Circuit Theory and Electronics Fundamentals

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T2 - RC Circuit Analysis

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1 Introduction

The purpose of this laboratory assignment is to study a circuit that can be observed in Figure 1 containing an independent voltage source V_s, a current-controlled voltage source V_d, a voltagecontrolled current source I_b, seven resistors, and one capacitor C. The voltage V_s varies in time according to the expression 1. (Note: in the resistors, the current was considered to go through from the right side to the left side, and downwards).

$$v_s(t) = V_s\,u(-t) + \sin(2\pi f\,t)\,u(t) \quad (1)$$

$$u(t) = \begin{cases} 0, & t < 0 \\ 1, & t \geqslant 0 \end{cases} \quad (2)$$

In Section 2, a theoretical analysis, using the software GNU Octave, of the circuit is presented. The first step in the analysis was to determine all the voltages in the nodes and the currents in the branches, for $t < 0$, using the nodal method. Secondly, the equivalent resistance seen by the capacitor, for $t = 0$, and the time constant, τ were calculated. The next step was to determine the natural response in node 6, for $t \in [0, 20]ms$. In step 4, the phasors of each node were calculated, using the nodal method and replacing C with its impedance Z_C. The fifth step, was to determine the final solution (natural+forced responses), for $f = 1kHz$ and the time interval $t \in [-5, 20]ms$. Finally, the frequency responses v_C and v_6 were computed for the frequency range 0.1 Hz to 1 MHz.

In Section 3, the circuit is analyzed by simulation using the software Ngspice, and the results are compared to the theoretical results obtained in Section 2.

In Section 4, is done a more detailed comparison between the results obtained by both analyses, theoretical and simulation.

The conclusions of this study are outlined in Section 5.

After running the program "t2_datagen.py", the following values were obtained in Table 1.

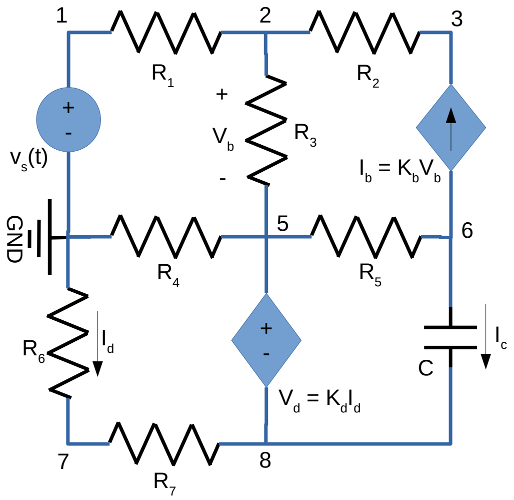

Figure 1: Circuit T2.

Components	
R_1	1.00934e+03 Ω
R_2	2.00297e+03 Ω
R_3	3.10903e+03 Ω
R_4	4.13001e+03 Ω
R_5	3.10841e+03 Ω
R_6	2.07408e+03 Ω
R_7	1.04985e+03 Ω
V_s	5.09750e+00 V
C	1.04885e-06 F
K_b	7.12347e-03 S
K_d	8.31335e+03 Ω

Table 1: Components characteristics.

2 Theoretical Analysis

In this section, the circuit shown in Figure 1 is analysed theoretically, according to the steps mentioned in Section 1.

2.1 Step 1

The circuit was analyzed for $t < 0$.

Resulting in the following matrix system:

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ G_1 & -G_1-G_2-G_3 & G_2 & G_3 & 0 & 0 & 0 \\ 0 & K_b+G_2 & -G_2 & -K_b & 0 & 0 & 0 \\ 0 & G_3 & 0 & -G_3-G_4-G_5 & G_5 & G_7 & -G_7 \\ 0 & -K_b & 0 & G_5+K_b & -G_5 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -G_6-G_7 & G_7 \\ 0 & 0 & 0 & 1 & 0 & K_dG_6 & -1 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_5 \\ V_6 \\ V_7 \\ V_8 \end{bmatrix} = \begin{bmatrix} V_s \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad (3)$$

Solving the system 3 results in the values in the Table 2.

