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The Role of a Degenerate Prior on the Posterior Distribution: A Bayesian RBC model

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Contents

List of Figures

List of Tables

List of Acronyms

DSGE	Dynamic Stochastic General Equilibrium
PDF	Probability density function
RBC	Real Business cycle

1 Introduction

Dynamic Stochastic General Equilibrium turned out to be a potent and flexible class of models aimed at approximating the economy's structure by focusing on agents' representative behavior. Their usage is of fundamental importance in many macroeconomics applications that are interested in forecasting and/or explaining business cycle fluctuations. However, a correct specification before any policy inference is of crucial importance. Both external studies and available time series data can be used to calibrate the fundamental parameters that drive the stylized model equations. Those parameters usually range from the depreciation of capital to the persistence of a technology shock and consumer discount factor.

Furthermore, Bayesian inference has largely provided additional tools to ease the challenge of a correct specification. In particular, beliefs about the value of the aforementioned parameters are used to foster what the data cannot capture for whatever reason. These beliefs come in the form of a parameter prior distribution cast by the analyst. The prior distribution, along with the likelihood function implied by the data is used to build an "updated" distribution of the parameter values, namely the posterior posterior.

The present work aims to shed light on how much the specification of the prior matters on the posterior distribution. The results suggest that an in-the-limit degenerate beta prior entirely dictates the posterior of the capital depreciation and technology shock persistence parameters, leaving little room for the data-implied ones. The work is organized as follows: Section 2 defines the structural model, its solution and the econometric approach along with the Montecarlo setup, Section 3 presents the results and Section 4 concludes.

2 Methodological Framework

To understand the role of the prior distribution as mentioned in the introduction, the results in the next section rely on a standard RBC model of a representative consumer optimal intertemporal consumption choice which is solved by employing a first-order perturbation and therefore expressed in terms of a linear state-space model. This is then used to simulate 5 series for investment of 200 quarters. Finally, the likelihood of the data is nested with a given prior distribution, and a posterior is derived using a Metropolis-Hastings algorithm. The following subsections present all the steps mentioned above in detail.

2.1 Structural Model

A representative household that derives utility from consumption is interested in maximizing his/her discounted utility over time given that he/she produces output from the capital and a technology factor that follows a Markov process. The produced output is available for consumption and to increase the net stock of capital in the next period.
Formally:

$$\max_{C_{t+s},K_{t+s+1}} \quad U_t = \mathbb{E}_t \sum_{s=0}^{\infty} \beta^s u_{t+s} \tag{1}$$

$$s.t. \quad Y_{t+s} = C_{t+s} + I_{t+s} \tag{2}$$

$$Y_{t+s} = e^{Z_{t+s}} F(K_{t+s}) \tag{3}$$

$$K_{t+s+1} = I_{t+s} + (1-\delta)K_{t+s} \tag{4}$$

Rearranging equations (2)-(4) in a unique constraint gives:

$$K_{t+s+1} = e^{Z_{t+s}} F(K_{t+s}) - C_{t+s} + (1-\delta)K_{t+s} \tag{5}$$

The constraint in equation (5) can then be used to obtain the following Lagrangian:

$$\begin{aligned}\mathcal{L} &= u(c_t) + \beta \mathbb{E}_t u(C_{t+1}) + \beta^2 \mathbb{E}_t u(C_{t+2}) + \dots \\ &+ \Lambda_t \left(e^{Z_t} F(K_t) - C_t + (1-\delta)K_{t+1}\right) \\ &+ \beta \mathbb{E}_t \Lambda_{t+1} \left(e^{Z_{t+1}} F(K_{t+1}) - C_{t+1} + (1-\delta)K_{t+2}\right) \\ &+ \beta^2 \mathbb{E}_t \Lambda_{t+2} \left(e^{Z_{t+2}} F(K_{t+2}) - C_{t+1} + (1-\delta)K_{t+2}\right) + \dots.\end{aligned} \tag{6}$$

with FOCs:

$$\frac{\partial \mathcal{L}}{\partial C_{t+s}} = \beta^s \mathbb{E}_t u'(c_{t+s}) = \beta^s \mathbb{E}_t \Lambda_{t+s}; \quad s = 0, 1, .., \infty \tag{7}$$

$$\frac{\partial \mathcal{L}}{\partial K_{t+s+1}} = \beta^s \mathbb{E}_t \Lambda_{t+s} = \beta^s \mathbb{E}_t \Lambda_{t+s} e^{Z_{t+s}} F'(K_{t+s}) + (1-\delta); \qquad s = 0, 1, .., \infty \tag{8}$$

