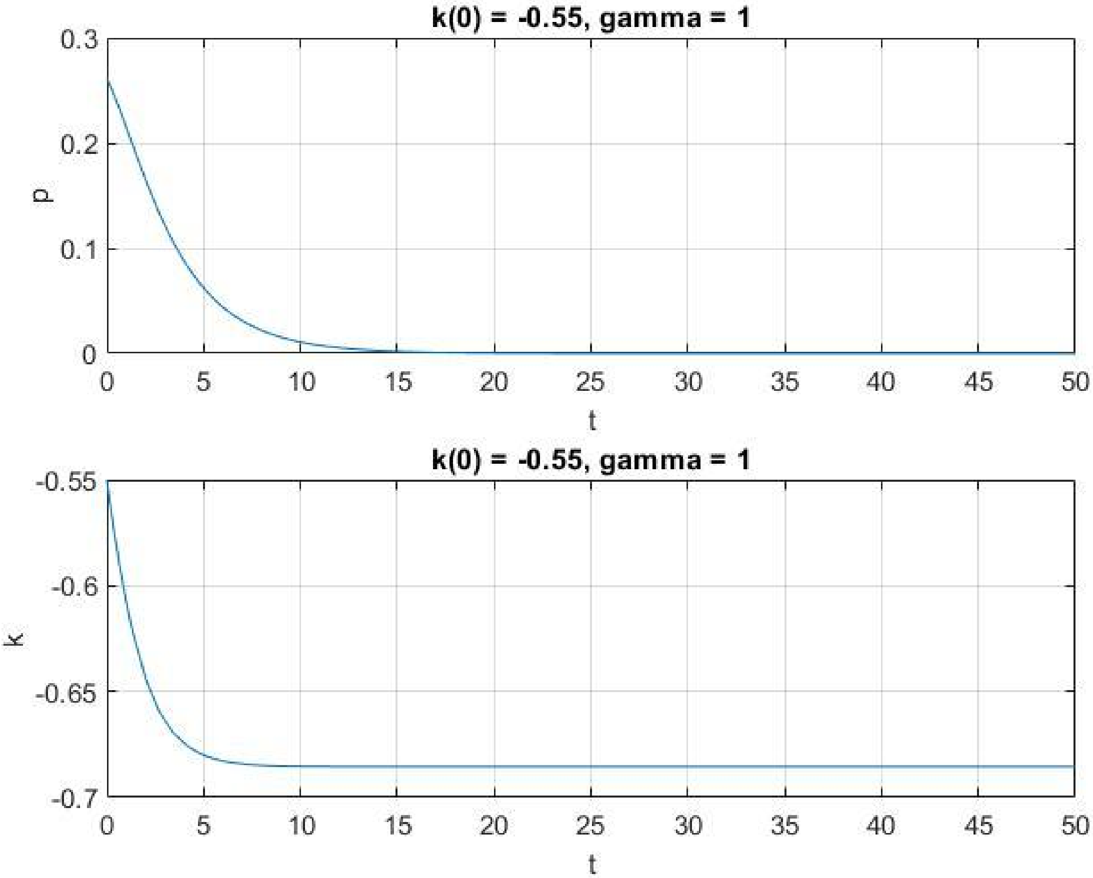
