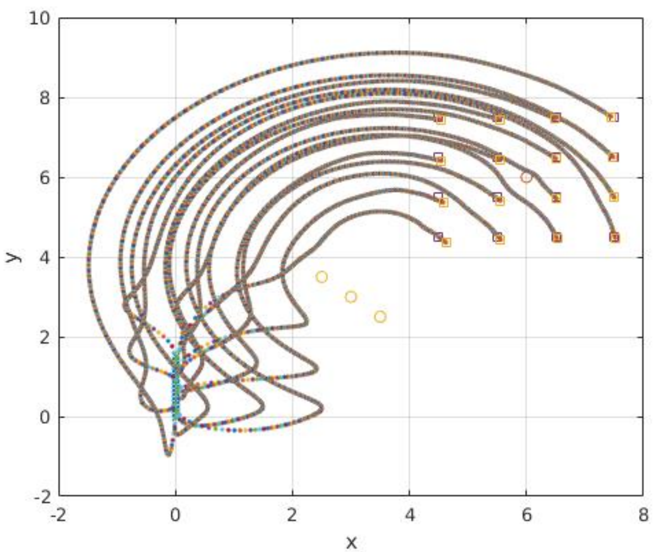
Translational and Rotational Dynamics (i-th agent)

$$\begin{cases} m_i \ddot{\boldsymbol{\rho}}_i = -m_i \mathcal{K}_{orb} \boldsymbol{\rho}_i - 2 m_i \boldsymbol{\Omega} \times \dot{\boldsymbol{\rho}}_i + \boldsymbol{f}_i + \boldsymbol{u}_i \\ \boldsymbol{J}_i \dot{\boldsymbol{\omega}}_i + \boldsymbol{\omega}_i \times (\boldsymbol{J}_i \boldsymbol{\omega}_i + \boldsymbol{h}_i) = \boldsymbol{g}_i + \boldsymbol{\tau}_i \end{cases}$$

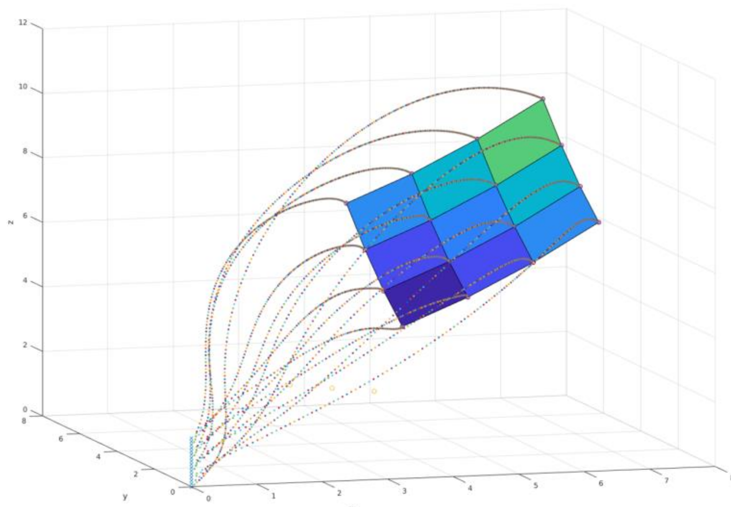
POTENTIAL FIELDS BASED TRANSLATIONAL CONTROL

Potential Fields are based on the linear **spring law** formulation that allows to define a target configuration for which the elements will be in equilibrium. The bodies will act as if immersed in a virtual force field, reordering according to the **spring graph**.

$$F_i = \sum_{(i,j)\in E} F_{ij} = \sum_{(i,j)\in E} k_{ij}(r_{ij} - l_{ij})\frac{p_j - p_i}{r_{ij}}$$