Importantly, the terminal condition will require that when s goes to infinity, no capital is left over in the subsequent period, otherwise it will be possible for the household to be better off, given that the unique argument of his utility is consumption.
A more convenient way to solve this problem is to formulate it in terms of the value function, once provided with the parametrization in (9), where: the utility function

is assumed to be logarithmic and as so concave in consumption; the production function depends on capital and α, the share of capital in output and the technology shock Z_t following an AR(1) process with conditional volatility ω.

$$u_t = ln(C_t); \qquad F(K_t) = K_t^{\alpha}; \quad Z_{t+1} = Z_t + \omega\epsilon_{t+1}; \quad \epsilon_{t+1} \sim \mathcal{N}(0,1). \tag{9}$$

The value function of the above problem is:

$$V(K_t, Z_t) = \max_{K_{t+1}} \mathbb{E}_t \sum_{s=0}^{\infty} \beta^s ln(C_{t+s}) \tag{10}$$

$$s.t. \;\; C_{t+s} = e^{Z_{t+s}} K_t^{\alpha} - K_{t+s+1} + (1-\delta)K_{t+s} \tag{11}$$

The algebraic manipulation provided in A allows us to equivalently state this problem as:

$$V(K_t, Z_t) = \max_{K_{t+1}} [ln(e^{Z_t} K_t^{\alpha} + (1-\delta)K_t - K_{t+1}) + \beta\mathbb{E}_t V(K_{t+1}, Z_{t+1})] \tag{12}$$

with FOC:

$$\frac{1}{\underbrace{e^{Z_t} K_t^{\alpha} + (1-\delta)K_t - K_{t+1}}_{C_t}} = \beta\mathbb{E}_t \left[\frac{\alpha e^{Z_{t+1}} K_{t+1}^{\alpha-1} - (1-\delta)}{\underbrace{e^{Z_{t+1}} K_{t+1}^{\alpha} + (1-\delta)K_{t+1} - K_{t+2}^{*}}_{C_{t+1}}} \right]. \tag{13}$$

The FOC along with the constraint in (11) constitute the canonical form [1] of the model, namely $\mathbb{E}_t\Gamma(\underbrace{O_t, X_t, Z_t}_{S_t}, \underbrace{O_{t+1}, X_{t+1}, Z_{t+1}}_{S_{t+1}}, \underbrace{\nu_{t+1}}_{\omega\epsilon_{t+1}})$. [2]

$$\mathbb{E}_t\Gamma[S_t, S_{t+1}, \nu_{t+1}] = \begin{bmatrix} \frac{1}{e^{Z_t} K_t^{\alpha} + (1-\delta)K_t - K_{t+1}} - \beta\mathbb{E}_t \left[\frac{\alpha e^{Z_{t+1}} K_{t+1}^{\alpha-1} - (1-\delta)}{e^{Z_{t+1}} K_{t+1}^{\alpha} + (1-\delta)K_{t+1} - K_{t+2}^{*}} \right] \\ e^{Z_t} K_t^{\alpha} - K_{t+1} + (1-\delta)K_t - C_t \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \tag{14}$$

The canonical form (14) is essentially made of the FOC of the maximization problem and the constraint (second row). The solution of this system concerning C_t and K_{t+1} would constitute the solution of the model in terms of the policy functions (i.e. the optimal amount of consumption given the amount of capital and the technology

[1] The notation of its arguments are provided in B.

[2] O_t is the equivalent of $\mathbf{y}$ not in ln in 1 of "*Create and solve a dynamic stochastic rational expectations model with the use of the lecture's code-Summer Semester 2024 Prof. Dr. Daniel Fehrle* (n.d.)".

shock). Namely:

$$C_t = C(K_t, Z_t);\ K_{t+1} = K(K_t, Z_t) \tag{15}$$

Unfortunately, unless there is a closed-form solution (not usually the case for complex models), one must rely on approximations and numerical methods. Many available methods are distinguishable in local and global ones. While the first linearly approximates the function we are interested in around a point of the state space (i.e. the steady state), the latter generally finds a solution on a chosen set of the state space. For simplicity, this work relies on local linear methods like perturbations of the first order. A perturbation of the first order consists of approximating the functions in (15) with a first-order Taylor expansion in a neighborhood of the steady state of capital and consumption[3] of the kind[4]:

$$\begin{aligned} \ln(C_t) &= \ln(C^*) + c_k(\ln(K_t) - \ln(K^*)) + c_z Z_t; \\ \ln(K_{t+1}) &= \ln(K^*) + k_k(\ln(K_t) - \ln(K^*)) + k_z Z_t \end{aligned} \tag{16}$$

Where c_k, c_z, k_k, and k_z are respectively the derivatives of the consumption and capital policy functions with respect to capital and technology shocks. The definition of the steady state in this setting comes as a moment in which the dynamics of the variables are not perturbed by a stochastic shock, so both state and control do not change quarterly.

