Making Sense of Everettian Quantum Measurement: A Study of Quantum Coin Flipping

Foundations of Probability [24.280] - Final Paper

Francisca Vasconcelos

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1 Introduction

At the heart of quantum mechanics lies the notion of the "collapse of the wavefunction." Although we model quantum systems as complex wavefunctions, we can only observe a portion of this full quantum state. The measurement process is probabilistic, which leads to a debate over the underlying processes at play and physical realization of the measurement itself.

In this paper, we focus on the arguments presented in the work *Self-locating Uncertainty and the Origin of Probability in Everettian Quantum Mechanics*, by Charles Sebens and Sean Carroll. Specifically, we introduce the reader to the fundamental postulates of quantum mechanics, the Many-Worlds Interpretation, the notion of self-locating uncertainty, the Indifference Principle/branch-counting, and the Epistemic Seperability Principle (ESP). We use these tools to study the Quantum Sleeping Beauty Paradoxes, which raise uncertainty in how we should assign probabilities to quantum measurements. We conclude with a brief discussion of the implications of this work for the field of Quantum State Tomography and of the Many-Worlds Interpretation for determinism.

2 A Brief Introduction to Quantum Mechanics

In this section we provide a brief overview of the foundations of quantum mechanics, in order to provide the necessary background for our later discussions.

2.1 The Fundamental Postulates of QM

There are six fundamental postulates of quantum mechanics[1]:

1. At each instant the state of a physical system is represented by a ket $|\psi\rangle$ in the space of states (a special space, known as a Hilbert space).

2. Every observable attribute of a physical system is described by an operator that acts on the kets that describe the system.

[1] MIT 8.05 course notes: `http://web.mit.edu/8.05/handouts/jaffe1.pdf`

3. The only possible result of the measurement of an observable A is one of the eigenvalues of the corresponding operator $\hat{A}$.

4. When a measurement of an observable A is made on a generic state $|\psi\rangle$, the probability of obtaining an eigenvalue a_n is given by the square of the inner product of $|\psi\rangle$ with the eigenstate $|a_n\rangle$, $|\langle a_n|\psi\rangle|^2$.

5. Immediately after the measurement of an observable A has yielded a value a_n, the state of the system is the normalized eigenstate $|a_n\rangle$.

6. The time evolution of a quantum system preserves the normalization of the associated ket. The time evolution of the state of a quantum system is described by $|\psi(t)\rangle = \hat{U}(t, t_0)|\psi(t_0)\rangle$, for some unitary operator $\hat{U}$.

From these six postulates and the Schrödinger equation, we can derive the majority of modern quantum mechanics theory. Of particular importance to our discourse is postulate (4), which is known as Born's Rule.

These fundamental postulates were developed to explain physically observed phenomena and have been widely accepted for their ability to precisely calculate observed experimental outcomes. This means that the underlying justification and physical intuition for the postulates is still an active area of research and debate. There exist several interpretations of quantum mechanics. The most prominent is the Copenhagen (or Heisenberg) Interpretation, which views the measurement of quantum states (collapse of the wavefunction) as a purely stochastic process, picking out the final state randomly in accordance to the probabilities assigned by Born's Rule.

In their work, *Self-locating Uncertainty and the Origin of Probability in Everettian Quantum Mechanics*, Charles Sebens and Sean Carroll argue for the Many-Worlds (Everettian) Interpretation, as formulated by Hugh Everett in 1957. Since we will be focusing on the arguments presented in this paper, we too will inherit this many-worlds view. This interpretation is explained in section [2.3].

2.2 Quantum Coin Notation and Born's Rule

Throughout this paper, we will assess thought experiments involving, what we call, quantum coins. This is meant to abstract some of the physics and will represent actual experiments one may perform in quantum mechanics, such as measuring the z-spin of a particle prepared in the x-spin eigenstate. In order to make use of our quantum coin, we introduce some notation following from quantum mechanics

$|\psi_{coin}\rangle$: the wavefunction representing all possible quantum coin toss outcomes

$|\phi_H\rangle$: the observable quantum coin toss outcome of Heads

$|\phi_T\rangle$: the observable quantum coin toss outcome of Tails.

In the case of a "fair" quantum coin, we can write our wavefunction as

$$|\psi_{coin}\rangle = \frac{1}{\sqrt{2}}\Big(|\phi_H\rangle + |\phi_T\rangle\Big). \tag{1}$$

It should be noted that $|\phi_H\rangle$ and $|\phi_T\rangle$ represent orthonormal basis functions, such that $\langle\phi_H|\phi_T\rangle = \langle\phi_T|\phi_H\rangle = 0$ and $\langle\phi_H|\phi_H\rangle = \langle\phi_T|\phi_T\rangle = 1$. From Born's Rule (fundamental postulate 4), we can calculate the probabilities of either outcome as

$$P(\text{Heads}) = \big|\langle\phi_H|\psi_{coin}\rangle\big|^2 = \left|\frac{1}{\sqrt{2}}\Big(\langle\phi_H|\phi_H\rangle + \langle\phi_H|\phi_T\rangle\Big)\right|^2 = \left(\frac{1}{\sqrt{2}}\right)^2\big|1+0\big|^2 = \frac{1}{2} \tag{2}$$

$$P(\text{Tails}) = \big|\langle\phi_T|\psi_{coin}\rangle\big|^2 = \left|\frac{1}{\sqrt{2}}\Big(\langle\phi_T|\phi_H\rangle + \langle\phi_T|\phi_T\rangle\Big)\right|^2 = \left(\frac{1}{\sqrt{2}}\right)^2\big|0+1\big|^2 = \frac{1}{2}. \tag{3}$$

Furthermore, we can represent a "biased" quantum coin as

$$|\psi_{coin}\rangle = \sqrt{\frac{1}{3}}\;|\phi_H\rangle + \sqrt{\frac{2}{3}}\;|\phi_T\rangle \tag{4}$$

from which it follows that we have measurement outcome probabilites

$$P(\text{Heads}) = \big|\langle\phi_H|\psi_{coin}\rangle\big|^2 = \left|\sqrt{\frac{1}{3}}\;\langle\phi_H|\phi_H\rangle + \sqrt{\frac{2}{3}}\;\langle\phi_H|\phi_T\rangle\right|^2 = \left(\sqrt{\frac{1}{3}}\right)^2\big|1+0\big|^2 = \frac{1}{3} \tag{5}$$

$$P(\text{Tails}) = \big|\langle\phi_T|\psi_{coin}\rangle\big|^2 = \left|\sqrt{\frac{1}{3}}\;\langle\phi_T|\phi_H\rangle + \sqrt{\frac{2}{3}}\;\langle\phi_T|\phi_H\rangle\right|^2 = \left(\sqrt{\frac{1}{3}}\right)^2\big|\sqrt{2}+0\big|^2 = \frac{2}{3} \tag{6}$$

