Quiz 6: Section 8.1 Practice Problems. **Name:**_________Xiangbo Meng_________________

10 points possible

Due date: Wednesday, 7/31, by 11:59 p.m.

Please show your **work** to solve the problem. Please organize your work as clearly as possible. **Clearly circle, box, mark, star, or highlight** the final answer in some way and include the random variable's support.

Hint: Pictures of the region of interest are helpful!

Worked out solutions provided at my YouTube channel at https://www.youtube.com/user/RoseHulmanOnline.

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1. An environmental engineer measures the amount (by weight) of particulate pollution in air samples of a given volume collected over the smokestack of a coal-operated power plant. X denotes the amount of pollutant per sample collected when a cleaning device on the stack is not in operation, and Y denotes the same amount when the cleaning device is operating. It is known that the joint probability density function of X and Y is:

$$f(x, y) = 1,\ 0 \le x \le 2,\ 0 \le y \le 1,\ x > 2y$$

(a) [+1] Determine the marginal density function for X, the amount of pollutant per sample collected when the cleaning device is not in operation. Set up the appropriate expression and evaluate it. Please include the marginal density function's support. Hint 1: Draw the region of interest first! Hint 2: Make sure the marginal density function integrates to 1 over its support.

Solution:

$f_x(x) = \int_0^{\frac{x}{2}} 1\, dy =$ **x/2 (0<x<2)**

(b) [+1] Determine the probability that the amount of pollutant with the cleaning device in operation is at most 1/3 of the amount without the cleaning device in operation. That is, determine $P(Y \le (1/3)X)$

Solution:

P(y<= 1/3 X) = $\int_0^2 \int_0^{\frac{x}{3}} 1\, dydx =$ **2/3**

2. A bank operates both a drive-up facility and a walk-up window. On a randomly selected day, let X be the proportion of time that the drive-up facility is in use (at least one customer is being served or waiting to be served) and Y be the proportion of time that the walk-up window is in use. Suppose the joint probability density function of X and Y is given by:

$$f(x, y) = \begin{cases} \frac{6}{5}(x + y^2) & 0 \le x \le 1, 0 \le y \le 1 \\ 0 & else \end{cases}$$

For each of these – set up the appropriate expressions and evaluate them.

(a) [+0.5] Determine the probability that neither facility is busy more than one-quarter of the time.

Solution: $\int_0^{\frac{1}{4}} \int_0^{\frac{1}{4}} \frac{6}{5}(x + y^2) dydx = 0.0109$

(b) [+0.5] For what proportion of the time is the walk-up busier than the drive-up?

Solution: the walk-up busier than the drive-up, mean that Y>X.
$\int_0^1 \int_0^y \frac{6}{5}(x + y^2)dxdy$ = **0.5**

(c) [+1] What's the expected proportion of time that the walk-up window is in use?

Solution: we are trying to solve for E(Y)
F(y) = $\int_0^1 \frac{6}{5}(x + y^2)dx$ = 0.6+1.2*y^2
E(x) = int((0.6+1.2*y^2)*y, y = 0..1) = **0.6**

3. Let X and Y have joint probability density function:

$$f(x,y) = \frac{1}{18} \qquad 0 < x < 3, \qquad x^2 < y < 9.$$

Please set up and evaluate the following.

(a) [+0.5] Determine $E(X)$.

f(x) = int(1/18, y = x^2..9) = ½ - 1/18 *x^2
E(x) = int((½ - 1/18 **x^2)**x, x = 0..3) = **9/8**

(b) [+0.5] Determine $E(Y)$.
f(y) = int(1/18, x = 0..sqrt(y)) = sqrt(y)/18
E(y) = int(sqrt(y)/18 * y, y = 0..9) = **27/5**

(c) [+1] Determine $E(X + Y)$.
E(x)+E(y) = 9/8 + 27/5= **261/40**

(d) [+1] Determine $E(XY)$. Hint: If X and Y are independent, then the following true: $E(XY) = E(X)\cdot E(Y)$. For this example, X and Y are not independent.

E(XY) = int(int((1/18)*x*y, y = x^2 .. 9), x = 0 .. 3) = 27/4

4. An advisor looks over the schedules of his 50 advisees to see how many math and science courses each has registered for in fall quarter. He summarizes his results in a table

		Y		
		0	1	2
	0	11	6	4
X	1	9	10	3
	2	5	0	2

where *X* is the number of science courses and *Y* is the number of math courses.

(a) [+0.5] What is the probability that one of his advisees selected at random is taking 2 math courses next quarter?
Solution: P(Y = 2) = (4+3+2)/50 = 9/50

(b) [+0.5] What is the probability that an advisee selected at random is taking 1 science course next quarter?
Solution: P(X =1) = 9/50

(c) [+0.5] What is the probability that an advisee selected at random is taking 2 math courses and 1 science course?
Solution: P(X = 1, Y= 2) = 3/50

(d) [+0.5] What's the probability that an advisee selected at random has registered for more math courses than science courses?
Solution: P(Y>X) = (6+4+3)/50 = 13/50

(e) [+1] Determine the marginal density function for each *X* (science) and *Y* (math) – piecewise defined is fine! Write them both below.
Solution:
P(x):
P(x = 0) = 21/50
P(x= 1) = 22/50
P(x = 2) = 7/50
P(y):
P(y=0) = 25/50
P(y=1) = 16/50
P(y=2) = 9/50

Bonus. [+1] You can determine the answer to the following bonus problem without integrating, but by using geometric probability. ***Geometric probability*** is discussed in the text in Section 8.1.

Geometric Probability. Let S be a bounded region in the plane, and let R be a region inside S. When a point is said to be randomly selected from S, then the probability that it is in the region R is (area of R)/(area of S).

Problem: Lindsey and Cody agree to meet at Java Haute for a birthday meal "sometime around 12:30." But neither of them is punctual – or patient. What will actually happen is that each will arrive at random sometime in the interval from 12:00 to 1:00. If one arrives and the other is not there, the first person will wait 15 minutes or until 1:00, whichever comes first, and then leave. What is the probability that the two will actually meet?

Hint: Assuming their arrival times are independent of each other, $f(x, y) = \frac{1}{3600}$ for $0 \leq x \leq 60$, $0 \leq y \leq 60$, where X is the number of minutes after noon that Cody arrives and Y is the number of minutes after noon that Lindsey arrives.

Happy Birthday Lindsey! (7/30)
Solution: