Circuit Theory and Electronics Fundamentals

Department of Electrical and Computer Engineering, Técnico, University of Lisbon

T1 - Circuit Analysis Methods

António Oliveira (96512), Daniela Cardoso (96517), Francisco Mendes (96529)

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Contents

1 Introduction **1**

2 Theoretical Analysis **2**
2.1 Mesh Analysis 2
2.2 Nodal Analysis 3
2.3 Results of both methods 4

3 Simulation Analysis **5**
3.1 Operating Point Analysis 5

4 Conclusion **6**

1 Introduction

The objective of this laboratory assignment is to study a circuit containing an independent voltage source V_a, an independent current source I_d, a current-controlled voltage source V_c, a voltage-controlled current source I_b, and 7 resistors. The circuit can be seen in Figure 1. (Note: in the resistors, the current was considered to go through from the right side to the left side, and downwards.)

In Section 2, a theoretical analysis, using the software GNU Octave, of the circuit is presented. In Section 3, the circuit is analyzed by simulation using the software Ngspice, and the results are compared to the theoretical results obtained in Section 2. The conclusions of this study are outlined in Section 4.

After running the program “t1_datagen.py”, the following values were obtained in Table 1.

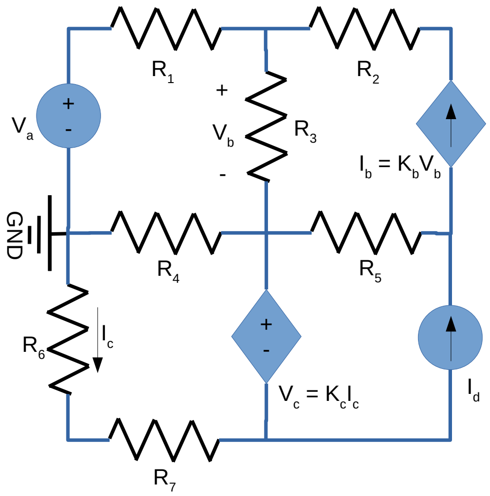

Figure 1: Circuit T1.

Components	
R_1	1.00934e+03 Ω
R_2	2.00297e+03 Ω
R_3	3.10903e+03 Ω
R_4	4.13001e+03 Ω
R_5	3.10841e+03 Ω
R_6	2.07408e+03 Ω
R_7	1.04985e+03 Ω
V_a	5.09750e+00 V
I_d	1.04885e-03 A
K_b	7.12347e-03 S
K_c	8.31335e+03 Ω

Table 1: Components characteristics.

2 Theoretical Analysis

In this section, the circuit shown in Figure 1 is analyzed theoretically, using the mesh method and the nodal method.

2.1 Mesh Analysis

The circuit was analyzed using the 4 elemental meshes, as shown in Figure 2

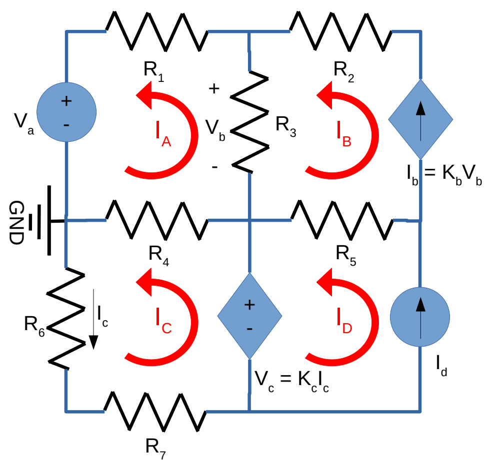

Figure 2: Currents through the elemental meshes.

Resulting in the following equations:

$$\begin{cases} -R_3(-I_A + I_B) - R_1(-I_A) + V_a + R_4(I_A - I_C) = 0 \\ I_B = I_b \quad (by\ observation) \\ -V_c - R_4(-I_C + I_A) + R_6 I_C + R_7 I_C = 0 \\ I_D = I_d \quad (by\ observation) \\ I_b = K_b V_b = K_b R_3(-I_A + I_B) \\ V_C = K_C I_C \end{cases} \tag{1}$$

Simplifying in the next matrix system:

$$\begin{bmatrix} R_3 + R_1 + R_4 & -R_3 & -R_4 & 0 \\ K_b R_3 & 1 - K_b R_3 & 0 & 0 \\ -R_4 & 0 & -K_C + R_4 + R_6 + R_7 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} I_A \\ I_B \\ I_C \\ I_D \end{bmatrix} = \begin{bmatrix} -V_a \\ 0 \\ 0 \\ I_d \end{bmatrix} \tag{2}$$

Solving the system 2 results in the values in the Table 2.

Mesh	(A)
I_A	-2.41670e-04
I_B	-2.53098e-04
I_C	9.42128e-04
I_D	1.04885e-03

Table 2: Currents of each mesh.