The steady states of the variables are [5]:

$$K^* = \left(\frac{1-\delta(1-\beta)}{\alpha\beta}\right)^{\frac{1}{\alpha-1}}; \quad Y^* = (K^*)^\alpha; \quad C^* = Y^* - \delta K^*; \quad I^* = Y^* - C^* \tag{17}$$

The system in (14) can be written as a function of the first-order linear approximation:[6]

$$\mathbb{E}_t \Gamma[C(K_t, Z_t), C(K(K_t, Z_t), Z_{t+1}), K_t, K(K_t, Z_t), Z_t] = F \tag{18}$$

To find the parameters of interest c_k, c_z, k_k, and k_z, the derivatives of F with respect to K_t and Z_t are set to zero. Formally:

$$F_k = c_k F_{c(k_t, Z_t)} + c_k k_k F_{c(k(k_t, Z_t), Z_{t+1})} + F_{k_t} + k_k F_{k(k_t, Z_t)} = 0 \tag{19}$$

[3] Here just the policy function of consumption and capital is considered for representing respectively the group of controls O and the group of X which is here composed only by capital. A policy function is also derived for the other controls.

[4] The logarithms of the variables are considered for a convenient interpretation. This does not necessarily come with the first-order perturbation.

[5] Those variables are transformed in natural logarithm in the computation of the Jacobian.

[6] C shows the system written in extensive form evaluated at the policy functions approximations

$$F_Z = c_z F_{c(k_t,Z_t)} + (c_k k_k + c_k) F_{c(k(k_t,Z_t),Z_{t+1})} + F_{k_t} + k_z F_{k(k_t,Z_t)} + F_{Z_t} = 0 \quad (20)$$

Equations (19) and (20) constitute a four system of four equations in four unknowns. Finally, solving this system for the unknowns allows us to write down the policy functions as stated in (16), provided the parametrization in Table 1[7].

Parameter	Description	Value
α	Capital share	0.3
β	Discount factor	0.99
δ	Rate of capital depreciation	0.02
ρ	AR(1) coefficient	0.9
ω	Conditional standard deviation of Z_t	0.01

Table 1: Parameters values

2.2 Econometric approach

The following steps in analyzing the role of a changing prior on the posterior distribution are: build up a likelihood function of the observations, make it dependent on the parameters of interest value, merge it with different prior specifications, simulate a posterior distribution, and compare its descriptive statistics with those of the prior.[8]
The way observations are simulated is discussed in the next section. The whole econometric approach can be summarized by the Bayes formula:

$$p(\theta|i_{1:T}) = \frac{p(i_{1:T}|\theta)p(\theta)}{\int p(i_{1:T}|\theta)p(\theta)d\theta} \quad (21)$$

where, in this work,

- θ is either ρ or δ.
- $i_{1:T}$ is one of the five simulated time series for investment.
- $p(\theta \mid i_{1:T})$ is the posterior probability density function.
- $p(i_{1:T} \mid \theta)$ is the likelihood function of the observations as a function of the parameter value.
- $p(\theta)$ is the prior probability density function, discussed in the second subsection.

7 D provides the parametrized state-space model

8 The codes relative to this section are `Posterior_delta.m`, `Exercise_delta.m`; `Posterior_rho.m`, and `Exercise_rho.m`

2.2.1 Likelihood

The Likelihood of the data is computed by means of the Kalman filter on a Gaussian Linear state-space model[9] of the form[10]:

$$i_t = H_k^i(\theta)k_t + H_z^i(\theta)Z_t; \tag{22}$$

$$\begin{bmatrix} k_{t+1} \\ Z_{t+1} \end{bmatrix} = \begin{bmatrix} H_k^k(\theta) & H_z^k(\theta) \\ 0 & (\theta = \rho) \end{bmatrix} \begin{bmatrix} k_t \\ Z_t \end{bmatrix} + \begin{bmatrix} 0 \\ \omega\epsilon_{t+1} \end{bmatrix}; \text{ with } \epsilon_{t+1} \sim \mathcal{N}(0,1). \tag{23}$$

The algorithm from which the Likelihood is derived employing the Kalman filter is the same one described by Fernandez-Villaverde et al. (2016) in Chapter 10.1. Hence, once provided with simulated data, the output from filter evaluation is $p(i_{1:T} \mid \theta)$.

