5.1

Let CFG G_0 be $G_0 = (V, \Sigma, R, S)$

$S \to cS \mid \varepsilon$ for c in Σ

Assume G_0 is decidable by Turing machine T_1. To make a contradiction we need to generate another turing machine T_2 that uses R of G_0 to decide ALL_{CFG} by running R on $\langle G_0, G_1 \rangle$. G_1 is another CFG generate Σ^*

if R accept, Accept grammar, else reject.

if T_2 deicde ALL_{CFG}, then it is impossible that T_1 decides EQ_{CFG}

5.2

To prove co-Turing-recognizable, we need to prove the complement of $\overline{EQ}_{CFG}$ is also Turning recognizable language.

Since Turing recognizable languages are closed under union, $\overline{EQ}_{CFG}$ is Turing recognizable. language.

Let $G_1 = L(G_1)$ $G_2 \neq L(G_2)$

Let turning machine that recognize $\overline{EQ}_{CFG}$ be M

To prove co-turing recognizable, we first test whether both G_1, G_2 are valid CFG. if not accept. ~~E~~lse, transform them to CNF and check any of G_1' or G_2' produces s on Σ^*. if yes, accept. Repeat.

At the end, $\overline{EQ}$ is proven as co-Turing-recognizable language

5.3

5.3

$$\left\{ \frac{aa}{a}, \frac{aa}{a}, \frac{b}{a}, \frac{ab}{abab} \right\}$$

$$\begin{array}{cccc} 4 & 4 & 2 & 1 \end{array}$$

Top $= aa + aa + b + ab = aaaabab$

bot $= a + a + a + abab = aaaabab$

So here is a match.

5.4

Assume that

$A = \{a^n \mid n \geq 0\}$ $B = \{a\}$

$A \leq_m B$

However B is finite so it is regular. while A is not finite.

therefore A is not regular

5.9

By theorem 4.11

$L = \{(w, M) : w \text{ is accepted by } M\}$

is undecidable

There must be a TM decides T

We need to prove L can be reduced to T.

5.15

$L = \{ \langle M, w \rangle \mid M$ moves its head left $\}$

Build a turing machine

$T = \langle M, w \rangle$

1. Run w + num of states + 1 steps

2. If head move to left accept
   else reject

There exist a turing machine

so it is decidable

5.17

Build a turing machine:

1. Check whether there are dominos with the same top and bot values. if so, accept.

2. check whether there is a domino is a multiple of another domino. if so, accept.

5.19

$$\left\{ \left[\frac{a_1}{b_1}, \frac{a_2}{b_2} \dots \frac{a_n}{b_n} \right] \right\}$$

Because it is silly post correspondence all as and bs are in the same length.

If a match is found, top length will be similar to bot length.

Therefore, it is easy to check whether these strings are the same.

If they are, it is decidable. while dominos are as deciders to find whether top and bot are the same