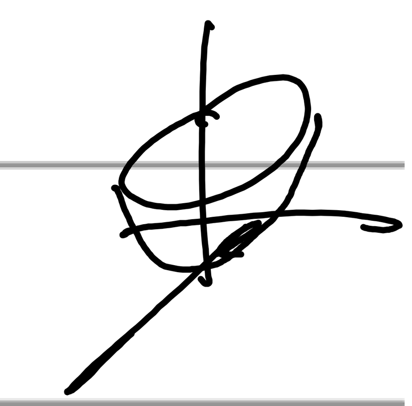
Hfst 6:

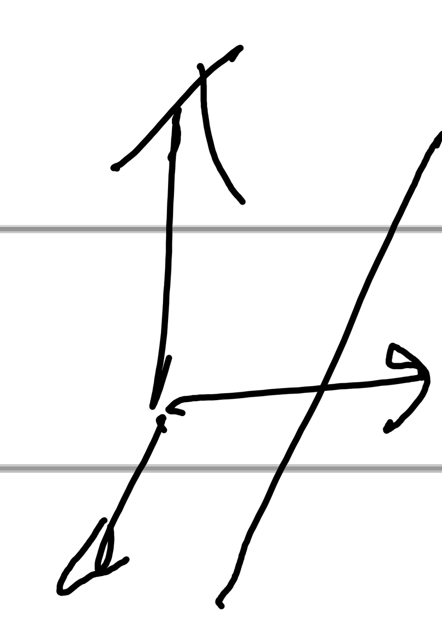
vectorvelden

Scalaire functies $z = f(x,y) \Rightarrow \mathbb{R}^2 \to \mathbb{R}$



Vector functie: $t \to \vec{r}(t) = (x(t), y(t), z(t))$

$\mathbb{R} \to \mathbb{R}^3$



Vectorvelden $(x,y,z) \to \vec{F}(x,y,z) \Rightarrow \mathbb{R}^3 \to \mathbb{R}^3$

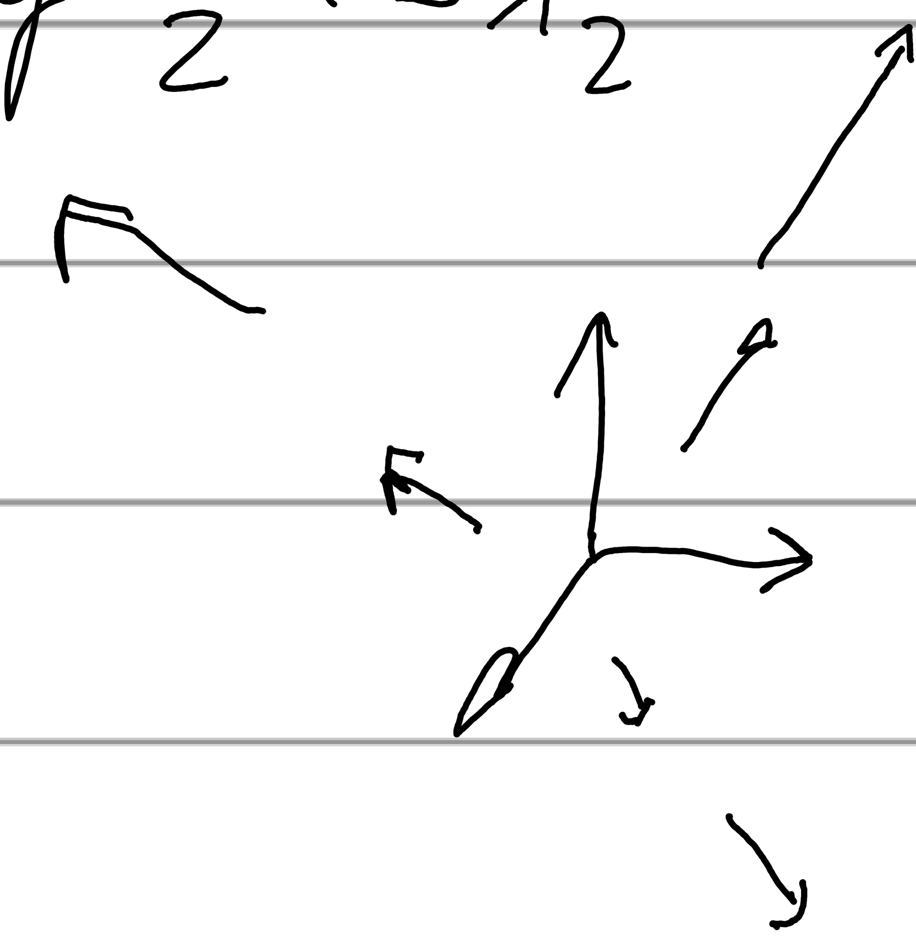
uitgebreid geschreven: $(F_x(x,y,z), F_y(x,y,z), F_z(x,y,z))$

$= (F_x, F_y, F_z)$

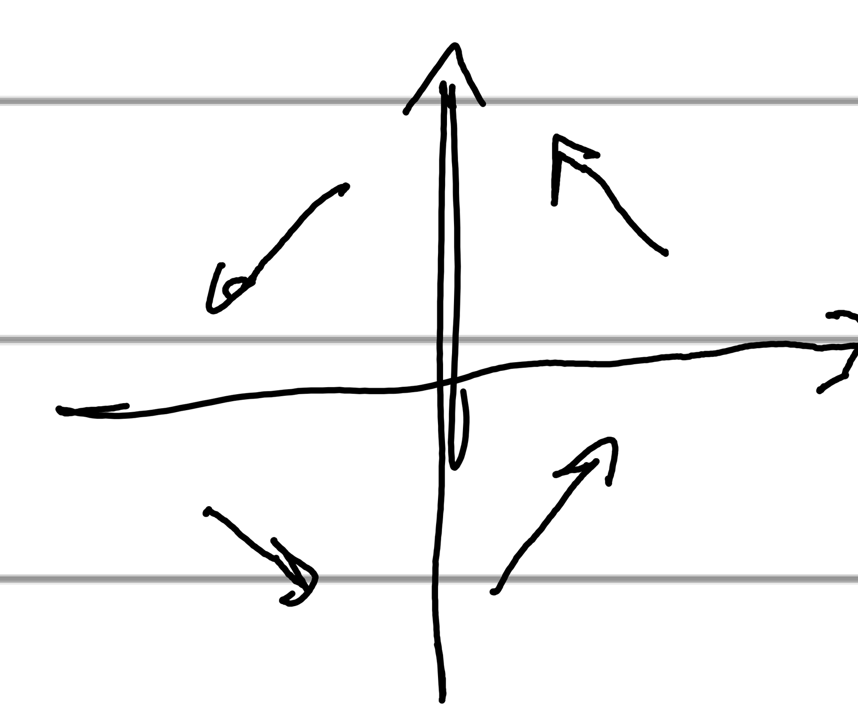
$= F_x \vec{1}_x, F_y \vec{1}_y, F_z \vec{1}_z$

VI 1. $\vec{F}(x,y,z)$ (plaatsvector $= \vec{r} = x\vec{1}_x + y\vec{1}_y + z\vec{1}_z$

wijst vanuit oorsprong en hoe verder hoe groter.



2. $\vec{F} = (-y, x)$ (rotatievector) $= -y\vec{1}_x + x\vec{1}_y$



3. $\vec{F} = \frac{GMm}{r^2}\vec{r}$ (gravitatieveld vector)

$m(x,y,z)$

4. $\vec{F} = \frac{qQ}{4\pi\varepsilon} \cdot \frac{\vec{r}}{r^3}$ (elektrostatische aantrekkingskracht

$q(x,y,z)$

$\vec{1}_r$: genormaliseerde plaatsvector $\frac{(x,y,z)}{\sqrt{x^2+y^2+z^2}}$

Operatie 4: divergentie

$\mathrm{Div}\,\vec{F} = \vec{\nabla} \cdot \vec{F}$

↳ inwendig product !

$$= \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right) \cdot \left(F_x, F_y, F_z\right)$$

$$= \frac{\partial F_x}{\partial x} + \frac{\partial F_y}{\partial y} + \frac{\partial F_z}{\partial z} \quad \in \mathbb{R}$$

Mate van totale verandering van vectorveld $\vec{F}$ in x, y, z richting

Vb $\vec{F} = (x^2 y, y^2 z, x z^2)$

$$\vec{\nabla} \cdot \vec{F} = \frac{\partial}{\partial x}(x^2 y) + \frac{\partial}{\partial y}(y^2 z) + \frac{\partial}{\partial z}(x z^2) = 2xy + 2yz + 2xz$$

(vb) $\vec{v}$: snelheidsvector stroming

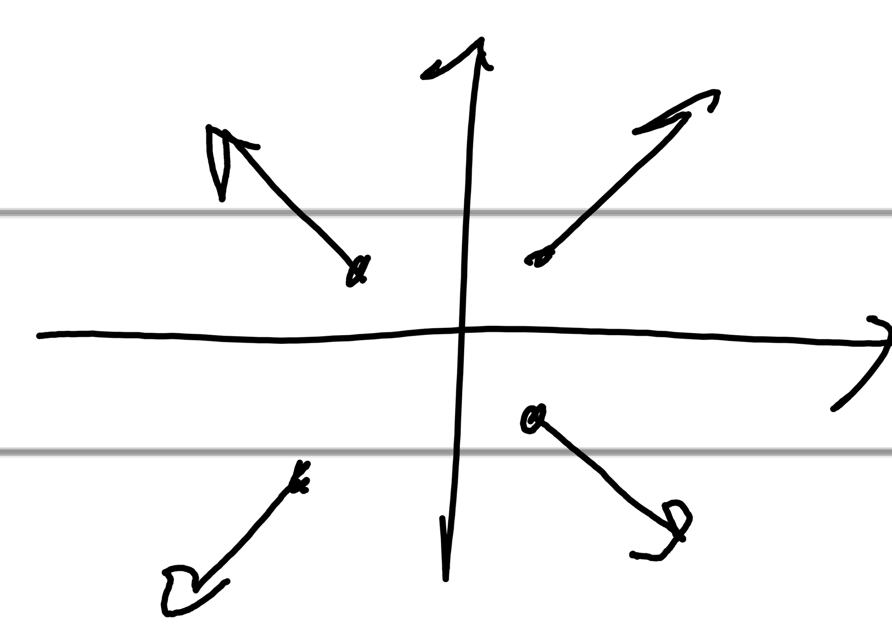
$$\vec{\nabla} \cdot \rho \vec{v} = -\frac{\partial \rho}{\partial t}$$ $\Leftrightarrow$ continuïteitsvergelijking uit vloeistofmechanica

ρ: dichtheid

(vb) $\vec{F} = (cx, cy)$

$c \in \mathbb{R}$

$c > 0$



$$\vec{\nabla} \cdot \vec{F} = \frac{\partial}{\partial x}(cx) + \frac{\partial}{\partial y}(cy) = 2c$$

met $c > 0$: uniforme expansie.

$c < 0$: inkrimpen

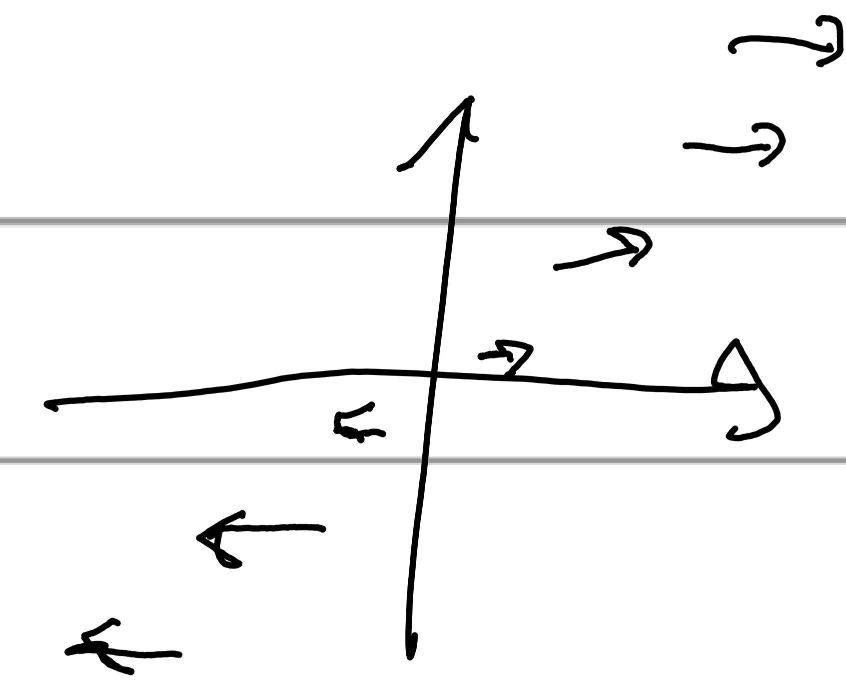
(vb) $\vec{F} = (-cy, cx)$

$c \in \mathbb{R}$

$c > 0$

$\vec{\nabla} \cdot \vec{F} = 0$: geen verandering (draait gewoon rond)

Vb $\vec{F} = (y, 0)$



$$\vec{\nabla}\cdot\vec{F} = \frac{\partial}{\partial x}(y) + \frac{\partial}{\partial y}(0) = 0$$: geen verandering

Vb $\vec{\nabla}\cdot\vec{E} = 0$ (elektrisch veld)

$\vec{\nabla}\cdot\vec{F} = 0$ (zwaartekracht veld)

$$\vec{\nabla}\cdot\frac{\vec{r}}{r^3}$$

$\vec{r} = (x, y, z)$

$r = \sqrt{x^2+y^2+z^2} = (x^2+y^2+z^2)^{1/2}$

$$\frac{\partial}{\partial x}\left[\frac{x}{(x^2+y^2+z^2)^{3/2}}\right] + \frac{\partial}{\partial y}\left[\frac{y}{(x^2+y^2+z^2)^{3/2}}\right] + \frac{\partial}{\partial z}\left[\frac{z}{(x^2+y^2+z^2)^{3/2}}\right]$$

$$\frac{\partial}{\partial x}\, x\cdot(x^2+y^2+z^2)^{-3/2} = (x^2+y^2+z^2)^{-3/2} - \frac{3}{2}\,x\,2x\,(x^2+y^2+z^2)^{-5/2}$$

$$= (x^2+y^2+z^2)^{-5/2}\cdot\left((x^2+y^2+z^2)^{1} - 3x^2\right)$$

zelfde voor de rest en als we alles optellen $= 0$

Def. $\vec{\nabla} \cdot \vec{F} = 0$ = solenoïdale vectorvelden

↳ vgl. je kan $\vec{G}$ vinden opdat $\vec{F} = \vec{\nabla} \times \vec{G}$

want $\vec{\nabla} \cdot (\vec{\nabla} \times \vec{G}) = 0$

Opper: rotatie

en $\begin{vmatrix} \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ F_x & F_y & F_z \end{vmatrix}$ gelijk dus det = 0

rot $\vec{F} = \vec{\nabla} \times \vec{F}$ = determinant

$$\begin{vmatrix} \vec{1}_x & \vec{1}_y & \vec{1}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ F_x & F_y & F_z \end{vmatrix} = \vec{1}_x \left(\frac{\partial F_z}{\partial y} - \frac{\partial F_y}{\partial z} \right) + \vec{1}_y \left(\frac{\partial F_x}{\partial z} - \frac{\partial F_z}{\partial x} \right) + \vec{1}_z \left(\frac{\partial F_y}{\partial x} - \frac{\partial F_x}{\partial y} \right)$$

In vlak

rot $\vec{F} = \vec{\nabla} \times \vec{F}$

$(F_x, F_y, 0)$

$$= \begin{vmatrix} \vec{1}_x & \vec{1}_y & \vec{1}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & 0 \\ F_x & F_y & 0 \end{vmatrix} = \left(0, 0, \frac{\partial F_y}{\partial x} - \frac{\partial F_x}{\partial y} \right)$$

rotatie in het vlak is een vector die er loodrecht opstaat betekenis : de mate van rotatie die een vector veld ondervindt

