9.1.21

$0, 4, 8, 12, 16, \ldots$

$a_n = ?$ met $n \geq 1$

Opl:

$a_n = 4n - 4$ want 0 is met $n = 1$ en niet aan 0

9.1.31

$a_n = 4 + (0.6)^n$ convergent of divergent.

$\lim_{n \to \infty} a_n = ?$ of diverge

$\lim_{n \to \infty} a_n = 4$

9.1.33

Converge or diverge?

$a_n = \frac{1-7n}{3+7n}$ $\lim_{n\to\infty} \Rightarrow$

Herschrijven volgens regels

$\lim_{n\to\infty} = \frac{1/n - 7}{3/n + 7} = \frac{0-7}{0+7} = -1$

9.1.35

Converge or diverge

$a_n = \frac{3-3n^4}{n^4+2n^3}$

Opl $\frac{\frac{3}{n^4}-3}{1+\frac{2\cancel{n^3}}{n^4}} = \frac{\frac{3}{n^4}-3}{1+\frac{2}{n}}$

$\lim \frac{3}{n^4} = 0$

$\lim \frac{2}{n} = 0$

$\rightarrow \frac{-3}{1} = -3$

9.1.51

lim $a_n = \frac{n}{10^n} = \frac{\infty}{\infty}$ L'Hôpital rule

$$\frac{1}{\ln 10 \cdot 10^n} = \frac{1}{\infty} = 0$$

9.2.1

find n^{th} partial sum of series

$$4 + \frac{4}{3} + \frac{4}{2\cancel{7}} + \dots + \frac{4}{3^{n-1}}$$

Opl said ratio olvr 2ᵈᵉ - net 1ˢᵗᵉ te olele.

$$r = \frac{\frac{4}{3}}{4} = \frac{1}{3}$$

n^{th} term is $\frac{4}{3^{n-1}}$ thus k^{th} term is $\frac{4}{3^{k-1}}$

General series $\sum_{k=1}^{\infty} a r^{k-1}$ an n^{th} partial sum $a_n = \frac{a(1-r^n)}{1-r}$

$$S_n = \frac{a(1-z^n)}{1-z}$$

$$= \frac{4\left(1-\left(\frac{1}{3}\right)^n\right)}{1-\frac{1}{3}} = \frac{4\left(1-\left(\frac{1}{3}\right)^n\right)}{\frac{3}{3}-\frac{1}{3}}$$

$$= \frac{12}{2}\left(1-\left(\frac{1}{3}\right)^n\right) = 6\left(1-\left(\frac{1}{3}\right)^n\right)$$

De serie convergeert als $|z| < 1$ en de som is $\frac{a}{1-z}$

dus de serie convergeert naar 6

9.2.7

$$\sum_{n=0}^{\infty} \frac{(-1)^n}{2^n} \qquad 1 - \frac{1}{2} + \frac{1}{4} - \frac{1}{8}$$

a

$$z = \frac{-\frac{1}{2}}{1} = -\frac{1}{2} \quad \rightarrow \text{Seriesom} = \frac{a}{1-z}$$

dus $\frac{1}{1--\frac{1}{2}} = \frac{1}{1+\frac{1}{2}} = \frac{1}{\frac{3}{2}} = \frac{2}{3}$

9.2.11

$$z = \frac{\left(\frac{8}{5} + \frac{2}{3}\right)}{(8+2)}$$ is $|z| < 1$? dan convergentie

$$a = (8+2)$$ niet juist want z = niet apart bekeken worden in een geometric serie

$$z = \sum_{n=0}^{\infty} \frac{8}{5^n} = \frac{1}{5}$$

en

$$z = \sum_{n=0}^{\infty} \frac{2}{3^n} = \frac{1}{3}$$

$$a = 8 \longrightarrow \frac{a}{1-z} = \frac{8}{1-\frac{1}{5}} = \frac{40}{4} = 10$$

$$a = 2 \longrightarrow \frac{a}{1-z} = \frac{2}{1-\frac{1}{3}} = \frac{6}{2} = 3$$

dan som dus 13 in totaal

9.2.25

Express repeating decimal as ratio of two integers

$0.\bar{7} = 0.7777\ldots$

Orf

$0.7777 = \frac{7}{10} + \frac{7}{10^2} + \frac{7}{10^3}$

$= \frac{7}{10}\left(1 + \frac{1}{10} + \left(\frac{1}{10}\right)^2 + \left(\frac{1}{10}\right)^3 + \ldots\right)$