We can also represent the state of Alice, the experimenter flipping the quantum coin, as $|A\rangle$ before she tosses the coin. Once she observes a coin toss outcome, we can update her state to be $|A_H\rangle$ if she observes Heads and $|A_T\rangle$ if she observes Tails. Note once again that $|A_H\rangle$ and $|A_T\rangle$ are orthonormal basis functions. Throughout the paper we will construct similar wavefunctions that additionally describe the environment and measurement devices that will display the experimental outcomes.

2.3 The Many-Worlds (Everettian) Interpretation

To explain the Many-Worlds Interpretation, we begin by formalizing the Copenhagen Interpretation. Imagine that experimenter Alice has a fair quantum coin, described by

$$|\psi_{coin}\rangle = \frac{1}{\sqrt{2}}\Big(|\phi_H\rangle + |\phi_T\rangle\Big) \tag{7}$$

Once she has flipped the coin, she can either observe an outcome of Heads or Tails. In the Copenhagen view, her quantum coin wavefunction will instantaneously collapse to either Heads or Tails

$$|\psi_{coin}\rangle_H = |\phi_H\rangle \tag{8}$$

$$|\psi_{coin}\rangle_T = |\phi_T\rangle \tag{9}$$

upon measurement and observation, as illustrated in [Fig 1]. For any future time, Alice will have full certainty in her observed outcome (assuming she does not flip the quantum coin again). Under this world-view, the process of measuring stochastically alters the physical reality.

$$|\psi_{coin}\rangle = \frac{1}{\sqrt{2}}(|\phi_H\rangle + |\phi_T\rangle) \xrightarrow{\text{measurement}} |\psi_{coin}\rangle_{H/T} = |\phi_{H/T}\rangle$$

Figure 1: The Copenhagen Interpretation of the measurement of a fair quantum coin toss.

The Many-Worlds Interpretation, however, argues that during the collapse of the wavefunction the world actually splits into several branches, corresponding to the different measurement outcomes and their relative likelihoods, as illustrated in [Fig 2]. In the case of a fair coin, the world will split into two branches, one in which the outcome of the toss is Heads

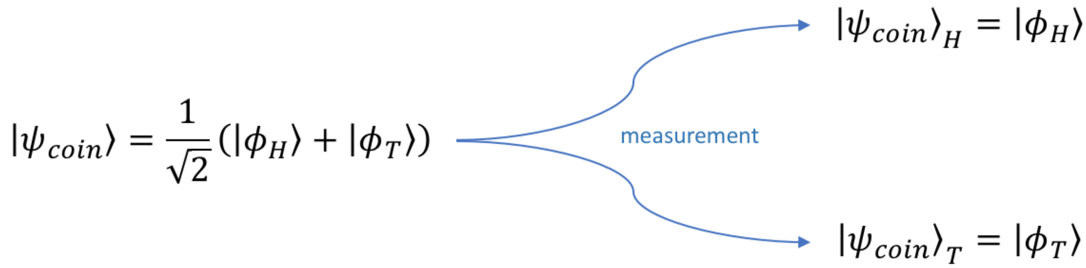

Figure 2: The Many-Worlds Interpretation of the measurement of a fair quantum coin toss.

and one in which it is Tails. Mathematically, this works out the same as in the Copenhagen Interpretation, since, within a given world branch, we proceed with the same equations. Although this interpretation is critical for the thought experiments we will perform, it creates some challenges, which will be described and explored in section [3]. Furthermore, the notion of branching can have several metaphysical interpretations in itself, which we will discuss in sections [4] and [5].

3 Quantum Mechanics and Probability

In this section we look into some key principles that unite the philosophy of quantum mechanics with the philosophy of probability.

3.1 Self-Locating Uncertainty

Let us now create a universal wavefunction which not only represents the state of our quantum coin $|\psi_{coin}\rangle$, but also the state of experimenter Alice $|A\rangle$, the measurement device $|D\rangle$, and the rest of the environment external to the experiment $|E\rangle$ (i.e. the rest of the universe).

$$|\Psi\rangle = |A\rangle|\psi_{coin}\rangle|D\rangle|E\rangle \tag{10}$$

Depending on whether the quantum coin lands Heads or Tails, the device and the environment will immediately, respectively take one of two forms: $|D_H\rangle$ and $|E_H\rangle$ or $|D_T\rangle$ and $|E_T\rangle$. Thus, at time t_0, after the coin flip and measurement, but before Alice observes the outcome of the experiment, the wavefunction becomes

$$|\Psi(t_0)\rangle = \frac{1}{\sqrt{2}}\,|A\rangle\Big(|\phi_H\rangle|D_H\rangle|E_H\rangle + |\phi_T\rangle|D_T\rangle|E_T\rangle\Big). \tag{11}$$

This equivalent to the expression

$$|\Psi(t_0)\rangle = \frac{1}{\sqrt{2}}\Big(|A\rangle|\phi_H\rangle|D_H\rangle|E_H\rangle + |A\rangle|\phi_T\rangle|D_T\rangle|E_T\rangle\Big). \tag{12}$$

Although mathematically equivalent, the two expressions have very different interpretations. Equation (11) makes it seem as though Alice is still one person, while the rest of the state branched into two branches. Equation (12), on the other hand, gives the impression that Alice has branched into two identical copies. This second equation embodies the Everettian Many-Worlds Interpretation discussed in section [2.3].

If we step back, this interpretation suggests an interesting, slightly discomforting notion. Since $|\Psi\rangle$ represents the state of the entire universe, everything in the universe that was not part of Alice's quantum coin toss experiment (i.e. Alice $|A\rangle$, the coin $|\psi_{coin}\rangle$, or the measurement device $|D\rangle$) is contained in the external environment state $|E\rangle$. Note how when Alice performed her coin toss, not only was everything in the experiment duplicated to two branches, but so was $|E\rangle$. This means that we were duplicated, MIT was duplicated, and the entirety of the universe was duplicated between branches, solely because of Alice's quantum coin toss. If we think about this in terms of all the quantum phenomena happening at any given instance in time, the universe is branching at an incredible rate and we are affected by measurements performed in distant galaxies. As we will discuss in the next section, this result prohibits the application of the Indifference Principle to quantum measurement, given no further confounding factors.

