UNIVERSAL QUANTUM GATE SETS & THE T-OPERATOR

6.S089 FINAL PROJECT

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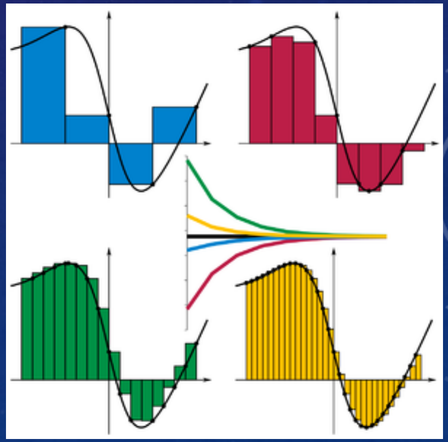
UNIVERSAL GATE SET DEFINITION

Linear operator whose inverse is its adjoint.
Product of unitary operators is unitary operators.

- Universal Gate Set: finite set of gates that can approximate any unitary matrix arbitrarily well
- Must be set to which ***any*** possible operation on a quantum computer belongs
  - In other words, any unitary operator can be expressed as finite sequence of gates from set
- Technically impossible: # of possible quantum gates is uncountable, whereas # of finite sequence from finite set is countable
- Only require that any quantum operation can be ***approximated*** by a sequence of gates from this finite set
  - **Solovay-Kitaev Theorem** guarantees quantum operations for unitaries on a constant # of qubits can be approximated efficiently
  - Arbitrary how accurate the approximation must be



CLIFFORD GROUP

PAULI

$$X = \sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad Y = \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \qquad Z = \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$|\mathbf{0}\rangle$

$|\mathbf{1}\rangle$

SUPERPOSITIONS

$$H = Hadmard = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

$$|+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$$

$$|-\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$$

[$\pi/2$ phase shift]

$$S = \begin{pmatrix} 1 & 0 \\ 0 & i \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\pi/2} \end{pmatrix} \qquad S^{\dagger} = \begin{pmatrix} 1 & 0 \\ 0 & -i \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & e^{-i\pi/2} \end{pmatrix}$$

$$|\circlearrowright\rangle = \frac{1}{\sqrt{2}}(|0\rangle + i|1\rangle)$$

$$|\circlearrowleft\rangle = \frac{1}{\sqrt{2}}(|0\rangle - i|1\rangle)$$

2-QUBIT

$$\oplus = C_{NOT} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

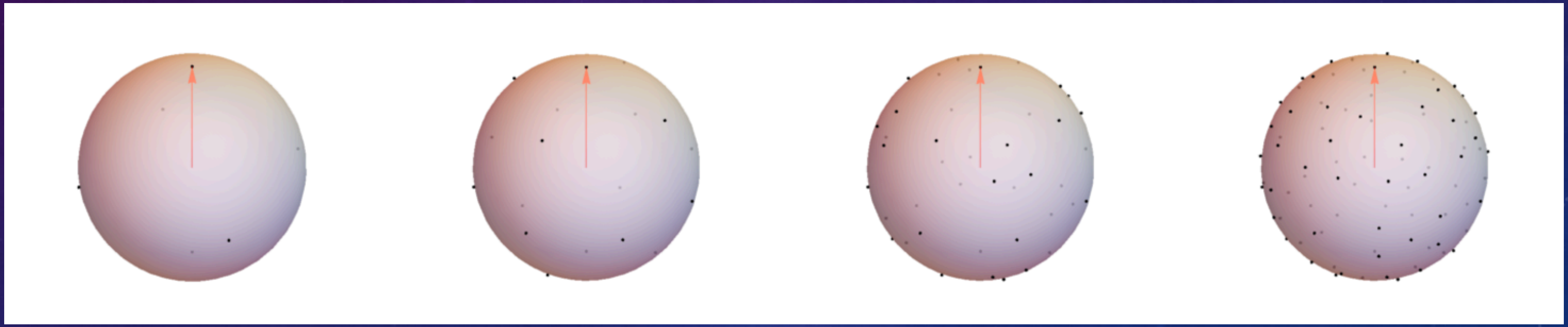
Can be simulated efficiently on classical computer $\Rightarrow$ NOT universal

GOTTESMAN-KNILL THEOREM

- Tells us that **stabilizer circuits** and even some highly entangled states can be efficiently simulated on a classical computer, meaning it is not universal
- Q: What is a stabilizer circuit?
- A: A quantum circuit with the following elements:
  - Preparation of qubits in computational basis states
  - Quantum gates from the Clifford group
  - Measurement in computational bases
- Cannot harness full power of quantum computation ⇒ **must include at least one non-Clifford gate in our circuits**

