Plan

Review

Logistic Regression

↳ Sigmoid

↳ Softmax

Cross Entropy Loss

Logistics

- Zoom!
- check in form
- scribed notes :)

Linear Regression

$(x^{(1)}, y^{(1)}), \dots, (x^{(n)}, y^{(n)})$

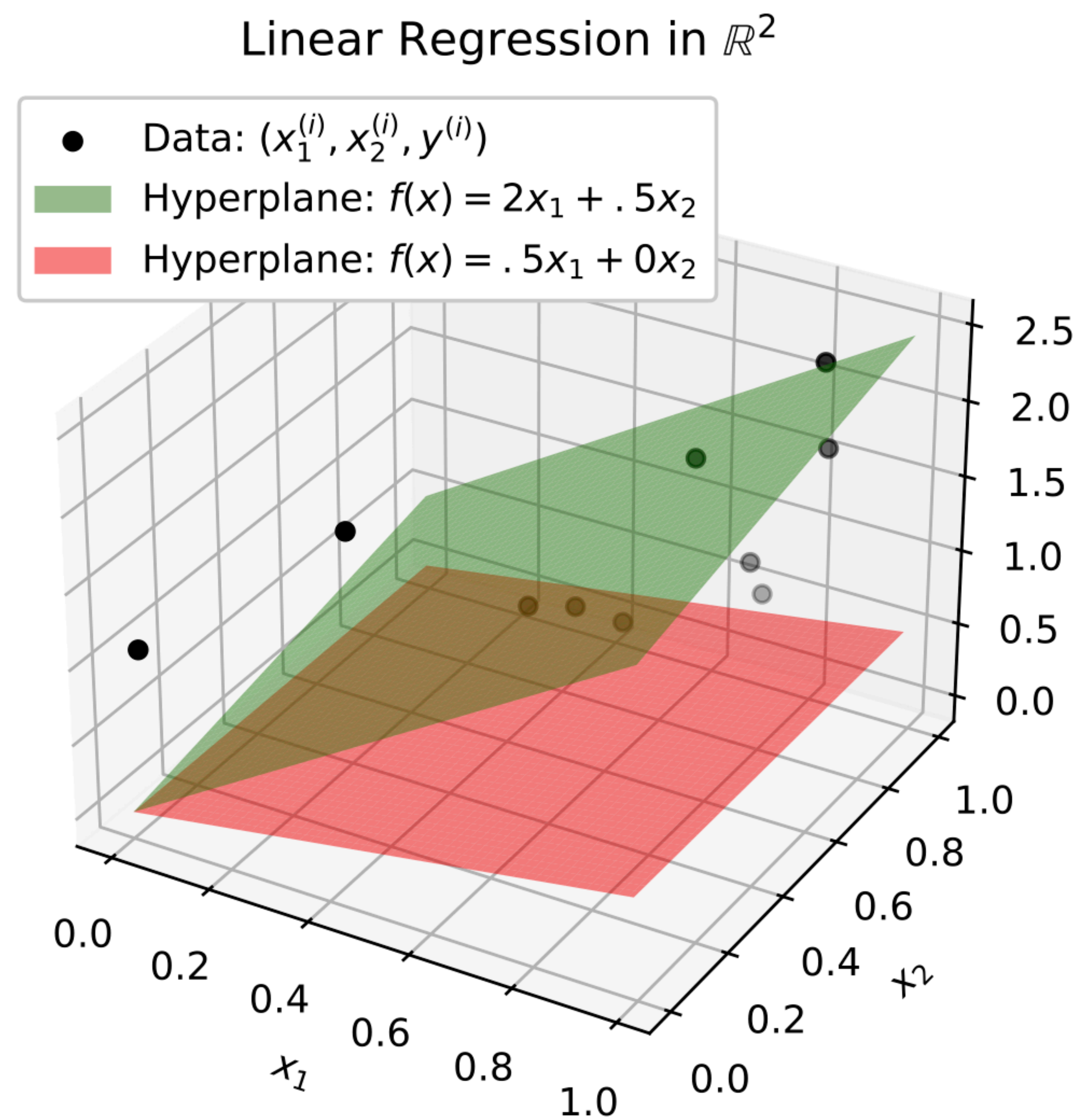
$x^{(i)} \in \mathbb{R}^d \qquad y^{(i)} \in \mathbb{R}$

