Fiscal Multipliers in a new fashion: the contribution of a break in volatility

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Abstract

Building on Rigbon's (2003) intuition, unconditional heteroskedasticity has become a powerful source of information for identifying structural shocks in SVAR analysis. This analysis specifically focuses on estimating US dynamic fiscal multipliers, leveraging the volatility break affecting the macroeconomic variables from 1984Q1 to 2006Q4. In particular, I employed two distinct approaches to integrate SVAR analysis with the extra information derived from this volatility shift. The first approach, presented by Lanne and Lütkepohl (2008), assumes constancy in structural parameters across periods, allowing for estimation without imposing any restrictions. By contrast, the approach proposed by Bacchiocchi and Fanelli (2015) starts from the more reasonable assumption that the change in the volatility reflects a simultaneous change in the structural parameters. Thus, this work aims to address the following three core questions. Which one of the two identification schemes provided by the literature is more reliable in estimating fiscal multipliers by exploiting the Volcker era's volatility break? What are the dynamic effects of tax and public expenditure shocks on GDP and how do they differ from the existing literature estimates? How did the fiscal multipliers change during the Great Moderation period?

1 Introduction

Estimating the effect of public expenditure and taxation on the economy has always been one of the main challenges for both empirical macroeconomists and policymakers. However, in the literature produced so far, there is a significant disagreement about the significance and the size of their effects. The variation affecting those estimations comes from the fact that they are recovered by exploiting complex identification schemes that rely on different assumptions. Furthermore, given this kind of uncertainty, the best scenario possible would be to get an over-identified model allowing to test the different assumptions made. Moreover, another problem affecting the estimated values for the US fiscal multipliers through classical SVAR analysis stems from the fact that those analyses often ignore the possibility of changes in the structural parameters. At the same time, the macroeconomic variables are used to show different regimes of volatility (See **Great Moderation** period). For this reason, in this paper we want to show that a specific statistical property characterizing our data, i.e. the unconditional heteroskedasticity, can be used, under specific assumptions, to define identification schemes allowing us to have either a model that does not rely on any specific restrictions or a model including a change in the structural parameters themselves. In fact, in a seminal contribution, Rigbon (2003) proposed for the first time the idea that unconditional heteroskedasticity can be a powerful tool for the identification of structural shocks. In particular, we can exploit the volatility breaks characterizing our data as a useful source of identifying information. Subsequently, this identification approach was applied to the classical SVAR analysis in two different ways. Firstly, it was adopted by Lanne and Lütkepohl (2008), who used the extra information, carried by different volatility regimes, to identify a SVAR model for monetary policy without additional restrictions. This apparently super-powerful identification scheme, which does not rely on any specific theory-driven restriction for the structural parameters, is actually based on the general idea that the structural parameters do not vary across different volatility regimes. Secondly, since this assumption is considered controversial while studying macroeconomic issues, Bacchiocchi and Fanelli (2015) derived new conditions for the identification of monetary policy shocks through volatility changes, allowing for changes also in the structural parameters. In this paper, after properly checking for the existence of a volatility break in our data, I adopt both methods to compute the US fiscal multipliers, enlightening the differences between the two approaches and the relative advantages and gains. However, as explained in the following sections, the method applied by Lanne and Lütkepohl, despite its great potential, is based on a very strong assumption considered low-reliable when focusing on the break in the macroeconomic time series caused by the "Great Moderation" period. In fact, that sudden change in the US monetary policy is widely considered a structural change, meaning that the observed reduction in volatility is strictly related to a change in the structural parameters themselves.

2 Data and reduced-form model

To perform the analysis I used the US macroeconomic data related to taxes (T), public expenditures (G), Gross Domestic Product (GDP), and the real 3-month T-Bill rate (RR). Moreover, the disposable observations cover the period from 1950Q1-2006Q4, with quarterly frequency.[1]. As a reduced-form model, I considered a VAR characterized by 4 lags, a constant term, and a time trend. Thus, the above-specified model is described in the following compact form:

$$W_t = \mathbf{\Gamma Z}(t) + \mathbf{u}_t$$

where $\mathbf{Z}(t) = (W_{t-1}, W_{t-2}, W_{t-3}, W_{-4}, D_t)$ is the set of matrices containing both the autoregressive and deterministic components, $\mathbf{\Gamma} = (\Pi_1, \Pi_2, \Pi_3, \Pi_4, \Phi)$ is the set of the coefficients associated with those components, and $\mathbf{u}_t$ is the 4-dimensional vector of the White Noise disturbances such that:

- E($\mathbf{u}_t$)=$\mathbf{0}$
- Var($\mathbf{u}_t$)=Σ_u
- Cov($\mathbf{u}_t$,$\mathbf{u}_{t-k}$)=$\mathbf{0}$, $k \neq t$

After estimating the reduced form specification, I computed the VAR innovations $\mathbf{u}_t$ and started to seek the existence of a break in their volatility.An exploration of the literature on the topic[2], suggests focusing the attention on the period that goes from 1979 to 1984, namely the Volcker era. Looking at the graphs reported in **Figure 1** one can notice that, after these years, the magnitude of the innovation of each series decreases.

[1]The original dataset is `DATI_FISCAL_CK.txt`. This dataset does not include the real interest rate, so it has been obtained by subtracting the inflation rate from the 3-month T-Bill rate.

[2]Kim and Nelson (1999) provide statistical evidence for a decline in volatility from 1984Q1.

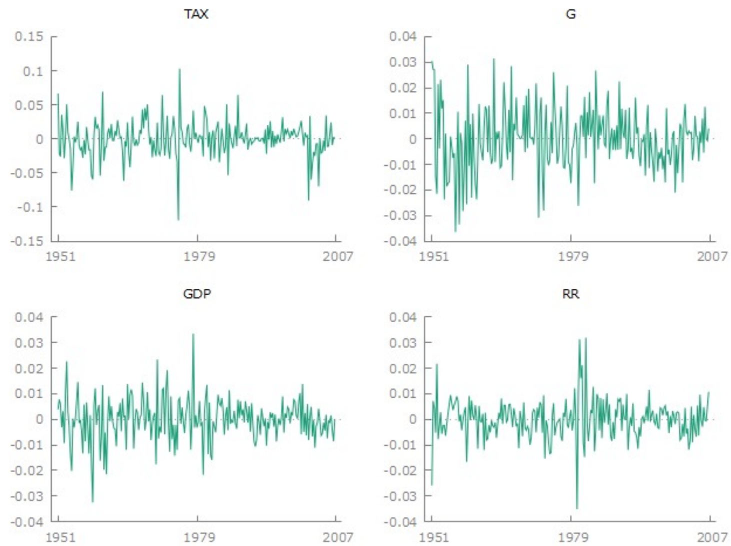


Figure 1: VAR innovations 1951Q1-2006Q4

Similarly to Lanne and Lütkepohl (2008) and Bacchiocchi, Castelnuovo and Fanelli (2018) I specifically test for the existence of a break in the volatility in the first quarter of 1984[3] To statistically check this assumption, the Likelihood-Ratio Chow-type test, characterized by the following hypothesis, is what suits the case:

- H_0: $\Sigma_{u1} = \Sigma_{u2} \cap \mathbf{\Gamma}_1 = \mathbf{\Gamma}_2$
- H_1: $\Sigma_{u1} \neq \Sigma_{u2} \cup \mathbf{\Gamma}_1 \neq \mathbf{\Gamma}_2$

Where Σ_{u1} and Σ_{u2} are the variance-covariance matrices of the innovations in the two periods and $\mathbf{\Gamma}_1(\Pi_{1,1} + ... + \Pi_{1,4}, \Phi_1)$, and $\mathbf{\Gamma}_2(\Pi_{2,1} + ... + \Pi_{2,4}, \Phi_2)$ represent the two sets of VAR coefficients of both autoregressive and deterministic components. The LR $-2[2757.17 - (1567.84 + 1273)]$ for this test is equal to 167.3988 and, under the null, it follows a χ^2_{80}. Thus, the p-value associated with the observed test statistics is equal to 0.000, implying that the existence of a break in 1984Q1 is confirmed by our data for every possible level of statistical significance. Moreover, I also test the null: $\Sigma_{u1} = \Sigma_{u2}$ against the alternative: $\Sigma_{u1} \neq \Sigma_{u2}$ under the assumption that the VAR coefficients are the same for both the periods considered (i.e. $\mathbf{\Gamma}_1 = \mathbf{\Gamma}_2$). This test has been performed to ensure that the break observed in 1984Q1 reflects also a shift in the volatility regime of the two periods, rather than being dependent only on the change in the VAR coefficients. The LR of this test is equal to$-2[2757.17 - (1618.28 + 907.8841)] = 462.01$, and, given its distribution under the null (χ^2_{10}),the corresponding p-value is 0.000. Similar to the previous test, this test specification also leads to state that the existence of a volatility break is statistically significant for every possible level. Thus, the data show a statistically significant break affecting the variance-covariance matrix Σ_u in 1984Q1.