T-GATE

- Non-Clifford
- Makes it possible to reach all different points of the Bloch Sphere
- By increasing the # of T-gates in our circuit (**T-depth**) we cover Bloch sphere more densely with states we can reach



$\boxed{T} = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\pi/4} \end{pmatrix}$

$\boxed{T^\dagger} = \begin{pmatrix} 1 & 0 \\ 0 & e^{-i\pi/4} \end{pmatrix}$

[$\pi/4$ phase shift]

S-gate = T^2

2-QUBIT UNIVERSAL GATE SET

- Simple set: { H ⊕ T }

- Barenco et. al 1995: any unitary matrix can be written as a combo of single- and 2-qubit gates, whereas classical reversible computing requires 3-bit gates (i.e. Toffoli)
    - In quantum world, a generic interaction between 2 qubits (that can be implemented accurately between any 2 qubits) can be used to calculate anything

TOFFOLI & DEUTSCH GATES

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}$$

- **Toffoli Gate** = CCNOT : universal classical reversible logic gate
  - 3-bit input & output
  - If first 2 bits are 1, inverts 3rd; otherwise all stay the same
  - Reversible ⇒ time-invertible, mapping from states to successors is 1-to-1
  - Can be used to build systems that perform any desired Boolean function computation, in reversible manner

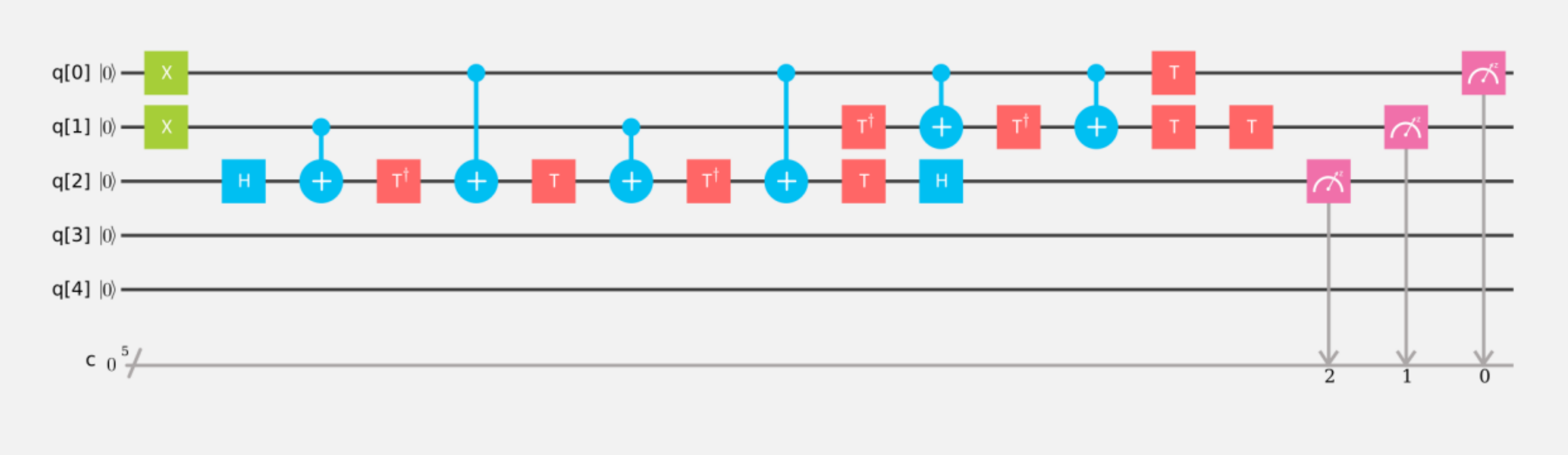
- **Deutsch Gate**: single-gate set of universal quantum gates
  - performs transformation:

$$|\,a, b, c\,\rangle \;\mapsto\; \begin{cases} i\cos(\theta)\,|\,a, b, c\,\rangle + \sin(\theta)\,|\,a, b, 1-c\,\rangle & for\ a = b = 1 \\ |\,a, b, c\,\rangle & otherwise \end{cases}$$

