TEACH-C
AN INTRODUCTION

PROJECT

The **continuous casting** machine is used in many metal industries, including the steel industry, to **manufacture metal billets or slabs** (rectangular, quadrangular or circular cross section with variable length) at **high temperature** – see illustrative images of **figure 1**.

These billets will subsequently be **cooled down** by natural (or, in some cases, by forced) **convection** at **atmospheric air**, either **suspended** or, for rectangular and quadrangular cross-sections, with **one face in direct contact with the ground** (that may be considered in this work as an adiabatic surface).

Consider the metal billet ***X*** in table 1 (***X*** is a dummy letter between A and T, and is to be defined for each group) schematized in figure 2, which has just been removed from a continuous casting machine at a uniform temperature, T_{in}.

Table 1 – Dimensions and materials of the metal billets. Initial temperatures and convection mechanisms and coefficients. Solid position.

Metal billet X (material)	H (cm)	W (cm)	D (cm)	L (cm)	T_{in} (ºC)	Convection mechanism; h [W/m^2K]	Solid Position
A (stainless steel)	40	40		400	900	Natural; 40	Suspended
B (brass)	30	30		500	1500	Forced; 250	Lower face on the ground

PROJECT

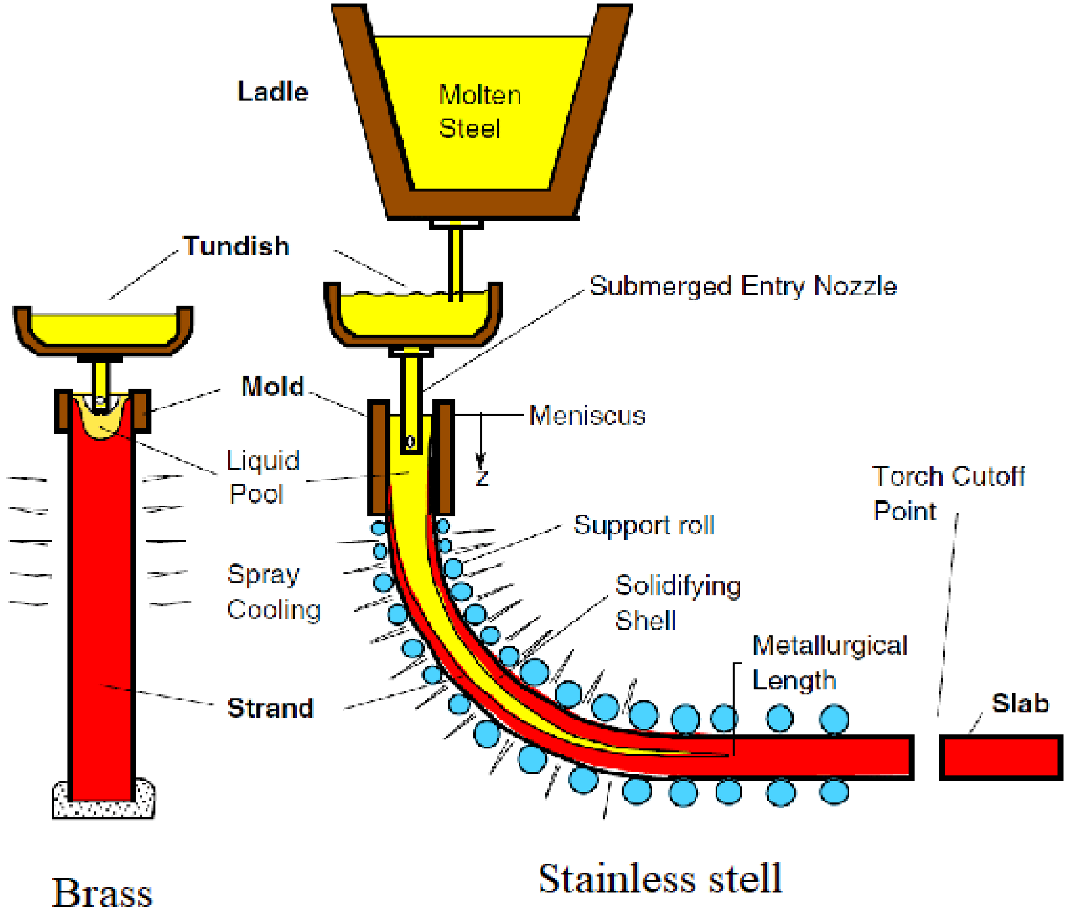


Figure 1 – Diagram of a continuous casting machine for circular brass and rectangular/quadrangular stainless steel billets and images of the steel billets manufactured in a continuous casting machine.

PROJECT

Table 2 – Properties of the metal billets materials.

Physical properties		
Material → Property ↓	Brass	Stainless steel
Density ρ [kg/m³]	7930	8500
Specific heat c [J/kg K]	385	460
Thermal conductivity [W/m K]	121	16,3
Thermal diffusivity [m²/s]	$39{,}6\times10^{-6}$	$4{,}2\times10^{-6}$

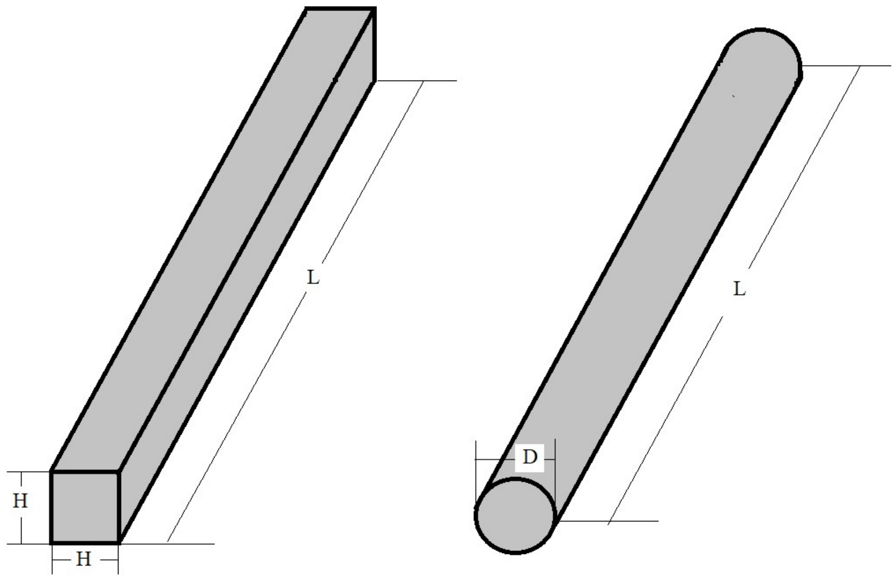
The solid is at its initial temperature (T_{in} with values displayed in Table 1) that is uniform. Then, the solid is cooled down by natural or forced convection, either suspended or with one surface on the ground (as displayed in Table 1) in atmospheric air at constant temperature (T_{air} = 20 ºC), with the axis along dimension *L* at the horizontal position.

PROJECT

Geometry/Solid	Area (cm^2)	Volume (cm^3)
A and B	184,1	100,5
C	69,1	31,4
D	108,4	70,7
Physical properties		
Property/material	Brass	Stainless steel
Density ρ [kg/m^3]	7930	8500
Specific heat c [J/kg K]	385	460
Thermal conductivity [W/m K]	121	16,3
Thermal diffusivity [m^2/s]	$39,6\times10^{-6}$	$4,2\times10^{-6}$

Table 1 – Areas, volumes and physical properties of brass and stainless steel solids

Figure 2 – Schematic of a stainless steel billet



PROJECT

a) Determine the **Biot number** of your case. Independently from the Bi value obtained, use the **lumped capacitance method** to determine the time temperature evolution of the solid, $T(t)$. Pay attention to the characteristic dimension, L_c, used by convention for the lumped capacitance method.

b) With the *h* (convection coefficient) value of Table 1 for your case, determine **the most adequate temperature distribution solution (analytical equation) as function of time and space.** For the parallelepiped-shaped billets determine the 3D solution $T3DP(x, y, z, t)$, and the 2D solution $T2DP(x, y, t)$ at the central vertical plane. For the cylindrical billets determine the 2D solution $T2DC(r, z, t)$ or the 1D solution $T1DC(r, t)$ and find which is the most adequate.