2D Deployment Problem

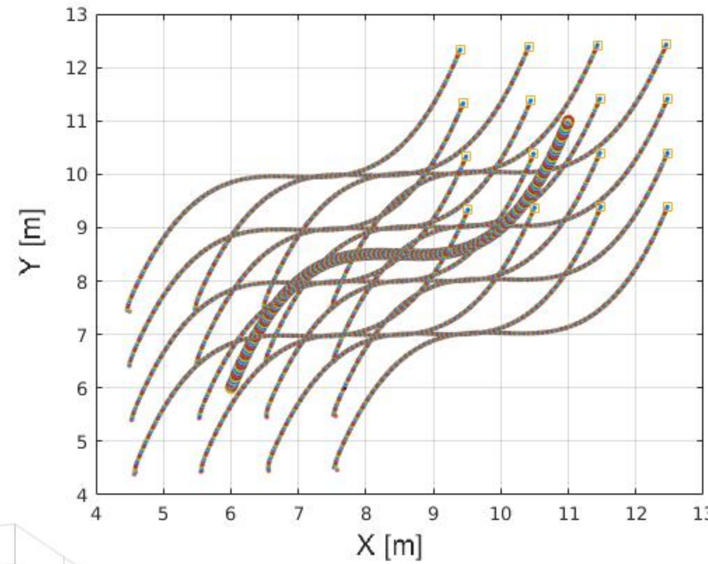


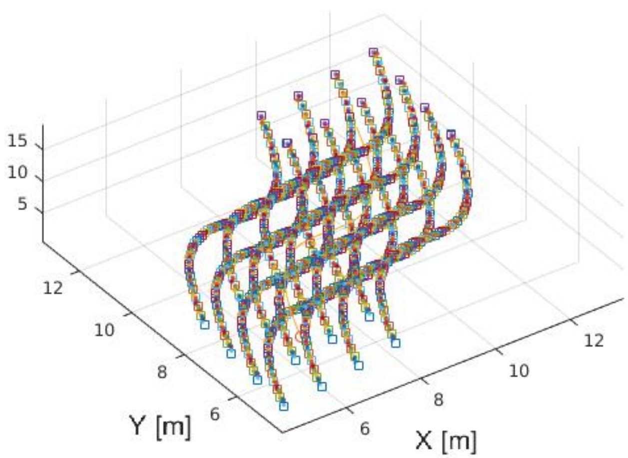
3D Deployment Problem

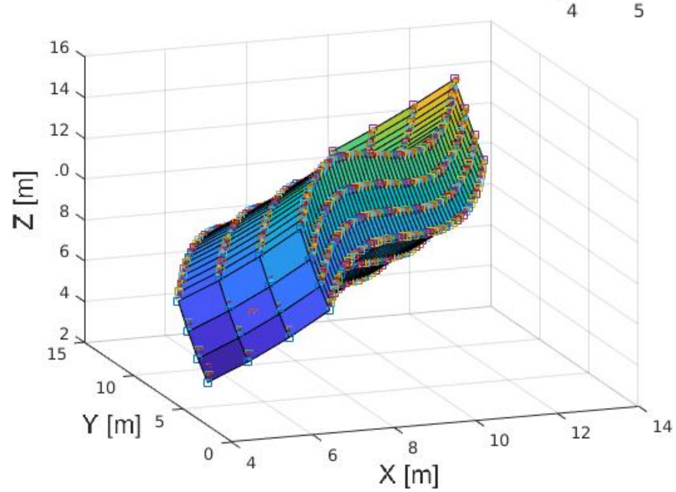
POTENTIAL FIELDS BASED TRANSLATIONAL CONTROL

MACRODYNAMICS: Rigid Movement of the Swarm

Once the structure is deployed, a trajectory can be generated for a few **landmark robots**, so that all the other ordinary robots will move accordingly, based on the spatial informations contained in the **spring graph**.







This is useful when the deployed swarm needs to be **rigidly** retargeted and oriented.

INDIVIDUAL LAGRANGIAN-BASED CONTROL

POTENTIAL FIELDS BASED TRANSLATIONAL CONTROL

- **Simple** implementation based on the linear spring law formulation.
- **Decentralized** algorithm, having collision avoidance characteristics.
- Defined with respect to the **ORF**.

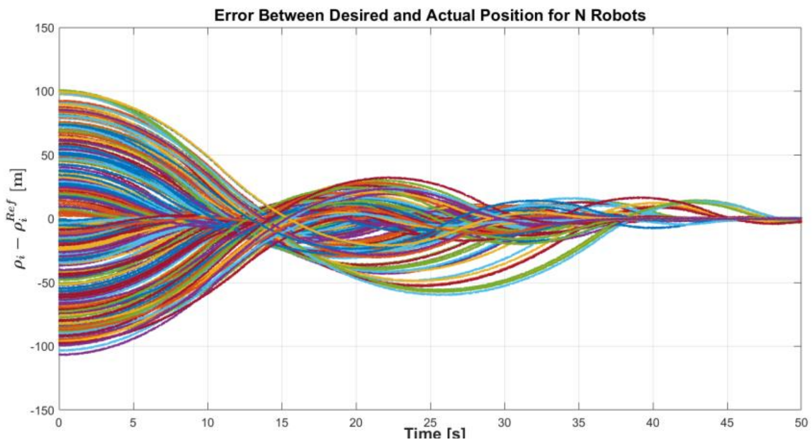
QUATERNION BASED ATTITUDE CONTROL

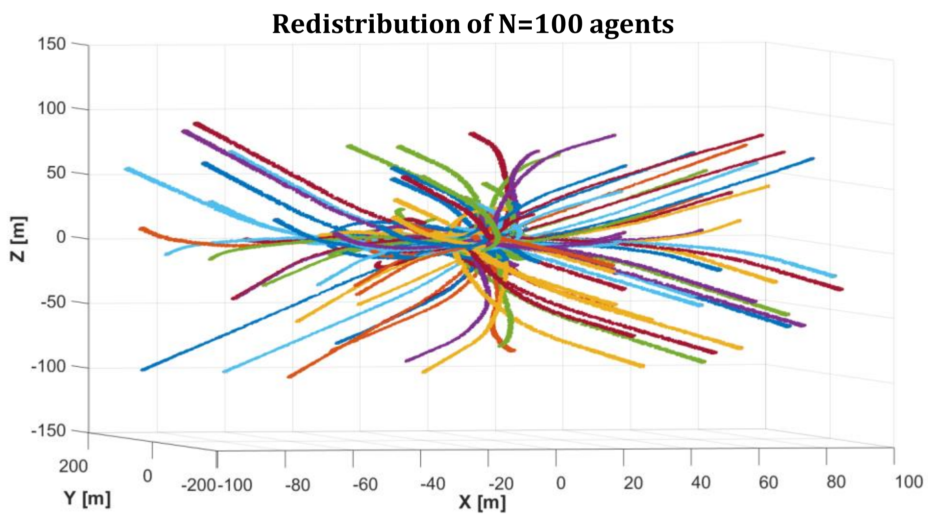
- **Quaternions** are defined with respect to the **inertial** reference frame.
- Asymptotically stable attitude control formulation.

