Entropy Projection Curved Gabor with Random Forest and SVM for Face Recognition

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Abstract. In this work, we propose a workflow for face recognition under occlusion using the entropy projection from the curved Gabor filter, and create a representative and compact features vector that describes a face. Despite the reduced vector obtained by the entropy projection, it still presents opportunity for further dimensionality reduction. Therefore, we use a Random Forest classifier as an attribute selector, providing a 97% reduction of the original vector while keeping suitable accuracy. A set of experiments using three public image databases: AR Face, Extended Yale B with occlusion and FERET illustrates the proposed methodology, evaluated using the SVM classifier. The results obtained in the experiments show promising results when compared to the available approaches in the literature, obtaining 98.05% accuracy for the complete AR Face, 97.26% for FERET and 81.66% with Yale with 50% occlusion.

Keywords: Face recognition · Face occlusion · Curved Gabor · Features selection

1 Introduction

One of the many goals of face recognition is the identification of individuals in crowds with security cameras. However, to create an ideal recognition system, the methodology should satisfy the following requirements: (1) effective differentiation of individuals (a large inter-class variation) while accepting variations between representations of the same individual (intra-class variation); (2) extraction of face images precisely through quick processing; and (3) low dimensional space to reduce computational costs as part of the classification process [17]. A task of major importance in these systems is detection under face occlusion [29].

This face variation is seldom easy or even impossible to recognize an individual through accessories and lighting conditions [26]. The problem presented by the occlusion linked to the difficulty of obtaining a feature vector with an ideal dimensionality are two of significant obstacles to the development of a robust system.

In this work, we propose a reduced feature vector face recognition workflow for images with face occlusions. The Entropy Curved Gabor Random Forest methodology consists in extracting the features using the Curved Gabor Entropy Projection. This allows a representation of the face in different degrees of orientation, scale, and translation. Recent proposals as in [14] used Curved Gabor Entropy Projection with vectors of high dimensionality, which made this face recognition algorithm unfeasible to recognize an individual in real time when using complex image databases. This paper describes how we use Random Forest (RF) [1] to select the most relevant features and make a representative and robust feature vector for real-time recognition. In the classification stage, we use the Support Vector Machine (SVM) [22] and evaluate the performance on three images database: AR-Face [18], FERET [20,25] and Yale B database [7]. The obtained results are compared with other approaches existing in the state-of-the-art.

In Sect. 2 several works that include potential solutions for face recognition in complex environments are presented. In Sect. 3 describes the proposed approach and tests performed as part of model evaluation, including all the parameters used to set up the curved Gabor filter, entropy, random forest, and SVM to obtain the results. We describe the image database used in the tests and present results and the experiments in Sect. 4. In Sect. 5 we discuss the results obtained by the proposed methodology. Lastly, Sect. 6 we show the potential limitations of this methodology and describe the perspectives of further research projects.

2   Related Work

Convolutional Neural Networks (CNNs) has integrated new workflows to solve tasks in image recognition, including faces, and objects recognition, and the diagnosis of medical images. In 2014, Parkhi et al. [19] proposed VGG-Faces, a CNN for the sole purpose of recognizing faces. However, after its creation, CNN's most powerful were presented, and at a specific instant, the creation of these powerful architectures stagnated. CNN's main problem is the large number of data to carry out the training process [9]. From this, techniques were proposed with the use of CNN's without performing all the training of the network. The approach proposed by Ghazi and Ekenel [8] presents a face recognition solution using transfer learning with pre-trained CNNs. The authors used the VGG-Faces and Lightened CNN by extracting the fully-connected layer of each model. In addition to face recognition, the authors proposed an evaluation of the deep learning techniques in different representations of the face in variations of illumination and occlusion.