PROJECT

c) Use the **TEACH-C code** with the convection heat transfer coefficients of Table 1 for your case to determine **the 2D spatial temperature distribution solution with time** (at the central vertical plane). **<u>Make sure that your numerical solution is grid-independent and obtained with the smallest computational effort</u>**. From such solution, extract the time evolution temperatures i) at the solid geometrical centre (both for cylindrical and parallelepiped-shaped geometries), and ii) at the centre of each of the largest faces (for the quadrangular cross-section), or at the centreline of the cylindrical surface. Note that in all cases with quadrangular cross-section, $L \gg H$ and, therefore, the problem may be treated as a two-dimensional case.

PROJECT

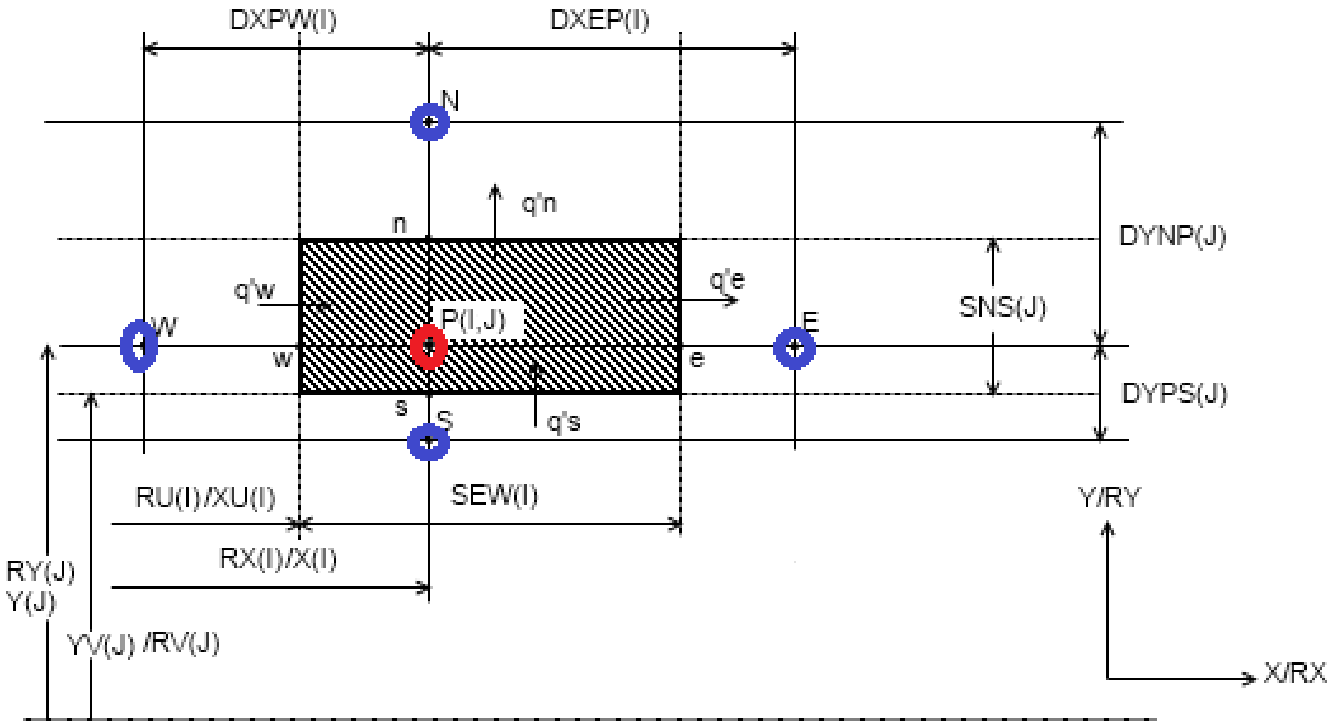
e) Plot $\boldsymbol{T(t)}$ **obtained in a), b) and c)** for the **centre of the largest faces of the solid** (quadrangular cross section) or for the **centre of the cylindrical surface** in a different graphic of the **non-dimensional temperature as a function of the Fourier number**.

f) Based on **questions d) and e) draw conclusions** about the different methods and the accuracy obtained.

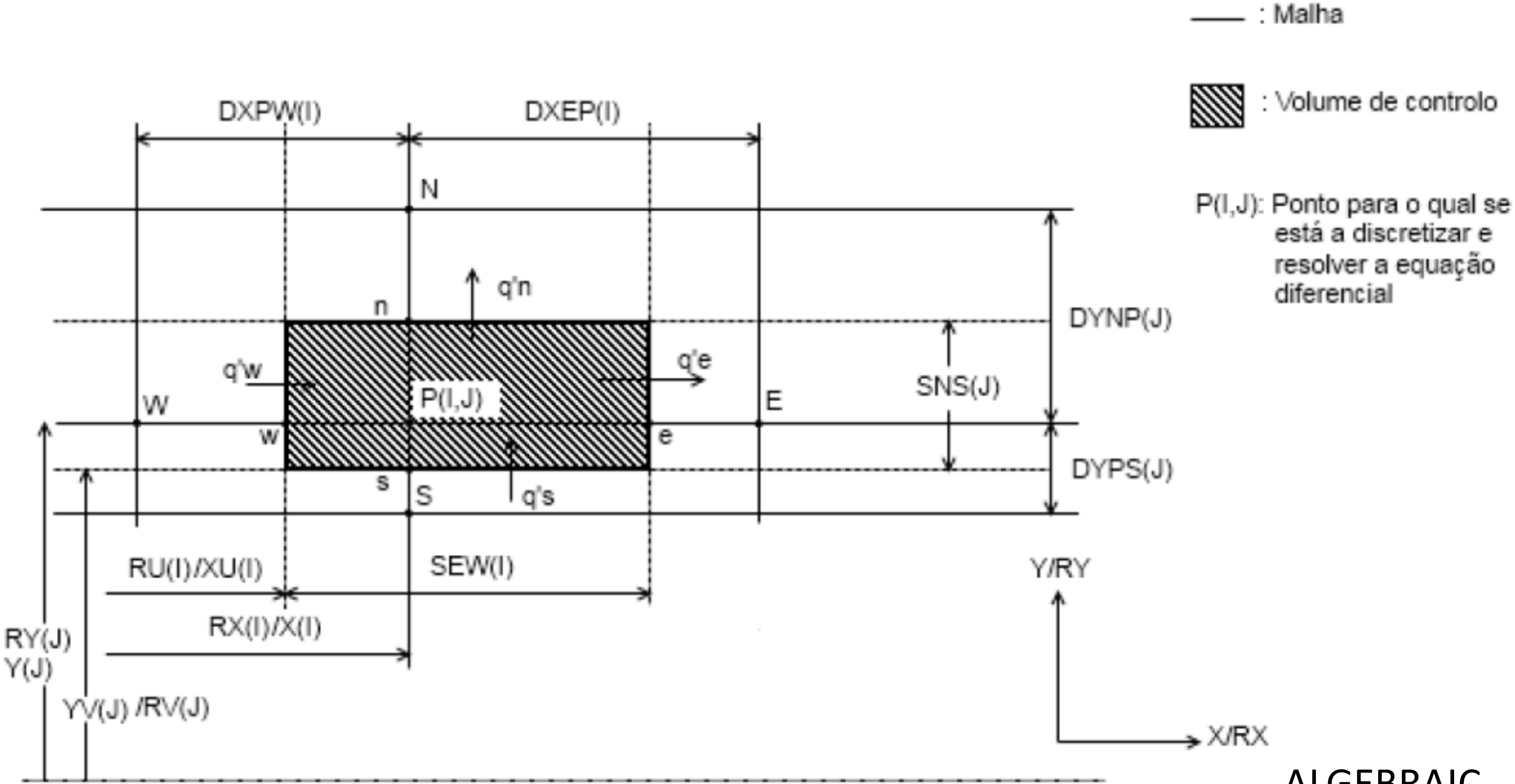
THE **TEACH-C** CODE IS BASED ON THE TRANSFORMATION OF DIFFERENTIAL EQUATIONS INTO ALGEBRAIC EQUATIONS

FOR THAT, THE INTEGRATION IN CONTROL VOLUMES (**THE GRID**) MAKES RECOURSE TO **FINITE DIFFERENCES**

EACH **NODE P** HAS AN ALGEBRAIC EQUATION FOR THE VARIABLE **T,** WHICH IS A FUNCTION OF AREAS IN 3D (**LENGTHS IN 2D**) AND **T AT THE NEIGHBOUR NODES** – THIS IS EXPRESSED BY THE COEFFICIENTS AN, AS, AE, AW.