[3] The test for the most significant structural change consisted in performing a series of the specified test for different dates in the range 1979Q1-1986Q4.

3 Exploiting unconditional heteroskedasticity

In SVAR analysis the presence of different volatility regimes characterizing the data has been considered a powerful source of identification information since Lanne and Lütkepohl (2008) applied in their work the intuition previously proposed by Rigbon (2003). In particular, these two authors integrated their SVAR analysis for monetary policy with the unconditional heteroskedasticity featured by the data under the assumption that the variance of the VAR innovations changes while the structural parameters remain constant across the two different volatility regimes. Subsequently, Bacchiocchi and Fanelli (2015) left this very strong and, at least, arguable assumption, for which the structural parameters are constant, and derived new necessary and sufficient conditions for local identification of structural shocks when there are simultaneous breaks in the variance-covariance matrix of the VAR innovations and in the structural parameters themselves. This more reasonable approach does not separate the change in the volatility regime from the structural parameters but allows for a connection between the two, implying, in the end, the estimation of different Impulse Response Functions (from now on denoted as IRFs). Moreover, if one expects a change in the structural parameters, estimating two different SVAR models, one for each of the two volatility regimes, would be inefficient because it would imply the loss of important statistical information carried by the volatility change allowing us to add new moment conditions. Therefore, since the aim is to exploit heteroskedasticity to compute the US fiscal multipliers, the two outlined approaches to the data will be adopted in the following sections. The general SVAR framework for this analysis is the B-model when performing the Lane and Lütkepohl approach, whereas an A-model best suits the theoretical reasoning when applying the Bacchiocchi and Fanelli identification strategy. Moreover, exploiting the volatility shift occurring in the 1984Q1 has been done in two different ways. In particular, for the first of the two approaches, only Σ_u changes as done by Lanne and Lütkepohl. In the second approach, instead, we work with a framework such that also Γ is allowed to change when we move from one regime to the other. For this reason, as demonstrated later, the differences in the IRFs and, consequently, in the dynamic fiscal multipliers of the two regimes can reflect the variation in the structural parameters and the change in the dynamics captured by different VAR coefficients. However, in the results section, also the dynamic fiscal multipliers for Bacchiocchi and Fanelli's approach estimated with constant VAR coefficients are presented.

4 Lanne & Lütkepohl scheme for constant structural parameters

Given the volatility break characterizing the data, the following decomposition is reliable:

$$\begin{cases} \Sigma_{\mathbf{u},1} = BB' & \text{for } t < 1984Q1 \\ \Sigma_{\mathbf{u},2} = BVB' & \text{for } t \geq 1984Q1 \end{cases}$$

where B is the MxM matrix of structural parameters characterizing the typical B-model and V is a MxM diagonal matrix with positive and distinct elements, which are also different from 1. This decomposition stems from the assumption that the change in volatility does not depend on the structural parameters included in matrix B. Instead, it is based on the idea that the volatility break results from a sudden change in the variance of the structural shock. In fact, in the new volatility regime, the variance of the structural shocks is no longer the identity matrix $\mathbf{I}_M$, but it becomes equal to the previously mentioned V matrix. Thus, thanks to this decomposition, one obtains $M^2 + M$ structural parameters (the ones included in the matrix B and the element on the diagonal of V) and $M(M+1)$ reduced form parameters (the free elements included in the two variance-covariance matrices). Indeed, since the number of structural parameters is equal to the number of reduced form ones, the former can be estimated without imposing any restrictions on matrix B. In this way, by exploiting the information carried by the unconditional heteroskedasticity, one can either avoid any restrictions on B or build an over-identified model allowing one to test different theories.In fact, since B can be recovered directly from the reduced form parameters without any restrictions, all the imposed restrictions are over-identifying and then can be tested. Moreover, the fact that the matrix B is constant across the two volatility regimes implies that the IRFs do not vary during the transition from the first period to the second one.

5 Bacchiocchi & Fanelli approach

Left the hypothesis that the structural parameters are not involved in the break characterizing the data from 1984Q1, I applied the approach outlined by Bacchiocchi and Fanelli(2015). This avoids estimating separately two specific SVAR models for each of the two volatility regimes detected. Given the statistical evidence reported in Section 2, the starting framework is the following one[4]:

$$W_t = \mathbf{\Gamma Z}(t) + \mathbf{u}_t$$

where:

$$\Sigma_{\mathbf{u}} = \begin{cases} \Sigma_{\mathbf{u},1}, \text{for } t < 1984Q1 \\ \Sigma_{\mathbf{u},2}, \text{for } t \geq 1984Q1 \end{cases} \qquad \mathbf{\Gamma} = \begin{cases} \mathbf{\Gamma}_1, \text{for } t < 1984Q1 \\ \mathbf{\Gamma}_2, \text{for } t \geq 1984Q1 \end{cases}$$

Since the starting SVAR model is the A-model and, for this scheme, the change in the volatility is the consequence of a change in the structural parameters:

$$\begin{cases} A\mathbf{u}_t = \boldsymbol{\epsilon}_t, & \text{for } t < 1984Q1 \\ (A+C)\mathbf{u}_t = \boldsymbol{\epsilon}_t, \text{for } t \geq 1984Q1 \end{cases}$$

where $\mathbf{u}_t$ is the 4-dimensional vector of VAR innovations, $\boldsymbol{\epsilon}_t \sim (\mathbf{0}, \mathbf{I}_4)$ is the 4-dimensional vector of structural shocks, and the 4x4 matrices A and $A + C$ are the matrices of the structural parameters defining the simultaneous relationships among the variables in the two volatility regimes. As visible from the system above, with this scheme, the structural parameters are allowed to vary from the first to the second period by adding the matrix C, which captures this change. Thus the following decomposition allows to exploit the break in volatility:

$$\begin{cases} \Sigma_{\mathbf{u},1} = A^{-1}(A^{-1})' & \text{for } t < 1984Q1 \\ \Sigma_{\mathbf{u},2} = (A+C)^{-1}(A+C)'^{-1} \text{ for } t \geq 1984Q1 \end{cases}$$

However, the values of the structural parameters cannot be recovered because their number is $2(MxM)$, in this case 32, while there are only $M(M+1)$, in our case 20, reduced form ones. Thus, to estimate the values of the structural parameters imposing at least 12 restrictions on the matrices A and C is needed. In the following subsections, different ways of restricting the matrices of the structural parameters are discussed. These differences come from specific hypotheses and considerations about the simultaneous relationships

[4]For comparison purposes, also results for Γ being constant across the two periods are shown.

affecting the variables included in the system. Therefore, after putting the restrictions, the estimation of the structural parameters is achieved by maximizing the following log-likelihood function:

$$
\begin{aligned}
l(a,c) = &-\frac{(T_1+T_2)M}{2}\log(2\pi) - \frac{T_1}{2}\log|A^{-1}(A^{-1})'| - \frac{T_1}{2}tr\{\hat{\Sigma}_{\mathbf{u},1}[A^{-1}(A^{-1})']^{-1}\} \\
&-\frac{T_2}{2}\log|(A+C)^{-1}(A'+C')^{-1}| - \frac{T_2}{2}tr\{\hat{\Sigma}_{\mathbf{u},2}[(A+C)^{-1}(A'+C')^{-1}]^{-1}\}
\end{aligned}
$$

where M is the number of variables in the VAR, in this setting is 4, and T_1 and T_2 are the number of observations for each period minus the number of lags.
Finally, as derived in Bacchiocchi and Fanelli (2015), I checked the necessary and sufficient conditions for the local identification of the structural parameters contained respectively in A and C. In particular, the necessary and sufficient condition for the local identification of the SVAR in the presence of a volatility break requires that the following $M(M+1)$-by-$\mathbf{q}$ matrix[5]:

$$
(I_2 \bigotimes \mathbf{D}_M^+)\begin{bmatrix} (A^{-1}\bigotimes I_M) & \mathbf{0}_{MxM} \\ (A+C)^{-1}\bigotimes I_M & (A+C)^{-1}\bigotimes I_M \end{bmatrix}\begin{bmatrix} \boldsymbol{H}_A & 0_{M^2xc} \\ 0_{M^2xa} & \boldsymbol{H}_C \end{bmatrix}
$$

has a full rank $\mathbf{q}$ (number of structural parameters to be estimated) when it is evaluated at $A = A_0$ and $C = C_0$. Besides this, the necessary order condition requires that the number of structural parameters free to be estimated is lower or equal to the number of free reduced-form parameters included in the variance-covariance matrices of the two regimes. Thus, when q is equal to $M(M+1)$, our model is exactly identified; by contrast, if $\mathbf{q} < M(M+1)$, the model becomes over-identified. Among the specifications with a theoretical interpretation, only the following ones (sections 5.1 and 5.2) respect the necessary and sufficient conditions outlined above.

