Intertemporal Theory of the Current Account

International Macroeconomics
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Introduction: Why do some countries borrow while others lend? Why do some economies consistently run trade deficits while others accumulate surpluses? In this chapter, we begin to answer these questions by developing an open economy model centered on optimal intertemporal decision-making. The fundamental idea is that countries, much like households, borrow or lend to smooth consumption over time when faced with fluctuating income or output.

Our analysis focuses on a small open economy—an economy that trades freely in goods and financial assets with the rest of the world, but whose actions do not affect international prices or interest rates. In such an environment, the country takes world prices and the international interest rate as given. This simplifying assumption allows us to focus on how external imbalances arise as the outcome of optimal consumption-savings decisions, without having to model general equilibrium effects.

1. The Household Maximization Problem

We consider a representative household that lives for two periods and receives an endowment of goods in each period, denoted by Q_1 and Q_2. The household can use its resources to consume or save via trade in a one-period bond.

In period 1, the household allocates income between consumption C_1 and net purchases of bonds $B_1 - B_0$. Bond holdings at the beginning of the period are denoted by B_0, and they yield a return r_0. Therefore, the period 1 budget constraint is:

$$C_1 + B_1 - B_0 = r_0 B_0 + Q_1$$

In period 2, the household again divides its resources between consumption C_2 and net bond purchases $B_2 - B_1$. Given the return on bonds between periods 1 and 2 is r_1, the second-period constraint is:

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$$C_2 + B_2 - B_1 = r_1 B_1 + Q_2$$

Since the household lives only two periods, it cannot leave behind unpaid debt (no-Ponzi condition), nor will it choose to save for a period that never arrives. Hence, the terminal condition is:

$$B_2 = 0$$

This is known as the transversality condition. Substituting this condition back into the second-period constraint and combining both budget equations, we eliminate B_1 and B_2 to derive the intertemporal budget constraint:

$$C_1 + \frac{C_2}{1+r_1} = (1+r_0)B_0 + Q_1 + \frac{Q_2}{1+r_1}$$

This expression states that the present value of consumption must equal the present value of income plus initial wealth. In other words, consumption plans must be affordable.

The household chooses C_1 and C_2 to maximize lifetime utility:

$$U = U(C_1) + \beta U(C_2)$$

where $\beta \in (0,1)$ is the subjective discount factor, representing the degree of patience. The consumer's problem is:

$$\max_{C_1, C_2} \quad U(C_1) + \beta U(C_2) \quad \text{subject to} \quad C_1 + \frac{C_2}{1+r_1} = \text{wealth}$$

We derive the first-order conditions to obtain our famous Euler equation:

$$U'(C_1) = (1+r_1)\beta U'(C_2)$$

This optimality condition is intuitive. Sacrificing one unit of consumption in period 1 reduces utility by $U'(C_1)$. If that unit is saved, it yields $1 + r_1$ units of consumption in period 2, each of which brings marginal utility $\beta U'(C_2)$. The right-hand side represents the utility gain from saving, and the consumer chooses C_1 and C_2 to equalize marginal cost and marginal benefit of intertemporal consumption shifts.

If the interest rate r_1 is high, future consumption becomes more attractive, pushing the consumer to save more and defer consumption. Conversely, a low interest rate favors higher present consumption.

To obtain a closed-form solution, we assume the utility function takes the logarithmic form:

$$U(C_t) = \ln(C_t)$$

The Euler equation then becomes:

$$\frac{1}{C_1} = (1 + r_1)\beta \cdot \frac{1}{C_2} \quad \Rightarrow \quad C_2 = (1 + r_1)\beta C_1 \tag{1}$$

Substituting this into the intertemporal budget constraint:

$$\begin{aligned} C_1 + \frac{(1 + r_1)\beta C_1}{1 + r_1} &= (1 + r_0)B_0 + Q_1 + \frac{Q_2}{1 + r_1} \\ C_1(1 + \beta) &= (1 + r_0)B_0 + Q_1 + \frac{Q_2}{1 + r_1} \\ C_1^* &= \frac{(1 + r_0)B_0 + Q_1 + \frac{Q_2}{1+r_1}}{1 + \beta} \end{aligned} \tag{2}$$

Using this, optimal second-period consumption is:

$$C_2^* = \beta(1 + r_1)C_1^* \tag{3}$$

Finally, we substitute C_1^* into the period-1 budget constraint to solve for optimal bond holdings at the end of period 1:

$$B_1^* = (1 + r_0)B_0 + Q_1 - C_1^*$$

2. The Trade Balance and the Current Account

Now, we introduce our open economy accounts. In this two-period economy, the trade balance is simply the gap between output and consumption in each period. Formally:

$$TB_1 = Q_1 - C_1 \tag{4}$$

$$TB_2 = Q_2 - C_2 \tag{5}$$

The current account in each period includes net investment income plus the trade balance. From the budget constraint:

$$CA_1 = r_0 B_0 + TB_1 = B_1 - B_0 \tag{6}$$

$$CA_2 = r_1 B_1 + TB_2 = -B_1 \tag{7}$$

The final equality in each case comes from combining the period-1 budget constraint with the terminal condition $B_2 = 0$. That is, any net foreign asset accumulation in period 1 reflects the current account surplus, and all outstanding positions are unwound in period 2.

