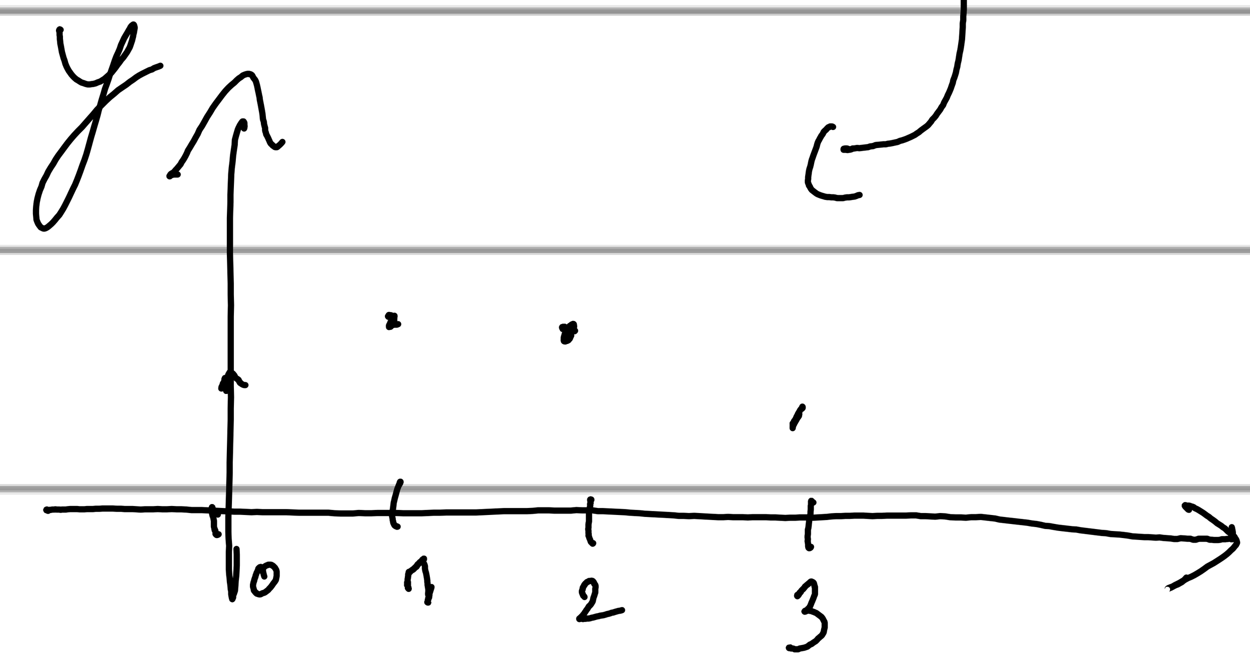
Reeksen

* Rijen

$$\{a_n\} = \{a_0, a_1, a_2, \dots\} \qquad n \in \mathbb{N}$$



Er kan ook functievoorschrift zijn

$$f(n) = \frac{1}{n}$$ BV

→ Convergentie: gedrag of oneindig?

$\lim_{n\to\infty} f(n)$ = Bepaald getal ⇔ convergentie naar dat getal

enkel naar $+\infty$

$$\lim_{n\to\infty} f(n) = \lim_{n\to\infty} a_n = L \to \in \mathbb{R} \to \text{convergent}$$

$\to \pm\infty \to$ divergent

limiet

$f(x)$ rationeel = hoogste graadscoefficient

regel d'hopital $\lim_{x \to \infty} = \frac{f(x)}{g(x)} \Rightarrow \frac{0}{0}$ of $\frac{\infty}{\infty}$

dan $\lim_{x \to \infty} \frac{f'(x)}{g'(x)}$

Vb $\lim_{x \to \infty} \frac{4-7x^6}{x^6+3} = -7$ rationeel

$\lim_{x \to \infty} \frac{1}{x^2} = 0$

$\lim_{x \to \infty} = \frac{\ln x}{x} = \frac{\frac{1}{x}}{1} \Rightarrow \frac{1}{x} = 0$

