Crime in a Lake: A primer

Francesco Tucci

1 Introduction and Motivation

The "Lake" model of the labour market, since its establishment as a valuable tool in macroeconomic theory (see Sargent & Ljungqvist), has had a wide range of applications in several diverse fields of economics (family economics, industrial organization and so forth). One of the most unexplored but interesting among these is the analysis of crime. Indeed, the flexibility of the framework and its ability to describe with a good level of approximation how economic agents take dynamic decisions, seem to fit properly the study of the determinants that can potentially push an individual to commit a criminal action. Hence, in the last ten years, a small but expanding number of researchers has turned to the modelling of such interactions.

So far the best attempt to introduce criminal decisions in the picture has been the one of Engelhardt, Rocheteau & Rupert (2008). They introduce crimes and punishments a la Becker (1968) in the baseline Pissarides's (2000) analysis of the labour market. Opportunities to commit a crime arrive to each individual with a given probability (which differs between employed and unemployed workers, the latters being more likely to undertake a crime). Crimes are assumed to differ in their values according to a distribution G. The individual can not choose a desired gravity of the illegal action and can only evaluate the different outcomes of committing a crime in terms of expectation. Given a probability of State's enforcement (independent from the value of the crime), he/she can be apprehended by the local force and sentenced to serve an amount of time in jail (assuming this does not vary across the range of criminal actions). Then, if lucky enough, he/she faces a determined likelihood of being released and join again the mass of unemployed workers. A key assumption of the model is that every individual, independently of him/her being employed, unemployed or in jail, faces the same probability of meeting a criminal and suffering the relative loss in terms of resources taken away. The rest of the model follows the basic framework of Pissarides (2000), with firms negotiating the optimal wage with workers according to a Nash-bargaining process, posting vacancies to fill-in empty positions and finding suitable matches on the job market. Employed agents face in each period an exogenous Poisson shock that renders their working contract with the firm unprofitable and thus leads to their dismissal. With this setting in mind, the authors compute the (unique) equilibrium of the system and then derive interesting policy insights running different simulations using several specifications for the structural parameter. First they analyse which effects some labour market reforms (in terms of modified unemployment benefits or improved workers' bargaining power in the negotiating the wage) might trigger in this economy. Secondly, they analyse what would change were the enforcement and the detention modified in a stricter or looser fashion.

As we can see from this short description of their work, it is possible to describe an economy that includes a third possibility on top of employment and unemployment without the need to change dramatically the baseline structure of the search and matching model. Then, one can analyse how this third component evolves besides the other two.

Simplifying the picture, we will try to see if it is possible to gain some insight about what would be an equilibrium distribution for the three shares of the population (namely employed, unemployed and incarcerated agents) in a simulation that starts with some plausible institutional parameters and assigns pre-determined spill-over dynamics for each type of agent. Once this has been accomplished, we will see how small policy changes captured by slight movements in the parameters of interest can result in different steady-states values for the optimal allocation of the population's shares. Suprisingly, we will see that, while the share of unemployed agents seems to be fairly robust across different specifications, those of employment and prison population seem to be more responsive to the institutional framework and to the specification at stake. As a last step, we try to include individual decisions as a way to optimally determine endogenously the parameters of the model, most notably when it comes to the job-finding rate and to the opportunity of committing a crime.

2 Model

2.1 The Lake

Our economy is inhabited by three types of agents: the employed, the unemployed and the incarcerated. We allow individuals to move from one state to the other, according to some probabilities that will determine whether the shift will take place or not.

Let us begin by describing the dynamic for the employees. In every period $t+1$, of the labour force employed at time t:

- a share $(1-\alpha)$ will remain employed.
- a share α will fall into unemployment.

We assume for simplicity that an employed agent is not offered any possibility to undertake a criminal action, because he/she is able to earn his/her living only with own resources.

On the contrary, unemployed agents do have the need to accumulate more resources and thus are offered the opportuntity of committing a crime in every period t with probability ϵ_1. We abstract here from incorporating different values for the crime and the individual decision of committing it. Hence, the probability ϵ_1 ultimately gives us the share of unemployed agents that resort to crime as a mean of rounding up unemployment benefits. Among those that resort to such activities, the local police force manages to apprehend a share π, either because of institutional inefficiencies or because of budget constraints.