2.2 Nodal Analysis

The circuit was analyzed using 7 nodes and the GND node, as shown in Figure 3

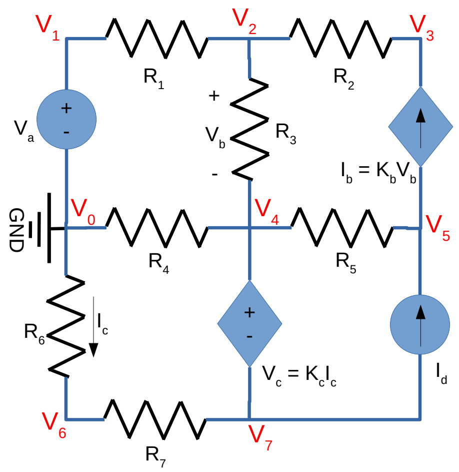

Figure 3: Voltage in each node.

Resulting in the following equations:

$$\begin{cases} KCL\ equations: \\ node\ 2 \rightarrow (V_1 - V_2)G_1 - (V_2 - V_3)G_2 - (V_2 - V_4)G_3 = 0 \\ node\ 3 \rightarrow (V_2 - V_3)G_2 + I_b = 0 \\ node\ 4 \rightarrow (V_2 - V_4)G_3 + (-V_4)G_4 - (V_4 - V_5)G_5 - I_{V_C} = 0 \\ node\ 5 \rightarrow (V_4 - V_5)G_5 + I_d - I_b = 0 \\ node\ 6 \rightarrow (-V_6)G_6 - (V_6 - V_7)G_7 = 0 \\ \\ Additional\ equations: \\ V_1 = V_a \\ I_b = K_b(V_2 - V_4) \\ I_{V_C} = I_d - (V_6 - V_7)G_7 \end{cases} \tag{3}$$

Simplifying in the next matrix system:

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ G_1 & -G_1 - G_2 - G_3 & G_2 & G_3 & 0 & 0 & 0 \\ 0 & K_b + G_2 & -G_2 & -K_b & 0 & 0 & 0 \\ 0 & G_3 & 0 & -G_3 - G_4 - G_5 & G_5 & G_7 & -G_7 \\ 0 & -K_b & 0 & G_5 + K_b & -G_5 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -G_6 - G_7 & G_7 \\ 0 & 0 & 0 & 1 & 0 & K_cG_6 & -1 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \\ V_6 \\ V_7 \end{bmatrix} = \begin{bmatrix} V_a \\ 0 \\ 0 \\ I_d \\ -I_d \\ 0 \\ 0 \end{bmatrix} \tag{4}$$

Solving the system 4 results in the values in the Table 3.

Node	(V)
V_1	5.09750e+00
V_2	4.85357e+00
V_3	4.34662e+00
V_4	4.88910e+00
V_5	8.93610e+00
V_6	-1.95404e+00
V_7	-2.94314e+00

Table 3: Voltage in each node.

2.3 Results of both methods

The voltage drop and currents of each resistor were the same of both methods and are shown in the Table 4.

Resistor	Voltage drop (V)	Current (A)
R_1	2.43927e-01	2.41670e-04
R_2	5.06949e-01	2.53098e-04
R_3	-3.55302e-02	-1.14281e-05
R_4	-4.88910e+00	-1.18380e-03
R_5	-4.04700e+00	-1.30195e-03
R_6	1.95404e+00	9.42128e-04
R_7	9.89095e-01	9.42128e-04

Table 4: Voltage drop and Currents of each resistor.

3 Simulation Analysis

3.1 Operating Point Analysis

Table 5 shows the simulated operating point results for the circuit under analysis. Compared to the theoretical analysis results, one notices no differences.

Name	**Value [A or V]**
@gib[i]	-2.53098e-04
@id[current]	1.048853e-03
@r1[i]	2.416704e-04
@r2[i]	2.530984e-04
@r3[i]	-1.14281e-05
@r4[i]	-1.18380e-03
@r5[i]	-1.30195e-03
@r6[i]	9.421277e-04
@r7[i]	9.421277e-04
v(1)	5.097498e+00
v(2)	4.853572e+00
v(3)	4.346623e+00
v(4)	4.889102e+00
v(5)	8.936101e+00
v(6)	-1.95404e+00
v(6b)	-1.95404e+00
v(7)	-2.94314e+00

Table 5: Operating point. A variable preceded by @ is of type *current* and expressed in Ampere; other variables are of type *voltage* and expressed in Volt.

There is an additional node 6b, between nodes 6 and 7, because the system needs a voltage source to know the current that is going through the resistor R_6 to insert the current-controlled voltage source V_c in the circuit.

4 Conclusion

In this laboratory assignment, the objective of analyzing a circuit with independent and dependent sources has been achieved. The simulation results matched the theoretical results precisely with both analysis methods. The reason for this perfect match is the fact that this is a straightforward circuit containing only linear components, so the theoretical and simulation models should not differ. For more complex components, the theoretical and simulation models could differ but this is not the case in this work.