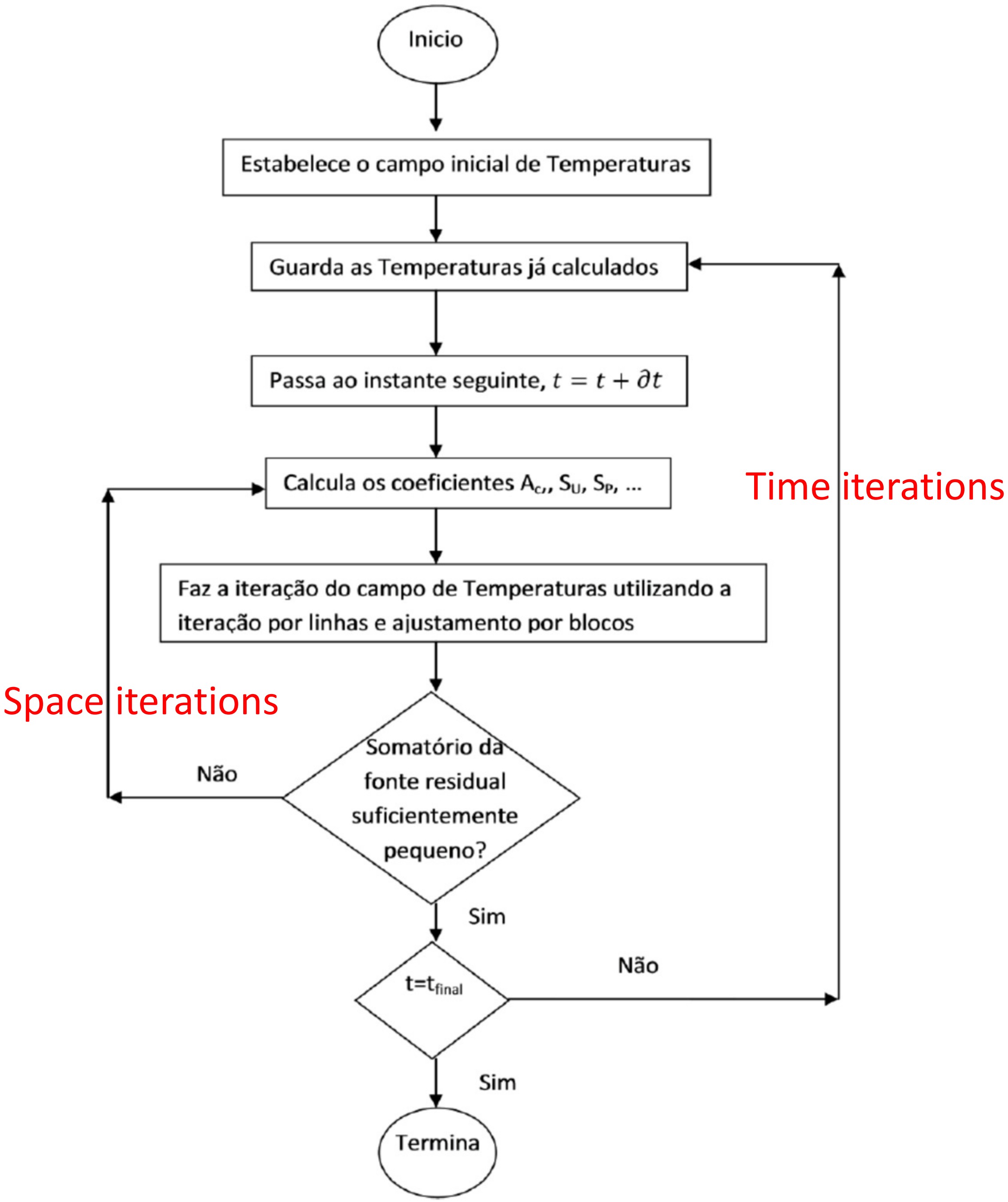
THE GRID



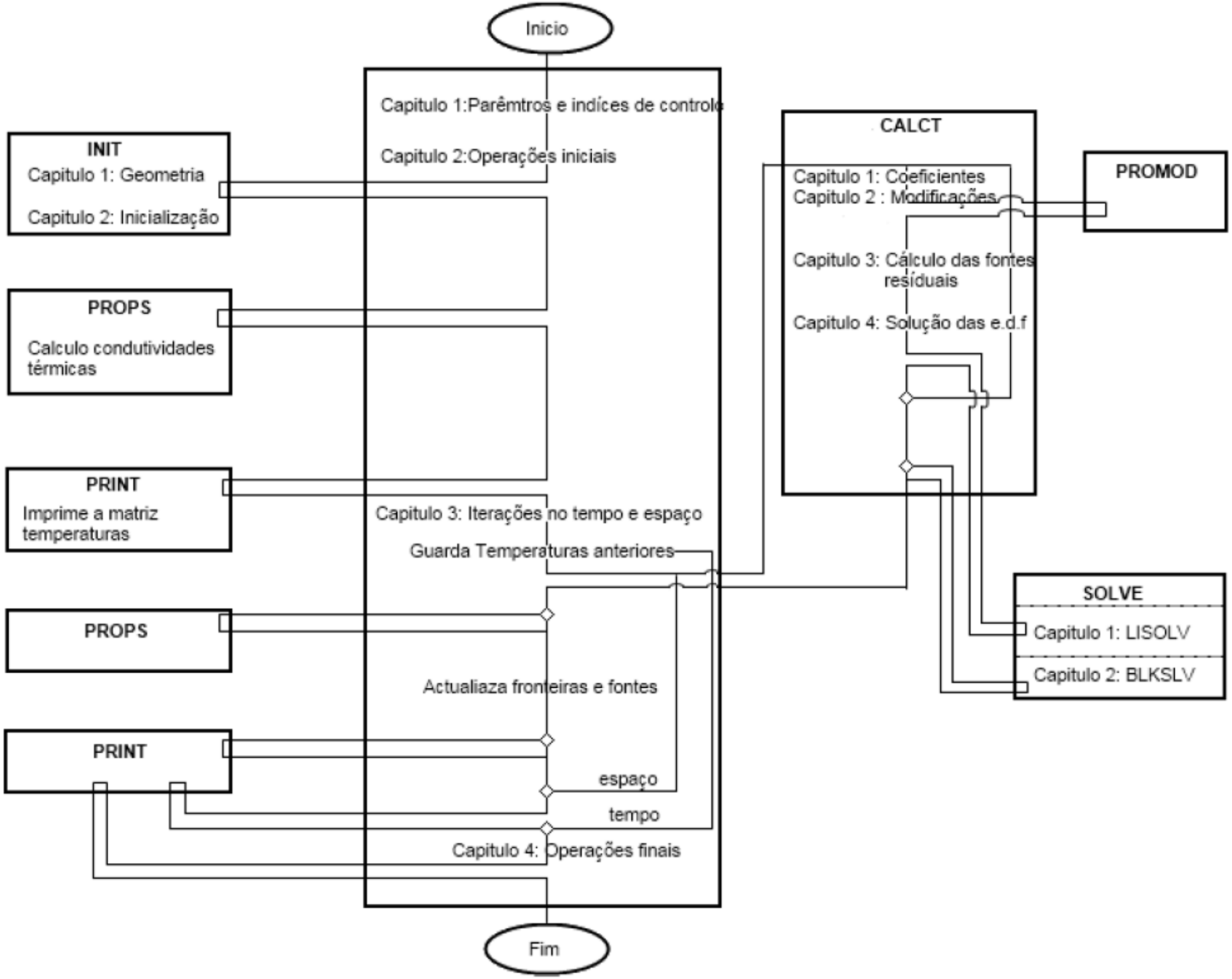
ALGEBRAIC EQUATION

$$\frac{(A_p - S_p)}{k} T_p = A_N T_N + A_s T_s + A_E T_E + A_W T_W + \boxed{S_U} + \frac{1-k}{k} (A_P + \boxed{S_P})^{n-1} T_P$$