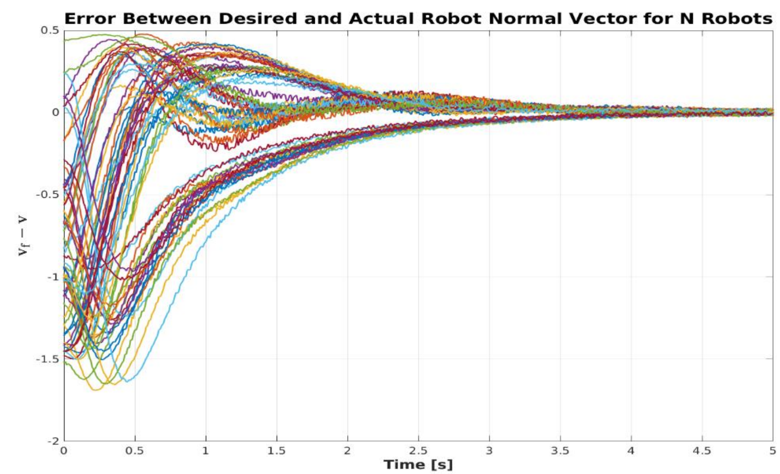
$$\boldsymbol{\tau}_i = \boldsymbol{J}_i^b \dot{\boldsymbol{\omega}}_i^{ref} + \boldsymbol{K}_\omega \boldsymbol{\omega}_{e,i} + \boldsymbol{K}_q \boldsymbol{q}_{e,i} + \boldsymbol{\tau}_{gyro,i}(\boldsymbol{q}_i, \boldsymbol{\omega}_i) - \hat{\boldsymbol{\tau}}_{ext,i}$$

NUMERICAL SIMULATIONS OF THE RECONFIGURATION PROBLEM

By applying the defined controllers on a swarm of N=100 agents, convergence is reached both in **position** and **orientation** to the desired target values.

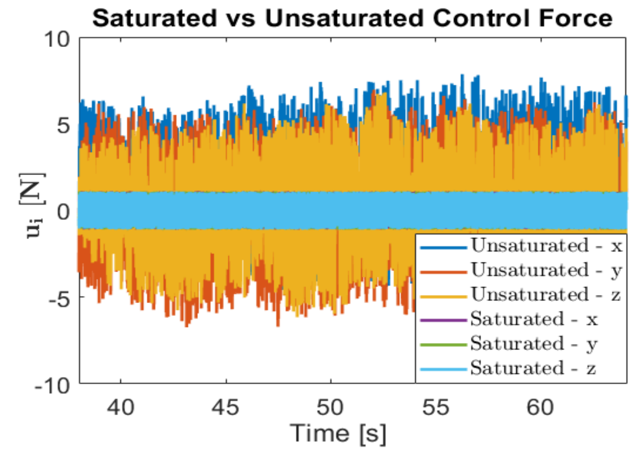




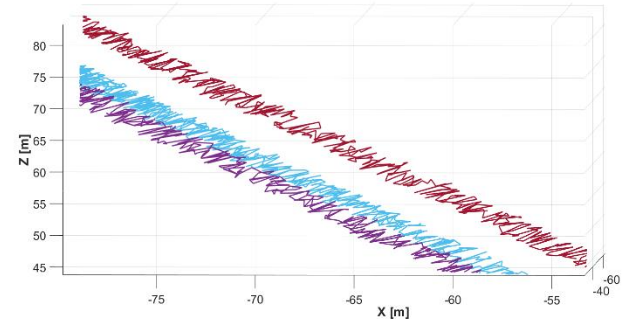


DOWNSIDES OF THE POTENTIAL FIELDS CONTROL

- Enlarging the **swarm dimensions**, the potential fields forces exerted among the particles increase, **saturating** the actuation capabilites.



- The precision loss due to the addition of **random noises** and unmodeled **disturbances** acting on the system degrades the quality of the control design.



- The limited **actuation strenghts** and **autonomies** of the small satellites must be considered, thus introducing **fuel efficient** trajectories.

The necessity for an alternative arises.

MODEL PREDICTIVE CONTROL APPROACH

Introduces a decentralized **real-time** control architecture, capable of individuating the **optimal trajectories** that **minimize** an **objective** function while satisfying some **constraints**. This is based on the **Model Predictive Control** (**MPC**) formulation, using a receding time horizon T_H, and adopting **Sequential Convex Programming** (**SCP**) in order to treat the non-convex problem.

Nonlinear Optimal Control Problem → Discretized and Decentralized (Approximated) Convex Problem → MPC-SCP Convex Optimization