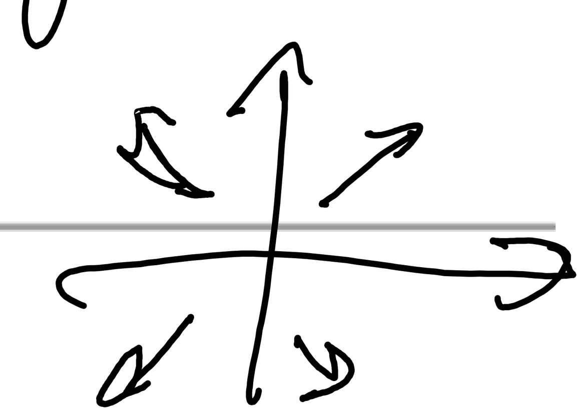
Vb

plaatsveld

1. $\vec{F} = (cx, cy)$

$c \in \mathbb{R}$

$$\vec{\nabla} \times \vec{F} = \left(0, 0, \frac{\partial (cy)}{\partial x} - \frac{\partial (cx)}{\partial y}\right) = 0$$



geen rotatie

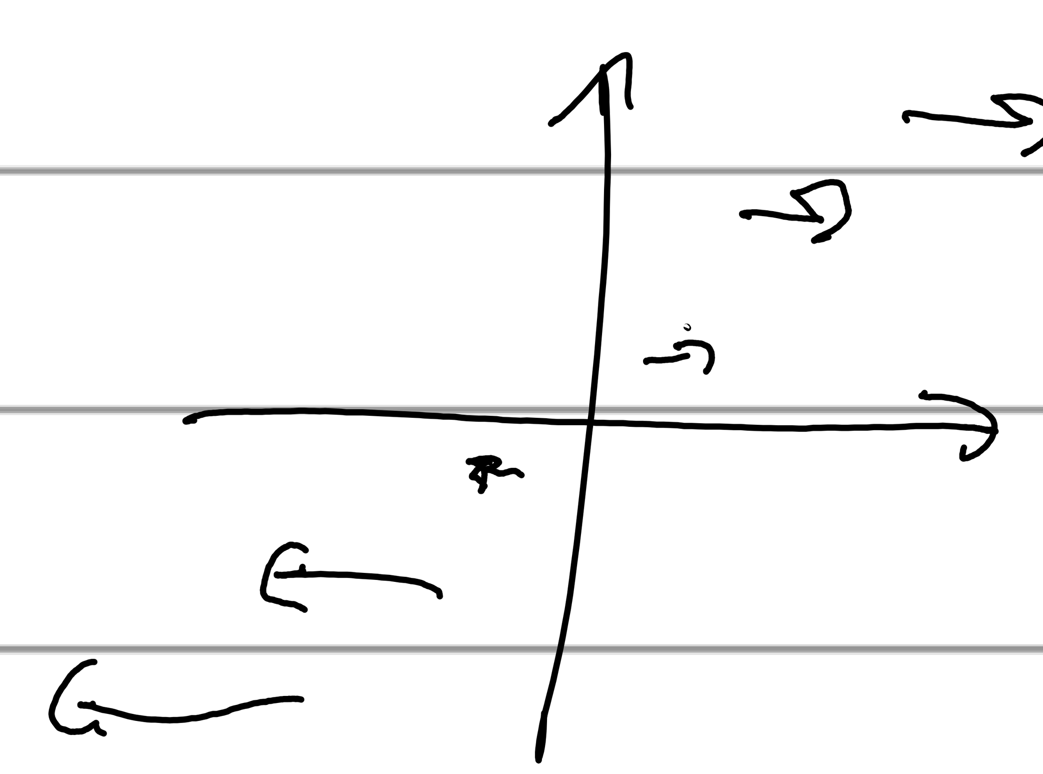
rotatieveld 2. $\vec{F} = (-cy, cx)$

$$\vec{\nabla} \times \vec{F} = \left(0, 0, \frac{\partial (cx)}{\partial x} - \frac{\partial (-cy)}{\partial y}\right) = (0, 0, 2c)$$

$c > 0$: tegen klok

Vectorveld 3. $\vec{F} = (y, 0)$

$$\vec{\nabla} \times \vec{F} = (0, 0, -1)$$



Def solenoidaal = divergentie = 0
Conservatief = $\vec{\nabla} \times \vec{F} = 0$

Elektrostatisch is zwaartekracht zijn conservatief

$\vec{\nabla} f$: gradiënt
$\vec{\nabla} \cdot \vec{F}$: divergentie
$\vec{\nabla} \times F$: rotatie

$\Delta f = \vec{\nabla} \cdot \vec{\nabla} f =$

↳ is een vectorveld en daarvan de divergentie

Laplaciaan.

$$\left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right) \cdot \left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z}\right) = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2}$$

Lijnintegralen

Vectorfunctie $t \to \vec{r}(t) = (x(t), y(t))$

Geg:

$$\vec{v}(t) = \frac{d\vec{r}}{dt} = \left(\frac{dx}{dt}, \frac{dy}{dt}\right)$$

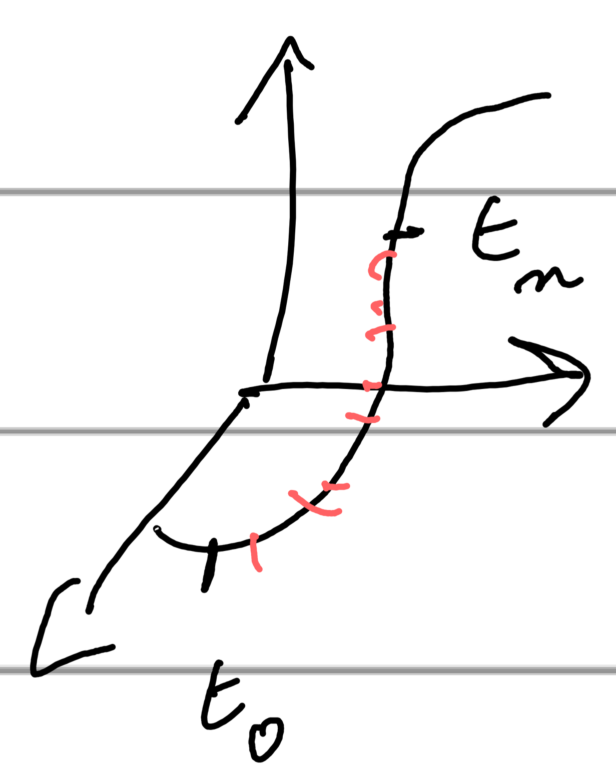
$$|\vec{v}| = \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}$$

$$ds^2 = \left(\frac{dx^2}{dt^2} + \frac{dy^2}{dt^2}\right) dt^2$$

$$ds = \sqrt{\frac{dx^2}{dt^2} + \frac{dy^2}{dt^2}}\, dt = |\vec{v}|\, dt$$

$$\vec{T} = \frac{\vec{v}}{|\vec{v}|} = \frac{dr/dt}{ds/dt} = \frac{d\vec{r}}{ds}$$

Lijnintegraal van scalaire functies



$$r(t) = (x(t), y(t), z(t))$$

$$\int_C f(x, y, z)\, ds \quad \left(\neq \int_a^b f(x)\, dx \right)$$

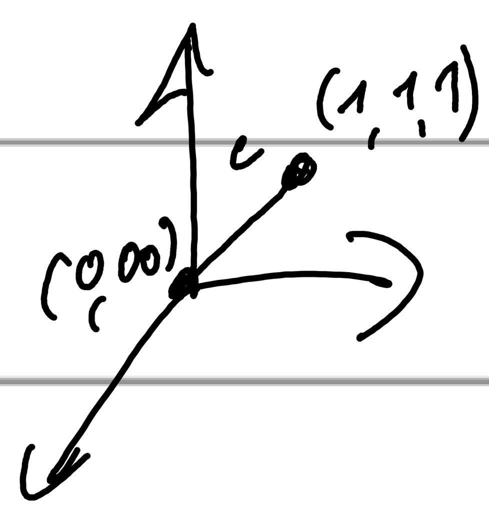
C ← curve

$$= \lim_{\substack{n \to \infty \\ \Delta k \to 0}} \sum_{k=1}^{n} f(x_k, y_k, z_k)\, \Delta x_k$$

In Praktijk

$$= \int_{t_0}^{t_1} f(x(t), y(t), z(t))\, \underbrace{|\vec{v}|\, dt}_{ds}$$

VI $\displaystyle\int_C (x-3y^2+z)\,ds$



$C \to \vec{r}(t) = (t, t, t) \to$ rechte lijn

$(0,0,0) \to (1,1,1)$

$\vec{r}(t) \to \vec{v}(t) = (1,1,1)$

$|\vec{v}| = \sqrt{3}$

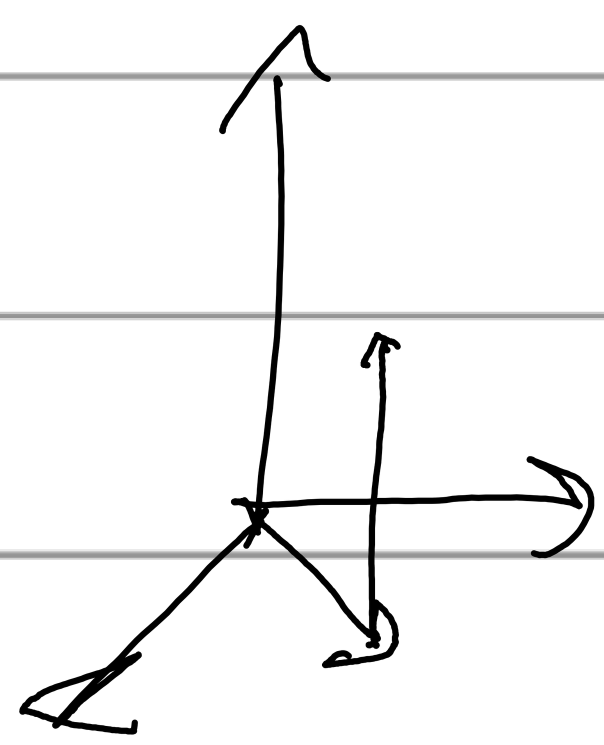
$$\int_0^1 (t-3t^2+\quad)\sqrt{3}\,dt = \int_0^1 (2t-3t^2)\sqrt{3}\,dt$$

$$\frac{\cancel{2}t^2}{\cancel{2}} - \frac{\cancel{3}t^3}{\cancel{3}} = t^2 - t^3$$

$$(0-0)\cdot\sqrt{3} = 0$$

Vb $\int_C (x-3y^2+z)\,ds$

net z Krane



$c = C_1 + C_2$

$C_1 = (0,0,0) \to (1,1,0) = \vec{r}(t,t,0)$

$C_1 = (1,1,0) \to (1,1,1) = \vec{r}(1,1,t)$

$$\int_{C_1} (x-3y^2+z)\,ds + \int_{C_2} (x-3y^2+z)\,ds$$

$C_1: \vec{r}(t,t,0) \to \vec{v}(1,1,0) \to |\vec{v}| = \sqrt{2}$

$C_2: \vec{r}(1,1,t) \to \vec{v}(0,0,1) \to |\vec{v}| = \sqrt{1} = 1$

$$\int_{C_1} \Rightarrow \int_0^1 (t-3t^2+0)\sqrt{2}\,ds = \sqrt{2}\cdot\left[\frac{t^2}{2}-\frac{\cancel{3}t^3}{\cancel{3}}\right]_0^1 = -\frac{1}{2}\sqrt{2}$$

$$\int_{C_2} \Rightarrow \int_0^1 (1-3\cdot 1^2+t)\sqrt{2}\,dt = -\frac{3}{2}$$

$$C_1 + C_2 = -\frac{\sqrt{2}}{2} - \frac{3}{2}$$

Toepassing: Massamiddelpunt van toren

$$M = \int_C \rho \, ds$$

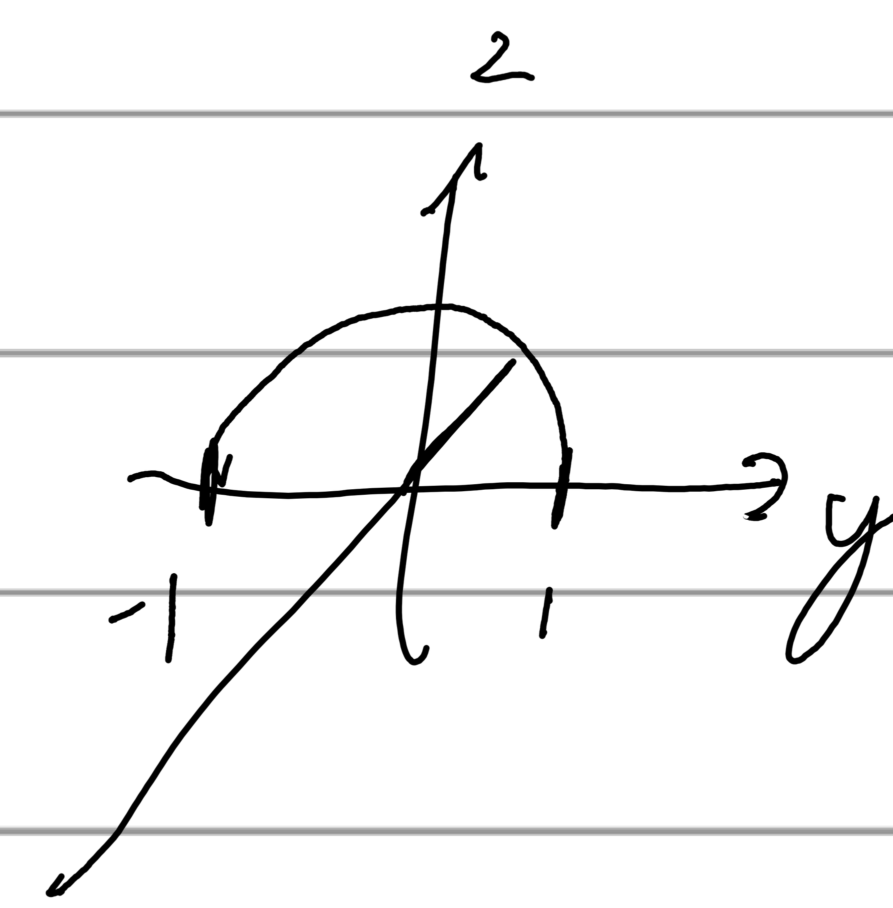
ρ massadichtheid, ds lengte

$$x_c = \frac{\int \rho x \, ds}{M}$$

idem voor y en z

Massamiddelpunt v/e boog in y/z vlak

dichtheid $\rho = 2 - z$



$x_c = 0$

$y_c = 0$

$\vec{r}(t) = (0, \cos t, \sin t)$

$\vec{v}(t) = (0, -\sin t, \cos t)$

$|\vec{v}| = 1 \quad (= \sqrt{\sin^2 + \cos^2})$

$$M = \int_C \rho \, ds = \int_0^{\pi} 2 - z \, ds = \int_0^{\pi} (2 - \sin(t)) \cdot 1 \, dt = 2\pi - 2$$

$$z_c = \int_0^{\pi} (2 - \sin t) \cdot \sin t \cdot 1 \, ds \text{ of } \int_0^{\pi} 2\sin t - \sin^2 t \, dt = \frac{8 - \pi}{2}$$

dus $\dfrac{\frac{8-\pi}{2}}{2\pi - 2} = \dfrac{8 - \pi}{4\pi - 4} = z_c$

Lijnintegralen van vectorvelden.

Arbeids integraal

$$\int_C \vec{F} \circ d\vec{r}$$

(F_x, F_y, F_z) (d_x, d_y, d_z)

$$= \int_C \vec{F} \cdot \frac{d\vec{r}}{ds}\, ds = \int_C \vec{F} \cdot \vec{T}\, ds$$

$$\int_C (\vec{F} \cdot \vec{T})\, ds = \lim_{n \to \infty} \sum_{k=1}^{n} \vec{F}(x_k, y_k, z_k) \cdot \vec{T}(x_k, y_k, z_k)$$

$$W = \int_C \vec{F} \cdot \vec{T}\, ds$$

ook $= \int \vec{F}\, d\vec{r}$ => vectornotatie

ook $= \int \vec{F}(\vec{r}(t)) \cdot \frac{d\vec{r}}{dt}\, dt$ => Parameter vectornotatie

$\int_C \left(F_x \frac{dx}{dt} + F_y \frac{dy}{dt} + F_z \frac{dz}{dt}\right) dt$ Parameter scalaire notatie

Theoretisch: scalaire differentiaal

$$\int_C F_x\,dx + F_y\,dy + F_z\,dz$$ dus dt weg te schrappen.

