Discuss the significance of the Fundamental Theorem of Calculus

Introduction

All Mathematics students know that differentiation is the inverse operation of integration, but without further thought the connection between these two operations seems arbitrary. Why should the gradient of a function relate to the area under its curve on a graph? The first part of the Fundamental Theorem of Calculus (FTC) tells us this connection between integration and differentiation, and also guarantees the existence of an anti-derivative for every continuous function (Cardiff University, 2014). Simply put, the first part of the FTC tells us that if a function is integrated and the result is then differentiated, we return to the original function.

$\frac{d}{dx}\int_a^x f(t)dt = f(x)$i.e. Part (1): Where f is continuous on [a, b], and $a \leq x \leq b$

The second part of the theorem then uses part one to show that the definite integral of a continuous function can be calculated using any anti-derivative of the function. This part of the theorem is key as it provides a much simpler way to evaluate continuous integrals (Cardiff University, 2014).

Part (2): $\int_a^b f(x)dx = [F(x)]_a^b = F(b) - F(a)$. i.e. F is an anti-derivative of f.

Discussion – Significance and applications

The FTC has many applications in the real world; in Optics, the Fresnel function can be used to measure the amount of reflection versus refraction of light, and can even be used to determine the colour of light reflected off a metal (Kokoska 2006).
Part one of the FTC tells us how to differentiate the Fresnel function, and allows us to analyse *S.*

$S(x) = \int_0^x (\sin((\pi t^2)/2))dt$The Fresnel function:
(Stewart 2012)

In Physics, the Sine Cardinal function can be used to model the damped harmonic motion of pendulums, and also has applications in signal processing and Fourier analysis (Kokoska 2006).

$Si(x) = \int_0^x (\sin(t)/t)dt$The Sine Cardinal function:

Furthermore, the error function is used extensively in Probability to determine the probability of error in a measure (Kokoska 2006).

$erf(x) = (2/\sqrt{\pi})\int_a^x (e^{-t^2})$The error function:

Each of these formulae depend on the FTC in their proofs. To be able to analyse any formula relating integral and differential calculus the FTC is needed. Even processes as

simple as differentiating a displacement/time curve to obtain velocity and acceleration (and integrating these to return to displacement) rely completely on the FTC.
The number of practical applications of the FTC is huge; this alone highlights the importance of the formula, but the true significance of the FTC is in it's meaning to pure mathematics. It establishes a deep connection between the geometrical concepts of differentiation (finding the derivative of a curve – it's gradient) and integration (finding the area under a curve), two operations which at first seem unrelated. The FTC gives us a precise inverse relationship between the integral and derivative, bridging the gap between these two branches of Calculus.

Conclusion

The significance of the FTC is that it gives us the easiest and simplest way to evaluate the definite integrals of a continuous function, not by computing the limit of a Riemann sum, but by using any one of the infinite anti-derivatives of the function. This relationship is so important to Calculus that the theorem is named the Fundamental Theorem of Calculus, as it is, ultimately, fundamental.

Bibliography

- Cardiff University (2014), MA1000 Calculus lecture notes

- Kokoska, S. 2006 *Examples to Reinforce Concepts That Are Connected to the Fundamental Theorem of Calculus.* In: Diefenderfer, C. et al. *Special Focus: The Fundamental Theorem of Calculus.* The College Board, p. 112

- Stewart, M. 2012 *The fundamental theorem of calculus* [Online].

Available at: http://math.ku.edu/~pgu/M121.Stewart.5.4.pdf
[Accessed: 28th November 2014].