With respect to the job search, we assume that unemployed agents find a job with probability λ. An important remark that needs to be done concerns the exact timing of job searching and crime decisions. Henceforth, we assume that an individual is first subjected to employment shocks and then, after having observed his/her labour outcome, is offered an opportunity to committ a crime. In other words, only those unemployed workers who already know that they are not going to fill any vacancy in the next period can resort to illegal activities. In a nutshell, among the unemployed at time t:

- a share λ finds a job.
- a share $(1-\lambda)(1-\epsilon_1\pi)$ commits a crime but gets away with it, thus stays unemployed.
- a share $(1-\lambda)(\epsilon_1\pi)$ commits a crime but is apprehended by the State and thus sent to jail.

As for the last component of our society, namely prison population, things are less complicated, in the sense that:

- with probability δ a prisoner is released at time t
- with probability $(1-\delta)$ a prisoner has to serve also in $t+1$

Furthermore, we make the (quite restrictive) assumption that this economy is composed exclusively of these three kinds of agents. Thus, we rule out every possible situation of labour force non partecipation.

Along the lines shown above, the dynamic of the economy between t and $t+1$ is characterized by the followings:

- $U_{t+1} = \alpha\, E_t + (1-\lambda)(1-\epsilon_1\pi)\, U_t + \delta\, P_t$
- $E_{t+1} = \lambda\, U_t + (1-\alpha)\, E_t$
- $P_{t+1} = (1-\lambda)(\epsilon_1\pi)\, U_t + (1-\delta)\, P_t$

Defining $Y_t = \begin{bmatrix} U_t \\ E_t \\ P_t \end{bmatrix}$, then the system of equations can be written as $Y_{t+1} = A\, Y_t$,

where

$$A = \begin{bmatrix} (1-\lambda)(1-\epsilon_1\pi) & \alpha & \delta \\ \lambda & (1-\alpha) & 0 \\ (1-\lambda)(\epsilon_1\pi) & 0 & (1-\delta) \end{bmatrix}$$

is the transition matrix for the three states.

The matrix A characterizes the transition for the employment, unemployment and prisoners rates over total population.

If we want to look at the stocks' evolution over time, assuming that total population grows at a rate g, the system becomes $Y_{t+1} = (1+g)\ AY_t$. While aggregate stocks will trivially diverge because of population's growth, it is possible to define a steady state for the rates as a Y^* such that: $Y^* = A\, Y^*$

2.2 Individual Dynamics

Summing up what we have said so far, each individual faces the possibility of being in one of the three states that characterize this economy. Defining by $(\psi_{tu}, \psi_{te}, \psi_{tp})$ the individual probability of being employed, unemployed or in prison respectively at time t, we can define the following transitional equation: $\psi_{t+1} = \psi_t\ P$, where P is given by:

$$\begin{bmatrix} (1-\lambda)(1-\epsilon_1\pi) & \lambda & (1-\lambda)(\epsilon_1\pi) \\ \alpha & (1-\alpha) & 0 \\ \delta & 0 & (1-\delta) \end{bmatrix}$$

and represents the transition matrix at an individual level. Given that P is aperiodic and irreducible if all the parameters are between 0 and 1, there exists a unique *stationary distribution* associated to P.

Moreover, P is also ergodic. Ergodicity is indeed a very desirable feature for the individual's transition matrix, because it allows us to establish a relationship between the stationary distribution of the probabilities of being in each state and the steady state population rates for the respective share of the population. In other words, when the number of periods is large enough, the amount of time an agent spend in each state will equal the stationary probability of that state. Moreover, as we can remark from the fact that P is the transpose of A, the average time an individual will spend in one state will equal the steady state rate for the respective share of the population.

In mathematical notation:

$$\overline{s_{i,N}} = 1/N \sum_{t=1}^{N} \mathbb{1} s_t = i \quad = \quad \psi_i^* \quad = \quad ss_i^*,\ when\, N \to \infty,\ i = (u, e, p)$$

Depending on the persistence of the matrix P, the model might take a very long period of time to satisfy this property.