2.2.2 Prior

The Beta distribution turns out to be particularly useful in modeling the prior of δ and ρ given that both are lying on the support [0,1]. Moreover, the change of its parameters allows for different specifications of the distribution, providing high flexibility, which is very useful to show the posterior changing of behavior. Formally, the probability density function of the beta distribution is:

$$f(\theta, a, b) = \frac{\theta^{a-1}(1-\theta)^{b-1}}{\mathbb{B}(a,b)}; \ \theta \in [0,1]; \ \mathbb{B}(a,b) = \frac{\Gamma(a)\Gamma(b)}{\Gamma(a+b)} \tag{24}$$

where Γ is the gamma function. The distribution has expected value and variance:

$$\mathbb{E}(\theta) = \frac{a}{(a+b)} \tag{25}$$

$$\mathbb{V}(\theta) = \frac{ab}{((a+b)^2(a+b+1)^2)} \tag{26}$$

The flexibility of Beta distribution is particularly interesting for the purpose of this study given the interesting behavior of the variance in the limit, in particular:

$$\lim_{a=b\to 0} \mathbb{V}(\theta) = 1/4; \quad \lim_{a=b\to\infty} \mathbb{V}(\theta) = 0 \tag{27}$$

---

[9] This representation comes directly from the solution of the model expressed in log deviations from the steady states.

[10] Importantly, θ is either δ or ρ. The parenthesis is to highlight that the space model here is conceived as a function of those parameters separately: one exercise keeps δ at the value in Table 1 and maximizes with respect to ρ, the other does the other way round

Therefore, when pushing the parameters to infinity the distribution of the prior exhibits a degenerate behavior. Another interesting behavior of the Beta distribution is its curvature depending on whether $a = b < 1$, $a = b = 1$, $a = b > 1$ as shown in Figure 1.

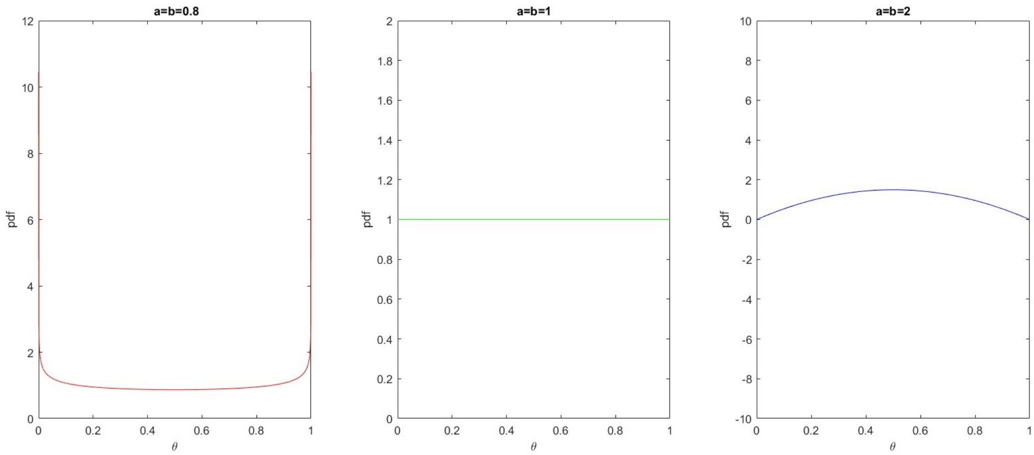

Figure 1: Curvature of the prior distribution for different values of a=b

2.3 Posterior maximization and Metropolis-Hasting Algorithm

Once the prior parameters have been specified, a proportional quantity of the posterior is maximized with respect to θ employing a numerical optimization:

$$\theta^* = argmax \; [p(i_{1:T}|\theta)p(\theta)] \tag{28}$$

The output of the numerical optimization is fundamental for two main reasons. First, under some regularity assumptions, it provides the value of the parameter more likely to be confirmed by the data and the initial beliefs of the prior. Second, the Hessian matrix from the numerical optimization is crucial when building the posterior distribution, which is not trivial to derive. Indeed, the probability density function (21) can be built by employing the Random Walk Metropolis-Hastings Algorithm suggested inFernandez-Villaverde et al. (2016) Chapter 12.2.2. Importantly, the covariance matrix for the algorithm is initialized at the inverse of the mentioned Hessian matrix (i.e. Hessian evaluated at the maximizer).
Finally, confronting the shape of the prior distribution with the sampled posterior ones will give an idea of how much and in which conditions the prior distribution dictates the posterior one.