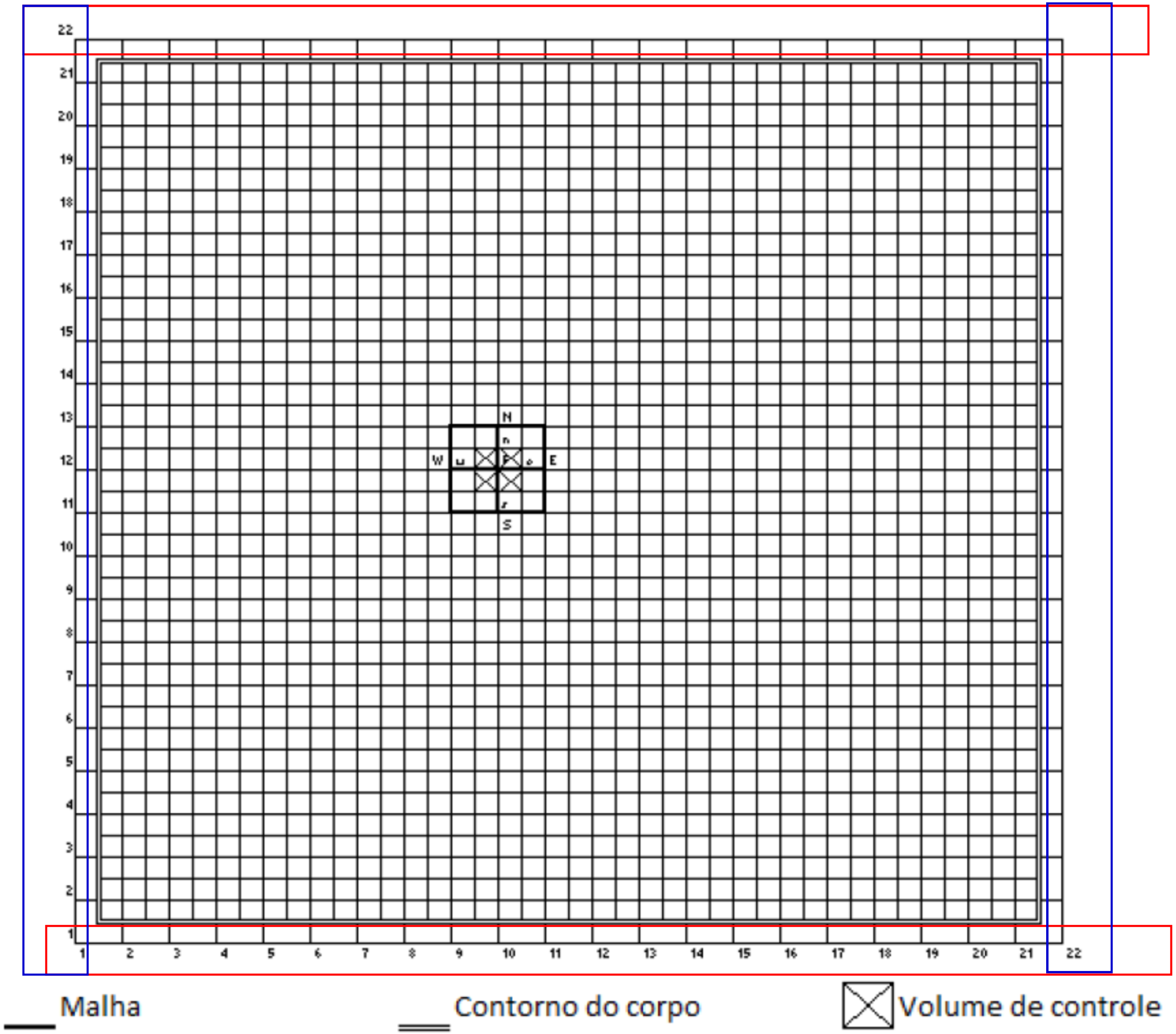
GLOBAL OVERVIEW



DETAILED OVERVIEW



THE GRID



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S
Malha
Contorno do corpo
Volume de controle

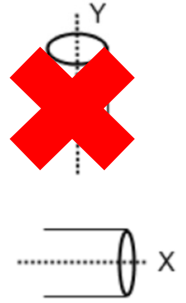
```
% Define o n\[UAcute]mero de n\[OAcute]s da malha segundo x e y
%%%%% Alterável %%%%%
IT=22;
JT=22;
NI=22;
NJ=22;
```

Must be equal (IT=NI; JT=NJ) – define the matrix sizes

```
% Define o tipo de coordenadas (cartesianas- 0 0;
% cilãndricas vertical 1 0; cilãndricas horizontais 0 1)
%%%%% Alterável %%%%%
INCYLX=0;
INCYLY=0;
```

INCYLX=1 AND INCYLY=1 – NOT ACCEPTED BY THE CODE!!!!

Coordenadas	INCYLX	INCYLY
Cartesianas	0	0
Cilíndricas (axissimétrico)		
	0	1




```
% Dimensões totais do domínio de solução, largura e altura [m]
%%%%% Alterável %%%%%
W=1;
H=1;
```

A nível de condições de fronteira o código está preparado para receber coordenadas cartesianas e cilíndricas com o eixo de simetria segundo xx (INCYLY=1 e INCYLX=0).

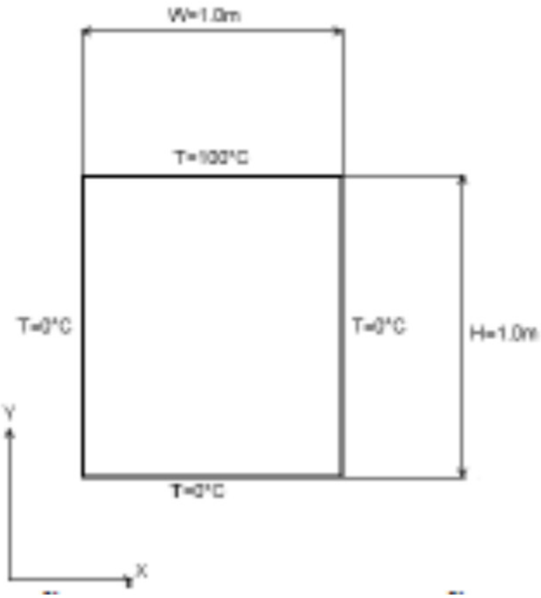
```
% Cálculo as abcissas na direcção xx
DX (1)=W/(NIM1-1);
X (1)=-0.5*DX (1);
for I=2:NI;
    DX (I)=DX (1);
    X (I)=X (I-1)+DX (I-1);

end

% Calcula as abcissas na direcção yy
DY (1)=H/(NJM1-1);
Y (1)=-0.5*DY (1);
for J=2:NJ;
    DY (J)=DY (1);
    Y (J)=Y (J-1)+DY (J-1);

end
```

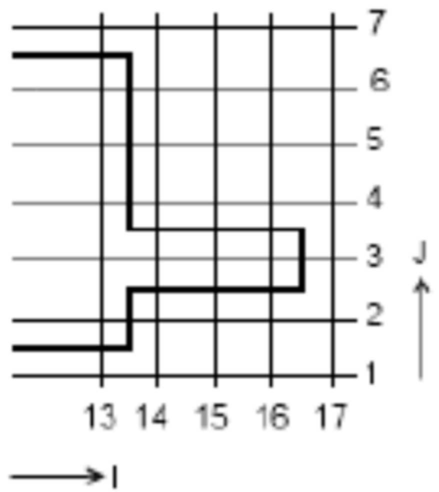
UNLESS YOUR GRID IS NOT UNIFORM, DO NOT CHANGE THIS!




```
% Estabelece limites do Domãnio
for I=1:NI;
    JS (I)=2;
    JN (I)=NJ-1;

end
```

NOTE THE EXCLUSION OF OUTSIDE NODES! UNLESS HAVE A GEOMETRY THAT REQUIRES SO, DO NOT CHANGE IT!



```
% Estabelece ponto monitor segundo x e y (IMON e JMON)
%%%%% Alterável %%%%%
IMON=6;
JMON=6;
```

POINT TO **OBSERVE TEMPERATURE EVOLUTION** (IN THE CONVERGENCE PROCESS IN SPACE AND IN TIME).
CHOOSE POINT IN ZONES WHERE YOU EXPECT **SPATIAL GRADIENTS**. ALSO WHERE **T VARIES WITH TIME**.

NITER	SOURCE	T(6,6)	TIME(s)	DT(s)	NSTEP
1	4.2E+001	1.188E-017	50	50	1
2	2.6E+000	2.394E-017	50	50	1
3	1.6E-001	3.043E-017	50	50	1
4	9.5E-003	3.289E-017	50	50	1
5	5.8E-004	3.363E-017	50	50	1

IS THIS THE BEST MONITORING POINT?
MAY BE NOT…

NITER	SOURCE	T(6,6)	TIME(s)	DT(s)	NSTEP
1	3.7E+001	3.153E-016	100	50	2
2	2.2E+000	4.432E-016	100	50	2
3	1.4E-001	4.876E-016	100	50	2
4	8.4E-003	5.002E-016	100	50	2
5	5.2E-004	5.033E-016	100	50	2

NITER	SOURCE	T(6,6)	TIME(s)	DT(s)	NSTEP
1	9.4E+000	1.178E-008	1000	50	20
2	6.5E-001	1.224E-008	1000	50	20
3	4.4E-002	1.229E-008	1000	50	20
4	2.9E-003	1.230E-008	1000	50	20
5	1.9E-004	1.230E-008	1000	50	20

```
% Propriedades do material (Tcond=cond,CV=calor esp,DENSIT=dens),
% Meio homogéneo
% Aço %%%%% Alterável %%%%%
for I=1:NI;
    for J=1:NJ;
        TCON (I,J)=14.68;
        CV (I,J)=485.67;
        DENSIT (I,J)=7800;
        if I==1 && J==1;
           BK=TCON (I,J);

        end
    end
end
```

THIS IS WHERE YOU DEFINE THE THERMAL PROPERTIES OF YOUR MATERIAL IN THE ENTIRE DOMAIN: DENSITY, SPECIFIC HEAT AND THERMAL CONDUCTIVITY.