Given that we accept this odd conclusion of the Many-Worlds Interpretation, we look back at Alice. Once Alice observes the outcome of the coin toss, her state will update to $|A_H\rangle$ or $|A_T\rangle$, respectively. However, in the period before she observes the experimental outcome, she is still in state $|A\rangle$ but knows that the quantum coin either landed Heads or Tails. During this post-measurement, pre-observation period, she knows she is in one of the two branches of $|\Psi\rangle$, but unsure as to which branch she actually, consciously lies in. We say that Alice has 'self-locating uncertainty.' This phenomena will play an important role in our later discussion of the Sleeping Beauty Paradox.

3.2 The Troubles of Indifference and Branch-Counting

Classically, the Indifference Principle states that if there are n mutually exclusive and collectively exhaustive possible outcomes, we should assign each outcome a probability of $\frac{1}{n}$. In the quantum realm, we can connect this to the Many-Worlds Interpretation through the notion of branch-counting. When we possess self-locating uncertainty, we can use the idea of branch-counting to assign likelihood $\frac{1}{\#\text{ branches}}$ to being in one of the given branches. In order to see why the Indifference Principle fails when describing a quantum measurement, we propose the Double Quantum Coin Flip Experiment, which we will abbreviate as DQCFE and is illustrated in [Fig 3].

In this thought experiment, suppose we now have two observers, Alice and Bob. Alice will begin by flipping her quantum coin at time t_A and then measure her coin. Bob, will observe the outcome and if Alice's coin lands Heads, he will flip his own at time t_B. He will measure and observe the outcome and the experiment will terminate. If Alice's coin lands Tails, the experiment will automatically terminate, without any coin flip by Bob. Alice will not be allowed to observe the outcome of either coin toss, but is aware of the experimental set-up. There are two variants of this experiment we will consider, to see where the problem emerges.

In the first version of the DQCFE, we will ask Alice for her credence that her quantum coin landed Tails following time t_A (after she flipped and measured it), but before time t_B (when Bob flips and measures his coin). As illustrated in [Fig 3], there will be two branches in the world at $t_A < t < t_B$, which Alice is also aware of. Thus, by the notions of indifference and branch-counting, Alice should have credence $\frac{1}{2}$ that her coin landed Tails.

In the second version of the DQCFE, we will only ask Alice for her credence after Bob flips and measures his coin, i.e. $t > t_B$. As illustrated in [Fig 3], there will be three branches to our wavefunction, which Alice is aware of. Thus, by the notions of indifference and branch-counting, Alice should have credence $\frac{1}{3}$ that her coin landed Tails.

So, what exactly happened at t_B for Alice to change her credence?

$$|\Psi(t=0)\rangle = |A\rangle|D\rangle_A|\psi_{coin}\rangle_A|B\rangle|D\rangle_B|\psi_{coin}\rangle_B|E\rangle_{AB} \tag{13}$$

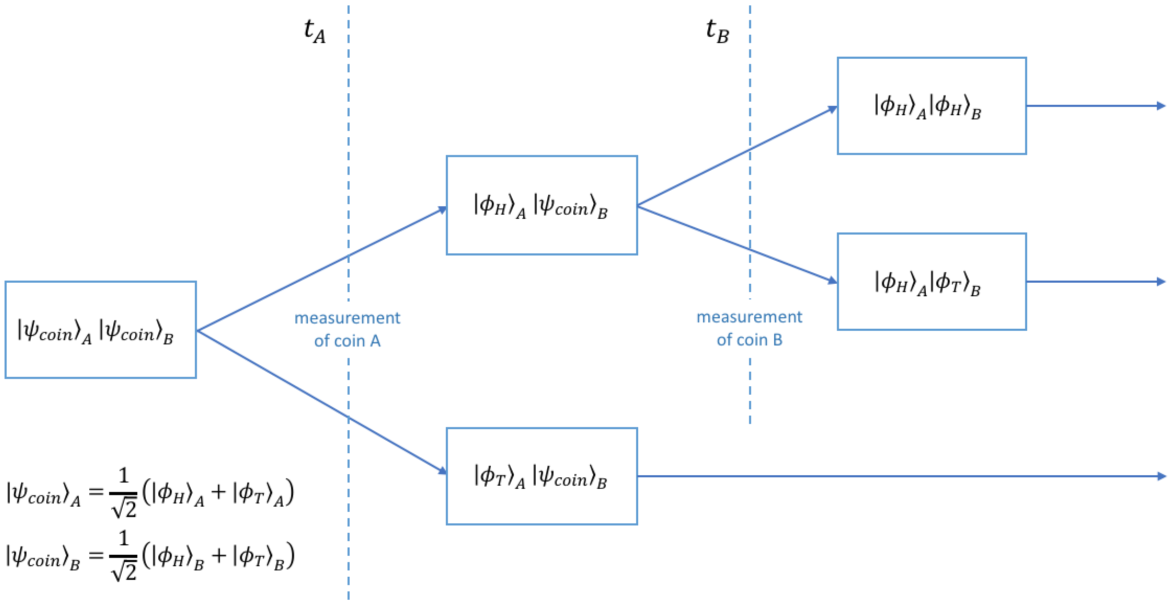

Figure 3: The Double Quantum Coin Flip Experiment (DQCFE).

$$\begin{aligned} |\Psi(t_A < t < t_B)\rangle \;&=\; \tfrac{1}{\sqrt{2}}\; |A\rangle|D_H\rangle_A|\phi_H\rangle_A|B\rangle|D\rangle_B|\psi_{coin}\rangle_B|E_H\rangle_{AB} \\ &+\; \tfrac{1}{\sqrt{2}}\; |A\rangle|D_T\rangle_A|\phi_T\rangle_A|B\rangle|D\rangle_B|\psi_{coin}\rangle_B|E_T\rangle_{AB} \end{aligned} \tag{14}$$

$$\begin{aligned} |\Psi(t > t_B)\rangle \;&=\; \tfrac{1}{2}\; |A\rangle|D_H\rangle_A|\phi_H\rangle_A|B_H\rangle|D_H\rangle_B|\phi_H\rangle_B|E_{HH}\rangle_{AB} \\ &+\; \tfrac{1}{2}\; |A\rangle|D_H\rangle_A|\phi_H\rangle_A|B_T\rangle|D_T\rangle_B|\phi_T\rangle_B|E_{HT}\rangle_{AB} \\ &+\; \tfrac{1}{\sqrt{2}}\; |A\rangle|D_T\rangle_A|\phi_T\rangle_A|B\rangle|D\rangle_B|\psi_{coin}\rangle_B|E_T\rangle_{AB} \end{aligned} \tag{15}$$