The authors in [15], highlight the main challenges related to face recognition in images. Among them are: distortions caused by lighting and variations of face

expression, head position and occlusion which all bring uncertainties. As this work is an approach proposed to solve such issues, a new regression model (Discriminative and Compact Coding - DCC) is developed that provides multi scale error measurement, compactness and interclass collaboration. This approach has demonstrated significant improvements in results from experiments using several image databases. However, the accuracies obtained for faces with occlusion were lower than those from other state of the art approaches.

A feature-sign search (FSS) in [16] presents an issue when using sparse coding in large image databases due to the high computational cost. The authors in this work developed a new algorithm, which iteratively solves two problems of convex optimization by searching for the best base vector for image representation. The approach performed robustly to occlusion problems in continuous blocks located on the inferior portion of the face. Accuracy values were 95.38% for recognition of individuals using the AR Face image database for scarf occlusion. However, the approach presented difficulties mostly when handling occlusions on the region around the eyes.

The Sparse Representation-based Classification (SRC) [24] and Correntropy-based Sparse Representation (CESR) [11] approaches do not acquire all the image features available but randomly select small amounts of information sufficient to rebuild the image. CESR was presented in [11] to deal with noise and occlusion problems in face recognition. In CESR, non-negative sparse representation is combined to the maximum correntropy criterion. Other works, which were used to compare with this approach, are founded upon linear regression. Linear Regression Classification (LRC) [24], FSS, Structured graphical lasso (SGLasso) [27] and DCC consider the test image as a linear combination of some of the training images.

Among the approaches mentioned above, all of them presented difficulties when performing face recognition dealing with continuous occlusions on regions around the eyes and mouth. That can be attributed to the holistic aspect of those approaches. In practice, holistic feature vectors do not perform very robustly to variations of lighting, face expression, head position and local deformation [23].

3   Proposed Methodology

In this paper, we propose a face recognition methodology based on feature selection using the Random Forest from a vector consisting of the Gabor Filter Bank response, entropy calculation, and classification with SVM. We present the flowchart of the proposed methodology in Fig. 1.

3.1   Features Extraction

In the feature extraction step, we subdivided the creation of the vector into three stages. In the first one, we used the result of the Gabor filter bank convolution with the input image to create an initial representation of the vector. In the second one, we apply block segmentation to subdivide the result from the previous
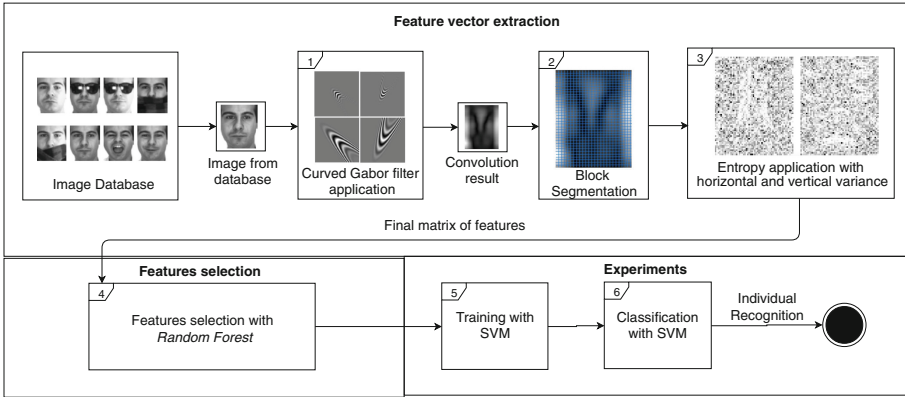
Fig. 1. General flowchart of the proposed methodology.

stage in order to reduce the feature vector dimensionality using entropy in the third stage.

Curved Gabor Filter Application: Initially proposed in [6] for 1D and later 2D signal processing [5]. Gabor has several configurations used in state of the art and among them, curved Gabor. This provides the extraction of contour features associated with regions that present curves on the face, such as the eyes, nose, mouth, and cheeks. Given this, we believe that curved Gabor is more appropriate than traditional Gabor [12].

