PEER TUTOR INTERVIEW QUESTIONS

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TOPIC: Inverse Matrices (MATH 1025)

(1) Let A be the following 3 x 3 matrix:

$$\begin{bmatrix} 3 & 1 & 1 \\ 2 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

Also, let C be some 3x3 matrix, $C \neq A$. Solve for C, given the following equation:

$$C * A^{-1} = det(A) * I$$

Easiest way to determine determinant: det(A)=(-1)$^{1+3}$$[(2)(1)-(1)(1)] + (-1)^{3+3}[(3)(1)-(2)(1)] = 1 + 1 = 2$

Observe that C*A^{-1}*A=C*I=C.

Therefore, $C = det(A) * I * A$=$det(A) * A$=

$$\begin{bmatrix} 6 & 2 & 2 \\ 4 & 2 & 0 \\ 2 & 2 & 2 \end{bmatrix}$$

(2) Let D be the matrix

$$\begin{bmatrix} cos^2(\theta) & 1 \\ sin^2(\theta) & 1 \end{bmatrix}$$

Also, let $-\pi \leq \theta \leq \pi$. Does D have an inverse matrix? Explain.

A square matrix is invertible if and only if its determinat does not equal 0.

$det(D)$=$cos^2(\theta) - sin^2(\theta) = cos(2\theta)$

If $cos(2\theta) = 0$, then the matrix is not invertible.

Recall that $cos(\theta) = 0$ whenever $\theta = \frac{\pi}{2} + n * \pi$,
n being any integer.

Therefore, it follows that $cos(2 * \theta) = 0$
whenever $\theta = \frac{\pi}{4} + \frac{n*\pi}{2}$.

Therefore, it follows that this matrix is not invertible if $\theta = \frac{-3\pi}{4}, \frac{-\pi}{4}, \frac{\pi}{4}, or \frac{3\pi}{4}$.

(3) Let G be the matrix

$$\begin{bmatrix} 2 & 1 & 1 \\ 4 & 1 & 1 \\ 2 & 3 & 2 \end{bmatrix}$$

(a) Find its adjugate matrix ($adj(G)$) and $det(G)$.

adj(G)=

$$\begin{bmatrix} (1)det(G_{1,1}) & (-1)det(G_{1,2}) & (1)det(G_{1,3}) \\ (-1)det(G_{2,1}) & (1)det(G_{2,2}) & (-1)det(G_{2,3}) \\ (1)det(G_{3,1}) & (-1)det(G_{3,2}) & (1)det(G_{3,3}) \end{bmatrix}^T$$

,

where

$$G_{1,1} = \begin{bmatrix} 1 & 1 \\ 3 & 2 \end{bmatrix}, G_{1,2} = \begin{bmatrix} 4 & 1 \\ 2 & 2 \end{bmatrix}, G_{1,3} = \begin{bmatrix} 4 & 1 \\ 2 & 3 \end{bmatrix}, G_{2,1} = \begin{bmatrix} 1 & 1 \\ 3 & 2 \end{bmatrix},$$

$$G_{2,2} = \begin{bmatrix} 2 & 1 \\ 2 & 2 \end{bmatrix}, G_{2,3} = \begin{bmatrix} 2 & 1 \\ 2 & 3 \end{bmatrix}, G_{3,1} = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}, G_{3,2} = \begin{bmatrix} 2 & 1 \\ 4 & 1 \end{bmatrix}, G_{3,3} = \begin{bmatrix} 2 & 1 \\ 4 & 1 \end{bmatrix}.$$

Therefore, adj(G)=

$$\begin{bmatrix} (1)(-1) & (-1)(6) & (1)(10) \\ (-1)(-1) & (1)(2) & (-1)(4) \\ (1)(0) & (-1)(-2) & (1)(-2) \end{bmatrix}^T = \begin{bmatrix} -1 & -6 & 10 \\ 1 & 2 & -4 \\ 0 & 2 & -2 \end{bmatrix}^T = \begin{bmatrix} -1 & 1 & 0 \\ -6 & 2 & 2 \\ 10 & -4 & -2 \end{bmatrix}$$

Also, det(G)=(1)[(4)(3)-(2)(1)]+(-1)[(2)(3)-(2)(1)]+(2)[(2)(1)-(4)(1)]
=10-4-4=2

(b) Find G^{-1} using $adj(G)$ and $det(G)$.

$$\mathrm{G}^{-1} = \frac{1}{det(G)} adj(G) = \frac{1}{2} * \begin{bmatrix} -1 & 1 & 0 \\ -6 & 2 & 2 \\ 10 & -4 & -2 \end{bmatrix}$$

(4) Let A and B be 2x2 matrices. Prove that $(AB)^{-1} = B^{-1}A^{-1}$.

This is one possible answer. Assume that the equation is true. Therefore, it follows that the inverse of AB is $B^{-1}A^{-1}$.

Observe that (AB)$(B^{-1}A^{-1})$
=A*B*B^{-1}*A^{-1}
=A*I*A^{-1}
=A*A^{-1}
=I,
as desired.