MA 381 Exam 1
Summer 2013, Dr. Evans
100 points possible Signature:_______Xiangbo Meng____________________
Time Limit: 2 hours I agree that I have not consulted anyone about this exam besides Dr. Evans.

- There are 12 problems with parts on this exam. The point values are listed next to each problem.
- You will have 2 hours once you download this exam to submit it to the Exam 1 Drop Box. When you download it, make sure you have 2 hours to work on it. I am setting ANGEL to report to me the time you download and upload the exam so that I can make sure your submission is within 2 hours of downloading it. You will be penalized for time over 2 hours.
- You must submit this exam as a single PDF document.
- You may use a calculator, Maple, prepared notes, textbooks, the internet ... basically anything except other people! I am thoroughly checking exams for collaboration. If there are any questions about collaboration on your part, I may require you to take follow up exams and/or the final exam at a proctored location.

- By Rose's Honor Code, I am requiring that the only living people who are allowed to look at your exam are you and me. If you do not follow this rule, you may receive a 0 on this portion of the exam and disciplinary action within and perhaps beyond the math department.
- You can leave your answers in "unreduced" (or combinatorics) form; e.g. $10 \cdot 9 \cdot 8$, or $\binom{5}{2}$, or 6^3.
- Please write scratch work on this exam. It's much easier for me to give you partial credit when I can see your work and thought process for each problem.

	Your Points
Page 1 (22 points)	
Page 2 (22 points)	
Page 3 (22 points)	
Page 4 (24 points)	
Page 5 (10 points)	
BONUS	
Total	

1. [+5] Javad has his Black Belt in Taekwondo (impressive!), and now he wants his Black Belt in Six Sigma as well. Unfortunately, he only has enough time (and money) to take the Six Sigma Black Belt Certification Exam at most 6 times. Let P represent a pass and F a fail. List the sample space S for his successive attempts at achieving his certification. For example, one possible outcome is *FFP*. Note that this sample space contains the possibility that he does not achieve certification.

Solution: S = {P,FP, FFP,FFFP,FFFFP,FFFFFP,FFFFFF}

2. [+6] A deck of four cards consists of two black cards: B1 and B2, and two red cards: R1 and R2. First, Mohammad draws a card at random and without replacement from the deck. Then Fahad draws a card at random and without replacement from the remaining cards. Let A be the event that Mohammad's card has a larger number on it than Fahad's card. Let B be the event that Fahad's card has a larger number on it than Mohammad's card.

Are A and B mutually exclusive events? Please explain why or why not. [If you prefer, your explanation could just be determining the sample spaces for events A and B and showing they are mutually exclusive.]

Solution: Yes, the intersection of A and B is an empty set so A and B are mutually exclusive.

3. [+11] In order to reach the next game level in "Angry Birds Play Cards," Lindsey must correctly determine the following two probabilities in (a) and (b) below. A and B are events such that $P(A) = 0.4$ and $P(B) = 0.5$.

(a) Assume A and B are mutually exclusive. Determine $P(A \cup B)$.

Solution: P(AUB) = P(A) + P(B) = 0.9

(b) Assume A and B are independent (and no longer mutually exclusive). Determine $P(A \cup B)$.

Solution: P(A∩B) = P(A) * P(B) = 0.4*0.5 = 0.2
P(AUB) = P(A)+P(B) – P(AB) = 0.4+0.5-0.2 = 0.7

4. [+16] Carly plays softball on the Rose-Hulman team and is an excellent pitcher and hitter. Suppose the probability that she pitches a "no hitter" is 0.2. Also, suppose the probability that she hits a home run is 0.3. The probability that she does both in a game is 0.05.

(a) What's the probability that she does pitch a "no hitter" and does not hit a home run?

Solution: Suppose A : she pitches a "no hitter"; B : she hits a home run.
P(A) = 0.2, P(AB) = 0.05. the probability A and not B = 0.2 – 0.05 = 0.15

(b) Given that she hits a home run (we know she hit a home run – we saw it happen!), what's the probability that she also pitches a "no hitter?" (Note: I'm not asking for P(home run ∩ "no hitter").)

Solution: we are trying to solve. $P(A|B) = \frac{P(AB)}{P(B)} = \frac{0.05}{0.3} = \frac{1}{6}$

(c) Are the events of pitching a "no hitter" and hitting a "home run" independent events according to the probabilities given? You must show independence or dependence using the mathematical definition of independence given in this course.

Solution: According to the definition of independence, if P(A | B) = P(A), we say that A is independent of B.

However, $P(A|B) = \frac{P(AB)}{P(B)} = \frac{0.05}{0.3} = \frac{1}{6} \neq 0.2 \neq P(A)$. So A and B are not independent.

5. [+6] Bilguun is an excellent cook! Suppose he is making a pasta dish and has 5 different ingredients that he could use. How many unique pasta dishes are possible (including the possibility that he does not use any of the ingredients)? Note: He could use none of the ingredients, 1 of the ingredients, 2 of the ingredients ...

Solution:

Every kind of ingredient has two choices either used or not used. So there are 2^5 choices = 32.