Looking at these equations and focusing specifically on the elements pertaining to Alice within the experiment ($|A\rangle$, $|\psi_{coin}\rangle_A$, and $|D\rangle_A$), nothing changes at the time t_B. One factor, however, that has a minimal dependence on Alice, which does change at time t_B, is the external environmental factor $|E\rangle_{AB}$. In fact, it is exactly this environmental component that causes branch-counting to contradict the likelihoods we observe experimentally and calculate using Born's Rule (as will be discussed in the next section). As eloquently stated by Sebens and Carroll,

> *It is tempting to think that the number of copies of Alice cannot change without her physical state changing–this is the way things work in classical physics. But in Everettian quantum mechanics, changes that purely affect her environment*

can change the number of copies of Alice in existence…Two intuitive constraints come into conflict: indifference, and the belief that Alice's probabilities should be unaffected by changes in the state of her environment.

Although it could be argued that the Indifference Principle is right and the Many-Worlds Interpretation is flawed, we will address such concerns by introducing the Epistemic Seperability Principle.

3.3 Epistemic Seperability Principle (ESP)

Using Born's Rule and (15), we calculate Alice's credences of her coin toss outcome in the DQCFE as,

$$P(\ |\phi_H\rangle_A \mid t_A < t < t_B\ ) = \left(\frac{1}{\sqrt{2}}\right)^2 = \frac{1}{2} \tag{16}$$

$$P(\ |\phi_T\rangle_A \mid t_A < t < t_B\ ) = \left(\frac{1}{\sqrt{2}}\right)^2 = \frac{1}{2} \tag{17}$$

$$P(\ |\phi_H\rangle_A \mid t > t_B\ ) = \left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2 = \frac{1}{4} + \frac{1}{4} = \frac{1}{2} \tag{18}$$

$$P(\ |\phi_T\rangle_A \mid t > t_B\ ) = \left(\frac{1}{\sqrt{2}}\right)^2 = \frac{1}{2} \tag{19}$$

Note that the credence does not change at t_B. From the form of (15), the key difference between this Born's Rule-based calculation and our indifference-based calculation in section [3.2] is whether we condition on the environmental factor $|E\rangle_{AB}$. In the case of indifference, or branch-counting, we consider the two branches in which Alice has outcome Heads, but the environment is different (because of the different outcomes of Bob's measurement), to be different branches. If we were to visualize the logic of Born's Rule, we would see that it assigns weights to the two Heads branches such that, relative to the Tails branch, they are equivalent to essentially one branch.

Thus, Bohr's Rule provides a mathematical justification that physically we should disregard changes in the external environment when assigning measurement credences. This is known as the Epistemic Seperability Principle (ESP). Mathematically,

$$P(B|\Psi_E) = P(B|\Psi), \tag{20}$$

where B is the likelihood of being in a given branch, Ψ_E is a wavefunction which solely describes the states in our experiment

$$|\Psi_E\rangle = |A\rangle|D\rangle_A|\psi_{coin}\rangle_A|B\rangle|D\rangle_B|\psi_{coin}\rangle_B, \tag{21}$$

and Ψ is the universal wavefunction which additionally accounts for the environment external to the experiment

$$|\Psi\rangle = |\Psi_E\rangle|E\rangle_{AB} = |A\rangle|D\rangle_A|\psi_{coin}\rangle_A|B\rangle|D\rangle_B|\psi_{coin}\rangle_B|E\rangle_{AB}. \tag{22}$$

Due to this conditional probability, it is clear that branch-counting does not hold generally for the universal wavefunction, but instead works for a modified wavefunction that solely accounts for the immediate measurement/experimental setup. In fact, there is an even stronger claim (Strong-ESP), which states that this conditioning is invariant to physically and temporally reallocating our experiment within the universe, given that $|\Psi_E\rangle = |\Psi'_E\rangle$.

4 The Quantum Sleeping Beauty Paradoxes

Although we made a strong argument for the use of Born's Rule to address branch-location induced uncertainty, it turns out that there are some other more complex situations in which Born's Rule does not provide the full picture. In this section we specifically consider two examples of temporal uncertainty (in addition to branch-location uncertainty) to show that, given an additional confounding factor, branch-counting may actually result in a credence that better describes our system. These thought experiments are similar to the "Sleeping Puzzle" discussed in 24.280. We will also briefly discuss the metaphysics behind the branching notions we use in the thought experiments, inherent to the Many-Worlds Interpretation, and the implications of these paradoxes for Quantum State Tomography.

4.1 The Two-Branch Quantum Sleeping Beauty

In the Two-Branch Quantum Sleeping Beauty Paradox, the subject, Beauty, will be put to sleep on Thursday night and sleep through Sunday, unless woken up. A quantum coin will be flipped and regardless of the outcome, we will wake Beauty up on Friday night. If the quantum coin lands Heads, we will also wake Beauty up on Saturday night, but she will have no recollection of waking up on Friday (she will only remember going to bed on Thursday). If the coin lands Tails, we will not wake Beauty up, but she will still remember waking up on Friday. Beauty is aware of this experimental set-up and will not forget it at any point in the experiment. This setup is illustrated in [Fig 4].

