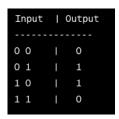
Exploring Non-Linearity and Dimensionality Expansion: Addressing the XOR Problem

Summary of Excercise:

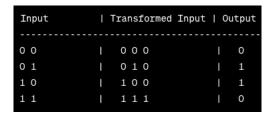
The excercise below demonstrate the XOR problem and how it can be separated by a line in higher dimensions. The XOR problem is a classic example of a non-linearly separable problem in two dimensions, but it becomes linearly separable in higher dimensions.

In a 2D space, the XOR problem looks like this:



In 2D space, there's no single straight line that can separate the 0s from the 1s. However, if you map these points into a 3D space, you can find a plane that separates them. One way to do this is by introducing a third dimension based on some non-linear transformation.

Let's consider a transformation $\varphi(x1, x2) = (x1, x2, x1 * x2)$, where (x1, x2) are the input features. Now, the transformed data looks like this:



Visualization using Python

```
1 import matplotlib.pyplot as plt
 2 from mpl_toolkits.mplot3d import Axes3D
3 import numpy as np
5 \text{ data} = \text{np.array}([[0, 0, 0],
                    [0, 1, 1],
6
7
                    [1, 0, 1],
8
                    [1, 1, 0]])
10
11 X = data[:, :2]
12 y = data[:, 2]
13
14 # Plot in 2D
15 plt.figure(figsize=(10, 5))
16
17 plt.subplot(121)
18 plt.scatter(X[y == 0][:, 0], X[y == 0][:, 1], color='r', marker='o', label='0')
19 plt.scatter(X[y == 1][:, 0], X[y == 1][:, 1], color='b', marker='^', label='1')
20 plt.title('2D Plot')
21 plt.xlabel('X1')
22 plt.ylabel('X2')
23 plt.legend()
25 # Plot in 3D
26 ax = plt.subplot(122, projection='3d')
27 ax.scatter(X[y == 0][:, 0], X[y == 0][:, 1], y[y == 0], color='r', marker='o', label='0'
28 ax.scatter(X[y == 1][:, 0], X[y == 1][:, 1], y[y == 1], color='b', marker='^', label='1'
29 ax.set_title('3D Plot')
30 ax.set_xlabel('X1')
31 ax.set_ylabel('X2')
```

```
32 ax.set_zlabel('X1 * X2')
33 ax.legend()
34
35 plt.tight_layout()
36 plt.show()
37
```