2.4 Monte Carlo setup

All the investment series used for the likelihood are derived by means of a Monte Carlo simulation based on the state-space model in (38)-(39). In particular, the mentioned state space is solved by having as a reference the parameters in Table 1. The investment series are simulated on the parametrized state space in D.

The recursion for all five series starts with 0 as the initial value for the transition equation (39) and is stopped after 700 steps. The values of the transition equation are then used in the measurement equation to generate the investment series. Lastly, the first 500 generated observations are burned to safely eliminate the starting value effect. Importantly, each series is conceived to be interpreted as quarterly investment data.

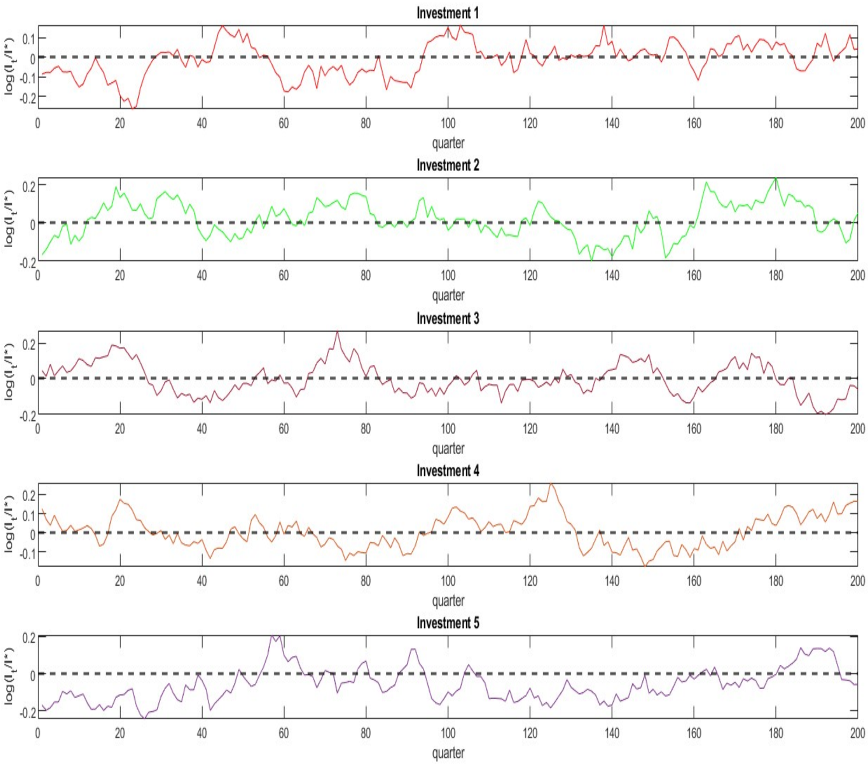

Figure 2: The five simulated series for quarterly-measured Investment

3 Results & discussion

The exercise on the prior-posterior comparison is separately conducted on the parameters δ and ρ. In particular, the comparison is made across all five simulated investment series to provide robustness to the central results: the change in the posterior distribution as the variance of the prior reduces. As mentioned before, the higher the value $a = b$ the lower the variance of the prior distribution.[11] This section presents and discusses the results for δ and ρ based posterior.