Vb arbeid door $\vec{F} = ((y-x^2), (z-y^2), (x-z^2))$

langs de weg $r(t) = (t, t^2, t^3)$

van $(0,0,0) \to (1,1,1)$

$$W = \int_C \vec{F} \cdot d\vec{r} = \int_0^1 \left(F_x \frac{dx}{dt} + F_y \frac{dy}{dt} + F_z \frac{dz}{dt} \right) dt$$

$$\vec{v} = (1, 2t, 3t^2)$$

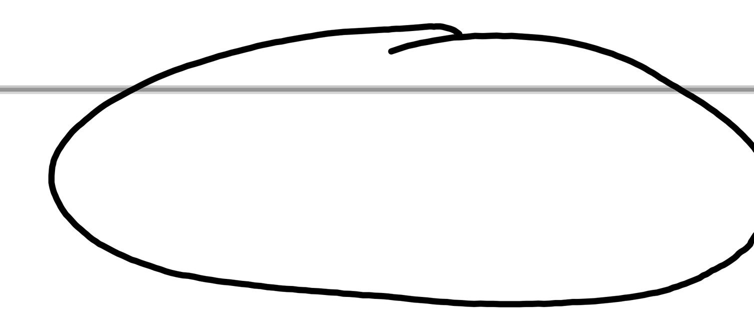
dus

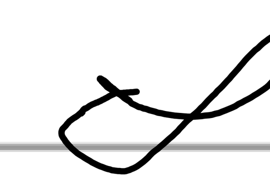
$$\int_0^1 (t^2 - t^2$$

WTF

Gesloten lijnintegraal

gesloten simpele kromme

wel 

niet 

dan spreken we over circulatie

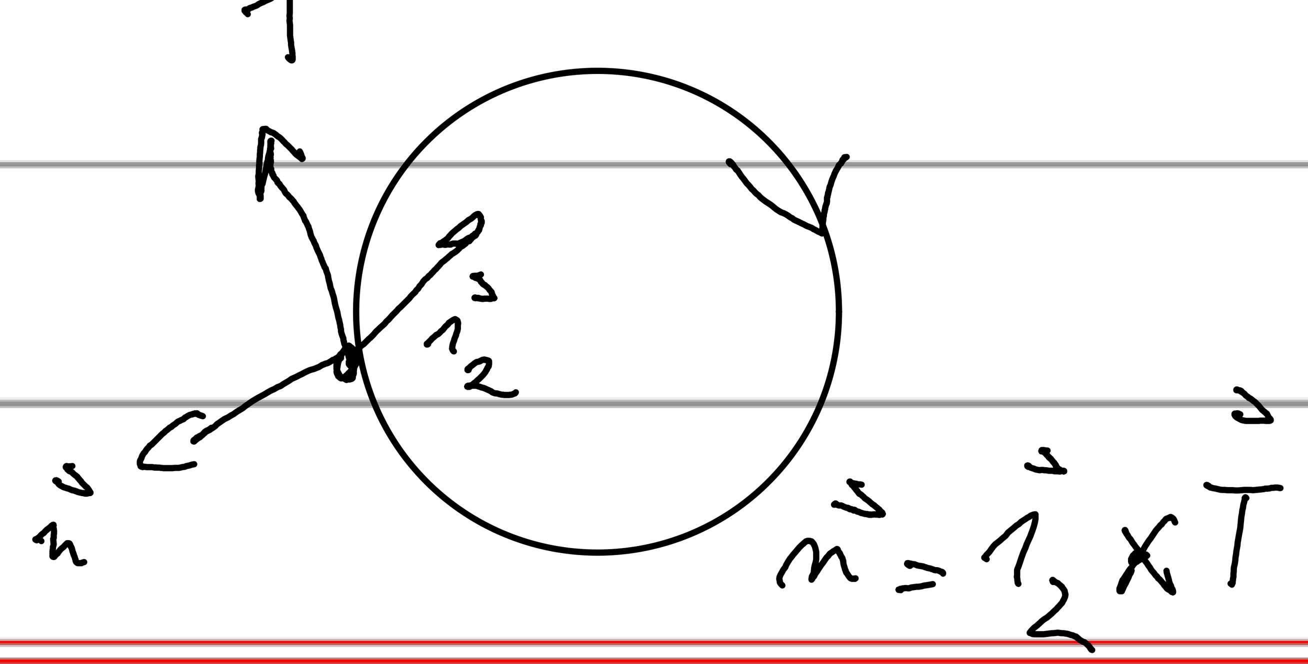
$$\oint_C \vec F \cdot \vec T \, ds$$ is arbeids integraal van geslote curve

<u>Flux</u>

$$\oint_C (\vec F \cdot \vec N)\, ds$$

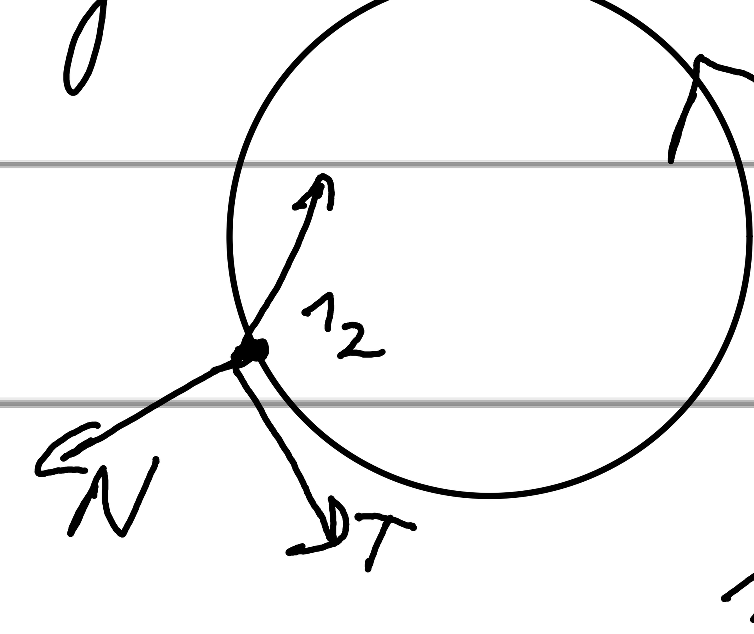
$$\vec N \perp \vec T \text{ en } \underbrace{\vec N \perp \vec 1_2}_{\text{als het in het vlak gebeurt}}$$

klok



$\vec n = \vec 1_2 \times \vec T$

tegenklok



$\vec n = \vec T \times \vec 1_2$

Met de klok

$$\begin{vmatrix} \vec{1}_x & \vec{1}_y & \vec{1}_z \\ 0 & 0 & 1 \\ \frac{dx}{ds} & \frac{dy}{ds} & 0 \end{vmatrix}$$

$$= \left(-\frac{dy}{ds}, \frac{dx}{ds}, 0\right)$$

Tegen de klok

Volgorde veranderd dus $0 = -0$

$$\left(\frac{dy}{ds}, \frac{-dx}{ds}, 0\right)$$

$$\oint_C \vec{F} \cdot \vec{N} \, ds = \int \left(F_x \cdot \frac{-dy}{ds} + F_y \cdot \frac{dx}{ds} \right) ds$$

$$\oint_C F_x \cdot \frac{-dy}{dt} + F_y \cdot \frac{dx}{dt} \, dt$$

(12) Bepaal de flux van het vectorveld ~~F~~ x_i:

$\vec{F} = (x - y, x)$ over cirkel $x^2 + y^2 = 1$ in tegenwijzerzin

$$\boxed{x = \cos t \quad y = \sin t}$$

$$z(t)\ (\cos t, \sin t)$$

$$\vec{v}(t)\ (-\sin t, \cos t)$$

$$\int_0^{2\pi} F_x \frac{dy}{dt} + F_y \frac{-dx}{dt}\, dt$$

$$\int_0^{2\pi} \underbrace{(\cos t - \cancel{\sin t}) \cos t}\ + \cancel{\cos t \cdot \sin t}\ dt$$

$$\int_0^{2\pi} \cos^2 t\, dt = \cancel{\sin t \cos t} + t = 2\pi$$

Lijnintegraal

↳ Scalair → $\int_C f \, ds = \int_C f \, |\vec{v}| \, dt$

↳ Vectorveld → $W = \int_C \vec{F} \cdot d\vec{r}$ of $\int_C \vec{F} \cdot \vec{T} \, ds$

$$= \int_C \left(F_x \frac{dx}{dt} + F_y \frac{dy}{dt} \right) dt$$

↳ fluxintegraal $\oint_C (\vec{F} \cdot \vec{N}) \, ds$

$$= \left(\frac{dy}{ds}, -\frac{dx}{ds}, 0 \right)$$

$$= \left(-\frac{dy}{ds}, \frac{dx}{ds}, 0 \right)$$

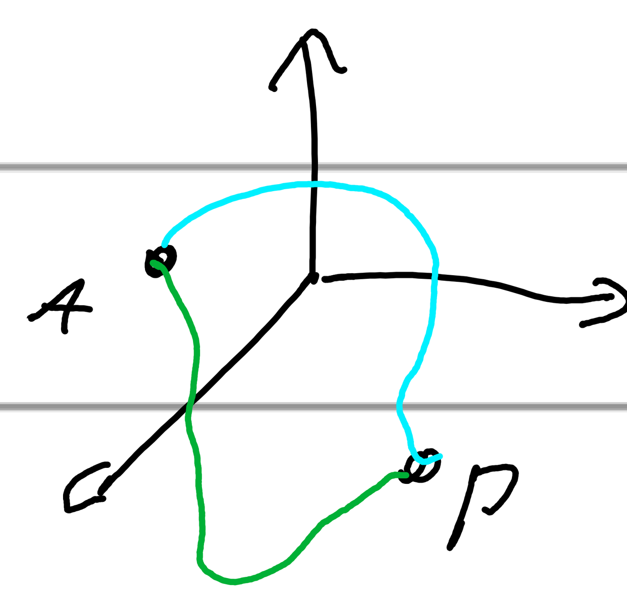
$$= \oint_C \left(F_x \frac{dy}{dt} + F_y \frac{-dx}{dt} \right) dt$$

Conservatieve vectorvelden

1) $\vec{F} = \vec{\nabla} f$ gegeven vectorveld $\vec{F} = (F_x, F_y, F_z)$

$$= \vec{\nabla} f \left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z} \right)$$

2) $\displaystyle\int_{\substack{C \\ A \to P}} \vec{F}\, d\vec{r} =$



Allemaal dezelfde uitkomst

onafhankelijk van de

$f(B)_{(x_1, y_1, z_1)} - f(A)_{(x_2, y_2, z_2)}$

Bewijs: $\displaystyle\int_{\substack{C \\ A \to B}} \vec{F}\, d\vec{r} = \int_{\substack{C \\ A \to B}} \vec{\nabla} f \cdot d\vec{r} =$

$$\left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y} \right) \cdot (dx, dy)$$

$$= \int \frac{\partial f}{\partial x} dx + \frac{\partial f}{\partial y} dy = \int_{\substack{C \\ A \to B}} df = \Big[f \Big]_A^B$$

$$f(B) - f(A)$$

Vb Arbeid van vectorveld $\vec{F}(yz, xz, xy)$

$$f = xyz$$

conservatief $\vec{F} = \vec{\nabla} f \Rightarrow \left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z}\right) = (yz, xz, xy)$

Verplaatsing langs $(-1, 3, 9) \rightarrow (1, 6, -4)$

dus f evalueren in $(1, 6, -4)$ en daarna $f(-1, 3, 9)$ aftrekken met $f = xyz$

gesloten

$$\oint_C \vec{F} \, d\vec{r} = 0$$

↳ kringintegraal enkel als kromme ligt op verbonden oppervlak

↳ $\vec{F}(F_x, F_y, F_z)$ continu afgeleiden op de curve

③ Component test

$\vec{F} = (F_x, F_y, F_z)$

$$\frac{\partial F_x}{\partial y} = \frac{\partial F_y}{\partial x}$$

$$\frac{\partial F_y}{\partial z} = \frac{\partial F_z}{\partial y}$$

$$\frac{\partial F_z}{\partial x} = \frac{\partial F_x}{\partial z}$$

④

$$\vec{F} = (F_x, F_y, F_z) = \vec{\nabla} f \left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z}\right.$$

$$\frac{\partial^2 f}{\partial y \partial x} = \frac{\partial^2 f}{\partial x \partial y}$$

$$\frac{\partial}{\partial y}\underbrace{\left(\frac{\partial f}{\partial x}\right)}_{= F_x} = \frac{\partial}{\partial x}\underbrace{\left(\frac{\partial f}{\partial y}\right)}_{= F_y}$$

5 $\vec{\nabla} \times \vec{F} = \vec{0}$

$$\begin{vmatrix} \vec{1}_x & \vec{1}_y & \vec{1}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ F_x & F_y & F_z \end{vmatrix} \quad 1_x \overbrace{\left(\frac{\partial F_z}{\partial y} - \frac{\partial F_y}{\partial z} \right)}^{0} - \vec{1}_y \overbrace{\left(\frac{\partial F_z}{\partial x} - \frac{\partial F_x}{\partial z} \right)}^{0}$$

$$- \vec{1}_z \underbrace{\left(\frac{\partial F_y}{\partial x} - \frac{\partial F_x}{\partial y} \right)}_{0}$$

$$= (0, 0, 0) = \vec{0}$$

Zwaartekracht (?)

elektrisch

$$\boxed{\begin{aligned}\vec{E} &= \frac{q}{4\pi\varepsilon_0 r^3}\vec{r} \\ &\text{Potentiaal functie} \\ V &= \frac{-q}{4\pi\varepsilon_0 r}\end{aligned}}$$

$$F = \frac{-GMm}{r^3}\vec{r}$$

Potentiaal

$$\Rightarrow V = \frac{-GMm}{r}$$

$\vec{r} = (x, y, z)$

$r =$ afstand tot oorsprong $= \sqrt{x^2+y^2+z^2}$

$$\vec{F} = -\vec{\nabla} g$$

$$F = \vec{\nabla} V$$

$$(E_x, E_y, E_z) = \left(\frac{\partial V}{\partial x}, \frac{\partial V}{\partial y}, \frac{\partial V}{\partial z}\right)$$ Symmetrie dus enkel voor x

$$E_x = \frac{\partial V}{\partial x} = \frac{\partial}{\partial x}\left(\frac{-q}{4\pi\varepsilon_0}(x^2+y^2+z^2)^{-1/2}\right)$$

(r invullen)

$$= \frac{-q}{4\pi\varepsilon_0}\left(-\cancel{\tfrac{1}{2}}\right)(x^2+y^2+z^2)^{-3/2}\,\cancel{2}x$$

$$= \frac{-q}{4\pi\varepsilon_0}(x^2+y^2+z^2)^{-3/2}(x)^?$$

(2.) $$\int_{P_1 (x_1, y_1, z_1)}^{P_2 (x_2, y_2, z_2)} \vec{E}\, d\vec{l} = V(P_2) - V(P_1)$$

$$= \frac{-q}{4\pi\varepsilon_0 \sqrt{x_2^2+y_2^2+z_2^2}} - \frac{-q}{4\pi\varepsilon_0 \sqrt{x_1^2+y_1^2+z_1^2}}$$

(3) component $\frac{\partial E_x}{\partial y} \stackrel{?}{=} \frac{\partial E_y}{\partial x}$

Wegens symmetrie bekijken tot x

$$\frac{\partial}{\partial y}\left(\frac{x}{(x^2+y^2+z^2)^{3/2}}\right) = \frac{\partial}{\partial y}\left(x \cdot (x^2+y^2+z^2)^{-3/2}\right)$$

Constantes worden weggelaten

$$= x\left(\frac{-3}{\cancel{2}}\right)(x^2+y^2+z^2)^{-5/2}\,\cancel{2}y$$

$$= -3xy\,(x^2+y^2+z^2)^{-5/2}$$

$$\frac{\partial}{\partial x}\left(\frac{y}{(x^2+y^2+z^2)^{3/2}}\right) = -3yx\,(x^2+y^2+z^2)^{-5/2}$$

(4) $\vec{\nabla} \times \vec{E} = \vec{0}$

Defn 1) is het een conservatief vectorveld
2) bepaal f

(A) $\vec{F} = (yz, xz, xy)$

1) $\rightarrow \dfrac{\partial F_x}{\partial y} \overset{?}{=} \dfrac{\partial F_y}{\partial x}$ $\Rightarrow z = z$

$\dfrac{\partial F_y}{\partial z} \overset{?}{=} \dfrac{\partial F_z}{\partial y}$ $\Rightarrow x = x$

$\dfrac{\partial F_z}{\partial x} \overset{?}{=} \dfrac{\partial F_x}{\partial z}$ $\Rightarrow y = y$

2) Bepaal f

Op basis van $\vec{F}(yz, xz, xy) = \left(\dfrac{\partial f}{\partial x}, \dfrac{\partial f}{\partial y}, \dfrac{\partial f}{\partial z}\right)$

$\dfrac{\partial f}{\partial x} = yz \Rightarrow f = \int yz\,dx + \varphi_1(yz)$ → Potentieel extra functie

$\dfrac{\partial f}{\partial y} = xz \Rightarrow f = \int xz\,dy + \varphi_2(xz)$

$\dfrac{\partial f}{\partial z} = xy = f \int xy\,dz + \varphi_3(xy)$

zvrelt oles

$$f = xy2 + \varphi(y,z)$$

$$f = xy2 + \varphi(x,z)$$

$$f = xy2 + \varphi(x,y)$$

uniche čim zveden

$$f = xy2 + C$$

(B) $(e^x\cos y + yz,\ xz - e^x\sin y,\ xy + z)$

(1) $e^x\sin y + z \overset{?}{=} z - e^x\sin y$ ok

$x = x$

$y = y$

(2)

$$\int e^x\cos y + yz \, dx = e^x\cos y + xyz + \varphi_1(y,z)$$

$$\int xz - e^x\sin y \, dy = xyz + e^x\cos(y) + \varphi_2(x,z)$$

$$\int xy + z \, dz = xyz + \frac{z^2}{2} + \varphi_3(x,y)$$

Campo: $xyz + \frac{z^2}{2} + e^x\cos(y) + C$

③ $\vec{F} = \left(\frac{-y}{x^2+y^2}, \frac{x}{x^2+y^2}, 0 \right)$

$$\oint_C \vec{F} \, d\vec{r}$$

↳ cirkel met straal 1

1. Is dit conservatief

$$\frac{\partial F_x}{\partial y} \stackrel{?}{=} \frac{\partial F_y}{\partial x} \Rightarrow \frac{y^2-x^2}{(y^2+x^2)^2} = \frac{(x^2-y^2)}{(x^2+y^2)^2}$$

= Conservatief

Problem gaat in vectorveld in (0,0) als teller of 0 wordt

$$\oint \vec{F} \, d\vec{r}$$

$x = \cos t$ $\quad x' = \sin t$

$y = \cos t$ $\quad y' = \sin t$

$0 \leq t \leq 2\pi$

$$\int_0^{2\pi} (-\sin t)(-\sin t) + (\cos t)(\cos t)\, dt = \int_0^{2\pi} dt = 2\pi$$

Def. exacte differentiaal

$$df = \frac{\partial f}{\partial x}\,dx + \frac{\partial f}{\partial y}\,dy + \frac{\partial f}{\partial z}\,dz$$

Stel $M\,dx + N\,dy + Z\,dz \iff$ Als en slechts als

(M, N, Z) componenten zijn van een conservatief vectorveld

dan is het differentiaalvorm exact.