2.3 Endogenous Parameters

In this extremely simplified setting, we can think of including several additional features. The next step is to endogenize some parameters of the model, as the job finding rate and the opportunity of committing a crime. In order to do this, we need to model how the individual carries out his/her future decisions. In other words, we need to set-up the individual value functions. Throughout this subsection, we assume that the utility derived from consuming either wages, unemployment, prison benefits or rewards from crimes assumes a linear form. Furthermore, we assume that future criminal opportunities and wages are discrete random variables, so that the individual can only infer about them using his/her expectations.

Let us start from the employed agent's one. We can write it in the following fashion:

$$V(w) = w + \beta((1-\alpha)V(w) + \alpha \sum_{c'} U(c')p_{c'})$$

note that, when the agent receives a Poisson shock that breaks his/her working contract, he/she has to take expectations over the distributions of crime opportunities he is likely to face in the next period, when unemployed.

For the unemployed agent that committed a crime of value c and got away with it, the value function is given by the following:

$$U(c) = c + b + \beta(\sum_{w'} max[V(w'), U(e)]p_{w'} + \sum_{c'} max[(1-\pi)U(c') + \pi P, U(c)]p_{c'})$$

where b is the value of the unemployed benefits the agent has access to.

For the incarcerated individual, the value function is the following:

$$P = x + \beta((1-\delta)P + \delta\sum_{c'} U(c')p_{c'})$$

where x is the value of every kind of benefits the agent receives in prison allowing him/her to earn a living.

Focus now on the value function of the unemployed worker. From this we can derive two tresholds that indeed will be very important in determining the dynamic of this kind of agent: the *reservation wage* and the *reservation level of crime*. The first is the wage value such that:

$$V(w) > U(c)$$

so that the agent accepts the job offer and starts being an employee again. The second is the crime opportunity value such that:

$$(1-\pi)U(c) + \pi P > U(c)$$

so that the agent prefers to profit from the opportunity and accepts the gamble of potentially be apprehended, more than stand still with what he/she already has.

Contrary to what we usually find in search and matching models, since we have as many unemployment value functions as there are crime values, the reservation wage and the reservation level of crime will not be unique but they will be vectors. Nonetheless, we could think of averaging them over the different value functions and then set endogenously the parameters λ and ϵ_1, using the followings:

$$\lambda = 1/C\sum_{c} P_c(w' \geq \overline{w}) \quad \textit{and} \quad \epsilon_1 = 1/C\sum_{c} P_c(c' \geq \overline{c})$$

where C represents the total number of possible crimes' values and $\overline{w}, \overline{c}$ are the reservation values for wage and crime respectively. Another possible extension would be to include a stochastic opportunity of receiving job offers, i.e. in every period only with probability γ the unemployed worker is able to scrutinize the different levels of wage. With proba. $(1-\gamma)$ instead, he/she stays unemployed and resorts to undertaking illegal actions. Indeed, this is the specification we will adapt in the numeri cal section of the exercise.

3 Why Computations

Computational methods are useful in this context because of two main reasons.

On the one hand, they allow us to simulate quickly and efficiently different specifications of our baseline model. This will be of particular interest when we will perform some sensitivity analysis related to small policy parameters' modifications.

Moreover, they enable us to check whether the condition relating the dynamic of each individual to that of the whole economy(see subsection 2.2) is verified for our model. By iterating the individual dynamic over different periods of time that are increasing over and over in lenght, we will show that indeed the proportion of time spent in each state by each individual asymptotically converges to the steady state rate of the respective state.

Nevertheless, when it comes to dynamic programming numerical methods show their usefulness in the most vivid way. In fact, the mathematical theory of dynamic programming tells us that, starting from an arbitrary value function value on the r.h.s of the Bellman equation and using the contraction mapping given by the maximization problem (with $\beta < 1$) and iterating the process, we will eventually reach a fixed point for the value function.

Among all the possible techniques to implement this in a computer(policy function iteration, EGM..) in the following we are going to rely on the simplest one, namely *Value Function Iteration*. While this method is the most intuitive, it is perhaps the less precise. Yet, because of time constraints, we chose this one to implement the individual value functions' section (unfortunately without even succeeding..)

4 Calibration

In the baseline specification's calibration of the model, we tried to follow as much as we could Engelhardt, Rocheteau & Rupert (2008).

