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Yet Another Try: Estimating Italian Labor Dynamic Response to a Technology Shock with a DSGE-Enhanced Proxy-SVAR

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Alla mia famiglia tutta e agli amici piu' cari.
To my whole family and my closest friends.

"Life ain't always empty."

- Fontaines D.C.

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Abstract

The dynamic response of labor to technology shocks has been a central focus of macroeconomic research, fueled by debates over various identification strategies. This work introduces a novel approach within the framework of a "DSGE Enhanced SVAR", aligning with the RBC hypothesis that technology shocks are the sole drivers of total factor productivity. Total factor productivity (TFP) is recovered from a theoretical laboratory represented by a calibrated RBC model for the Italian economy. The so-obtained TFP measure is then incorporated as an external instrument in a proxy-SVAR estimated on Italian quarterly data over the period 1998Q1-2019Q4.
Empirical results reveal that a positive technology shock implies modest yet significant increases in hours worked and labor productivity. In line with the literature, I document that technology shocks explain a limited portion of business cycle fluctuations in Italy.

Contents

Chapter 1

Introduction

Among many others, one of the classical questions that economics professors love to ask their undergraduate students when introducing their lectures is surely "What is productivity? How would you measure it?".
The correct answer to this question is usually general because the concept of productivity is. From one of the most important objects of analysis in micro-based studies on firms' performances to a pivotal milestone of the growth theory, productivity also plays a crucial role in business cycle analysis.
In this context, unexpected changes in productivity (productivity shocks) have long been regarded as the primary drivers of aggregate fluctuations. In particular, these productivity shocks- often referred to as technology shocks- are considered exogenous changes in the quantity of output produced from a given quantity of inputs[1].
The very early contributions in this respect are from *Kydland and Prescott (1982)*, *Long and Plosser (1983)* and *Prescott (1986)*.These models depart from the Ramsey growth model in two key ways. First, they introduce random fluctuations around a balanced growth path to better capture the short-term dynamics observed in the data. Second, they account for endogenous rather than constantly growing labor supply (per capita hours worked/per capita employment). All the subsequent models closely related to the mentioned seminal works were focused on real rather than nominal or monetary types of shocks, hence the denomination of Real Business Cycle models (RBC).
The preferred performance evaluation methodology (i.e. how good the model is in explaining aggregate fluctuations in the data) has mostly relied

[1] *Romer (2019)*.

on the model's ability to match some main unconditional moments of the data. According to this criterion, the RBC framework did in general a good performance in explaining the key macroeconomic time series fluctuations of the postwar U.S. economy, despite a then increasing skepticism in the literature[2].

An early questioning of the RBC model performance was raised in *Hansen and Wright (1992)* concerning the actual close-to-zero correlation found in the US data between labor productivity and hours worked and the model's predicted high positive one. The acknowledgment of this failure led to the hypothesis that non-technology shocks could have a negative role in hours/employment fluctuation and offset the positive technology ones. Accounting for additional shocks led the RBC framework closer to the actual labor-market dynamics in *Christiano and Eichenbaum (1992)*.

However, proposing a more detailed way of testing the prediction of a theoretical model focused on conditional (on technology shocks) rather than unconditional moments in the data, *Galí (1999)* opened the door for a New Keynesian over an RBC based explanation for the business cycle of the US and the remaining G7 countries. In particular, he showed with Structural Vector Autoregression (SVAR) methods[3] that hours/employment respond negatively to a positive technology shock and positively to all non-technology shocks. This result seemed to invalidate the view of the previously mentioned RBC with multiple shocks, where technology was the leading actor. However, these latest conclusions have been strongly questioned by *McGrattan (2004)* in the sense that a SVAR is not a valid tester for theoretical models. In particular, the author shows that hours fall after a technology shock as in *Galí (1999)* even if the data generating process is an RBC model. The strand of these critics mostly questions the validity of the long-run restrictions in the technology shock identification.

To overcome this problem, much of the following contributions mostly relied on sign-restriction identification schemes or external variables accounting for technology shocks. Exploiting sign restrictions consistent with a popular class of Dynamic Stochastic general Equilibrium (DSGE) models with both real and nominal frictions, the *Dedola and Neri (2007)* contribution shifted the debate from the unstable ground of long-run restrictions to a

[2] *King and Rebelo (1999)*.

[3] The main identifying assumption is that labor productivity is persistently influenced just by the technology shock. This restriction is also called "long-run restriction".

specification-robust one[4]. The main findings of this latter study, not sensitive to specification assumptions, show that hours worked are likely to increase and technology shocks still have a sound role in US business cycle fluctuations yet not being significant drivers of hours worked, which would require considering other sources of disturbance. In an exercise opposing long-run against sign restrictions with different VAR specifications, *Peersman and Straub (2009)* shows for European data how the response of hours is sensitive to the identification scheme. Similar to *Dedola and Neri (2007)*, the response of hours worked to a technology shock is positive and significant while its relative importance in explaining both output and hours fluctuations lies around 20% [5].
Alexopoulos (2011) approach, instead, relies on constructing a new measure of technical change out of books published in the field of technology to identify the impact of technology shocks on economic activity. The author shows that these changes in information technology are important sources of fluctuations driving an increase in all main aggregates, labor included, even if to a lesser extent.
However, a strand of the literature did not reject the long-run restriction identification scheme while acknowledging that structural changes in the US economy over the years could have a major role in determining whether hours respond positively or negatively. *Galí and Gambetti (2009)* and *Cantore et al. (2017)* contribution are relevant in this latter respect. While the former is more focused on the SVAR methodology with limited structural interpretation, the latter extends the structural interpretation of the change in hours response showing that it depends crucially on the value of the capital-labor substitution [6]. In particular, they find that an over-time increase in the parameter governing how much capital and labor are perfect substitutes is responsible for the change in hours worked following a labor-augmenting technology shock.

Between the sign-restriction-based methods typically used in DSGE-enhanced SVAR models and the approach of using an external source of

[4]If the series used in *Galí (1999)* are assumed to be stationary and then applied in levels rather than first difference, the response of hours worked to a technology shock are positive (see *Christiano et al. (2003)*). Moreover, *Galí and Rabanal (2005)* show that the effect on hours worked changes drastically (from a decline to a rise) from a pre to a post-Volker era subsample.

[5]*Dedola and Neri (2007)* findings for U.S. data shows that technology shocks account for an average 20% of output but less than 10% of hours fluctuations.

[6]*Nucci and Riggi (2011)* shows that the overtime reduction in the negative response of hours worked is related to an increase in performance-related pay schemes in the 1980s.

identification, as proposed by *Alexopoulos (2011)*, lies a space where the present contribution fits. This work leverages latent total factor productivity (TFP) from a calibrated DSGE model to identify and estimate the response of Italian labor dynamics to a technology shock within a proxy-SVAR consistent framework. Furthermore, the adopted strategy also allows for the recovery of non-targeted shocks.
The results reveal a small yet significant increase in both hours worked and labor productivity, consistent with the calibrated RBC model's prediction. However, they also highlight the greater influence of other shocks, challenging the primacy of the technology ones.

The remainder of the paper is organized as follows. Chapter 2 presents the theoretical framework of the DSGE model and proxy-SVAR employed in this study, along with an explanation of how these models are integrated. In Chapter 3 an empirical application for the Italian economy is presented with a discussion of the main results. Last Chapter 3.4 concludes suggesting future research directions.

Chapter 2

Theory and Methodology

The structure of the following chapter is organized as follows. Section 2.1 briefly introduces the different ways DSGE and SVAR have been combined in macroeconomic research. Section 2.2 and Section 2.3 present the DSGE model setting of this paper and the SVAR technique employed, respectively. Section 2.4 presents how the combination of the two methods can achieve a technology shock identification.

2.1 DSGE and SVAR

In business cycle research both DSGE and SVAR still maintain a sound role when it comes to identifying and analyzing the impact of shocks on main aggregates. In a strong theoretical framework, the former derives the equilibrium conditions of an economy from microfundations of usually households and firms deciding how much to consume, invest, work, and produce given a resource constraint. It is the combination of values of the so-called structural parameters, whether estimated or imposed, that accounts for the characterization of different economies.

In a more data-driven setting, the latter uses aggregate time series as endogenous variables in a VAR framework where innovations (also called residuals) play a crucial role. In particular, the so-called structural shocks are mapped to the innovations through a matrix of structural parameters usually characterized by numerical or sign restriction.

It is not by chance that both models imply the crucial role of structural parameters. Under certain conditions[1], a solved DSGE model, or part of it,

[1] For the conditions to hold when moving from a DSGE to SVAR see *Fernández-Villaverde et al. (2007)*

can be represented as a Structural VAR of finite order, allowing to compare the respective impulse responses to shocks.

Apart from this strictly technical ground, the relationship between DSGE and VAR has been extensively explored in the literature in two ways: DSGE evaluation with SVAR and DSGE-enhanced SVAR. The former is more focused on verifying and validating the theoretical assumptions of the DSGE through the empirical setting offered by SVAR. Among the many, *Galí (1999)* contribution also mentioned in Chapter 1 is of relevance as well as some criticism on this way[2] of proceeding (*McGrattan (2004)*). The latter is interested, as an example, in estimating a wide range of DSGE models from different theoretical assumptions (New Keynesian and RBC) to derive broad and consistent sign restrictions to support the SVAR identification scheme[3]. However, the range of DSGE-enhanced SVAR is broader and includes also Bayesian applications as *Negro and Schorfheide (2004)*.