① Model: $f(x) = w \cdot x$
for $w \in \mathbb{R}^d$

② Loss: $\mathcal{L}(w) = \frac{1}{n} \|Xw - y\|_2^2$

③ optimization: $\nabla_w \mathcal{L}(w^*) = 0$

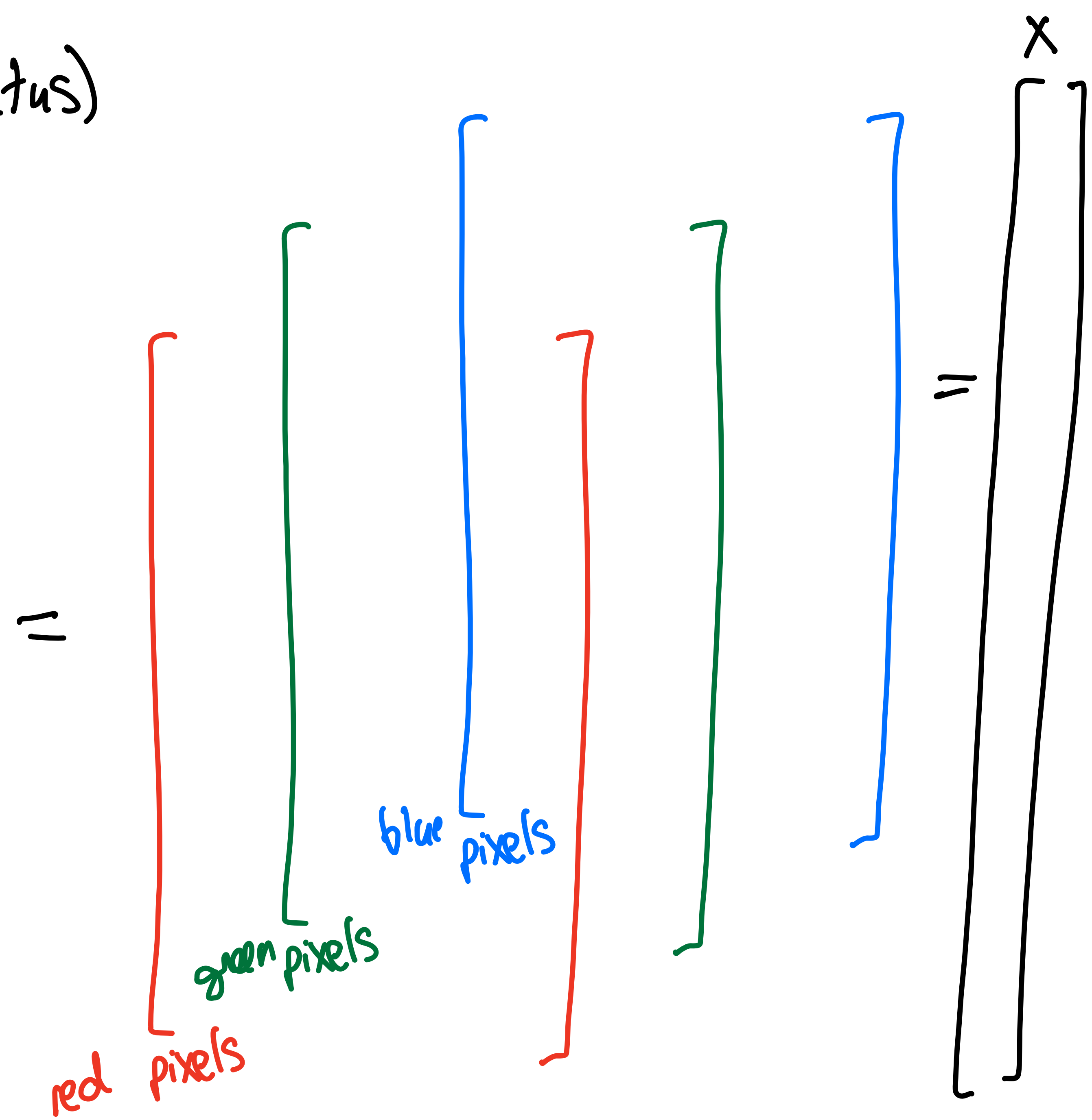
$$\Longleftrightarrow \quad w^* = \underbrace{\underset{d\times n\; n\times d}{(X^T X)^{-1}}\, \underset{d\times n}{X^T}\, \underset{n\times 1}{y}}_{d\times 1}$$



Motivation

- What if labels are classes (rather than values)?
- What happens when we can't find the exact optimal?