Vb
$$\int_{(1,1,1)}^{(2,3,-1)} y\,dx + x\,dy + 4\,dz$$

Is $(y, x, 4)$ conservatief?

$$\left.\begin{array}{l} 0 = 0 \\ 0 = 0 \\ 0 = 0 \end{array}\right\} \text{ dus conservatief vectorveld}$$

$$\frac{\partial f}{\partial x} = y = \int y\,dx = yx + \varphi(y, z)$$

$$\frac{\partial f}{\partial y} = x \qquad = xy + \varphi(x, z)$$

$$\frac{\partial f}{\partial z} = 4 \qquad = 4z + \varphi(x, y)$$

$$xy + 4z$$

$$f = xy + 4z + C$$

$$\Rightarrow \int_{(1,1,1)}^{(2,3,-1)} df$$

$$(xy+4z)_{(2,3,-1)} - (xy+4z)_{(1,1,1)} = -3$$

Belangrijk : Def   Examen!

① $\vec{F} = \vec{\nabla} f$

Indien zo dan

② $\int_{P_1}^{P_2} \vec{F}\, d\vec{r} = f(P_2 - P_1)$

gevolg

↳ $\oint \vec{F}\, d\vec{r} = 0$

⇐ onder voorwaarde gesloten

③ Component test

$$\frac{\partial F_x}{\partial y} = \frac{\partial F_y}{\partial x}, \quad \frac{\partial F_y}{\partial z} = \frac{\partial F_z}{\partial y}, \quad \frac{\partial F_z}{\partial x} = \frac{\partial F_x}{\partial z}$$

④ $\vec{\nabla} \times \vec{F} = 0$

Exacte differentiaalvorm

$M\,dx + N\,dy + P\,dz$ is exact enkel als het componenten zijn van een conservatief veld.

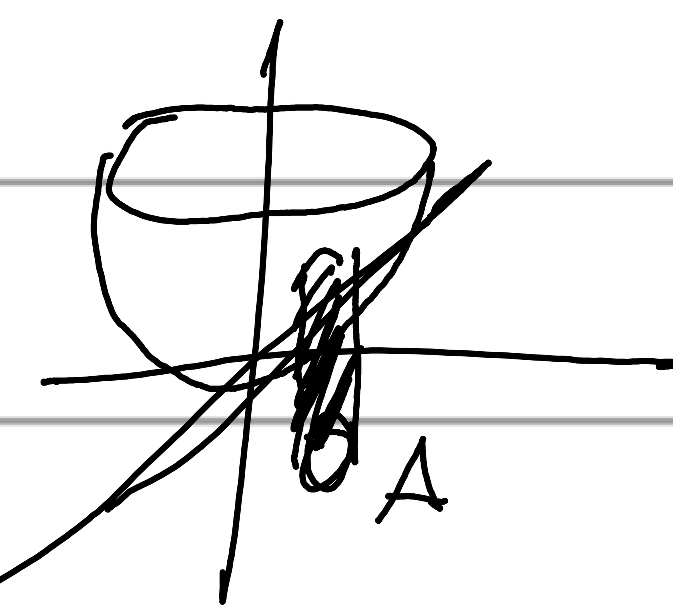
$$\vec{\nabla} f = \left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z}\right)$$

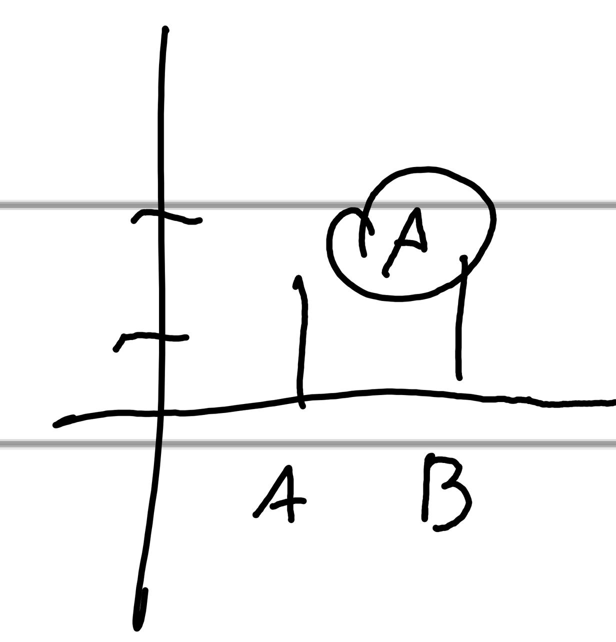
↳ Herschrijven als $\frac{\partial f}{\partial x}dx + \frac{\partial f}{\partial y}dy + \frac{\partial f}{\partial z}dz = df$

Stelling van Green

Verband tussen dubbelintegraal en lijnintegraal

$$\iint_A f(x,y)\,dx\,dy$$





$$= \int_{x_A}^{x_B} dx \int_{f(x)_C}^{f(x)_B} f(x,y)\,dy$$

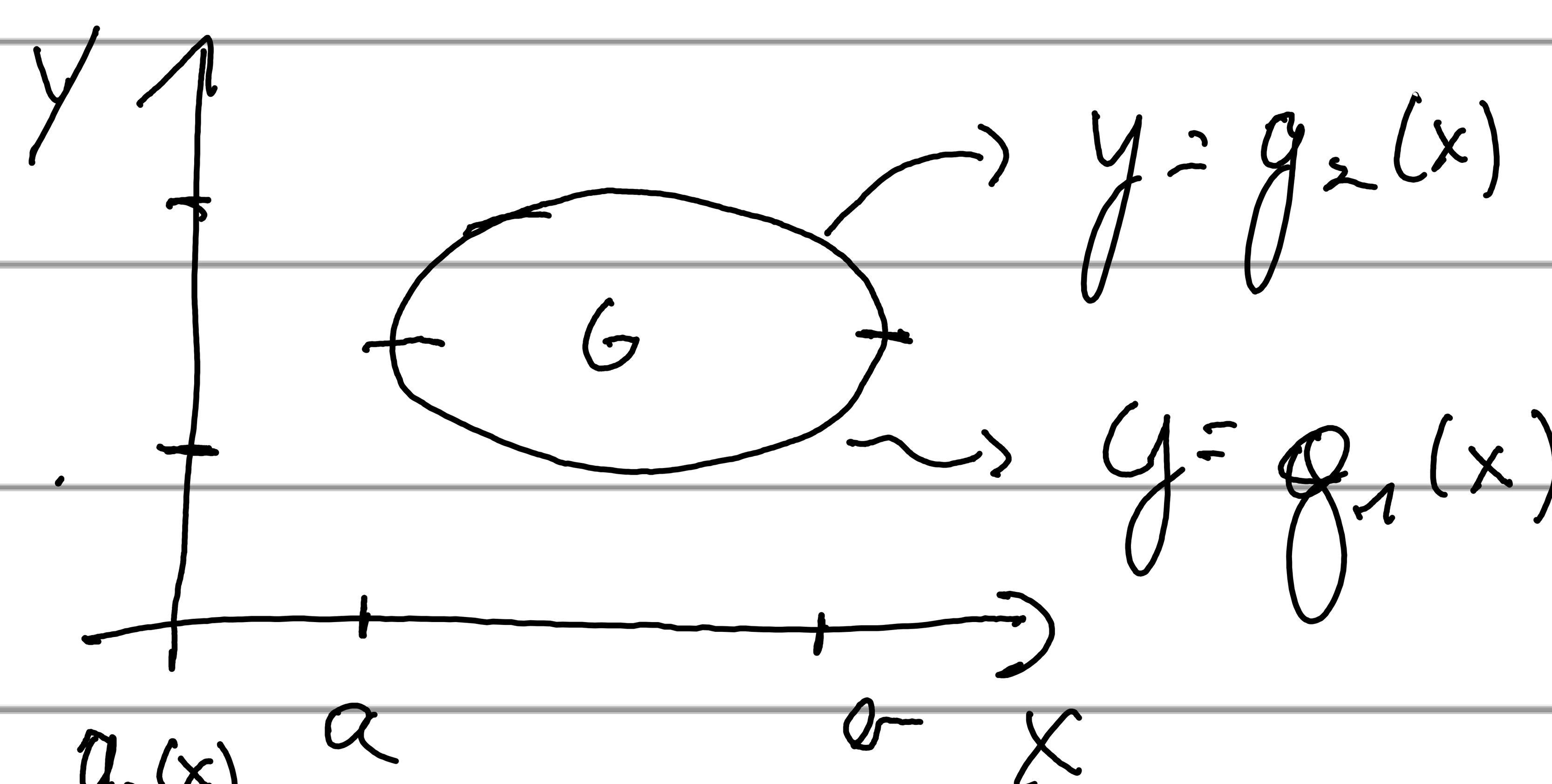
$$\int_C \vec{F}\, d\vec{r} = \int F(x,y,z)\left(\frac{\partial x}{\partial t}, \frac{\partial y}{\partial t}, \frac{\partial z}{\partial t}\right) dt = \int F_x\,dx + F_y\,dy + F_z\,dz$$

Stelling

$$\oint_{k_f} P\,dx + Q\,dy = \iint_G \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \right) dx\,dy$$

$\left.\begin{array}{l} P(x,y) \\ Q(x,y) \end{array}\right\}$ Moeten continue en afleidbaar zijn

Bewijs Blauw



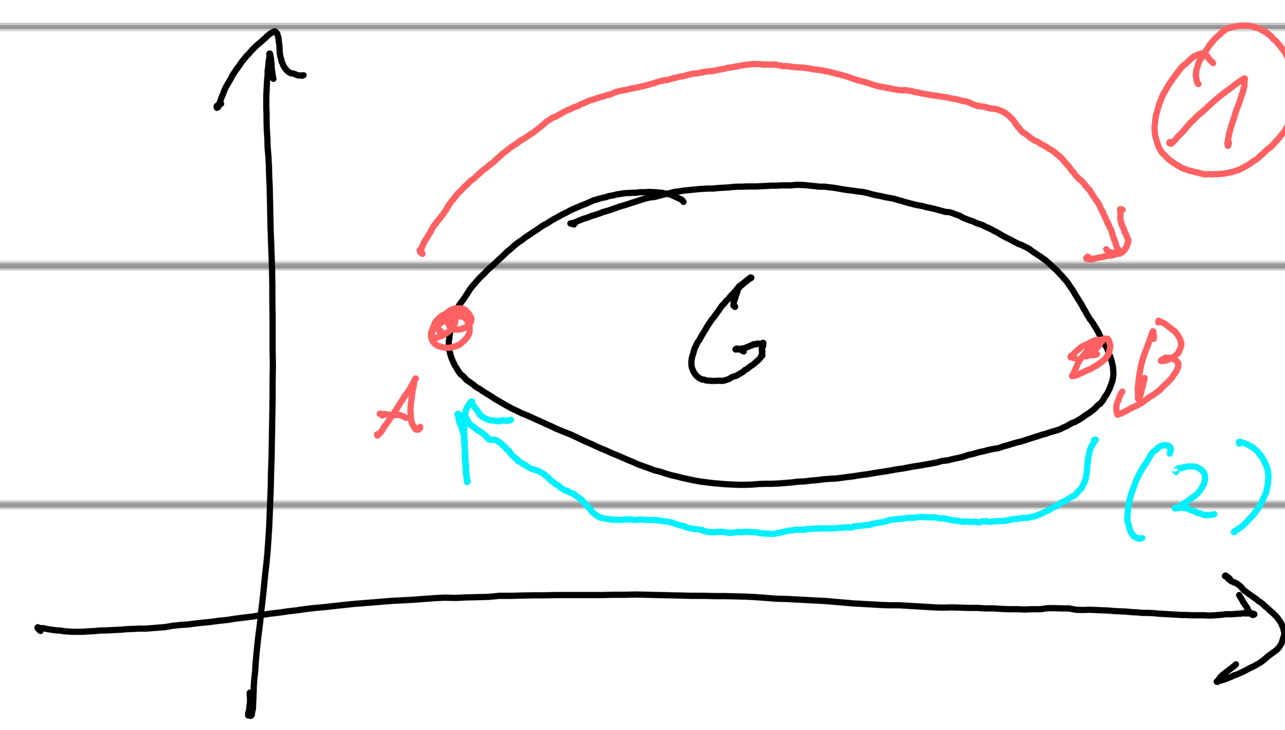
$$\iint_G \frac{\partial P}{\partial y}\,dx\,dy = \int_a^b dx \int_{y_1(x)}^{g_2(x)} \frac{\partial P}{\partial y}\,dy$$

$$= \int_a^b \left[P(x,y) \right]_{y=G_1(x)}^{y=G_2(x)} dx$$

$$= \int_a^b \left[P(x, y_2(x)) - P(x, y_1(x)) \right] dx$$

opsplitsen $= \int_a^b (P(x, g_2(x)))dx + \int_{\cancel{a}\, b}^{\cancel{b}\, a} (P(x, g_1(x)))dx$

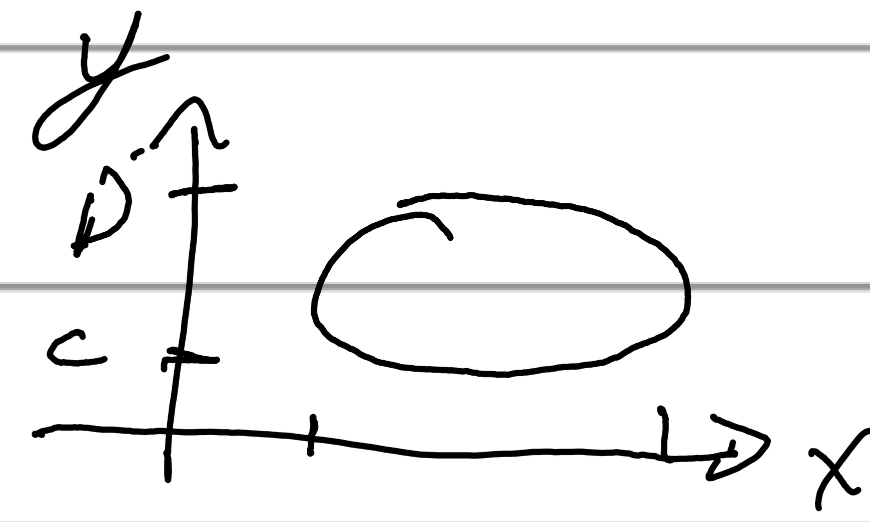
zodoende krijgen we gesloten integraal van $A \to B$



$$= \oint_{K_-} P(x, y)\, dx$$

$$= -\oint_{K^+} P(x, y)\, dx$$

Bewijs zool



$$\iint \frac{\partial Q}{\partial x}\, dx\, dy = \int_c^d dy \int_{f_1(y)}^{f_2(y)} \frac{\partial Q}{\partial x}\, dx$$