5.1 General over-identified specification

The baseline specification for the restrictions imposed on the matrices of structural parameters is the following one:

$$
\left(\begin{bmatrix} a_{11} & 0 & a_{13} & 0 \\ 0 & a_{22} & 0 & 0 \\ a_{31} & a_{32} & a_{33} & 0 \\ 0 & a_{42} & a_{43} & a_{44} \end{bmatrix} + \begin{bmatrix} c_{11} & 0 & c_{13} & 0 \\ 0 & c_{22} & 0 & 0 \\ c_{31} & c_{32} & a_{33} & 0 \\ 0 & c_{42} & c_{43} & c_{44} \end{bmatrix} \mathbb{1}_4(t \geq 1984Q1)\right)\begin{bmatrix} u_t \\ u_g \\ u_{gdp} \\ u_{rr} \end{bmatrix} = \begin{bmatrix} \epsilon_t \\ \epsilon_g \\ \epsilon_{gdp} \\ \epsilon_{rr} \end{bmatrix}
$$

To better understand the meaning of these restrictions, it is useful to display them through the following simultaneous system of equations:

$$
t < 1984Q1 \begin{cases} u_t = -\underbrace{\frac{a_{13}}{a_{11}}}_{\theta_1} u_{gdp} + \frac{1}{a_{11}}\epsilon_t \\ u_g = \frac{1}{a_{22}}\epsilon_g \\ u_{gdp} = -\frac{a_{31}}{a_{33}}u_t - \frac{a_{32}}{a_{33}}u_g + \frac{1}{a_{33}}\epsilon_{gdp} \\ u_{rr} = -\frac{a_{42}}{a_{44}}u_g - \frac{a_{43}}{a_{44}}u_{gdp} + \frac{1}{a_{44}}\epsilon_{rr} \end{cases} ; \; t \geq 1984Q1 \begin{cases} u_t = -\underbrace{\frac{a_{13}+c_{13}}{a_{11}+c_{11}}}_{\theta_2} u_{gdp} + \frac{1}{a_{11}+c_{11}}\epsilon_t \\ u_g = \frac{1}{a_{22}+c_{22}}\epsilon_g \\ u_{gdp} = -\frac{a_{31}+c_{31}}{a_{33}+c_{33}}u_t - \frac{a_{32}+c_{32}}{a_{33}+c_{33}}u_g + \frac{1}{a_{33}+c_{33}}\epsilon_{gdp} \\ u_{rr} = -\frac{a_{42}+c_{42}}{a_{44}+c_{44}}u_g - \frac{a_{43}+c_{43}}{a_{44}+c_{44}}u_{gdp} + \frac{1}{a_{44}+c_{44}}\epsilon_{rr} \end{cases}
$$

[5]where D_M^+ is the Moore-Penrose inverse of the duplication matrix and H_A and H_C are the selection matrices with a one indicating the position of the parameter to be estimated when multiplied with the vector of those parameters.

In determining this first scheme, the main references are the restrictions adopted by Blanchard and Perotti (2002). Starting from their model, I first removed the restrictions they put on the main diagonal since not working with an AB-model, but just with an A-model. Besides this, any assumption about the elasticity of taxes to GDP has been made[6]. While Blanchard and Perotti (2002) assumed it to be equal to 2.08, Martens and Ravn (2014) estimated it to be equal to 3.13. Given this difference in its possible value, the interest is in estimating it and its variation when moving from one period to the other. Thus, to do so, a_{13} and c_{13} are left free to be determined through the estimation process. Moreover, I assume that a_{12}, c_{12}, a_{21} and c_{21} are equal to zero. These assumptions, as shown by the system of equations, reflect the idea that there are no simultaneous relationships between taxes and public expenditure. According to Blanchard and Perotti (2002), also a_{23} and c_{23} are assumed to be equal to zero, implying that the elasticity of public expenditures to GDP is zero in both regimes. These assumptions make public expenditure the most independent variable of the system resulting in it being explained only by its innovations. Finally, further assumptions about its simultaneous relationships with the other variables are needed as this work also includes the real interest rate. While assuming for both periods that the real interest rate does not affect the other variables, it is assumed to be dependent on both public expenditures and GDP. As shown by the matrix form, these assumptions are reflected by the three zeros placed on the last column of both A and C and in the fact that in the last row of the two matrices, only a_{41} and c_{41}, i.e. the determinants of the elasticity of real rate to taxes, are equal to zero. There are two main features of this baseline scheme. On one hand, I allow for a change in the structural parameters which are not equal to zero in the first period, implying that the structural break does not introduce any new simultaneous relationship among the variables. On the other hand, the model is over-identified since enclosing 2 restrictions more than the minimum amount required. These over-identifying restrictions will be particularly useful in the next subsection when exploiting them to test the significance of further assumptions about the structural parameters and their changes after the structural break.

5.2 Identification of structural shocks with non-null elasticity of public expenditure to GDP

The assumption that the elasticity of public expenditure is equal to zero is widely considered in the literature. For example, Blanchard and Perotti (2002) asserted that the time needed by the government to approve new spending laws is too long to make the public expenditure react to GDP in the same period particularly when working with quarterly intervals. This assumption, implying no simultaneous linear dependence of G to GDP, was subsequently made also by Martens and Ravn (2013). However, the aim is to exploit the two over-identifying restrictions imposed on the baseline model previously shown to test the assumption that the elasticity of public expenditure to GDP is equal to zero. Indeed, the presence of automatic stabilizers such as US unemployment insurance mechanisms may let public spending increase when unemployment increases during economic downturns. Therefore, the public expenditure is expected to show a negative, therefore anti-cyclical, simultaneous relationship (elasticity) with the GDP. Thus, this underlying economic reasoning

[6] From now on the elasticity of tax in this setting is equal to -θ_1 or -θ_2. This specification is made in order to distinguish between the estimated value of θ and how it enters the system of equations

leads to change A and C in the following way[7]:

$$\left(\begin{bmatrix} a_{11} & 0 & a_{13} & 0 \\ 0 & a_{22} & a_{23} & 0 \\ a_{31} & a_{32} & a_{33} & 0 \\ 0 & a_{42} & a_{43} & a_{44} \end{bmatrix} + \begin{bmatrix} c_{11} & 0 & c_{13} & 0 \\ 0 & c_{22} & c_{23} & 0 \\ c_{31} & c_{32} & c_{33} & 0 \\ 0 & c_{42} & c_{43} & c_{44} \end{bmatrix} \mathbb{I}_4(t \geq 1984Q1)\right) \begin{bmatrix} u_t \\ u_g \\ u_{gdp} \\ u_r \end{bmatrix} = \begin{bmatrix} \epsilon_t \\ \epsilon_g \\ \epsilon_{gdp} \\ \epsilon_r \end{bmatrix}$$

Given this matrix form, the only equation that changes from the previous system of is the one describing the behavior of public expenditure. In fact, while previously the innovation of G is dependent only on itself, this scenario allows for a non-null elasticity of G to GDP, which is defined as -γ[8]. This change is shown by the two new equations reported below:

$$t < 1984Q1 : u_g = -\underbrace{\frac{a_{23}}{a_{22}}}_{\gamma_1} u_{gdp} + \frac{1}{a_{22}}\epsilon_g \qquad ; t \geq 1984Q1 : u_g \qquad = -\underbrace{\frac{a_{23}+c_{23}}{a_{22}+a_{22}}}_{\gamma_2} u_{gdp} + \frac{1}{a_{22}+c_{22}}\epsilon_g$$

The expected result for this elasticity is a negative value in both regimes since the assumption of an anti-cyclical behavior of G. However, after estimating this new setup, I performed a quasi-likelihood ratio test for over-identifying assumptions (see section 6.3).