The curved Gabor filter banks were created with frequencies (u) and orientations (v), where the number of orientations varies according to the degree of curvature of c, as seen in the state-of-the-art works. For the degree of curvature a set $c = \{0; 0.05; 0.1; 0.2\}$ was defined, which produces a curved Gabor filter bank [10, 12]. To determine the values of c, a series of experiments were performed, varying $c = 0$ to 0.5. For $c > 0.2$, the curved Gabor filter bank presented acute deformations that reduced the quality of the generated feature vectors. For c different from 0, the number of orientations should be 16, twice the number of no curved filters due to the asymmetric features of the curved filter.

The curved Gabor bank uses 16 orientations while the traditional filter uses only 8. This is due to the asymmetry of the curved filter bank, which requires twice the amount of filters.

After the curved Gabor filter bank is built, the convolution of each filter belonging to the bank, through the input image. Thereby the magnitude of the curved Gabor filter bank responses is generated, and contains the initial set of features of the face. This set of responses from the curved Gabor is the final product in the first stage (Fig. 1.1).

Block Segmentation: Since images of different individuals can have the same histogram sequence and share the same log energy entropy, both images can be incorrectly identified as belonging to the same person [28]. Another possible approach to solve this problem is the segmentation of curved Gabor filter magnitude responses, obtained in stage 1, into small non-overlapping blocks (Fig. 1.2). Then each block, although from the same original image, will have different intensities. Computationally, this segmentation technique is fast, simple, and is not based on detection of essential areas of the face like the eyes, nose and mouth [2].

Entropy: The third stage uses the combination of Local Variance Projection Log Energy Entropy (LVP-LEE) features. The concept of entropy introduced by Shannon in [21] was initially used in studies on communication systems. After considering the components of those systems as probabilistic components, it was then used in other fields such as image processing [3]. The LVP-LEE as presented in [28], its function is to work as a feature extractor and a dimensionality reducer for feature vectors obtained in stage 2.

After the segmentation of the curved Gabor filter magnitude responses into non-overlapping blocks, a dimensionality reduction technique can be applied. Here, LVP-LEE was chosen, because at the same time as it extracts features it creates more representative features vectors. For each block in curved Gabor magnitude response, the method based on the entropy is applied. By applying the LVP-LEE on each curved Gabor filter bank response, two new data matrices are produced, representing the entropy of variance projection in the horizontal and vertical directions. After they are concatenated into only one feature vector, they are able to be used by a classifier to recognize a face (Fig. 1.3).

3.2   Features Selection with Random Forest

The features extraction step provides many features. In this way, it is necessary to reduce the features vector size using face image representation. For this it is necessary to identify the most relevant elements of the vector, making use of a features selection technique.

Among several algorithms that provide the relevance of each element to the classification, we use Random Forest [1] in this work. Its choice is due to its extensive use in the selection of variables in problems involving large amounts of data [13]. The feature vector, provided by step 1, is divided into s sub-vectors of size t. The k elements of each sub-vector most relevant to the classification are selected using the Random Forest. In this process, the number of trees for 250 was defined, according to tests performed with the parameter varying in 1, 25, 50, 250, and 1000.

According to our experiments, there was no significant increase with values above 250 for the number of trees. Then, we concatenate the elements then selected in each sub-vector, forming a new candidate vector that is subjected to the same procedure described. Thus, in the second iteration a new vector is got, thus being considered the final representation of the individual.

The choices of the values of the parameters s and k are of fundamental importance to get a relevant feature vector. For the best choice of these values, we performed a set of experiments on face databases. The experiments have to change the values of s and k and then execute step 2 of the methodology, getting the value of the accuracy. In the execution of the procedure, we observed the maximization of precision and minimization of the vector size. We identify that for the proposal, the best values in the first iteration are $k = 100$ and $s = 32$, totalizing a features vector of size 3200. In the second iteration, the best values are $k = 100$ and $s = 28$. The vector produced in this iteration is considered the face representation.