Name	Value [A or V]
I_c	0.00000e+00
I_b	-2.53098e-04
I_1	2.41670e-04
I_2	2.53098e-04
I_3	-1.14281e-05
I_4	-1.18380e-03
I_5	-2.53098e-04
I_6	9.42128e-04
I_7	9.42128e-04
V_1	5.09750e+00
V_2	4.85357e+00
V_3	4.34662e+00
V_5	4.88910e+00
V_6	5.67584e+00
V_7	-1.95404e+00
V_8	-2.94314e+00
I_{V_d}	-9.42128e-04
I_{V_s}	-2.41670e-04

Table 2: Values computed for $t < 0$.

2.2 Step 2

The circuit was analyzed, for $t = 0$, as shown in Figure 2. In order to do that, V_s was considered to be zero and the capacitor was replaced with a voltage source $V_x = V(6) - V(8)$, being V_6 and V_8 the voltages in the nodes 6 and 8. In this case, V_x can be interpreted as a Thévenin voltage, therefore I_x, the Norton current equivalent, has to be from the negative node to the positive node of the voltage source V_x.

A nodal analysis in the circuit resulted in the following matrix system:

$$\begin{bmatrix} -G_1-G_2-G_3 & G_2 & G_3 & 0 & 0 & 0 & 0 \\ K_b+G_2 & -G_2 & -K_b & 0 & 0 & 0 & 0 \\ G_3 & 0 & -G_3-G_4-G_5 & G_5 & G_7 & -G_7 & -1 \\ -K_b & 0 & G_5+K_b & -G_5 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & -G_6-G_7 & G_7 & 0 \\ 0 & 0 & 1 & 0 & K_dG_6 & -1 & 0 \\ 0 & 0 & 0 & 1 & 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} V_2 \\ V_3 \\ V_5 \\ V_6 \\ V_7 \\ V_8 \\ I_x \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ V_x \end{bmatrix} \quad (4)$$

Solving the system 4 results in the values in the Table 3. The equivalent resistor R_{eq} was obtained by calculating $\frac{V_x}{I_x}$ and the time constant $\tau = R_{eq}C$.

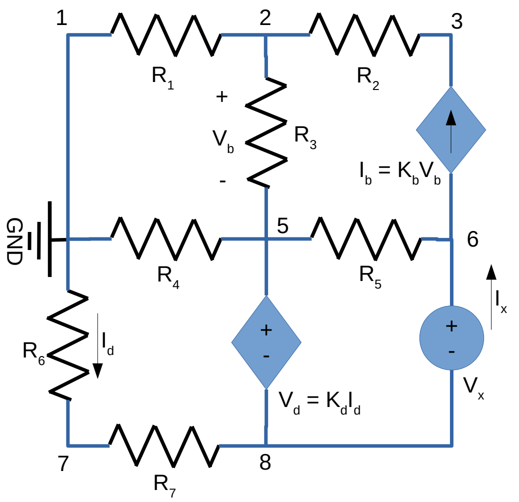

Figure 2: Circuit, for $t = 0$.

Node	Voltage (V)
V_1	0.00000
V_2	0.00000
V_3	0.00000
V_5	0.00000
V_6	8.61897
V_7	0.00000
V_8	0.00000
	Value
V_x	8.61897e+00 V
I_x	2.77279e-03 A
R_{eq}	3.10841e+03 Ω
τ	3.26027e-03 s^{-1}

Table 3: Values computed, for $t = 0$.

The time constant τ can only be calculated if we know the resistor seen by the capacitor terminals when all independent sources (in this case V_s) are switched off. In the moment of transition $t = 0$, V_s is considered to be zero even though $u(t)$ is 1 because that is the value it will take at $t = \varepsilon$, for a very small $\varepsilon > 0$.

2.3 Step 3

The circuit was analyzed to determine the natural response in node 6, $v_{6n}(t)$, for $t \in [0, 20]ms$. The obtained plot can be observed in Figure 3.

