Homework Set 2 **NAME:**____

MA 381, Probability, Summer 2013

15 points possible; each problem is worth 1 point

Submission: Submit one single pdf file with all of your work into Homework Set 2 Drop Box. Please name the document as HW2_LastName; e.g. HW2_Evans.

Due date: Tuesday, 6/11 at 11:59 p.m. in the Homework Set 1 Drop Box
Coverage: Sections 2.3, 2.4, 3.1, 3.2, 3.3

Please write or type your answers in the open spaces provided on the following pages.

- You may work together on these problems, but **what you turn in must be your own write up and work**. Do not directly copy work from another source, which includes another student's paper! Points will be deducted if copying is obviously noticeable or discovered.
- Please **show as much work as possible**, where work may include words, graphs, diagrams, Maple code, etc. You need something written to show how you are solving a problem. Please organize your work as clearly as possible. It's much easier to give partial credit when the work is all available, especially writing down the expressions you are working with, e.g., integrals.
- Clearly **circle, box, star, or highlight** your final answer.

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1. In how many ways can 6 A's, 5 B+'s, 10 B's, 2 C+'s, and 2 C's be randomly assigned to this Summer MA 381 class of 25 students?

Solution: So we are trying to fine the number of permutations of 6 A's, 5 B+'s, 10 B's, 2 C+'s, and 2 C's. The number of permutation is $\frac{25!}{6!*5!*10!*2!*2!}$.

2. Seven letters are randomly selected without replacement from the English alphabet. What is the probability that "middle letter" or "median letter" (in terms of ordering from A to Z) chosen is M?

Solution: the total combination is $\binom{26}{7}$. In order to make M as the middle letter we have to choose 3 from the other 25 letters so it will be $\binom{25}{3}$ and choose 3 from the remaining 22 letters so it will be $\binom{22}{3}$. So the total possible combination is $\binom{25}{3} * \binom{22}{3}$. And the probability is $\frac{\binom{12}{3}*\binom{13}{3}}{\binom{26}{7}}$.

3. From a group of 5 different women and 7 different men, how many different committees consisting of 2 women and 3 men can be formed if 2 of the men are feuding and refuse to serve on the committee together? [Been there, saw that.]

Solution: there are totally $\binom{5}{2}$ ways to choose wemen. There are totally $\binom{7}{3}$ ways to choose men. However there are two guys don't want to work together so if the two guys are already in the 3 men we choose, there are 4 other choices for the other guy. So there are 5 cases are not avalible. So the total valid cases is $\binom{5}{2} * (\binom{7}{3} - 5) =$ 300

4. The integers 1, 2, 3, ..., 9 are arranged in a row, resulting in a 9-digit number. What is the probability that the resulting 9-digit integer is odd AND the digits 6 and 4 are next to each other (in either order)?

Solution: the total arrangement is 9!. Last digit has to be a odd number so last digit has 5 choices and second last digit has 8 third last has 7 … there are totally 5*8! arrangements are odd. And 6 and 4 cannot be on the last digit so we can treat the first 8 digit as a permutation of the remaining 8 numbers. If 6 and 4 are adjacent, we can treat them as a number so there are 7! permutations, as it can be 64 and 46 so there are totally 4 * 7! permutations. So there are totally 5*2*7!numbers are odd and 6, 4 are adjacent. so the probability is (5*2*7!)/9!

BONUS. [+1] What is the probability that the resulting 9-digit integer is divisible by 3?
Solution: the sum of the resulting 9 digits are 1+2+3+4+5+…+9 = 45. 45 is divisible by 3 so the resulting 9 digit number is always divisible by 3 so the probability is 1.

5. In drawing seven cards ***with replacement*** from an ordinary deck of 52 cards, what is the probability of getting three aces of spades, two queens of hearts, and two kings of clubs?

There are totally 52^7 ways of drawing 7 cards with replacement. Are totally $\frac{7!}{3!*2!*2!}$ ways to have three aces of spades, two queens of hearts, and two kings of clubs in the 7 cards that are drawn. So the probability is $\frac{7!}{3!*2!*2!}/52^7$

6. Heidi Isaia (Dr. Isaia's wife) wants to give her son, Stefan, 10 different Angry Bird Collector Cards within a 7-day period. If Heidi gives Stefan cards once a day, in how many ways can this be done? Assume that there can be days when he gets no cards or even that he could get all of the cards on one day. That is, on any given day, he can receive from 0 to 10 cards.

Hint 1: There is an example in the textbook in this section that's similar to this problem.
Hint 2: Try an example with fewer cards – perhaps start with 2 cards, then 3 cards, etc.

Solution: suppose x_1 to x_7 represent the number of cards that has been given to Stefan. So we have equation $x_1 + x_2 + x_3 + \ldots + x_7 = 10$. So the total number of ways to arrange x_1 to x_7 are $\binom{10+7-1}{10} = \binom{16}{10}$.