$$\int_c^d \left[Q(x,y) \right]_{x=f_1(y)}^{x=f_2(y)} dy$$

$$\int_c^d Q(f_2(y),y) - Q(f_1(y),y)\, dy$$

Opplitin → $\displaystyle\int_c^d Q(f_2(y),y) + \int_d^c Q(f_1(y),y)\, dy$

↑ nrrolt + onolt relyerob irrselt

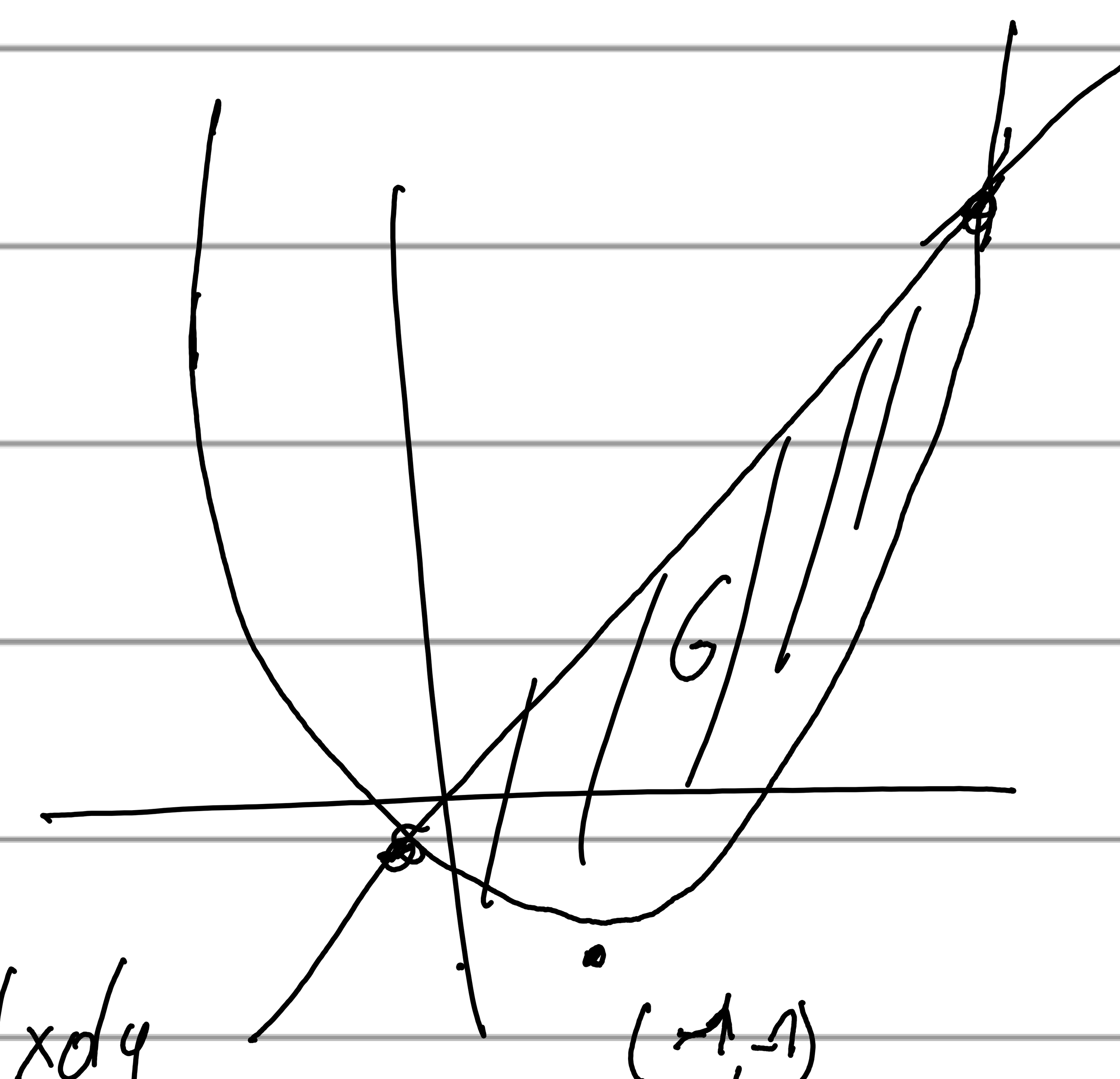
$$= \oint_K Q(x,y)\, dy$$

Praktijk: lijnintegraal die veel werk vraagt omzetten naar dubbel integraal

Vb

$$\oint_{C^+} 3xy\,dx + 2x^2\,dy$$

$$C\begin{cases} y = x \\ y = x^2 - 2x \\ \quad = (x-1)^2 - 1 \end{cases}$$



$$\oint_{C_+} P\,dx + Q\,dy = \iint_G \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right) dx\,dy$$

$P = 3xy$, $Q = 2x^2$, $\frac{\partial Q}{\partial x} = 4x$, $\frac{\partial P}{\partial y} = 3x$

$$= \int_0^3 dx \int_{x^2-2x}^{x} (4x - 3x)\,dy$$

Snijpunten?

$x = 0$ of $x = 3$

$y = 0$ $\quad$ $y = 3$

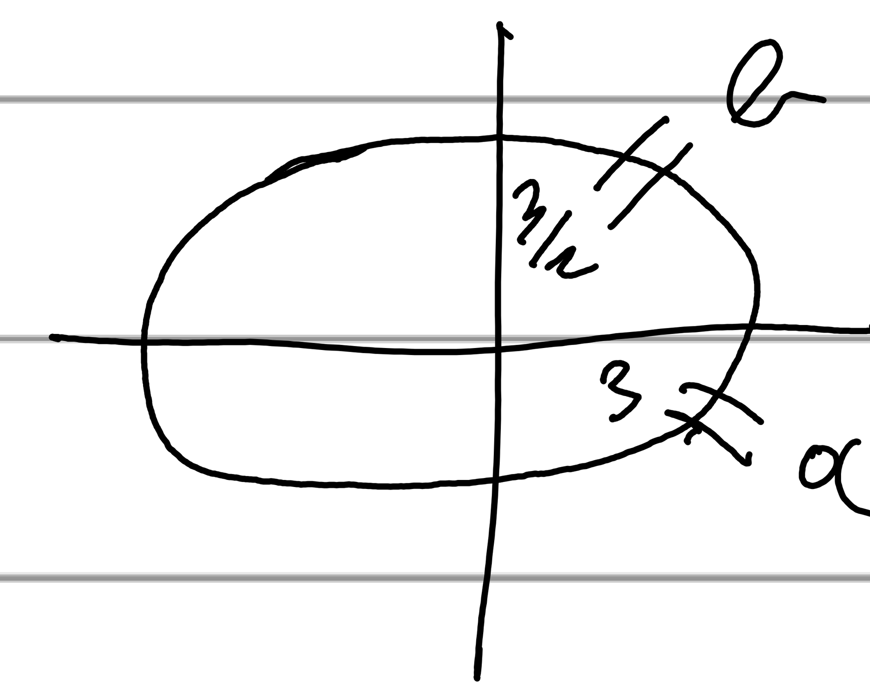
$$= \int_0^3 x[y]_{x^2-2x}^{x}\,dx$$

$$= \int_0^3 x(x - x^2 + 2x)\,dx = \int_0^3 (-x^3 + 3x^2)\,dx = \frac{27}{4}$$

12 $\oint_{K} (3x-y)\,dx + (x+2y)\,dy$

K rand G: $x^2+4y^2=9$ $\rightarrow$

$\frac{x^2}{9}+\frac{y^2}{\frac{9}{4}}=1$



Elips $x=3\cos t$

$y=\frac{3}{2}\sin t$

$$\iint \frac{\partial(x+2y)}{\partial x}-\frac{\partial(x+2y)}{\partial y}\,dx\,dy$$

$\frac{\partial(x+2y)}{\partial x}=1$, $\frac{\partial(x+2y)}{\partial y}=-1$

$$\iint -2\,dx\,dy$$

$$2\iint dx\,dy=9\pi$$

Gevolg van stelling van Green

Oppervlakte gebied G omsloten door Kromme K

$$S = \frac{1}{2}\oint_{K^+} x\,dy - y\,dx = -\oint_{K^+} y\,dx$$

$$\oint_{K^+} P\,dx - Q\,dy = \iint_G \underbrace{\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}}_{\overset{\shortparallel}{1 \text{ dan geld}}} dx\,dy$$

Dat $= 1$ als 1) $P = 0$, $Q = x \Rightarrow \oint_{K^+} x\,dy$

2) $P = y$, $Q = 0 \Rightarrow \oint_{K^+} -y\,dx$

3) $\frac{\partial Q}{\partial x} = \frac{1}{2}$ $\quad \frac{\partial P}{\partial y} = -\frac{1}{2}$ $\quad \oint_{K^+} -y\,dx + x\,dy$

$Q = \frac{x}{2}$ $\quad P = -\frac{y}{2}$

(vi) Bepaal opp van Ellips $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

↳ dubbelintegraal

$$\oint_{K^+} x\,dy$$

$x = a\cos t$

$y = b\sin t$

$$= \int_0^{2\pi} \underline{a\cos t}$$

$$\oint_0^{2\pi} x \frac{dy}{dt}\,dt \Rightarrow \int a\cos t \cdot b\cos t\,dt$$

$\frac{dy}{dt} = y'$

$$ab\int_0^{2\pi} \cos^2 t\,dt = ab\sin 2\pi$$

Vectorinterpretatie

$$\oint_{K^+} P\,dx + Q\,dy = \iint_G \frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\,dx\,dy$$

(1) $\vec{F} = (F_x, F_y) = (P, Q)$

$$\oint_K \vec{F}\,d\vec{r} = \int (\vec{F} \cdot \vec{T})\,ds$$

$$= \iint_G \left(\frac{\partial F_y}{\partial x} - \frac{\partial F_x}{\partial y}\right) dx\,dy$$

∇ rotatie van vectorveld

$(\vec{\nabla} \times \vec{F})_z$

$$\oint (\vec{F} \cdot \vec{T})\,ds = \iint_G (\vec{\nabla} \times \vec{F})_z$$

Vlakke variant van de stelling van Stokes

Verband lijn $\Leftrightarrow$ opp.-integraal

② $\oint_{K^+} (\vec{F} \cdot \vec{N})\, ds$

$\hookrightarrow \vec{T} \times \vec{1}_2 = \left(\frac{dy}{ds}, \frac{-dx}{ds}, 0\right)$

$= \oint_{K^+} \left(F_x \frac{dy}{\cancel{ds}} + F_y \frac{-dx}{\cancel{ds}}\right) \cancel{ds}$

$= \oint_{K^+} \underbrace{F_x\, dy}_{Q} - \underbrace{F_y\, dx}_{P}$

SG $\quad \underbrace{\iint \left[\frac{\partial F_x}{\partial x} + \frac{\partial F_y}{\partial y}\right]}\, dx\, dy$

$\iint (\vec{\nabla} \cdot \vec{F})\, dx\, dy$

$\parallel$

divergence

Velké rozšíření na Gauss-Ostrogradsky

$\oiint \Leftrightarrow$ 3 rozměrný integrál

Samengevat

→ Bewijs + tekening

→ gerdy nlt opp

→ toepassen → $\oint_K$ → conservatieve functie

→ Stelling van green

→ 3 ...

→ Vectorpotentialen

Oppervlakte integraal

Scalaire functies als vectorvelden

↳ Fluxintegralen

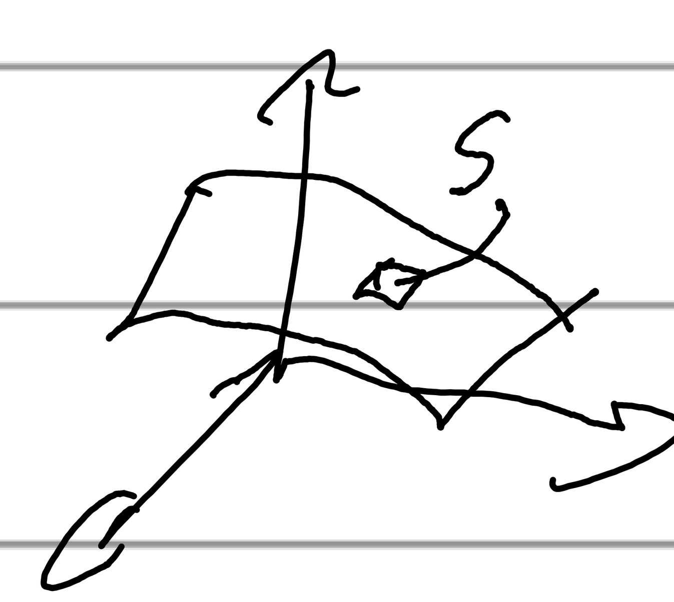
$$\int_K f\, ds$$

↳ $y = y(x)$

$$ds = \sqrt{dx^2 + dy^2} = \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$

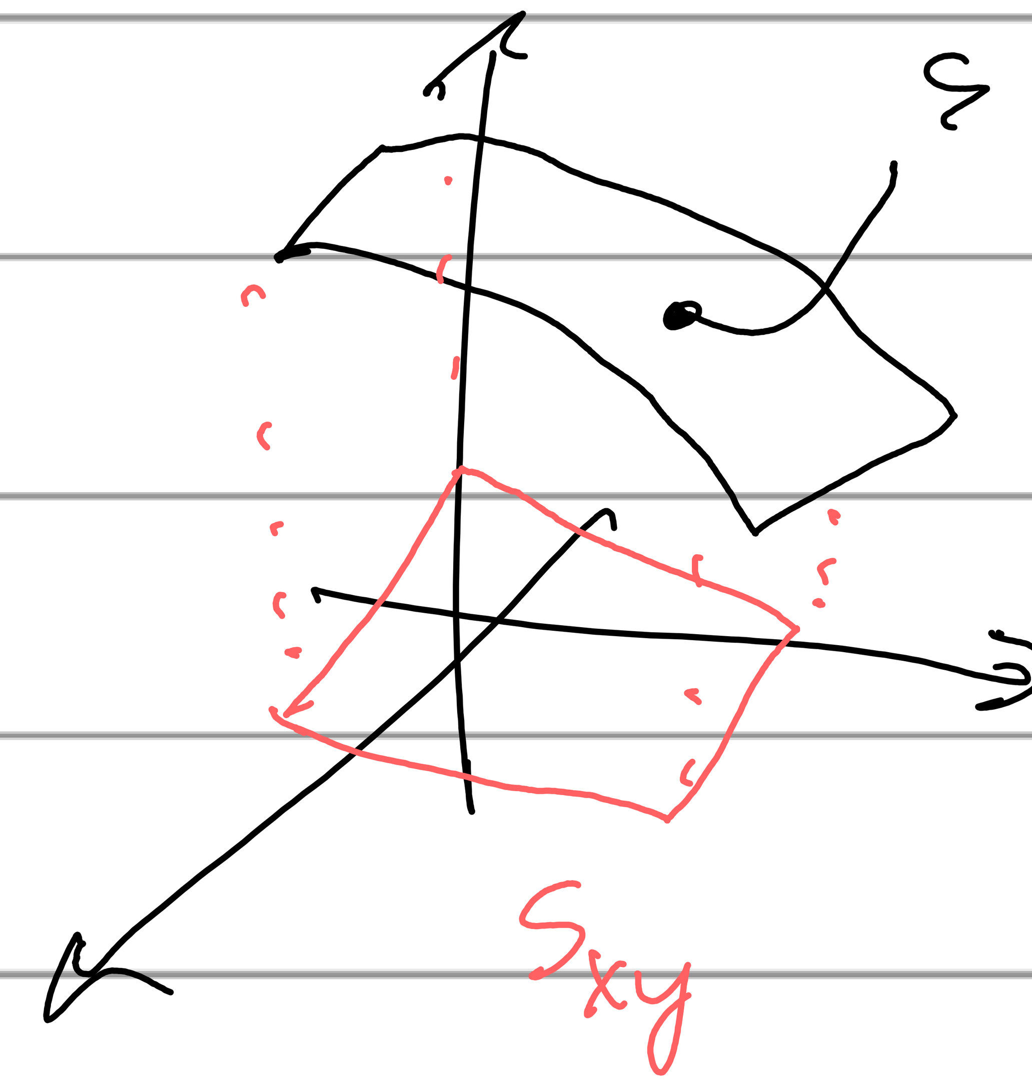
$$\int_{x_0}^{x_1} f \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$

$$\iint_S f\, dS$$



$$dS = \sqrt{1 + \left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2}\, dx\, dy$$

$$\iint f \sqrt{1+\left(\frac{\partial z}{\partial x}\right)^2+\left(\frac{\partial z}{\partial y}\right)^2}\, dx\, dy$$



$$\lim_{\substack{n\to\infty \\ \Delta x_k \to 0 \\ \Delta y_k \to 0}} \sum_{k=1}^{n} f(x_k, y_k, z_k)\, \Delta x_k\, \Delta y_k$$

Tvr.

$$f=1 \Rightarrow \iint_S dS = \text{pp}(S)$$

$$f=\rho \Rightarrow \iint_S \rho\, dS = \text{masa}(S)$$

masenosredište na [illegible]

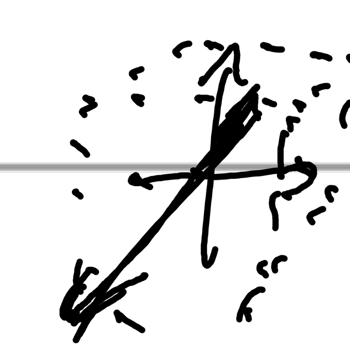
$$m_x = \iint \frac{\rho x\, dS}{M}$$

$$\oint f\, ds \iff \oiint f\, dS$$

Gesloten opp: bol, kubus

Vb

$$\iint_S (x+y+z)\, \underbrace{dS}_{\downarrow} \qquad S \begin{cases} x+2y+4z=8 \\ \text{1ste octant} \end{cases}$$