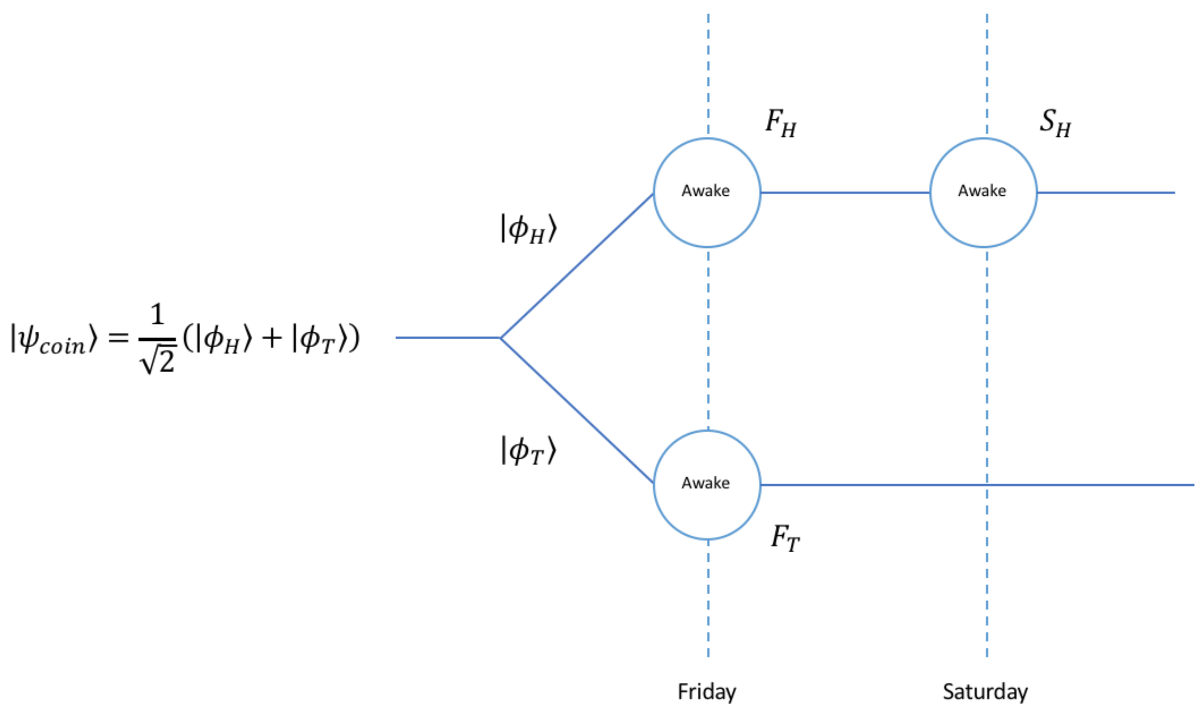

Figure 4: The Two-Branch Quantum Sleeping Beauty Experiment.

Now, imagine that Beauty wakes up and only remembers going to sleep on Thursday. This means that it could be Friday for either quantum coin toss outcome or Saturday for a Heads outcome. What credence should she assign the quantum coin having landed Heads?

According to Born's Rule, it does not matter what day it is. The amplitudes of $|\phi_H\rangle$ and $|\phi_T\rangle$ in the $|\psi_{coin}\rangle$ wavefunction were both $\frac{1}{\sqrt{2}}$, meaning Beauty would have a credence $\frac{1}{2}$ of the coin landing Heads. This, however, does not seem to make intuitive sense. Since the quantum coin is fair, it is twice as likely that Beauty will wake up on a day corresponding to Heads outcome, than a day corresponding to Tails outcome, if we are to repeat the experiment several times. Instead of asking for a credence, imagine that each time we woke Beauty up, we asked her to predict the quantum coin toss outcome (Heads or Tails) and gave her \$1 for each correct guess. If we ran the experiment 1000 times and Beauty was to guess Heads every time, she would have an expected earning of \$1000 (she has a 50% chance of being right, but will earn \$2 each time she is right). Meanwhile, if she was to guess Tails every time, she would have an expected earning of \$500 (she has a 50% chance of being right, but will only earn \$1 each time she is right). Thus, it does not seem to make sense to have a credence of $\frac{1}{2}$, but instead seems that Beauty should have a higher credence in Heads.

It turns out that recourse to ESP and the Indifference Principle gives a better description of this situation. Branch-counting tells us that $P(F_H) = P(F_T)$, while strong-ESP tells us that $P(F_H) = P(S_H)$. Thus $P(F_H) = P(F_T) = P(S_H)$, which by indifference means that we should assign a credence of $\frac{1}{3}$ to each of the three possible day-outcome combinations. This means that Beauty should have a credence $\frac{2}{3}$ in the quantum coin toss outcome being Heads, which is what we expect.

4.2 The Three-Branch Quantum Sleeping Beauty

The Three-Branch Quantum Sleeping Beauty Paradox is very similar to the Two-Branch Paradox, except we now have two coins. In this experiment, Beauty will again go to sleep on Thursday night. If the first quantum coin lands Tails, she will wake up only on Friday. However, if the first quantum coin lands Heads, a second quantum coin will be flipped. If this second coin has an outcome of Heads, Beauty will wake up only on Friday. However, if the second coin lands Tails, Beauty will wake up only on Saturday. The set-up is illustrated in [Fig 5]. Once again, the question is, what credence should Beauty assign to Heads?

This experiment appears very similar to the Double Quantum Coin Flip Experiment (DQCFE) described in section [3.2]. In fact, if we were to remove the temporal component of waking up on Friday versus Saturday, it would be exactly the same. Thus, in this case, the temporal uncertainty has no effect on the credence Beauty should have of Heads. As

