# CS543 / ECE 549: Computer Vision, Homework 1

**Semester:** Spring 2021

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## 1 Vanishing Points and Vanishing Lines

Without losing generality, we choose a pair of parallel lines from plane $\mathbf{N}^T\mathbf{X} = d$, A: $Y = kX, Z = d - N_xX - N_yY$, B: $Y = kX + b, Z = d - N_xX - N_yY$. From basic geometry knowledge we know that the projection of the lines in a 2D plane are still lines. Thus, we select $(0, 0, d), (t, kt, z_a)$ from A and $(0, b, z_0), (t, kt + b, z_b)$ from B where $z_0 = Z(0, b), z_a = Z(t, kt), z_b = Z(t, kt + b)$ to compute the equations of the projected lines in 2D plane.
Converting from camera coordinate to the image coordinate, we get the selected points from A are $(0, 0)$ and $(\frac{ft}{z_a}, \frac{fkt}{z_a})$, the selected points from B are $(0, \frac{fb}{z_0})$ and $(\frac{ft}{z_b}, \frac{f(kt+b)}{z_b})$.
Thus we get:

$$
\begin{aligned}
&A : y = kx \\
&B : y = (k + \frac{b(z_0 - z_b)}{z_0 t})x + \frac{fb}{z_0}
\end{aligned}
$$

Solve the intersecting point of line A and line B in the image plane, we get:

$$
\begin{aligned}
x' &= \frac{ft}{z_b - z_0} = \frac{ft}{\frac{d - N_xt - N_y(kt+b) - d + N_yb}{N_z}} = -\frac{fN_z}{N_x + N_yk} \\
y' &= kx' = -\frac{fN_zk}{N_x + N_yk}
\end{aligned}
$$

Substitute k back with $\frac{y}{x}$, we get:

$$
x' = -fN_z\frac{x}{N_xx + N_yy}, y' = -fN_z\frac{y}{N_xx + N_yy}
$$

Thus we have:

$$
N_xx' + N_yy' = -fN_z
$$

Thus we know the intersecting point of line A and line B in the image plane always lies on the line $N_xx + N_yy + fN_z = 0$

## 2 Rectangle and Cylinder under Perspective Projection

1. Suppose the right side of the projection on the image plane is x distance away from Z-axis, then from the basic geometry knowledge, we have:

$$\frac{f}{Z} = \frac{x}{d}$$
$$\frac{f}{Z} = \frac{l+x}{L+d}$$

Thus we have:

$$x = \frac{df}{Z}$$
$$l = \frac{f(L+d)}{Z} - x = \frac{fL}{Z}$$

2. If we know the corresponding L in question 1, then we can easily get the result from question 1. Suppose L is broken into $L_1$ and $L_2$ which are the left part and right part of L broken by center of the circle, then we have:

$$\frac{r}{L_1} = \frac{Z}{\sqrt{Z^2 + (d+L_1)^2}}$$
$$\frac{r}{L_2} = \frac{Z}{\sqrt{Z^2 + (d-L_2)^2}}$$

Then we get:

$$L_1 = \frac{r^2 d + Zr\sqrt{Z^2 - r^2 + d^2}}{Z^2 - r^2}$$
$$L_2 = \frac{-r^2 d + Zr\sqrt{Z^2 - r^2 + d^2}}{Z^2 - r^2}$$

Hence, $L = L_1 + L_2 = \frac{2Zr\sqrt{Z^2-r^2+d^2}}{Z^2-r^2}$. Then according to question 1 we have that:

$$l = \frac{fL}{Z} = \frac{2fr\sqrt{Z^2 - r^2 + d^2}}{Z^2 - r^2}$$

## 3 Phong Shading Model

The generated figures are shown in **Figure 1-4**
In the code, the first thing I do is to use a Numpy 3D array to contain the camera coordinates in each pixel coordinates, computed by $X = \frac{(x-cx)Z}{f}, Y = \frac{(y-cy)Z}{f}$. Z coordinates can be obtained directly from the given Z matrix.
Surface normal vector $\hat{n}$ can be derived directly from matrix N (N[y, x, :]).
$\hat{v_i}$ contains both point light direction, which is derived by point_light_loc - camera_coordinates, and directional light direction, which is given as directional_light_dirn.
$\hat{v_r}$ is computed by directly using the negative of the camera coordinates tensor because we assume the camera is at the origin point.
$\hat{s_i}$ also contains both point light reflection direction and directional light reflection direction. The reflection light direction vector can be computed by $2(x \cdot N) - x$, where x is the incident light vector and both N and x are unit vector.