For instance, we assume the job finding rate λ to be 0.2 and the the job disruption rate α to be 0.05, in line with what they do in their work. However, some of their calibrated estimates were somehow puzzling and difficult to insert in our framework(as for example when they set the enforcement rate π equal only to 0.019). Since fully adopting their estimates represented a major problem for our computations, often we departed from them. Hence, we assume that the baseline enforcement rate π is equal to 0.7, that the annual probability of being released from prison (δ) is 0.25 and that the probability of committing a crime ϵ_1 is equal to 0.1.

Although these estimates are surely arbitrary, they are not so far from matching some real world features when it comes to justice and crime. It would be interesting for further research to see how well they eventually fit with the data.

For population growth, we assume an annual rate for g of 0.005, in line with what we observe today in most developed economies. Table 1 summarizes the benchmark calibration for the model.

As for the starting conditions for the population, we set the starting employment rate to equal 0.89 (perhaps a level which is too high compared to the calibration of the model, and we will see that indeed the model immediately tends towards a lower level for this variable), the unemployment rate to 0.09 and prison population rate to 0.02.

This being said, the numerical strategy is composed of the following steps: we determine the steady state for the benchmark economy, the dynamic of aggregate stocks and the convergence of the rate towards steady state values. Second, we will check that the ergodicity conditions is satisfied, and that indeed the time spent in each state by each individual does indeed converge to the steady state. Third, we will analyse how the equilibrium steady state would change if we were to trigger some small perturbations in the economy, such as modifying the enforcement rate or the rate of release from prison. Fourth, and this part is still pretty much work in progress (it will be interesting to go back to it for further studies), we will try to endogenize the job finding rate and crime rate. Yet, due to the fact that our value function iteration does not converge, we are not able to infer any result from this attempt.

Before presenting the results, it seems to us fair enough to say that, in building the code, we relied a lot on the Lake model of the job market provided in Julia format by Thomas Sargent and John Stachursky in their set of lectures QuantEcon.

5 Results

Figure 1 shows the dynamic of the aggregate stocks for each different component of the population. As previously remarked, it is interesting that, starting from our initial calibration for the employment rate, the economy sharply adjusts the level of employment and unemployment towards a pattern that is more compatible with the underlying structural parameters. Hence, employment falls while unemployment rises dramatically in the very first periods.

Trivially, since total population is increasing at a constant annual rate of g, these curves explode over time.

Figure 2 instead shows the dynamic of the unemployment, employment and prison population rates with respect to total population. We can see that the steady state values for unemployment and prison population is sensibly higher than what we input as starting values in the framework, at the expense of the employment rate, whose relative magnitude is eaten away by the other two components.

After having determined the steady states for this economy, we moved to the analysis of whether the ergodicity condition is satisfied, i.e. the longer the period, the more the average time spent in each state by the individuals converges towards the steady state value for that state. This should hold true independently of the individual having started as employed, unemployed or in prison.

To achieve this, we simulate our economy over different increasingly longer periods of time and then compute the average time spent in each position by every single agent. Eventually, we would expect this average time to get closer and closer to the steady state values.

Results are in line wiht our prior, as shown by Figure 3,4,5, in which we plot the average time spent in each state for an individual respectively unemployed, emloyed and in prison at time 0.

Figure 6 begin with sensitivity analysis. The first policy change we would like to study is what happens to the steady state values of this economy if we consider different levels of the enforcement rate π, either below or above our starting benchmark of 0.7. Figure 6 shows what happens to the ss variables along the range $(0.2, 0.8)$. Results are in line with our intuition, i.e. that increasing the enforcement rate increases prison population. Yet, it is surprising that this raise is triggered by a decrease in the ss rate of employment and not by a fall in unemployment, i.e. the component whose members are the ones who might potentially undertake criminal activities. This feature would be worth a deeper scrutiny in order to understand why it is indeed the case.

Figure 7 applies the same concept to changes in the probability of committing a crime ϵ_1, which we consider over a very wide range, from 0.05 to 0.5 (the latter being a very extreme case, but still worth analysing). As expected, raising the opportunity of committing a crime increases prison population (keep in mind that in Figure 7 π is fixed at its benchmark value of 0.7): more and more unemployed agents resort to crime in order to accrue further resources, but they still face a very high probability of being apprehended, thus may very likely end up in prison. Once again, also the ss rate of unemployment dwindles as we increase the crime rate, perhaps because of the shrinking in the unemployed mass. Note that, in the extreme case (ϵ_1=0.5), agents are so likely to commit a crime that the rate of prison population outpaces the unemployment one.