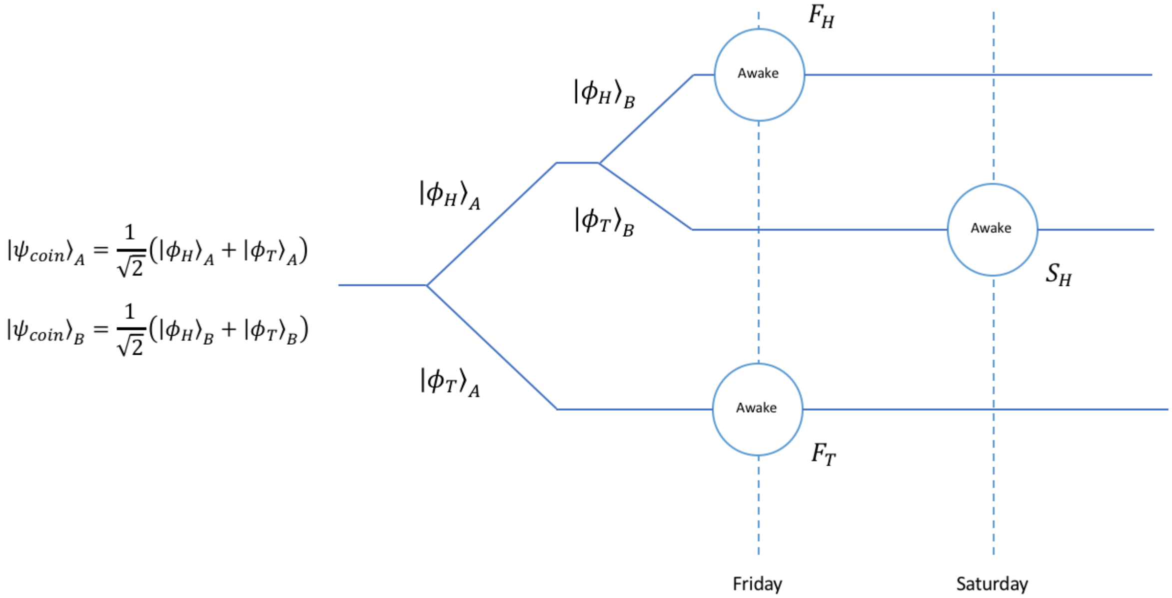

Figure 5: The Three-Branch Quantum Sleeping Beauty Experiment.

shown in section [3.2], she should use Born's Rule to assign a credence of $\frac{1}{2}$.

4.3 The Metaphysics of Branching

In the Two- and Three-Branch Sleeping Beauty Paradoxes, we wanted to calculate Beauty's credence in the quantum coin landing Heads or Tails. However, given that we are using the Many-Worlds Interpretation of quantum mechanics, is this even a valid credence for Beauty to have? According to the Everettian view, at the toss/measurement of the quantum coin, the world is split between two branches, one in which the outcome is Heads and one in which the outcome is Tails. Thus, could it not be argued that since Beauty is present in both worlds, experiencing both outcomes simultaneously, she should assign a credence of 1 to both Heads and Tails? Where exactly does the uncertainty lie? We propose two different metaphysical interpretations (Many-Beauties and Duplicator) of the branching notion at hand, to locate the potential underlying source of uncertainty in these two paradoxes. In section [5] we will discuss the implications of these two different interpretations for determinism.

4.3.1 The Many-Beauties Interpretation

In the Many-Beauties Interpretation of Everettian quantum mechanics, we adopt the view that in fact the universe is not splitting with each quantum coin toss. Instead, all the possible quantum coin tosses in the universe have been predetermined. For each possible combination of coin toss outcomes, a different universal path exists in which that sequence of events occurs. However, the universal paths do not interact.

As illustrated in [Fig 6], for the Two-Branch Quantum Sleeping Beauty Paradox, before the quantum coin is even tossed, there already exist two separate "branches," or paths: one in which the coin is guarenteed to land Heads and one in which it is guaranteed to land Tails. This is clarified even further in the Three-Branch Quantum Sleeping Beauty Paradox,

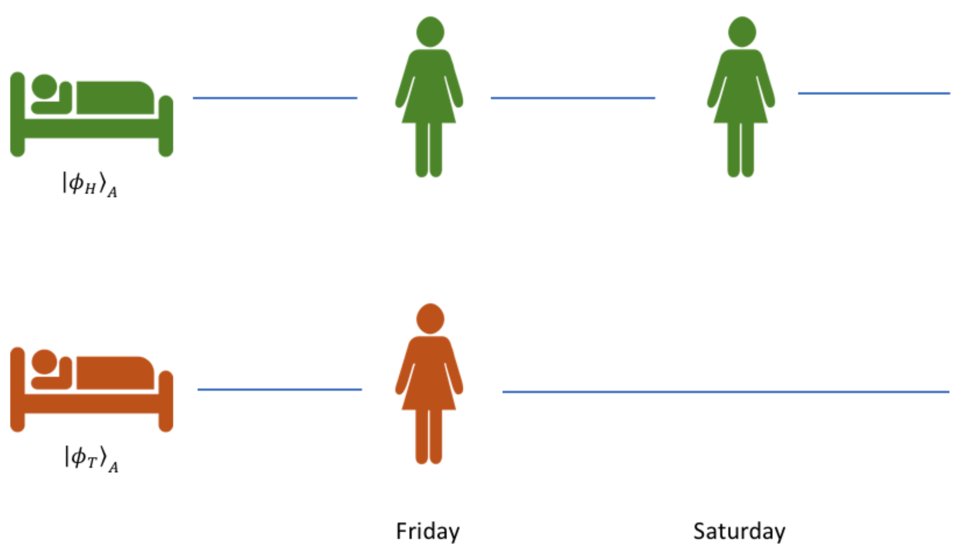

Figure 6: The Thwo-Branch Quantum Sleeping Beauty Experiment Many-Beauties Interpretation.

where three branches exist, corresponding to the three possible coin combination outcomes, as illustrated in [Fig 7].

In this interpretation, there is no connection between the "many worlds." In fact, they are *distinct* worlds. We could get into semantics about where and how exactly these worlds start, but, for now, let us just assume that all the worlds start at the Big Bang, or origin of the universe, and are exactly the same for all intents and purposes up until the quantum

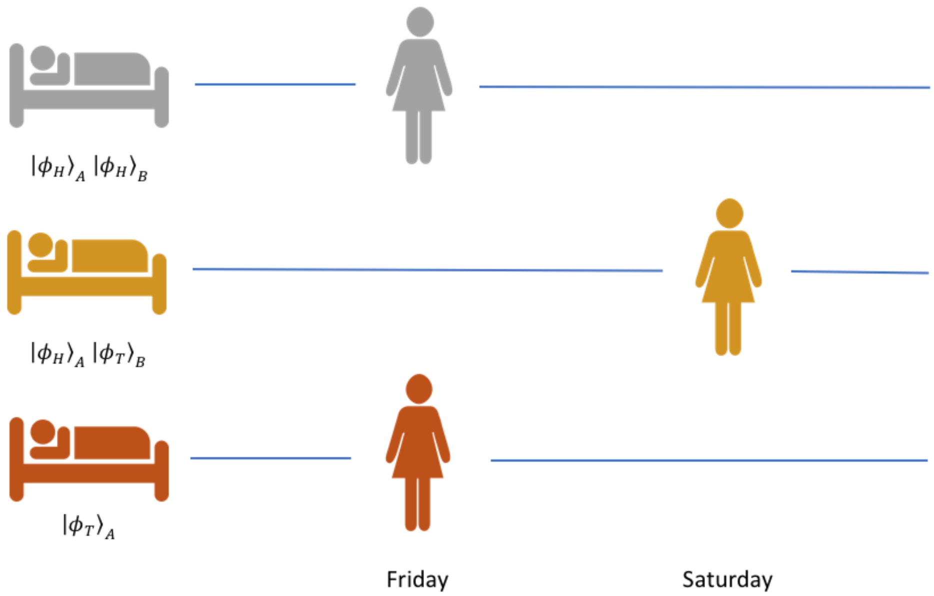

Figure 7: The Three-Branch Quantum Sleeping Beauty Experiment Many-Beauties Interpretation.

