MA 381 Exam 2
Summer 2013, Dr. Evans
100 points possible
Time Limit: 2.5 hours

Name:__________**Xiangbo Meng**_________________________

Signature:___Xiangbo Meng__________________________
I agree that I have not consulted anyone about this exam besides Dr. Evans.

♣ There are 11 problems with parts on this exam, and each problem is worth 10 points. **Choose** your favorite **10 problems** and put a big **X** through the problem number that you **don't want me to grade***:

1   2   3   4   5   6   7   8   9   10   X**11**

* If you don't exclude a problem, then I'll just exclude the final problem #11.

♣ You will have 2.5 hours once you download this exam to submit it to the Exam 2 Drop Box. When you download it, make sure you have 2.5 hours to work on it. I am setting ANGEL to report to me the time you download and upload the exam so that I can make sure your submission is within 2.5 hours of downloading it. You will be penalized for time over 2.5 hours.

♣ You must submit this exam as a single PDF document.

♣ You may use a calculator, Maple, prepared notes, textbooks, the internet ... basically **anything except other people!** I am thoroughly checking exams for collaboration. If there are any questions about collaboration on your part, I may require you to take follow up exams and/or the final exam at a proctored location or on campus when you return from summer break.

♣ By Rose's Honor Code, I am requiring that the only living people who are allowed to look at your exam are you and me. If you do not follow this rule, you may receive a 0 on this portion of the exam and disciplinary action within and perhaps beyond the math department.

♣ You can leave your answers in "unreduced" (or combinatorics) form; e.g. $10 \cdot 9 \cdot 8$, or $\binom{5}{2}$, or 6^3.

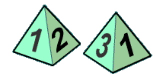
♣ **Please include the supports** with the probability mass functions and cumulative distribution functions for a random variable.

♣ Please write scratch work on this exam. It's much easier for me to give you partial credit when I can see your work and thought process for each problem.

	Your Points		**Your Points**
Problem 1		**Problem 7**	
Problem 2		**Problem 8**	
Problem 3		**Problem 9**	
Problem 4		**Problem 10**	
Problem 5		**Problem 11**	
Problem 6		***Subtotal 2***	
Subtotal 1		***Subtotal 1***	
		Bonus	
		TOTAL	

1. Yashi has a set of 3 fair four-sided dice. The sides of the dice contain the numbers (equally likely) 1, 2, 3, or 4 (see figure – the dice are pyramid shaped with 4 sides). She makes up the following game for you to play:

- All 3 dice are rolled.
- If Yashi rolls the numbers 1, 2, 3 (in any way – no die shows a 4 and no die duplicates another), then you give her $1.
- If Yashi does not roll all three numbers 1, 2, 3, then she'll give you $0.10.



How much money would you expect to win or lose if you play this game with her 100 times?

Solution: Suppose Yashi wins X games out of 100.

The probability of Yashi win the game is ¾ * 2/4 *1/4 = 6/64

P(x) = $\binom{100}{x} * \left(\frac{6}{64}\right)^{x} * \left(\frac{58}{64}\right)^{100-x}$ x= 0,1,2,3…100

E(x) = $\sum_{x=0}^{100} \binom{100}{x} * \left(\frac{6}{64}\right)^{x} * \left(\frac{58}{64}\right)^{100-x} * x$ = 9.375

Expect income is 9.375 * (-1) + (100-9.375)*0.1 = **-0.3125**

2. Let's use 2 of Yashi's fair four-sided dice. The sides of the dice contain the numbers (equally likely) 1, 2, 3, or 4. Roll the two fair four-sided dice, and let *X* be the maximum of the two numbers obtained. [As an example, if we roll the two dice and one is a 4 and the other is a 2, then the maximum of the two rolls is 4.]

(a) [+5] Write down the probability mass function *p*(*x*) of *X* and include its support. You can write it as a piecewise function or a more "compact" function.

X = 1,2,3,4
There are totally 4*4 =16 different combinations.
If x =1, there is 1 case, (1,1).
P(x=1) = 1/16.
If x =2, there are 3 cases (1,2),(2,1),(2,2)
P(x=2) =3/16.
If x = 3 there are 5 cases(1,3),(3,1),(2,3),(3,2)(3,3)
P(x=3) = 5/16. Which means one die is 3, the other one can be either 1,2,3.
If x = 4 there are 7 cases (1,4)(4,1)(2,4)(4,2)(3,4)(4,3)(4,4)
P(x=4) = 7/16

(b) [+5] Write down the cumulative distribution function *F*(*x*) of *X*. Make sure that you define *F*(*x*) for all values of *x*.

$$f(x) = \begin{cases} 0, & x < 1 \\ 1/16, & 1 \le x < 2 \\ \frac{1}{4}, & 2 \le x < 3 \\ \frac{9}{16}, & 3 \le x < 4 \\ 1, & x \ge 4 \end{cases}$$

3. In the Fall quarter for MA 381, I'm going to institute "Probability Gateway exams" on counting techniques that students must pass in order to get credit for the course. Here are the rules for the Gateway exams:

- Each Gateway exam will be very similar in content. The probability that a "B" student can pass any one of the exams (independent of any other) is 0.75.
- A student must pass two of these exams (in any order) to get credit for the course.
- There are an unlimited number of these exams, and students may take them an unlimited number of times (in theory) in order to pass two exams.