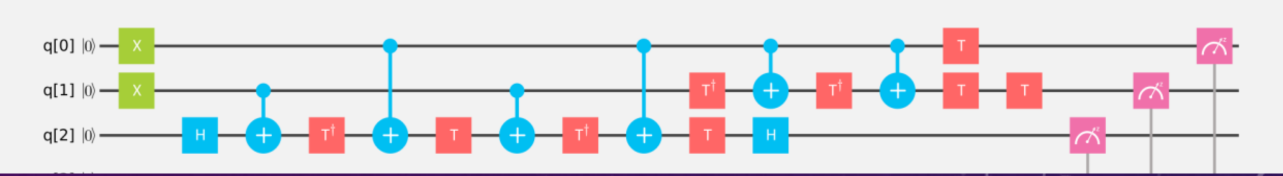
  - Classical Toffoli gate is reducible to quantum D($\boldsymbol{\pi/2}$)
  - Meaning all classical logic operations can be performed on universal quantum computer

IBM QUANTUM EXPERIENCE

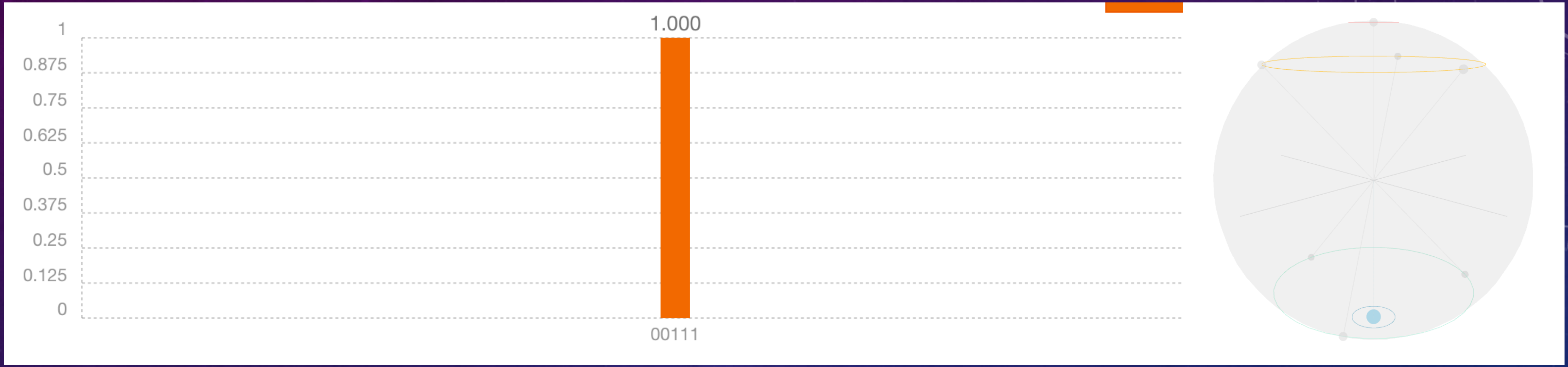
- Implementing a Toffoli Gate using H, CNOT, and T
- Run on the IBM Quantum Computer, with 1024 shots (3 units)