6 Empirical Results

In the following subsections, I present the results obtained by estimating and testing, when possible, the SVAR models outlined in the previous section. Since the objective of the analysis is the empirical study of the impact of fiscal spending and taxes on GDP, I consider the fiscal dynamic multipliers associated with a one-dollar increase in both taxes (T) and public expenditure (G). Moreover, for each model, I report in the Appendix the graph representing all the IRFs characterizing the systems.
The tax and spending multipliers, computed in the way described below, respectively represent the dollar response of GDP to a one-dollar increase in tax revenues or public expenditures from their average levels. [9].

$$\text{Dynamic Tax multiplier} \longrightarrow \qquad \boldsymbol{M}_{T,h} = \frac{\left(\frac{\partial GDP_{t+h}}{\partial \epsilon_t^{\mathrm{T}}}\right)}{\frac{\partial \mathrm{T}_t}{\partial \epsilon_t^{\mathrm{T}}}} \Bigg/ \left(\frac{\overline{\mathrm{T}}}{\overline{\mathrm{GDP}}}\right), \quad h = 1, \ldots, 20.$$

$$\text{Fiscal spending multiplier} \longrightarrow \qquad \boldsymbol{M}_{G,h} = \frac{\left(\frac{\partial GDP_{t+h}}{\partial \epsilon_t^{\mathrm{G}}}\right)}{\frac{\partial \mathrm{G}_t}{\partial \epsilon_t^{\mathrm{G}}}} \Bigg/ \left(\frac{\overline{\mathrm{G}}}{\overline{\mathrm{GDP}}}\right), \quad h = 1, \ldots, 20.$$

For each model, the 90% confidence bands are computed by making use of the Moving Block Bootstrap method, so as not to incur the poor approximation of analytic ones, which in the finite sample can lead to over-rejecting the null of no significant effects. In particular, this bootstrap scheme is well-suited for

[7]Also a scheme with a_{23} equal to 0 and c_{33} free is supported by the necessary and sufficient condition. However, there are no significantly different results for these data

[8]the reasoning is the same reported in footnote 6

[9]The average ratios are obtained on the unlogged data and within each subsample

time series analysis, as it allows to account for Garch-type VAR innovations. As described by Brüggemann et al. (2016), to guarantee consistency, the confidence bands are computed as Hall's percentile intervals. In particular, I opt for blocks of length equal to 4 and 1000 replications. Moreover, in the application of this bootstrap technique, I also consider the structural break affecting the VAR coefficients $\mathbf{\Gamma}$ and variance-covariance matrix of innovations $\Sigma_{\mathbf{u}}$. For this reason, in the simulation of new samples, I use two different pools of innovation blocks (one for each of the two periods).

In the first subsection, I present and comment on the multipliers out of the Lanne and Lütkepohl approach, whereas the second is devoted to the General over-identified specification. The third subsection discusses the setting with Non-null public expenditure elasticity to GDP. Finally, the last subsection is devoted to testing and commenting on the best model.

6.1 Lanne and Lütkepohl approach results

Estimating the model accordingly to Lanne and Lütkepohl I get the following values for the structural parameters included in matrices B and V:

$$B = \begin{bmatrix} 0.0051 & 0.0144 & 0.00017 & 0.0243 \\ 0.000278 & 0.0057 & -0.0128 & -0.0008 \\ -0.002 & 0.0096 & 0.00056 & 0.00083 \\ -0.0081 & -0.00011 & 0.00052 & 0.0028 \end{bmatrix}; V = \begin{bmatrix} 0.3988 & 0 & 0 & 0 \\ 0 & 0.2697 & 0 & 0 \\ 0 & 0 & 0.4171 & 0 \\ 0 & 0 & 0 & 0.6596 \end{bmatrix}$$

As shown by matrix B and **Figure 2**, the on-impact effect of tax on GDP is negative (-2.15 dollars) while the effect of public expenditure is positive (8.24 dollars). However, the large uncertainty characterizing the point estimates is reflected by very large confidence bands. For this reason, I conclude that the effect of tax and public expenditure shocks on GDP is not statistically significant for each period included in our time horizon. Moreover, to get a comparison with the following A-model specifications results, as done by Martens and Ravn (2014), I derived the implied A matrix from the previously estimated B matrix. Thus, the estimated value of θ is -1.36, which, considering the system of equations, implied by an A-model, can be interpreted as a positive elasticity of tax to GDP. In absolute terms, this value is slightly lower than the one selected by Blanchard and Perotti(2002). Whereas, the elasticity of public expenditure to GDP is negative and equal to -29.59, suggesting an anti-cyclical behavior of this variable.

6.2 General over-identified specification results

For the general over-identified specification, two settings have been explored: the first assumes a change also in the VAR coefficients $\mathbf{\Gamma}$. In contrast, the second considers them constant across the volatility regime change.

In the first case, the estimated values of θ_1 and θ_2, are respectively -1.412 and 0.66. The values can be compared with the -2.08 constraint estimated externally by Blanchard and Perotti (2002) and the one (-3.14) implied by the proxy SVAR method applied by Mertens and Ravn (2014). Importantly, by considering the parameters in the system of equations shown in the theoretical framework in subsection 5.1, the on-impact effect of GDP on tax revenues is positive in the first period and negative for the second one. However, the latter is not statistically significant as shown by the associated IRFs (see Appendix).
Figure 3 shows, in the two different periods, the results of this first setting. Surprisingly, by looking at

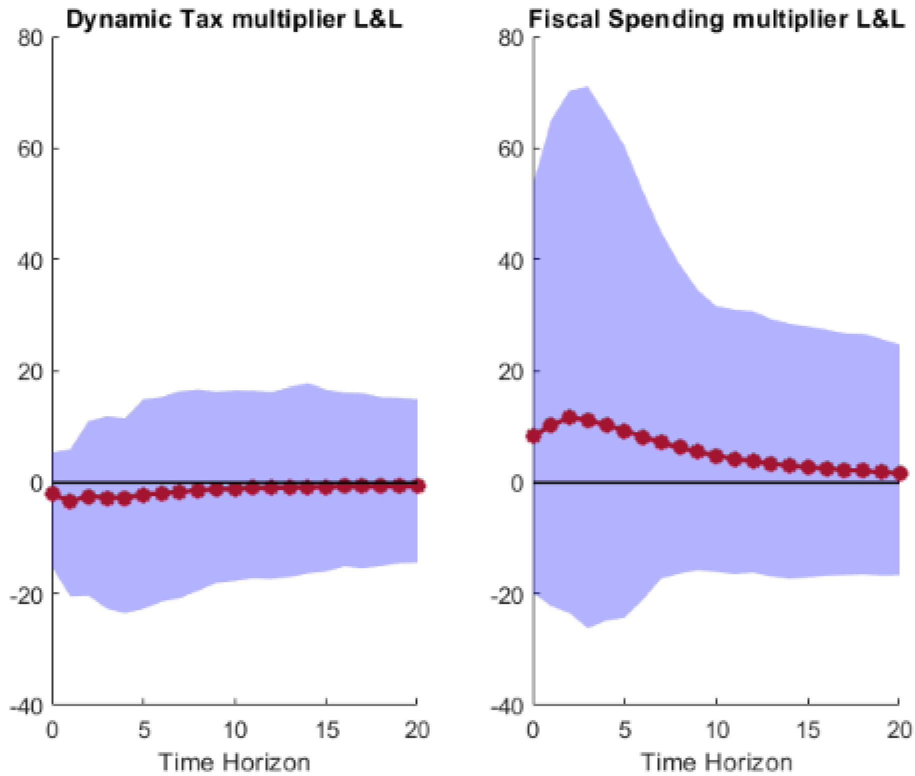


Figure 2: Fiscal multipliers derived from the Lanne and Lütkepohl B-model

the point estimates (red line) a one-dollar increase in tax revenues average level has respectively a positive impact effect of 0.05 (pre-Volcker) and 0.50 (Great Moderation) dollars. However, the 90 percent confidence interval depicted with the blue area, suggests that the on-impact effect is not statistically different from zero in both periods. A significant negative effect is seen starting from six quarters ahead of the tax increase for the pre-Volcker era, while a significant positive effect (0.37 dollars) is observed almost two years from the increase for the Great Moderation era. What remains consistent with the most prominent literature on the multipliers are the Fiscal Spending multipliers with a significant positive effect of 1.18 and 1.10 dollars following a one-dollar increase respectively in the pre-Volcker and Great Moderation period, with peaks of 2.14 and 1.69 both reached six months ahead the spending increase. Noticeably, the lower variability associated with the estimates for the pre-Volcker era results in a longer-lasting positive and significant effect than the Great Moderation ones.

In the second case, the one with a common reduced form across the two periods, the estimated values of θ_1 and θ_2, are respectively -1.3288 and -1.0132. This implies, that in both periods the tax elasticity is always positive, differently from the changing reduced form parameters' case. However, this approximation of a common reduced form across the two periods, while resulting in more reasonable point estimates of the multipliers (**Figure 4**), shows very large 90% confidence intervals not leaving any room for significant effects, except for the on-impact increase of 1.08 dollars following a one-dollar increase in public expenditure for the pre-Volker era.