$\lim_{x \to \infty} \underbrace{\left(1+\frac{x}{n}\right)^n}_{e^{\ln}} = 1^\infty \Rightarrow$ afgeschaft

$\Rightarrow e^{\overbrace{n \ln\left(1+\frac{x}{n}\right)}^{n}} = e^{\lim_{n \to \infty} n \left(1+\frac{x}{n}\right)}$

$$\lim_{n\to\infty} n \ln\left(1+\frac{x}{n}\right) = +\infty \cdot 0 \Rightarrow \lim_{n\to\infty} \frac{\ln\left(1+\frac{x}{n}\right)}{\frac{1}{n}} = \frac{0}{0}$$

$$\overset{H}{=} \lim_{n\to\infty} \frac{\frac{\frac{-x}{n^2}}{\left(1+\frac{x}{n}\right)}}{\frac{-1}{n^2}} = \frac{x}{1+\frac{x}{n}} = x$$

$$= e^{x}$$

Reeksen.

$$\sum_{n=0}^{\infty} a_n = a_0 + a_1 + a_2 + \dots$$

$$\left.\begin{aligned} S_0 &= a_0 \\ S_1 &= a_1 + a_0 \\ S_2 &= a_1 + a_2 + a_0 \end{aligned}\right\} \text{Partieelsommen}$$

Als dit convergeert naar een bepaald getal dan gaat de reeks convergeren. $\lim_{n \to \infty} = L \in \mathbb{R} \rightarrow$ reeks convergeert

VB Meetkundige rij: $a,\ a \cdot z,\ a \cdot z^2,\ a \cdot z^3$

$\times z \quad \times z \quad \times z$ = z is de reden

De som = $\boxed{\dfrac{a}{1-z}}$

Voorwaarde: de absolute waarde van $|z| < 1$ dan convergentie

$$\sum_{n=0}^{+\infty} \frac{4}{2^n} = 4 \sum_{n=0}^{+\infty} \frac{1}{2^n} = \frac{4}{1-\frac{1}{2}} = 8$$

$\left(\frac{1}{2}\right)^n$

$\frac{1}{1} + \frac{1}{2} + \frac{1}{4} + \dots$

Ve $\frac{1}{9} + \frac{1}{27} + \frac{1}{81}$

$\times \frac{1}{3}$ $\quad \times \frac{1}{3}$ = rația

$$\frac{1/3}{1-\frac{1}{3}} = \frac{1}{6} \text{ și e convergentă}$$

Ve

$$\sum_{n=1}^{\infty} \frac{3^{n-1}-1}{6^{n-1}} = ?$$

Opsplite $\displaystyle \underbrace{\sum_{n=1}^{\infty} \left(\cancel{\tfrac{3}{6}}\tfrac{1}{2}\right)^{n-1}} - \sum_{n=1}^{\infty} \left(\frac{1}{6}\right)^{n-1}$

$$\frac{1}{1-\frac{1}{2}} - \frac{1}{1-\frac{1}{6}} = \frac{4}{5}$$

Ve $\displaystyle \sum_{n=0}^{\infty} \frac{4}{2^n} \Rightarrow 4 \sum_{n \geq 0} \left(\frac{1}{2}\right)^n = 4 \cdot \frac{1}{1-\frac{1}{2}} = 8$

* Machtreeksen

$$\sum_{n=0}^{+\infty} c_n x^n = c_0 + c_1 x + c_2 x^2 + c_3 x^3 + \dots$$

ook genoteerd $\sum_{n=0}^{+\infty} c_n (x-a)^n$

↳ machtreeks rond a

* Stelling van D'Alembert

0

$$\lim_{n\to+\infty} \left| \frac{c_{n+1} x^{n+1}}{c_n x^n} \right| < 1 \Rightarrow \text{convergent}$$