Let X represent the number of Gateway exams that a B student will need to take to pass two of them. [As an example, a student would be done in 2 exams if they have a pass, and a pass: PP. A student would be done in 3 exams if they have FPP or PFP.]

(a) [+6] Write down the probability mass function $p(x)$ of X. Please include its support.
X = 2,3,4,5…
P(x) = $\binom{\boldsymbol{x-1}}{\boldsymbol{1}} * \mathbf{0.75^2} * \mathbf{0.25^{x-2}}$

(b) [+4] On average, how many Gateway exams will a B student need to take in order to pass two of them?

E(x) = $\sum_{x=2}^{infinity} \binom{x-1}{1} * 0.75^2 * 0.25^{x-2} * x \cong \mathbf{2.67}$

4. Do you remember the problems from an earlier homework set based on Marilyn vos Savant's *Parade Magazine* column? Here's a problem that she got wrong, and your job is to correct her. Here's the scenario:

On January 22, 2012 Marilyn vos Savant admitted a mistake in her column. In the original column, published on December 25, 2011, a reader asked:

> "I manage a drug-testing program for an organization with 400 employees. Every three months (or four times a year), a random-number generator selects 100 names for drug-testing. Afterward, these names go back into the selection pool. Obviously, the probability of an employee being chosen in one quarter is 25 percent. But what is the likelihood of an employee being chosen over the course of a year?"
>
> —Jerry Haskins, Vicksburg, Miss.

Marilyn's incorrect response was:

"The probability remains 25 percent, despite the repeated testing. One might think that as the number of tests grows, the likelihood of being chosen increases, but as long as the size of the pool remains the same, so does the probability. Goes against your intuition, doesn't it?"

Yes – it goes against intuition because this answer is WRONG.

What is the actual probability that an employee gets selected at least once for drug-testing at this organization?

Assumptions:

- There are 400 employees and each has an equally likely chance of getting chosen during each testing period.
- Testing occurs four times in a given year.
- We want the probability that a given employee gets selected at least once for testing.

Please set up the correct expression to determine this probability and evaluate it.

X is the number of times that one employee has been chosen.
X = 0,1,2,3,4
P(x) = $\binom{4}{x}$0.25^x*0.75^(4-x)
P(x > = 1) = **0.6835937500**

5. Jeff likes warm weather, and hopefully he's getting his fill of it in Houston, Texas! In Indiana in the month of July, there is a 23% chance that a given day will have a maximum temperature over 90 degrees Fahrenheit.

(a) [+4] Let *X* represent the number of days in July (out of 31 days) that the maximum temperature in Indiana will be over 90 degrees Fahrenheit. What's the probability that we'll have 12 or more days (out of 31) with a maximum temperature of 90 degrees Fahrenheit or more in Indiana? Please set up the appropriate expression and evaluate it.

Solution: X = 0,1,2…31
P(x) = $\binom{31}{x}(23\%)^x(77\%)^{31-x}$
P(12<=x<=31) = $\sum_{x=12}^{31}\binom{31}{x}(23\%)^x(77\%)^{31-x}$ = 0.0363
The probability of 12 or more days with a maximum temperature of 90 degrees is **0.0363**

(b) [+3] Out of 31 days in July, how many days should we expect to have a maximum temperature of over 90 degrees Fahrenheit in Indiana?

Solution: E(x) = $\sum_{x=0}^{31}\binom{31}{x}(23\%)^x(77\%)^{31-x} * x$ = **7.13**

(c) [+3] Assuming independence from year to year, what's the probability of seeing 12 or more days with a maximum temperature of 90 degrees Fahrenheit or more in Indiana in July 2014 and then again 12 or more in July 2015?
Solution: as year to year is independent, P(>=12 days in 2014 $\cap >=$ 12 days in 2015) = P(>=12 days in 2014) * P(>=12 days in 2015) = **0.0363^2** $\cong \mathbf{0.00132}$

6. One of my alma maters is The Ohio State University. Usually two or three times a year, I get a call from the alumni association asking me to donate money to the school.

Suppose a student volunteers to make calls to alumni asking for donations. The probability that a given caller (independent of any other caller) will make a donation (especially to a state school) is 0.08.

(a) [+4] What is the probability that a student volunteer will receive a donation from a caller in 10 or less calls? Please set up the expression you would use to calculate this value and then evaluate it.

Solution:
Suppose student volunteer make X calls and get a donation from a caller
P(x) = 0.92^(x-1)*0.08
P(x <=10) = $\sum_{x=1}^{10} 0.92^{x-1} * 0.08 \cong \mathbf{0.566}$

(c) [+6] Given that the student has not received a donation in his or her first 8 calls (that is, we know the number of calls needed for a donation is at least 9 or more), what's the probability that it will take the student a total of 15 calls or more to receive a donation?
Solution: P(X >= 15 | P(X > 8)) = (P(X >= 15)∩P(X > 8))/ P(X>8) = P(X>=15)/P(x>8) = (1-P(x<=14))/p(X>8)
≅(1- 0.689)/0.92^8 ≅ 0.606

7. Let X be a random variable with cumulative distribution function (CDF) given by:

$$F(x) = \begin{cases} 0 & x < 1 \\ 0.4 & 1 \le x < 3 \\ 0.75 & 3 \le x < 6 \\ 1 & x \ge 6. \end{cases}$$

Determine the following.