In the spirit of the second strand more than the first, the present work exploits the contribution from a plain vanilla RBC in two ways. First, a reasonable calibration of the RBC permits the recovery of the sample period latent total factor productivity (TFP) through the methodology explained in Section 2.4. Second, the RBC framework provides theoretical support to the assumption of technology shock being the only structural shock affecting the on-impact TFP[4].

2.2 A Plain Vanilla RBC

This section presents the structure of the standard DSGE model used to estimate the TFP. The DSGE model employed is a standard RBC with endogenous labor and one exogenous shock, the technology one, as the only one responsible for the business cycle fluctuations. In particular, this model looks at the economy as being populated by a large number of identical agents. This latter assumption allows us to consider what the choice of a representative agent (household or firm) is. For this reason, each variable is expressed in per capita terms. Importantly, the model does not admit growth.

[2]The ability of SVAR in evaluating a DSGE model has been deeply explored in *Christiano et al. (2006)*. They find that especially short-run restriction SVARs can be useful for discriminating among competing economic models (DSGE validation).

[3]An extensive list of examples relevant for this analysis is also mentioned in Chapter 1.

[4]See (2.38) for a formal definition.

Households

The representative household seeks to maximize his expected lifetime utility which takes the form:

$$U = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{c_t^{1-\theta} - 1}{1-\theta} + A \frac{l_t^{1-\gamma} - 1}{1-\gamma} \right] \; ; \; l_t + h_t = 1 \tag{2.1}$$

where:

- $\{c_t\}_{t=0}^{\infty}$, $\{l_t\}_{t=0}^{\infty}$ and $\{h_t\}_{t=0}^{\infty}$ are, respectively, the sequences of consumption leisure and work (hours-worked) to choose optimally as to maximize (2.1).
- β^t is the discount factor.
- θ is the consumer coefficient of relative risk aversion.
  In particular, for $\theta = 0$, utility is not curved and directly increases in consumption, this rules out any consumption smoothing. For $\theta \to 1$, consumption contributes in logarithm to intertemporal utility (moderate risk aversion). Values of $\theta > 1$ indicate a household's higher willingness for over-time consumption stabilization.
- γ is the inverse of the Frisch elasticity of labor supply.
  The Frisch elasticity reflects the shifts in labor supply choices following a change in the wage rate, so that the higher this value the higher people's willingness to work if the wage rate increases (lower leisure). Accordingly, for small values of γ (i.e. high values of Frisch elasticity), the household's choice for leisure (labor) is highly reactive to a change in the wage rate, whereas for $\gamma \to 1$ the contribution of leisure (labor) to the intertemporal utility is logarithmic[5].
- A is a scale factor weighing the importance of leisure relative to consumption in the utility function.

Firms and Resource Constraint

Provided with labor and capital, firms produce output according to the following constant return to scale Cobb-Douglas technology:

$$y_t = z_t \, k_t^{\alpha} \, h_t^{1-\alpha} \tag{2.2}$$

[5]The interpretation is similar for both consumption and leisure.

$$ln\ z_{t+1} = \rho\ log z_t + \sigma \epsilon_{t+1}^{tech} \quad \epsilon_t \overset{\text{iid}}{\sim} \mathcal{N}(0,1) \tag{2.3}$$

$$k_{t+1} = (1-\delta)\ k_t + i_t; \quad t = 0, ..., \infty \tag{2.4}$$

where:

- z_t is a "productivity disturbance" following an AR(1) process driven by a sequence of independent and identically distributed shocks (i.e. ϵ_t) with standard deviation σ.
- α is the output elasticity to capital or the share of output that is attributed to capital.
- k_t is the per capita capital employed in the production technology. Given a positive value of k_0, the next period capital is what has not yet undergone depreciation (δ is the quarterly depreciation rate of capital) plus investment.
- h_t is the per capita fraction of hours worked on total available time.

The final good can either be consumed or invested by households, so the following equation represents the whole economy resource constraint:

$$c_t + i_t = z_t\ k_t^{\alpha}\ h_t^{1-\alpha} \tag{2.5}$$

Equilibrium Conditions

In every period, the representative firm demands capital and labor given the wage rate and the cost of capital borrowed from the representative household, by facing the following static problem:

$$\max_{k_t,\, h_t} \quad \pi_t = y_t - w_t h_t - R_t k_t \tag{2.6}$$

$$\text{s.t.} \quad y_t = z_t\ k_t^{\alpha}\ h_t^{1-\alpha} \tag{2.7}$$

The demands for labor and capital in this perfectly competitive market are consistent with the marginal cost of each factor being equal to its marginal product[6]:

$$R_t = \alpha z_t k_t^{\alpha-1} h_t^{1-\alpha} \tag{2.8}$$

$$w_t = (1-\alpha) z_t k_t^{\alpha} h_t^{-\alpha} \tag{2.9}$$

[6] The derivation is provided in Appendix A.

Given the resource constraint, the representative household faces the following optimization problem:

$$\max_{c_t,\, l_t} \quad U_t = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{c_t^{1-\theta} - 1}{1-\theta} + A \frac{l_t^{1-\gamma} - 1}{1-\gamma} \right] \tag{2.10}$$

$$\text{s.t.} \;\; c_t = z_t\, k_t^{\alpha}\, h_t^{1-\alpha} - k_{t+1} + (1-\delta) k_t \tag{2.11}$$

Household choices of consumption and labor also underpin the choice of labor and capital, as the remaining time from leisure is devoted to production and what is not consumed is invested, fueling capital stock innovation. The F.O.C's of the above problem can be written as [7]:

$$1 = \beta\, \mathbb{E}_t \left[\left(\frac{c_t}{c_{t+1}} \right)^{\theta} [R_{t+1} + (1-\delta)] \right] \tag{2.12}$$

$$\frac{w_t}{c_t^{\theta}} = A(1-h_t)^{-\gamma} \tag{2.13}$$

The set of optimal conditions and the resource constraints constitute the system of equations describing the general equilibrium of the model.
The system is characterized by 9 equations with 9 endogenous variables[8] and an exogenous one:

$$1 = \beta\, \mathbb{E}_t \left[\left(\frac{c_t}{c_{t+1}} \right)^{\theta} [R_{t+1} + (1-\delta)] \right]; \tag{2.14}$$

$$\frac{w_t}{c_t^{\theta}} = A(1-h_t)^{-\gamma}; \tag{2.15}$$

$$R_t =_t k_t^{\alpha-1} h_t^{1-\alpha}; \tag{2.16}$$

$$w_t = (1-\alpha) z_t k_t^{\alpha} h_t^{-\alpha}; \tag{2.17}$$

$$log\; z_{t+1} = \rho\; log z_t + \sigma \epsilon_{t+1}^{tech}; \tag{2.18}$$

$$k_{t+1} = (1-\delta)\; k_t + i_t; \tag{2.19}$$

$$y_t = z_t\; k_t^{\alpha}\; h_t^{1-\alpha}; \tag{2.20}$$

$$i_t = c_t - y_t; \tag{2.21}$$

$$h_t = 1 - l_t \tag{2.22}$$

[7]A detailed derivation is provided in Appendix A.2.
[8]For the empirical analysis, labor productivity will be added later.

Given the system (2.14)-(2.22), the interest is to find a solution that tracks the response of each endogenous variable to changes in the so-called state variables k_t and z_t and their transition from a period to another. Namely:

$$\mathbf{u_t} = \mathbf{g}(\mathbf{x_t}, \xi) \tag{2.23}$$
$$\mathbf{x_{t+1}} = \mathbf{h}(\mathbf{x_t}; \xi) + \xi\sigma\epsilon_{t+1} \tag{2.24}$$

where $\mathbf{u_t}$ is the vector of control (endogenous) variables and x_t is the vector of state variables.[9] As the model does not exhibit a closed-form solution, one of the approaches is to log-linearize the system around a non-stochastic steady state employing a first-order perturbation method[10].
This method consists of finding a Taylor series expansion of the functions **g** and **h** around the steady state point of each variable. Namely:

$$\bar{\mathbf{u}} = \mathbf{g}(\bar{\mathbf{x}}, \mathbf{0}); \tag{2.25}$$
$$\bar{\mathbf{x}} = \mathbf{h}(\bar{\mathbf{x}}; \mathbf{0}); \tag{2.26}$$

Steady States and Solution Form

The following system describes the steady states of the model depending on its structural parameters, that is when the model is fully calibrated, one knows exactly the steady states of all the variables to find a linear solution of the kind seen above.

$$\bar{z} = 1; \tag{2.27}$$
$$\bar{R} = \frac{1}{\beta} - \frac{1}{\delta}; \tag{2.28}$$
$$\bar{k} = \left(\frac{\bar{R}}{\alpha\bar{z}\bar{h}^{1-\alpha}}\right)^{\frac{1}{\alpha-1}}; \tag{2.29}$$
$$\bar{y} = \bar{k}^{\alpha}\bar{h}^{1-\alpha}; \tag{2.30}$$
$$\bar{i} = \delta\bar{k}; \tag{2.31}$$
$$\bar{c} = \bar{y} - \bar{i}; \tag{2.32}$$
$$\bar{l} = 1 - \bar{h}; \tag{2.33}$$
$$\bar{w} = (1-\alpha)\bar{z}\bar{k}^{\alpha}\bar{h}^{-\alpha} \tag{2.34}$$

[9] As in *Fernandez-Villaverde et al. (2016)* Section 4.2.3 pp.24. The perturbation parameter in *Fernandez-Villaverde et al. (2016)* here is ξ.
[10] The perturbation parameter in *Fernandez-Villaverde et al. (2016)* here is ξ.