MPC-SCP CONVEXIFIED AND DISCRETIZED PROBLEM

Convexified and Discretized Optimization Problem

$$\min_{\mathbf{u}_j} \sum_{k=k_0}^{T-1} ||\mathbf{u}_j[k]||_1 \Delta t$$

$$subject\ to$$

$$\mathbf{x}_j[k+1] = A_j[k]\mathbf{x}_j[k] + B_j[k]\mathbf{u}_j[k] + c_j[k], \quad k = k_0, ..., T-1 \quad j = 1, ..., N$$

$$||\mathbf{u}_j[k]||_\infty \leq U_{max} \qquad k = k_0, ..., T-1, \qquad j = 1, ..., N$$

$$\mathbf{x}_j[0] = \mathbf{x}_{j,0} \qquad j = 1, ..., N$$

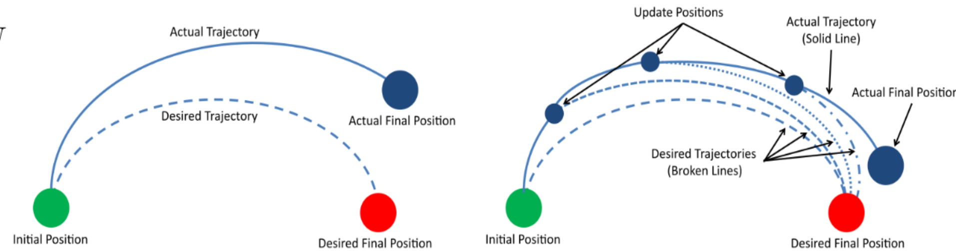
$$\mathbf{x}_j[T] = \mathbf{x}_{j,f} \qquad j = 1, ..., N$$

$$(\bar{\mathbf{x}}_j[k] - \bar{\mathbf{x}}_i[k])^T G^T G(\mathbf{x}_j[k] - \bar{\mathbf{x}}_i[k]) \geq R_{col}||G(\bar{\mathbf{x}}_j[k] - \bar{\mathbf{x}}_i[k])||_2,$$

$$k = k_0, ..., T \qquad i \in \mathcal{N}_{[j]}, \quad j = 1, ..., N-1$$

which requires a **linearization** of the system defined before.

SCP algorithm uses the **new trajectories** identified in the optimization problem as **nominal** ones in the following iteration step, until convergence.



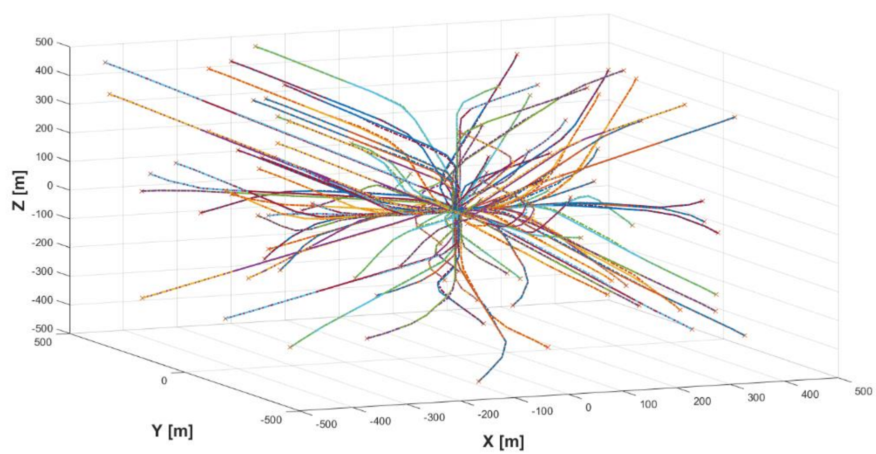
a) Trajectory from running the SCP algorithm at the initial time only

b) Trajectory from running the MPC implementation of the SCP algorithm

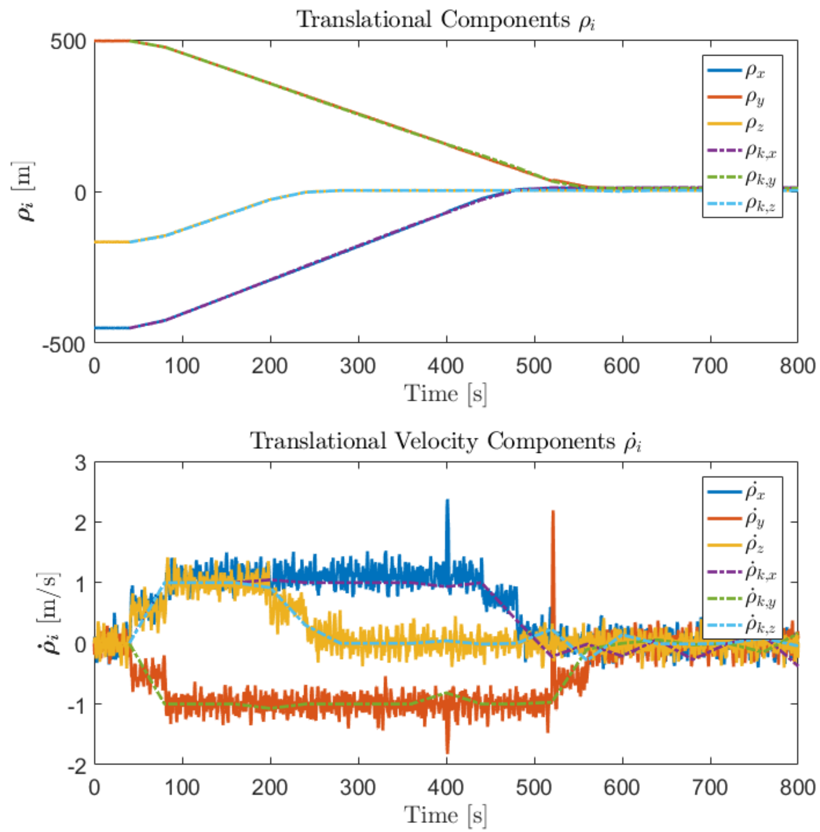
The **MPC** approach exploits the new measurements of the real system in order to provide a **robust** control strategy against unmodeled **disturbances** and **errors**.