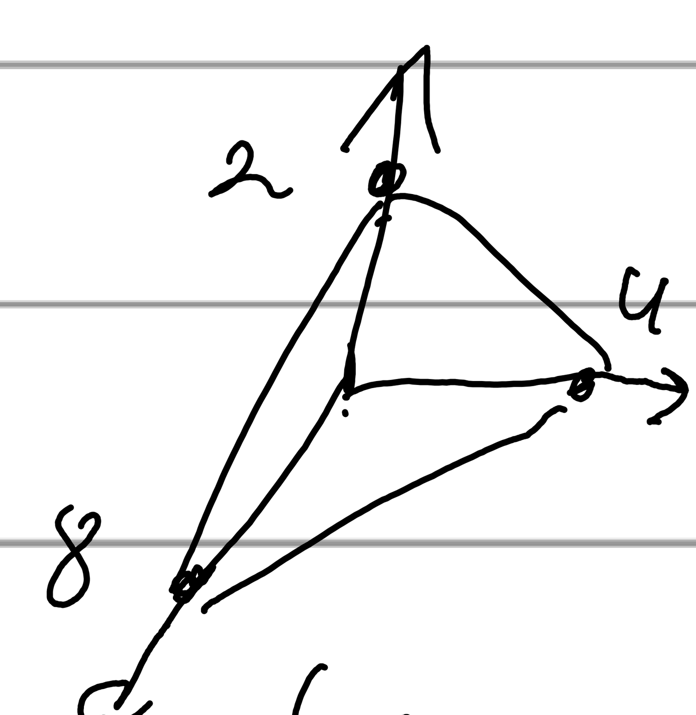
$\hookrightarrow x>0$

$y>0$

$z>0$

$$\sqrt{1+\left(\frac{\partial z}{\partial x}\right)^2+\left(\frac{\partial z}{\partial y}\right)^2}\; dx\, dy$$

$$\hookrightarrow z = \frac{8-x-2y}{4}$$



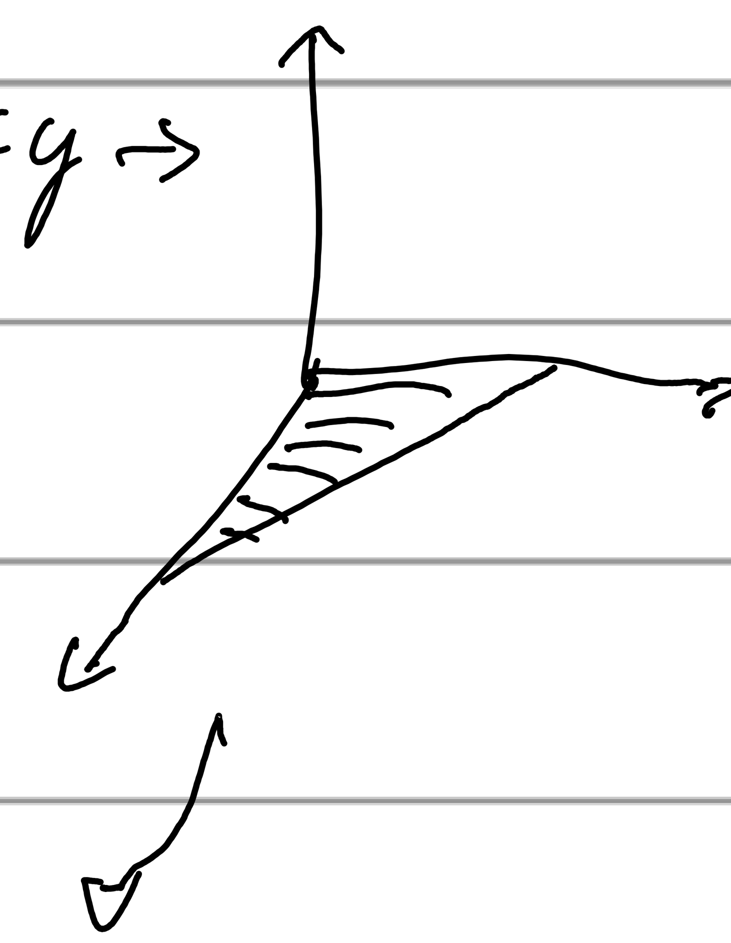
dus $x, y, z \geq 0$

$$\frac{\partial z}{\partial y} = -\frac{1}{2}$$

$$\frac{\partial z}{\partial x} = -\frac{1}{4} \quad \Rightarrow \quad \sqrt{1+\left(-\frac{1}{2}\right)^2+\left(-\frac{1}{4}\right)^2} = \frac{\sqrt{21}}{4}$$

$$\iint\limits_{S_{xy}}$$

$S_{xy} \rightarrow$



$x+2y=8$

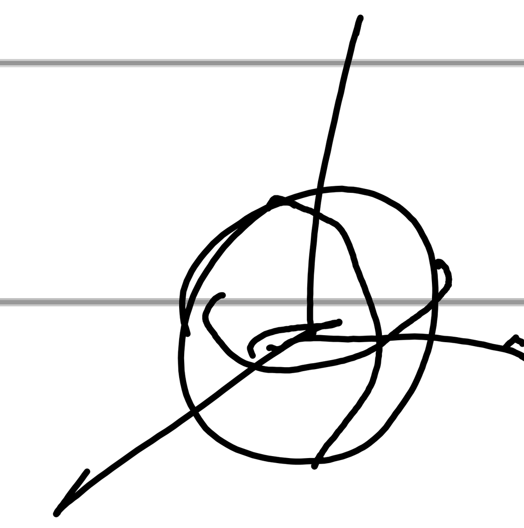
(z gelijk aan 0)

$$\int_0^8 dx \int_0^{\frac{8-x}{2}} \left(x+y+2-\frac{x}{4}-\frac{y}{2}\right)\frac{\sqrt{21}}{4}\,dx\,dy$$

$$=\frac{56}{3}\sqrt{21}$$

VI $\displaystyle\oiint_S z^2 \, dS$

$S \Rightarrow x^2+y^2+z^2=4$



$z = \sqrt{4-x^2-y^2}$

$$\frac{\partial z}{\partial x} = \frac{-x}{\sqrt{4-x^2-y^2}}$$

Julien wenn nicht Explicit kennen

$$\frac{\partial z}{\partial y} = \frac{-\partial F/\partial x}{\partial F/\partial z} = \frac{-\cancel{2}x}{\cancel{2}z}$$

$$\frac{\partial z}{\partial y} = \frac{-y}{\sqrt{4-x^2-y^2}}$$

$$dS \sqrt{1 + \frac{x}{4-x^2-y^2} + \frac{y}{4-x^2-y^2}} = \frac{2}{\sqrt{4-x^2-y^2}}$$

dS is opgesteld voor $+\sqrt{\ }$ dus voor kwarts

$$2\iint_S z^2 \cdot \frac{2}{\sqrt{4-x^2-y^2}}\,dx\,dy = 4\iint_{S_{xy}} \frac{2\,dx\,dy}{\sqrt{4-x^2-y^2}}$$

$4-x^2-y^2$

$$= 4\iint_{S_{xy}} \sqrt{4-x^2-y^2}\,dx\,dy$$

$x = 2\cos\theta$

$y = 2\sin\theta$

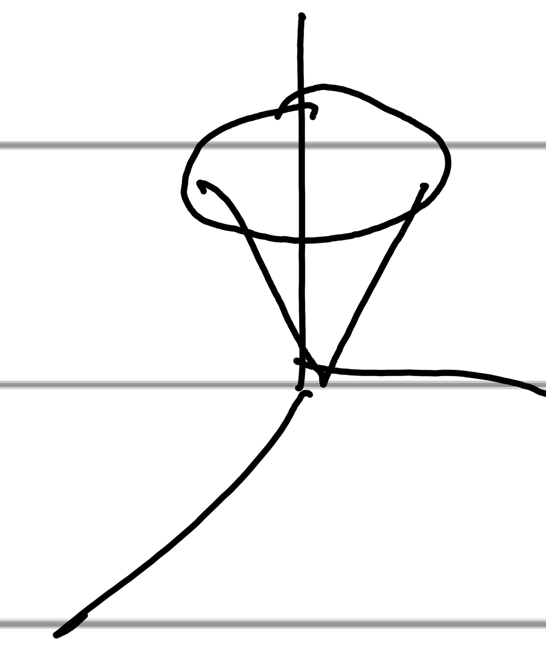
$0 \leq \theta \leq 2\pi$

$$4\int_0^{2\pi} d\theta \int_0^2 \sqrt{4-r^2}\, r\, dr$$

$$= \frac{64\pi}{3}$$

30 Massa van Kegel $\quad z=\sqrt{x^2+y^2} \quad 0\leq z\leq 1$

$\rho = x^2$



$$\iint \rho \, dS$$

$$\frac{\partial z}{\partial x} = \frac{\cancel{1}\cdot \cancel{2}x}{\cancel{2}\sqrt{x^2+y^2}}$$

$$\frac{\partial z}{\partial x} = \frac{x}{\sqrt{x^2+y^2}}$$

$$dS = \sqrt{1+\frac{x^2+y^2}{x^2+y^2}} = \sqrt{\frac{2(x^2+y^2)}{x^2+y^2}} = \sqrt{2}$$

$$\iint_{S_{xy}} x^2\sqrt{2}\, dx\, dy$$

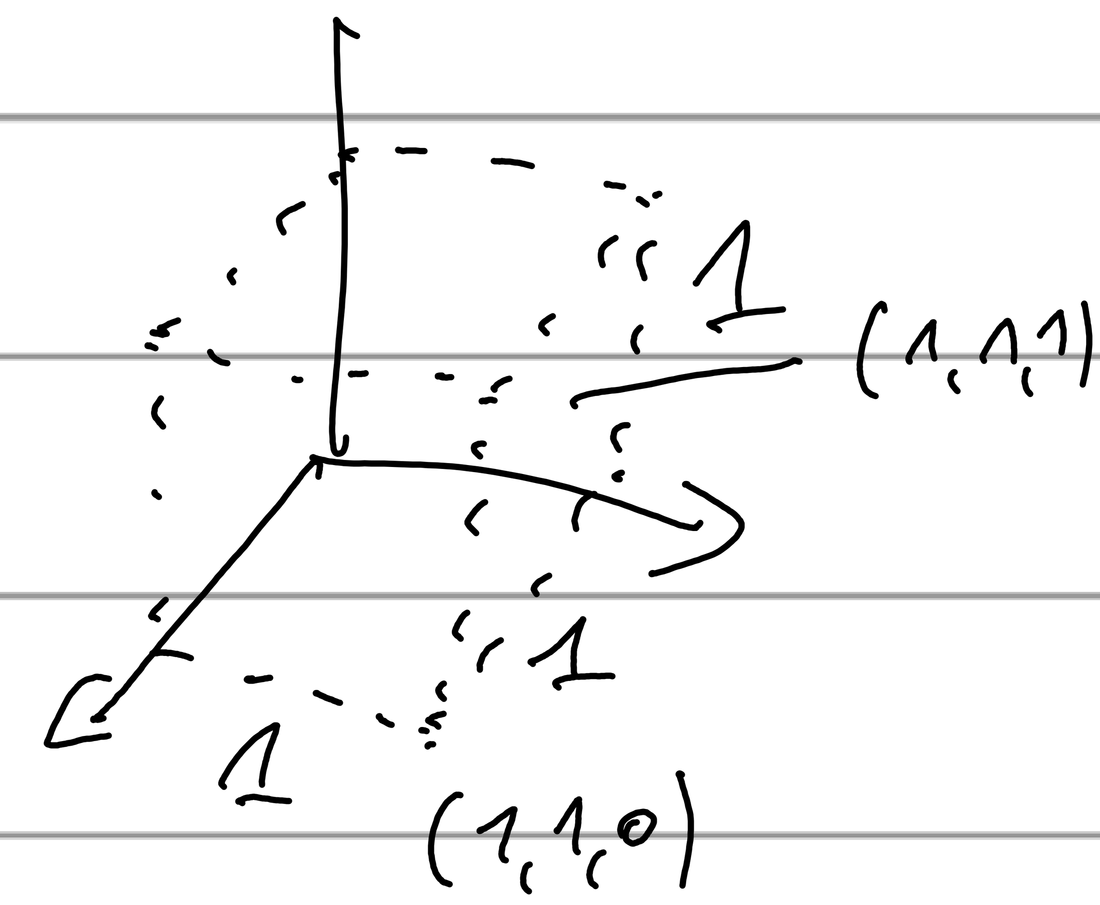
$r = 1$

$x = 1\cos\theta$

$y = 1\sin\theta$

$$\sqrt{2}\int_0^{2\pi} d\theta \int_0^1 r^2\cos^2(\theta)\underset{\substack{\uparrow\\ \text{Jacobiaan}}}{r}\, dr = \frac{\pi\sqrt{2}}{4}$$

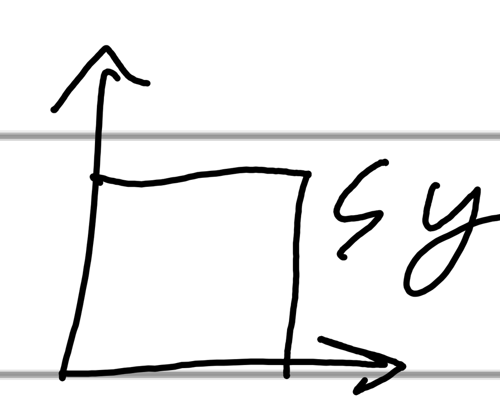
Vb $\displaystyle\oiint_S xyz\,dS$



Dat zijn dus 6 vlakken
maar alles dat in x, y en z vlak ligt op 0 komt
en dan is er symetrie

Stel enkel $z=1$

$$dS \sqrt{1+\underbrace{\left(\frac{\partial z}{\partial x}\right)^2}_{=0}+\underbrace{\left(\frac{\partial z}{\partial y}\right)^2}_{=0}} = 1$$

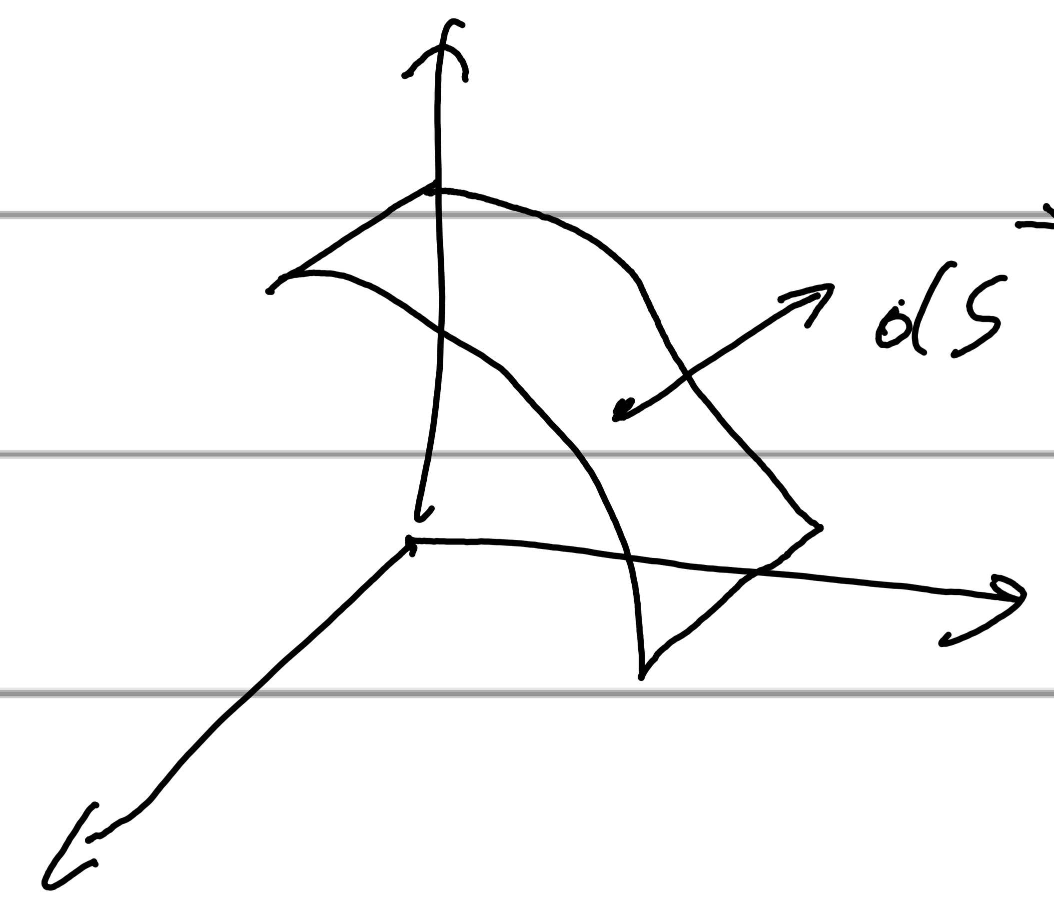


$$\iint_{S_{xy}} xy\,dx\,dy$$

$$\int_0^1 dx \int_0^1 xy\,dy = \frac{3}{4} \Rightarrow 3\cdot\frac{3}{4} = \frac{9}{4}$$

Oppervlakte-integraal van een vectorveld

$$\iint_S \vec F \cdot d\vec S$$



$$\iint_S \vec F \cdot \vec 1_n \cdot dS$$

Mate van veldlijnen die door een oppervlak lopen

$\vec F \,//\, dS$: max fluxie

$\vec F \perp dS$: 0

Positief naar buiten

negatief naar oorsprong

$\vec 1_n$ Als S als expliciet gegeven wordt $z = f(x,y,z) = z(x,y) - z$

$$\vec 1_n = \frac{\vec\nabla f}{|\vec 1_n|} = \frac{\left(\frac{\partial z}{\partial x}, \frac{\partial z}{\partial y}, -1\right)}{\sqrt{\left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2 + 1}}$$

$$\frac{dS}{dx\,dy}$$

$$\iint_S (\vec{F} \cdot \vec{1}_n)\, dS$$

$$= \iint \overbrace{(F_x, F_y, F_z)}^{\vec{F}} \cdot \overbrace{\frac{\left(\frac{\partial z}{\partial x}, \frac{\partial z}{\partial y}, -1\right)}{\cancel{\sqrt{\left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2 + 1}}}}^{\vec{1}_n} \overbrace{\cancel{\sqrt{\left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2 + 1}}}^{dS} \, dx\, dy$$

$$= \iint \left(F_x \frac{\partial z}{\partial x} + F_y \frac{\partial z}{\partial y} - F_z\right) dx\, dy$$

