Term Project Report
Intro to Computer Programming
360-420-DW
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How Rough is Canada's Border?

Introduction

For our term project we wanted to base our code and research on the more mathematical side of science, so instead of taking a physics based problem we found a problem that almost solely focuses on mathematics. The problem we wanted to showcase is how roughness equates to a fractal dimension. A fractal is a shape with a non-integer dimension. The best way we thought of showing the relationship between fractal dimensions and roughness was to answer the question "How rough is Canada's Border?" So we set out to find away to calculate the fractal dimension of Canada's border.

Method

The goal of our term project is to find the roughness of Canada using Fractal dimension. Before we can find the roughness of Canada we must first determine how to calculate a fractal dimension. An easy example that shows how to find a fractal dimension is by using a self similar shape such as the Sierpinski Triangle.

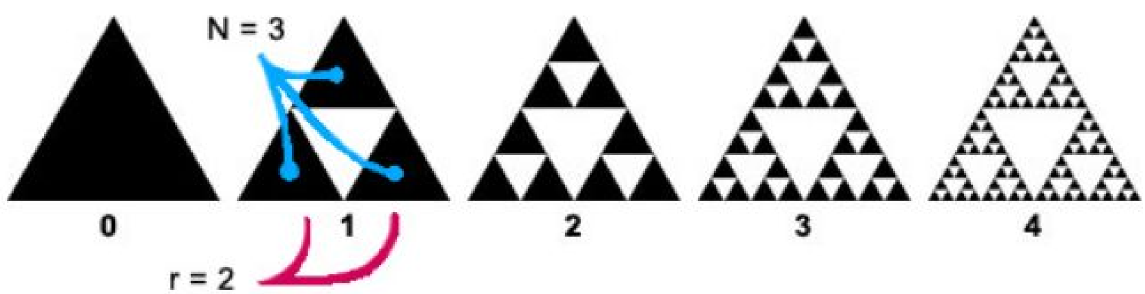


Figure 1

We can determine the fractal dimension of the Sierpinski triangle, using the notation in figure 1.
r = How much smaller is each triangle in order i than order i-1?
N = The factor of how much the number of triangles increased by.
D = Fractal Dimension

$$D = \frac{\log(N)}{\log(r)} = \frac{\log(3)}{\log(2)} = 1.585 \qquad (1)$$

With this method we can see that the fractal dimension of the Sierpinski Triangle fractal is 1.585.

The outline of canada is not a self-similar shape, so to find its fractal dimension we would have to get creative. The approach that we will use is placing the border on a grid and counting how many boxes the border touches. We then scale the image by some factor and count again how many boxes the outline touches.This method can be visualized in figure 2. The dimension could then be found by the following relationship.

S = Scaling factor
D = Fractal Dimension
N = Number of Boxes
C = Some Constant

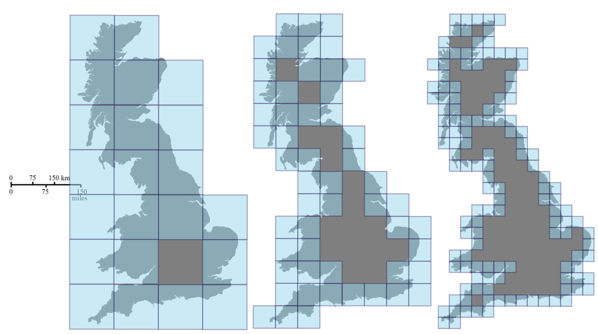
$N = C * (S^D)$ (2)

or

$Log (N) = Log C + D * Log (S)$ (3)

$D = Log(N)/Log(S)$ (4)

Figure 2



Finally, the fractal dimension found using this method is equal to the roughness of the border. Using equation 4, which is the slope of the relationship between the log of the number of boxes and the log of the scaling factor, we will find our fractal dimension.

Test Cases

To make sure that our code is executing properly, there are 3 ways that we can test our code. First, we will test our code against shapes with non-fractal dimensions, for example we can test it against a 2 dimensional shape like a square and our code should return a dimension of 2. Next, we will test it against a fractal shape that is self-similar, meaning it has a known fractal dimension, for example, we will test it against a sierpinski triangle to see if we can get close to the dimension of 1.585. Thirdly, we will test it against a border with a known fractal dimension, in our case we will be testing it against the known border of the UK which should have a fractal dimension of 1.21. Once we know that our code works we can insert the border of Canada and find its roughness.