(image, cat status)



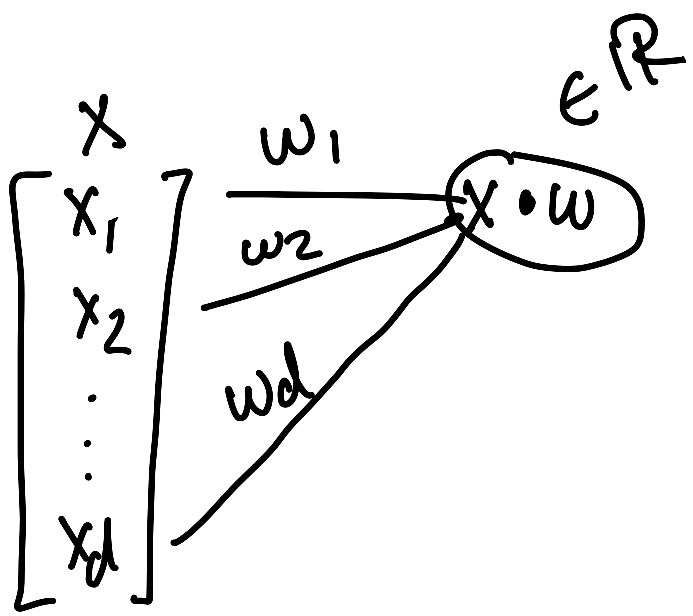
$y \in \{0,1\}$

Supervised Binary Classification

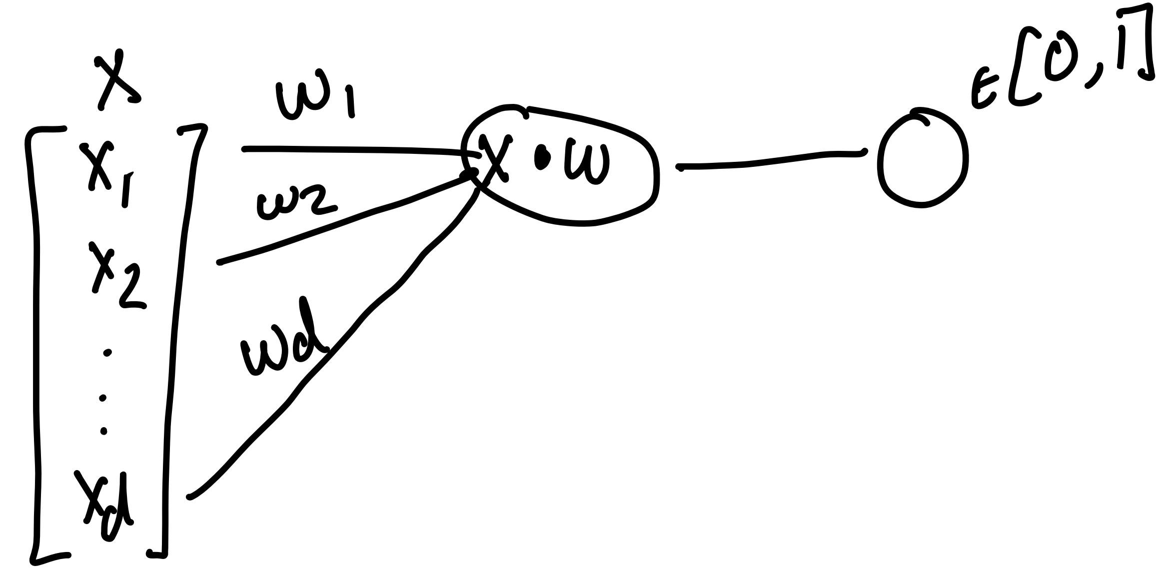
$\ldots (x^{(i)}, y^{(i)}) \ldots$

$x^{(i)} \in \mathbb{R}^d \qquad y^{(i)} \in \{0,1\}$

Goal: $f(x^{(i)})$ = probability of positive class



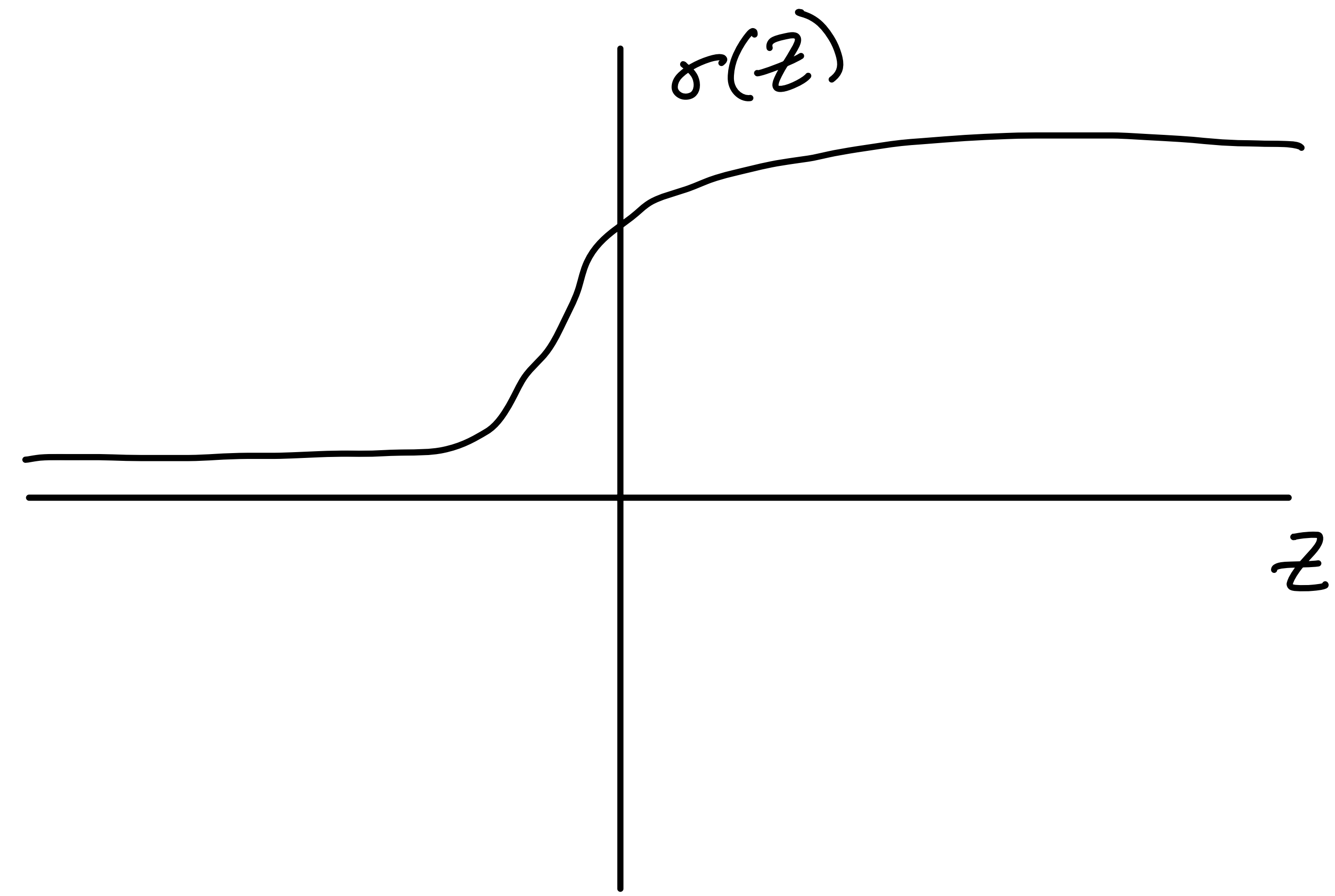
Linear Regression

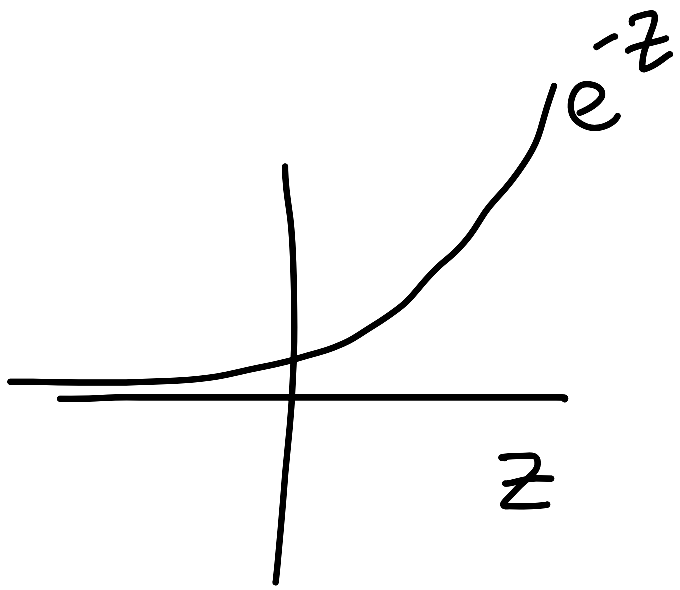


Logistic Regression

Sigmoid

$$\sigma(z) = \frac{1}{1+e^{-z}}$$





$$\lim_{z\to\infty} \frac{1}{1+e^{-z}} = \frac{1}{1+0} = 1$$