Concluding the comparative statics related to crime policies, Figure 8 shows how the ss react to different rates of release from prison δ, from very restrictive regimes ($\delta \leq 0.1$) to more mild ones ($\delta \geq 0.4$). As we expected, ss prison population falls with the release rate. Yet, surprisingly again, it seems to be the mass of employed agents which profits the most from the incoming flows of redeemed agents, and not the one in which they flow directly, i.e. unemployment.

Turning the attention to the last two variables we can make exogenously change in our economy, λ and α, results from comparative static do not contradict the intuition.

From Figure 9, we can clearly see how the unemployment rate falls in λ, at the benefit of the working population. Keeping in mind that the job finding rate affects as well the opportunity of committing a crime, also prison population decreases in λ.

From Figure 10 instead, we can see the opposite dynamic being at work if we consider increasing changes in α,i.e. the possibility of losing the job.

6 Further Research and Conclusion

While the accuracy of the obtained results and the speed are tested explicitly in Julia, we would like to conclude stressing the lines along which it would be possible to develop this field of research. It would be very interesting to endogenize some parameters of this model, along the lines already mentioned in the introductory section. This means that one should be able to solve the individual problem and compute the relative value functions, something that, in spite of several trials with a wide range of different specifications, we were not able to do. As showed by Sargent and Stachursky in their simulation on QuantEcon, once this is accomplished, a researcher could think of including in the picture also the analysis of some fiscal policy settings, which are compatible with the optimality conditions derived from the solution of the individual problem.

We believe this a very promising field for future research. Indeed, on top of Engelhardt, Rocheteau & Rupert (2008), very little has been accomplished so far and the room for futher insights is still very large.

Table 1:

λ	α	π	δ	ϵ_1	g
0.2	0.05	0.7	0.25	0.1	0.005

Figure 1: *Dynamic for the aggregate stocks*

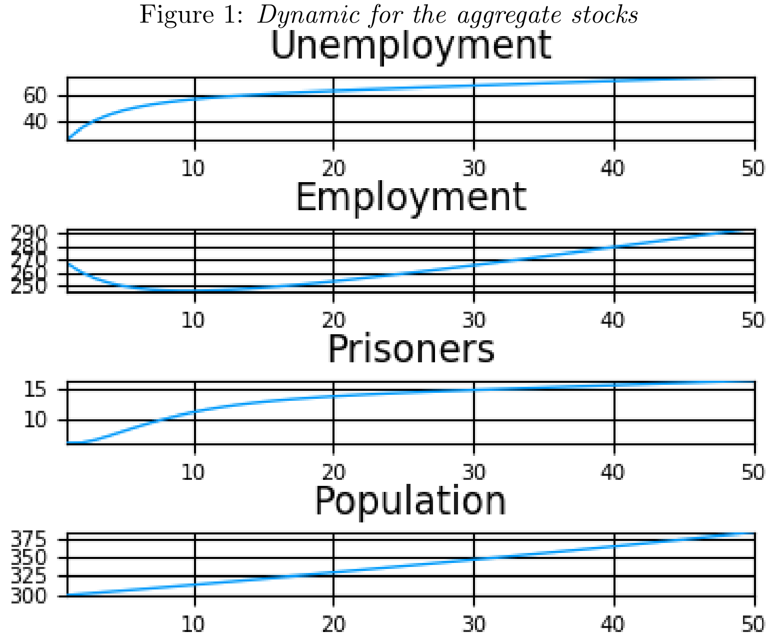

Figure 2: *Dynamic for the population rates(steady states in dashed red)*

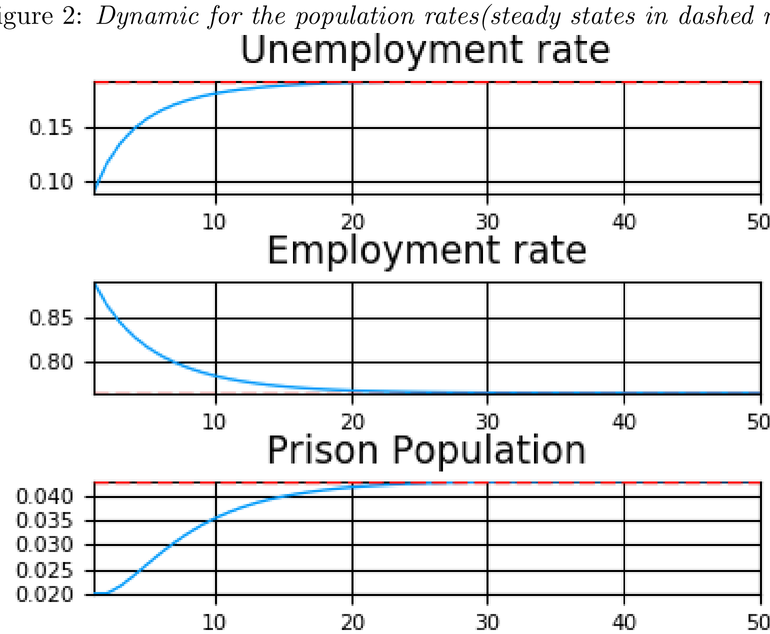

Figure 3: *Average time spent in each state and convergence to ss*