According to Fractal Foundation, here are some known fractal dimensions of certain borders...
South Africa: D = 1.05
Australia: D = 1.13
Great Britain: D = 1.21
Norway: D = 1.52

Coding Method

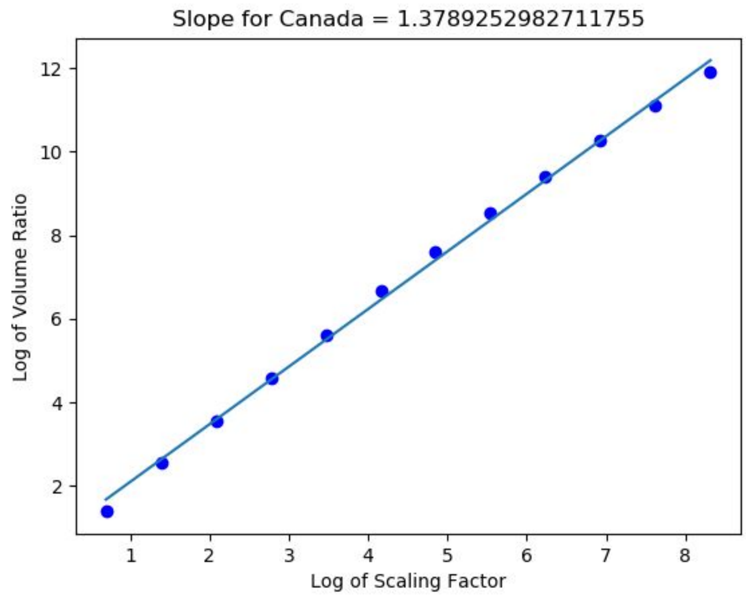
For this project we will be using Python and a program that helps us use and manipulate images is Pillow also known as PIL. Instead of making a virtual grid to see what boxes are being touche by the shapes, we decided it would be easier to go through each pixel of the image and determine whether the pixels were touching the shape., With this program we'll be counting the number of pixels that are touched by our images, whether they be a border or a regular shape. We will determine if the pixel is touching the border by looking at its colour, if the pixel is black then it is touching the border, if the pixel is white it is not touching the border. This may very from image to image, depending on the colour of the image. Once we have that number we will scale our image up and run it through the same loop of counting how many pixels are touched by the image. Since the image is a vector image, when we scale these images up they will not lose resolution which makes our calculated fractal dimension much more accurate as the pixels are touching smooth lines. Since our images get increasing larger every time we scale it up, our code running time will increase as well. 256x256 pixels takes about 1 minute, 2048x2048 pixels takes about 1 hour.

Results

Before we inserted an image of Canada into our program we needed to test it against all of our test cases. When we tested our code, all of our test cases gave us an answer with an error of much less than 1%. This showed us that the code worked and we were prepared to insert our border of Canada.

Shape	Theoretical Fractal Dimension	Experimental Fractal Dimension
Square	2.0	1.999
Sierpinski Triangle	1.585	1.5848
Great Britain’s Border	1.21	1.208
Canada’s Border	-	1.378

The graph to the right is the graph we received when putting the border of Canada through our code. It is not a perfectly straight line because border's are not technically fractal shapes so there is some estimation. Since our other values were so close to the theoretical value we can be confident in saying that the fractal dimension of the border of Canada is 1.378.



Discussion

Since we know that the roughness of a shape is equal to its fractal dimension we can answer our question of how rough is Canada's border, and the answer is the roughness of Canada's border is equal to 1.378. This means that the border of Canada is rougher than that of the border of the UK.this is most likely because of all this islands that make up most of the north of canada. According to the Fractal Foundation "The lower the dimension, the straighter and smoother the coastline. The higher the dimension the more jagged and wiggly the coastline is".

References

1. 3Blue1Brown, "Fractals are typically not self-similar", Published on Jan 27, 2017, Consulted on April 2nd, 2019, https://youtu.be/gB9n2gHsHN4

2. "Fractal Dimension." *Fractal Foundation Online Course - Chapter 1 - FRACTALS IN NATURE*, http://fractalfoundation.org/OFC/OFC-10-3.html

3. "Fun Science: Fractals in Nature and Fractal Measurement." *Vironevaeh*, 23 Nov. 2012, vironevaeh.com/2012/11/23/fun-science-fractals-in-nature-and-fractal-measurement/.