The steady-state value for hours worked is usually set based on an average value of the fraction of per capita hours worked on total available time[11]. Given the steady state of the variables, the first-order perturbation solution[12] will have the following form:

$$ln(\mathbf{u_t}/\overline{\mathbf{u}}) = \mathbf{u_k}\; ln(k_t/\bar{k}) + \mathbf{u_z}\; ln(z_t/\bar{z}) + \mathbf{d}\sigma\epsilon_t^{tech} \tag{2.35}$$

$$ln(\mathbf{x_{t+1}}/\overline{\mathbf{x}}) = \mathbf{x_k}\; ln(k_t/k) + \mathbf{x_z}\; ln(z_t/z) + \mathbf{b}\sigma\epsilon_t^{tech} \tag{2.36}$$

Where $\mathbf{u}$ is the vector of control variables, $\mathbf{u_k}$ and $\mathbf{u_z}$ are the vectors that contain the partial derivatives of the control variables with respect to capital and the TFP component, respectively. Equation (2.36) is the so-called transition equation describing the dynamics of the state variables. Lastly, the vectors $\mathbf{d}$ and $\mathbf{b}$ are the responses of the control variables and of the state variables to the technology shock.

2.3 Proxy-SVARs

Among the many ways to identify macroeconomic shocks in an SVAR setting, one has been imported from a largely used microeconometric technique: instrumental variables.
Fundamental in the estimation of causal effects in many quasi-experimental settings, the idea of exploiting an external source of variation able to capture the randomness of the treatment we are interested in, has conquered a sound position also in structural macroeconometrics.
The ways by which an external variable can contribute to SVAR[13] identification was first introduced by *Stock and Watson (2008)*, *Stock and Watson (2012)*, *Mertens and Ravn (2013)* and *Caldara and Kamps (2017)*.[14]

This section will first present the framework of a standard and so-called IV-SVAR or Proxy-SVAR and then present a full-information scheme in the sense of *Angelini and Fanelli (2019)*. Importantly, both approaches will be presented in a way that is consistent and closely related to the purpose of this work.

[11] In the empirical analysis in 3 its setting is shown for Italy.

[12] For an extensive discussion see *Fernandez-Villaverde et al. (2016)* Section 4.2.4.

[13] The use of external instruments in macroeconometrics is not exclusively related to SVAR. Also direct estimation of the structural impulse response is relevant and known as 'local projections-IV'. See *Jorda et al. (2015)* for an early contribution

[14] This section review is based on *Stock and Watson (2018)* and *Angelini et al. (2024)*.

2.3.1 Standard Proxy-SVAR

Given the SVAR model:

$$Y_t = \Gamma_{y,t} D_t + \sum_{j=1}^{l} \Pi_j Y_{t-j} + u_t, \qquad u_t = B\epsilon_t \ \ (t = 1, ..., T) \tag{2.37}$$

where:

- Y_t is an $n \times 1$ vector of endogenous variables;
- D_t is a 2×1 vector of deterministic components (constant and linear trend) with $\Gamma y, t$ being the associated $n \times 2$ matrix of coefficients;
- Y_{t-j} is the $n \times 1$ vector of lagged endogenous variables with parameters Π_j;
- u_t is an $n \times 1$ vector of reduced form innovations with covariance matrix $\Sigma_u := \mathbb{E}(u_t u_t')$;
- ϵ_t is a 2×1 vector of structural shocks and B is a $n \times 2$ matrix of parameters mapping the structural shocks to the reduced form innovations.

The 2×1 vector of structural shocks contains a technology (ϵ_t^{tech}) and a non-technology shock (ϵ_t^{notech}) are normalized such that $\Sigma_\epsilon = \mathbb{E}(\epsilon_t \epsilon_t') = I_n$. If the target shock is the technology one, then we are interested in the first column of the B matrix (partial identification), $B_{.,1}$ afterward.
A standard proxy-SVAR approach would assume that there is at least an external variable z_t that is correlated to the target shock and uncorrelated with the non-target shock.
That is, the conditions

- **Relevance**: $\mathbb{E}[z_t \epsilon_t^{tech}] = \phi$;[15]
- **Exogeneity**: $\mathbb{E}[z_t \epsilon_t^{notech}] = 0$;

imply:

$$z_t = \phi \ \epsilon_t^{tech} + \omega_{z,t} \tag{2.38}$$

Where ϕ is the relevance parameter and $\omega_{z,t}$ is a measurement error independent of the vector of shocks.

[15] For simplicity $\mathbb{E}[z_t] = 0$ is assumed.

A reasonable question could be, what is a non-technology shock? The contributions that usually apply this differentiation usually employ long-run restrictions. Therefore, the usual interpretation is that technology shock is permanent while all the others (non-technology) have transitory effects on labor productivity.
This work assumes that a technology shock is the only shock driving the on-impact total factor productivity (as the plain vanilla RBC predicts), while the non-technology is just an aggregation of the remaining shocks.

Multiplying equation (2.38) by the VAR innovations (u_t') it can be shown that:

$$\mathbb{E}[u_t z_t] = \phi B_{.,1} \tag{2.39}$$

A natural estimator of $\mathbb{E}[u_t z_t] = \Sigma_{u,z}$ would be the moment one, easily derived from the VAR residuals and the proxy series. Formally:

$$\hat{\Sigma}_{u,z} = \frac{1}{T-l} \sum_{t=1}^{T-l} \hat{u}_t z_t \xrightarrow{p} \mathbb{E}[u_t z_t] \tag{2.40}$$

Therefore, $\hat{\Sigma}_{u,z}$ is a consistent estimator of $\phi B_{.,1}$. However, to compute impulse response functions, that is obtain $B_{.,1}$ one would need to separate it from ϕ. This can be achieved either by (i) imposing additional moment conditions or (ii) a unit effect normalization as suggested by *Stock and Watson (2018)*, section 2.1.

2.3.2 Full-Information Scheme

One of the limits of the standard approach is that it recovers only target shocks on-impact response. However, under certain conditions, the information from the external variable can also be exploited to recover the non-target shock responses.
For this work, the following SVAR is considered in the full-information setting:

$$\begin{pmatrix} Y_t \\ z_t \end{pmatrix} = \begin{pmatrix} \Gamma_{y,t} \\ 0 \end{pmatrix} D_t + \begin{pmatrix} \sum_{j=1}^{l} \Pi_j & 0 \\ 0 & \rho_1 \end{pmatrix} \begin{pmatrix} Y_{t-j} \\ z_{t-1} \end{pmatrix} + \begin{pmatrix} u_t \\ \nu_{z,t} \end{pmatrix} \tag{2.41}$$

where Y_t is a 2×1 vector containing hourly labor productivity and hours worked. The only difference with the standard specification in (2.37) is that the reduced form also includes the external variable and some restrictions

on the autoregressive matrix are applied.
With this setting in mind, it is possible to conceive this latest SVAR as a "large B-model" of the kind:

$$\begin{pmatrix} u_{1,t} \\ u_{2,t} \\ \nu_{z,t} \end{pmatrix} = \underbrace{\begin{pmatrix} b_{1,1} & b_{1,2} & 0 \\ b_{2,1} & b_{2,2} & 0 \\ \phi & 0 & \omega_{z,t} \end{pmatrix}}_{B} \begin{pmatrix} \epsilon_t^{tech} \\ \epsilon_t^{notech} \\ 1 \end{pmatrix} \tag{2.42}$$

where the parameters ϕ and $\omega_{z,t}$ are the same as seen in the previous setting [16].
Let $\theta_0 = [b_{1,1}, b_{2,1}, \phi, b_{1,2}, b_{2,2}, \omega_{z,t}]'$ be the 6×1 vector of parameters to be estimated, then, we can claim that (i) the necessary order condition for identification is always verified in this setting[17] and (ii) that the necessary and sufficient rank condition for local identification[18] should be:

$$rank\{D_n^+(B_0 \otimes I_n)S_B\} = 6 \tag{2.43}$$

where D_n^+ is the Moore-Penrose pseudo inverse of the duplication matrix, B_0 is the B matrix evaluated at the true parameters θ_0 and S_B is the matrix selecting the parameters to be estimated out of the vectorized matrix B_0.
The reduced form estimation of the residuals variance-covariance matrix $\hat{\Sigma}_u$ allows to recover the structural parameters in B through maximum likelihood, as $\Sigma_u = BB'$[19].