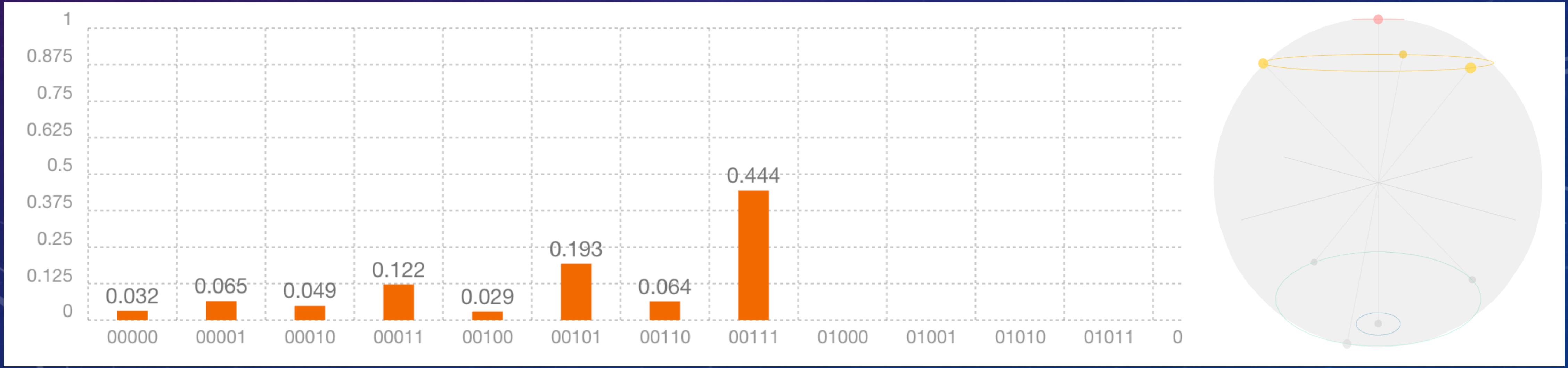
RESULTS



SIMULATION

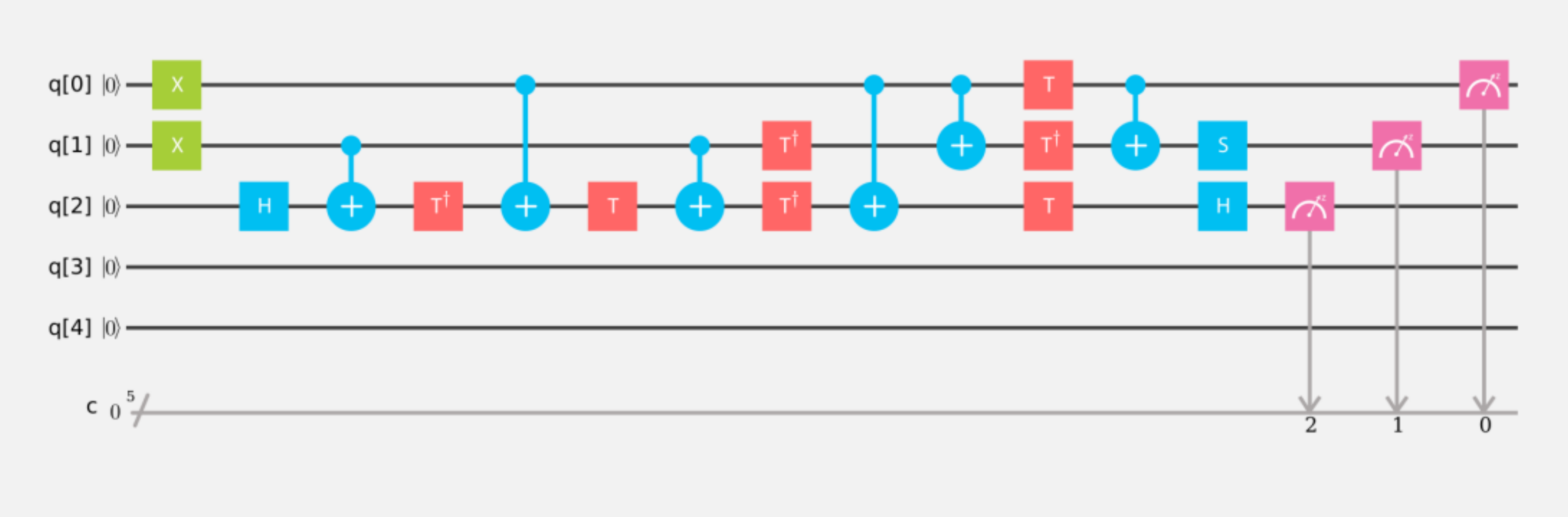


EXPERIMENT

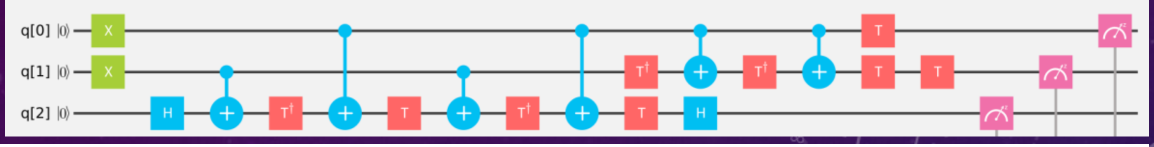


EXPERIMENT #2

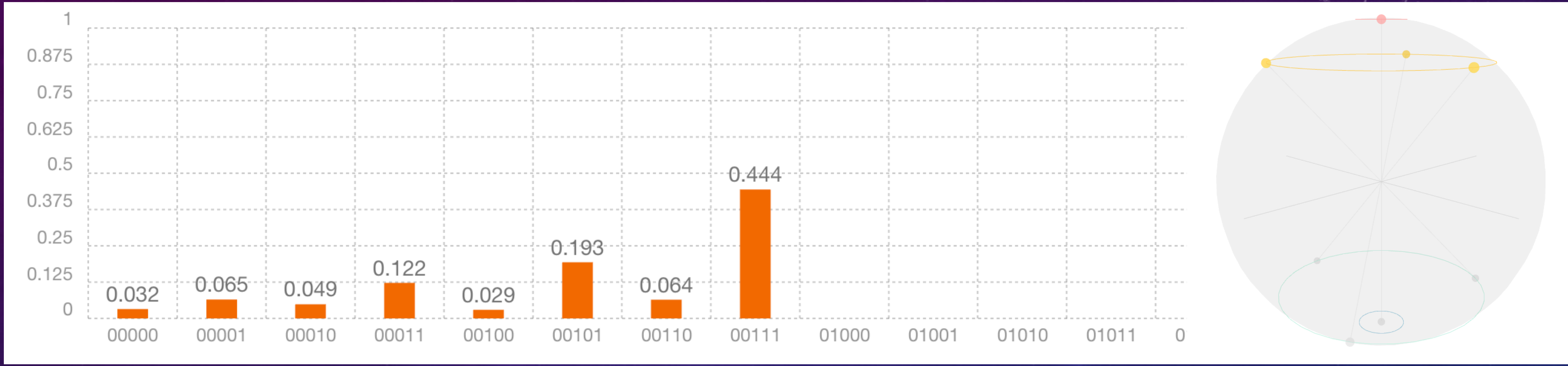
- Trying to improve the results from the first experiment
- Reducing T-depth from 7 to 4 (would be T-depth 5, but we use S gate instead of T^2)
- Run on the IBM Quantum Computer, with 1024 shots (3 units)