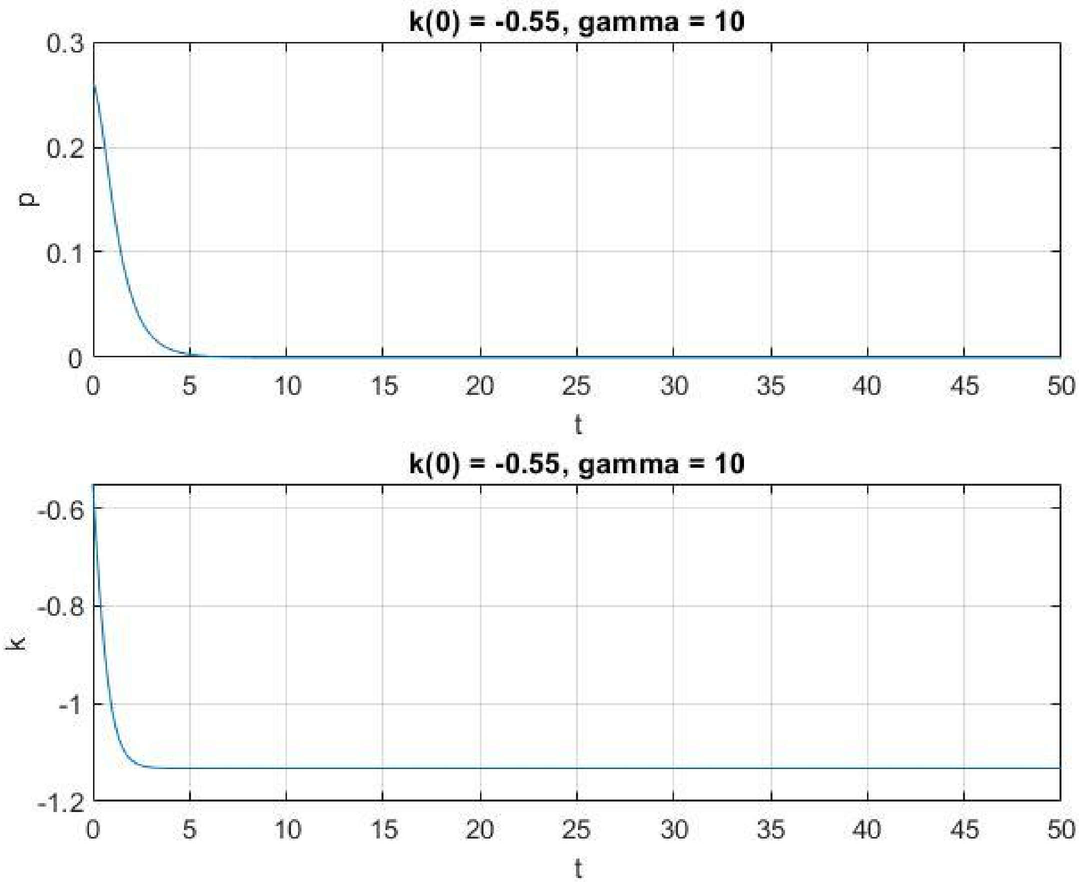
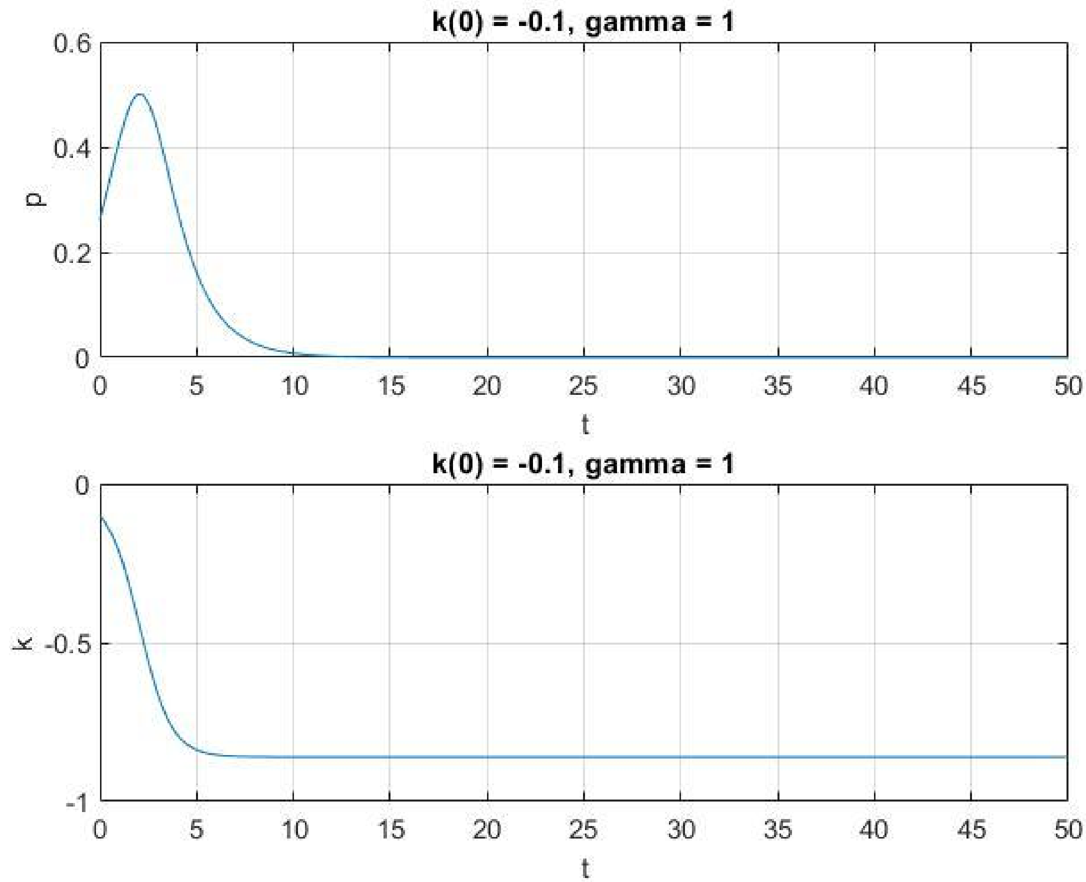