$= -1, 1 \Rightarrow$ Manueel te verifiëren

$$\lim_{n\to+\infty} \left| \frac{c_{n+1} x}{c_n} \right| < 1$$

$$|x| < \lim_{n\to+\infty} \left| \frac{c_n}{c_{n+1}} \right| = R$$

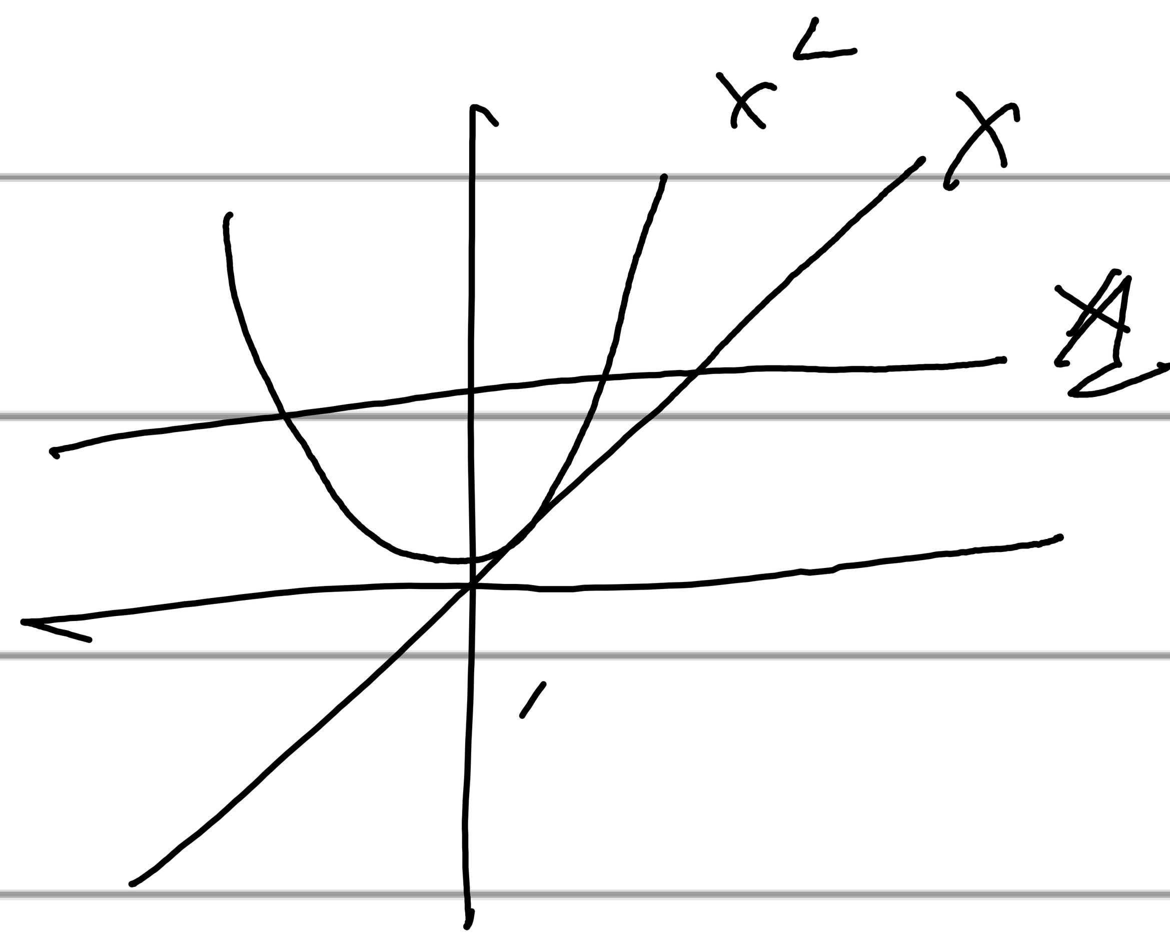