MPC APPLICATION ON THE NONLINEAR SYSTEM

Considering N=100 agents, and relevant noises affecting the system, we expect an increase in **computational times** mainly due to the increasing number of **collision avoidance constraints**.



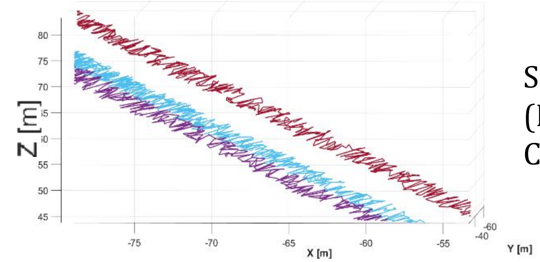
Swarm reconfiguration in the workspace



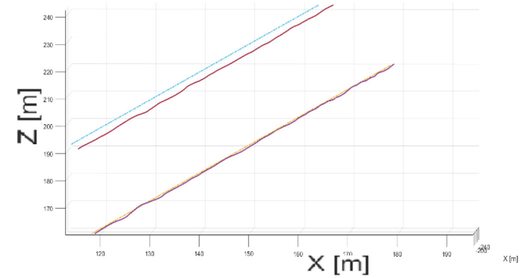
i-th agent position and velocity convergence

IMPROVEMENTS DUE TO THE MPC-SCP INTRODUCTION

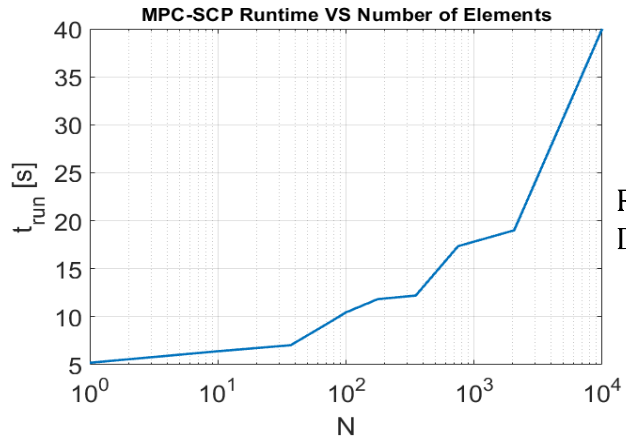
- **Minimization** of the **control** efforts required to reorder the elements
  - ↳ Important for the limited **autonomies** of the nano and **femtosats**.
- Higher **precision** in the **reference tracking** problem, caused by the reduced fuel consumption.
- Improved **robustness** against external unmodeled disturbances and parameter uncertainties or noises.
- **Scalability** is still poor with respect to the **increase** in the cloud **dimensions**.



Spatial trajectories (Potential Fields Control).



Spatial trajectories (MPC-SCP Control).



Runtime vs Dimension

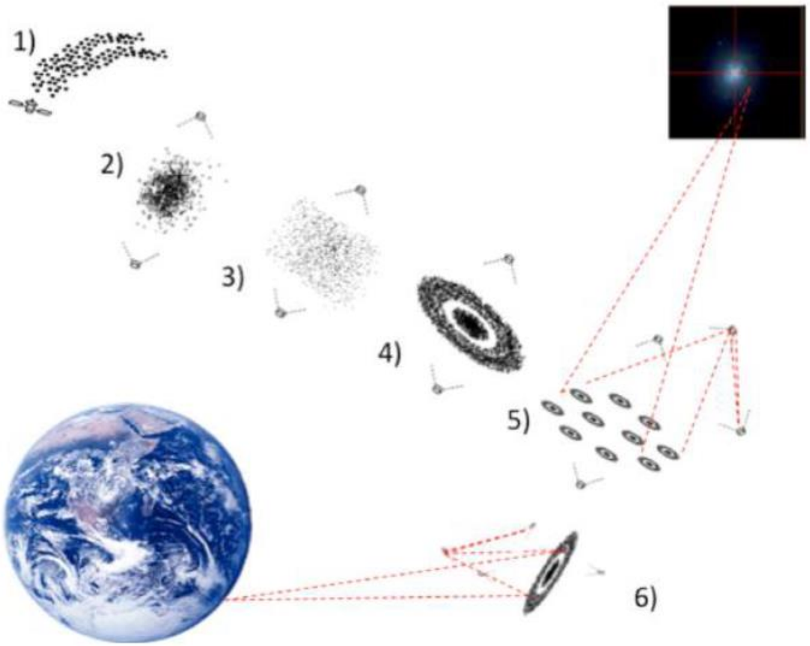
EULERIAN APPROACH FOR THE CONTROL OF A DISTRIBUTED SYSTEM

ORBITING RAINBOWS

- Orbiting Rainbows is an example of distributed **non-contacting granular media** forming a space-based observatory.