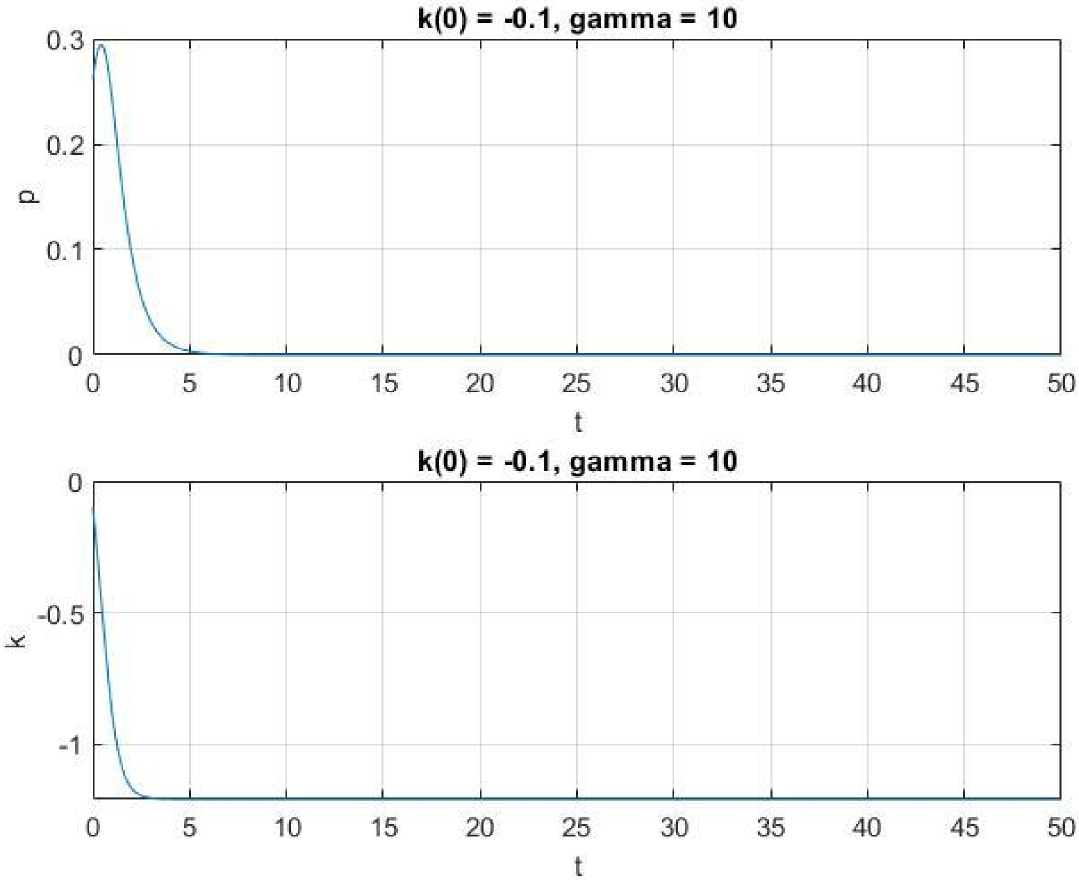
Problem 1