7. There are *N* types of drugs sold at CVS to help with sinus congestion. A random sample of *n* of these drugs is taken with replacement. What is the probability that Brand A, Advil Sinus, is included?

Solution: there is totally $\binom{N}{n}$ ways of choosing n drugs from total N types. Number of cases that A is not in n is $\binom{N-1}{n}$ (N-1 represent all the type of drug except for A) so the number of way to choose n drugs from N and A is in n. is $\binom{N}{n} - \binom{N-1}{n}$ so the probability is $\frac{\binom{N}{n}-\binom{N-1}{n}}{\binom{N}{n}}$.

8. A window dresser has decided to display 10 different dresses in a circular arrangement on a clothes rack. How many unique displays are possible? Hint: Look at a smaller example to get the idea.

Solution: there are totally10! Permutation however, every kind has 10 permutaion considered as the same rotation. So there are totally 10!/10 = 9!

9. In last summer's MA 381 class, I had five students, including Victor and Brandon. If the students arrived in random order, what is the probability that Victor did *not* arrive immediately after Brandon? (For example, Student 1, Student 2, Victor, Student 3, Brandon)

Solution: 1 represent student 1, 2 represent student 2, 3 represent student 3, V represent Victor, B represent Brandon. There are totally 5! Olders. We can treat BV as one person so there are 4! number of permutation that Victor arrived immediately after Brandon. So there are 5! – 4! = 4*4! Number of cases Victor did *not* arrive immediately after Brandon. So the probability is 4*4!/5! = 4/5;

10. A 3-digit number is selected at random, such as 952, or 124, or 809. What is the probability that its ones place is less than its tens place and its tens place is less than its hundreds place, such as 952, or 531, or 640?

Note: The first digit of the 3-digit number must be non-zero.

There are totally 9*10*10 = 900 3-digit numbers. We can randomly choose any three numbers and rearrange it from large to small. So there are $\binom{10}{3} = 120$. The probability is 120/900 = 2/15.

11. In throwing two fair dice, what is the probability of an absolute value difference of 1 **given** the two dice land on different values?

Solution: if the value of the first die is larger than second die. First die can be 1,2,3,4,5 and the second die will be 2,3,4,5,6, there are 5 cases. If the first die is smaller than second die, first die can be 2,3,4,5,6 and second die will be 1,2,3,4,5 so there are 5 cases. There are totally 10 cases. Total combination 6*6 = 36. So the probability is 10/（36－6） = 1 / 3.

12. Let's consider Rose-Hulman as being made up of 3 mutually exclusive and complementary groups: Students, Staff, and Faculty. In statistical surveys at Rose-Hulman where individuals are randomly selected and are asked questions, experience has shown that only 25% of students, 18% of staff, and 12% of faculty will respond. I'm planning on sending a questionnaire to a group of randomly selected individuals from Rose about their understanding of Black Belts in Six Sigma. If 70% of the Rose population is students and 20% is staff, what percent will answer my questionnaire?

Solution: 70% * 25% + 20% * 18% + 10%*12% = 22.3%

13. In a trial, the judge is 60% sure that Waldo, the accused, has committed a crime. Sponge Bob is a witness who knows whether Waldo is innocent or guilty. However, Sponge Bob is Waldo's close friend (because Sponge Bob can always find Waldo whenever he's hiding) and will lie with probability 0.75 if Waldo is guilty. He will tell the truth if she is innocent. What is the probability that Sponge Bob will commit perjury?

Solution: A: Sponge Bob will lie if Waldo is guilty.
B : Waldo commit a crime.
P(A|B) = P(AB)/ P(B)
We are trying to get P(AB) = P(A|B)*P(B) = 0.75*60% = 0.45.

14. Exits A (staircase to Crapo/Moench parking lot), B (staircase to Hulman Union), and C (Greenhouse in G219) are the only escape routes from Crapo. Diligently kept Crapo records show that in a blackout, of the students who try escape, 50% used Exit A, 30% used Exit B, and 20% used Exit C. These records also show that 80% of those who tried to escape via A, 75% of those who tried to escape via B, and 92% of those who tried to escape via C got trapped. What is the probability that a student succeeds in escaping Crapo in a blackout?

Solution: 50%*20% + 30%*25%+20%*8% = 19.1%

15. **Political Problem!** A child called Mittens gets lost in the White House. The child's parents believe that the probability of his being lost in the east wing is 0.35 and in the west wing is 0.65. The security department sends an officer to the east and an officer to the west to look for the child. If the probability that a security officer who is looking in the correct wing finds the child is 0.3, find the probability that the child is found.

Solution: A: the officer find the kid;
B1: the kid is lost in the east wing;
B2: the kid is lost in the west wing;
P(A) = P(A|B1)*P(B1)+P(A|B2)*P(B2) = 0.3*0.35 + 0.3*0.65 = 0.3