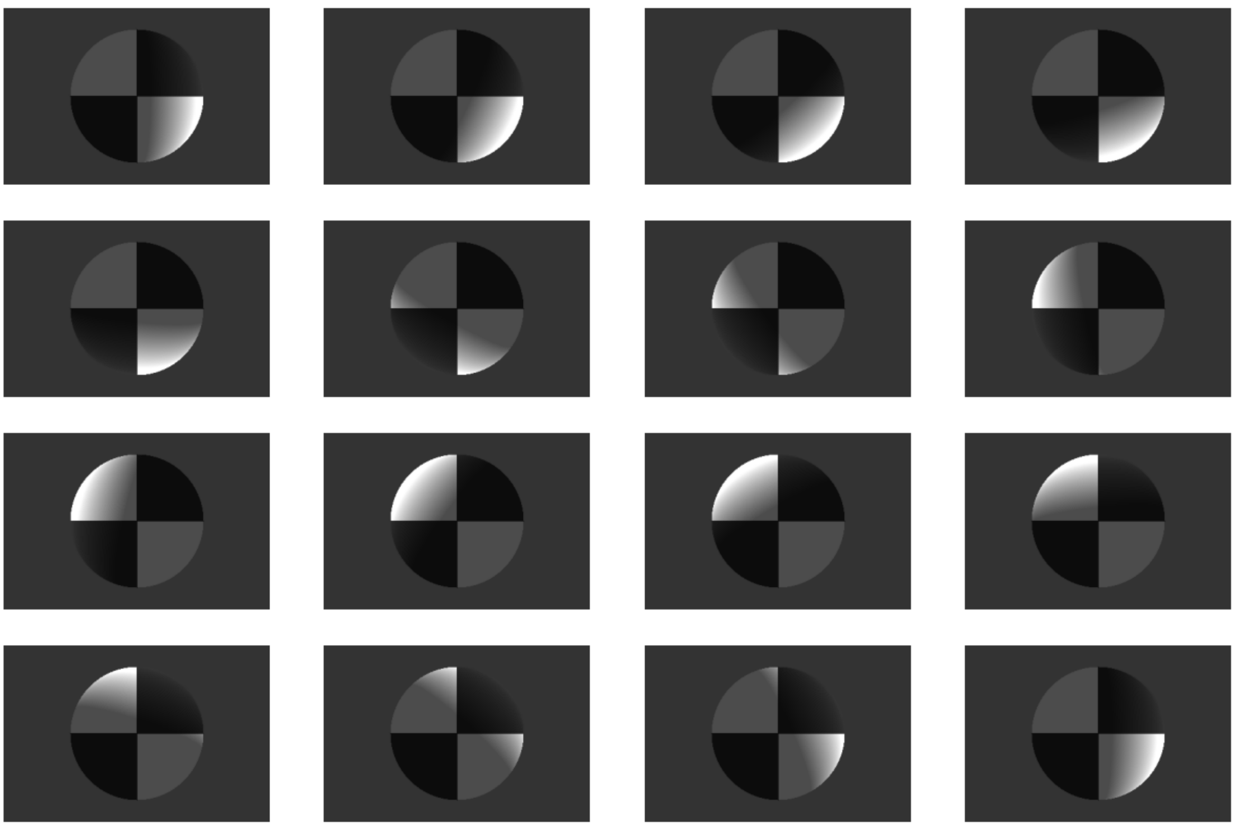

Figure 1: Specular 0 move direction

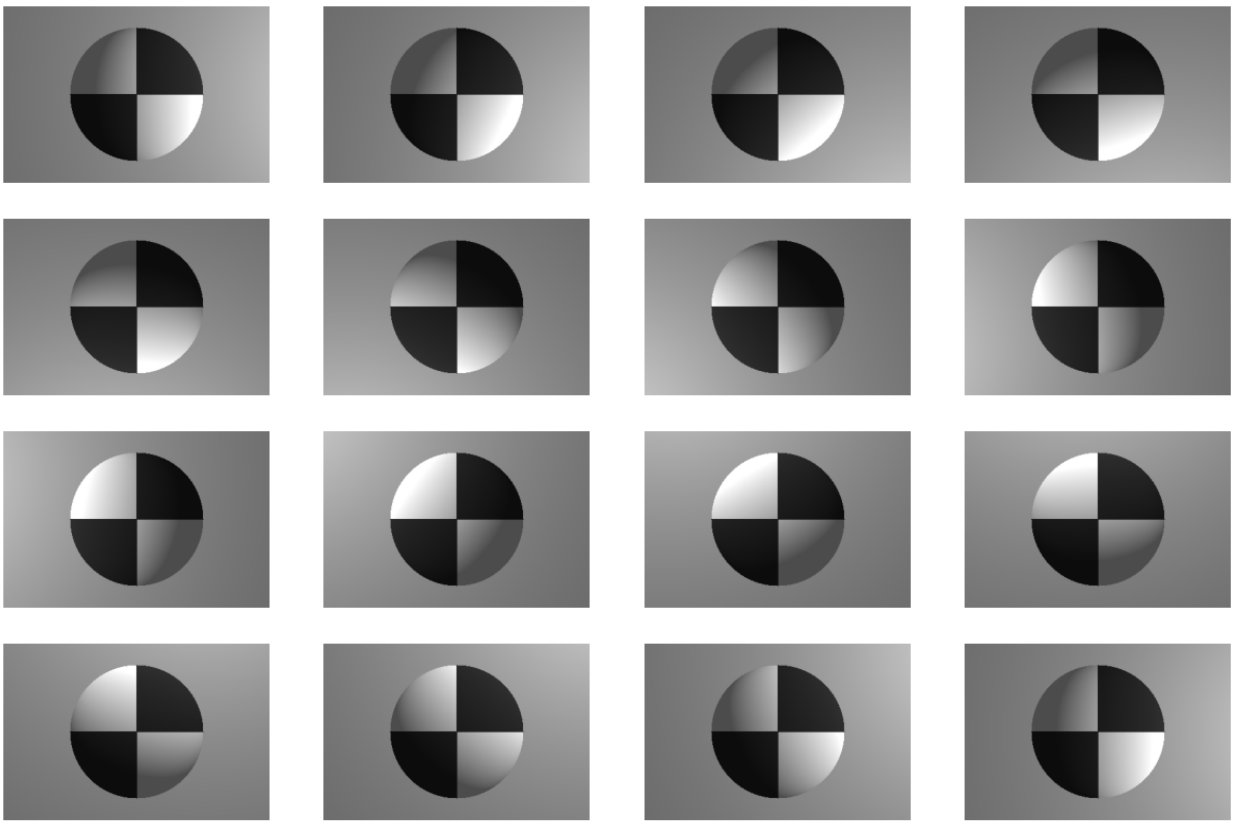

Figure 2: Specular 0 move point

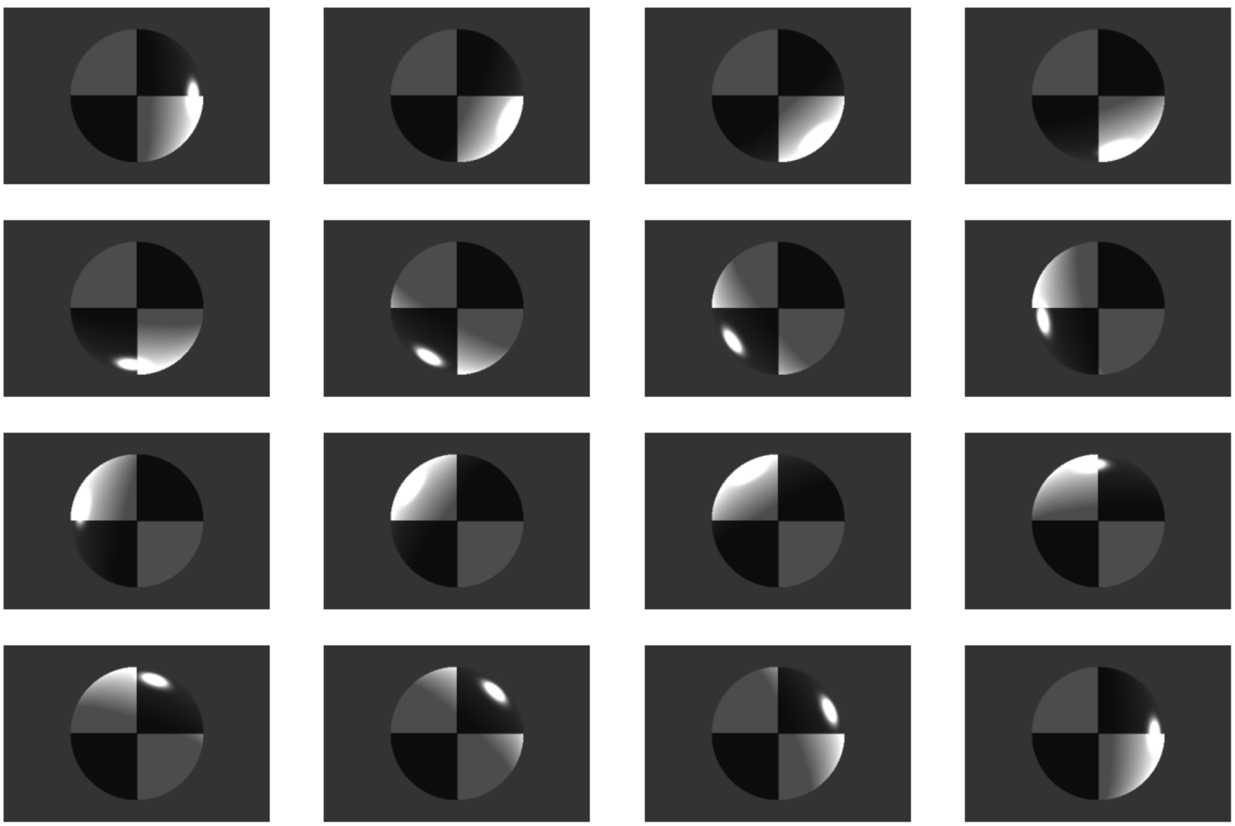