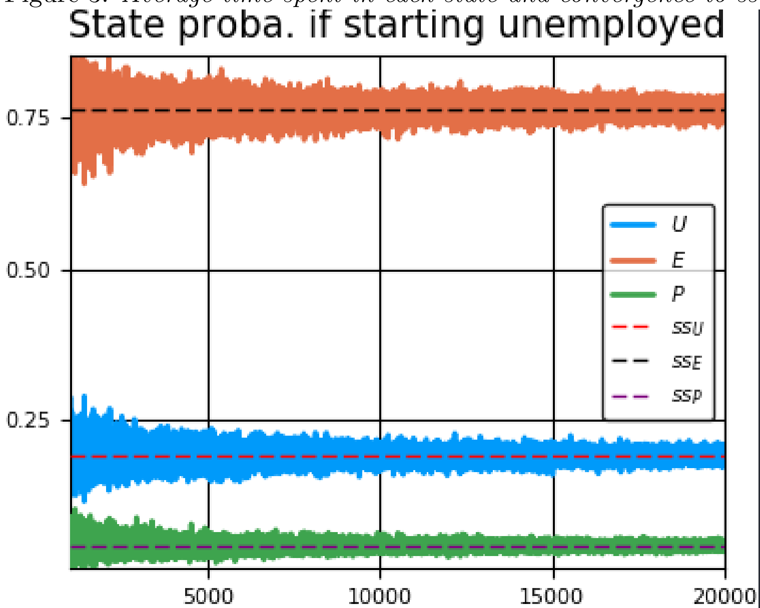

Figure 4: *Average time spent in each state and convergence to ss*

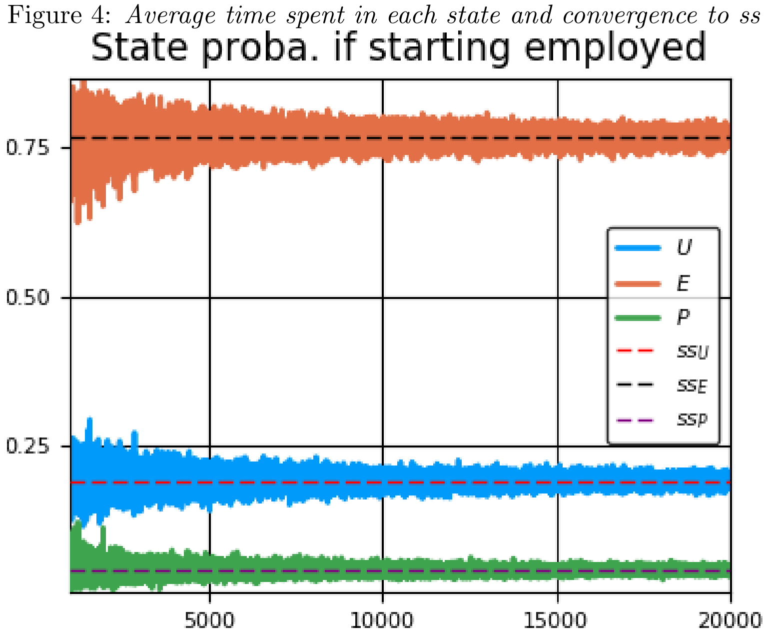

Figure 5: *Average time spent in each state and convergence to ss*

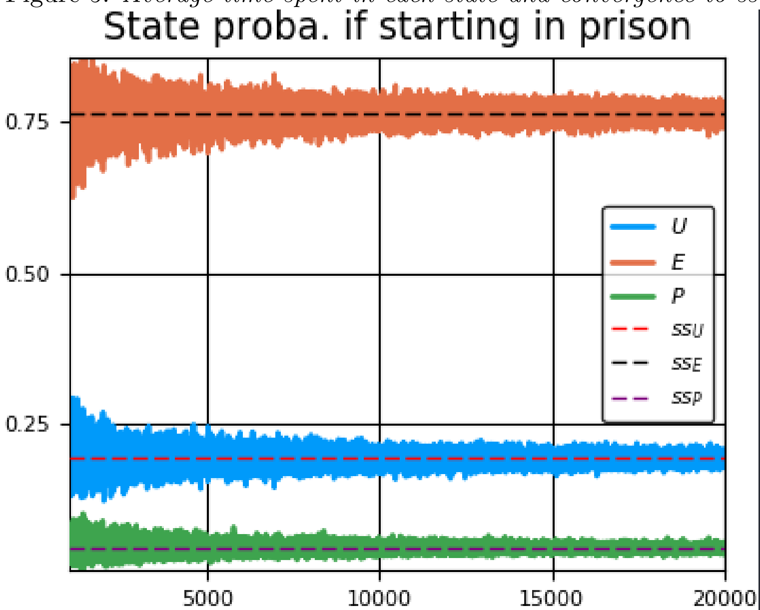

Figure 6: *SS for different enforcement rates*

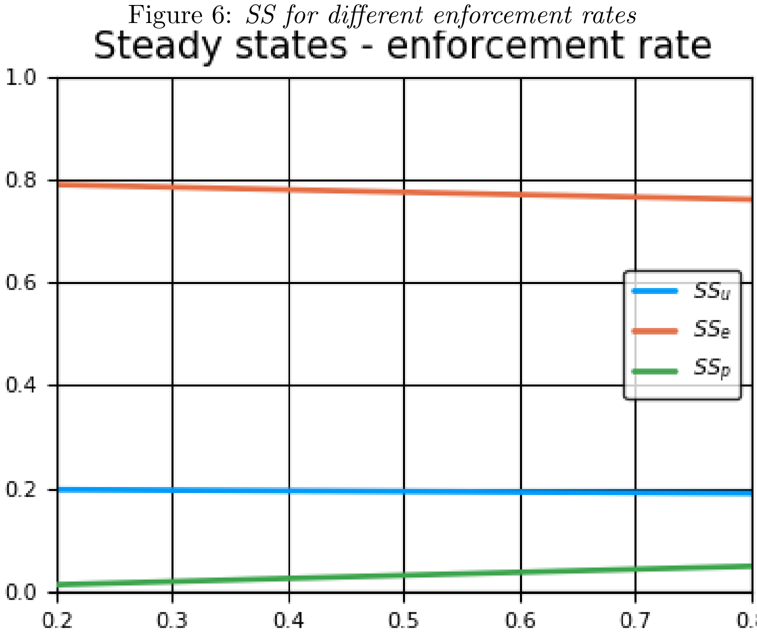

