Folha 4

1) $E_x = 0$ $E_y = 30\cos\left[(2\pi\times10^8)t - \frac{2\pi}{3}x\right]$ $E_z = 0$

a. $\omega = 2\pi f \rightarrow f = \frac{\omega}{2\pi} = \frac{\cancel{2\pi}\times10^8}{\cancel{2\pi}} = 1\times10^8$ Hz

$K = \frac{2\pi}{\lambda} \rightarrow \lambda = \frac{2\pi}{K} = \frac{2\pi}{\frac{2\pi}{3}} = 3$ m

b. a equação devia ser $E_m\cos(Kx - \omega t)$, logo propaga-se segundo $-x$

2) $\vec{E} = E_m \,\text{sen}\,(10^{14}t - Kz)\vec{x}$ NC^{-1}

$\vec{B} = 10^{-6}\,\text{sen}\,(10^{14}t - Kz)\vec{y}$ T

b. $\frac{\vec{E_m}}{\vec{B_m}} = c \Rightarrow \vec{E_m} = c \cdot \vec{B_m} = 3,0\times10^8 \times 10^{-6} = 3,0\times10^2\ Vm^{-1}$

d. $I = \frac{1}{2\mu_0} E_m B_m = \frac{1}{2\times4\pi\times10^{-7}} \times 3,0\times10^2 \times 10^{-6} \approx 119,4\ W m^{-2}$

c. $\omega = 2\pi f$ $c = \lambda f \Rightarrow \lambda = \frac{c}{f} = \frac{3,0\times10^8}{1,59\times10^{13}} \approx 1,89\times10^{-5}$ m → Infravermelho

$10^{14} = 2\pi f \Rightarrow f = \frac{10^{14}}{2\pi} \approx 1,59\times10^{13}$ Hz

3) ondas AM → $[520 ; 1610]$ kHz $\Rightarrow [520\times10^3 ; 1610\times10^3]$ Hz

ondas FM → $[88,0 ; 108,0]$ MHz $\Rightarrow [88,0\times10^6 ; 108,0\times10^6]$ Hz

a. AM → $\lambda \in [186 ; 577]$ m

$\lambda_{min} = \frac{c}{f_{max}} = \frac{3,0\times10^8}{1610\times10^3} \approx 186$ m $\lambda_{max} = \frac{c}{f_{min}} = \frac{3,0\times10^8}{520\times10^3} \approx 577$ m

FM → $\lambda \in [2,8 ; 3,4]$ m

$\lambda_{min} = \frac{c}{f_{max}} = \frac{3,0\times10^8}{108,0\times10^6} \approx 2,8$ m $\lambda_{max} = \frac{c}{f_{min}} = \frac{3,0\times10^8}{88,0\times10^6} \approx 3,4$ m

b. $\lambda_{antena} = \frac{1}{4}\lambda$ $f = 1000\times10^3$ Hz $\lambda = \frac{c}{f} = \frac{3,0\times10^8}{1000\times10^3} = 300$ m $\lambda_{antena} = \frac{1}{4}\times300 = 75$ m

direção vertical (⊥ à antena)

c. $\lambda_{antena} = \frac{1}{2}\lambda = 246$ m $\lambda = 2\times246 = 492$ m $f = \frac{c}{\lambda} = \frac{3,0\times10^8}{492} = 610$ kHz

direção horizontal (⊥ à antena)

d. $f = 900\times10^6$ Hz $\lambda = \frac{c}{f} = \frac{3,0\times10^8}{900\times10^6} \approx 0,33$ m → Antena pequena

4) $P = 50\times10^3$ W $d = 100\times10^3$ m $I = \frac{P}{A} = \frac{50\times10^3}{4\pi(100\times10^3)^2} = 3,98\times10^{-7}$

$I = \frac{1}{2\mu_0} E_m B_m$ $B_m = \frac{E_m}{c}$

$I = \frac{1}{2\mu_0 c} E_m^2 \Rightarrow E_m = \sqrt{2\mu_0 c I} = \sqrt{2\times4\pi\times10^{-7}\times3,0\times10^8\times3,98\times10^{-7}} = 1,73\times10^{-2}\ Vm^{-1}$

5) $430\ THz < f < 760\ THz \Rightarrow 430 \times 10^{12}\ Hz < f < 760 \times 10^{12}\ Hz$

$\lambda_{min} = \frac{c}{f_{max}} = \frac{3,0 \times 10^{8}}{760 \times 10^{12}} \approx 3,95 \times 10^{-7}\ m \rightarrow$ azul

$\lambda_{max} = \frac{c}{f_{min}} = \frac{3,0 \times 10^{8}}{430 \times 10^{12}} \approx 6,98 \times 10^{-7}\ m \rightarrow$ vermelho