3.1 Results for δ

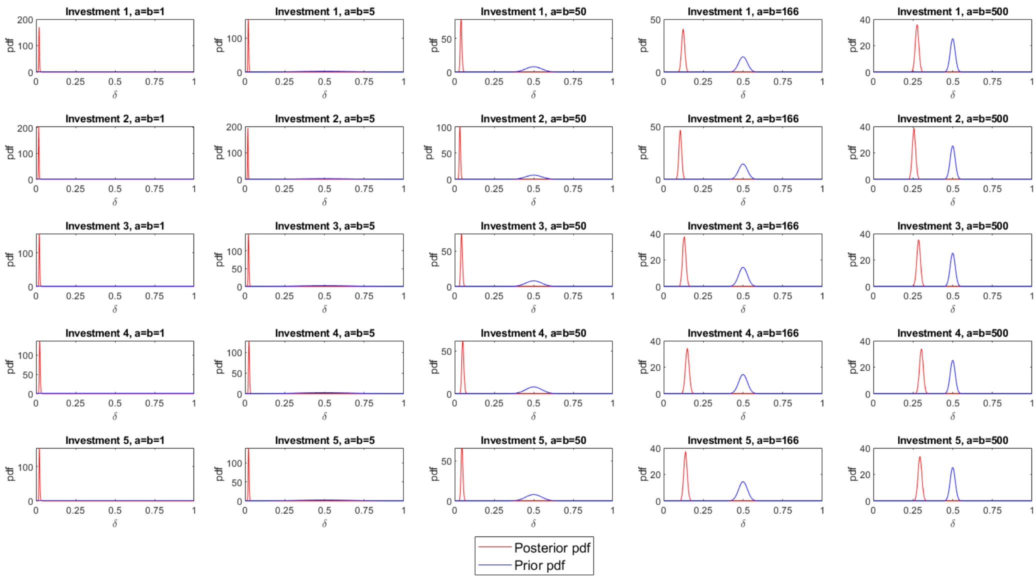

Figure 3: Prior (blue) and Posterior (red) probability density functions comparison for δ.

Figure 3 shows the posterior-prior comparison for the parameter delta across all the different Investment series and Beta distribution parameters. In particular, the first column considers the case in which the Beta is collapsed into a uniform distribution (see Figure 1). In this first case, the results consistently show that a uniform distribution between 0 and 1 does not have a relevant effect in shaping the posterior distribution of the parameter delta. However, columns 2 to 5 show that this is not the case when the variance of the prior significantly decreases (a and b increasing). Indeed, the posterior distribution mode converges toward the prior one.

[11] The exercise is conducted for $a = b = [1, 50, 166, 500]$

Interestingly, also the variance of the posterior[12] changes with the prior parameters, keeping increasing from the minimum reached at $a = b = 1$.
The following Table 2 shows the results for maximizers of the proportional posterior as presented in (28) for different levels of prior parameters. The value of δ (column 1) is stable around the one used to simulate the series when the prior distribution is collapsed to a uniform between 0 and 1. However, as shown in Figure 3 it converges toward the prior mode (0.5) as $a = b$ increases.

	$\theta^* = \delta^*$				
Series	$a = b = 1$	$a = b = 5$	$a = b = 50$	$a = b = 166$	$a = b = 500$
Investment 1	0.0191	0.0202	0.0399	0.1221	0.2749
Investment 2	0.0168	0.0176	0.0325	0.1041	0.2555
Investment 3	0.0203	0.0215	0.0428	0.1286	0.2849
Investment 4	0.0229	0.0244	0.0505	0.1481	0.3022
Investment 5	0.0208	0.0221	0.0453	0.1367	0.2932

Table 2: The maximizers of equation (28) for δ.

3.2 Results for ρ

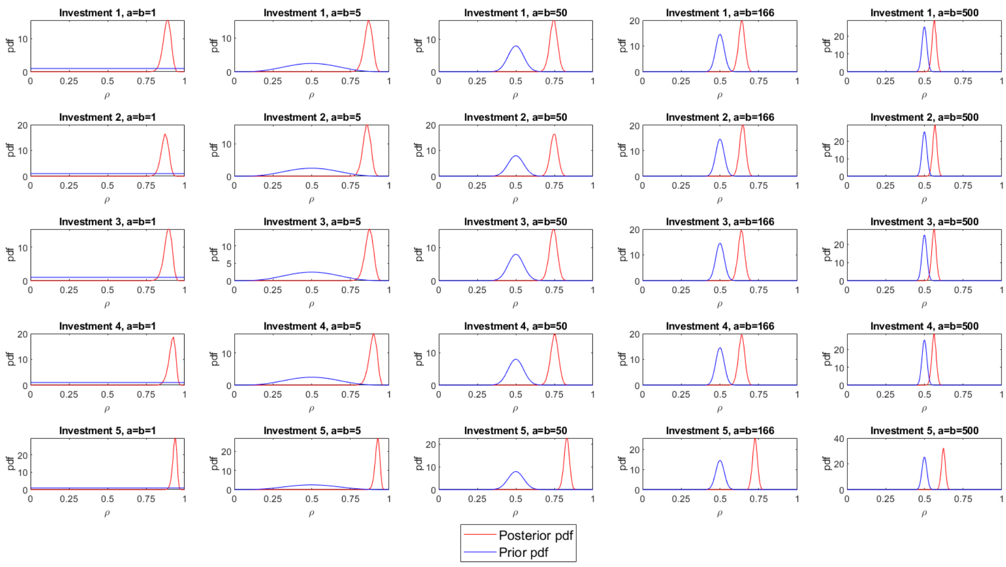

Figure 4: Prior (blue) and Posterior (red) probability density functions comparison for ρ.