(x, y, z) met $z = z(x, y)$

Indien Impliciet

$$f(x, y, z) = 0$$

$\dfrac{\partial z}{\partial x} = \dfrac{-\partial F/\partial x}{\partial F/\partial z} =$ en van de andere

Impliciet

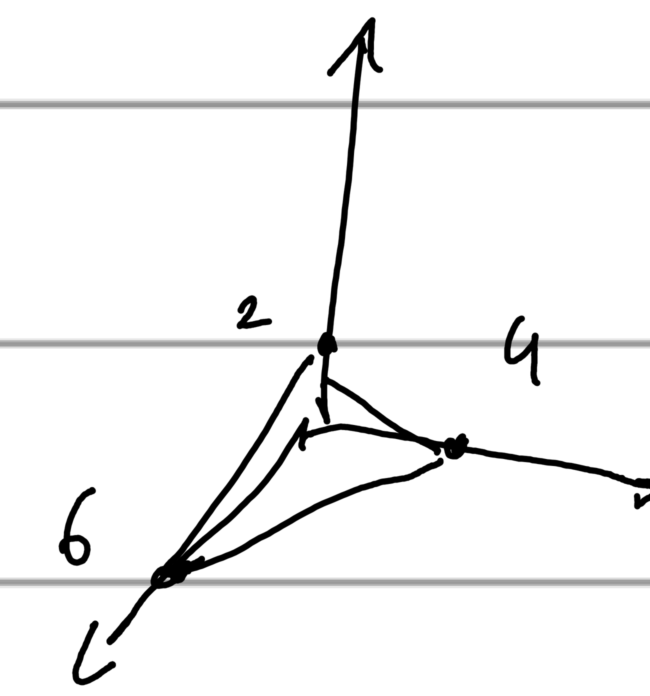
$$\iint_{S_{xy}} \left(F_x \frac{\partial f}{\partial x} + F_y \frac{\partial f}{\partial y} + F_z \frac{\partial f}{\partial z} \right) dx\,dy$$

(VI) Bepaal Flux van $\vec{F} = (18z, -12, 3y)$

$S: 2x + 3y + 6z = 12$

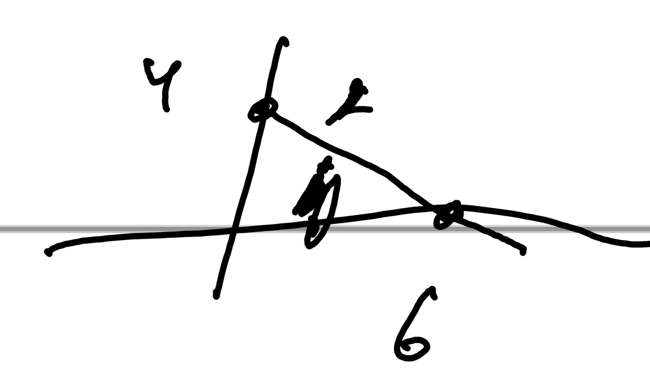
1ste octant

→ S expliciet met



$$z = \frac{12 - 2x - 3y}{6}$$

$$z = 2 - \frac{x}{3} - \frac{y}{2}$$



$$y = -\frac{2}{3}x + 4$$

$$\frac{\partial z}{\partial x} = -\frac{1}{3}$$

$$\frac{\partial z}{\partial y} = -\frac{1}{2}$$

$$\int_0^6 dx \int_0 \left(F_x \frac{\partial z}{\partial x} + F_y \frac{\partial z}{\partial y} - F_z \right) dy$$

"

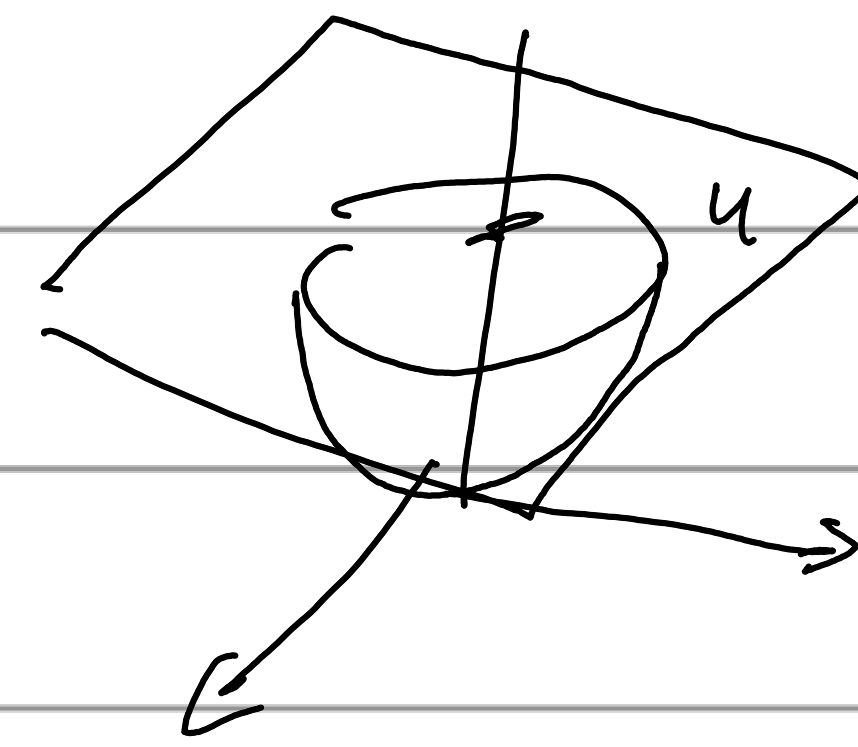
$$\int_0^6 dx \int_0^{4-\frac{2}{3}x} \left(18 \overbrace{\left(2 - \frac{x}{3} - \frac{y}{2}\right)}^{z} \cdot -\frac{1}{3}\right) + \left(-12 \cdot -\frac{1}{2}\right) - 3y \; dy$$

$= 24$

Vb

$\iint \vec{F} \, ds \qquad z = 4$

$z = x^2 + y^2$



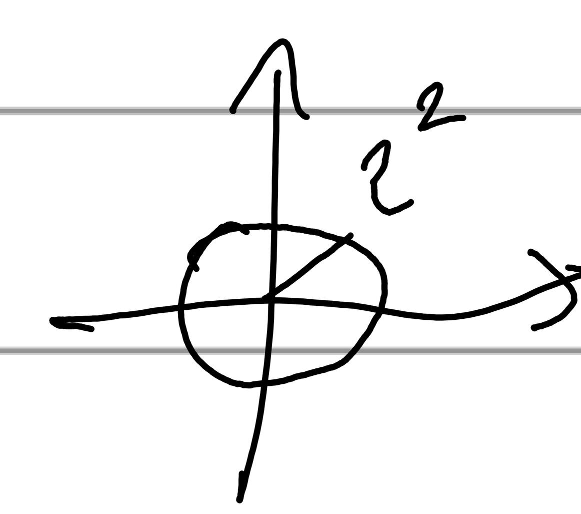
$\vec{F}(x, y, 3)$

$z = 4$ = gelied 1

$\frac{\partial z}{\partial x} = 0 \qquad = \iint -F_z \, dx\,dy$

$\frac{\partial z}{\partial y} = 1$

$= \iint\limits_{S_{xy}} -3 \, dx\,dy$



$-3 \, Opp(S) = -\pi \cdot \underset{=2}{r^2} \cdot 3 = -12\pi$

$z = x^2 + y^2$

$\frac{\partial z}{\partial x} = 2x$

$\frac{\partial z}{\partial y} = 2y$

$\iint\limits_{S_{xy}} (x \cdot 2x + y \cdot 2y - 3) \, dx\,dy$

$x = 2\cos\theta$

$y = 2\sin\theta$

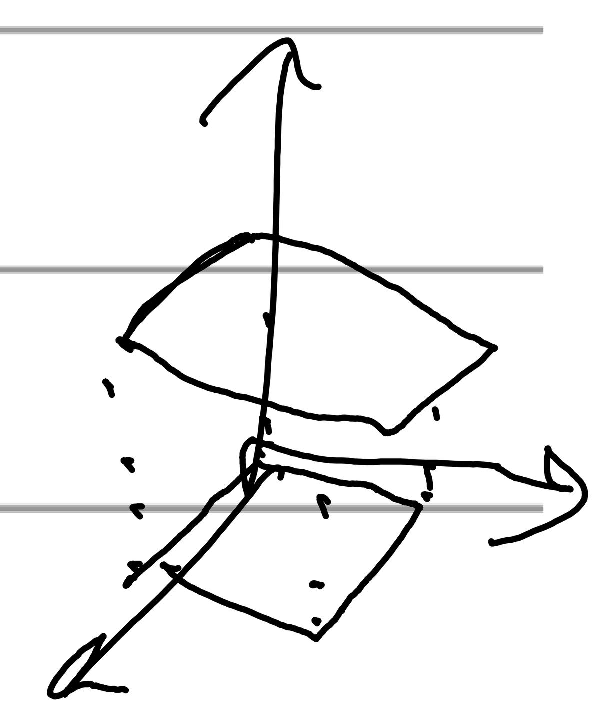
$$\int_0^{2\pi} d\theta \int_0^{2} (2z^2 - 3)\cdot 2\,dz = 4\pi$$

Samenvatting

Oppervlakte integraal van scalaire functie

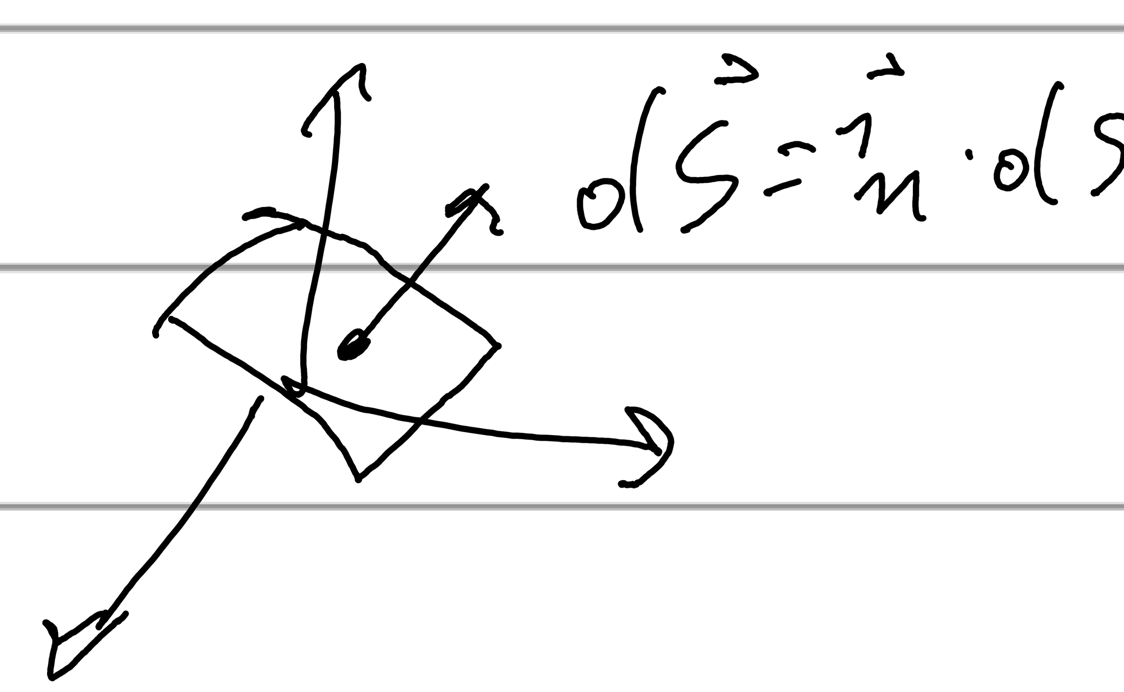
$$\iint_S f\, dS \Rightarrow \iint f \sqrt{\left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2 + 1}\; dx\, dy$$

z in functie van z substitueren



Oppervlakte Integraal van vectorveld

$$\iint_S \vec{F} \cdot d\vec{S}$$



$$\iint_S \vec{F} \cdot \vec{1}_n\, dS$$

$z = (x,y)$

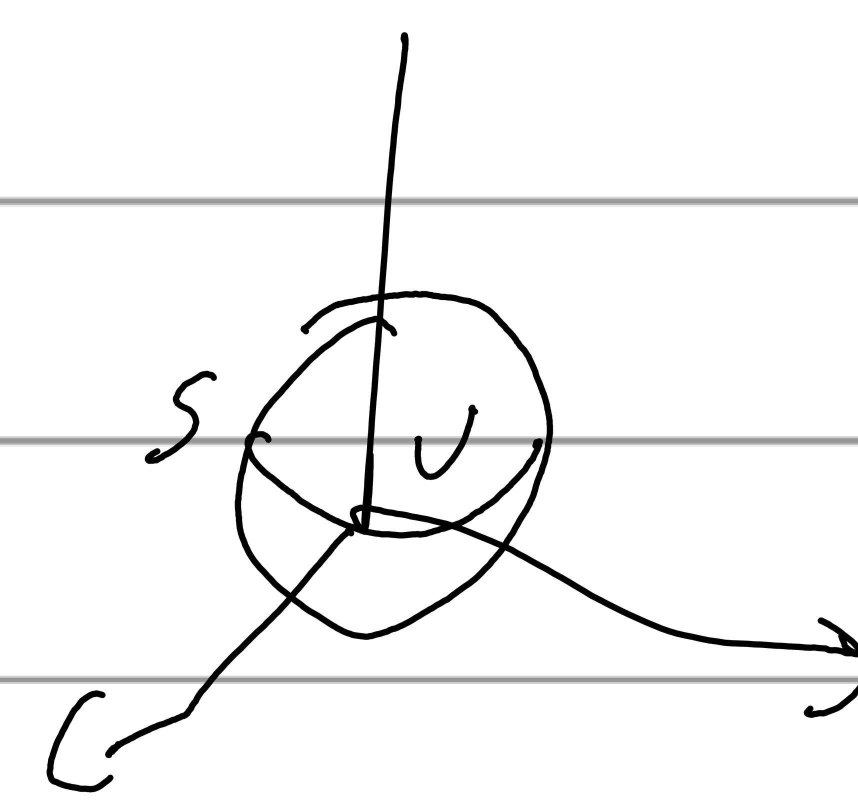
$$= \iint_S F_x(x,y,z)\frac{\partial z}{\partial x} + F_y(x,y,z)\frac{\partial z}{\partial y} - F_z\, dx\, dy$$

Integratiestellingen

Gauss-Ostrogradski : divergentiestelling

$$\oiint_S \vec{F}\cdot d\vec{S} = \iiint_V \vec{\nabla}\cdot\vec{F}\, dV$$

$d\vec{S} = \vec{1}_n \cdot dS$

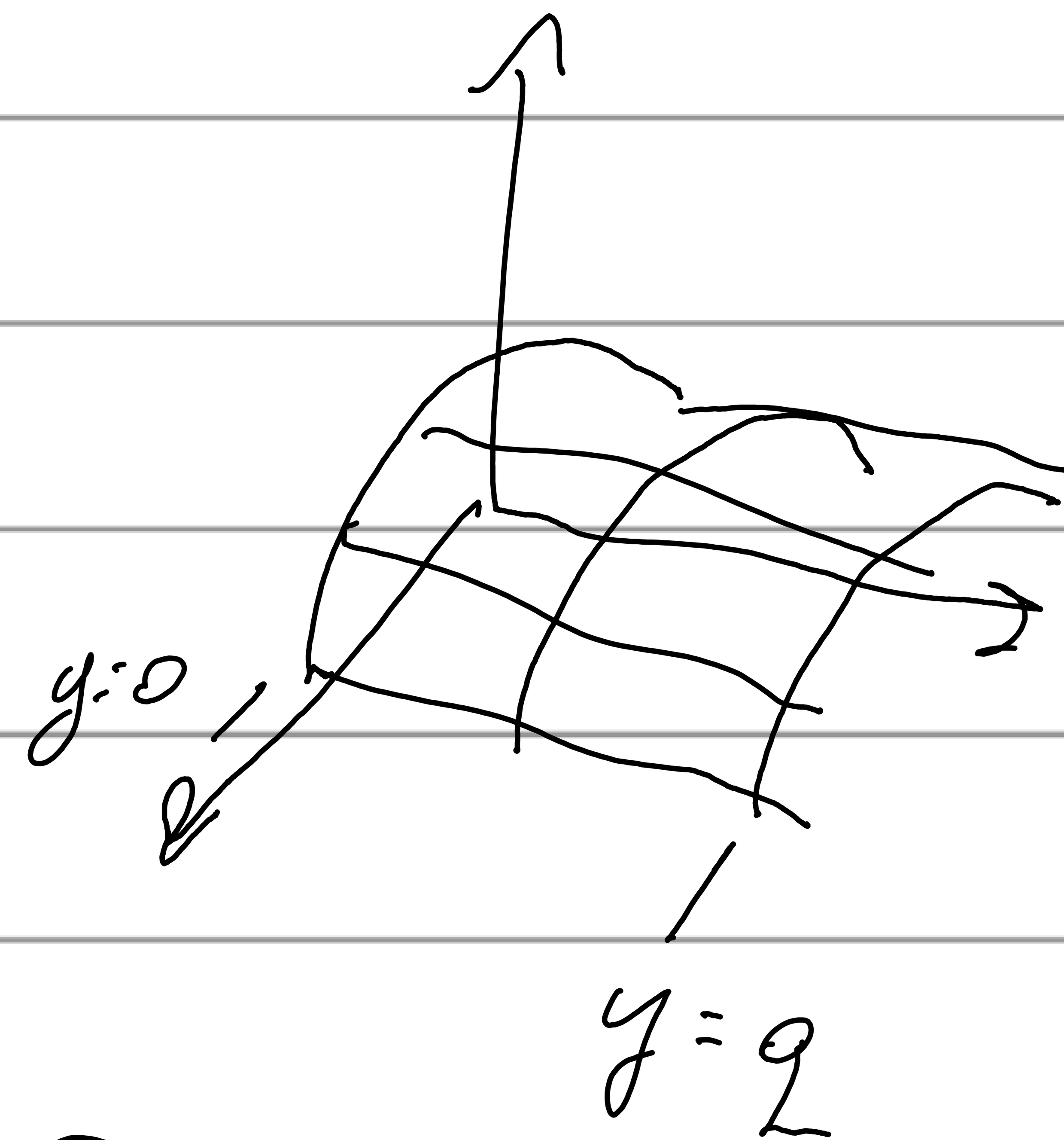


Volume bol

S : oppervlak bol

∀ Enkel gesloten opp.