6) $t = 431$ anos-luz   taxa $= 2,2 \times 10^{3}$ $P_{sol} = 3,90 \times 10^{26}\ W$

$\hookrightarrow d = vt = (3,0 \times 10^{8}) \times 431\,(360 \times 24 \times 3600) \approx 4,02 \times 10^{18}\ m$

$I = \frac{P}{A} = \frac{(2,2 \times 10^{3})(3,90 \times 10^{26})}{4\pi (4,02 \times 10^{18})^2} = 4,22 \times 10^{-9} \Rightarrow \frac{1}{2\mu_0 c} E_m^2$

$E_m = \sqrt{(4,22 \times 10^{-9})(2 \times 4\pi \times 10^{-7} \times 3,0 \times 10^{8})} \approx 1,78 \times 10^{-3}\ V/m$

$c = \frac{E_m}{B_m} \Rightarrow B_m = \frac{E_m}{c} = \frac{1,78 \times 10^{-3}}{3,0 \times 10^{8}} \approx 5,93 \times 10^{-12}\ T$

Deve responder ao campo elétrico ($\vec{E}$), pois este tem uma amplitude maior

7) $I = 20\ Wm^{-2}$ $f = 1 \times 10^{6}\ Hz$

$I = \frac{1}{2\mu_0 c} E_m^2$ $E_m = \sqrt{2\mu_0 c\, I} = \sqrt{2\,(4\pi \times 10^{-7})(3,0 \times 10^{8}) \times 20} \approx 123\ Vm^{-1}$

$B_m = \frac{E_m}{c} = \frac{123}{3,0 \times 10^{8}} \approx 4,1 \times 10^{-7}\ T$ $\omega = 2\pi f = 2\pi \times 10^{6}\ rad/s$

$\lambda = \frac{c}{f} = \frac{3,0 \times 10^{8}}{1 \times 10^{6}} = 300\ m$ $K = \frac{2\pi}{\lambda} = \frac{2\pi}{300} \approx 2,09 \times 10^{-2}\ m^{-1}$

$E = E_m \cos(Kx - \omega t) \rightarrow E = 123 \cos(2,09 \times 10^{-2} x - 2\pi \times 10^{6} t)\ V/m$

$B = B_m \cos(Kx - \omega t) \rightarrow B = 4,1 \times 10^{-7} \cos(2,09 \times 10^{-2} x - 2\pi \times 10^{6} t)\ T$

8) Aplicando a lei de Malus a cada polarizador: $I_t = I_i \cos^2\theta$

→ Como no primeiro polarizador a radiação incidente não é polarizada, fica com polarização vertical:

$I_t = \frac{1}{2} I_0$

→ No segundo polarizador: $I_t = \frac{1}{2} I_0 \cos^2 60$

→ No terceiro polarizador: $I_t = \left(\frac{1}{2} I_0 \cos^2 60\right) \cos^2 30$

Logo, a fração de luz emergente do sistema será:

$\frac{1}{2} \cos^2 60 \cos^2 30\ I_0 \approx 0,094\ I_0$

9) Segundo a Lei de Snell-Descarte: $n_1 \, sen \, \theta_B = n_2 \, sen \, \theta_R$

a. $\theta_B + 90° + \theta_R = 180° \Rightarrow \theta_B + \theta_R = 90° \Rightarrow \theta_R = 90° - \theta_B$

Logo, $sen \, \theta_R = sen(90° - \theta_B) = \cos \theta_B$

$$n_1 \, sen \, \theta_B = n_2 \, sen \, \theta_R \Rightarrow \frac{sen \, \theta_B}{\cos \theta_B} = \frac{n_2}{n_1} \Rightarrow tg \, \theta_B = \frac{n_2}{n_1} \Rightarrow \theta_B = tg^{-1}\left(\frac{n_2}{n_1}\right) \text{ cqm.}$$

b. Segundo a lei de Malus: $I_t = I_o \cos^2 \theta \rightarrow \theta = 90°$, logo, $I_t = I_o \cos^2 90 = 0$

10) $\theta_i = 45°$ $\quad n_{ar} = 1,00$

Lei de Snell-Descartes

(A) $n_{ar} \, sen \, 45 = n_{vidro} \, sen \, \alpha$

(B) $n_{vidro} \, sen \, \beta = n_{ar} \, sen \, 90 \rightarrow$ Reflexão total
$\rightarrow \beta = 90° - \alpha$

$$sen \, \alpha = \frac{n_{ar} \, sen \, 45}{n_{vidro}} = \frac{\sqrt{2}}{2 \, n_{vidro}}$$

$$n_{vidro} = \frac{n_{ar} \, sen \, 90}{sen(90 - \alpha)} = \frac{1}{\cos \alpha} \Rightarrow \cos \alpha = \frac{1}{n_{vidro}}$$

$$sen^2 \alpha + \cos^2 \alpha = 1 \Leftrightarrow \frac{2}{4 n_{vidro}^2} + \frac{1}{n_{vidro}^2} = 1 \Rightarrow \frac{6}{4 n_{vidro}^2} = 1 \Rightarrow n_{vidro} = \sqrt{\frac{6}{4}} \approx 1,22$$