2.4 Methodology

In the spirit of providing external support to the shock identification strategy, the present work relies on the recovery of a latent variable from a calibrated DSGE model to instrument a technology shock and estimate impulse response functions on labor productivity and hours worked.
The steps involved in this new strategy are:

I. Exploit model consistent data to calibrate the theoretical model pre-

[16] The exogeneity condition seen in the previous approach is still present here in the form of the zero restriction in the third-row second-column of matrix B.

[17] The number of parameters to be estimated should be less or equal than $\frac{1}{2}n(n+1)$ where n is the number of the VAR endogenous variables.

[18] For a detailed derivation of this condition see *Angelini and Fanelli (2019)*.

[19] See *Angelini and Fanelli (2019)*.

sented in Section 2.2;

II. Obtain a state-space representation of the theoretical model;

III. Employ the Kalman smoother to recover the latent state variable z_t, also known as Total Factor Productivity;

IV. Use the estimated latent variable as an instrument in the full-information scheme SVAR;

V. Estimate the impulse response functions on labor productivity and hours worked.

The remaining part of this section will delve deep into explaining steps II and III in the framework of Kalman smoother, as the treatment of steps I, II, and V will be left to the empirical discussion in Chapter 3, while step IV fully relies on Subsection 2.3.2.

2.4.1 Kalman Smoother

In the case of linear solutions as the ones stated in equations (2.35) and (2.36) it is possible to cast the model into the state-space representation:

$$\mathbf{y_t} = H\mathbf{x}_t + \zeta; \quad \zeta \sim \mathcal{N}(0_{n(y)\ x\ 1}, M_{n(y)\ x\ n(y)}) \tag{2.44}$$

$$\mathbf{x}_{t+1} = F\mathbf{x}_t + \mathbf{b}\nu_t; \tag{2.45}$$

where, in this setting, $\mathbf{y_t}$ is the $n \times 1$ vector of observables, generally a subset of the control variables $\mathbf{u_t}$, ζ is a measurement error, $\mathbf{x_t}$ is the 2×1 vector of state variables as in Section 2.2, H is the $n \times 2$ matrix of parameters derived in (2.35) and F is the 2×2 matrix of parameters from the transition equation (2.36), $\nu_t = \sigma\epsilon_t^{tech}$ and $\mathbf{b}$ is the same as in (2.36).

Given this state-space representation, obtaining an updated forecast of the state variables from a set of observables $\mathbf{y_t}$ is possible through the Kalman filter.

The Kalman filter can be summarized in the following steps:

1. Let $\mathbf{x_{0|0}}$ be the state variables at the unconditional mean and $P_{0|0}$ be the matrix collecting their unconditional second moments

2. And the conditional forecast be $\mathbf{x_{1|0}} = F\mathbf{x_{0|0}}$ with mean squared forecast error $P_{1|0} = FP_{0|0}F' + \mathbf{b}Q\mathbf{b}'$[20]

3. The conditional forecast so obtained allows to get the predicted value of the observable by $\mathbf{y_{1|0}} = H\mathbf{x}_{1|0}$ with mean square prediction error $P^y_{1|0} = HP_{1|0}H' + M$.

4. The update $\mathbf{x_{1|1}}$ is obtained through the observation $\mathbf{y_1}$ with the following formula:

$$\mathbf{x_{1|1}} = \mathbf{x_{1|0}} + P_{1|0}H'(P^y_{1|0})^{-1}(\mathbf{y_1} - \mathbf{y_{1|0}}) \tag{2.46}$$

The update $\mathbf{x_{1|1}}$ enters the right-hand side of the conditional forecast in step 2 to obtain $\mathbf{x_{2|1}}$ and so on until the last observation $\mathbf{y_T}$ is used.
The recursion generates the sequences:

- $\{x_{t|t}\}_{t=1}^T$, $\{P_{t|t}\}_{t=1}^T$;
- $\{x_{t+1|t}\}_{t=0}^{T-1}$, $\{P_{t+1|t}\}_{t=0}^{T-1}$

The interest is to recover a latent variable in a defined historical period, given a theoretical model and at least one observable. Therefore, information from the full set of data observed rather than the quarterly-updated forecast better fits the purpose of the analysis. This kind of inference is called the smoothed estimation of the states $\mathbf{x_t}$, denoted $\mathbf{x_{t|T}}$ [21].
Given the sequences derived from the Kalman filter, the smoothed states estimation follows:

$$\mathbf{x_{T-1|T}} = \mathbf{x_{T-1|T-1}} + P_{T-1|T-1}F'P^{-1}_{T|T-1}(\mathbf{x_{T|T}} - F\mathbf{x_{T|T-1}}) \tag{2.47}$$

until $\mathbf{x_{1|T}}$.

[20] $Q{=}\sigma\mathbb{E}[\epsilon_t^{tech}\epsilon_t^{tech\prime}] = \sigma$.

[21] For a more general version of both Kalman filter and Kalman smoother see *Hamilton (1994)* ch.13

Chapter 3

Empirics

The structure of the chapter is organized as follows. Section 3.1 motivates the choice of the country object of the analysis, Section 3.2 discusses the calibration of the RBC model presented 2.2 for the Italian economy and the smoothed TFP. Section 3.3 presents the estimation of the impulse response functions from the full-information proxy-SVAR and discusses the relative importance of the technology shock.

3.1 Italian Labor Productivity Dynamics

Among the countries of the Euro Area (EA afterward), Italy is widely recognized as experiencing a consistent lag in real output growth. In a sample period similar to the one adopted in this analysis, *Greco (2023)* shows the real output growth of 0.3% per year for Italy against an average of 1.20% for the whole EA. In the same study, an output growth decomposition analysis shows that the main reason for this performance gap is hourly labor productivity, followed by a consistent decrease in hours worked per person employed. In a closer subsample analysis, the author shows that productivity growth accelerated from a weak performance only during the Global Financial Crisis as a result of the strong decrease in hours worked.
The decomposition of the shocks in technology and non-technology as the only shocks operating in the economy could (i) quantify the benefits of a technology improvement and (ii) help understand the relative contribution of different shocks on Italian hourly labor productivity, apart from providing further evidence on hours worked response.

3.2 Calibration

The calibration of the model's parameters presented in Section 2.2 for the Italian economy relies on model-consistent data and values collected mainly from Poveledo (2004) as the theoretical setting is very similar. The present section will first show and discuss the derivation of model-consistent data, how they are used to estimate part of the RBC structural parameters, and then motivate the adopted main calibration.

3.2.1 Model Consistent Data

All the notation of the per capita real variables in this section will refer to the one adopted in Section 2.2. Among all, two series can contribute effectively to capturing the model underlying business cycle for the Italian economy: the quarterly real per capita output y_t and the quarterly per capita fraction of hours worked h_t.

In the closed economy RBC model, the final good (or output) can be consumed or invested, as $y_t = c_t + i_t$. Given the resource constraint, the output series is recovered from summing up the series on the final consumption of households and gross fixed capital formation[1]. In particular, the original nominal series have been deflated using the GDP implicit deflator and converted in per capita terms through the working age population series for the sample 1998Q1-2019Q4. Although exhibiting a weak trend, the output series should be de-trended to distinguish its cyclical components.

The variables can be de-trended through (i) the Hodrick-Prescott filter, (ii) the bandpass filter, (iii) linear trend, and (iv) first differences. The choice of the filter is based on an evaluation of the second moments of the retrieved data being compliant with some stylized facts as $\sigma_c < \sigma_y < \sigma_i$ with $\frac{\sigma_c}{\sigma_y}, \frac{\sigma_i}{\sigma_y} < 2$, $Corr(y, c) > 0.5$ and the idea of business cycle object of the analysis.

	HP filter	Bandpass filter	Linear trend	First differences
$100\sigma_y$	1.6427	1.7442	4.2058	1.0143
σ_c/σ_y	0.80824	0.78974	0.64154	0.81765
σ_i/σ_y	1.7624	1.7998	2.176	1.9294
$Corr(y, c)$	0.97309	0.97918	0.98501	0.93258
$Corr(y, i)$	0.94457	0.9567	0.98688	0.88509
$Corr(y_t, y_{t-1})$	0.84332	0.9376	0.97078	0.36315

Table 3.1: Summary statistics for different filters

[1]More on data sources source in Appendix B

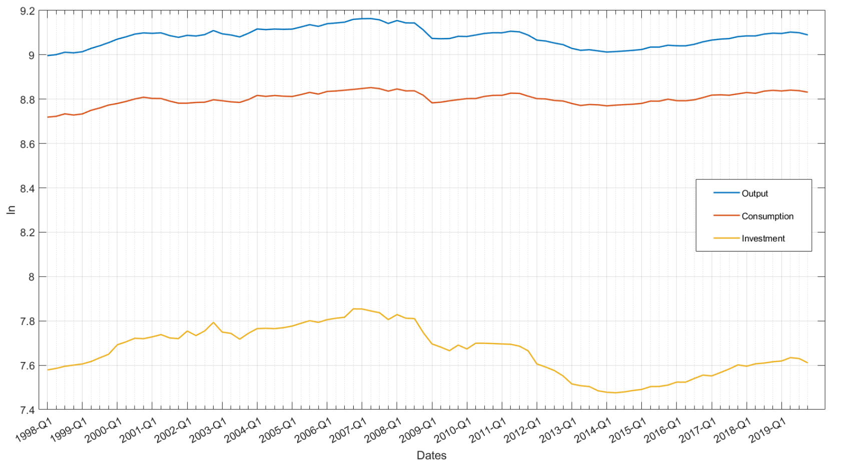

Figure 3.1: The real per capita output (blue), consumption (orange), and investment (yellow) series in natural logarithm terms for Italy. Output is the sum of consumption and investment.