3.3   Classification

A Support Vector Machine is a powerful technique for pattern classification [22]. A SVM creates a hyperplane or a set of hyperplanes in a high-dimensional space, which can be used for classification by supervised learning. In step 2, the features representing diverse individuals from the image database are submitted to the classifier. After the training, the SVM generates the result as a data prediction model for the image classification.

Lastly, a vector generated is classified by the SVM and, consequently, the identification of the individual is made. The LIBSVM library [4], with a kernel Radial function, was used as a tool to support the SVM performance as a classifier. At this step, the k-fold cross-validation technique was used to estimate any error generated by the classifier. At the end of this process, the accuracy for face recognition of this approach, as well as the parameters used in SVM, is obtained. At the end of cross-validation, the best values given by the proposed approach for the following parameters of SVM were: penalty $C = 8$ and parameter $\gamma = 3.0517578125 \cdot 10^{-5}$ of the RBF kernel.

4   Datasets and Experimental Results

In order to compare our results, we used three public databases: AR-Face Database, FERET, and Extended Yale B. We evaluated the proposed methodology using k-fold cross-validation with $k = 10$ in three public databases that present occlusion. Among the various techniques of face recognition are the follow approaches: SRC [24], LRC [16], CESR [11], FSS [16], SGLasso [27], DC, CC and DCC [15] that were used to compare the approach proposed in this work. For the AR-Face database, we include results obtained by Ghazi and Ekenel [8], and the methodology with only the Curved Gabor + Entropy (CGEP), without the features selection. For the execution of the experiments, we used MATLAB and the Scikit-learn library in an Intel®Core™ i5-8350U CPU @ 1.70 GHz × 8 computer with 16 GB RAM.

4.1 AR Face

The AR Face Database, which consists of over 4,000 colored images of 126 people (70 men and 56 women) in total. The images in this database are frontal pictures of faces with different facial expressions, lighting conditions and occlusions. The main feature of this face database is the occlusion caused by glasses and scarves worn by the individuals. The pictures were resized to 48×56 pixels for our experiments in order to turn the dimensions similar to other algorithms used as comparison. This database was partitioned into 6 different subsets in order to group different features in the tests. Face images with different facial expressions, lighting and occlusion are grouped to enable a more robust experiment of the proposed approach in different scenarios.

In Table 1, we present the results obtained by the proposed methodology in the AR Face database. For the lighting variations subset, the best result was obtained by the CGEP methodology. However, due to the features vector of high dimensionality, the proposed methodology in this work becomes more interesting because it presents an accuracy above 99% and a vector with only 2800 features. In occlusion by sunglasses subset, we observed that the best result was obtained by the proposed methodology, with 96%. This result demonstrates the robustness of our methodology for several types of occlusion and presents the gain obtained through the selection of attributes.

Table 1. Comparisons between the results and state-of-the-art methodologies in AR Face database

Database	LRC	SRC	CESR	FSS	SGLasso	DC	CC	DCC	Ghazi and Ekenel [8]	CGEP [14]	Proposed methodology
Lighting variations	31.37	45.80	48.74	44.96	38.24	45.24	79.83	71.01	x	100.00	99.50
Occlusions by sunglasses	25.21	28.99	68.49	28.99	21.85	74.79	3.78	72.69	35.45	76.00	96.00
Occlusion by scarf	94.96	95.38	96.64	95.38	93.28	97.06	68.49	97.06	89.09	84.50	96.50
Illumination + sunglasses	8.19	15.13	20.80	14.50	11.55	23.95	3.15	22.48	x	94.75	96.50
Illumination + occlusion by scarf	18.28	29.41	36.76	27.31	21.22	29.20	63.87	45.80	x	98.25	99.25
Facial expressions variations	x	x	x	x	x	x	x	x	x	98.25	99.00
Complete basis	x	x	x	x	x	x	x	x	x	99.14	98.05

The occlusion created in the occlusion by scarf subset occurs throughout the lower part of the image, being cut at 50% of its height, getting a new image that in the classification. In this experiment, all the methodologies presented got

high settling rates, of which six presented values above 94%. DC and DCC got the best accuracy with 97.06%, and the proposed methodology reached 96.50%. Such metrics are justified by the significant reduction in the size of the final image used in the tests. Therefore, suggesting that the level of dimensionality reduction applied to the images should be adjusted regarding the image size. The occlusion created in the occlusion subset per scarf occurs throughout the lower part of the image, being cut at 50% of its height, obtaining a new image that is used in the classification.