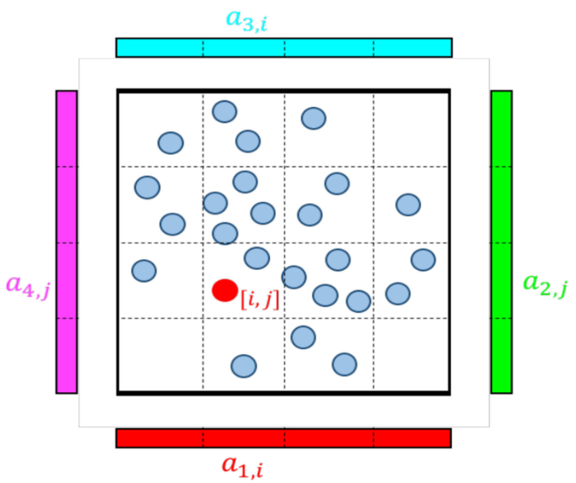
- The dimensions of the charged grains of dust are in the order of microns.

- If N becomes consistent, a continuum-based **Eulerian** framework is often preferred, where the collective properties of the swarm (e.g., its **probability distribution**) are controlled.



OPTIMAL TRANSPORT

- The **passive** nature of the particles entails the introduction of **external** electric-field based actuators, making the system **under-actuated**.
- Modeling the cloud as a **probability distribution**, the control challenge is formulated as an optimal transport problem.
- **Optimal Transport (OT)** is based on **convex optimization**, used to plan the optimal transference from an initial to a desired probability distribution, with respect to the given cost function.



$$\boldsymbol{p}_{k+1}^{l} = \boldsymbol{p}_{k}^{l} + \Delta \boldsymbol{f}_{c}$$

TIME EXTENSION OF THE OT BASED PROBLEM

Optimal Transport based Convex Optimization Problem

$$\min_{\mu_k, a_{1,i}, a_{2,j}, a_{3,i}, a_{4,j}} D_{\mathcal{L}_2}\left(\mu_{k+1}, \nu\right)$$

Time Extention of the Optimal Transport based Problem

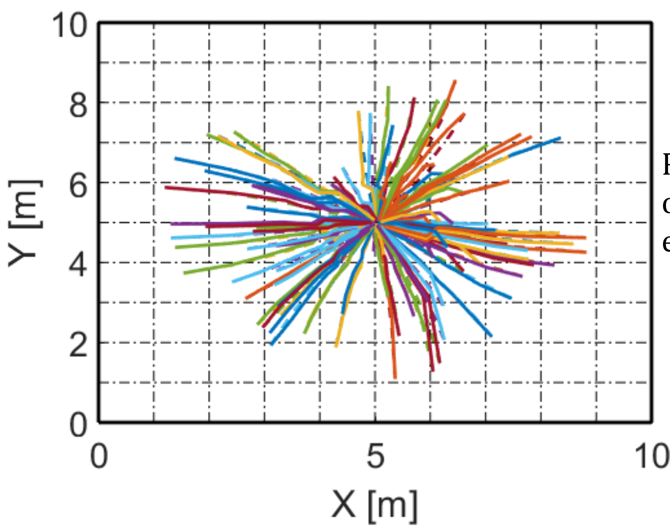
$$\min_{\mu_k, a_{1,i}, a_{2,j}, a_{3,i}, a_{4,j}} \sum_{k=k_0}^{T-1} \left(D_{\mathcal{L}_2}\left(\mu_{k+1}, \nu\right)\right) \Delta t$$

$$+ \ Particles \ Propagation \ Constraint:$$

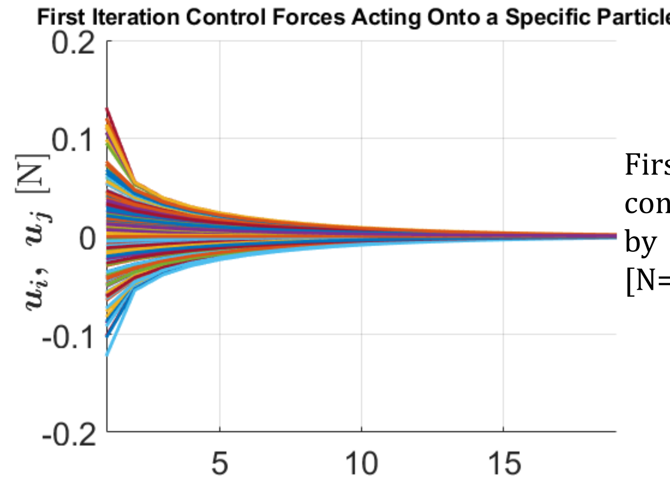
$$\boldsymbol{p}_{k+1}^l = \boldsymbol{p}_k^l + \Delta\left(\frac{a_{1,i}}{dis(l, a_{1,i})}\hat{y} - \frac{a_{2,j}}{dis(l, a_{2,j})}\hat{x} - \frac{a_{3,i}}{dis(l, a_{3,i})}\hat{y} + \frac{a_{4,j}}{dis(l, a_{4,j})}\hat{x}\right) +$$

$$+\Delta \sum_{m \in N, m \neq l} a_{rep} \frac{\boldsymbol{p}_k^l - \boldsymbol{p}_k^m}{\left\|\boldsymbol{p}_k^l - \boldsymbol{p}_k^m\right\|_2^3}, \quad l = 1, ..., N, \quad k = k_0, ..., T-1$$