Figure 3: Specular 1 move direction

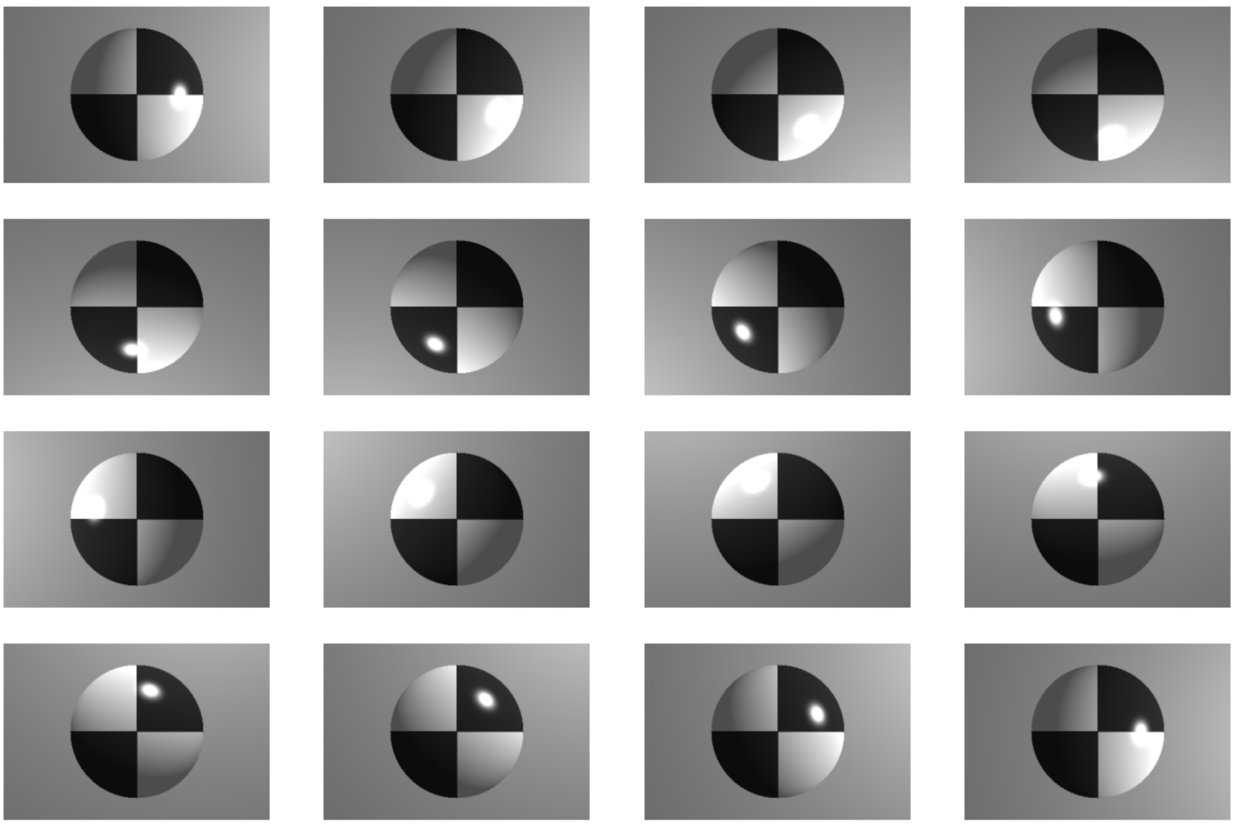

Figure 4: Specular 1 move point

# 4 Dynamic Perspective

1. First we have that:

$$x = \frac{fX}{Z}$$
$$y = \frac{fY}{Z}$$

   Thus take the derivative we have:

$$\dot{x} = \frac{f(\dot{X}Z - \dot{Z}X)}{Z^2}$$
$$\dot{y} = \frac{f(\dot{Y}Z - \dot{Z}Y)}{Z^2}$$