k(0) = -0.55, gamma = 1
0.3
0.2
0.1
0
p
0
5
10
15
20
25
30
35
40
45
50
t
k(0) = -0.55, gamma = 1
-0.55
-0.6
-0.65
-0.7
k
0
5
10
15
20
25
30
35
40
45
50
t


k(0) = -0.55, gamma = 10
p
0.3
0.2
0.1
0
0 5 10 15 20 25 30 35 40 45 50
t
k(0) = -0.55, gamma = 10
k
-0.6
-0.8
-1
-1.2
0 5 10 15 20 25 30 35 40 45 50
t

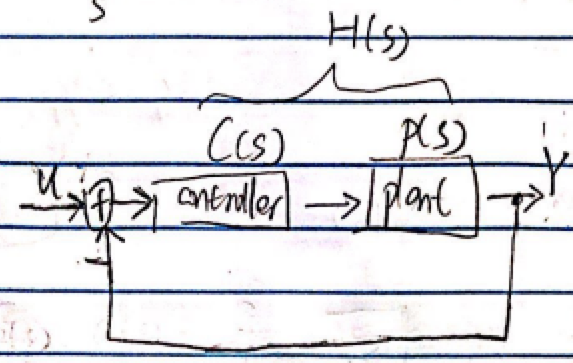


k(0) = -0.1, gamma = 1
p
0.6
0.4
0.2
0
0
5
10
15
20
25
30
35
40
45
50
t
k(0) = -0.1, gamma = 1
k
0
-0.5
-1
0
5
10
15
20
25
30
35
40
45
50
t


k(0) = -0.1, gamma = 10
p
0.3
0.2
0.1
0
0
5
10
15
20
25
30
35
40
45
50
t
k(0) = -0.1, gamma = 10
k
0
-0.5
-1
0
5
10
15
20
25
30
35
40
45
50
t

Problem 2

1. $P(s) = \frac{2}{s^2}$ $\quad C(s) = k + k_d s$



$$H(s) = \frac{C(s) \cdot P(s)}{1 + C(s) \cdot P(s)}$$

$$= \frac{(k + k_d \cdot s) \cdot \frac{2}{s^2}}{1 + (k + k_d s) \cdot \frac{2}{s^2}}$$

$$= \frac{\left(\frac{2k}{s^2} + \frac{2k_d}{s}\right) \cdot s^2}{\left(1 + \frac{2k}{s^2} + \frac{2k_d}{s}\right) \cdot s^2}$$

$$= \frac{2k + 2k_d \cdot s}{s^2 + 2k + 2sk_d}$$

2.

let $x = \begin{bmatrix} v \\ \dot{v} \end{bmatrix}$ $\quad \begin{cases} \dot{x} = Ax + Bu \\ y = Cx + Du \end{cases}$