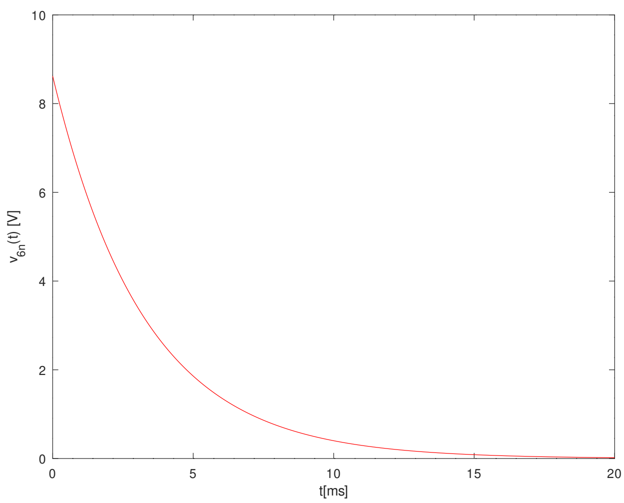

Figure 3: Natural response, $v_{6n}(t)$, for $t \in [0, 20]ms$.

2.4 Step 4

Using the phasor voltage $v_s = \exp{(-j\frac{\pi}{2})}$, and replacing C with its impedance Z_C,

$$Z_C = \frac{1}{jwC} \tag{5}$$

the circuit was analyzed to calculate the phasors in each node, for the time interval $t \in [0, 20]ms$ and the frequency $f = 1kHz$. Solving the matrix system 6 the results accquired are shown in Table 4.

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ Y_1 & -Y_1 - Y_2 - Y_3 & Y_2 & Y_3 & 0 & 0 & 0 \\ 0 & K_b + Y_2 & -Y_2 & -K_b & 0 & 0 & 0 \\ 0 & Y_3 & 0 & -Y_3 - Y_4 - Y_5 & Y_5 + \frac{1}{Z_C} & Y_7 & -Y_7 - \frac{1}{Z_C} \\ 0 & -K_b & 0 & Y_5 + K_b & -Y_5 - \frac{1}{Z_C} & 0 & \frac{1}{Z_C} \\ 0 & 0 & 0 & 0 & 0 & -Y_6 - Y_7 & Y_7 \\ 0 & 0 & 0 & 1 & 0 & K_d Y_6 & -1 \end{bmatrix} \begin{bmatrix} \tilde{V}_1 \\ \tilde{V}_2 \\ \tilde{V}_3 \\ \tilde{V}_5 \\ \tilde{V}_6 \\ \tilde{V}_7 \\ \tilde{V}_8 \end{bmatrix} = \begin{bmatrix} \tilde{V}_s \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{6}$$

Phasor	Value
$\tilde{V}_1$	0.00000 -1.00000 j
$\tilde{V}_2$	0.00000 -0.95215 j
$\tilde{V}_3$	0.00000 -0.85270 j
$\tilde{V}_5$	0.00000 -0.95912 j
$\tilde{V}_6$	-0.08234 +0.57335 j
$\tilde{V}_7$	0.00000 +0.38333 j
$\tilde{V}_8$	0.00000 +0.57737 j

Table 4: Phasors in each node, for $t \in [0, 20]ms$ and $f = 1kHz$.

2.5 Step 5

The circuit was analyzed to determine the total solution, given by the sum of the natural and forced solutions, in node 6, $v_6(t)$, for the time interval $t \in [-5, 20]ms$ and frequency $f = 1kHz$. In Figure 4 the plot for $v_s(t)$ an $v_6(t)$ can be observed.

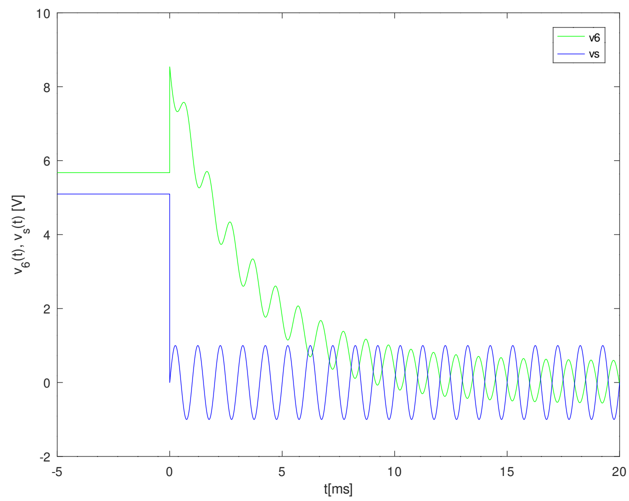

Figure 4: Total solution, $v_6(t)$ and $v_s(t)$, for $t \in [-5, 20]ms$ and $f = 1kHz$.