From the lighting variations and occlusions generated in the first subsets of the database. Two new subsets have been created that combine the features presented and increase the difficulty in recognizing the individual. The proposed methodology obtained the best results in both subsets, demonstrating Gabor's ability to extract features in images that have light variation. In illumination + occlusion by sunglasses we obtained 96.50%, and in illumination + occlusion by scarf, it reached 99.25%.

The last subset presented has variations of facial expressions. In this subset segregate the faces that represent neutral expressions, smile, anger, and fright, enabling the verification in an environment closer to the real world. State-of-the-art approaches do not mention the results obtained for this subset. However, we conducted experiments in order to determine the applicability of our methodology in this type of problem. The proposed methodology reached 99.00%.

4.2   FERET

The FERET database consists of 1400 face images. This database has as main feature the occlusion of the face through changes in lighting. The imaging database was the result of the US Department of Defense's same-name program aimed at developing a standard image database for testing and evaluating face recognition algorithms. In the experiment presented, the face region of the original image was manually cut, and the images were resized to 48×56 pixels, similar to those used in the other comparison approaches.

Table 2 presents the results of the accuracy of the various approaches in the FERET database. The proposed approach obtains the best results with the accuracy of 97.26%, followed by DCC with 92.84%. As a result, precision and recall of 97.26% are obtained indicating that the proposed approach has the same ranking of individuals as false positives and false negatives, a characteristic appropriate for a balanced database.

Table 2. Comparisons between the results and state-of-the-art methodologies in FERET database

Methodology	LRC	SRC	CESR	FSS	SGLasso	DC	CC	DCC	Proposed
Accuracy	79.22	86.37	92.35	86.27	82.35	92.55	64.02	92.84	**97.26**

4.3 Extended Yale B

Extended Yale B database corresponds to 2496 frontal facial images of 38 people, photographed in gray-scale on 64 controlled variations of illumination. From the base, 30 faces per subject were randomly selected and overlaid with a fixed image of a baboon. The size of the occlusion generated ranges from 10%, 20%, 30%, 40%, 50%, 60%, 70% and 80% while the position of the occlusion occurs randomly. The new base generated is exemplified in Fig. 2. Due to the random characteristic of occlusion positioning, the Yale B database used in each experiment are different regarding the positioning of the occlusion.

(a) 10% occlusion (b) 20% occlusion (c) 30% occlusion (d) 50% occlusion (e) 70% occlusion (f) 80% occlusion
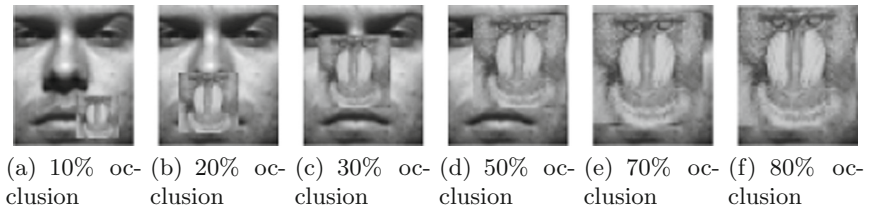
Fig. 2. Faces from Extended Yale B database.