(a) $P(X < 3) = 0.4$

(b) $P(1 < X \le 3)$ = P(x<=3) – P(x<=1) = 0.75-0.4 = 0.35

(c) $P(X = 4)$ = P(x<=4) - P(x<4) = 0

(d) $P(X \geq 6)$ = 1- P(X<6) = 1-0.75 = 0.25

(e) $E(X)$
P(x = 1) = 0.4
P(x = 3) = 0.35
P(x = 6) = 0.25
E(x) = 1*0.4+3*0.35+6*0.25 = **2.95**

8. Recall that out of our 21 current class members, 6 of them have birthdays in July (unusual!) Suppose I put a slip of paper with each class member's name and his or her birthday on it in a hat. I'm going to select 4 names (without replacement) from the hat and throw them birthday parties. Let X be the number of students who I select that actually have July birthdays.

(a) [+4] Determine the probability mass function $p(x)$ of X. Include the support of X with your probability mass function.

Solution: there are totally $\binom{21}{4}$ ways to select.

$P(x) = \frac{\binom{6}{x}\binom{15}{4-x}}{\binom{21}{4}}$ x = 0,1,2,3,4

(b) [+3] On average, how many of the selected 4 will actually have July birthdays?

$E(x) = \sum_{x=0}^{4} \frac{\binom{6}{x}\binom{15}{4-x}}{\binom{21}{4}} * x \cong \mathbf{1.14}$

(c) [+3] Determine the standard deviation of X.

Var(x) = E(x^2) – E(x)^2 ≅ 0.693

Standard deviation of X is sqrt(Var(x)) ≅ **0.833**

9. Ever since Honey Boo Boo made "Go Go Juice" famous, Circle K convenient store can't keep enough Red Bull on their shelves to meet current demand.

Daily demand for Red Bull cans at Circle K follows a Poisson distribution with a mean demand of 35 cans per day.

(a) [+5] Determine the probability that more than 40 cans are demanded at Circle K in a randomly selected day.
Solution: lambda = 35
P(x <= 40) = $\sum_{n=0}^{40} \frac{e^{-35} 35^n}{n!} \cong 0.825$
So P(x >40) = 1-0.825 = **0.175**

(b) [+5] Determine the probability that more than 150 cans are demanded at Circle K in a randomly selected work week, where a work week is 5 days (Monday – Friday).

Solution: lambda = 35*5 = 175
P(x > 150) = $\sum_{n=151}^{infinity} \frac{e^{-175} 175^n}{n!} \cong$ **0.970**

10. De Song is currently volunteering in a remote area of China called Sichuan. His internet connection is rather unreliable. When he is able to connect to the internet, the length of his session *X* before he is disconnected (in minutes) is given by the following probability density function:

$$f(x) = \begin{cases} \frac{1}{5}e^{-\frac{x}{5}} & x > 0 \\ 0 & \text{otherwise.} \end{cases}$$

(a) [+3] Suppose De Song is able to connect to the internet this afternoon. What's the probability that the length of his internet session is less than 3 minutes?

Solution: P(x>=3) = $\int_{x=3}^{infinity} \frac{1}{5}e^{-\frac{x}{5}}dx \cong 0.549$

P(x<3) = 1-P(x>=3) = 1- 0.549 = **0.451**

(b) [+3] On average, how long is a randomly selected internet session for him?

E(x) = $\int_{x=0}^{infinity} \frac{1}{5}e^{-\frac{x}{5}} * x\, dx = \mathbf{5\ min}$

(c) [+4] Given that he has been able to be on the internet more than 5 minutes, what's the probability he'll be able to be on for more than a total of 12 minutes?

Solution: P(more than 12 min | more than 5 min) = P(more than 12 min)/P(more than 5 min) = $\int_{x=12}^{infinity} \frac{1}{5}e^{-\frac{x}{5}}\, dx / \int_{x=5}^{infinity} \frac{1}{5}e^{-\frac{x}{5}}\, dx$ = **0.247**

11. Jiaren enjoys photography. The time X (in minutes) that it takes him to position himself for a good shot is given by the following probability density function:

$$f(x) = \begin{cases} x & 0 < x < 1 \\ 2 - x & 1 \leq x < 2 \\ 0 & \text{otherwise.} \end{cases}$$

(a) [+4] Determine the average amount of time that it takes him to position himself for a good shot. That is, determine the expected value $E(X)$ of X. Write out the appropriate expression and evaluate it.

Solution: $\int_{x=0}^{1} x * x \, dx + \int_{x=1}^{2} (2 - x) * x \, dx = 1$

(b) [+6] Determine the cumulative distribution function $F(x)$ of X. Please clearly list the support values for all values of x.