[12] The value set for c in the Metroplis-Hasting Algorithm is 1.8. Check Fernandez-Villaverde et al. (2016) Chapter 12.2.2.

Figure 4 shows the comparison for ρ between the prior distribution and the Metropolis-Hastings generated posterior distribution across different investment series and beta distribution parameters. Although the previously highlighted pattern in this comparison is confirmed, some noteworthy differences are discussed. First, the posterior distribution for the shock autoregressive parameter ρ shows more variability around the 0.9 value (column 1) than the equivalent for δ. Second, the posterior mode shows a generally higher sensitivity to the increase in the a and b, therefore converging rapidly to the prior mode than δ.

Table 3 shows this faster convergence from column 1 maximizers being close to the value on which the simulated series relies (see 1) to the prior mode value of 0.5 for $a = b = 500$.

$\theta^* = \rho^*$					
Series	$a = b = 1$	$a = b = 5$	$a = b = 50$	$a = b = 166$	$a = b = 500$
Investment 1	0.8895	0.8691	0.7453	0.6405	0.5631
Investment 2	0.8748	0.8581	0.7482	0.6462	0.5672
Investment 3	0.8969	0.8745	0.7436	0.6377	0.5613
Investment 4	0.9248	0.9009	0.7515	0.6398	0.5616
Investment 5	0.9386	0.9271	0.8306	0.7262	0.6226

Table 3: The maximizers of equation (28) for ρ.

4 Conclusion

In the setting of a Real Business Cycle DSGE model with a first-order perturbation solution, the present work shows how the behavior of a parameter distribution is when its prior beliefs are degenerate in the limit while keeping all the others at a fixed deterministic level of calibration.

Interestingly, the results not only show that a prior becoming degenerate dictates the posterior distribution shape and mode but also that the way it happens is closely related to the parameter under analysis, as shown for δ and ρ.

These results were obtained using a simulated time series generated by the model solution to compute the likelihood. However, valuable insights can also be gained by using real data and analyzing multiple observables individually. This approach would allow for a more thorough comparison of the information content each time series provides regarding that parameter.

References

Fernandez-Villaverde, Jesus et al. (2016). *Solution and Estimation Methods for DSGE Models*. NBER Working Papers 21862. National Bureau of Economic Research, Inc. URL: https://EconPapers.repec.org/RePEc:nbr:nberwo:21862.

Create and solve a dynamic stochastic rational expectations model with the use of the lecture's code-Summer Semester 2024 Prof. Dr. Daniel Fehrle (n.d.).

Code repertory

`Solution&simulation.m` based on `CreatModel_bench.m` from Prof. Dr. Daniel Fehrle.

`SolveD.m` from Prof. Dr.Daniel Fehrle.

`SolveS.m` from Prof. Dr. Daniel Fehrle.

`Series Plot.m`

`Posterior_delta.m` based on `baRBC.m` from Prof. Dr. Daniel Fehrle.

`Posterior_ro.m` based on `baRBC.m` from Prof. Dr. Daniel Fehrle.

`Exercise_delta.m` based on `baRBC.m` from Prof. Dr. Daniel Fehrle.

`Exercise_ro.m` based on `baRBC.m` from Prof. Dr. Daniel Fehrle.

`NumHessian.m` from Prof. Dr. Daniel Fehrle.

`delta_exercise.fig`

`ro_exercise.fig`

`Exercise_d_work.mat`- Matlab workspace.

`Exercise_r_work.mat`- Matlab workspace.