$$\lim_{z\to-\infty} \frac{1}{1+e^{-z}} = \frac{1}{\infty} = 0$$

Multiple Classes

$\ldots (x^{(i)}, y^{(i)}) \ldots$

$x^{(i)} \in \mathbb{R}^d \qquad y^{(i)} \in \{0, 1, \ldots, k\}$

$f : \mathbb{R}^d \to [0,1]^k$

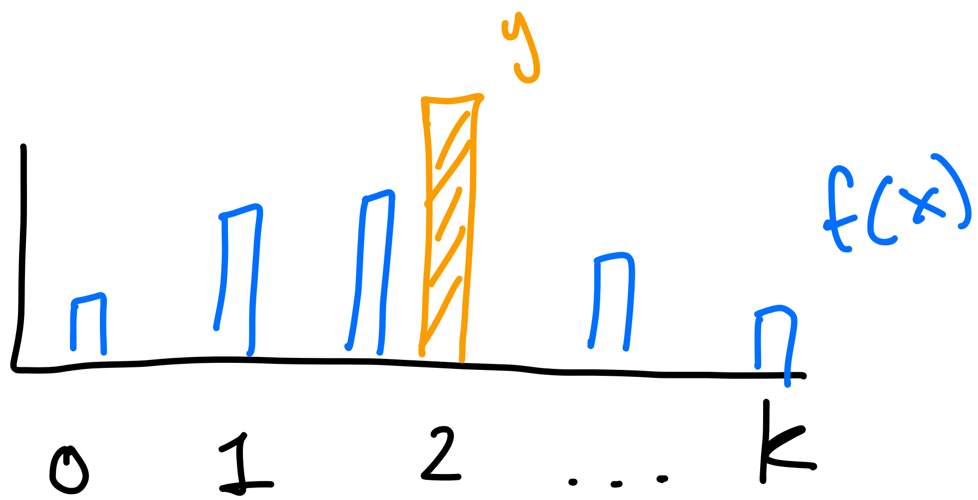
Probability distribution

1. non-negative
2. sums to 1

Goal: $f(x^{(i)})_\ell =$ prob of class ℓ

What should the architecture be?

Loss



Goal: Measure distance between <u>distributions</u> y and $f(x)$