Relevant computed moments for each filter are reported in Table 3.1. Two main features help in the final choice. First, the linear trend filter displays investment volatility exceeding the threshold of 2 with respect to output one. Second, the first differenced series, shows a very low degree of output autocorrelation, making it more suitable for more short-run oriented analysis. As both the remaining HP and bandpass filters suit the stylized facts, the choice between the two is made on the one that is relatively less persistent in output, the HP filter[2], as to focus the analysis on a not-so-long cycle[3]. Figure 3.2 provides a visualization of the length of the cycles captured by different filters.

The steady states of the model specified in the system of equations (2.27)-(2.34) require, among the others, the calibration of the long-run value $\bar{h}$. Relying on a methodology similar to *Poveledo (2004)*, the steady state value of hours worked in the sample 1998Q1-2019Q2 is the average of the ratio of the per capita hours worked series by the quarterly amount of disposable time in a day, excluding eight hours of sleeping time[4]. The long-run fraction of quarterly hours worked $\bar{h}$ in the considered sample is 0.1951 (19%

[2]With the value of the parameter $\lambda = 1600$ for quarterly data.

[3]The choice of the HP filter is also motivated by comparison reasons as it is a standard in the literature.

[4]The denominator of the fraction is $\frac{16hours}{day} \times \frac{7days}{week} \times \frac{4.3weeks}{month} \times \frac{3months}{quarter}$.

Figure 3.2: Output de-trended series with different filters

of the available time devoted to work)[5].

3.2.2 Estimation and final calibration

The estimation of the parameters is carried out by the maximization of the likelihood function derived from the Kalman filter (*Fernandez-Villaverde et al. (2016)* Chapter 10). In particular, the de-trended output series enters as the only observable in the state-space representation (see Equation 2.44), given that the theoretical model conceives only one shock[6]. However, estimating the full set of parameters is not feasible, as the data may adjust the parameters to comply only with the observable and ignore the patterns in other series.
A common way of proceeding in the literature is to rely on partial estimation, while some other parameter values are imposed from external and more specific data analysis or similar studies. This analysis is no exception. Moreover, the validity of the adopted calibration is focused on getting consistent results for the TFP estimation rather than forcing the model to fit the data.

The main calibration relies on estimating the standard deviation of the TFP σ and imposing the other parameter values on literature-consistent ones.

[5]The value computed in *Poveledo (2004)* is 0.12.

[6]For the reason why the number of observables should always be equal to the number of shocks see *Fernandez-Villaverde et al. (2016)*, section 11.1.3.

Parameter	Description	Value	Source / Method
	Estimated Parameters		
σ	Std. deviation of TFP shock	0.0080 (0.0006)	Maximum Likelihood
	Imposed Parameters		
α	Output elasticity to capital	0.33	Standard in RBC literature
β	Household discount factor	0.99	Standard in RBC literature
δ	Capital depreciation rate	0.025	Standard in RBC literature
ρ	Autocorrelation of TFP	0.99	Standard in RBC literature
θ	Household Relative risk aversion	1	Imposed
γ	Inverse of labor supply elasticity	0	Imposed
$\bar{h}$	Long-run fraction of hours worked	0.1951	Data on hours worked

Table 3.2: Main Calibration

Table 3.2 shows the estimated (standard error in parenthesis) and the imposed parameter values. The chosen value for α is a standard in the RBC literature on advanced economies as well as the household discount factor β, and Italy is no exception[7]. Equal to 0.0088 in *Poveledo (2004)*[8], the capital quarterly depreciation rate is here set at the value of 0.025 as it is also a standard choice in the literature[9].Difficulties in the estimation of parameter ρ led to setting it at 0.99 to account for the maximum possible level of persistence of technology shocks[10]. Lastly, the household relative risk aversion and the inverse of the Frisch elasticity are set to match the ratio between the standard deviation of the fraction of hours worked and output (both HP filtered and in log terms). The mentioned ratio is in the data 0.59 and 0.5 from the model simulated variables[11]. This calibration implies a logarithmic contribution of consumption in the household utility and an infinite intertemporal elasticity of substitution in the labor input, implying volatility of hours increasing relatively to output[12].
Appendix C provides the solutions, the steady state values, and the impulse response functions implied by the main calibration[13]. Figure 3.3 shows the latent TFP implied by the main calibration of the model estimated with

[7] See *Poveledo (2004)*.

[8] Computed as the ratio of Capital consumption/ Net stock of capital

[9] Final TFP results are robust to $\delta = 0.0088$.

[10] Other models allow to distinguish between temporary and persistent technology shocks as the one in *Lindé (2009)*.

[11] The simulation of the model data is carried out with Dynare by replicating 1000 times a sample of 288 observations, cutting out the first 200 for each variable and computing moments as average over repetitions.

[12] The calibration with γ and θ equal to 1, all other parameters constant, implies a reduction in the mentioned hours-output volatility ratio.

[13] The solution of the model, the estimation of the parameter, and the smoothed estimates of the TFP are computed by Dynare 6.1 .

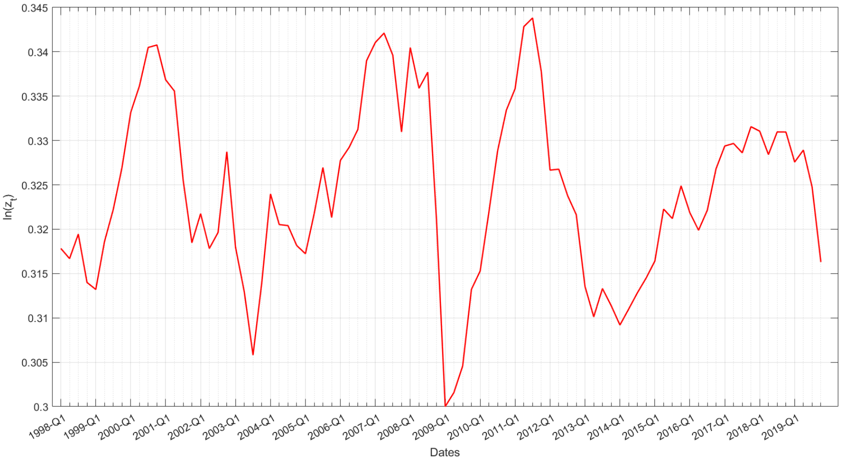

Figure 3.3: Smoothed estimates of the model implied quarterly Total factor productivity series 1998Q1-2019Q4 for the Italian economy.

the Kalman smoother seen in Subsection 2.4.1.

3.3 Findings

This section first specifies the variables entering the full-information SVAR outlined in Subsection 2.3.2, discusses their specification, and then presents the results on impulse response functions and variance decomposition obtained for the Italian economy.

3.3.1 VAR specification

The vector Y_t in the reduced form of the SVAR in (2.41) is $Y_t = [\Delta l\_prod_t, \Delta hw_t]'$, where Δl_prod_t is the first difference of the logarithm of hourly labor productivity index and Δhw_t is the first difference[14] of the logarithm of per capita hours worked[15]. The variable z_t is the first difference estimated TFP [16]. All the restrictions in the auto-regressive matrix in (2.41) are applied and the number of lags chosen is one, based on likelihood methods. The only difference with the specification (2.41) is that the vector of deterministic components $D_{t,y}$ only includes a constant term. The sample ranges from 1998Q2 to 2019Q4 with a size of $T = 87$[17].
Section D.2 and Section D.3 show the estimates of the reduced form and the SVAR on-impact matrix of the shocks.
The estimated parameter $\hat{\phi}$ under the mentioned reduced form specification is 0.0038[18], implying a correlation between the technology shock and the TFP of 67.85%. This result suggests that the relevance condition for the proxy-SVAR in Equation 2.38 is satisfied. The necessary and sufficient rank condition (2.43) is also satisfied.

3.3.2 Italian Labor Dynamic Response to Technology Shock

The estimated impulse response functions for both a technology and a non-technology shock, identified under the full-information scheme assumptions, are presented in Figure 3.4.

[14] With this specification all the eigenvalues of the auto-regressive part lie inside the unit disk. No evidence for a co-integrating relationship between the variables in Y_t specified in levels is detected.

[15] Notice that this time hours worked are not expressed as a fraction of available time.

[16] As it is close to a unit-root process.

[17] One observation is lost due to first-differencing.

[18] The parameter ϕ is a measure of covariance. The correlation is computed by dividing the covariance by the product of the standard deviation of the first-differenced TFP (0.0056) and the standard deviation of the shock, which is assumed to be 1.

The figure shows that a positive technology shock results in an increase in both labor productivity and hours worked[19]. In particular, the labor productivity index significantly increases by 0.35% following a one-standard-deviation increase in technology shock. However, the significant effect vanishes after one period.

