SESSION 2: MATLAB NUMERICS

htp://uiuc-cse.github.io/matlab-sp17/

OUTLINE

- Review: Matlab basics
  - Control Flow, Function
- *Example: Heat conduction*
- Example: Radioactive decay chain
  single ODE, systems of linear ODE
- *Example: Systems of non-linear ODE*

FOR LOOP, IF-ELSE

- for loop :

```
for loop_index = vector
        block of code
end
```

- if-elseif – else statement:

```
if condition 1
        do something 1
elseif condition 2
        do something 2
else
        do something 3
end
```

RECAP - FUNCTION

- Method 1: Explicitly define a function through m file.
- Method 2: Use a function handle

(ex)

```
fun = @(x) sin(x);
f1 = @(f,a,b,c) c*f(a+b);
f1(fun,2,3,4)
```

EXAMPLE: HEAT CONDUCTION (1)

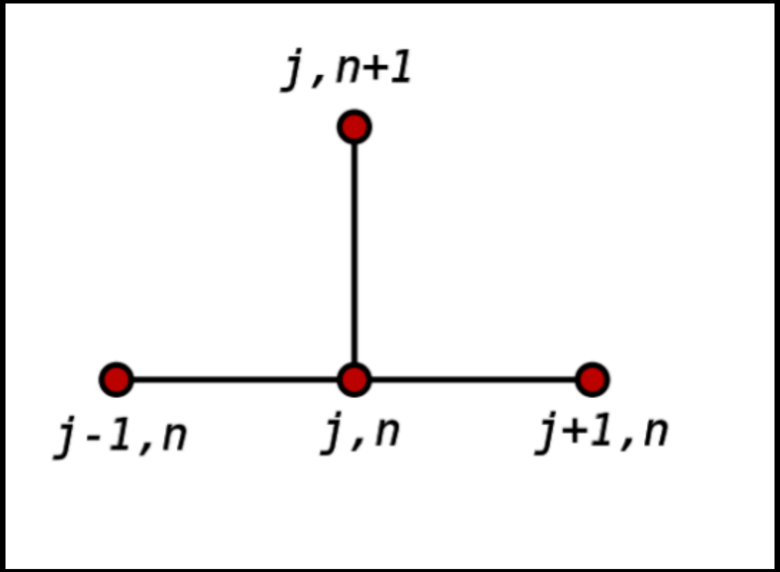
- Transient heat conduction problem

Initial condition u(x,t=0) = sin(πx/L),
Boundary conditions u(x=0,t) = u(x=L,t) = 0 and length L = 1

$$\boxed{\frac{1}{\alpha}\frac{du}{dt}=\frac{d^2u}{dx^2}} \text{ thus } \boxed{\frac{1}{\alpha}\frac{u_i^m}{\delta t}=\frac{u_{i-1}^{m-1}-2u_i^{m-1}+u_{i+1}^{m-1}}{\delta x^2}}$$

Finite difference approximation

$$f'(x)\approx\frac{f(x+h)-f(x)}{h} \quad \text{thus} \quad f''(x)\approx\frac{f(x+h)-2\mathrm{f}(x)+f(x-h)}{h^2}$$



EXAMPLE: RADIOACTIVE DECAY CHAIN (1)

- $\frac{dN}{dt} = -\lambda N$

where "N" is the number of atoms and λ is the decay constant of the material.

- Now, this system has an analytic solution of the following form:

$$N(t) = N_0 e^{(-\lambda t)}$$

Write a function (dNdt) to return the analytical value for N given N0, λ, t.

- Matlab's built-in solvers: ode23, ode45, etc…
- Compare the results visually:
  - analytical vs. ode23
  - ode23 vs. ode 45

EXAMPLE: RADIOACTIVE DECAY CHAIN (2)

- Now let's introduce another species.
- System of Linear ODE:

$$
\begin{aligned}
\frac{dc_1}{dt} &= -\lambda_1\, c_1 \\
\frac{dc_2}{dt} &= \lambda_1\, c_1 - \lambda_2\, c_2 \\
\frac{dc_3}{dt} &= \lambda_2\, c_2 - \lambda_3\, c_3
\end{aligned}
$$

Matrix Form

$$
\begin{bmatrix} c_1^{(1)} \\ c_2^{(1)} \\ c_3^{(1)} \end{bmatrix} = \begin{bmatrix} -\lambda_1 & 0 & 0 \\ \lambda_1 & -\lambda_2 & 0 \\ 0 & \lambda_2 & -\lambda_3 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix}
$$

- Re-write dNdt function with vectorized equations with two given decay constants as follows:
  - lambda_A = log(2);
  - lambda_B = log(2)/20;
- Evaluate using ode 45 with N0=[100;0]; T=[0 50]

SYSTEMS OF NONLINEAR ODE(1)

- System of Lorenz equations:

$$
\begin{aligned}
\frac{dx}{dt} &= -\sigma x + \sigma y \\
\frac{dy}{dt} &= \rho x - y - xz \\
\frac{dz}{dt} &= -\beta z + xy,
\end{aligned}
$$

- Solving a system of nonlinear ODE in MATLAB is quite similar to solving a single equation, but we must define a function for a system of equation as an m-file (not by function control)

SYSTEMS OF ODE (2)

- Set the initial condition and parameters:

  $\sigma = 10$, $\beta = 8/3$, and $\rho = 28$, as well as $x(0) = -8$, $y(0) = 8$, and $z(0) = 27$
- Define a function as m file:

  function xprime = *lorenz(t,x);*

  …
- Evaluate with ode45
  - >>x0=[-8 8 27]; tspan=[0,20];
  - >>[t,x]=ode45(@lorenz,tspan,x0)
- Plot the Lorenz strange attractor, which is a plot of z versus x:
  - >>plot(x(:,1),x(:,3))
- Plot each component of the solution as a function of t,
  - >>plot(t,x(:,1))