In Fig. 3, we present the results obtained by the methodology in the Extended Yale B database with occlusion generated by the baboon. These data point out that in all compared approaches, as occlusion increases, the accuracy decreases. However, in our work, we obtained results ranging from 96.22% with 10% occlusion to 50.87% with 80% occlusion. These values surpass all the approaches used in the comparison for all occlusion variations. This demonstrates the robustness of the proposed methodology in high percentages of occlusion.
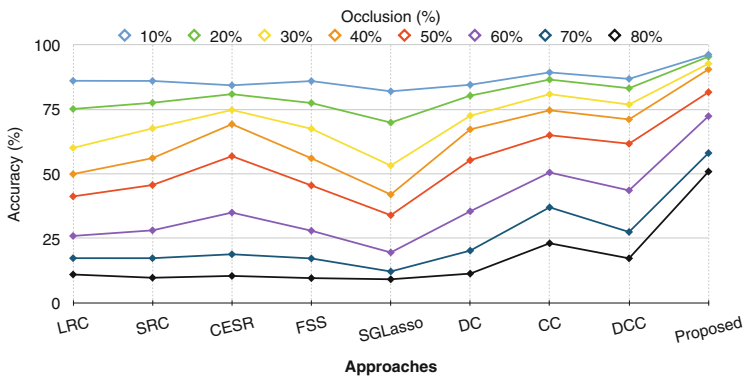

Fig. 3. Comparison of the results of the proposed methodology with approaches on the Yale base with occlusion.

During the execution of the experiments, we calculated the time required to extract the features for a single image. Given this, we obtained 3.98 s with sequential processing of the images. We believe that with the use of parallelism, this time will decrease considerably.

5   Discussion

The feature vector created after the extraction stage results in 94080 elements for images of size 48×56 pixels. From this representation, we use Random Forest for feature selection in stage 4. As a result, the new generated vector has a smaller size than the initial one, with 2800 elements. The reduction in size presented a percentage of 97.024%. The low dimensionality of the image representation provides the use of less storage space and fewer times for image retrieval and identification.

The results obtained by the methodology proposed for the AR Face base were superior in three of the four subsets that presented partial occlusion of the face. These results show the robustness of the methodology in situations that present this type of uncontrolled condition. For the other subsets, the results obtained proved superior or equivalent to those of state of the art. In situations involving light variation and change of facial expressions, the results obtained were higher than 98%. The result presented for the complete base allows us to infer that the created vector is representative enough to classify the faces in different conditions although they are in the same set.

The experiments with occlusion performed at the Yale base have the objective of creating conditions of uncontrolled environments in a controlled database. From the results obtained, we can evaluate positively the results achieved. These experiments were performed with the random overlap of the baboon in each image. Since in the work of [15], the authors do not specify whether the image of the baboon is in the same position, varying only the percentage of occlusion. Thus, the methodology had to adapt to occlusions in different positions and from this, to identify the individual. Among the state-of-the-art methodologies, CC achieved the best results, followed by DCC. However, the proposed methodology exceeded the accuracy values of all state-of-the-art approaches.

6   Conclusions

In this work, we propose a pipeline methodology for face recognition under occlusion conditions. The Gabor curved filter bank forms the methodology as a descriptor, local variance projection function entropy as dimensionality reducer, Random Forest as attribute selector and SVM as the classifier. From three consolidated public bases, several experiments were created, and their results were compared to those of state-of-the-art methodologies.

Among the main contributions of the methodology we have, the robustness in images of faces that present occlusion and the creation of a more representative vector and with lower dimensionality. With the use of the entropy and the bank

of curved Gabor filters, we were able to create a robust vector with the presence of occlusion. This is demonstrated by their results that when compared to state of the art, were superior. As for the dimensionality of the characteristics vector, the selection through Random Forest allowed the most relevant and most representative characteristics to be kept in the vector and the rest discarded. Such disposal provided the increase or stability of the accuracy depending on the base used. The selection also allowed for vector reduction by a little over 97% when compared to that provided by Curved Gabor + Entropy.

However, limitations are still found, and some ideas are proposed to complement this study. Among them, we perform experiments on non-controlled image databases and the implementation of other classifiers, as well as the use of GPUs in the feature extraction stage.

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