2.6 Step 6

The quantities $v_c(f)$ and $v_6(f)$ depend on the frequency. In order to study their behaviour for the frequency range 0.1 Hz to 1 MHz, the linear equation system 6 can be manipulated to obtain the system 7.

$$\left[\begin{array}{cccc|cc} K_b+Y_2 & -Y_2 & -K_b & 0 & 0 & 0 \\ Y_3-K_b & 0 & K_b-Y_3-Y_4 & -Y_6 & 0 & 0 \\ K_b-Y_1-Y_3 & 0 & Y_3-K_b & 0 & 0 & 0 \\ 0 & 0 & 1 & K_dY_6-\frac{Y_6}{Y_7}-1 & 0 & 0 \\ \hline K_b & 0 & -Y_5-K_b & 0 & Y_5+\frac{1}{Z_C} & -\frac{1}{Z_C} \\ 0 & 0 & 0 & -Y_6-Y_7 & 0 & Y_7 \end{array}\right] \begin{bmatrix} \tilde{V}_2 \\ \tilde{V}_3 \\ \tilde{V}_5 \\ \tilde{V}_7 \\ \tilde{V}_6 \\ \tilde{V}_8 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ -Y_1\tilde{V}_s \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad (7)$$

Analyzing the system 7, the vector $\begin{bmatrix} \tilde{V}_2 & \tilde{V}_3 & \tilde{V}_5 & \tilde{V}_7 \end{bmatrix}^T$ can be determined independently of the frequency, given that all the elements of the upper left matrix are constants. From there, utilizing the equation 5 to get the impedance Z_C for each frequency, the remaining values $\tilde{V}_6$ and $\tilde{V}_8$ can be obtained directly from the expressions given in that system, as shown in equation 8, for each frequency.

$$\begin{cases} \tilde{V}_8 = \frac{(Y_6+Y_7)\tilde{V}_7}{Y_7} \\ \tilde{V}_6 = \frac{-K_b\tilde{V}_2+(Y_5+K_b)\tilde{V}_5+\frac{\tilde{V}_8}{Z_C}}{Y_5+\frac{1}{Z_C}} \end{cases} \quad (8)$$

After determining $\tilde{V}_6$ and $\tilde{V}_8$, $\tilde{V}_c$ is given by $\tilde{V}_c = \tilde{V}_6 - \tilde{V}_8$. The data obtained can be seen in Figures 5 and 6, where the plot for the magnitude and phase, respectively, of $v_s(f)$, $v_c(f)$ an $v_6(f)$ can be observed.

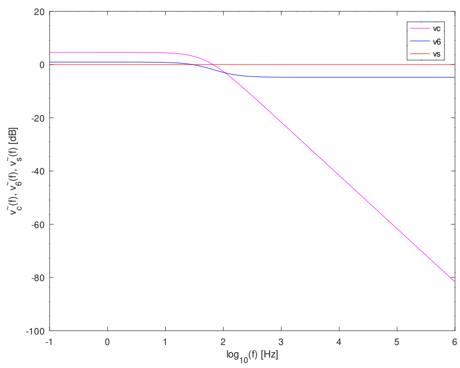

Figure 5: Frequency logarithmic scale, magnitude in dB.

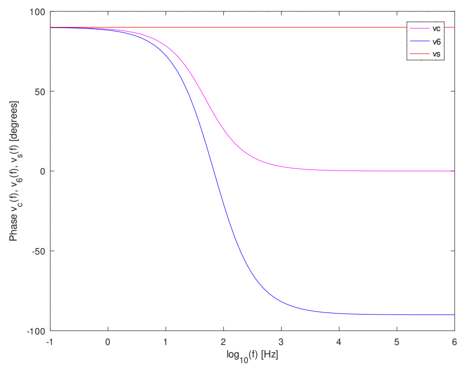

Figure 6: Frequency logarithmic scale, phase in degrees.