$a = 1$

$r = \frac{1}{10}$

$= \frac{7}{10}\left(\frac{1}{1-\frac{1}{10}}\right) = \frac{7}{9}$

9.2.~~8~~3

Find the Values of x where the series converges + Sum

$$\sum_{n=0}^{\infty} (-1)^n (x-3)^n$$

find $z \Rightarrow (-1)^0 (x-3)^0 = 1$

$(-1)^1 (x-3)^1 = -(x-3) = -x+3$

$$z = \frac{-x+3}{1} = -x+3$$

z moet tussen -1 en 1 liggen

$-1 < -x+3 < 1$

dus x: $-x+3 = -1 \Rightarrow x = 4$

$-x+3 = 1 \Rightarrow x = 2$

dus $2 < x < 4$

Sum of series $\frac{a}{1-z} \Rightarrow \frac{1}{1-(-x+3)}$

9.7.15

$$\sum_{n=0}^{\infty} \frac{(x-5)^n}{10^n}$$

Find the radius of convergence

1. $\lim_{n \to \infty} \left| \frac{U_{n+1}}{U_n} \right|$

$\rightarrow \left| \frac{x-5}{10} \right|$ waarbij < 1 dus $\frac{|x-5|}{10} < 1$

$x \Rightarrow 15$ of -5

$-5 < x < 15$

radius is 2 of ratio of 10

9.7.11

$$\sum^{\infty} \frac{(-1)^n (x+1)^{n+1}}{n!}$$

Voor welke waarde x convergeert en welke radius

$$\lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right|$$ if $\rho < 1$: absoluutly convergentie

$\rho > 1$ of $\rho = \infty$: divergentie

if $\rho = 0$: ?

$$\lim_{n\to\infty} \left| \frac{(x+1)^{n+1}}{(n+1)!} \cdot \frac{n!}{(x+1)^n} \right| = \left| \frac{x+1}{n+1} \right| = \frac{|x+1|}{n+1} \simeq 0$$

$\Downarrow$

$0 < 1$

$\Downarrow$

$-\infty < x < \infty$

en $R = \infty$ aangezien elke x kan.

9.8.5

Taylor polynoom voor orde ~~0~~, 1, 2, 3 voor f at $x=a$

$$f(x) = \frac{1}{x}, \quad a = 3$$

Opl

$P_0(x) = \frac{1}{3}$ want $\frac{f^{(k)}(a)}{(k!)} \cdot x-a)^k$ met $k=0$

$P_1(x) =$

$$P_0(x) + f'(a)(x-a)$$

$$\frac{1}{3} + \left(-\frac{1}{a^2}\right)(x-3)$$

$$\frac{1}{3} - \frac{1}{(3)^2}(x-3)$$

$$\frac{2}{3} - \frac{x}{9}$$

2de orde

$$\frac{f''(a)}{2!}(x_1-a)^2$$

$$\frac{d}{dx}\left(-\frac{1}{x^2}\right)=\frac{2}{a^3}$$

$$\frac{1}{3}+\left(\frac{2}{3}-\frac{x}{9}\right)+\left(\frac{\frac{2}{3^3}}{2}(x-3)^2\right)$$

0de 1ste 2de

$$=\frac{x}{27}-\frac{x}{3}+1$$

3de orde

$$\frac{f'''(a)}{3!}(x-a)^3$$

$$\frac{d}{dx}\left(\frac{2}{a^3}\right)=-\frac{6}{a^4} \quad \text{dus} \quad \frac{\frac{-6}{3^4}}{6}(x-3)^3$$

$$\text{t. vergaand} = \frac{-x^3}{81}+\frac{4x^2}{27}-\frac{2x}{3}+\frac{4}{3}$$

9.9.7

General expression in the taylor series $x=0$ for

$\ln(1+6x^4)$

Normal: $$\sum_{n=1}^{\infty} \frac{(-1)^{n-1} x^n}{n} = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \dots$$

$$\frac{(-1)^{n-1}(6x^4)^n}{n} = 6x^4 - \frac{(6x^4)^2}{2} + \frac{(6x^4)^3}{3} \dots$$

$$= 6x^4 - \frac{6^2x^8}{2} + \frac{6^3x^{12}}{3} - \dots$$

9.9.25

taylor $x = 0$ for $x^5 \tan^{-1} x^4$

$$\sum_{n=0}^{\infty} \frac{(-1)^n x^{8n+9}}{2n+1} \quad \text{T Fuche}$$

$x = 0$ for $x^5 \tan^{-1} x^3$

$$\sum_{n=0}^{\infty} \frac{(-1)^n x^{6n+8}}{2n+1} \leftarrow (2 \times 3)n + (5+3)$$

