Quiz 5 **Name: Xiangbo Meng**

MA 381, Probability, Summer 2013
10 points possible
Due date: Wednesday, 7/17, by the **11:59 p.m.**

Please show your **work** to solve the problem. Please organize your work as clearly as possible. **Clearly circle, box, mark, star, or highlight** the final answer in some way.

Suggestion: Try to treat this as an "in-class quiz" and see if you can do these problems in 30 minutes or less with your prepared notes and/or probability distribution chart. This will be good practice for Exam 2.

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1. [+4 total points] Roll three fair six-sided dice. Let *X* be the number "4's" obtained.

(a) [+1] Write down the probability mass function $p(x)$ for *X*. Make sure to include its support (or "domain").

X= {0,1,2,3}
P(X=0) = (5/6)^3 = 125/216
P(X=1) = $\binom{3}{1}$(1/6)(5/6)^2 = 25/72
P(X=2) = $\binom{3}{2}$*(1/6)^2(5/6) = 5/72
P(X=3) = $\binom{3}{3}$*(1/6)^3 = 1/216

(b) [+1] Write down the ***cumulative distribution function*** $F(x)$ for **all** values of x. Don't forget that $F(x)$ is a piecewise defined function and we write $F(x)$ for all real numbers x.

$$f(x) = \begin{cases} 0, & x < 0 \\ \frac{125}{216}, & 0 \leq x < 1 \\ \frac{25}{27}, & 1 \leq x < 2 \\ \frac{215}{216}, & 2 \leq x < 3 \\ 1, & x \geq 3 \end{cases}$$

(c) [+0.5] Determine the expected value of *X*; that is, determine $E(X)$.

E(x) = 0*125/216 + 1*25/72+2*5/72+3*1/216 = **1/2**

(d) [+0.5] Determine the expected value of X^2; that is determine $E(X^2)$.

E(x^2) = 0*125/216 + 1*25/72+4*5/72+9*1/216 = **2/3**

(d) [+0.5] Determine the variance of *X*.

Var(x) = E(x^2) – E(x)^2 = 2/3 – (1/2)^2 = **5/12**

(e) [+0.5] You may have completed this problem without considering that *X* is one of our "special discrete distributions." Which one is it? You can check all of answers above knowing what type of random variable *X* is.
Solution: **Binomial Random Variable**

2. [+3 points total] Angela loves to play soccer. When she takes a shot on goal in a given game, her probability of making the goal is 0.05. Let's assume any one shot is independent of any other given shot.

(a) [+1] What's the probability that it will take her 25 or more shots to make one goal? Please set up the correct expression and evaluate it.

Solution: suppose X is the number of games that take her to make a goal. G represent goal. F represent Fail
P(x>=25) = 1- P(x<=24)
P(x<=24) = 0.05*(0.95)^x, x = {0,1,2,3,4,5…23}
P(x<=24) = 0.708
So P(x>=25) = 1-0.708 = 0.292

(b) [+1] What's the probability that it will take her 55 or more shots to make two goals? Please set up the correct expression and evaluate it.
Solution: X is the total shots that she takes to make 2 goals
X = 2 P(X) = 0.05^2
X = 3 P(X) = 0.95*0.05^2*2 because she get shoot like FGG or GFG
X = 4 P(X) = 0.95^2*0.05^2*3 because she get shoot like FFGG or FGFG or GFF
As the last shot has to remain goal, so multiplier at last should be C(X-1,1) = X-1
So P(X<55) = $\sum_{n=2}^{54}$ 0.05^2*0.95^(n-2)*(n-1) = 0.759
So P(X >= 55) = 1- 0.759 = **0.241**

(c) [+1] How many shots on goal would you expect her to take in order to make 3 goals?
As last shot has to be goal, so every number of X there are C(X-1,2)number of permutations
So P(x) = $\sum_{n=3}^{infinity}$ 0.05^3*0.95^(n-3)*C(n-1,2)
E(x) = $\sum_{n=3}^{infinity}$ 0.05^3*0.95^(n-3)*C(n-1,2) *n = 60

3. [+3 points total] My new black lab/mastiff puppy has a hard time calming down when she comes in the house from being outside. The time *T* (in minutes) that it takes her to "relax" can be described quite well by the probability density function:

$$f(t) = \begin{cases} \frac{1}{35}e^{-\frac{t}{35}} & t > 0 \\ 0 & \text{otherwise.} \end{cases}$$

(a) [+1] Suppose that I have just brought her inside the house after being outside. What's the probability that it will take her more than 45 minutes to relax? Please set up the appropriate expression and evaluate it.
$solution$: $\int_{45}^{infinity} \frac{1}{35}e^{-\frac{t}{35}}$ = **e^(-9/7) = 0.276**

(b) [+1] On average, when I bring her in from outside, how long does it take her to relax? In other words, determine *E*(*T*). Set up the appropriate expression and evaluate it.

Solution: E(x) = $\int_{45}^{infinity} \frac{1}{35}e^{-\frac{t}{35}} * t$ = 80*e^(-9/7) = 22.1 min

(c) [+1] Suppose again that I have just brought her inside the house after being outside. Given that I've waited more than 15 minutes and she's not relaxed, what's the probability that it will take her a total time of 45 minutes or more to relax? Hint: This problem requires you to compute a **conditional probability.**
Solution: A :it will take her a total time of 45 minutes or more to relax
B: I've waited more than 15 minutes and she's not relaxed
P(A|B) = P(AnB)/P(B) = P(A)/P(B) = ($\int_{45}^{infinity} \frac{1}{35}e^{-\frac{t}{35}}$)/ ($\int_{15}^{infinity} \frac{1}{35}e^{-\frac{t}{35}}$) = 0.679