2. See **Figure 5-9**

   - In scenario 1, we have $t_z = 1$ and use road image
   - In scenario 2, we have $t_x = 1$ and use wall image
   - In scenario 3, we have $t_z = 1$ and use wall image
   - In scenario 4, we have $t_x = 1, t_y = 1, t_z = 1$ and use wall image
   - In scenario 5, we have $\omega_y = 1$ and use wall image

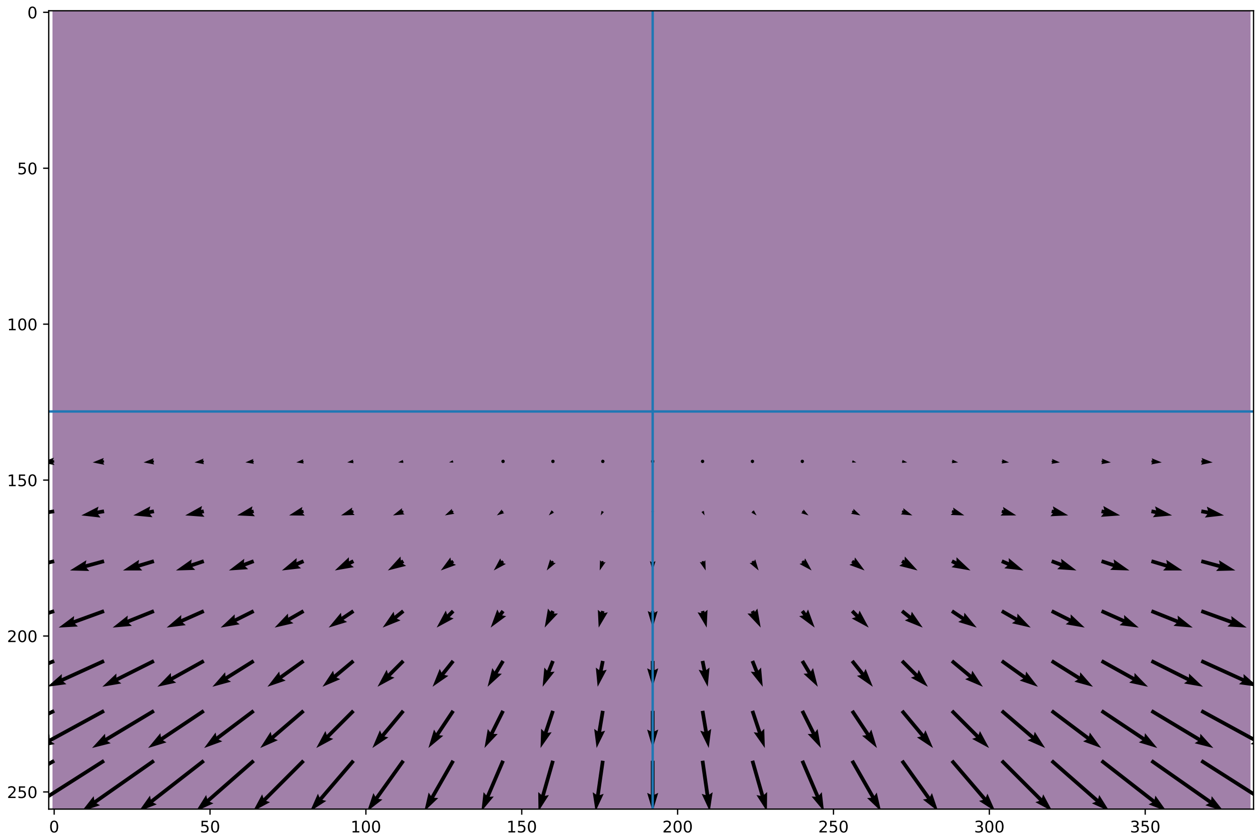


Figure 5: 4.2.1

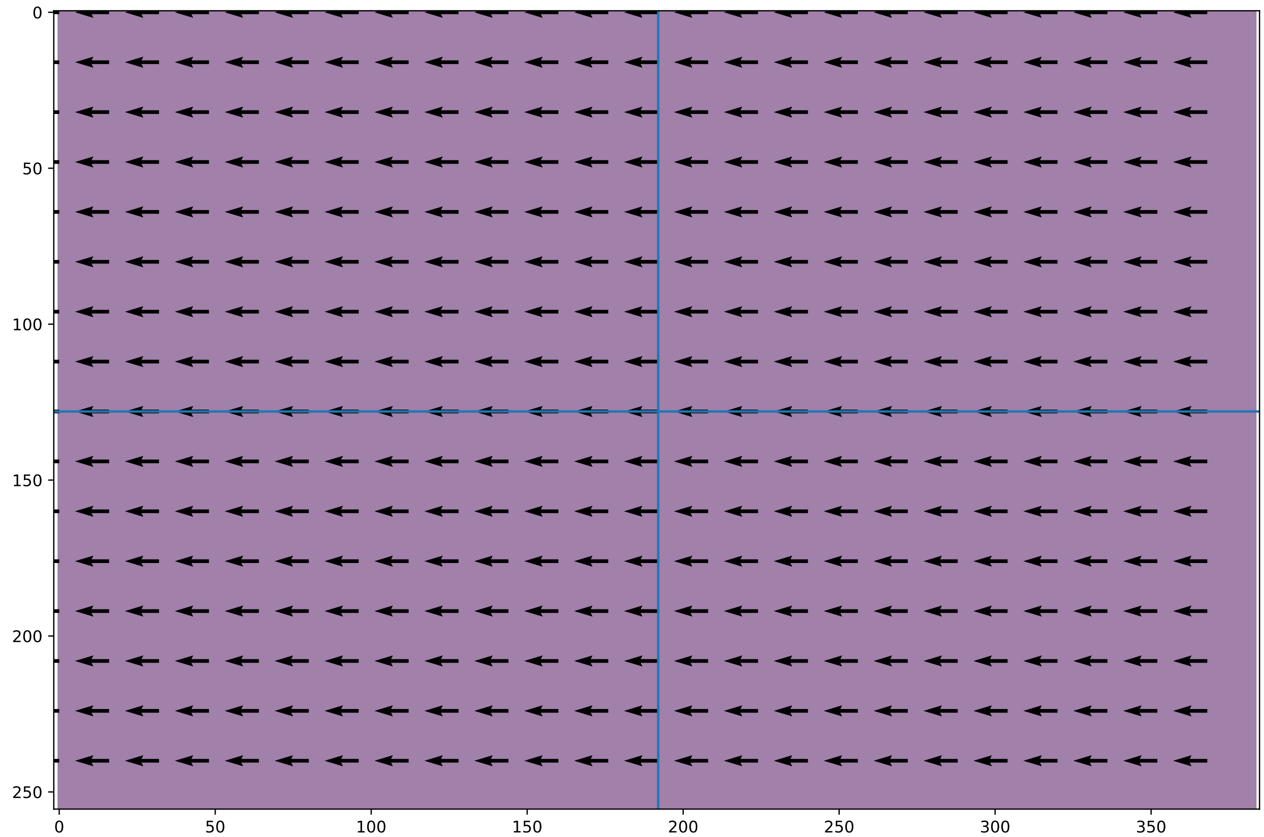


Figure 6: 4.2.2