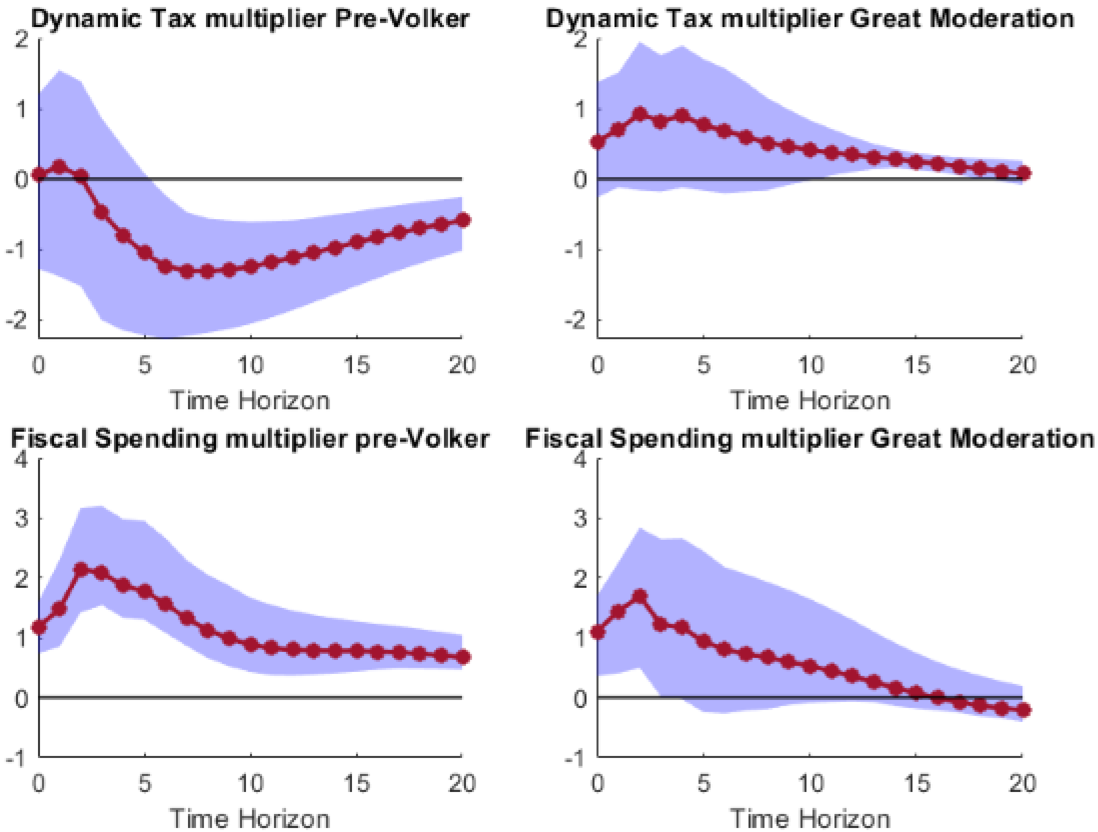


Figure 3: General over-identified specification set of multipliers with changing VAR coefficients.

6.3 Non-null elasticity of public expenditure to GDP results

Also for this specification, I report the resulting multipliers for both changing and constant reduced forms across the two periods.

As shown in **Figure 5**, the non-null elasticity scheme with changing reduced form parameters exhibits similar behavior to the corresponding one analyzed in the previous scheme, with some differences that we discuss below. First, there is an uncertainty reduction in this new setting, especially for the on-impact value of the tax multiplier and for the spending multiplier accounting for the Great Moderation period. Second, the values of the on-impact spending multipliers are for the two periods respectively equal to 2.70 and 2.96 dollars with peaks of 3.96 and 4.17 dollars two quarters after the shock. This means that a spending increase has a greater effect on GDP than in the previous case. Third, the surprisingly positive tax multiplier computed for the Great Moderation period exhibits lower levels with more spread uncertainty across all time horizons considered. Moreover, consistent with the assumption of an anti-cyclical response of government spending as a reaction to an innovation in GDP, both the values of γ_1 and γ_2 are respectively 0.9624 and 1.2468 (they are consistent because entering negatively in the system of equations specified in the theoretical section). The values of θ_1 and θ_2 for this model are -1.4379 and -0.4891, implying positive tax revenues to GDP elasticities.

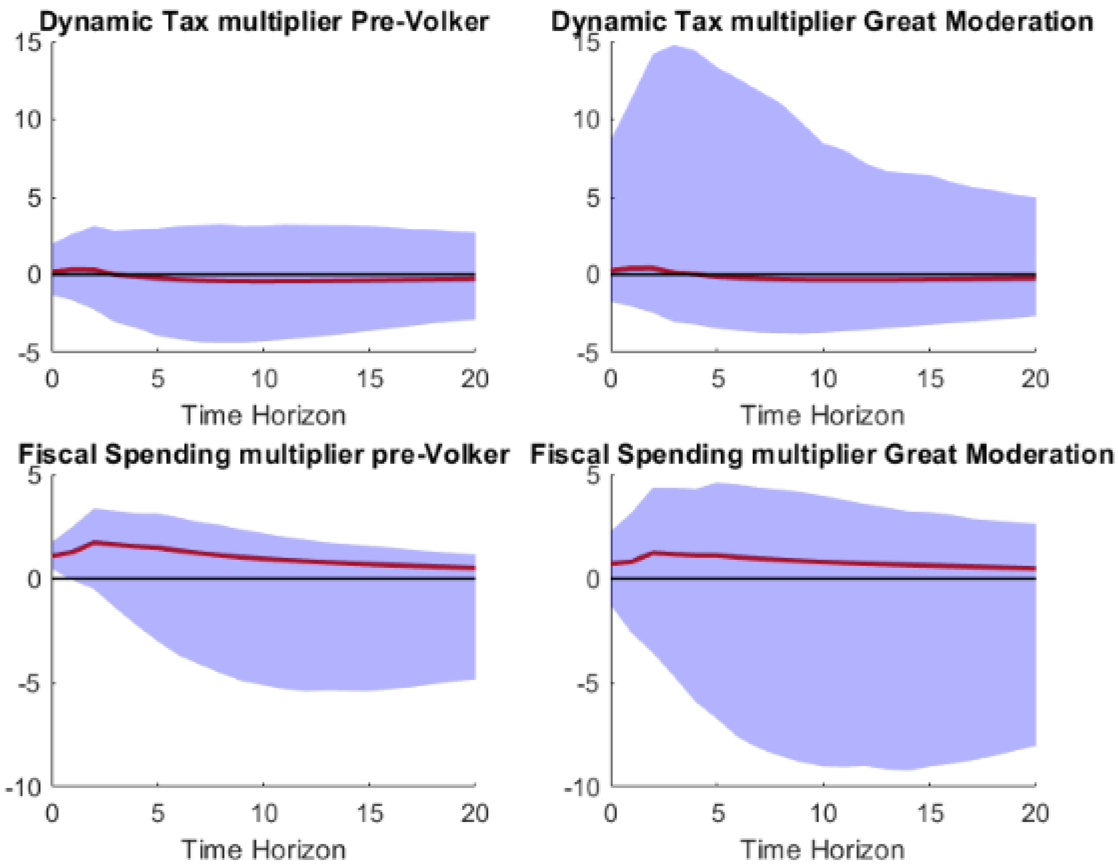


Figure 4: General Over-identified specification set of multipliers with constant VAR coefficients.

In the model specification with a constant reduced form, the estimated values for θ_1 and θ_2 are respectively -1.5776 and -1.8662, still implying positive tax revenues elasticities to GDP. The estimated values of γ_1 and γ_2 are respectively 1.9970 and 2.6064, implying that the responses of G to an increase in GDP are negative in both periods. As seen for the previous specification, the constant reduced form setting (Figure 5) is characterized by vast confidence intervals that also in this case exclude almost any significant impact of an increase in the policy variables (G or T) on GDP, except for an initial significant value of the fiscal spending multiplier accounting for the pre-Volcker era. Being 6.46 dollars on impact, this effect lasts just for two quarters ahead of the shock.

6.4 Which specification best fits the data?

To assess which model best fits the data I first compare the two approaches (L&L and Bacchiocchi Fanelli) in an empirical and computational shade. Although L&L gives at least more reasonable point estimates (negative tax multiplier and positive spending one), the large confidence bands and the unreasonable assumption of constant structural parameters other than constant VAR coefficients lead to consider it not so exhaustive. Moreover, the maximization algorithm employed for this approach estimation presents very high instabilities in the results. Second, focusing on the other approach, recall that the Chow-type quasi-