← convergentiestraal

$? \;]-R \quad 0 \quad R[\; ?$

$x = -R$ $x = R$

Vb

$$\sum_{n=0}^{+\infty} x^n = 1 + x + x^2 + x^3 = \frac{1}{1-x}$$



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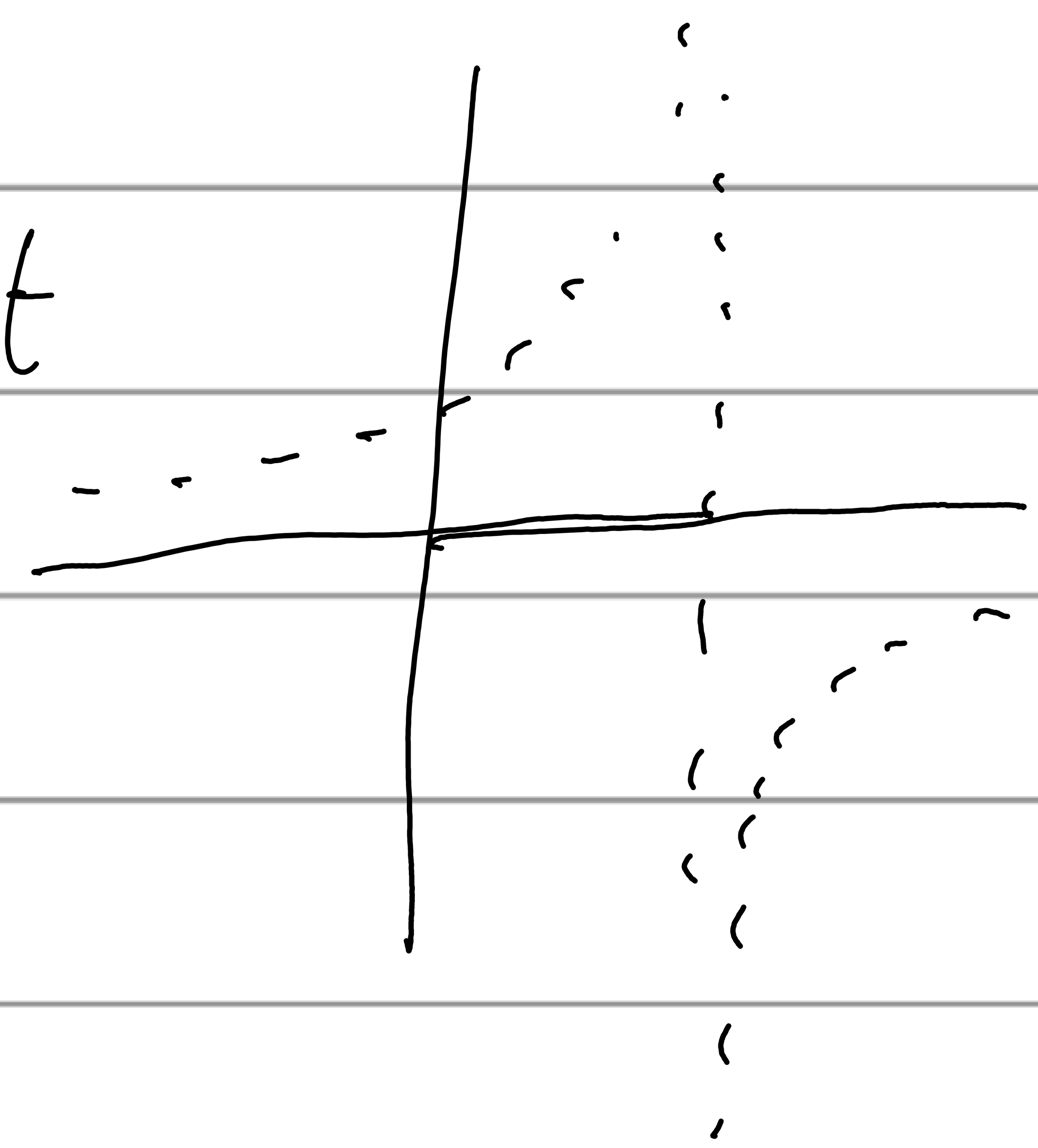