VI) V. $\begin{cases} z=0 \\ y=0 \\ y=2 \\ z=1-x^2 \end{cases}$



$$\vec{F}=(x+\cos y,\ y+\sin z,\ z+e^x)$$

$$\oiint_S \vec{F}\,dS = \iiint_V \underbrace{\vec{\nabla}\cdot\vec{F}}\,dV$$

$$\frac{\partial F_x}{\partial x}+\frac{\partial F_y}{\partial y}+\frac{\partial F_z}{\partial z}$$

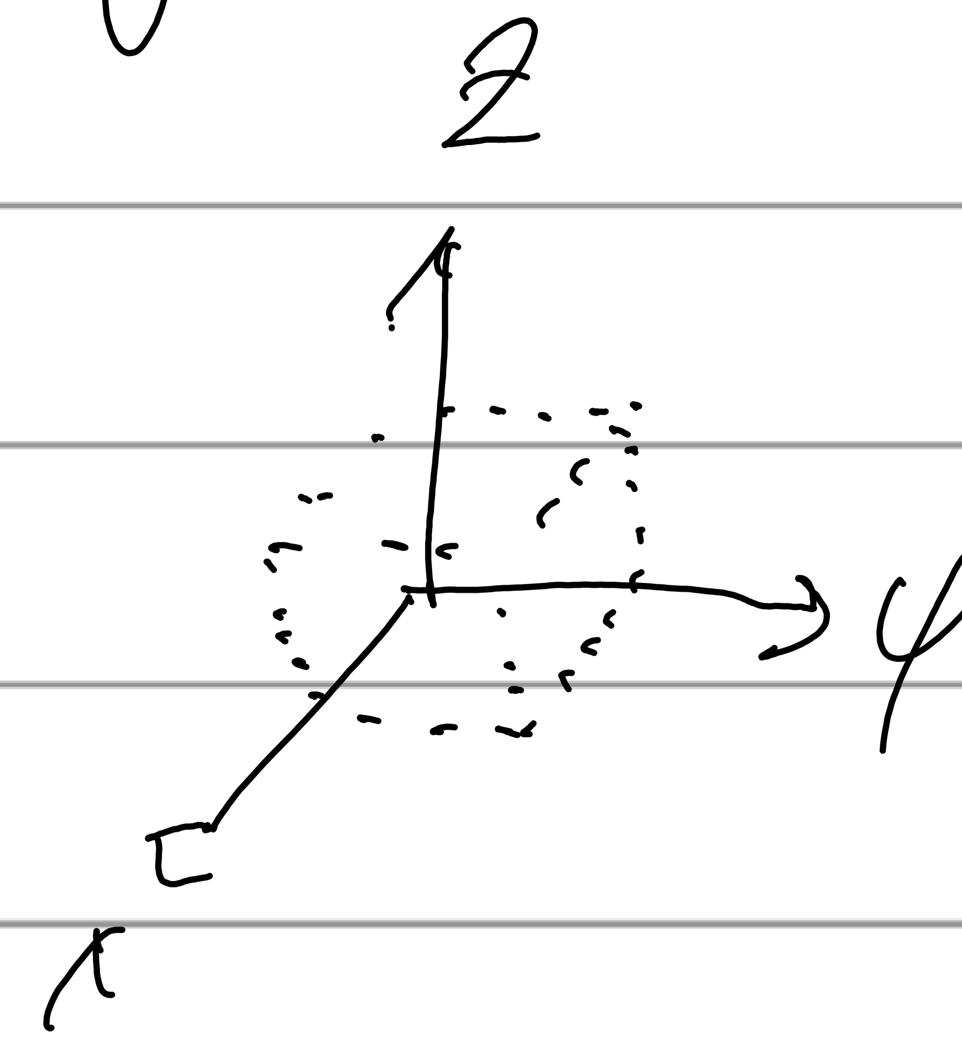
$$1 \quad 1 \quad 1$$

$$\iiint 3\ dx\,dy\,dz$$

$$\Rightarrow \int_{-1}^{1} dx \int_0^2 dy \int_0^{1-x^2} dz = 8$$

(iv) $\oiint_S \vec{F} \cdot d\vec{S} \qquad \vec{F} = (4xz, -y^2, yz)$

S { Cubus



$$\iiint \frac{\partial F_x}{\partial x} + \frac{\partial F_y}{\partial y} + \frac{\partial F_z}{\partial z} \; dx\,dy\,dz$$

$$= \int_0^1 dx \int_0^1 dy \int_0^1 4z - 2y + y \; dx\,dy\,dz$$

$4z - y$

$= \frac{3}{2}$

Toep continuïteitsvergelijking

$\rho \vec{v}$ = stromingsdichtheid

S = gesloten oppervlak

$$\oiint_S \rho \vec{v} \, dS =$$

Verschil van de hoeveelheid massa per tijdseenheid

$$= -\frac{dM}{dt} \iiint_V \rho \, dV$$

$$\iiint_V (\vec{\nabla} \cdot \rho \vec{v}) \, dV$$

Stelling van Leibniz: je mag complete integraal binnenhalen als een partiële

$$\iiint_V \vec{\nabla} \cdot \rho \vec{v} \, dV = \iiint_V \frac{\partial \rho}{\partial t} \, dV$$

Теор

$$\oiint_S \vec{E} \cdot d\vec{S} = \frac{q}{\varepsilon_0}$$

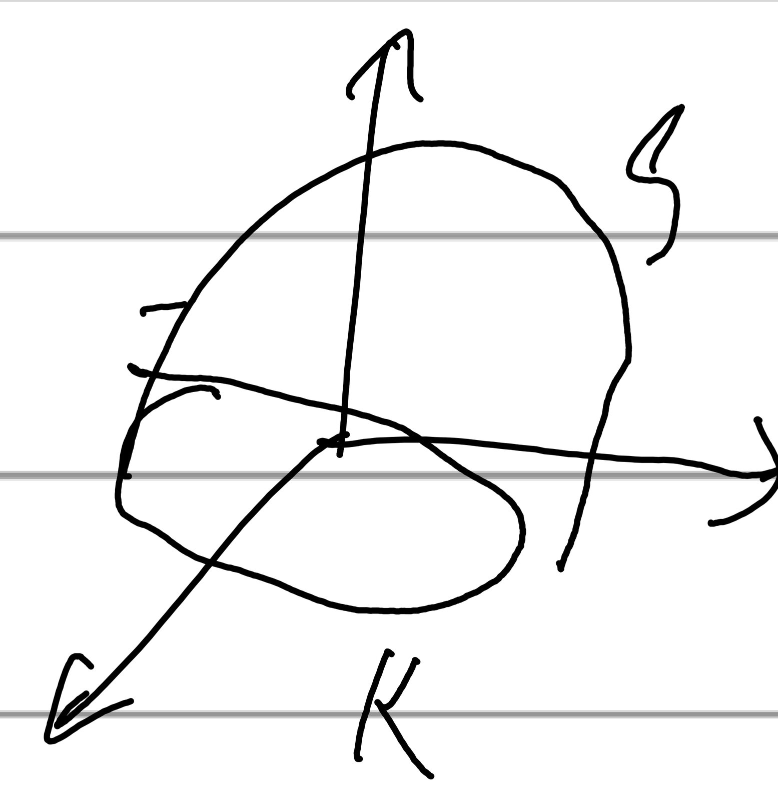
$$\Downarrow$$

$$\iiint \vec{\nabla} \cdot \vec{E} \, dV = \frac{1}{\varepsilon_0} \iiint \rho \, dV$$

Stelling van Stokes

Verband lijnintegraal en oppervlakte integraal

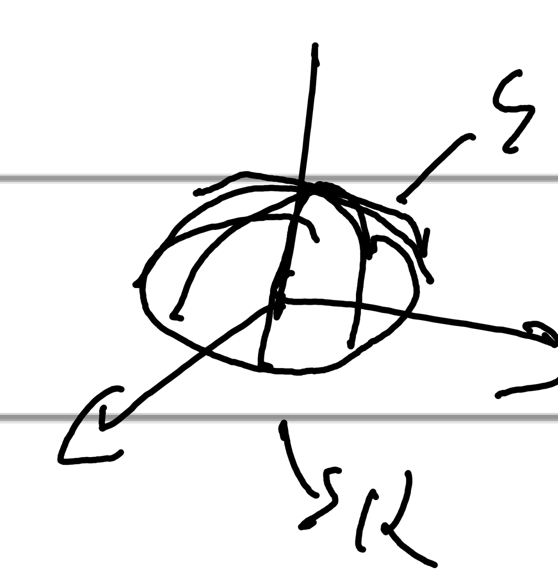
$$\oint_K \vec{F} \cdot \vec{1}_z \, ds = \iint_S (\vec{\nabla} \times \vec{F}) \, d\vec{S}$$



vb $\vec{F} = (2x - y, -yz^2, -y^2 z)$

S = halve bol $x^2 + z^2 + y^2 = 1$

K = kromme die bol omsluit in xy vlak



$$\iint (\vec{\nabla} \times \vec{F}) \, d\vec{S}$$

$$\begin{vmatrix} \vec{1}_x & \vec{1}_y & \vec{1}_z \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ 2x-y & -yz^2 & -y^2 z \end{vmatrix} \qquad \vec{1}_x \left(\frac{\partial (-y^2 z)}{\partial y} - \frac{\partial (-yz^2)}{\partial x} \right) \dots$$

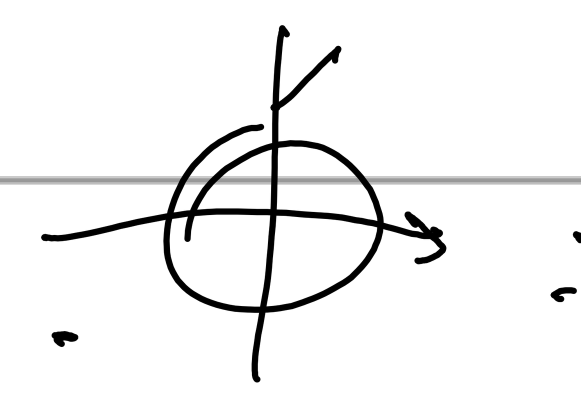
$$= (0, 0, 1)$$

$$\iint_S (0,0,1)\, dS$$

We weten dat

$$\iint_S \vec{F}\, d\vec{S} = \iint_S \Big(\underbrace{F_x \frac{\partial z}{\partial x}}_{=0} + \underbrace{F_y \frac{\partial z}{\partial y}}_{=0} - \underbrace{F_z}_{=1} \Big)\, dx\, dy$$

$$\iint_S -1 \, dx\, dy$$



Opp $= -\pi$

$$\oint_K \vec{F} \cdot d\vec{z}$$

$\hookrightarrow x = \cos t \qquad 0 \leq t \leq 2\pi$

$y = \sin t$

$z = 0$

$$\int_R \vec{F} \, dz = \int \left(F_x \frac{dx}{dt} + F_y \frac{dy}{dt} + F_z \frac{dz}{dt} \right) dt$$

$x = \cos t \Rightarrow \frac{dx}{dt} = -\sin t$

$y = \sin t \Rightarrow \frac{dy}{dt} = \cos t$

$z = 0 \Rightarrow \frac{dz}{dt} = 0$

$$\oint 2x - y, \; -yz^2, \; -y^2 z \; d\vec{z}$$

$$\oint (2\cos t - \sin t) \cdot -\sin t - 0 \cdot \cos t - 0, 0)$$

$$\int_0^{2\pi} -2\sin t \cos t + \sin^2 t = \pi$$

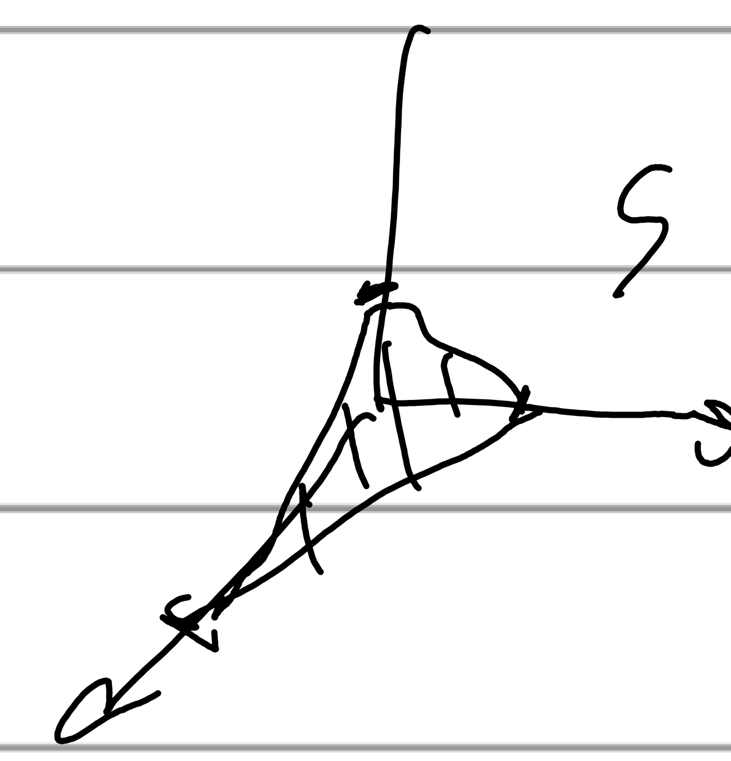
Vb

$$\oint_K 2xy^3\,dx + 3x^2y^2\,dy + (2z+x)\,dz$$

$A = (2,0,0)$

$B = (0,1,0)$

$C = (0,0,1)$



3 lijnintegralen

$$\iint_S \vec{\nabla}\times\vec{F}\,dS$$

$\rightsquigarrow z = z(x,y)$

$\hookrightarrow (2xy^3, 3x^2y^2, 2z+x)$

$\vec{\nabla}\times\vec{F}$

$\downarrow$

notatie

$$\begin{vmatrix} 1_x & 1_y & 1_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 2xy^2 & 3x^2y^2 & 2z+x \end{vmatrix} = (0, -1, 0)$$

Opp bepalen

$$ax + by + cz + d = 0$$

$$\hookrightarrow A \to 2a + d = 0$$

$$B \to b + d = 0$$

$$C \to c + d = 0$$

$$\frac{1}{2}x + y + z = 1$$

$$z = -\frac{1}{2}x - y - 1$$

$$\iint_{S_{xy}} \left(\underset{\substack{\parallel \\ 0}}{F_x} \frac{\partial z}{\partial x} + \underset{\substack{\parallel \\ -1}}{F_y} \frac{\partial z}{\partial y} - \underset{\substack{\parallel \\ 0}}{F_z} \right) dx\,dy$$

$$\frac{\partial z}{\partial y} = -1$$

dus

$$\iint_{S_{xy}} 1\,dx\,dy = \frac{1 \cdot 2}{2} = 1$$

(Wet) Wet van Faraday

$$\mathcal{E} = \underbrace{\frac{-d\,\Phi_m}{dt}}$$ → magnetische flux

$$\Phi = \iint_S \vec{B}\, dS$$

$$\oint_K \vec{E}\, d\vec{l} = \iint_S (\vec{\nabla} \times \vec{E})\, dS = \iint -\frac{\partial \vec{B}}{\partial t}\, dS$$

$\vec{\nabla} \times \vec{E}$ = verandering magnetisch veld(afgeleide) van de tijd

Stokes

$$\underbrace{\oint_C \vec{F} \cdot d\vec{l}}_{0 = \text{lijn conservatief vectorveld}} = \underbrace{\iint_S \vec{\nabla} \times \vec{F} \, d\vec{S}}_{0 \text{ lijn conservatief vectorveld}}$$

$\Rightarrow$ Voorwaarde van conservatief vectorveld

Gauss: divergentie: oppervlakte en volume integraal

$$\oiint \longrightarrow \vec{\nabla} \cdot \vec{F}$$

Stokes: lijn en oppervlakte integraal

$$\vec{\nabla} \times \vec{F}$$