11) $n_{nucleo} = 1,58$ $\quad n_{bainha} = 1,53$

Segundo a Lei de Snell-Descartes

$1,58 \, sen \, \theta_i = 1,53 \, sen \, 90 \rightarrow$ reflexão total

$$sen \, \theta_i = \frac{1,53}{1,58} \Rightarrow \theta_i = sen^{-1}\left(\frac{1,53}{1,58}\right) \approx 75,55°$$

$$\beta = 90° - 75,55° = 14,45°$$

12)

a. $n_{ar} \, sen \, \theta_i = n_n \, sen \, \theta_R$

$sen \, \theta_R = sen(90 - \theta_i) = \cos \theta_i$

$\Big\} \; n_{ar} \, sen \, \theta_i = n_n \cos \theta_i \Rightarrow n_{ar}^2 \, sen^2 \theta_i = n_n^2 \cos^2 \theta_i$

$$\cos^2 \theta_i = 1 - sen^2 \theta_i = 1 - \frac{n_b^2}{n_n^2} \rightarrow \underset{\substack{\downarrow \\ 1}}{n_{ar}} \, sen^2 \theta_i = n_n^2 \left(1 - \frac{n_b^2}{n_n^2}\right) \Leftrightarrow sen^2 \theta_i = n_n^2 - n_b^2 \Rightarrow sen \, \theta_i = \sqrt{n_n^2 - n_b^2}$$

b. $n_n = 1,486$ $\quad \frac{n_n - n_b}{n_n} = 0,010 \Leftrightarrow \frac{1,486 - n_b}{1,486} = 0,010 \Rightarrow n_b = 1,486 - 0,010 \times 1,486 \approx 1,471$

$$sen \, \theta_{max} = \sqrt{(1,486)^2 - (1,471)^2} \Rightarrow \theta_{max} = sen^{-1}\left(\sqrt{(1,486)^2 - (1,471)^2}\right) \approx 12,2°$$

13) $f = 2,0 \times 10^4$ Hz $\quad v_1 = 1,0 \times 10^3\ ms^{-1}$ $\quad \theta_i = 40°$ $\quad \theta_R = 90° - 30° = 60°$

a. $n_1 \text{ sen } 40 = n_2 \text{ sen } 60 \Leftrightarrow \frac{n_2}{n_1} = \frac{\text{sen } 40}{\text{sen } 60} \approx 0,74$

b. $\theta_c = \text{sen}^{-1}\left(\frac{n_2}{n_1}\right) = \text{sen}^{-1}(0,74) \approx 47,73°$

c. $\lambda_1 = \frac{v}{f} = \frac{1,0 \times 10^3}{2,0 \times 10^4} = 0,05$ m $\quad n = \frac{c}{v} = \frac{c}{\lambda f}$

$\frac{n_2}{n_1} = \frac{\frac{c}{\lambda_2 f}}{\frac{c}{\lambda_1 f}} \Leftrightarrow \frac{n_2}{n_1} = \frac{\lambda_1}{\lambda_2} \Rightarrow \lambda_2 = \lambda_1 \times \frac{1}{\frac{n_2}{n_1}} = 0,05 \times \frac{1}{0,74} \approx 6,76 \times 10^{-2}$ m

14) $\lambda_{ar} = 633 \times 10^{-9}$ m $\quad \lambda_{sf} = 474 \times 10^{-9}$ m

a. $\frac{n_{ar}}{n_{sf}} = \frac{\lambda_{sf}}{\lambda_{ar}} \Rightarrow n_{sf} = n_{ar} \times \frac{\lambda_{ar}}{\lambda_{sf}} = 1 \times \frac{633 \times 10^{-9}}{474 \times 10^{-9}} \approx 1,34$

b. $f = \frac{c}{\lambda} = \frac{3,0 \times 10^8}{633 \times 10^{-9}} \approx 4,74 \times 10^{14}$ Hz $\rightarrow$ A frequência é igual em ambos os meios

c. Lei de Snell-Descartes para o ângulo crítico

$n_{sf} \text{ sen } \theta_i = n_{ar} \text{ sen } 90 \Rightarrow n_{sf} \text{ sen } \theta_i = 1 \Rightarrow \theta_i = \text{sen}^{-1}\left(\frac{1}{n_{sf}}\right) \approx 48,3°$

15) $n_n = 1,542$ $\quad \theta_{max} = \frac{32°}{2} = 16°$

a. $n = \text{sen } \theta_{max} = \text{sen } 16 \approx 0,276$

$n = \sqrt{(n_n)^2 - (n_b)^2} \Rightarrow n^2 = (n_n)^2 - (n_b)^2 \Rightarrow (n_b)^2 = (n_n)^2 - n^2 \Rightarrow n_b = \sqrt{(n_n)^2 - n^2} = \sqrt{1,542^2 - 0,276^2} \approx 1,517$

b. $v = \frac{c}{n_b} = \frac{3,0 \times 10^8}{1,517} \approx 1,98 \times 10^8\ ms^{-1}$

16)

a. Assumindo a potência e a distância dadas na alínea b.

