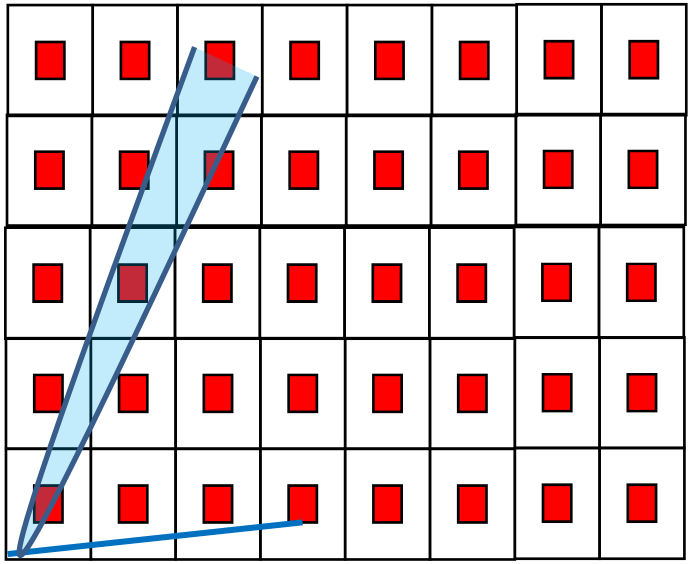
Pistes récentes pour les coureurs

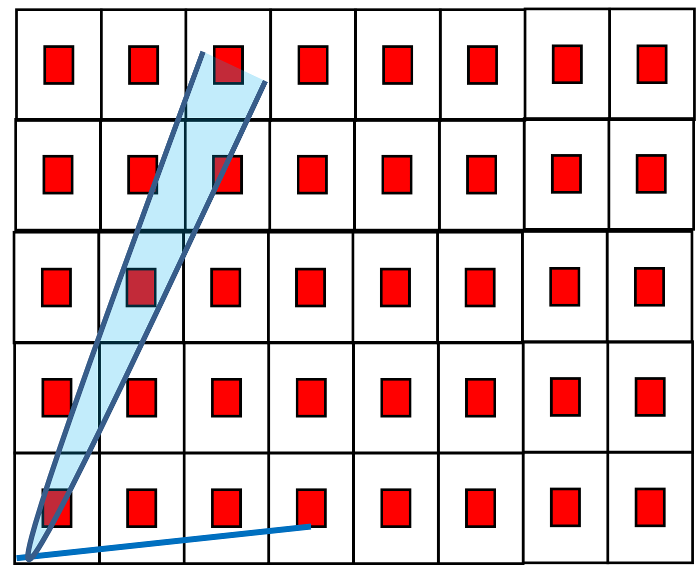
1. Put boxes into cones



2. **Thm** (Terence Tao, May 2015): it is possible to bound $|v|$ as a function of k. **Q**: Could there be a better, simpler solution more useful for proving LRC?

3. Average min distance from START on intervals

1. The cones



Equivalent formulations :

∃ integers $\zeta_1, \ldots, \zeta_k \geq 0$ so that

rescaling v by a real, denote it t,

are in a « red » box, that is,

$\zeta_i + \frac{1}{k+1} \leq v_i t \leq \zeta_i + \frac{k}{k+1}$, **so each facet of the cone above a fixed box** involves only two variables :

$$\frac{\zeta_i}{v_i} + \frac{1}{(k+1)v_i} \leq \frac{\zeta_j}{v_j} + \frac{k}{(k+1)v_j} \quad \forall\, i, j = 1, \ldots k$$

that is, $\exists$ nonneg integers $\zeta_1, \ldots, \zeta_k$

$$\max_i \frac{\zeta_i}{v_i} + \frac{1}{(k+1)v_i} \leq \min_j \frac{\zeta_j}{v_j} + \frac{k}{(k+1)v_j}$$

is equivalent to LRC, and handles all possible t …

One has to show that *every v is in such a cone* !

Opens some visualizations, inequalities, … 54 pages

2. $|v|$ as a function of k

Thm : (Terence Tao, May 2015) There exists $f(k)$ so that it is sufficient to check LRC (v) with $\max v \leq f(k)$.

Towards a simpler proof, possibly part of a solution …

Let $v_1 < v_2 < \ldots < v_k$; if $v_k/v_{k-1} \geq k$, then easy induction.

Question : Can't $v_i/v_{i-1} \leq k^2$ (or any $f(k)$) for all $i=2, \ldots, k$ be supposed ? If $\exists\, i$: $v_i/v_{i-1} > k^2$, can't we separate into two?

3. Average min distance of START

v=(1,2,3) , $\frac{1}{k+1}$ = max > 2 mean

Definite integral:

More digits

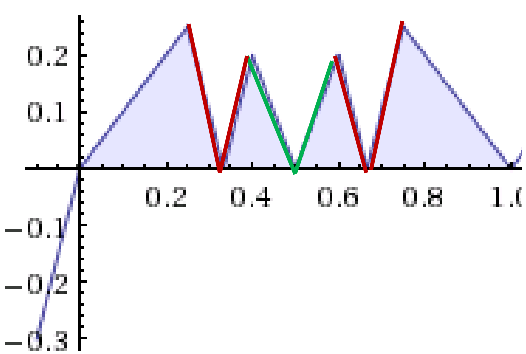
$$\int_0^1 \min(\mathrm{frac}(t), 1-\mathrm{frac}(t), \mathrm{frac}(2t), 1-\mathrm{frac}(2t), \mathrm{frac}(3t), 1-\mathrm{frac}(3t))\, dt =$$

$$\frac{7}{60} \approx 0.116667 < \frac{1}{2(k+1)} = 0.125 .$$ Refine the method !

$\mathrm{frac}(x)$ is the fractional part function

$\min(x_1, x_2)$ is the minimal value function

Visual representation of the integral:



1

2

3

Mean on some subintervals?

Here $[0, \frac{1}{3}]$, or $[\frac{1}{3}, 1]$is good!

Bound on Max in terms of Mean?