A Bellman equation

The substitution of the constraint inside the value function gives:

$$V(K_t, Z_t) = \max_{K_{t+1}} \mathbb{E}_t \sum_{s=0}^{\infty} \beta^s ln(e^{Z_{t+s}} K_t^{\alpha} - K_{t+s+1} + (1-\delta)K_{t+s}) \tag{29}$$

Given the optimal sequence K^*_{t+s+1}, the value function becomes:

$$\begin{aligned} V(K_t, Z_t) = {} & ln(e^{Z_t} K_t^{\alpha} + (1-\delta)K_t - K^*_{t+1}) + \\ & \beta \mathbb{E}_t ln(e^{Z_{t+1}} {K^*_{t+1}}^{\alpha} + (1-\delta)K^*_{t+1} - K^*_{t+2} + \\ & \beta^2 \mathbb{E}_t ln(e^{Z_{t+2}} {K^*_{t+2}}^{\alpha} + (1-\delta)K^*_{t+2} - K^*_{t+3} + \\ & \dots \end{aligned} \tag{30}$$

$$V(K_t, Z_t) = \; ln(e^{Z_t} K_t^{\alpha} + (1-\delta)K_t - K^*_{t+1}) + \beta \mathbb{E}_t V(K^*_{t+1}, Z_{t+1}) \tag{31}$$

Equations (30) and (31) let us visualize that given an optimal sequence K^*_{t+s+1} a very long problem reduces to a one-period optimization (i.e. what is the optimal choice of capital tomorrow given the starting amount?). Formally:

$$V(K_t, Z_t) = \; \max_{K_{t+1}} \; ln(e^{Z_t} K_t^{\alpha} + (1-\delta)K_t - K_{t+1}) + \beta \mathbb{E}_t V(K_{t+1}, Z_{t+1}) \tag{32}$$

Notice that (32) can be written as:

$$\begin{aligned} V(K_t, Z_t) = \; & \max_{K_{t+1}} [ln(e^{Z_t} K_t^{\alpha} + (1-\delta)K_t - K^*_{t+1}) + \\ & \beta \mathbb{E}_t ln(e^{Z_{t+1}} K_{t+1}^{\alpha} + (1-\delta)K_{t+1} - K^*_{t+2}) + \\ & \beta^2 \mathbb{E}_t V(K_{t+2}, Z_{t+2})] \end{aligned} \tag{33}$$

Then the FOC is:

$$\frac{\partial V(K_t, Z_t)}{\partial K_{t+1}} = -\frac{1}{e^{Z_t} K_t^{\alpha} + (1-\delta)K_t - K_{t+1}} + \beta \mathbb{E}_t \left[\frac{\alpha e^{Z_t} K_{t+1}^{\alpha-1} - (1-\delta)}{e^{Z_t} K_{t+1}^{\alpha} + (1-\delta)K_{t+1} - K^*_{t+2}} \right] = 0 \tag{34}$$

B Canonical form notation

The set of control variables is $O_t = [Y_t, C_t, I_t]'$.
The set of state variables is $X_t = [K_t]'$.
The set of stochastic variables is $Z_t = [Z_t]'$.

C Canonical form with approximations

$$\mathbb{E}_t\Gamma[C(K_t,Z_t),C(K(K_t,Z_t),Z_{t+1}),K_t,K(K_t,Z_t),Z_t] = \begin{bmatrix} \frac{1}{C(K_t,Z_t)} - \beta\mathbb{E}_t\left[\frac{\alpha e^{Z_{t+1}}K(K_t,Z_t)\alpha-1-(1-\delta)}{C(K(K_t,Z_t),Z_{t+1})}\right] \\ e^{Z_t}K_t^{\alpha} - K(K_t,Z_t) + (1-\delta)K_t - C(K_t,Z_t) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \tag{35}$$

D Parametrized state-space model

$$y_t = 0.3000\ k_t + Z_t; \tag{36}$$

$$i_t = -0.7735\ k_t + 4.1387\ Z_t; \tag{37}$$

$$c_t = 0.5672\ k_t + 0.2186\ Z_t; \tag{38}$$

$$\begin{bmatrix} k_{t+1} \\ Z_{t+1} \end{bmatrix} = \begin{bmatrix} 0.9645 & 0.0828 \\ 0 & 0.9 \end{bmatrix} \begin{bmatrix} k_t \\ Z_t \end{bmatrix} + \begin{bmatrix} 0 \\ 0.01\ \epsilon_{t+1} \end{bmatrix}; \text{ with } \epsilon_{t+1} \sim \mathcal{N}(0,1). \tag{39}$$

Affirmation

I hereby declare that I have composed my Seminar paper *"Degenerate Prior"* independently using only those resources mentioned, and that I have as such identified all passages which I have taken from publications verbatim or in substance. I agree that the work will be reviewed using plagiarism testing software. Neither this paper, nor any extract of it, has been previously submitted to an examining authority, in this or a similar form.

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