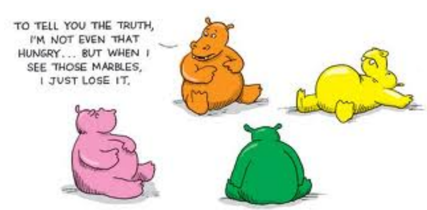
6. [+5] Yilin loves math; in fact she is a math major! She writes a letter to her best friend posing the following true/false problem.

"Let A and B be any two events. Then $P\left((A \cup B)^{\mathrm{C}}\right) = P\left(A^{\mathrm{C}}\right) \cdot P\left(B^{\mathrm{C}}\right)$, where C represents complement."

Is the above statement true or false? [No proof is necessary.]

Solution: false;

7. [+7] Aaron enjoys playing board games. When he was a child, his Mom started him out with a game called "Hungry Hungry Hippos." How many arrangements of the letters in HUNGRYHUNGRYHIPPOS are possible if we want to keep all of the H's together?



Solution: we can put all H together and treat them as one letter. As there are 18 total letters if we treat all H as 1 letter we have 18-3+1 = 16 letters.
There are 2 Us, 2Ns, 2 Gs, 2 Rs, 2Ys, 2Ps;
So there are totally $\frac{16!}{2!*2!*2!*2!*2!*2!}$ arrangement.

8. [+10] April enjoys Swing Dancing. She would like to choreograph a dance for next year's Engineers in Concert. She decides that she wants to use the following dance moves without repetition.

1. Handswitch
2. Hesitation Swingout
3. Lindy Circle
4. Airplane
5. Count Cradle
6. Texas Tommy
7. Sugarfoot
8. Chassez

If she randomly selects 4 of the moves to start her opening sequence, what is the probability that she will:

(a) use an Airplane, Lindy Circle, Sugarfoot, and Texas Tommy in that exact order?

Solution: |S| = $\binom{8}{4}$ * 4!. So the probability of use Airplane, Lindy Circle, Sugarfoot, and Texas Tommy in that exact order is $\frac{1}{\binom{8}{4} * 4!}$.

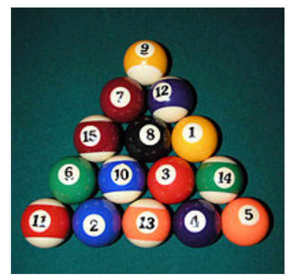
(b) use the Airplane, Lindy Circle, Sugarfoot, and Texas Tommy in any order possible?

Solution: |S| = $\binom{8}{4}$. The probability of use those 4 movement in any order possible is that $\frac{1}{\binom{8}{4}}$

9. [+7] Danielle is a hostess (greets and seats customers) at T.G.I. Friday's restaurant in Terre Haute. A group of 4 men and 4 women want to be seated at a round table. How many seating arrangements are possible if the men and women alternate seats?

Solution: Suppose M represents Male, F represents Female. So there arrangement look like this: MFMFMFMF. First M has 4 choices, first F has 4 choices, second M has 3 choices, second F has 4 choices. So there are totally 4!*4! Choices. And the arrangement can also be FMFMFMFM so there are totally 4!*4!*2. As there are 8 people and table is rounded so there are 8 same circular permutation for one linear permutation. So the total arrangement is $\frac{4!*4!*2}{8} = \frac{4!*4!}{4} = 144$

10. [+7] Francis loves playing pool. In the game of pool, there are 7 striped balls and 8 solid balls. If 4 balls are currently sitting in the pockets, what's the probability that 3 are striped and 1 is solid?



Solution: |S| = $\binom{7+8}{4} = \binom{15}{4}$. There are totally $\binom{7}{3} * \binom{8}{1}$ ways to have 3 striped 1 solid in the pocket. So the probability that 3 are striped and 1 is solid $\frac{\binom{7}{3}*\binom{8}{1}}{\binom{15}{4}}$.

11. [+10] Keri is on the Rose-Hulman rifle team. 70% of the time the team practices at an outdoor range, while 30% of the time they practice at an indoor range. At the outdoor range, Keri hits her target 95% of the time, while at the indoor range, her precision is 80%.

(a) During a given season, what's the probability that Keri hits her target? [Assume she shoots outdoors and indoors according to the percentages I gave in the problem description.]

Solution: the total percentage is 70% * 95% + 30% * 80% = 90.5%

(b) Given that Keri is at a shooting range and hits her target, what's the probability that she is shooting at an outside range?

Solution: the percentage of she is shooting at and outside range is that P(Outside| Target) = $\frac{P(Outside \cap Target)}{P(Target)}$ $\frac{70\% * 95\%}{90.5\%}$ = 73.5% (shooting outside and make target are independent)

12. [+10] Dan loves to travel; in fact, he's in Germany right now. Let's say he decides to travel about Europe during an upcoming holiday weekend. He's not sure where he's going to go when he reaches the train station. There's a 60% chance that he'll decide to go to Spain, 30% Slovakia, and 10% France. If he goes to:

- Spain, there's a 90% chance that he'll have a good time.
- Slovakia, there's a 95% chance that he'll have a good time.
- France, there's a 99% chance that he'll have a good time.

When he returns from his trip, he reports to us that he had a good time. What's the probability that he went to Slovakia?

Solution: there are 60%*90% + 30% *95% + 10%* 99% = 92.4% that he will have a good time. So the probability that he went to Slovakia is $\frac{30\% * 95\%}{92.4\%}$ = 30.8%