Figure 7: *SS for different crime rates*

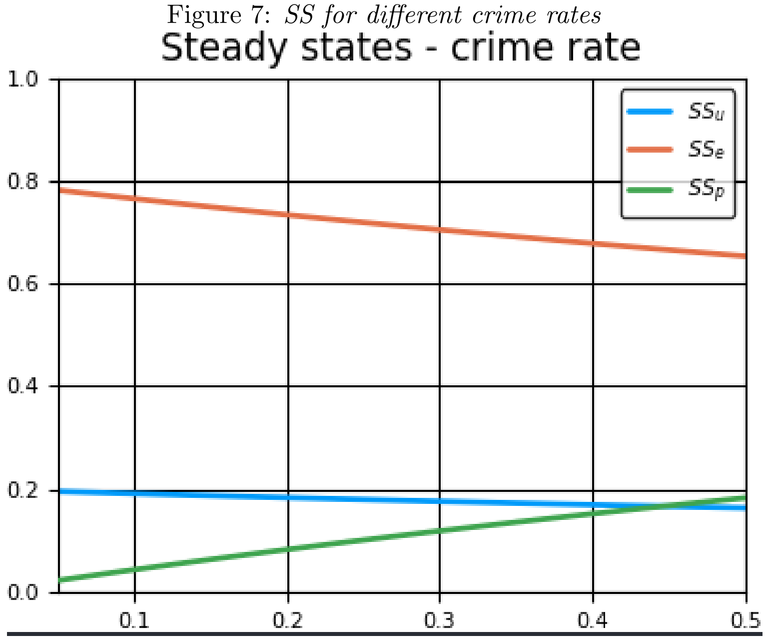

Figure 8: *SS for different release rates*

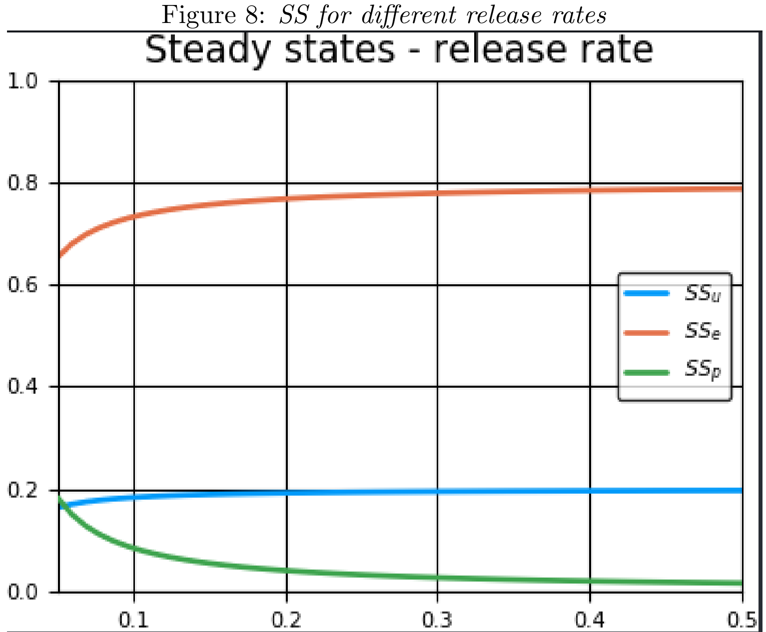

Figure 9: *SS for different job finding rates*

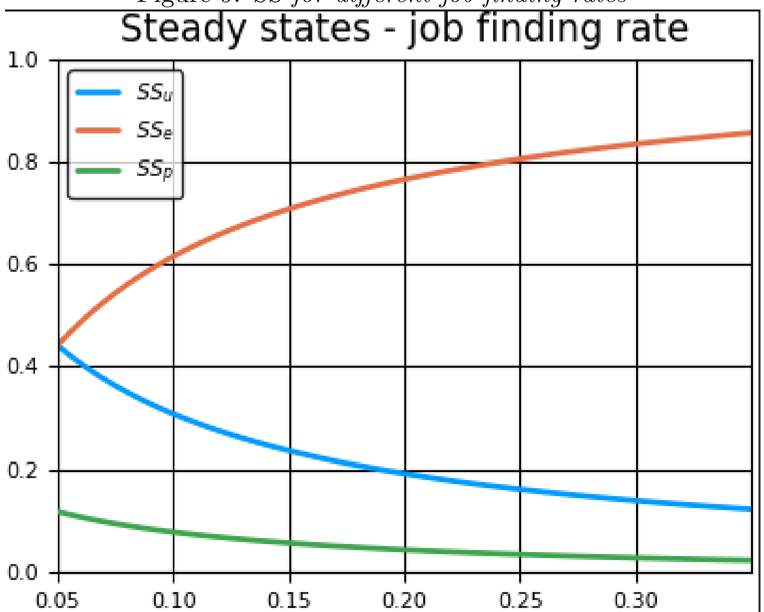

Figure 10: *SS for different job loss rates*