Exploring Non-Linearity and Dimensionality Expansion: Addressing the XOR Problem

Summary of Excercise:

The excercise below demonstrate the XOR problem and how it can be separated by a line in higher dimensions. The XOR problem is a classic example of a non-linearly separable problem in two dimensions, but it becomes linearly separable in higher dimensions.

In a 2D space, the XOR problem looks like this:

Input	Output
0 0	0
0 1	1
1 0	1
1 1	0

In 2D space, there's no single straight line that can separate the 0s from the 1s. However, if you map these points into a 3D space, you can find a plane that separates them. One way to do this is by introducing a third dimension based on some non-linear transformation.

Let's consider a transformation φ(x1, x2) = (x1, x2, x1 * x2), where (x1, x2) are the input features. Now, the transformed data looks like this:

Input	Transformed Input	Output
0 0	0 0 0	0
0 1	0 1 0	1
1 0	1 0 0	1
1 1	1 1 1	0

Visualization using Python

```
import matplotlib.pyplot as plt
from mpl_toolkits.mplot3d import Axes3D
import numpy as np

data = np.array([[0, 0, 0],
                 [0, 1, 1],
                 [1, 0, 1],
                 [1, 1, 0]])


X = data[:, :2]
y = data[:, 2]

# Plot in 2D
plt.figure(figsize=(10, 5))

plt.subplot(121)
plt.scatter(X[y == 0][:, 0], X[y == 0][:, 1], color='r', marker='o', label='0')
plt.scatter(X[y == 1][:, 0], X[y == 1][:, 1], color='b', marker='^', label='1')
plt.title('2D Plot')
plt.xlabel('X1')
plt.ylabel('X2')
plt.legend()

# Plot in 3D
ax = plt.subplot(122, projection='3d')
ax.scatter(X[y == 0][:, 0], X[y == 0][:, 1], y[y == 0], color='r', marker='o', label='0'
ax.scatter(X[y == 1][:, 0], X[y == 1][:, 1], y[y == 1], color='b', marker='^', label='1'
ax.set_title('3D Plot')
ax.set_xlabel('X1')
ax.set_ylabel('X2')
```

```
32 ax.set_zlabel('X1 * X2')
33 ax.legend()
34
35 plt.tight_layout()
36 plt.show()
37
```