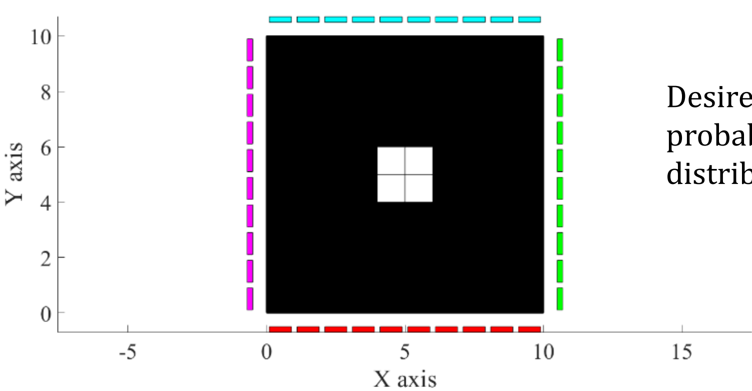
TIME EXTENSION OF THE OT BASED PROBLEM – NUMERICAL SIMULATIONS



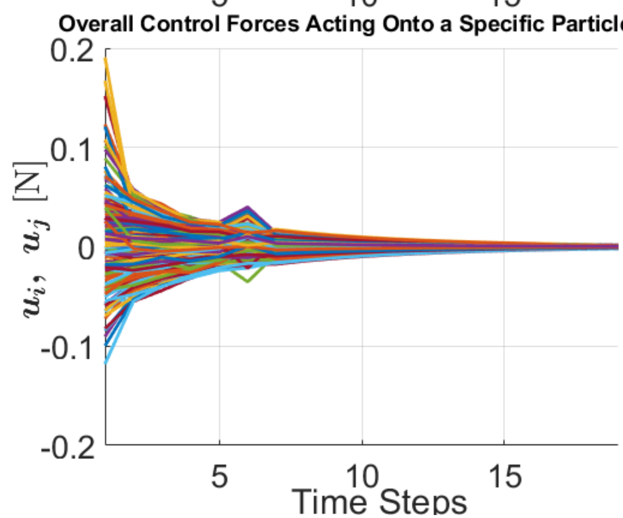
Reconfiguration of N=100 grain elements.



First iteration control actions felt by each particle [N=100].



Desired probability distribution.



Overall control actions felt by each particle [N=100].

OBJECTIVE FUNCTION MODIFICATION

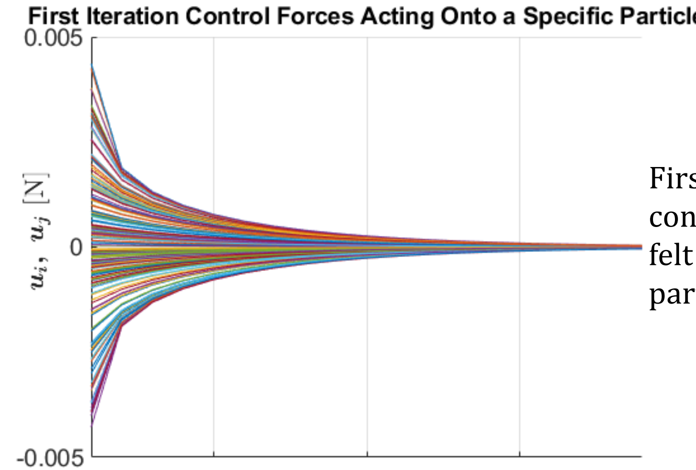
In order to reduce the control effort, the **objective function** is modified:

$$\min_{\mu_k, a_{1,i}, a_{2,j}, a_{3,i}, a_{4,j}} \sum_{k=k_0}^{T-1} \left(D_{\mathcal{L}_2}\left(\mu_{k+1}, \nu\right)\right) \Delta t + \sum_{k=k_0}^{T-1} ||\boldsymbol{a}_{i,j}[k]||_1 \Delta t$$

↳ It includes the **fuel efficiency** problem, by minimizing the control actions used throughout the redistribution **timeline**.

↳ For N=100 particles it is possible to **save** up to 5% of the consumed **energy**.

Iteration	F-count	f(x)	Norm of Step
0	2761	3.911960e+02	
1	5522	2.726680e+02	5.174e+00
2	8283	1.572102e+02	4.879e+00
3	11051	1.294424e+02	1.110e+00
4	13813	1.036929e+02	1.760e+00
5	16575	9.038210e+01	9.585e-01
6	19337	8.407526e+01	4.880e-01
7	22099	8.190265e+01	2.297e-01
8	24862	8.143897e+01	7.443e-02
9	27626	8.131868e+01	2.901e-02