BK IS A DUMMY VARIABLE TO BE USED LATER IN THE CALCULATION OF THE RESIDUALS NORMALIZATION (SNORM).

IF PROPERTIES ARE FUNCTION OF T, DO NOT EXPRESS THAT HERE!

```
% Temperatura inicial
TINIC=0;

% Par\[AHat]metros de controlo do programa
%%%%% Alterável %%%%%
% N\[UAcute]mero máximo de iteraç\[OTilde]es
MAXIT=10;
% N\[UAcute]mero máximo de interaç\[OTilde]es no tempo
MAXSTP=20;
% O output deverá conter os valores de T em intervalos de
NITPRI=110;
% "NITPRI" para "NSTPRI" iteraç\[OTilde]es no tempo
NSTPRI=1;

% Factor de sub relaxação, Máximo resãduo e intervalo de tempo (s)
%%%%% Alterável %%%%%
URFT=1;
SORMAX=0.001;
DT=50;

% Selecciona o Regime---Estacionário->INTIME=0, Transiente->INTIME=1
%%%%% Alterável %%%%%
INTIME=1;

if INTIME==0;
    MAXSTP=1;

end

% Indica se as propriedades são constantes --- constantes->INPRO=0,
% variáveis->INPRO=1
%%%%% Alterável %%%%%
INPRO=0;
```

VALUE OF INITIAL TEMPERATURE. **T MATRIX NOT INITIALIZED HERE**!

YOU MAY NEED TO CHANGE THIS. MAXIMUM SPACE ITERATIONS (**MAXIT**) AND MAXIMUM TIME STEPS (**MAXSTP**)

PRINT CONTROL VARIABLES. T MATRIZ PRINTED INTERVALS IN SPACE ITERATIONS (**NITPRI**) AND IN TIME STEPS (**NSTPRI**)

```
% Temper
TINIC=0;
```

$$\frac{(A_P-S_P)}{k}T_P = A_N T_N + A_S T_S + A_E T_E + A_W T_W + S_U + \frac{1-k}{k}(A_P + S_P)^{n-1} T_P$$

```
% Par\[AHat]metros de controlo do programa
%%%%% Alterável %%%%%
% N\[UAcute]mero máximo de iteraç\[OTilde]es
MAXIT=10;
% N\[UAcute]mero máximo de interaç\[OTilde]es no tempo
MAXSTP=20;
% O output deverá conter os valores de T em intervalos de
NITPRI=110;
% "NITPRI" para "NSTPRI" iteraç\[OTilde]es no tempo
NSTPRI=1;

% Factor de sub relaxação, Máximo resãduo e intervalo de tempo (s)
%%%%% Alterável %%%%%
URFT=1;
SORMAX=0.001;
DT=50;

% Selecciona o Regime---Estacionário->INTIME=0, Transiente->INTIME=1
%%%%% Alterável %%%%%
INTIME=1;

if INTIME==0;
    MAXSTP=1;

end

% Indica se as propriedades são constantes --- constantes->INPRO=0,
% variáveis->INPRO=1
%%%%% Alterável %%%%%
INPRO=0;
```

UNDER-RELAXATION FACTOR AND CONVERGENCE CRITERIA. **IN PRINCIPLE, YOU DO NOT NEED TO CHANGE IT.**

TIME STEP

CHOSES REGIME: **STEADY (INTIME=0)** OR **UNSTEADY (INTIME =1)**

CHOSES PROPERTIES VARIATION: **CONSTANT (INPRO=0)** OR **VARIABLE (INPRO =1).**

EQUATION NOT EXPRESSED HERE!

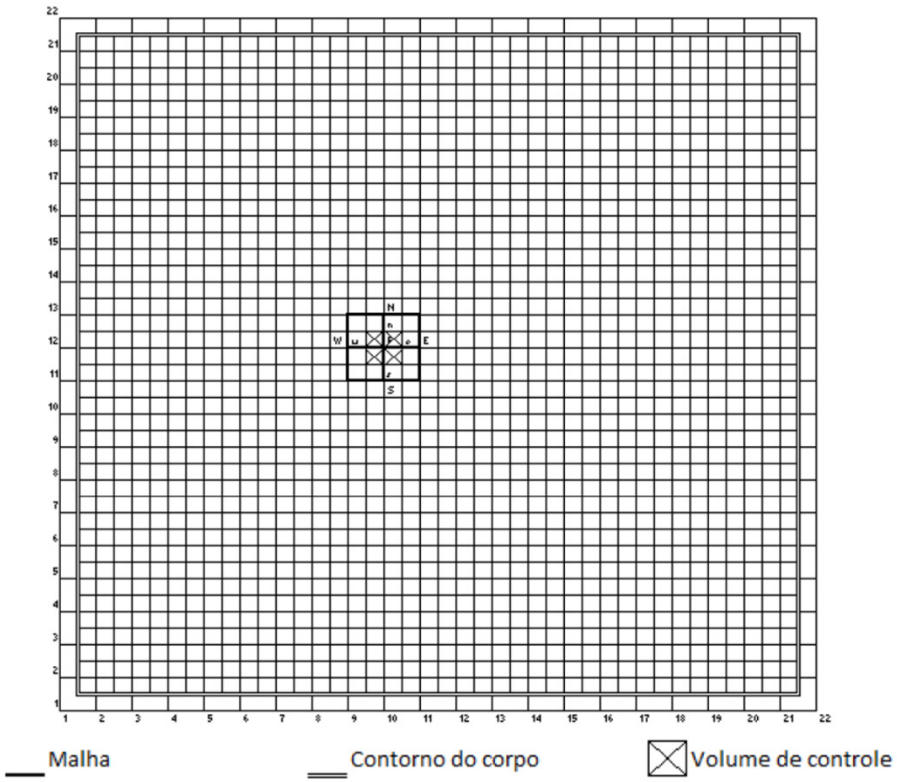
```matlab
% Imp\[OTilde]e valores de fronteira e inicializa variável dependente

% Valores de fronteira
%%%%% Alterável %%%%%
TTOP=100;
TBOT=0;
TLEFT=0;
TRIGHT=0;

for I=2:NIM1
    T (I,1)=TBOT;
    T (I,NJ)=TTOP;
end

for J=2:NJM1
    T (1,J)=TLEFT;
    T (NI,J)=TRIGHT;
end
```

DEFINES **VALUES** OF BOUNDARY CONDITIONS. BUT **DOES NOT IMPLEMENT** THEM HERE!!!




```matlab
% Cálculo do factor de normalização do resãduo
AK=BK;
SNORM=AK*(TTOP-TBOT)*W/H;
SNORM=abs (SNORM);
```

HOW TO DEFINE THIS?

SUBROUTINES THAT YOU **MUST NOT** CHANGE!

SUBROUTINE INIT
DO NOT CHANGE IT!

SUBROUTINE PRINT
DO NOT CHANGE IT!

SUBROUTINE SOLVE
DO NOT CHANGE IT!

SUBROUTINES WHERE **YOU MAY HAVE TO CHANGE** INTRUCTIONS

SUBROUTINE **PROPS**

<u>CHANGE ONLY IF PROPERTIES VARY</u>!

```
%------------- Capitulo 1 - Actualização das propriedades
 ----------------%

for I=1:NI;
    for J=1:NJ;
        GAMH(I,J)=TCON(I,J);
    end
end
```

IF A PROPERTY CHANGES WITH T, IT IS HERE THAT YOU MUST EXPRESS IT

SUBROUTINE **CALCT**

CHANGE ONLY IF YOU HAVE A SOURCE TERM!

```
%Termos de fonte quando existentes
%%%%% Alterável %%%%%
SU(J)=0;
SP(J)=0;
```

S=4-5T

```
BP=-5
CP=4
SU(J)=SU(J)+CP
SP(J)=SP(J)+BP
```

$$S = B_P T_P + C_P$$

LINEARIZED!!!!!!