VB Check convergentie van

$$x - \frac{x^2}{2} + \frac{x^3}{3} - \dots$$

Hoe ziet het eruit $\sum_{n=0}^{+\infty} c_n x^n$?

$$= \sum_{n=1} (-1)^{n+1} \frac{x^n}{n}$$

(we kunnen niet starten van $n=0$ wat deling door 0)

$$\lim_{n \to +\infty} \left| \frac{\frac{x^{n+1}}{n+1}}{\frac{x^n}{n}} \right| = \lim_{n \to +\infty} \left| \frac{x^{\cancel{n+1}}}{n+1} \cdot \frac{n}{\cancel{x^n}} \right| < 1 \quad \text{voorwaarde convergentie}$$

$$\Downarrow$$

$$|x| < \lim_{n \to +\infty} \frac{n+1}{n} = 1$$

Stel $x = -1$ dan $-1 - \frac{1}{2} - \frac{1}{3} - \frac{1}{4} - \dots \to$ divergent

$x = 1$ dan $1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \dots \to$ convergent dus $]-1, 1]$ convergent

$$1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots$$

Bespreek convergentie : eerst model vinden

$$\sum_{n=0}^{+\infty} C_n x^n$$

$$\sum_{n}^{+\infty} \left(\frac{1}{n!}\right) x^n$$

$$\lim_{n \to +\infty} \left| \frac{\frac{x^{n+1}}{(n+1)!}}{\frac{x^n}{n!}} \right| = \lim_{n \to +\infty} \left| \frac{x^{\cancel{n+1}}}{(n+1)!} \cdot \frac{\cancel{n!}}{\cancel{x^n}} \right|$$

$$(n+1)!$$
$$\shortparallel$$
$$(n+1)\cancel{n!}$$

$$= \lim_{n \to +\infty} \left| \frac{x}{n+1} \right| < 1$$

$$|x| < \lim (n+1) = +\infty$$

$\hookrightarrow x \in \mathbb{R}$

VI) check convergentie

$$x - \frac{x^3}{3} + \frac{x^5}{5} - \dots$$

$$\lim \left| \frac{c_{n+1} x^{n+1}}{c_n x^n} \right|$$

n-de term en n+1-de term?

$n = 1$: dan x

$n = 2$: dan $-\frac{x^3}{3}$ $\rightarrow (-1)^{n-1} \frac{x^{2n-1}}{(2n-1)}$

$n = 3$ dan $\frac{x^5}{5}$

$$\lim_{n \to +\infty} \left| \frac{\frac{x^{2n+1}}{2n+1}}{\frac{x^{2n-1}}{2n-1}} \right| = |x^2| \left| \frac{2n-1}{2n+1} \right| < 1$$

$$|x^2| < 1$$

$$|x| < 1$$

~~stel~~ $x = -1$: $-1 + \frac{1}{3} - \frac{1}{5}$ → convergent

$x = 1$: $1 - \frac{1}{3} + \frac{1}{5}$ → convergent

* Extra stelling

$$f(x) = \sum_{n=0}^{+\infty} c_n (x-a)^n$$

$$\hookrightarrow f'(x) = \sum_{n=1}^{+\infty} c_n\, n (x-a)^{n-1} \rightarrow |x| < R \Rightarrow \frac{1}{1-x} = 1 + x + x^2$$

f' inderdaad

$$\hookrightarrow \frac{1}{(1-x)^2} = 1 + 2x + 3x^2 +$$

$$\hookrightarrow \int f(x) = \sum_{n=0}^{+\infty} \frac{c_n (x-a)^{n+1}}{n+1}$$

$$\rightarrow \frac{1}{1+x} = \cdot - x + x^2 - x^3$$

$$\hookrightarrow \int \frac{dx}{1+x} = \ln(1+x) =$$

$$x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \ldots$$

* Taylor.