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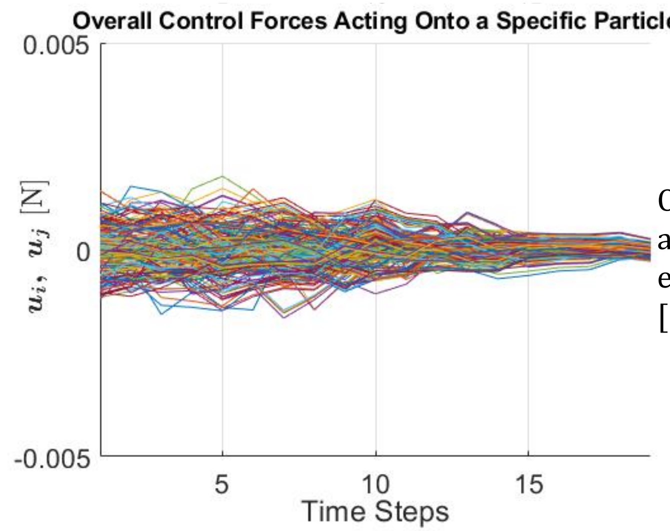
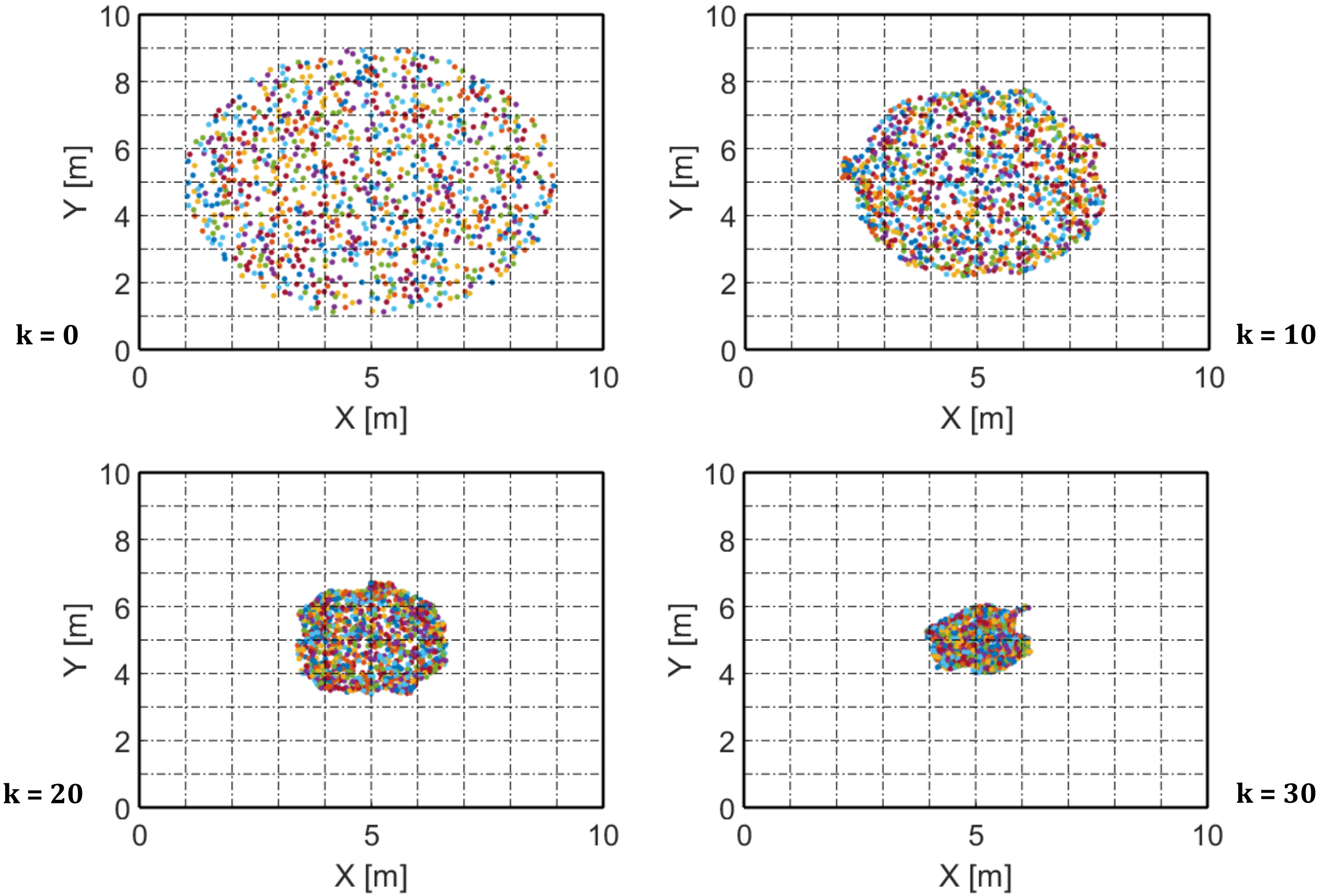


Overall control actions felt by each particle [N=100].

1000 GRAIN CLOUD SIMULATION



CONCLUSIONS – LAGRANGIAN AND EULERIAN FRAMEWORKS COMPARISONS

LAGRANGIAN FRAMEWORK

- **Deterministic Approach**
  Higher precision and possibility of formation flying applications.
- **Classic Implementation**
  Abundant literature available.
- **Fuel Efficiency**
  The optimization allows to minimize the actuation energy and to improve the robustness to external disturbances.

EULERIAN FRAMEWORK

- **Increasing Dimensions**
  When the number of elements increases, the Eulerian framework can grant better **scalability**.
- **Flexibility**
  Reduced sensitivity to the issue of **loss of bodies** and changes in the amount of constituents.
- **Real Time**
  Difficult real-time implementation of the MPC based logic for very large clouds.

CONCLUSIONS – HYBRID DISTRIBUTED CONTROL

Hybrid VLSR System

- Swarm of active robots
- Addition of external actuators

Euler Based Control Framework

Used in the initial **deployment** phase when the requirement is to regroup and coarsely **confine** the cloud in a specific region.

- **Avoid** generating **individual control actions** for each robot.
- **Limited** amounts of **energy** and **autonomy** available.
- The greater efforts of the redistribution task would be absorbed by the **non-contacting actuators.**

Lagrangian Based Control Framework

- Used for controlling the specific element during the final stage of **precision** manoeuvering and **formation flying**.

THANKS FOR THE ATTENTION