BP ≤ 0

$$\frac{(A_p - S_p)}{k} T_p$$

S=4-5T^3

$$S = S^* + \left(\frac{dS}{dT}\right)^* (T_p - T_P^*)$$

$$S^* = 4 - 5T_P^3$$

* - Means “previous iteration”

$$\left(\frac{dS}{dT}\right)^* = -15T_P^{*2}$$

$$S = 4 - 5T_P^3 - 15T_P^{*2}(T_p - T_P^*) = 4 - 5T_P^3 + 15T_P^{*3} - 15T_P^{*2}T_p$$

$$S = 4 + 10T_P^{*3} - 15T_P^{*2}T_p$$

$$\begin{cases} CP = 4 + 10T_P^{*3} \\ BP = -15T_P^{*2} \end{cases}$$

```
%Termos fonte
CP=4+10*T(I,J)^3;
BP=-15*T(I,J)^2;
SU(J)=SU(J)+CP;
SP(J)=SP(J)+BP;
```

SUBROUTINE PROMOD

USUALLY NEEDS CHANGES FOR BOUNDARY CONDITIONS!

```
%O Utilizador define o tipo de condição de fronteira (1-Temperatura,
%2-Fluxo, 3-Convecção e 4-simetria para a fronteira Norte, Sul, Este e
%Oeste (CFN,CFS,CFW,CFE)
%%%%% Alterável %%%%%
CFN=1;
CFS=1;
CFW=1;
CFE=1;
```

OUTPUT

```
CONDUCTION IN RECTANGULAR BAR WITH PRESCRIBED SURFACE TEMPERATURE

HEIGHT, H [M]-----------------------------------=      1.000
WEIGHT, W [M]-----------------------------------=      1.000
SPECIFIC HEAT, CV [J/KG.K]----------------------=    485.670
THERMAL CONDUCTIVITY, TCON [W/M.K]--------------=     14.680
DENSIT, DENSIT [KG/M3] -------------------------=    7800.00
INITIAL TIME STEP, DT [S] ----------------------=       50.0
SOURCE NORMALIZATION FACTOR, SNORM -------------= 1.468E+003
NUMBER OF NODES IN X DIRECTION, NI -------------=         22
NUMBER OF NODES IN Y DIRECTION, NJ -------------=         22
```

TEMPERATURE (°C)

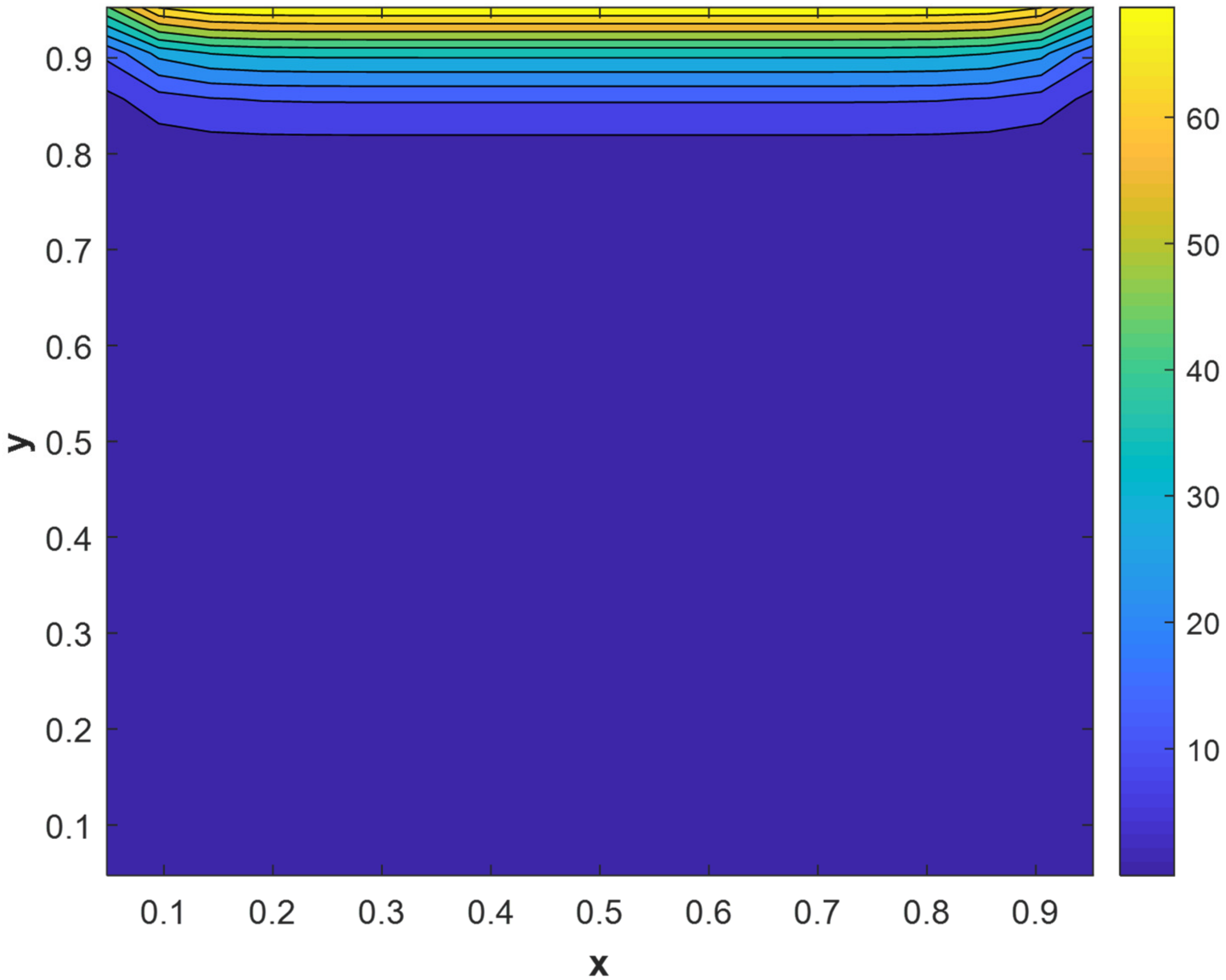
J \ I=	1	2	3	4	5	6	7	8	Y
22	0.00E+000	1.00E+002	1.00E+002	1.00E+002	1.00E+002	1.00E+002	1.00E+002	1.00E+002	1.0000
21	0.00E+000	4.70E+001	6.88E+001	7.40E+001	7.52E+001	7.55E+001	7.55E+001	7.55E+001	0.9750
20	0.00E+000	1.55E+001	2.97E+001	3.46E+001	3.60E+001	3.64E+001	3.65E+001	3.65E+001	0.9250
19	0.00E+000	4.95E+000	1.09E+001	1.33E+001	1.42E+001	1.44E+001	1.45E+001	1.45E+001	0.8750
18	0.00E+000	1.46E+000	3.46E+000	4.39E+000	4.73E+000	4.83E+000	4.86E+000	4.87E+000	0.8250
17	0.00E+000	3.94E-001	9.71E-001	1.26E+000	1.38E+000	1.41E+000	1.42E+000	1.43E+000	0.7750
16	0.00E+000	9.69E-002	2.45E-001	3.24E-001	3.57E-001	3.69E-001	3.72E-001	3.73E-001	0.7250
15	0.00E+000	2.19E-002	5.64E-002	7.58E-002	8.42E-002	8.72E-002	8.82E-002	8.84E-002	0.6750
14	0.00E+000	4.59E-003	1.20E-002	1.63E-002	1.82E-002	1.90E-002	1.92E-002	1.93E-002	0.6250
13	0.00E+000	8.99E-004	2.37E-003	3.26E-003	3.67E-003	3.83E-003	3.89E-003	3.90E-003	0.5750
12	0.00E+000	1.66E-004	4.42E-004	6.11E-004	6.94E-004	7.27E-004	7.38E-004	7.42E-004	0.5250
11	0.00E+000	2.90E-005	7.78E-005	1.08E-004	1.24E-004	1.30E-004	1.32E-004	1.33E-004	0.4750
10	0.00E+000	4.83E-006	1.30E-005	1.83E-005	2.10E-005	2.22E-005	2.26E-005	2.27E-005	0.4250
9	0.00E+000	7.70E-007	2.09E-006	2.95E-006	3.41E-006	3.60E-006	3.68E-006	3.70E-006	0.3750
8	0.00E+000	1.18E-007	3.22E-007	4.58E-007	5.30E-007	5.63E-007	5.75E-007	5.79E-007	0.3250
7	0.00E+000	1.75E-008	4.78E-008	6.83E-008	7.95E-008	8.46E-008	8.66E-008	8.73E-008	0.2750
6	0.00E+000	2.50E-009	6.87E-009	9.86E-009	1.15E-008	1.23E-008	1.26E-008	1.27E-008	0.2250
5	0.00E+000	3.47E-010	9.56E-010	1.38E-009	1.62E-009	1.73E-009	1.78E-009	1.80E-009	0.1750
4	0.00E+000	4.68E-011	1.29E-010	1.88E-010	2.21E-010	2.37E-010	2.44E-010	2.47E-010	0.1250
3	0.00E+000	6.14E-012	1.70E-011	2.48E-011	2.93E-011	3.16E-011	3.25E-011	3.29E-011	0.0750
2	0.00E+000	6.91E-013	1.92E-012	2.81E-012	3.33E-012	3.59E-012	3.70E-012	3.75E-012	0.0250
1	0.00E+000	0.00E+000	0.00E+000	0.00E+000	0.00E+000	0.00E+000	0.00E+000	0.00E+000	0.0000
X=	0.0000	0.0250	0.0750	0.1250	0.1750	0.2250	0.2750	0.3250	