Formule of formularium: examenvraag: hoe kom je er aan

$$f(x) = a_0 + a_1(x-a) + a_2(x-a)^2 + \dots$$

? ? ?

① $\xrightarrow{\text{substitueer}} f(a) = a_0$

② $f'(x) = a_1 + 2a_2(x-a) + 3a_3(x-a)^2$

$\xrightarrow{\text{substitueer}} f'(a) = a_1$

③ $f''(x) = 2a_2 + 3\cdot 2(x-a) + 4{,}3(x-a)^2$

$$f''(a) = 2a_2 \Leftrightarrow a_2 = \frac{f'(a)}{2!}$$

$$a_3 = \frac{f'''(x)}{3!}$$

$$= f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \frac{f'''(a)}{3!}(x-a)^3 + \dots$$

$$= f(a) + f'(a)x + \frac{f''(a)}{2!}x^2 + \frac{f'''(a)}{3!}x^3$$

Bij Taylor is het rond $a \neq 0$

Bij Maclaurin is het rond $\emptyset$

Vb $y = e^x$

		$x=0$	$c_n = \frac{f^n(0)}{n!}$	
f	e^x	1	1	
f'	e^x	1	1	x
f''	e^x	1	$1/2!$	x^2
f'''	e^x	1	$1/3!$	x^3

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots$$

is ook ontwikkelbaar

Eksamenvraag

VI $y = \cos x$

		$x=0$	
$f(x)$	$\cos x$	1	1
$f'(x)$	$-\sin x$	0	$0\,x$
f''	$-\cos x$	-1	$-\frac{1}{2!}x^2$
f'''	$\sin x$	0	$\frac{0}{3!}x^3$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \dots$$

Convergentiestraal : d'Alembert

$$\lim \left| \frac{C_{n+1}x^{n+1}}{C_n x^n} \right| \;? \quad \dots \quad \forall x$$

Pag 14 : basisreeks ontwikkeling
: Convergentie stralen

Vb $\ln(1+x) = \quad -1 < x \leq 1$

$\hookrightarrow 1+x>0$] moet je niet checken.
$x>-1$

→ olemen checken $\quad |x|>0$

* Stelling van de resttem (enkel toepassen, niet kunn

$$f(x) \sum_{k=0}^{n} \frac{f^{(k)}(a)(x-a)^k}{k!} + R_{n+1}$$

$+ R_{n+1}$ = restterm

↓

$$\frac{f^{(n+1)}(c)}{(n+1)}(x-a)^{n+1}$$

$\exists c \in [a, x[$

→ lim van resttem = 0

dan convergeert de taylorreeks

Vb $\sin(x) = x - \frac{x^3}{3!}$ (afgekapt tot 2 termen)

Voor welke x is dit geldig als de fout kleiner moet zijn dan $3 \cdot 10^{-4}$

Maar hoe ver van 0 kan je weggaan

volgende term: $\frac{x^5}{5!}$ welke de fout afschat

$$\frac{x^5}{5!} < 3 \cdot 10^{-4}$$

$$|x| < \sqrt[5]{3 \cdot 10^{-4} \cdot 5!} = 0{,}514$$ is de max afstand van 0

* Binomiaal Geen theorie, wel of-

Statistiek

$$(1+x)^n = \sum_{k=0}^{\infty} \binom{n}{k} x^k = \binom{n}{0} + \binom{n}{1} x + \binom{n}{2} x^2 + \text{ect}$$