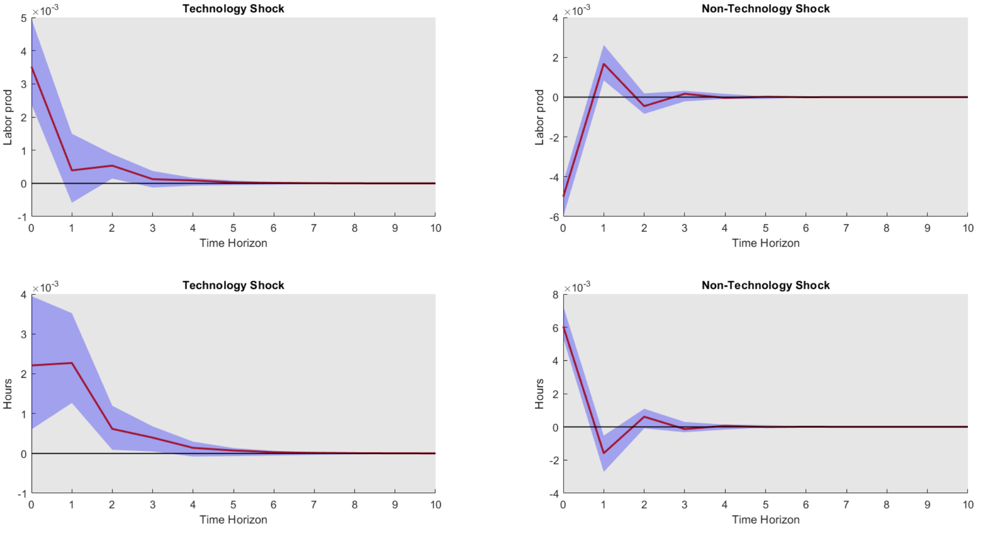

Figure 3.4: Impulse response to positive technology and non-technology shock of Italian labor productivity and hours worked. Hall's percentile 90% confidence bands based on 1000 iid[20]bootstrap repetitions. The sample period is 1998Q2-2019Q4.

The response on hours worked is less relevant (0.22% on-impact growth) and displays a wider confidence interval. Nevertheless, it displays more persistence as the positive effect vanishes after less than one year.

These findings seem to be consistent with the RBC prediction (see Figure C.1 for similarity in signs and magnitude) on the effects of technology shock on labor supply. In particular, a positive technology shock implies one hour of work being more productive. This means that the household's lifetime wealth is expected to increase, therefore, she will consume more and reduce labor supply. However, the marginal product of labor also increases, making it more profitable to supply labor. Additionally, an incentive to work more and produce more also comes from the necessity to increase investment, now more profitable given that the marginal product of capital,

[19]The signs in the estimated B matrix are normalized along the main diagonal so that a positive technology shock impacts positively on labor productivity and a non-technology shock impacts positively on hours worked.

or equivalently, the real interest rate, has also risen after the shock. The standard RBC model predicts the willingness to increase labor supply for the reasons outlined to dominate over the wealth effect.
As aggregating different shocks other than technology, the interpretation of the effects of non-technology ones on both labor productivity and hours worked is not straightforward. Nonetheless, the shape of the impulse response functions suggests a negative conditional (on the non-technology shock) correlation between labor productivity and hours worked. Following a reasoning similar to *Galí (1999)*, this negative conditional correlation could be responsible for the negative unconditional one observed[21] between Δl_prod_t and Δhw_t of -46.7%. The computed[22] values of the conditional correlation for the technology and the non-technology shock are 77.67% and -99.77%, respectively. In summary, according to these results, on one hand, the relationship between the quarterly change in hours worked and labor productivity is driven by a positive correlation, on the other by a negative one (stronger).
The identification used so far also allows for tracking the importance of the technology shock relative to the aggregation of all the others in the economy, assumed to be uncorrelated with the TFP. In this respect, the forecast error variance decomposition acts as a powerful tool.

Figure 3.5 presents the forecast error variance decomposition for hours worked and labor productivity up to two years and six months. Three facts are important to be noticed. First, the aggregation of all other shocks in the economy dominates unexpected productivity improvements. Second, technology shock explains labor productivity variability more than it does for hours worked, with 30% importance against less than 20% on average, respectively. Third, the relative importance of the technology shock on hours worked experiences a steep rise from the impact to one quarter after, while slightly declining for labor productivity in the same time interval. These results are close to the evidence in *Peersman and Straub (2009)*, where the authors find technology improvements accounting for around 20% importance in hours worked variability for the Euro Area. *Dedola and Neri (2007)* shares similar results in a DSGE-enhanced sign-restricted SVAR estimated with US data.
The consistency of these results, while not hurting the key RBC model predictions on hours worked dynamics, still, challenges the fundamental

[21]See Section D.1
[22]See *Galí (1999)* section II.B

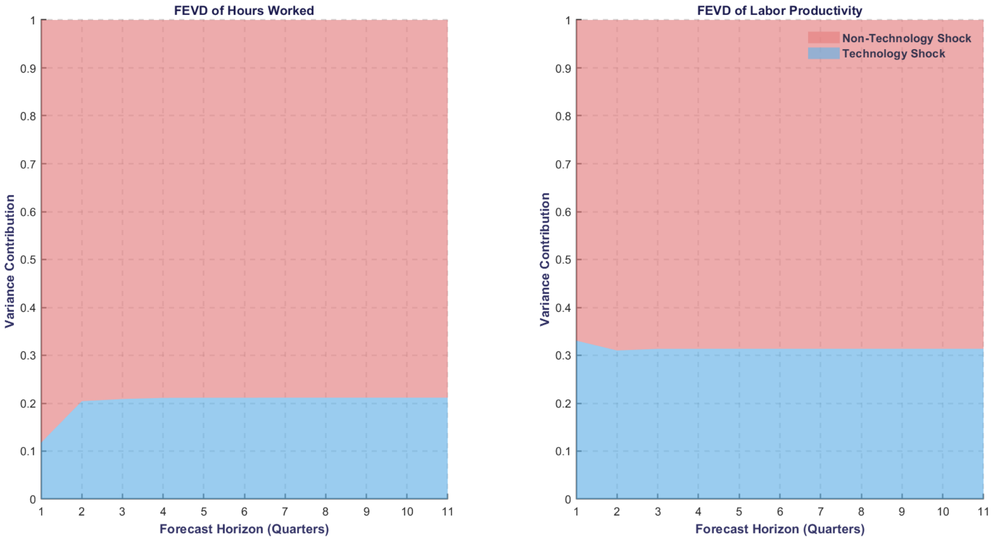

Figure 3.5: Forecast error variance decomposition of hours worked and labor productivity: technology (blue) and non-technology (red) shocks relative importance.

hypothesis that technology shocks- the only source of TFP- are the main drivers of business cycle fluctuations.
Lastly, a comparison between the DSGE smoothed estimates series of technology shocks and the SVAR implied ones[23] helps to assess how well the full-information scheme technology shocks track their DSGE counterpart. Figure 3.6 shows the similarity between the DSGE and the SVAR-implied technology shocks. In particular, the two series display a correlation of 67%, and, most importantly, the DSGE technology shock is uncorrelated with the non-technology ones implied by the SVAR.

3.4 Robustness

The robustness checks carried out for this work involve the estimation of the SVAR reduced form, the on-impact matrix B. Regarding the former, the restriction on the quarterly growth of TFP not Granger-causing the variables in vector Y_t has been relaxed, leaving the results virtually unchanged, as shown in Figure D.2. The latter considers an over-identifying restriction scheme on matrix B. In particular, the coefficient $b_{1,2}$ in (2.42)

[23]The multiplication of the inverse of the estimated B (3×3) matrix by the $3{\times}T{-}l = 86$ matrix of estimated residuals allows to recover the $3 \times T - l = 86$ matrix of structural shocks.

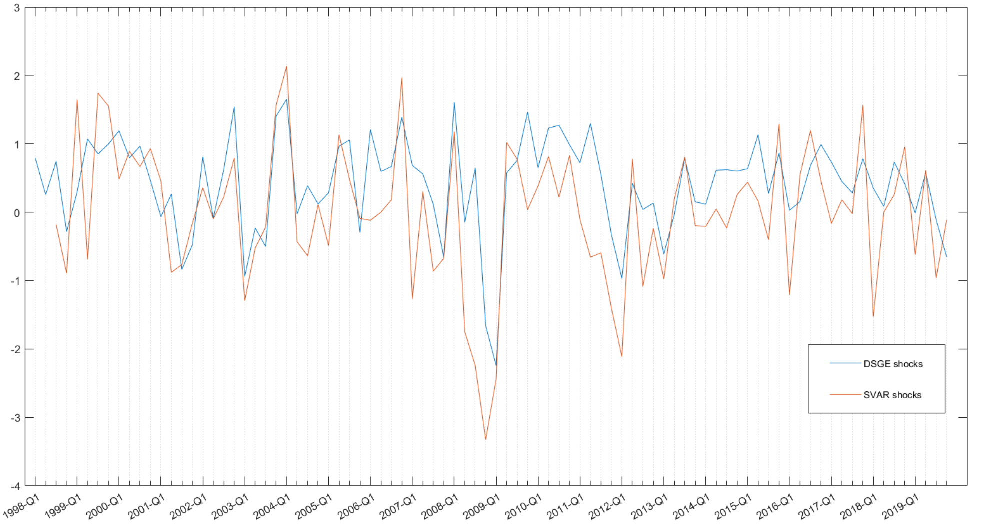

Figure 3.6: Smoothed DSGE (blue) and SVAR (orange) implied technology shocks

would be restricted to zero assuming that the non-technology shock has no on-impact effect on labor productivity. This allows us to test if the model better represents the data under the extra restriction placed, in this case, $b_{1,2} = 0$. The likelihood ratio test for over-identifying restrictions $LR = -2[L_{restr} - L_{un-restricted}]$ is distributed, under the null hypothesis of restriction validity, as a χ^2_q where q is the number of extra-restrictions, in this case 1. The test $LR = 42.30$ rejects the null hypothesis with more than 99% confidence level.