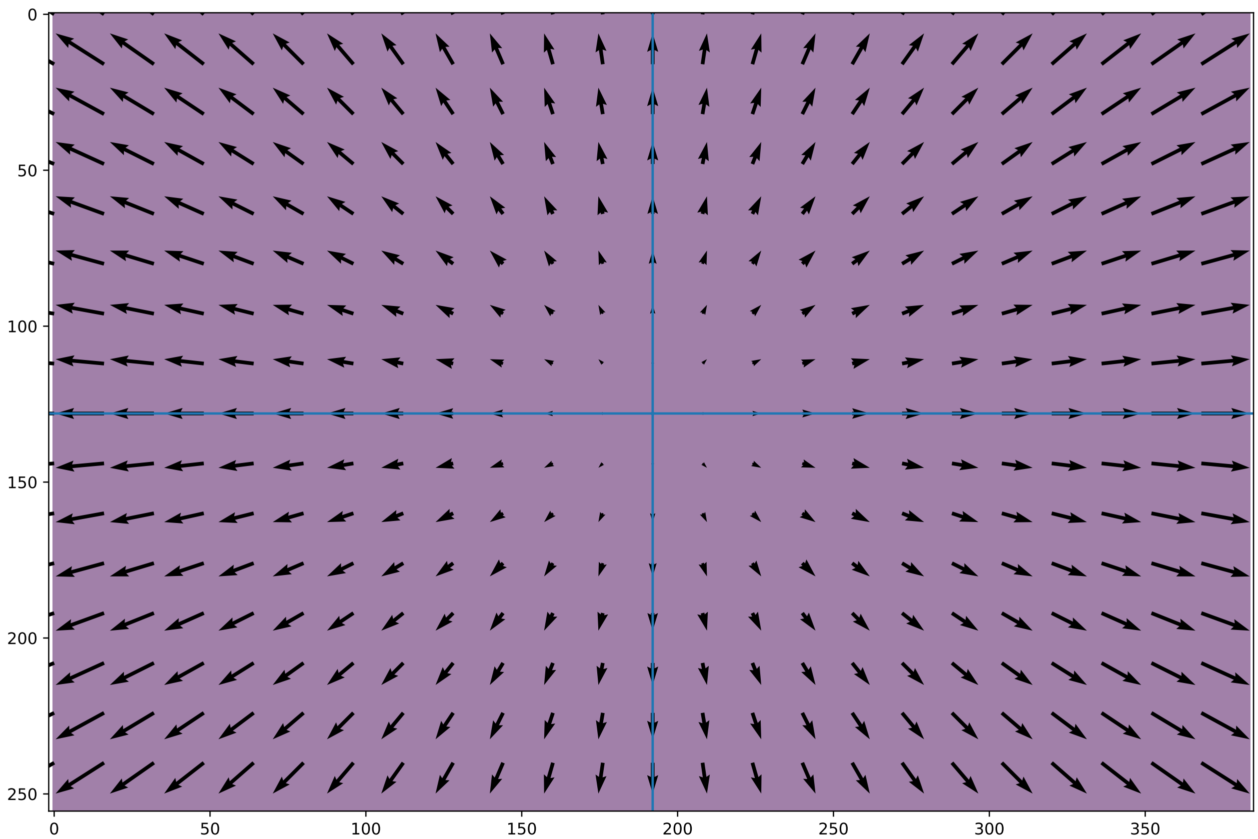


Figure 7: 4.2.3

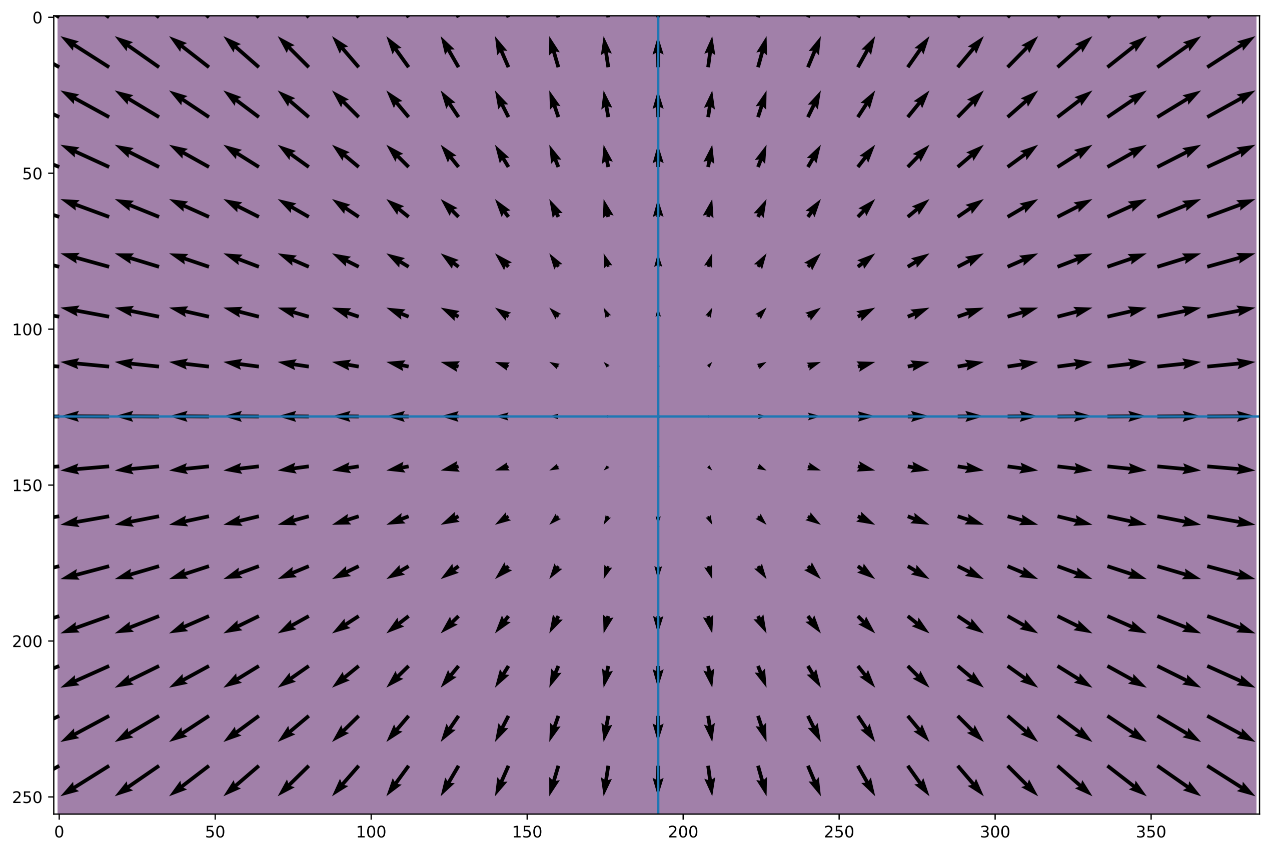


Figure 8: 4.2.4

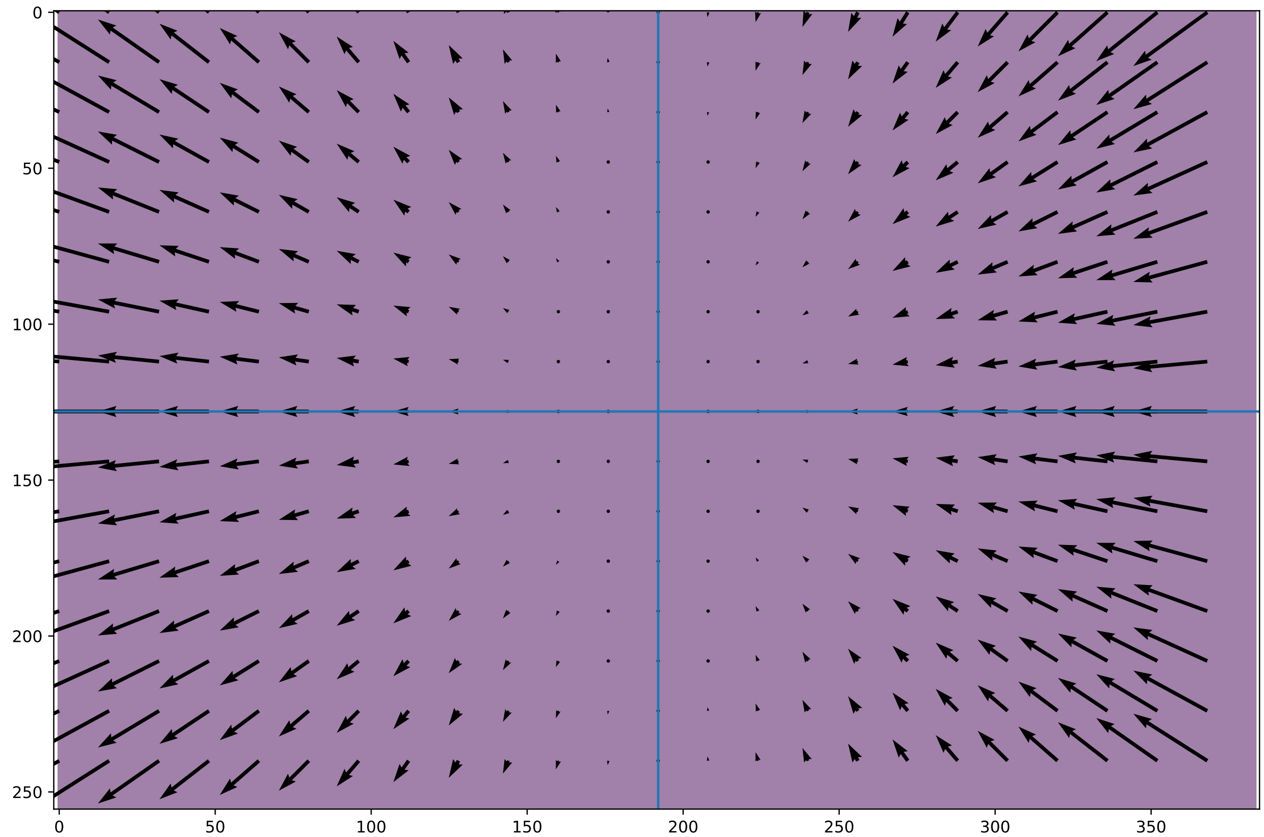


Figure 9: 4.2.5