3 Simulation Analysis

3.1 Step 1

Table 5 shows the simulated operating point results for the circuit under analysis, for $t < 0$.

3.2 Step 2

Table 6 shows the simulated operating point results for the circuit under analysis, for $t = 0$.

Here, $I_x = -vx\#branch$ because it is not possible to define the current through the voltage source V_x as going from the node n- to the node n+. It may seems that there is current through all the branches and voltages in most of the nodes, but, after observing and considering that the values given by the "datagen" file, they have precision till 10^{-11}, therefore it is possible to say that voltages lesser $10^{-11}V$ and currents lesser than $10^{-11}A$ are indistinguishable from 0.

Name	**Value [A or V]**
@c[i]	0.000000e+00
@gib[i]	-2.42062e-04
@r1[i]	2.310332e-04
@r2[i]	2.420620e-04
@r3[i]	-1.10288e-05
@r4[i]	-1.23547e-03
@r5[i]	-2.42062e-04
@r6[i]	1.004440e-03
@r7[i]	1.004440e-03
v(1)	5.168854e+00
v(2)	4.934507e+00
v(3)	4.430769e+00
v(5)	4.968812e+00
v(6)	5.700690e+00
v(7)	-2.05136e+00
v(7b)	-2.05136e+00
v(8)	-3.09838e+00
hvd#branch	-1.00444e-03
vr6#branch	1.004440e-03
vs#branch	-2.31033e-04

Table 5: Operating point. A variable preceded by @ or has # in its name is of type *current* and expressed in Ampere; other variables are of type *voltage* and expressed in Volt.

Name	**Value [A or V]**
@gib[i]	0.000000e+00
@r1[i]	0.000000e+00
@r2[i]	0.000000e+00
@r3[i]	0.000000e+00
@r4[i]	0.000000e+00
@r5[i]	-2.91021e-03
@r6[i]	4.336809e-19
@r7[i]	2.558565e-18
v(1)	0.000000e+00
v(2)	0.000000e+00
v(3)	0.000000e+00
v(5)	0.000000e+00
v(6)	8.799066e+00
v(7)	-8.85705e-16
v(7b)	-8.85705e-16
v(8)	-3.55271e-15
hvd#branch	2.910213e-03
vr6#branch	4.336809e-19
vs#branch	0.000000e+00
vx#branch	-2.91021e-03

Table 6: Operating point. A variable preceded by @ or has # in its name is of type *current* and expressed in Ampere; other variables are of type *voltage* and expressed in Volt.

3.3 Step 3

Using as initial condition $V_s = 0V$ and $V_c = V_6 - V_8$, it was possible to simulate the natural response in the node 6, $v_{6n}(t)$. The plot can be seen in Figure 7, for $t \in [0, 20]ms$.

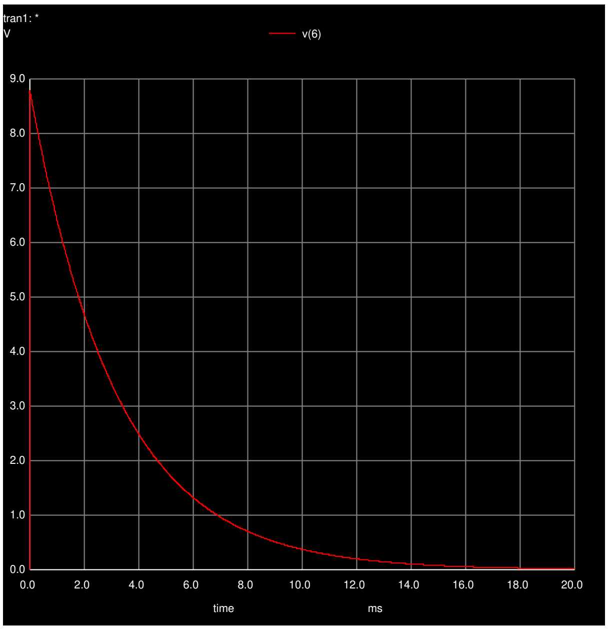

Figure 7: Natural response in node 6, for $t \in [0, 20]ms$.