coin toss that causes their physical realities to diverge. One could ask, then, what is the difference between Beauty in path one (B_1) and Beauty in path two (B_2) before the physical realities of their universes begin to diverge. In principle, everything about them is exactly the same: their hair, their previous life experiences, their environments. Every atom in their body and in the world surrounding them is exactly the same. In response to this, we argue that there is something different, since they exist simultaneously and distinctly. Exactly how they differ is beyond the scope of this paper, both philosophically and mathematically. Thus, for now, we consider B_1 and B_2 to be clones living in two distant galaxies of the universe that will never interact and have the same exact physical reality, up until the toss of the quantum coin that determines when they will wake up.

Let us now consider the Many-Beauties Two-Branch Sleeping Paradox. B_1 will live in the universal path in which the quantum coin will land Heads and she will wake up on both Friday and Saturday. B_2 will live in the universal path in which the coin will land tails and she will wake up only on Friday. As exterior universal observers, we are aware of this arrangement. However, $B1$ and $B2$ have no idea in which particular universal path they lie. Thus, we argue that Beauty is not creating a credence of a Heads outcome, but is instead creating a credence that she is $B1$ and lives in the first universal path.

This view leads us to a more general, interesting conclusion. Assuming that the universe contains a finite number of particles and that the length of the universe is finite, there is a large but finite number of quantum measurements that can occur in the duration of the universe. Thus, the number of universal paths is finite. This means that, over time, as Beauty observes more and more coin toss outcomes, she eliminates possible paths she could be in. For example, if in the Two-Branch Beauty case, she finds out after the experiment that she only woke up on Monday, meaning a Tails outcome, then she has ruled out the possibility of being $B1$. However, even in the very last quantum coin toss of Beauty's life, having eliminated billions upon billions of other possible universal life paths, Beauty still cannot be certain if she is B_{1e100} or $B_{1e100+1}$ and will only know after she observes the outcome of the toss.

4.3.2 The Duplicator Interpretation

In the Duplicator Interpretation of Everettian quantum mechanics, the world actually does divide. More precisely, the world is duplicated. Everything is the same between the two worlds, except for the outcome of the quantum coin toss. How then does the uncertainty arise? Once again, does Beauty not just experience both outcomes? In this interpretation, we argue that while the physical reality can be copied, conscience cannot. Thus, in the process of "duplicating," we are creating the same physical reality, but all the people in it will have a new, distinct conscience from themselves in the original branch. This is illustrated in [Fig 8] for the Two-Branch Quantum Sleeping Beauty Paradox. Specifically, we depict the process of consciousness transfer by drawing a brain above the Beauties in the branch where the original Beauty lands. At the beginning of the experiment, we can think of it as if two Beauties share the same body and life and physical reality. However, at the toss of the coin, one of the two bodies will end up in the Heads Branch and the other in the Tails branch. Only one of these two resulting Beauties can inherit the original shared conscience. Thus, the body in which the concscience lands corresponds to the original Beauty. This is similarly illustrated for the Three-Branch Quantum Sleeping Beauty Paradox, as illustrated in [Fig 9]. In this case there are three bodies in the original Beauty, but once again, only one can inherit the original conscience. We argue that in this view, in assigning a credence of quantum coin

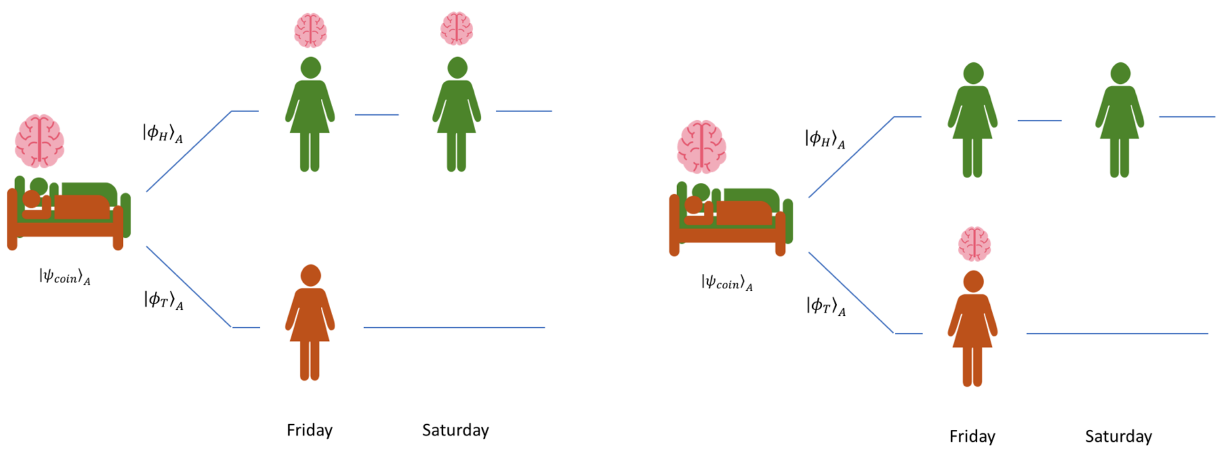

Figure 8: The Two-Branch Quantum Sleeping Beauty Experiment Duplicator Interpretation.