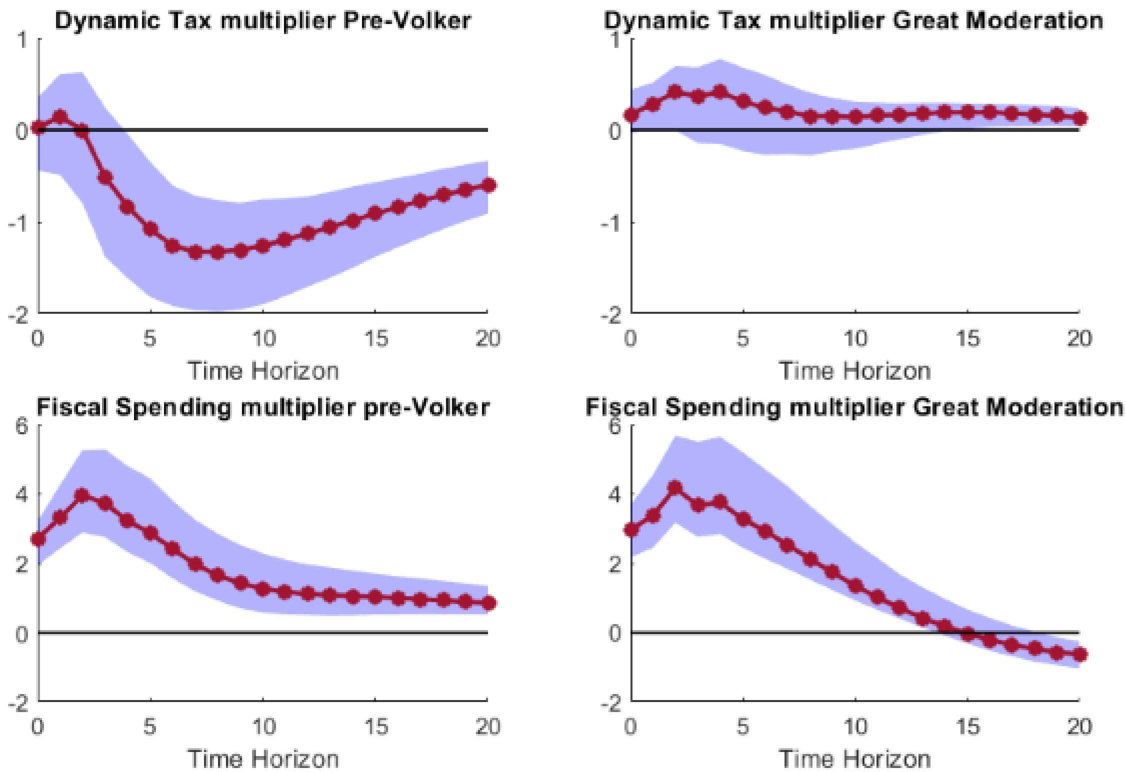


Figure 5: Non-null elasticity specification set of multipliers with changing VAR coefficients.

likelihood ratio test, shown in section 2, already suggests that a specification with changes in the reduced form parameters is supported by the data with a very high confidence level. Nonetheless, I presented the results for both specifications for three main reasons. The first is to show that when moving from constant to changing VAR coefficients there is a remarkable reduction in the uncertainty related to the multipliers point estimates. Secondly, the results with the change in VAR coefficients are less reliable as the time horizon increases, since the VAR estimated in the second period exhibits a unit root, whereas a constant reduced form exhibits inverse roots all inside the unit circle. Finally, by presenting the results with both changing and constant VAR coefficients, one can better appreciate the factors contributing to the variation in fiscal multipliers between the two periods. Specifically, one can see the differences originated from a change in the structural parameters affecting the on-impact effect and the differences that are due to a variation in the dynamics (VAR coefficient). Moreover, the fact that one of the SVAR schemes is over-identified opens the possibility of testing it against an exactly identified scheme, the one called non-null elasticity of G to GDP. Again, I make use of a quasi-likelihood ratio test specified as follows $LR = -2 * [L_1 - L_2]$ where L_1 is the quasi-likelihood of the exactly identified specification, and L_2 is the quasi-likelihood of the over-identified SVAR. Under the null hypothesis of the restricted (over-identified) model being the "true" one, the LR statistic is distributed as a χ_2. The computed result of the test is 0.0091 with a p-value of 0.9955, so the null is not rejected. As a result, the over-identified scheme with a changing reduced form seems to best fit the data. Moreover, this implies that the assumption of the elasticity of G to GDP being equal to zero is confirmed by the data.

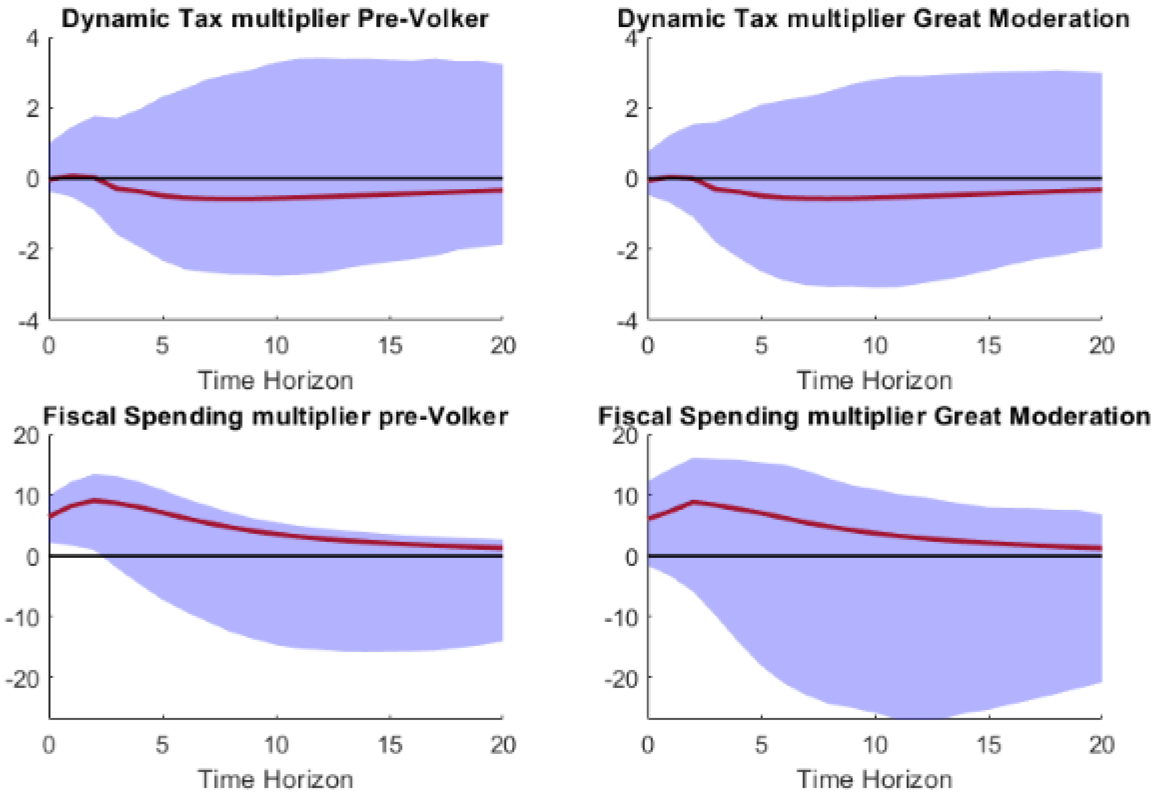


Figure 6: Non-null elasticity specification set of multipliers with constant VAR coefficients.

7 Conclusions

This study employs a well-known methodology, typically used in monetary policy studies, to give its contribution to the wide and debated literature about the estimation of fiscal multipliers. This work, mainly focused on two approaches of the same methodology, is performed both in the theoretical and empirical part under the guideline of the common literature attention in two crucial parameters: the elasticity of both taxes and public expenditure to GDP, as done in Caldara and Kamps (2017). Concerning the two mentioned elasticities, I conclude for a general reduction of the tax elasticity when moving to the Great Moderation period and confirm a non-statistically different from zero public expenditure elasticity to GDP. Moreover, I always point out by comparison the improvements achieved by allowing both the structural parameters and the VAR coefficients to change across two different volatility regimes. Surprisingly, I find that the model that best fits the data also produces non-common-shaped dynamic tax multipliers. They differ from the common literature for two main reasons. First, the results for the two periods present a very high uncertainty on impact. Secondly, they show some positive point estimates of GDP following a tax increase. By contrast, the spending multiplier findings are very consistent with the common literature and act as refinements by shedding light on how a volatility break affects both levels and uncertainty characterizing the dynamic effects of a public expenditure increase. Being unconditional heteroskedasticity a source of information also for this topic, a possible improvement could surely stem from a sample expansion, as new volatility breaks that is new sources of information, may arise by the continuously shape-changing US economy.

8 Bibliography

1. Bacchiocchi, E. and Fanelli, L. (2015). Identification in Structural Vector Autoregressive models with structural changes, with an application to U.S. monetary policy. *Oxford Bulletin of Economics and Statistics* 77, 761-779.

2. Bacchiocchi, E., Castelnuovo, E.,and Fanelli, L. (2018). Give me a break! Identification and estimation of the macroeconomic effects of monetary policy shocks in the U.S. *Macroeconomic Dynamics* 22(6), 1613-1651.

3. Blanchard,O and Perotti, R.(2002). An Empirical Characterization of the Dynamic Effects of Changes in Government Spending and Taxes on Output. *Quarterly Journal of Economics* 117(4),1329-1368.

4. Brüggeman,R.,Jentsch, C. and Trenkler C. (2016). Inference in VARs with conditional heteroskedasticity of unknown form. *Journal of Econometrics* 191(2016),69-85.

