ECE6554: Homework #5

Due: Apr. 01, 2022

Problem 1: Scalar System. [15 pts] Continue with the scalar direct MRAC problem with additive Gaussian noise. Presuming that the measurement noise was zero mean and 0.25 standard deviation, modify the adaptive system to include one of the following strategies: (continuous) deadzone, σ-, or e-modification. Make sure to plot all important states. Compare with the unmodified version, in terms of outcomes and behavior.

Note: Measurement noise means that you do not use the true state x, but rather a *measured* version of the state x with additive noise, $x_m = x + \epsilon$, just like in the earlier parameter drift homework problem.

Problem 2: Scalar System. [15 pts] Continue with the scalar direct MRAC problem with additive Gaussian measurement noise. Presuming that the noise was zero mean and 0.25 standard deviation, modify the adaptive system to include the Projection Operator in the adaptive law to prevent parameter drift. Please let me know what adaptive and projection parameter settings you used. Comment on this solution versus the unmodified version and the x-modified version (deadzone, σ-mod, or e-mod).

Problem 3: PerfSpec. [15 pts] In prior homeworks, you were asked to solve a control synthesis problem for the following control system using error feedback,

$$P(s) = \frac{2}{s^2},$$
$$C(s) = K + K_d s.$$

as well as derive an adaptive controller when the true system plant was actually

$$P(s) = \frac{2}{s(s-0.5)}, \tag{1}$$

Using the MRAC design from the previous homework, add noise to the measurements $x_{\text{meas}}(t) = x(t) + d(t)$, and compensate for the noise by using a properly compensating adaptive update law of your choice (σ-mod, e-mod, or projection operator). Compare the results to a step input and to a sinusoidal input for the cases without noise and with noise (in the measurement).

Problem 4: Scalar System. [15 pts] Read up on the section covering Composite Adaptation, which will not be covered in lecture due to there being sufficient foundation to not require it. Redo the scalar, direct MRAC problem from earlier homeworks using Composite Adaptation; for both the step and the sinusoidal reference signals. Play around with the setup and discuss the results; especially as regards the transient behavior which is what composite adaptation is supposed to improve. Comment on the convergence of the full composite adaptive system (state + adaptive parameters) versus the standard D-MRAC adaptive controller.

File: A set of notes for Composite Adaptation should be linked with this assignment.

Note: In Prof. Hovakimyan's notes, the Lyapunov analysis shows that an improved rate of convergence may be obtained for Composite Adaptation. Other researcher's have also argued that it provides improved parameter convergence. In fact, a recent paper in IEEE TAC has employed composite adaptation ideas for parameter convergence without requiring PE. Usually papers that ensure convergence without requiring PE prove that the Lyapunov rate is negative definite, or becomes so under some condition, and therefore no longer require excitation to be persistent (in the case of *always* holding).

Problem 5: Indirect MRAC. [18 pts] Direct MRAC has been studied for scalar systems in an earlier homework. Consider instead applying the indirect MRAC method, where the system model is

$$\dot{x}(t) = 3x(t) + 2u(t), \qquad x(0) = 0,$$

and the reference model is

$$\dot{x}_m(t) = -2x_m(t) + 3r(t), \qquad x_m(0) = 0.$$

Track the reference signals $r(t) = 5$ and $r(t) = 4\sin(3t)$. Assume that your estimate of the plant model is

$$\dot{\hat{x}}(t) = \hat{x}(t) + 2u(t), \qquad x(0) = 0,$$

and use these parameters to define the initial conditions for the adaptation parameters $\hat{a}(0)$ and $\hat{b}(0)$. For uniformity, we should all use $\underline{b} = 0.75$.

Play around with the adaptation gains and provide some observations as to the resulting behavior. In particular, what have you observed regarding the intial transient behavior and the final tracking state of the entire adaptive system?

Problem 6: PWMN. [10 pts] Let's revisit linear system of the CARE problem, but with a slightly different control effectiveness,

$$A = \begin{bmatrix} 1 & 3 \\ -1 & 2 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 0 \\ 2 \end{bmatrix}. \tag{2}$$

This homework will explore an alternative control method compared to the LQR approach. The LQR approach uses state feedback and an infinite time horizon to synthesize the stabilizing policy. An alternative approach is to use point-wise minimum norm control laws. These are control laws that are synthesized to meet a given Lyapunov convergence rate, at that instant, through the use of control Lyapunov function methods. This homework problem will compare the two strategies in preparation for their conversion to adaptive control laws. Use the same initial conditions as beforee, $x_0 = (1.5, 2)^T$.

Stabilization of the system should use the following positive definite matrix

$$Q = \begin{bmatrix} 4.5 & 2.8 \\ 2.8 & 2.5 \end{bmatrix} \tag{3}$$

and its associated P as determined by Matlab's CARE solver (`care`). Use the P, Q pair to determine the LQR gain K_{LQR} and feedback control law u_{LQR}, as well as to determine the piece-wise minimum norm control law u_{PWMN}. Run the PWMN controller with $\epsilon \in \{0.25, 0.5, 1, 1.5\}$.

a) Provide plots of the state convergence for $\epsilon = 1$ (e.g., plots of x versus t).

b) Provide plots of the control signal versus time for all ϵ tested and for the LQR controller. These should be on different plots for the various ϵ. It is OK if the LQR controller is plotted together with that one, as its outcome provides a means to compare the other outcomes.

c) Provide plots of V versus t for all ϵ tested and for the LQR controller. These can most likely be on the same plot.

d) Provide plots of $\dot{V}$ versus t for all ϵ tested and for the LQR controller. In each plot include the upper bound convergence (versus time) rate as determined by the Lyapunov derived strategy. Each controller result should probably be on a different plot to visualize the difference.

e) Plot $\log(V(t))$ versus t to show that the convergence rate actually changes as a function of ϵ. For sure plot the LQR controller result too. Exponentially convergence should have a linear decrease in time for $\log(V)$, though it may be faster on ocassion.

f) Report the L_2 norm of the control effort for all of the controllers (LQR plus different ϵ). Probably best if you do so in a table.

g) Comment on the control law for the various ϵ. How does it vary as ϵ varies. Why does the behavior you see happen? Explain why the $\log(V)$ curves make sense.

File: Linked with the homework, there should be a code stub file. It helps provide consistent results for evaluation. You can either figure out how to save the outcomes for the requested plots in order to plot them together, or judiciously use Matlab's `hold` on select plots.

Note: Linked in the course wiki *Lecture Notes* section is a short write-up about the PWMN Controller. The PWMN controller is supposedly a continuous controller, but when numerically integrated it can have switching type behavior. Plus, the denominator can get small, which leads to numerical sensitivity in the integrator. These properties mean that numerical integration of PWMN controllers can be dicey and not always work out. The chosen ϵ are known to work. It is best to start with the LQR controller, then work from the least aggressive PWMN controller up to the most aggressive.

Starting with this homework, the last problem will be associated to the project efforts/work. The activity is to be done on your own or with one other person. No discussion with others. Solutions will not be posted as you should have the basic foundation to follow through on the activities without them, plus this is to simulate working on your own through the entire sequence. The deliverables attached to the homework are to ensure consistent progress.

Problem 7: Project. [25 pts] Pick one of the canned projects and work out Step #1. That will involve implementing the baseline linear controller for it, plus the associated baseline adaptive controller for the linear system. Provide the same type of plots as in the homeworks and solutions. They should show that the adaptive controller is better able to meet the performanec specifications under an incorrect plant model. There might be some follow-up work related to the nonlinear system. Address the requested action items and information.

If you are working on a custom problem, then perform the equivalent first pass as above.