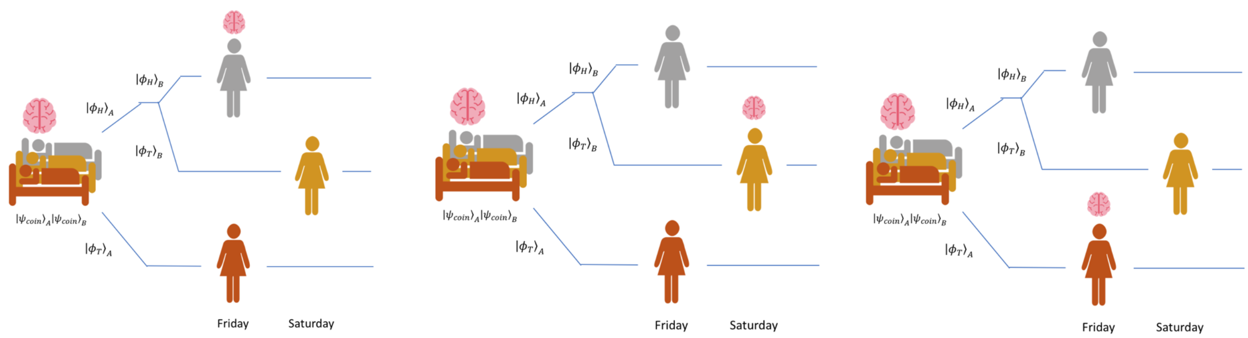

Figure 9: The Three-Branch Quantum Sleeping Beauty Experiment Duplicator Interpretation.

toss outcome of Heads for the Two-Branch Paradox, Beauty is in fact assigning a credence of her conscience transferring to the Beauty in the Heads branch. Any Beauty that does not inherit the original Beauty conscience will create a new conscience.

Under this Duplicator Interpretation, when Beauty was born there simultaneously existed billions of physical duplicates of her that shared a single conscience. With each quantum coin flip she experienced, some of those duplicates branched off and formed their own conscience. This poses an interesting thought. Since only one Beauty can repeatedly inherit her original conscience from the time of her birth to her death, this means that essentially every Beauty in existence is not the original or true Beauty. Thus, under this interpretation, you can claim with very high certainty that you are not actually you.

4.4 Implications for Quantum State Tomography

From the two forms of the Quantum Sleeping Beauty Paradox as well as our discussion in section [3], a quantum state measurement proves to be a challenging philosophical issue, forcing us to question fundamental principles of probability and quantum mechanics that are generally taken for granted. There are several factors we must consider and, as it turns out, neither indifference nor Born's Rule is an end-all-be-all solution to our quantum measurement needs. Even in the simple case of flipping a quantum coin or two, we see the emergence of paradoxes that could have a variety of solutions, depending on our interpretation and the mathematical tools at hand.

These challenges are deeply rooted in the field of Quantum State Tomography (QST), which is dedicated to the experimental and statistical determination of quantum states based on measurement of a repeatedly created wavefunction. In its simplest form, this is equivalent to being handed a quantum coin, without being told whether it is fair or biased. Given this coin, we want to discover what its exact Born's Rule amplitudes are, allowing us to make predictions of the likelihood of the coin landing Heads or Tails in our world. If this is not challenging enough, we have the additional limitation that, once we flip our quantum coin, it will be stuck forever in whatever outcome it lands on the first flip. Thus, if we want to perform repeated measurements, we must try to reconstruct a perfect replica of our original quantum coin, which can generally only be done to a certain precision.

In the case of QST, we not only deal with the uncertanties of the notion of measurement itself. We attempt to use this poorly understood measurement mechanism to determine the weighting of our quantum coin, which changes slightly each time we try to recreate it.

5 Quantum Determinism?

From a deterministic view of the classical world, we use probability and statistics to describe phenomena that are too complex to simulate, given our computational resources. However, removing these computational barriers, we could, in theory, predict any outcome exactly. Hence, probability serves as a mathematical simplification of complex, but solvable problems.

For instance, take the example of flipping a classical coin. If we knew the initial velocity, trajectory, shape, size, and weight of the coin, as well as the surface it was landing on, we could arguably use classical mechanics to create a decent approximate model of whether the coin would land Heads or Tails (given the starting conditions of our toss). If we wanted to create an exact model, we would potentially need to account for factors such as air resistance and interactions with neighboring particles. One might even argue that we would need to know the state of all the particles in the universe. However, given the assumption of a *classical* world, in which all the acting bodies are deterministic, we could in theory create an exact model of how our coin would land. Such a model would be far more precise than simply stating an expectation or likelihood of Heads/Tails. This, however, is not necessarily the case in quantum mechanics, depending on the interpretation.

As described in section [2.1], the Copenhagen Interpretation views measurement as an inherently stochastic process. This indeterministic view differs dramatically from our deterministic classical view of the world. In fact, several great scientists were so surprised by and doubtful of this result that they proposed the EPR (Einstein-Podolsky-Rosen) Paradox, a thought experiment which aimed to show that the physical reality described by quantum mechanics is incomplete in this world-view. They argued that particles could act in such a way that their position and momentum could be predicted simultaneously, with greater certainty than allowed by the Heisenberg Uncertainty Principle ($\Delta x \Delta p \geq \frac{\hbar}{2}$). The only way to circumvent violation of the Uncertainty Principle was if the particles could somehow break their counterpart's measurement accuracy, via communication faster than the speed of light (which is prohibited by relativity). At the time, it was believed that a local hidden variables theory could resolve this paradox. The local hidden variable theory, however, was disproved by the Bell test, and we now regard the seemingly instantaneous interaction between particles as quantum entanglement. Nonetheless, the debate over this indeterministic view is still very active and there are several non-local hidden variable theories that remain to be disproven.

One of the benefits of the Many-Worlds Interpretation is that it is, on paper, an entirely deterministic theory. As described in section [2.3], whenever we perform a measurement of a quantum system, our world branches into several worlds, each of which contains a different outcome of the measurement. Thus, we are no longer using probabilities to determine a physical outcome, but rather to determine the number of branches our world will break into. In section [4.3] we then proceeded to describe different possible metaphysical interpretations of this branching phenomena. Between the two proposed interpretations, only the Many-Beauties Interpretation is truly deterministic. In this view, each possible life path has been determined from the origin of the universe and if you were to somehow figure out which path you fall in, you would know the outcome of every future quantum coin toss. In this case, your credence in the coin landing Heads would truly be either zero or one. Under the Duplicator Interpretation, however, our conscience transferred to one of multiple possible branches and there was no knowledge as to which branch it would transfer. In the remaining branches, physical realities would be duplicated, but new consciences would spawn. If we are to follow our conscience, there is no certainty into which branch we fall or which outcome of the quantum coin toss we will observe. Under the Duplicator Interpretation, the Many-Worlds interpretation is ultimately indeterministic. Measurement makes way for uncertainty in your conscious experience and life path. Thus, could the toss of a quantum coin, perhaps, be the key to free will?