Substituting the optimal consumption C_1^* into the trade balance expression:

$$TB_1^* = Q_1 - C_1^* = Q_1 - \frac{(1 + r_0)B_0 + Q_1 + \frac{Q_2}{1+r_1}}{1 + \beta}$$

$$TB_1^* = \frac{(1+\beta)Q_1 - (1+r_0)B_0 - Q_1 - \frac{Q_2}{1+r_1}}{1+\beta} = \frac{\beta Q_1 - (1+r_0)B_0 - \frac{Q_2}{1+r_1}}{1+\beta}$$

The optimal current account in period 1 is:

$$CA_1^* = B_1^* - B_0 = [(1+r_0)B_0 + Q_1 - C_1^*] - B_0 = r_0 B_0 + TB_1^*$$

The model reveals how intertemporal preferences, interest rates, and expected output influence a country's trade and external financial position. If optimal consumption in period 1 exceeds endowment Q_1, then:

$$C_1^* > Q_1 \quad \Rightarrow \quad TB_1^* < 0$$

The economy must borrow from abroad to finance its excess consumption, implying a trade deficit and a reduction in the current account balance. Conversely, if the economy saves (i.e., $C_1^* < Q_1$), it will run a trade surplus and accumulate foreign assets. It is worth noting that as we are assuming identical households (same preferences and parameters), then households do not borrow from one another, but they borrow in aggregate from abroad.

This simple two-period framework captures the essence of current account dynamics: countries borrow and lend internationally to smooth consumption across periods in response to output shocks, interest rates, and intertemporal preferences. Let's us review how some of this shock propagate through our small open economy model.

3. Adjustment to Temporary and Permanent Output Shocks

A fundamental principle in intertemporal economics is that forward-looking agents respond differently to temporary and permanent changes in income. A temporary shock affects only one period's endowment, while a permanent shock affects income across both periods. This distinction is key to understanding how optimal consumption and external balances (savings, trade balance, and current account) adjust.

3.1. Adjustment to Temporary Output Shocks

Consider a positive temporary income shock in period 1, so that Q_1 increases while Q_2 remains unchanged.

From the optimal consumption rule:

$$C_1^* = \frac{(1+r_0)B_0 + Q_1 + \frac{Q_2}{1+r_1}}{1+\beta} \quad \Rightarrow \quad \frac{\partial C_1^*}{\partial Q_1} = \frac{1}{1+\beta} < 1$$

That is, the increase in Q_1 raises consumption in period 1, but by less than one-to-one due to consumption smoothing. The rest of the shock is saved.

- $C_1^* \uparrow$ but $\Delta C_1^* < \Delta Q_1$
- $C_2^* = \beta(1 + r_1)C_1^* \uparrow$
- From $B_1^* = (1 + r_0)B_0 + Q_1 \uparrow -C_1^* \uparrow$, savings $B_1^* \uparrow$
- Trade balance improves: $TB_1^* = Q_1 \uparrow -C_1^* \uparrow$
- Since $CA_1^* = r_0 B_0 + TB_1^* \uparrow$, $CA_1^* \uparrow$
- In period 2, $TB_2^* = Q_2 - C_2^* \uparrow$, $TB_2^* \downarrow$
- $CA_2^* = r_1 B_1 \uparrow +TB_2^* \downarrow$

Conclusion: A temporary positive output shock leads to a trade surplus and current account surplus in period 1, as the economy saves a portion of the temporary windfall to increase future consumption.

3.2. Adjustment to Permanent Output Shocks

Now suppose the positive output shock is permanent: both Q_1 and Q_2 increase by the same amount. Let the increase be ΔQ.

From the consumption rule:

$$C_1^* = \frac{(1 + r_0)B_0 + Q_1 + \frac{Q_2}{1+r_1}}{1 + \beta} \quad \Rightarrow \quad \Delta C_1^* = \frac{\Delta Q + \frac{\Delta Q}{1+r_1}}{1 + \beta}$$

Since both periods' endowments rise, the total present value of income increases permanently, and the household adjusts consumption upward in both periods proportionally.