NITER	SOURCE	T(6,4)	TIME(s)	DT(s)	NSTEP
1	6.537E-002	6.700E+002	0	0	1
2	2.019E-003	6.735E+002	0	0	1
3	7.789E-005	6.737E+002	0	0	1

RUN **ORIGINAL CASE**

UNSTEADY, t=**20iterations** x 50s/iteration = **1000s**

22 x 22

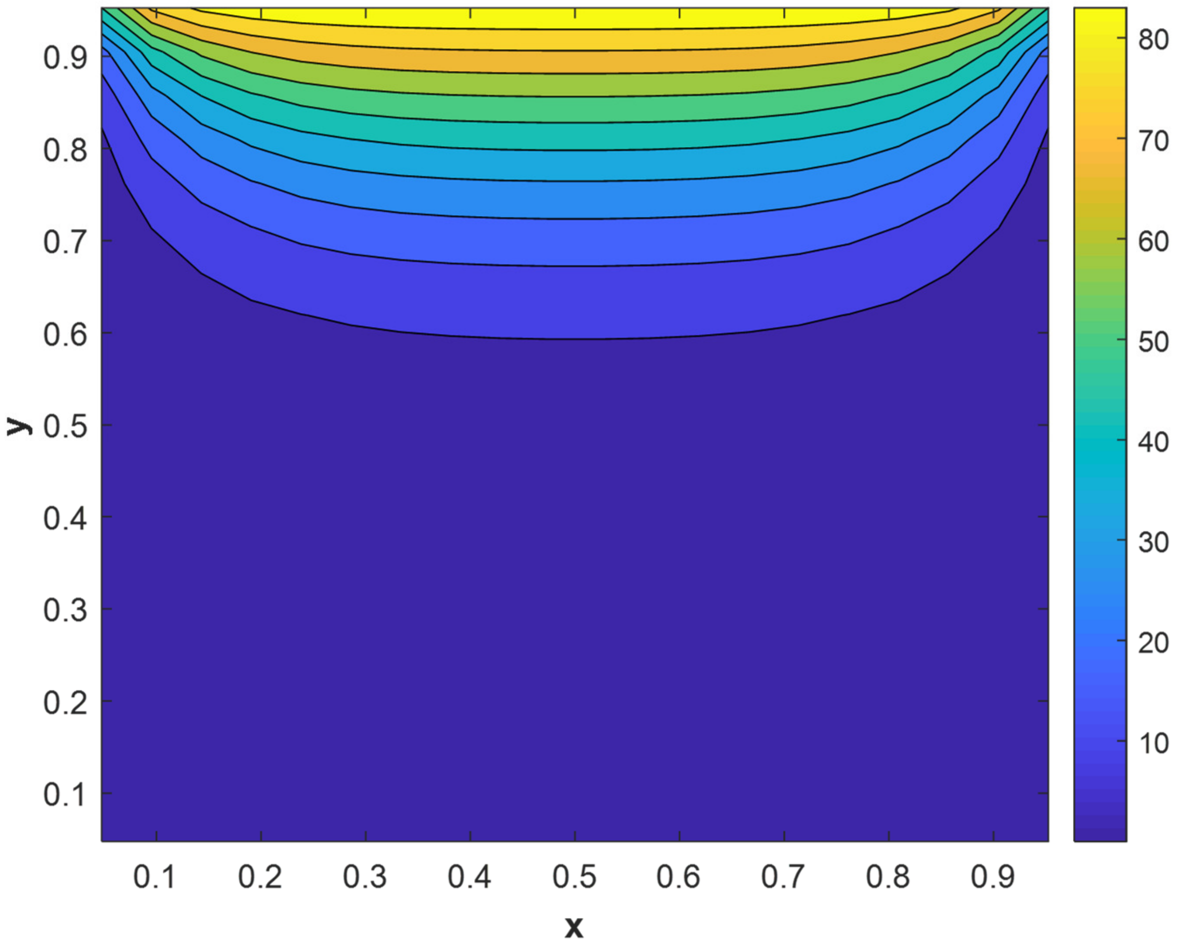


RUN **ALMOST ORIGINAL CASE (MORE TIME!)**

UNSTEADY, t=140iterations x 50s/iteration = **7000s**

MAXSTP = 140

22 x 22



RUN **STEADY CASE**