3.4 Step 4

Repeating the previous step to simulate the total response on node 6, considering $v_s(t) = \sin(2\pi ft)$ and frequency $f = 1kHz$. The plot containing the stimulus and the response can be seen in Figure 8.

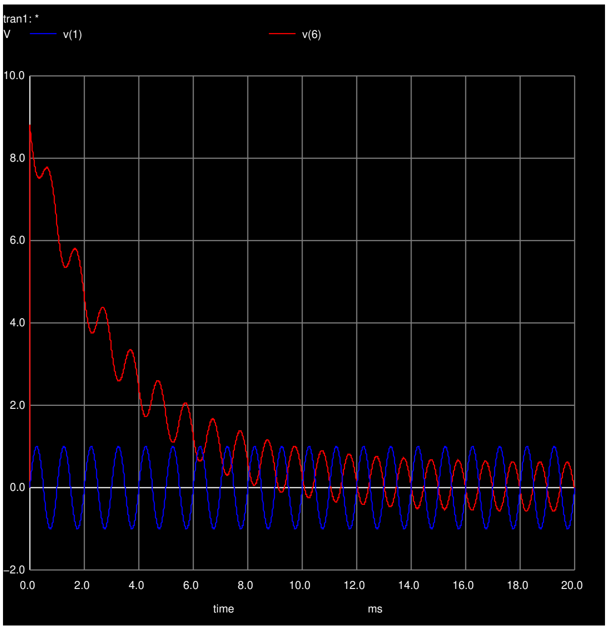

Figure 8: Total response in node 6, for $t \in [0, 20]ms$ and $f = 1kHz$.

3.5 Step 5

3.5.1 Magnitude Response

Figure 9 shows the magnitude of the frequency response for the circuit under analysis, for the voltages $v_c(f),\ v_6(f)$ and $v_s(f)$, with f from 0.1 *Hz* to 1 *MHz*.

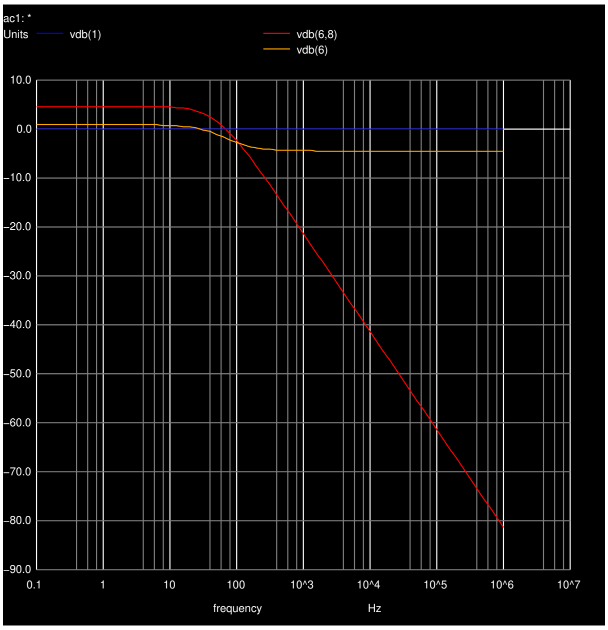

Figure 9: Magnitude response.

3.5.2 Phase Response

Figure 10 shows the phase in degrees of the frequency response for the circuit under analysis.

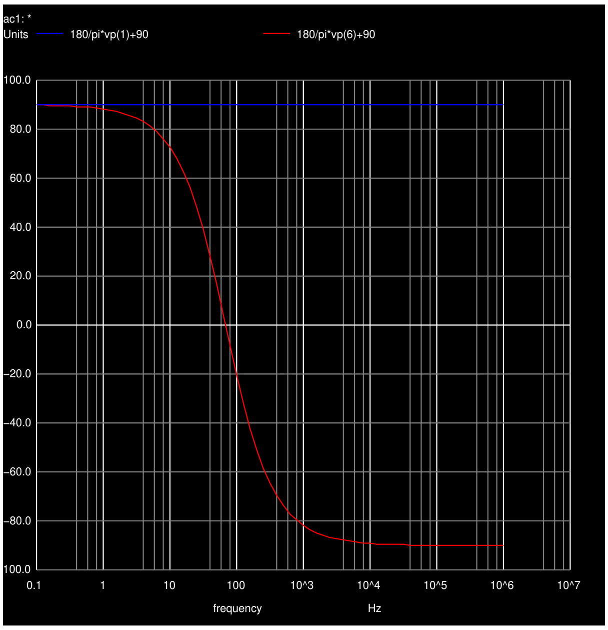

Figure 10: Phase response.