- $C_1^* \uparrow$, $C_2^* \uparrow$
- Increase in C_1^* is roughly proportional to ΔQ
- $B_1^* = (1 + r_0)B_0 + Q_1 - C_1^*$: ambiguous depending on how much of the increase in Q_1 is saved. However, we know the variation will be minimal
- $TB_1^* = Q_1 - C_1^*$: ambiguous and minimal variation too
- $CA_1^* = r_0 B_0 + TB_1^*$, so the change in current account is also limited

Conclusion: A permanent income shock leads to proportional changes in consumption across both periods. There is little change in saving, so the trade balance and current account remain roughly unchanged. The economy internalizes the shock immediately by adjusting consumption upward.

The main insight is that *Temporary shocks* lead to intertemporal smoothing via borrowing/lending, causing external imbalances. In contrast, *permanent shocks* are absorbed through proportional changes in consumption, with little effect on the current account.

4. Adjustment to Interest Rate Shocks

Now consider a shock to the international interest rate r_1 faced by the small open economy, assuming it takes this rate as given. Let us analyze the effects of an increase in r_1 on consumption, savings, the trade balance, and the current account.

From the Euler equation:

$$U'(C_1) = (1 + r_1)\beta U'(C_2) \quad \Rightarrow \quad \frac{C_2^*}{C_1^*} = (1 + r_1)\beta$$

An increase in r_1 raises the return to saving, thereby inducing households to substitute future consumption for present consumption. Thus:

$$\frac{\partial C_1^*}{\partial r_1} < 0, \quad \frac{\partial C_2^*}{\partial r_1} > 0$$

Substituting the Euler equation into the intertemporal budget constraint and solving for C_1^* and C_2^*, we find that the increase in r_1 reduces C_1^*, increases B_1^*, and hence improves the trade balance and the current account in period 1.

- $C_1^* \downarrow$, $C_2^* \uparrow$ due to intertemporal substitution
- $B_1^* \uparrow$: more saving today to take advantage of higher returns
- $TB_1^* = Q_1 - C_1^* \downarrow$, $TB_1^* \uparrow$
- $CA_1^* = r_0 B_0 + TB_1^* \uparrow$, $CA_1^* \uparrow$
- In period 2, $CA_2^* = -B_1^* \uparrow$, $CA_2^* \downarrow$: accumulated savings are consumed

Conclusion: A rise in the world interest rate leads to lower present consumption, increased saving, and a current account surplus in period 1. This is reversed in period 2 when savings are drawn down to finance higher consumption. Take into account that the intertemporal substitution only accounts for the *substitution effect*. There is also an *income effect* that will rise or decrease consumption in both periods if the country is a creditor o debtor, respectively. Given our log preferences (and as it is in real life), the substitution effect dominates the income effect.

5. Adjustment to Anticipated Output Shocks

Consider now the case where households learn in period 1 that their period 2 income Q_2 will be higher. This constitutes an anticipated (but not yet realized) output shock.

The present discounted value of the endowment increases, which shifts the intertemporal budget constraint outward. Households feel wealthier today, despite the increase in income occurring only tomorrow.

From the optimal consumption rules:

$$C_1^* = \frac{(1+r_0)B_0 + Q_1 + \frac{Q_2+\Delta Q}{1+r_1}}{1+\beta} \quad \Rightarrow \quad \frac{\partial C_1^*}{\partial Q_2} = \frac{1}{(1+\beta)(1+r_1)} > 0$$

Hence, consumption in *both* periods increases despite the fact that the extra resources are only received in period 2.

- $C_1^* \uparrow$, $C_2^* \uparrow$
- Because Q_1 is unchanged, the increase in C_1^* must be financed through borrowing: $B_1^* \downarrow$
- $TB_1^* = Q_1 - C_1^* \uparrow$, $TB_1^* \downarrow$: the trade balance worsens
- $CA_1^* = r_0 B_0 + TB_1^* \downarrow$, $CA_1^* \downarrow$: the current account also worsens
- In period 2, the higher income $Q_2 + \Delta Q$ is used to repay the debt, so $CA_2^* = -B_1^* \downarrow$, $CA_2^* \uparrow$

If the same output shock were not anticipated, then consumption in period 1 would remain unchanged and all the adjustment would take place in period 2, leading to a spike in C_2 and no effect on the current account in period 1.

These cases underscore the importance of three critical distinctions when analyzing shocks:

1. **Is the shock temporary or permanent?**
   Temporary shocks prompt saving/dissaving behavior; permanent shocks adjust consumption proportionally.
2. **Is the shock current or future?**
   Future (anticipated) shocks cause preemptive consumption adjustments, affecting today's external balances.
3. **Is the shock anticipated or unexpected?**
   Anticipated shocks affect behavior today via forward-looking consumption planning. Unanticipated shocks affect only future choices.

Together, these insights form the basis for modeling trade and current account dynamics in open economies under rational expectations and intertemporal optimization. Once we have out setup, we can start to elaborate in our open economy model and add extensions and more advanced concepts.