5. Caldara, D. Kamps, C. (2017).The Analytics of SVARs: A Unified Framework to Measure Fiscal Multipliers.*Review of Economics Studies* 84(2017),1015-1040

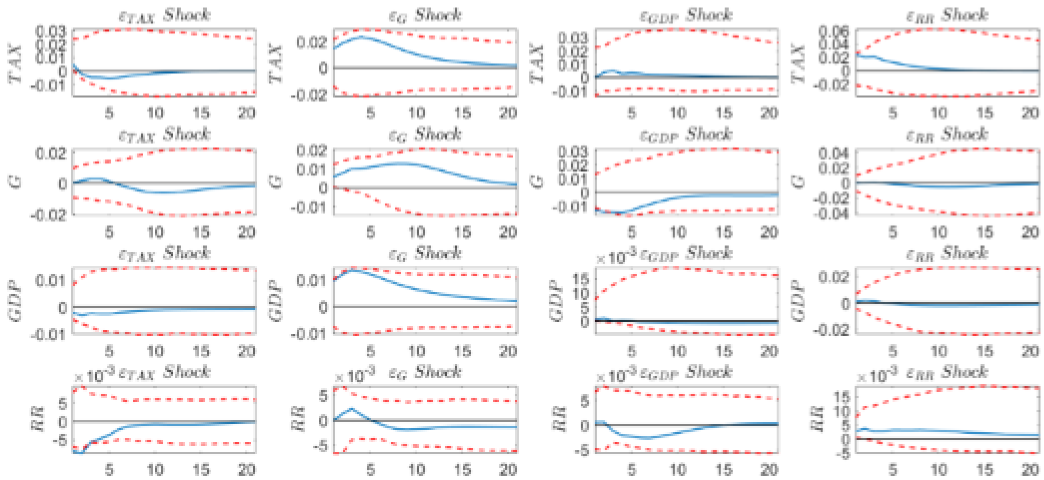
6. Lanne,M and Lütkepohl, H.(2008). Identifying monetary policy shocks via changes in volatility. *Journal of Money, Credit and Banking* 40, 1131-1149.

7. Mertens, K. and Ravn, M. (2014), A reconciliation of SVAR and narrative estimates of tax multipliers. *Journal of Monetary Economics* 68, S1-S19.

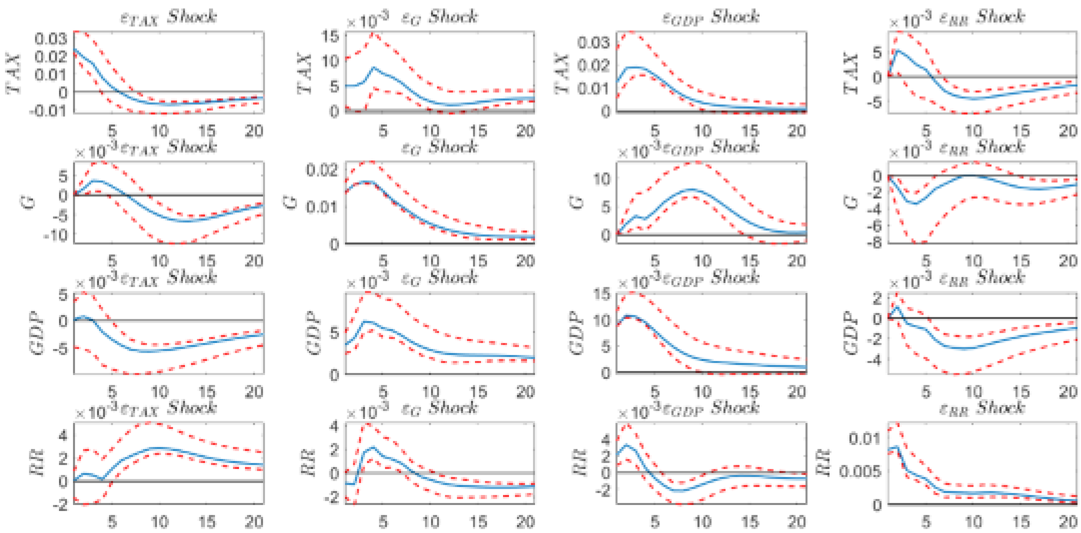
8. Kim, C. and C. Nelson (1999) Has the U.S. economy become more stable? A Bayesian approach based on a Markov-switching model of the business cycle. *Review of Economics and Statistics* 81, 608–616.

9. Rigobon, R. (2003) Identification through heteroskedasticity. *Review of Economics and Statistics* 85, 777–792.

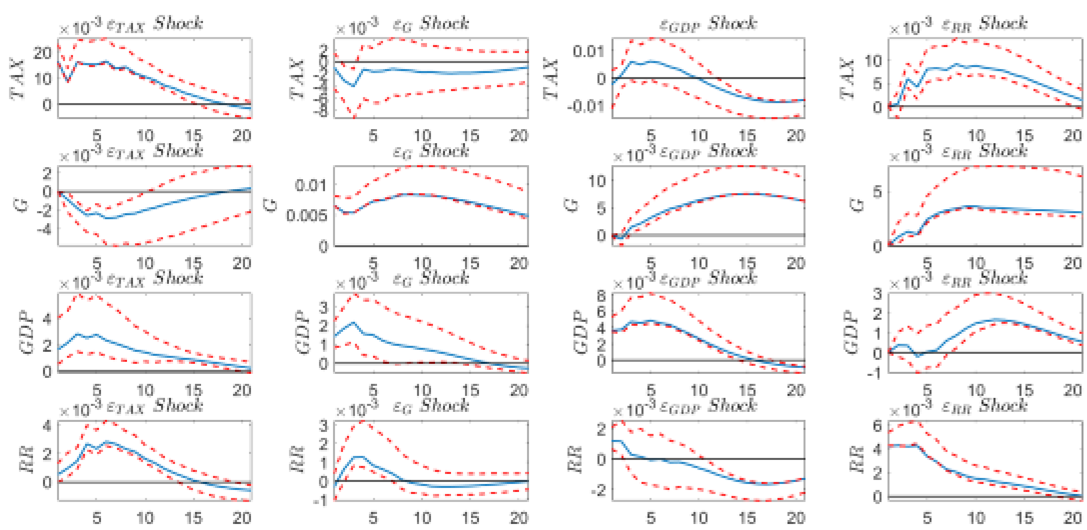
9 Appendix



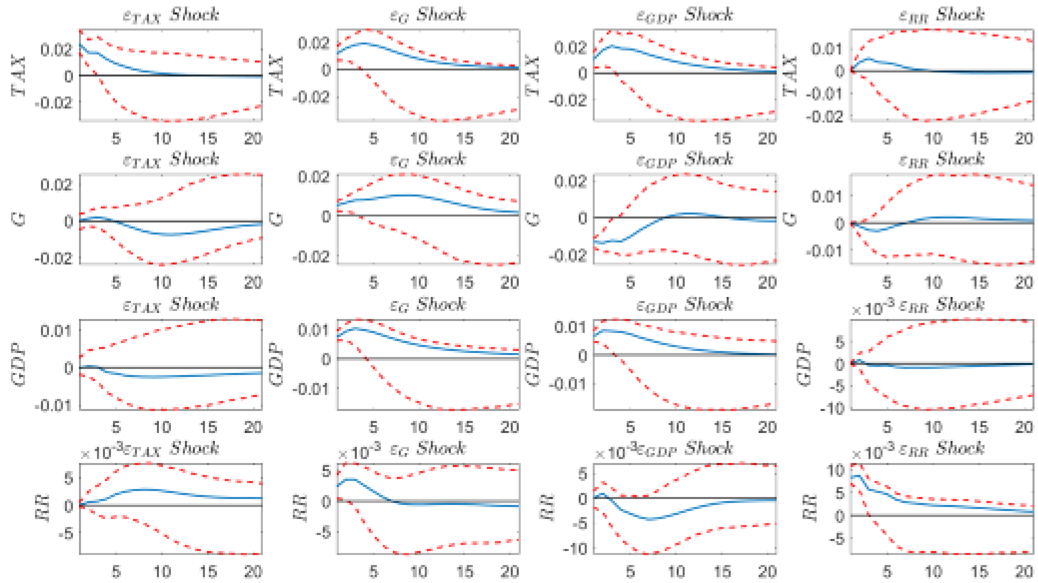
Set of IRFs for the L&L approach



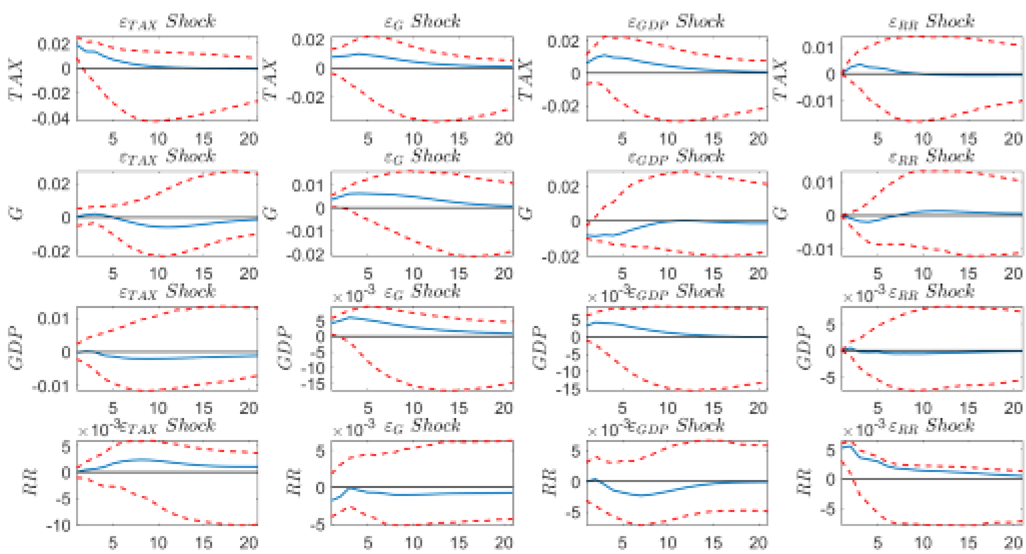
Set of IRFs for the General over-identified model with changing VAR for the pre-Volcker period