$n \in \mathbb{N}$

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}$$

zeedsreeemel $\frac{n!}{2!(n-2)!} x^2$

$$= 1 + nx + \frac{n(n-1)}{2} x^2 + \dots$$

$n \in \mathbb{R}$

$n = \frac{1}{2} \Rightarrow \sqrt{1+x}$

Taylorreeks ontwikkeling misbruiken

$$\Rightarrow 1 + \frac{1}{2}x + \frac{1}{2}\left(-\frac{1}{2}\right)\frac{x^2}{2} +$$

$$\int_0^1 \sin x^2 \, dx$$ tussen 0 en 1 met precisie 0,001

we: $\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!}$

$x \leftarrow x^2$

we: $\sin x^2 = \underbrace{x^2 - \frac{x^6}{3!} + \frac{x^{10}}{5!} - \dots}$

$$\int_0^1 (\dots)\, dx$$

$$\left[\frac{x^3}{3} - \frac{x^7}{7 \cdot 3!} + \frac{x^{11}}{11 \cdot 5!} + \right]_0^1$$

$$\frac{1}{3} - \frac{1}{42} - \frac{1}{11 \cdot 5!}$$

$\frac{1}{3} = 0{,}33$ $\qquad$ $\frac{1}{11 \cdot 5!} = 0{,}00076$

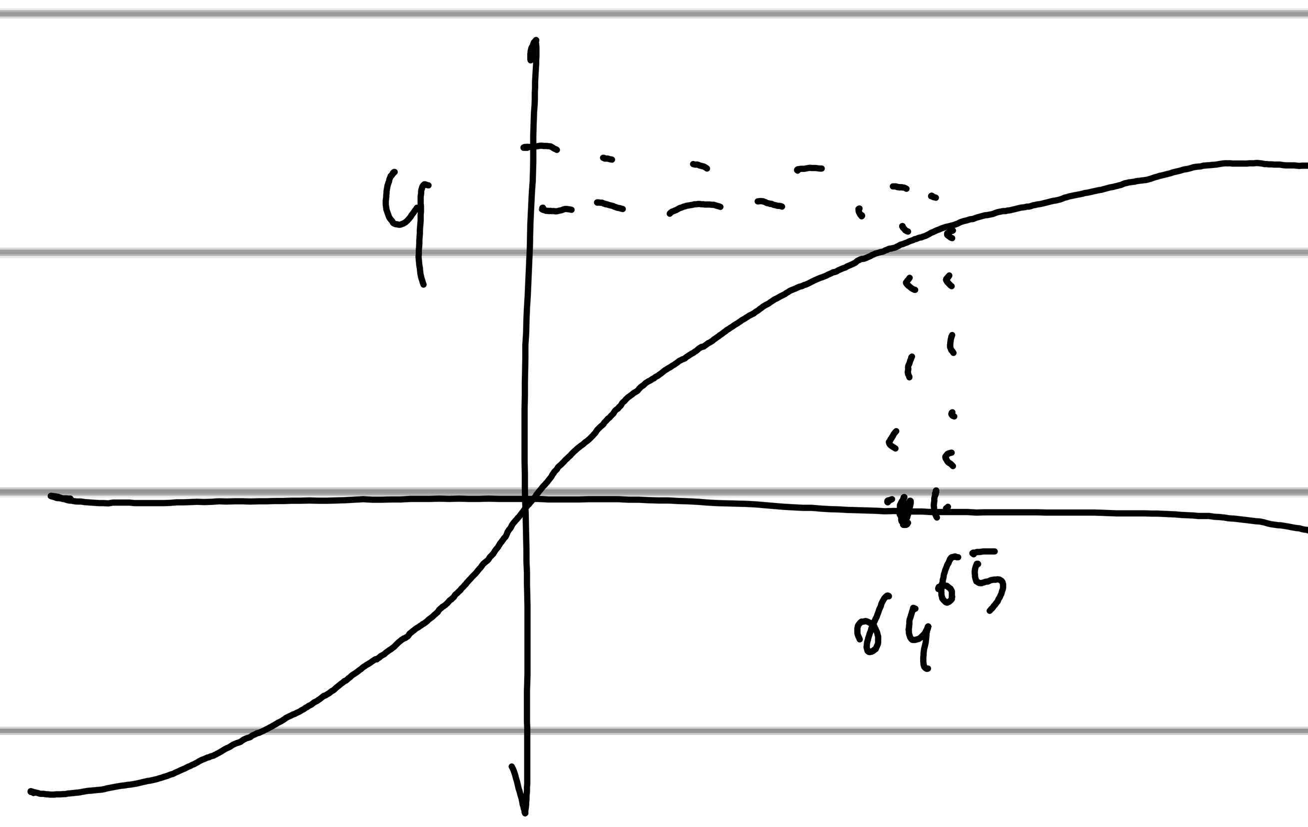
↳ kleiner dan 3 cijfers na de komma dus mag verwaarloosd worden.