$x = 0$ for $x^2 \tan^{-1} x^5$

$$\sum_{n=0}^{\infty} \frac{(-1)^n x^{10n+7}}{2n+1} \leftarrow (5 \times 2)n + (5+2)$$

9.9.41

Sin x replace by $x - \frac{x^3}{6}$ with error les than $4 \cdot 10^{-4}$

→ $x - \frac{x^3}{6}$ ob twee eerste termen van ob Mclaurin series

voor sin : de volgende term $\left|\frac{x^5}{5!}\right| = \frac{|x^5|}{120}$

$$4 \cdot 10^{-4} > \left|\frac{x^5}{120}\right|$$

9.10.1

first four terms of $(1+9x)^{1/2}$ binomial $(1+x)^m$

eerste term is steeds 1

$$\binom{m}{k} = \frac{m(m-1)(m-2)\dots(m-k+1)}{k!}$$

$$\begin{array}{l} m = \\ k = \end{array} \binom{1/2}{1} = \frac{1}{2} \cdot 9x = \frac{9}{2}x$$

↑ tweede term

$$\begin{array}{l} m \\ k \end{array} \binom{1/2}{2} = \frac{m(m-1)}{2!} = -\frac{1}{8} \cdot (9x)^2 = \frac{-81x^2}{8}$$

↑ tweede term

obwide tem $\begin{matrix} n \\ k \end{matrix}\binom{1/2}{3} = \dfrac{\frac{1}{2}\left(\frac{1}{2}-1\right)\left(\frac{1}{2}-2\right)}{3!} = \dfrac{1}{16}$

$$\frac{1}{16}\cdot(9x)^3 = \frac{729\,x^3}{16}$$

& obwide tem

Allmecol sanen

9.10.5

first four terms binomial $\left(1+\frac{x}{7}\right)^{-2}$

first tem $k=0 \Rightarrow 1$

$\binom{-2}{1} = n = -2$ over de 2^{de} net $\frac{x}{7}$ dees

$-\frac{2x}{7}$ wor 2de tem

$\binom{-2}{2} = 3 \rightarrow \quad 3\left(\frac{x}{7}\right)^2 = \frac{3x^2}{49}$

$\binom{-2}{3} = -4 \quad\quad -4\left(\frac{x}{7}\right)^3 = \frac{-4x^3}{343}$

9.10.15

$$\int_0^{0.3} \sin x^2 \, dx$$ met serie en precisie 10^{-5}

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \ldots$$

$$\sin x^2 = x^2 - \frac{(x^2)^3}{3!} + \frac{(x^2)^5}{5!} - \ldots$$

stel $\frac{(x^2)^5}{5!}$ met $0.3 = 0.0000000049208$

$4.92 \cdot 10^{-8}$

stel $\frac{(x^2)^3}{3!} = 0.0001215$ of $1.215 \cdot 10^{-4}$

$$\int x^2 - \frac{(x^2)^3}{3!} + \frac{(x^2)^5}{5!} \, dx = \frac{x^3}{3} - \frac{x^7}{7 \cdot 3!} + \frac{x^{11}}{11 \cdot 5!}$$

en dan pas checken met de factoren

9.10.20

$$\int_0^{0.3} e^{-x^2}\,dx \approx$$

met precisie 10^{-8}

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \dots$$

$$1 e^{-x^2} = 1\left[1 + (-x^2) + \frac{(-x^2)^2}{2!} + \frac{(-x^2)^3}{3!} + \frac{(-x^2)^4}{4!} \dots\right]$$

$$1\left[1 - x^2 + \frac{x^4}{2!} - \frac{x^6}{3!} + \frac{x^8}{4!}\right.$$

$$1\int_0^{0.3}$$

$$1\int_0^{0.3} x - \frac{x^3}{3} + \frac{x^5}{5\cdot 2!} - \frac{x^7}{7\cdot 3!} + \frac{x^9}{9\cdot 4!}$$

error zoeken met $x = 0{,}3$ " $5{,}2 \cdot 10^{-6}$ $\quad 9{,}11 \cdot 10^{-8}$

dus tot hier...

$= 0{,}29123789$