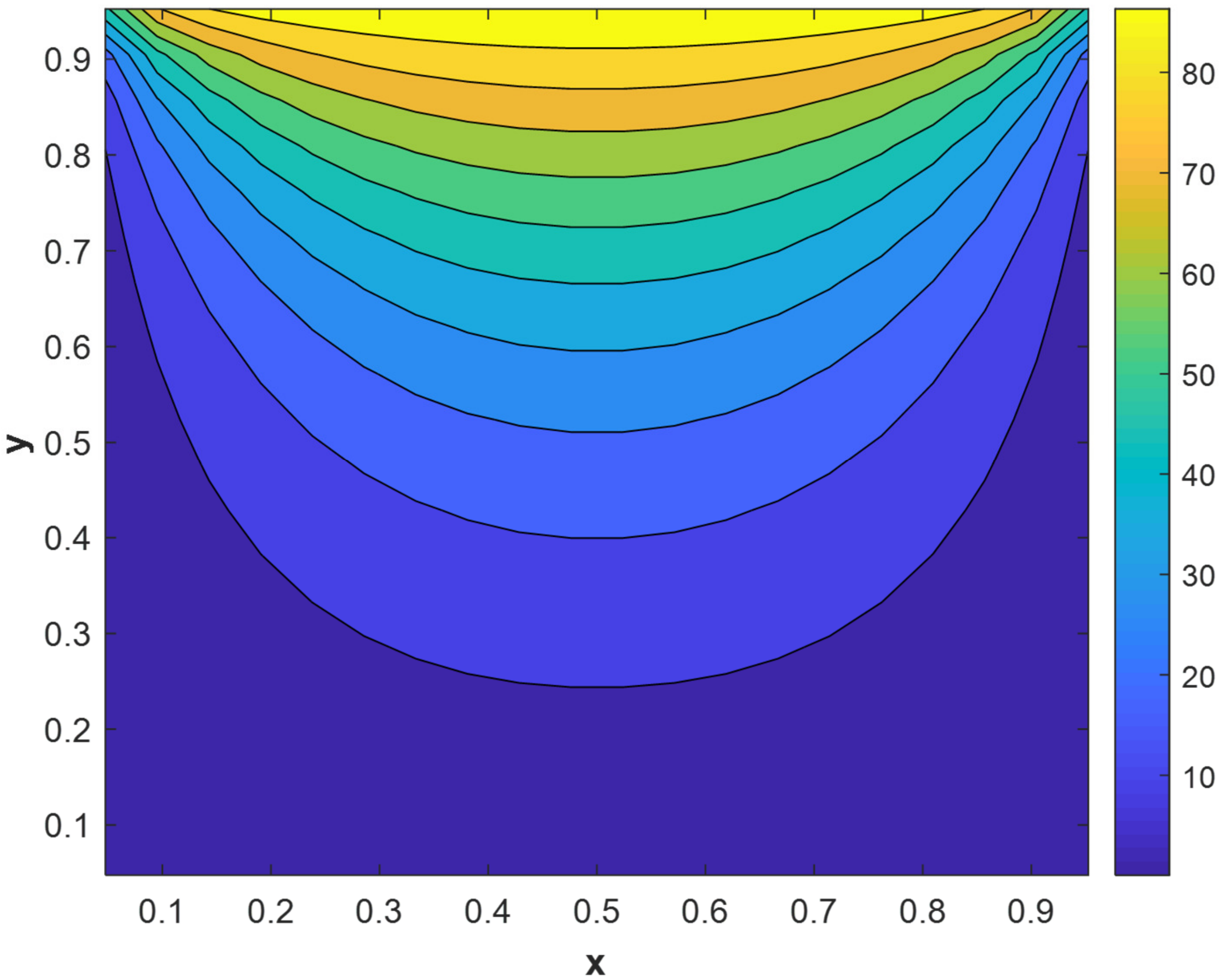
MAXIT =1000

NITPRI=1100

INTIME = 0

22 x 22

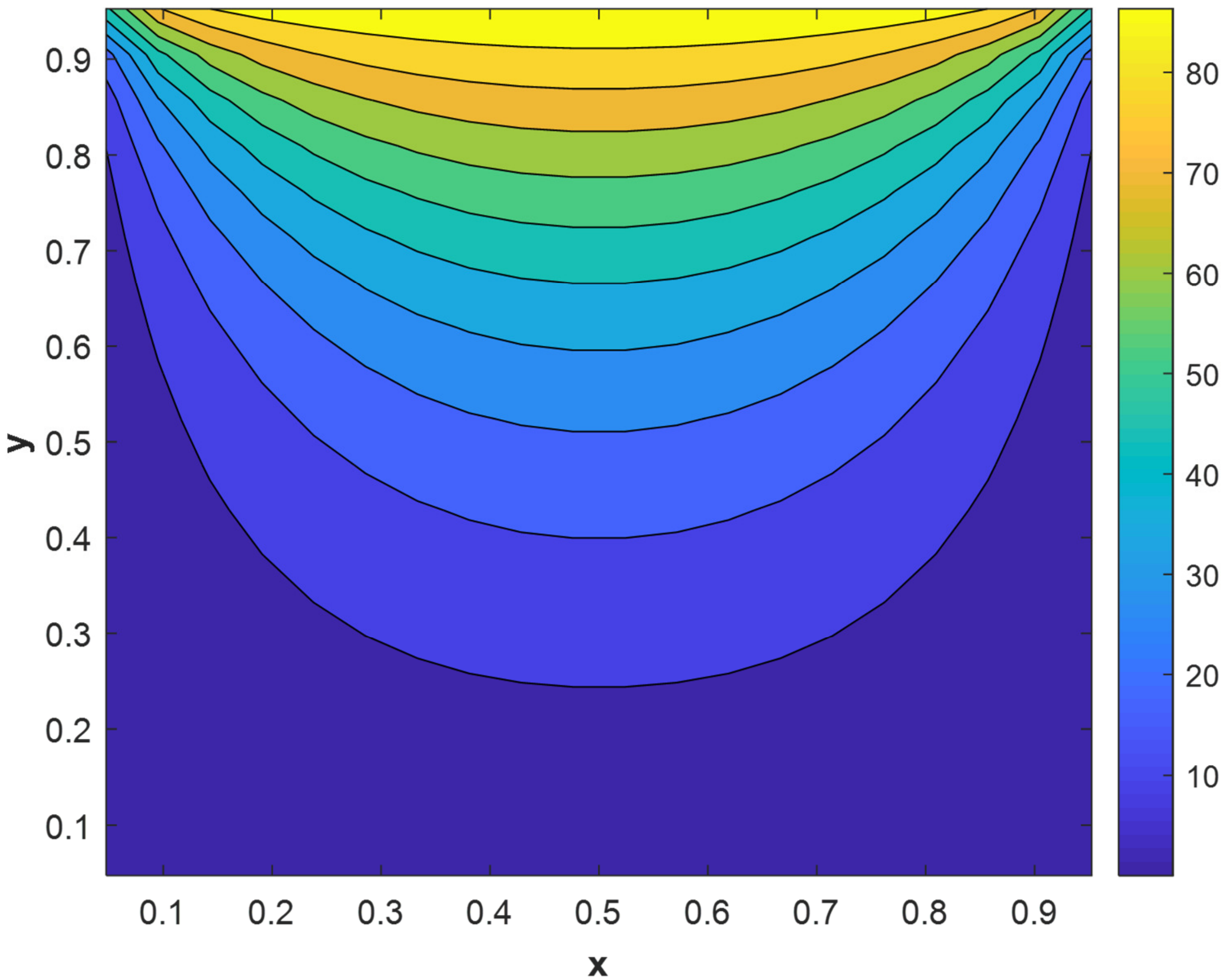
180 iterations



RUN STEADY CASE **(BETTER CONVERGENCE)**

SORMAX=0.00001

274 iterations



RUN **STEADY CASE**

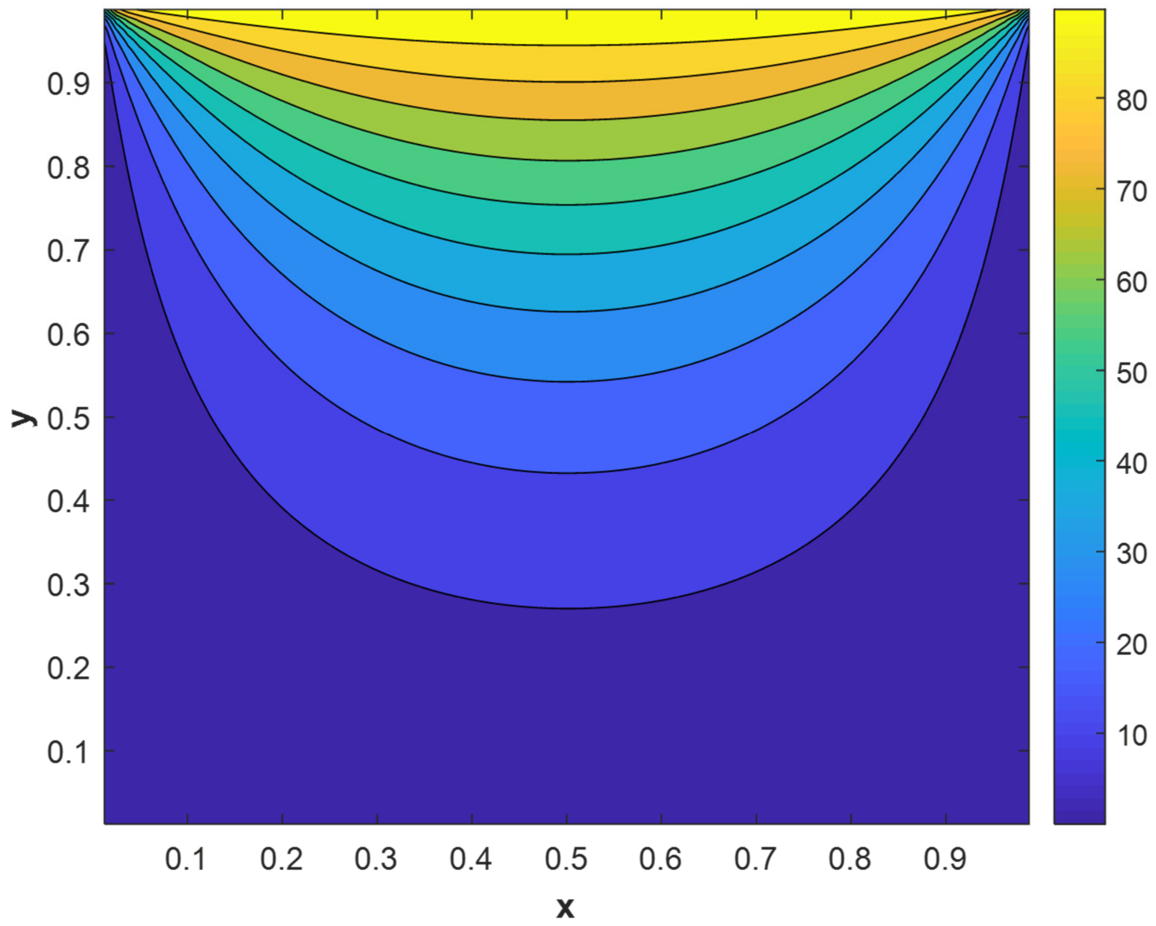
MAXIT =1000

NITPRI=1100

INTIME = 0

82 x 82

DID NOT CONVERGE!!!!!



RUN **STEADY CASE**

MAXIT =10000

NITPRI=11000

INTIME = 0

82 x 82

2848 iterations