$\frac{1}{3} - \frac{1}{42} = 0{,}310$

Bereken $\sqrt[3]{65}$ tot 3 cijfers na de komma

Welke functie ligt er op: $\sqrt[3]{64} = 4$

$\sqrt[3]{x}$ ontwikkelen rondom $a = 64$



Taylor rond $a = 64$

		$a = 64 = 2^6$	$\frac{f^{n}(a)}{k!}(65-a)^k$ $(x = 65)$
f	$x^{1/3}$	$2^{6/3} = 2^2$	4
f'	$\frac{1}{3}x^{-2/3}$	$\frac{1}{3}x^{-\frac{12}{3}} = \frac{1}{3}x^{-4}$	
f''	$\frac{1}{3}\left(-\frac{2}{3}\right)x^{-5/3}$	$\frac{1}{3\cdot 16} = \frac{1}{48}$	$\frac{1}{48}\cdot(x-64)$
		$\frac{1}{3}\cdot\left(-\frac{2}{3}\right)2^{-10}$	$\frac{1}{9}2^{-10}(x-64)^2$

$$-\frac{2}{9}\cdot 2^{-1} < 0{,}001$$

$$= 4 + \frac{1}{48}$$

Tcep

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots$$

$$e^{i\theta} = \cos\theta + i\sin\theta$$

$x \to i\theta$

$$e^{i\theta} = 1 + i\theta + \frac{(i\theta)^2}{2!} + \frac{(i\theta)^3}{3!} + \dots$$

info

$i^2 = -1$

$i^2 = -i$

$i^3 = 1$

~~$i^4 = -1$~~

$$= 1 + i\theta + \frac{-1\theta^2}{2!} + \frac{-i\theta^3}{3!} + \dots$$

$$= 1 - \frac{\theta^2}{2!} - \frac{\theta^4}{4!} \dots + i\theta - \frac{i\theta^3}{3!} + \frac{i\theta^5}{5!} \dots$$

$$= \cos\theta \qquad + i\sin\theta$$

Bsp

$$\lim_{x \to 1} \frac{\ln x}{x-1} = \lim_{x \to 1} \frac{(x-1) - \frac{(x-1)^2}{2} + \frac{(x-1)^3}{3}}{x-1}$$

$$= \lim_{x \to 1} 1 - \frac{1}{2}(x-1) + \frac{(x-1)^2}{3} = 1$$

Samenvatting

<u>meetkundige</u>

$$a + az + az^2 + az^3 \dots = \frac{a}{1-z} \qquad |z|<1$$

$\times z \qquad \times z \qquad \times z$

<u>machtreeksen</u>

$$\sum c_n (x-a)^n$$

↳ D'Alembert

$$\lim_{n\to\infty} \left| \frac{c_{n+1} x^{n+1}}{c_n x^n} \right| < 1$$

$$R = \lim_{n\to\infty} \left| \frac{c_n}{c_{n+1}} \right| \Rightarrow |x| < R$$

Taylor + Mclaurinreeks

$$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!} (x-a)^n$$

↳ Toepassing

Binomiaalreeks: speciaal geval