Conclusions

As argued in Chapter 1 much of the debate surrounding the response of hours worked to technology shocks stems from the varied restrictions imposed in the shock identification strategy. In this context, I presented a new channel for a linear DSGE model to provide external support in SVAR identification, other than sign restriction. Furthermore, I show that the common RBC assumption, which considers technology shocks as the only drivers of total factor productivity, contributes in two key ways. First, it permits the recovery of non-target shocks (i.e. non-technology shocks) in a full-information proxy SVAR framework. Second, the responses to hours worked and labor productivity quarterly change in Italy are positive, aligning with RBC theory. Finally, aligned with the sign-restriction literature, the forecast error variance decomposition results cast doubt on the RBC assumption that technology shocks are the primary drivers of business cycle fluctuations. Put differently, observing a positive response of hours worked to a technology shock may come at the expense of conceding that technology shocks do not dominate other sources of economic fluctuations.

These results, however, cannot be considered definitive as, although the presented robustness checks, the space for improvements is large. On top of the others, it can be noticed that the external instrument is estimated from a theoretical model with its degree of flexibility. This would lead to assessing how sensitive is the TFP estimation to extensions in the theoretical model framework, including those differentiating technology shocks by their temporary or permanent effects, as in *Lindé (2009)*. Furthermore, a model that accounts for multiple shocks would enable the use of additional observables, resulting in more precise estimates of the structural parameters, including those relevant to TFP estimation. Further research should apply the same method to estimate impulse responses and forecast error variance decomposition with US data, to check for broader consistency of the final results. Lastly, improving the number of endogenous macroeconomic variables in the SVAR would provide broader insights into the macroeconomic effects of a technology shock.

Appendices

Appendix A

A.1 Firm Optimization

The F.O.C's for the problem in (2.6) and (2.7) are:

$$\frac{\partial \pi_t}{\partial k_t} = \alpha z_t k_t^{\alpha-1} h_t^{1-\alpha} - R_t = 0 \tag{A.1}$$

$$\frac{\partial \pi_t}{\partial h_t} = (1-\alpha) z_t k_t^{\alpha} h_t^{-\alpha} - w_t = 0 \tag{A.2}$$

Rearranging:

$$R_t = \alpha z_t k_t^{\alpha-1} h_t^{1-\alpha} \tag{A.3}$$

$$w_t = (1-\alpha) z_t k_t^{\alpha} h_t^{-\alpha} \tag{A.4}$$

A.2 Household Optimization

The dynamic optimization problem of the household in (2.10) and (2.11) can be expressed in terms of the following dynamic programming problem:

$$V(k_t, z_t) = \max_{k_{t+1};\, h_t} \left[\frac{c_t^{1-\theta} - 1}{1-\theta} + A \frac{(1-h_t)^{1-\gamma} - 1}{1-\gamma} \right] + \beta \mathbb{E}_t V(k_{t+1}, z_{t+1}) \tag{A.5}$$

Adding the constraint in (2.11), (A.5) becomes:

$$V(k_t, z_t) =$$

$$\max_{k_{t+1};\, l_t} \left[\frac{\left(z_t\, k_t^{\alpha}\, h_t^{1-\alpha} - k_{t+1} + (1-\delta) k_t\right)^{1-\theta} - 1}{1-\theta} + A \frac{(1-h_t)^{1-\gamma} - 1}{1-\gamma} \right] + \beta \mathbb{E}_t V(k_{t+1}, z_{t+1}) \tag{A.6}$$

with F.O.C.'s:

$$\frac{\partial V(k_t, z_t)}{\partial k_{t+1}} = -\underbrace{\left(z_t\ k_t^{\alpha}\ h_t^{1-\alpha} - k_{t+1} + (1-\delta)k_t\right)}_{c_t}{}^{-\theta} + \beta \mathbb{E}_t \frac{\partial V(k_{t+1}, z_{t+1})}{\partial k_{t+1}} = 0 \tag{A.7}$$

$$\frac{\partial V(k_t, z_t)}{\partial h_t} = \underbrace{\left(z_t\ k_t^{\alpha}\ h_t^{1-\alpha} - k_{t+1} + (1-\delta)k_t\right)}_{c_t}{}^{-\theta} \underbrace{(1-\alpha)(z_t\ k_t^{\alpha}\ h_t^{-\alpha})}_{w_t} - A(1-h_t)^{-\gamma} = 0 \tag{A.8}$$

The application of the envelope theorem allows us to write:

$$\frac{\partial V(k_t, z_t)}{\partial k_t} = (z_t\ k_t^{\alpha}\ h_t^{1-\alpha} - k_{t+1} + (1-\delta)k_t)^{-\theta}[\underbrace{\alpha(z_t\ k_t^{\alpha-1}\ h_t^{1-\alpha})}_{R_t} + (1-\delta)] \tag{A.9}$$

That shifted for one period, give a solution for:

$$\frac{\partial V(k_{t+1}, z_{t+1})}{\partial k_{t+1}} = \underbrace{\left(z_{t+1}\ k_{t+1}^{\alpha}\ h_{t+1}^{1-\alpha} - k_{t+2} + (1-\delta)k_{t+1}\right)}_{c_{t+1}}{}^{-\theta}[\underbrace{\alpha(z_{t+1}\ k_{t+1}^{\alpha-1}\ h_{t+1}^{1-\alpha})}_{R_{t+1}} + (1-\delta)] \tag{A.10}$$

Hence, the F.O.C in (A.7) becomes:

$$c_t^{-\theta} = \beta\ \mathbb{E}_t \left[c_{t+1}^{-\theta}[R_{t+1} + (1-\delta)]\right] \tag{A.11}$$

And the one in (A.8):

$$c_t^{-\theta} w_t = A(1-h_t)^{-\gamma} \tag{A.12}$$

Appendix B

B.1 Data Description

- **Nominal Gross Fixed Capital Formation/Investment** : Gross Fixed Capital Formation, Quarterly, Current prices, million euro, Seasonally and Calendar adjusted, Italy, Eurostat.
- **Nominal Final Consumption of households**: Final Consumption expenditure of household, Quarterly Current prices, million euro, Seasonally and calendar adjusted data, Italy, Eurostat.
- **Working age population 15-64**: Working-Age Population Total, From 15 to 64 Years, Persons, Quarterly, Seasonally adjusted, Italy, FRED.
- **GDP implicit deflator**: GDP implicit deflator, Quarterly, Price index, 2015=100, euro, Seasonally and calendar adjusted data, Italy, Eurostat.
- **Hours worked**: Total Employment, Quarterly, Thousands of Hours worked, Seasonally and calendar adjusted data, Statistical Office of the European Commission (Eurostat), Dataset: ENA, downloaded from ECB Employment Hours Dataset.
- **Hourly Labor Productivity**: Real labor productivity per hour worked; Quarterly, Index, 2010=100,Seasonally and calendar adjusted data, Italy, Eurostat.