Cross entropy between p and q:

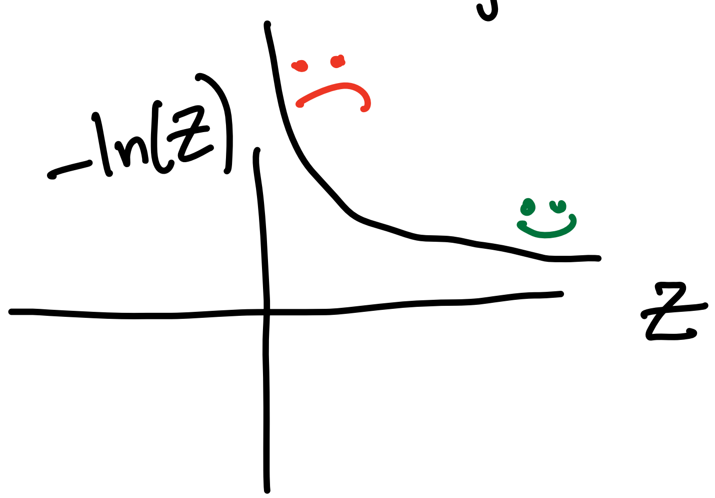
$$H(p,q) = -\mathop{\mathbb{E}}_{j \sim p}\left[-\ln(q(j)) \right] = -\sum_{j=1}^{K} p(j) \ln(q(j))$$

When p is "one-hot" i.e., $\exists\, j^*$ s.t. $p(j^*) = 1$ then

$$H(p,q) = -\, p(j^*) \ln(q(j^*)) = -\ln(q(j^*))$$

Cross entropy between p and q:

$$H(p,q) = -\sum_{j=1}^{K} p(j)\ln(q(j)) \overset{\text{"one-hot"}}{=} -\ln(q(j^*))$$



Optimization

- Exact is not doable (you'll see on problem), but yet we can still find good weights. How?