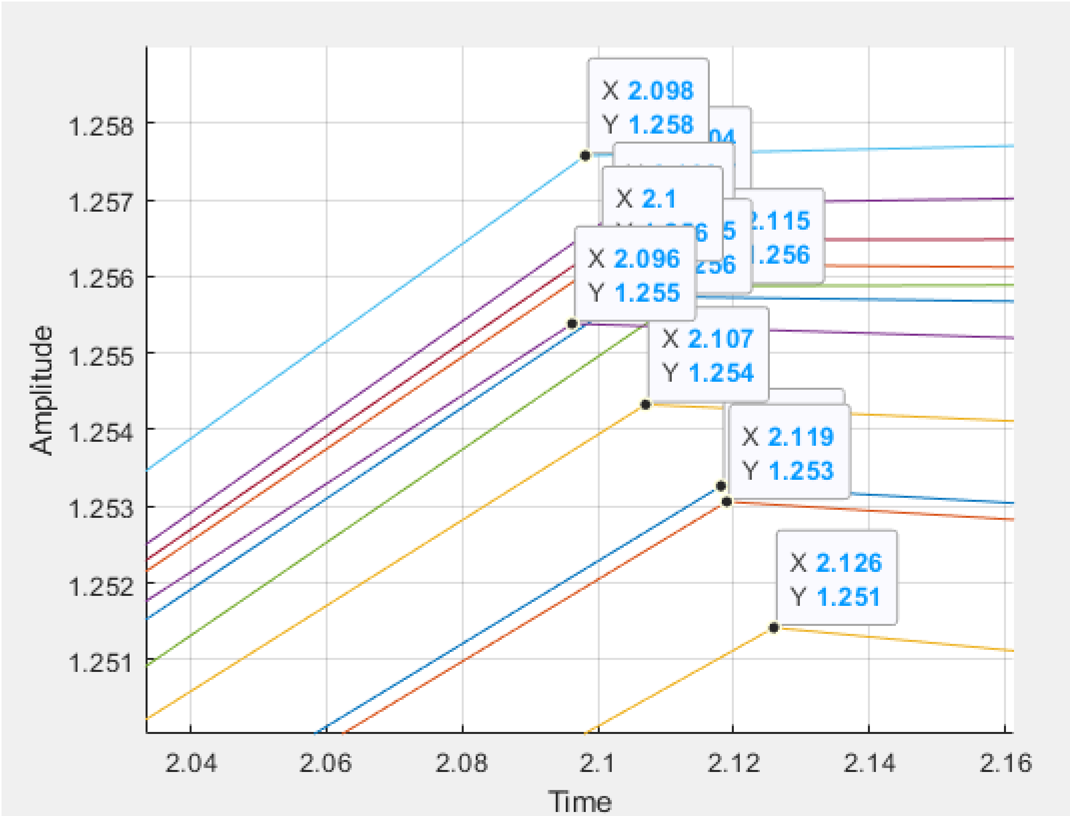
$$H(s) = \frac{Y(s)}{U(s)} = \frac{2k_d s + 2k}{s^2 + 2k_d s + 2k} = \frac{N(s)}{M(s)}$$

$$H(s) = \frac{Y}{V} \cdot \frac{V}{U} \quad \text{let} \begin{cases} \frac{Y}{V} = 2k_d s + 2k & \cdots ① \\ \frac{V}{U} = \frac{1}{s^2 + 2k_d s + 2k} & \cdots ② \end{cases}$$

From ② we get $u = \ddot{v} + 2k_d \dot{v} + 2kv$

$$\begin{cases} y = 2k_d \dot{v} + 2kv = \begin{bmatrix} 2k & 2k_d \end{bmatrix} x \\ \dot{x} = \begin{bmatrix} \dot{v} \\ \ddot{v} \end{bmatrix} = A \begin{bmatrix} v \\ \dot{v} \end{bmatrix} + B \cdot u = \begin{bmatrix} 0 & 1 \\ -2k & -2k_d \end{bmatrix} \begin{bmatrix} v \\ \dot{v} \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u. \end{cases}$$

We found K = 0.565, Kd = 0.625 after a round of trial and errors as the graph below shows:



At this K and Kd value, the system's peak time is 2.096 seconds and 25.5% overshoot.
Here's the code:

Function definition:

```
function xdot = f(~, x)
u = heaviside(sym(1));

K = 0.575;
Kd = 0.625;

xdot = zeros(2,1);

xdot(1) = x(2);
xdot(2) = u - 2*K*x(1) - 2*Kd* x(2);
end
```

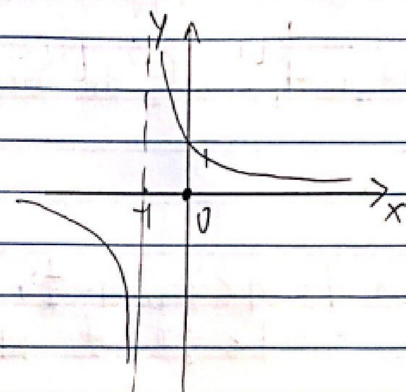
Main code:

```
K = 0.575;
Kd = 0.625;

tspan = [0, 20];
```

```
x0 = [0; 0];

[t,x] = ode45( @q2func, tspan, x0);

figure(1); hold on;
y = 2*K*x(:,1) + 2*Kd*x(:,2);

plot(t, y);
xlabel('Time');
ylabel('Amplitude');
grid on;
```

Problem 3

$f(x) = \frac{1}{x+1}$



Given by the problem, for $x \in [0, \infty)$, $|f(x) - f(y)| \leq L\,|x-y|$, $L \geq \frac{|f(x)-f(y)|}{|x-y|}$

as y gets infinitely close to 0 as x goes to $+\infty$. we can tell $L=1$

Problem 4

$|(f_1(x) - f_1(y)) + (f_2(x) - f_2(y))| = |(f_1(x) + f_2(x)) - (f_1(y) + f_2(y))|$

$\leq |f_1(x) - f_1(y)| + |f_2(x) - f_2(y)| \leq L_1\,|x-y| + L_2\,|x-y| = (L_1 + L_2)\,|x-y|$

There fore, we proved above that if $f_1 + f_2$ is locally Lipschitz, then f_1 and f_2 are locally Lipschitz.

Problem 5

Let $|f_1(x) - f_1(y)| \le L_1 |x-y|$, $|f_2(x) - f_2(y)| \le L_2 |x-y|$

Given $f_1 \circ f_2(x) = f_1(f_2(x))$

we have $|f_1 \circ f_2(x) - f_1 \circ f_2(y)|$

$= |f_1(f_2(x)) - f_1(f_2(y))| \le L_1 |f_2(x) - f_2(y)|$

since $|f_2(x) - f_2(y)| \le L_2 |x-y|$

we have $|f_1 \circ f_2(x) - f_1 \circ f_2(y)| \le L_1 \cdot L_2 |x-y|$

Hence, we proved if f_1 and f_2 are locally Lipschitz $f_1 \circ f_2$ is locally Lipschitz

Problem 6

a) Given $|\sin(x)| < |x| < 0.\pi$, $|x| < \frac{1}{2}\pi$
we can deduce $|\sin(x)| < \frac{1}{2}\pi$ if $|x| < \frac{1}{2}\pi$

since $\lim_{x \to 0} |x| = 0$. we can also prove $\lim_{x \to 0} |\sin(x)| = 0$

Hence, the function is continuous at 0.

b) Using trig identity $\cos(2x) = 1 - 2\sin^2 x$

$\lim_{x \to 0} \cos(2x) = 1 - 2 \cdot \lim_{x \to 0} \sin(x) \lim_{x \to 0} \sin(x)$

From part a, we know $\lim_{x \to 0} |\sin(x)| = 0$ $\begin{cases} \lim_{x \to 0^+} \sin(x) = 0 \\ \lim_{x \to 0^-} \sin(x) = 0 \end{cases}$

Hence $\lim_{x \to 0} \cos(2x) = 1$, Cosine function is continuous at 0

c. According to trig identity $\sin(x+h) = \sin x \cos h + \cos x \sin h$

$$\lim_{x \to 0} \sin(x+h) = \lim_{x \to 0} (\sin h) = \sin h$$

Hence, sine function is continuous at any real x.