Appendix C

C.1 Steady States: Main Calibration

The following steady states implied from the main calibration are also used to initialize the model on Dynare.

$$\bar{z} = 1; \tag{C.1}$$

$$\bar{R} = 0.035101 \tag{C.2}$$

$$\bar{k} = 5.53078 \tag{C.3}$$

$$\bar{y} = 0.58829 \tag{C.4}$$

$$\bar{i} = 0.138269 \tag{C.5}$$

$$\bar{c} = 0.450021 \tag{C.6}$$

$$\bar{l} = 0.8049 \tag{C.7}$$

$$\bar{w} = 2.02027 \tag{C.8}$$

$$A = 4.4893 \tag{C.9}$$

The parameter A seen in Section 2.2 is:

$$A = \frac{(1-\bar{h})^{\gamma}}{\bar{c}^{\theta}} \frac{\bar{y}}{\bar{h}}(1-\alpha) \tag{C.10}$$

C.2 Solution: Main Calibration

This section shows the first-order perturbation solution of the RBC model in the form presented in equations 2.35 and 2.36.

$$\begin{bmatrix} ln(y_t/\bar{y}) \\ ln(c_t/\bar{c}) \\ ln(h_t/\bar{h}) \\ ln(R_t/\bar{R}) \\ ln(w_t/\bar{w}) \\ ln(i_t/\bar{i}) \\ ln(l_t/\bar{l}) \\ ln(Hprod_t/(H\bar{p}rod) \end{bmatrix} = \begin{bmatrix} -0.005649 & 1.630030 \\ 0.495320 & 0.674761 \\ -0.500968 & 0.955268 \\ -1.005649 & 1.630030 \\ 0.495320 & 0.674761 \\ -1.636134 & 4.739111 \\ 0.12430 & -0.231548 \\ -0.121430 & -0.231548 \end{bmatrix} \begin{bmatrix} ln(k_t/\bar{k}) \\ ln(z_t/\bar{z}) \end{bmatrix} + \begin{bmatrix} 0.013119 \\ 0.005431 \\ 0.007688 \\ 0.013119 \\ 0.005431 \\ 0.038142 \\ -0.001864 \\ 0.005431 \end{bmatrix} \epsilon_t^{tech} \tag{C.11}$$

$$\begin{bmatrix} ln(k_{t+1}/\bar{k}) \\ ln(z_{t+1}/\bar{z}) \end{bmatrix} = \begin{bmatrix} -0.9340. & 0.118478 \\ 0 & 0.99 \end{bmatrix} \begin{bmatrix} ln(k_t/\bar{k}) \\ ln(z_t/\bar{z}) \end{bmatrix} + \begin{bmatrix} 0.000954 \\ 0.007968 \end{bmatrix} \epsilon_t^{tech} \tag{C.12}$$

C.3 Parametrized State-space Model: Main Calibration

The following is the parametrized state space model as expressed in (2.44) (no measurement error is considered) and (2.45)

$$\begin{bmatrix} ln(y_t/\bar{y}) \end{bmatrix} = \begin{bmatrix} -0.005649 & 1.630030 \end{bmatrix} \begin{bmatrix} ln(k_t/\bar{k}) \\ ln(z_t/\bar{z}) \end{bmatrix} \tag{C.13}$$

$$\begin{bmatrix} ln(k_{t+1}/\bar{k}) \\ ln(z_{t+1}/\bar{z}) \end{bmatrix} = \begin{bmatrix} -0.9340. & 0.118478 \\ 0 & 0.99 \end{bmatrix} \begin{bmatrix} ln(k_t/\bar{k}) \\ ln(z_t/\bar{z}) \end{bmatrix} + \begin{bmatrix} 0.000954 \\ 0.007968 \end{bmatrix} \epsilon_t^{tech} \tag{C.14}$$

C.4 RBC Impulse Response Functions: Main Calibration

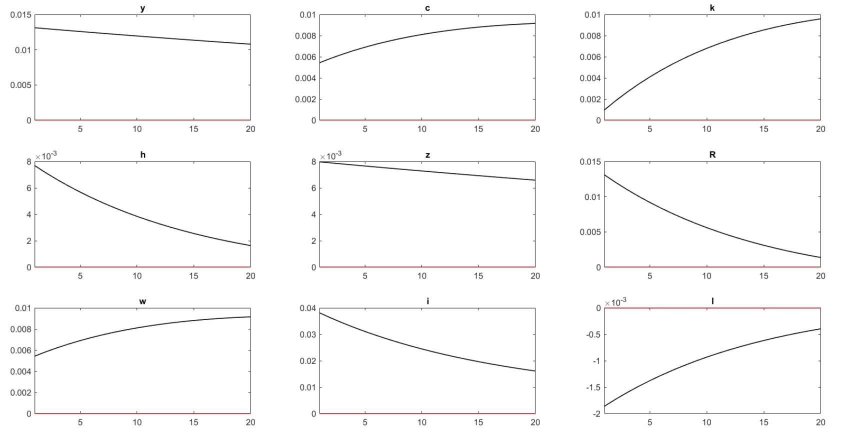

Figure C.1: RBC Impulse Responses following a technology shock

Appendix D

D.1 Plot of First Differenced Series

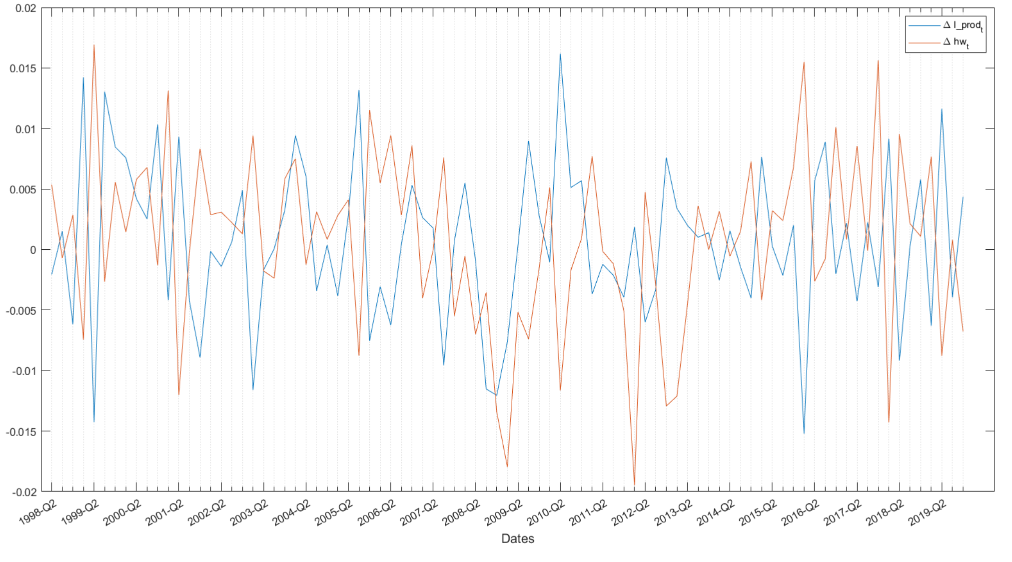

Figure D.1: Plot of the first-differenced series in Yt

D.2 Estimated VAR Reduced-form

$$\begin{pmatrix} Y_t \\ z_t \end{pmatrix} = \begin{pmatrix} \begin{bmatrix} 0.0005 \\ 0.0002 \end{bmatrix} \\ 0 \end{pmatrix} D_t + \begin{pmatrix} \begin{bmatrix} -0.0410 & 0.2429 \\ 0.5335 & 0.1789 \end{bmatrix} & 0 \\ 0 & 0.2216 \end{pmatrix} \begin{pmatrix} Y_{t-j} \\ z_{t-1} \end{pmatrix} + \begin{pmatrix} u_t \\ \nu_{z,t} \end{pmatrix} \tag{D.1}$$

with:

$$\hat{\Sigma}_u = \begin{pmatrix} 3.7362e^{-5} & - & - \\ -2.24870e^{-5} & 4.14680e^{-5} & - \\ 1.33717e^{-5} & 8.40584e^{-5} & 2.97319e^{-5} \end{pmatrix} \tag{D.2}$$

D.3 Estimated SVAR On-impact Matrix B

$$\begin{pmatrix} u_{1,t} \\ u_{2,t} \\ \nu_{z,t} \end{pmatrix} = \underbrace{\begin{pmatrix} 0.0035 & -0.0050 & 0 \\ (8.2273e^{-}4)^{1} & (5.5945e^{-}4) & - \\ 0.0022 & 0.0060 & 0 \\ (9.4321e^{-}4) & (5.4678e^{-}4) & - \\ 0.0038 & 0 & 0.0039 \\ (5.2433e^{-}4) & 0 & (3.2179e^{-}4) \end{pmatrix}}_{\hat{B}} \begin{pmatrix} \epsilon_t^{tech} \\ \epsilon_t^{notech} \\ 1 \end{pmatrix} \tag{D.3}$$

D.4 IRFs without TFP Restrictions

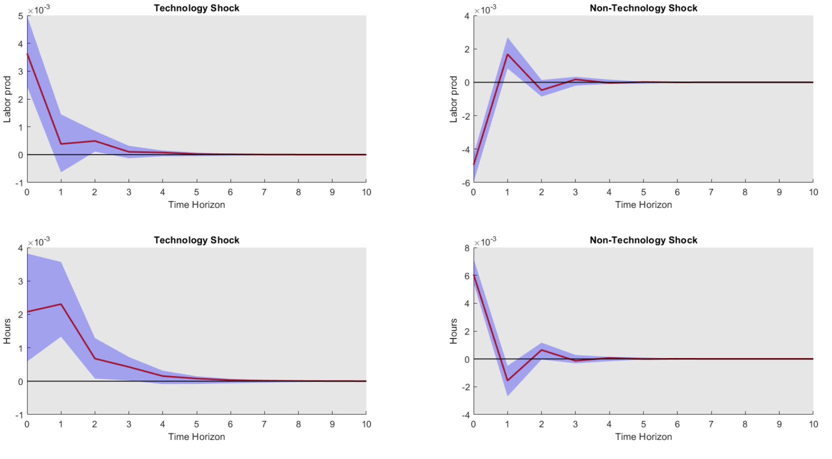

Figure D.2: Impulse response to positive technology and non-technology shock of Italian labor productivity and hours worked. Hall's percentile 90% confidence bands based on 1000 iid bootstrap repetitions. The sample period is 1998Q2-2019Q4. Reduced-form without TFP restrictions on Y_t

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