4 Comparison

4.1 Step 1

We can now compare the results obtained for step 1 with both theoretical analysis and simulation analysis presented previously in Tables 2 and 5.

Name	Value [A or V]
I_c	0.00000e+00
I_b	-2.53098e-04
I_1	2.41670e-04
I_2	2.53098e-04
I_3	-1.14281e-05
I_4	-1.18380e-03
I_5	-2.53098e-04
I_6	9.42128e-04
I_7	9.42128e-04
V_1	5.09750e+00
V_2	4.85357e+00
V_3	4.34662e+00
V_5	4.88910e+00
V_6	5.67584e+00
V_7	-1.95404e+00
V_8	-2.94314e+00
I_{V_d}	-9.42128e-04
I_{V_s}	-2.41670e-04

Table 7: Octave values for $t < 0$

Name	Value [A or V]
@c[i]	0.000000e+00
@gib[i]	-2.42062e-04
@r1[i]	2.310332e-04
@r2[i]	2.420620e-04
@r3[i]	-1.10288e-05
@r4[i]	-1.23547e-03
@r5[i]	-2.42062e-04
@r6[i]	1.004440e-03
@r7[i]	1.004440e-03
v(1)	5.168854e+00
v(2)	4.934507e+00
v(3)	4.430769e+00
v(5)	4.968812e+00
v(6)	5.700690e+00
v(7)	-2.05136e+00
v(7b)	-2.05136e+00
v(8)	-3.09838e+00
hvd#branch	-1.00444e-03
vr6#branch	1.004440e-03
vs#branch	-2.31033e-04

Table 8: Ngspice values for $t < 0$

4.2 Step 2

We can now compare the results obtained for step 2 with both theoretical analysis and simulation analysis presented previously in Tables 3 and 6.

Node	Voltage (V)
V_1	0.00000
V_2	0.00000
V_3	0.00000
V_5	0.00000
V_6	8.61897
V_7	0.00000
V_8	0.00000
	Value
V_x	8.61897e+00 V
I_x	2.77279e-03 A
R_{eq}	3.10841e+03 Ω
τ	3.26027e-03 s^{-1}

Table 9: Octave values, for $t = 0$

Name	Value [A or V]
@gib[i]	0.000000e+00
@r1[i]	0.000000e+00
@r2[i]	0.000000e+00
@r3[i]	0.000000e+00
@r4[i]	0.000000e+00
@r5[i]	-2.91021e-03
@r6[i]	4.336809e-19
@r7[i]	2.558565e-18
v(1)	0.000000e+00
v(2)	0.000000e+00
v(3)	0.000000e+00
v(5)	0.000000e+00
v(6)	8.799066e+00
v(7)	-8.85705e-16
v(7b)	-8.85705e-16
v(8)	-3.55271e-15
hvd#branch	2.910213e-03
vr6#branch	4.336809e-19
vs#branch	0.000000e+00
vx#branch	-2.91021e-03

Table 10: Ngspice values, for $t = 0$

We can see that the theoretical and simulation results for both steps are reasonably similar given what was stated in Section 3, which was expected.

5 Conclusion

In this laboratory assignment, the objective of analyzing the RC circuit shown in Figure 1 has been achieved. Time and frequency analyses have been performed both theoretically using the Octave maths tool and by circuit simulation using the Ngspice tool. The simulation results matched the theoretical results, apart from some voltages calculated in Step 2. Nonetheless, even this difference can be attributed to the precision of the data used that renders voltages as low as the ones obtained null. Having that into consideration, the simulation and theoretical results matched in every step, observed by comparing each step's tables and/or plots.