Set of IRFs for the General over-identified model with changing VAR for the Great Moderation period



Set of IRFs for the General over-identified model with common VAR coefficients for the pre-Volcker period



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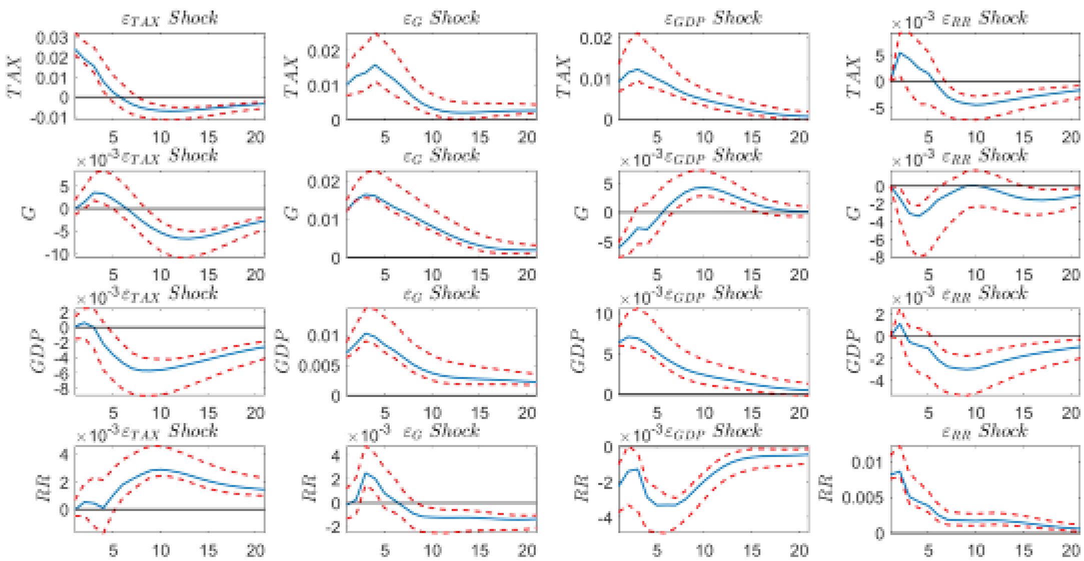
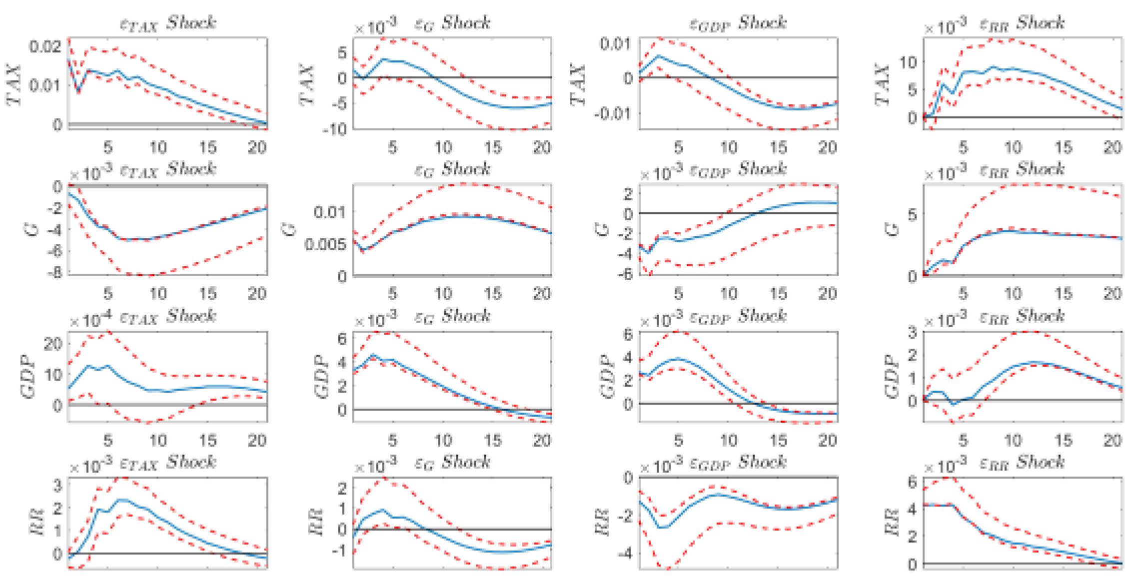


Set of IRFs for the Non-null elasticity model with changing VAR coefficients for the pre-Volcker period



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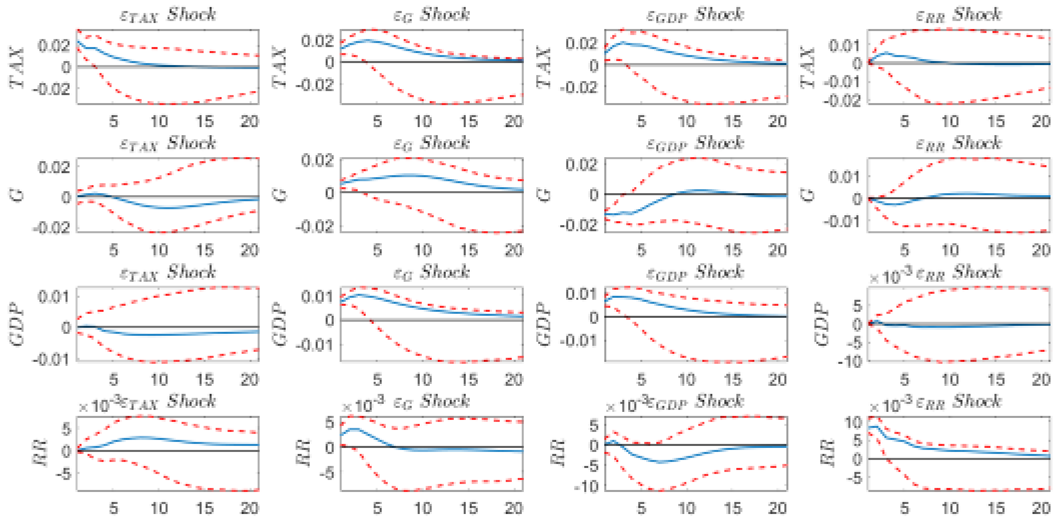

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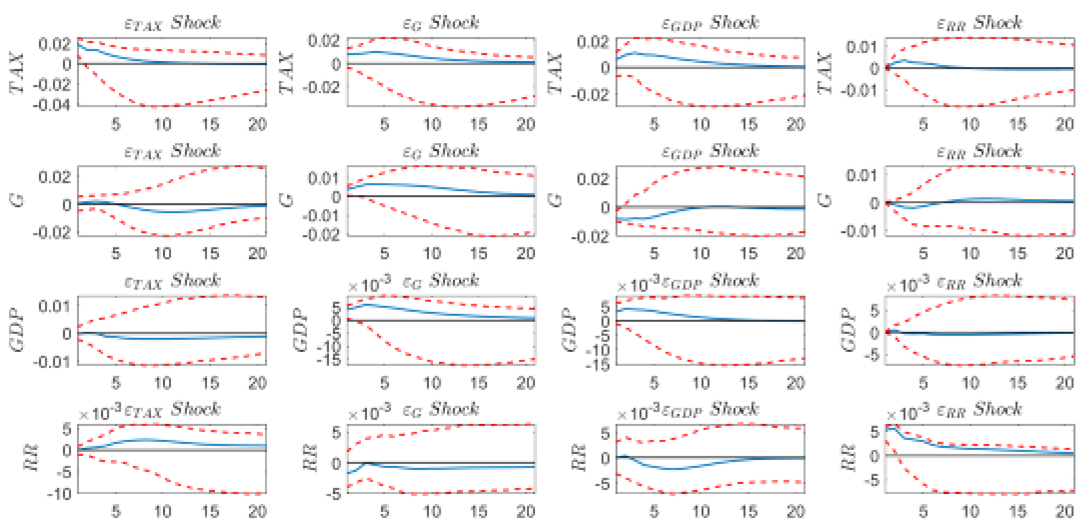

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