$I = \frac{P}{A} = \frac{50}{4\pi(3^2)} \approx 0,442\ Wm^{-2}$

b. $I = \frac{1}{2\mu_0 c} E_m^2 \Rightarrow E_m = \sqrt{2\mu_0 c\, I} = \sqrt{2 \times 4\pi \times 10^{-7} \times 3,0 \times 10^8 \times 0,442} \approx 18,3\ Vm^{-1}$

$B_m = \frac{E_m}{c} = \frac{18,3}{3,0 \times 10^8} = 6,1 \times 10^{-8}$ T

17) $n_n = 1,475$ $\quad n_b = 1,460$ $\quad R_n = 25 \times 10^{-6}$ m

a. Trata-se do ângulo crítico.

$n_n \text{ sen } \theta_c = n_b \text{ sen } 90 \Rightarrow \text{sen } \theta_c = \frac{n_b}{n_n} \Rightarrow \theta_c = \text{sen}^{-1}\left(\frac{1,460}{1,475}\right) \approx 81,8°$

b) $\alpha = 90 - 81,8 = 8,2°$

$n_{ar} \operatorname{sen} \theta = n_n \operatorname{sen} \alpha \Rightarrow \theta = \operatorname{sen}^{-1}\left(\dfrac{n_n \operatorname{sen} \alpha}{n_{ar}}\right) = \operatorname{sen}^{-1}\left(\dfrac{1,475 \operatorname{sen}(8,2)}{1}\right)$

$\Rightarrow \theta \approx 12,1°$

c) $\theta = 12,1°$ $\quad d = 1 \times 10^3 \, m$ $\quad n = ?$

$x = \dfrac{R_n}{tg\,\alpha} = \dfrac{25 \times 10^{-6}}{tg(8,2)} \approx 1,73 \times 10^{-4} \, m \rightarrow$ distância entre reflexões

Logo, $n = \dfrac{d}{x} = \dfrac{1 \times 10^3}{1,73 \times 10^{-4}} \approx 5,78 \times 10^6$

18) $n_{ar} = 1$ $\quad \theta_1 = 45°$ $\quad \theta_2 = 14°$ $\quad \theta_3 = 60°$

a. (A) $n_{ar} \operatorname{sen} 45 = n_p \operatorname{sen} \alpha \Rightarrow n_p \operatorname{sen} \alpha = \dfrac{\sqrt{2}}{2}$

(B) $n_p \operatorname{sen}(90 - \alpha) = n_\ell \operatorname{sen}(90 - 14) \Rightarrow n_p \cos \alpha = 1,41 \operatorname{sen} 76$

(C) $n_p \operatorname{sen} \alpha = n_\ell \operatorname{sen} 30 \Rightarrow \dfrac{\sqrt{2}}{2} = n_\ell \times \dfrac{1}{2} \Rightarrow n_\ell = \sqrt{2} \approx 1,41$

b. $n_p \operatorname{sen} \alpha = \dfrac{\sqrt{2}}{2}$

$n_p \cos \alpha = 1,41 \operatorname{sen} 76 \approx 1,37$

$n_p^2 = \operatorname{sen}^2 \alpha + \cos^2 \alpha \therefore n_p = \sqrt{\left(\frac{\sqrt{2}}{2}\right)^2 + (1,37)^2} \approx 1,54$

c. $L = 50 \times 10^{-2} \, m$

$1,54 \operatorname{sen} \alpha = \dfrac{\sqrt{2}}{2} \Rightarrow \alpha = \operatorname{sen}^{-1}\left(\dfrac{\sqrt{2}}{2 \times 1,54}\right) \approx 27,33°$

$y = \dfrac{50 \times 10^{-2}}{tg(27,33°)} \approx 0,967 \, m \Rightarrow d = \sqrt{(50 \times 10^{-2})^2 + (0,967)^2} \approx 1,09 \, m$

Como percorre d 2 vezes, $s = 2 \times 1,09 = 2,18 \, m$

$t = \dfrac{s}{v} = \dfrac{s}{\frac{c}{n_p}} = \dfrac{s\, n_p}{c} = \dfrac{2,18 \times 1,54}{3,0 \times 10^8} \approx 1,12 \times 10^{-8} \, s$