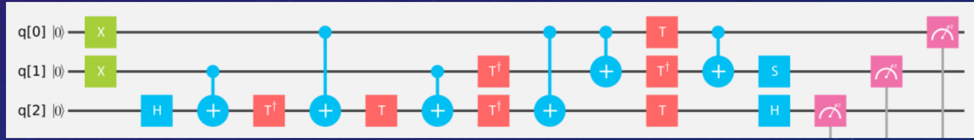


EXPERIMENT RESULTS

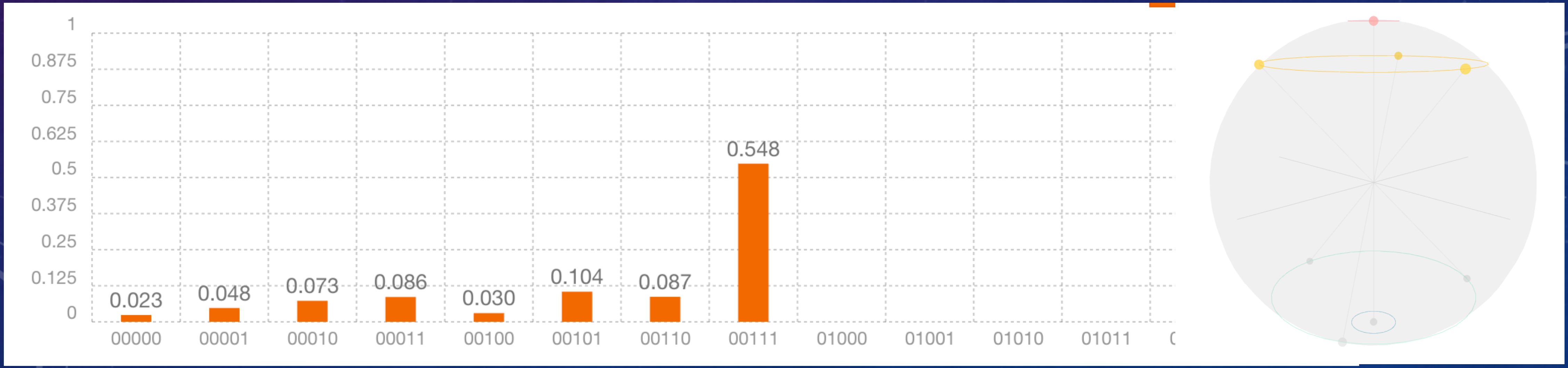


EXPERIMENT #1





EXPERIMENT #2



SOURCES

- Elementary Gates for Quantum Computation, Barenco et. al (1995)
- Quantum Circuits of T-Depth One, Selinger (2013)
- IBM Quantum Experience User Guide
- Wikipedia
- Caltech Quantum Computation (Physics/CS 219) Course Notes, Preskill