Production in an Open Economy

International Macroeconomics
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Introduction: In this chapter, we extend our analysis of the current account by introducing production and investment decisions into the framework of an open economy. Unlike previous chapters, where output was assumed to be exogenously given as an endowment, we now allow firms to produce output using physical capital. As a result, the current account will not only be influenced by household saving decisions, but also by firms' optimal investment choices. This richer setting allows us to explore how shocks to productivity, interest rates, or policy can affect both investment and savings, and thereby shape the behavior of the current account.

1. The Investment Decision of Firms

Consider a small open economy populated by a representative household and a representative firm. As before, the economy lasts for two periods, denoted period 1 and period 2. The main difference is that output is now produced by firms through investment in physical capital, rather than being exogenously given.

Suppose that in period 1, firms invest an amount I_1 in physical capital, which becomes productive in period 2. Output in period 2, denoted Q_2, is produced according to the following production function:

$$Q_2 = A_2 F(I_1),$$

where $A_2 > 0$ represents productivity, and $F(\cdot)$ is a strictly increasing and concave function. We assume the following properties:

- $F(0) = 0$: no investment yields no output,
- $F'(I_1) > 0$: capital increases output,
- $F''(I_1) < 0$: diminishing marginal returns to capital.

Investment I_1 is financed through borrowing. Let D_1^f be the amount borrowed in period 1. We then have:

$$D_1^f = I_1$$

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Firms repay their loans in period 2. If the world interest rate is r_1, total repayment is $(1 + r_1)D_1^f$. Thus, profits in period 2 are given by:

$$\Pi_2 = A_2 F(I_1) - (1 + r_1)I_1$$

The firm chooses I_1 to maximize Π_2, taking r_1 and A_2 as given. The first-order condition is:

$$A_2 F'(I_1) = 1 + r_1$$

This condition states that firms invest up to the point where the marginal product of capital equals the marginal cost of capital. Intuitively, if $A_2 F'(I_1) > 1 + r_1$, then the firm can increase profits by raising investment. Conversely, if $A_2 F'(I_1) < 1 + r_1$, the firm will reduce investment.

It follows that the optimal level of investment, I_1^*, is decreasing in the interest rate. A higher r_1 raises the cost of capital and makes some previously profitable investment projects unviable, reducing the incentive to invest. Thus, changes in the international interest rate can have a direct and immediate impact on the economy's current account through their effect on firm investment, even before considering any adjustment in savings.

Let us now analyze the effect of a productivity shock on the investment decision of firms. Recall that firms choose investment I_1 in period 1 to maximize profits in period 2, which depend on the production function $A_2 F(I_1)$. Suppose the efficiency factor A_2 increases, as a result, the marginal product of capital increases at every level of I_1. Investment projects that were already profitable become more attractive, and some projects that previously had a marginal product below the cost of capital $1+r_1$ now become profitable. Therefore, firms optimally increase their level of investment, and we conclude that investment is an increasing function of the productivity factor A_2.

So far, we have focused on the firm's behavior in period 1, when it decides how much to invest in order to produce output in period 2. What about profits in period 1? These are determined by investment decisions made in the past—specifically, before period 1—and are thus not under the firm's control at the time.

Let I_0 denote the amount of capital installed prior to period 1. This capital generates output $Q_1 = A_1 F(I_0)$ in the first period, and was financed by borrowing $D_0^f = I_0$ at an interest rate r_0. Therefore, period-1 profits are given by:

$$\Pi_1 = A_1 F(I_0) - (1 + r_0)I_0.$$

Since I_0, r_0, and A_1 are all predetermined or exogenous from the perspective of the firm in period 1, these profits are taken as given. In particular, there is nothing the firm can do in period 1 to change them. Nonetheless, the qualitative effect of changes in these parameters is the same as the ones studied previously. Specifically, a higher interest rate r_0 lowers profits, as debt repayment increases, and a higher productivity level A_1 raises profits, as it increases output for a given level of capital I_0.

These effects are important for determining the income received by households from firm ownership, and will influence consumption and savings decisions, and therefore the current account, as we will now explore in the next section.

2. The Consumption-Saving Decision of Households

Households in the present production economy behave similarly to those in the endowment model studied in chapter 2, but with one key difference: they are now the owners of firms. Hence, instead of receiving exogenous endowments, they receive profit payments. Specifically, they earn profits $\Pi_1(r_0, A_1)$ in period 1 and $\Pi_2(r_1, A_2)$ in period 2. In addition, households may hold bonds and receive interest income from them.

Let B_0^h denote the household's initial bond holdings at the beginning of period 1. These bonds yield interest income $r_0 B_0^h$. In contrast to earlier models, we now distinguish between the household's individual asset position B_t^h and the aggregate net foreign asset position of the country B_t, since firms also borrow to finance investment.

The household's budget constraint in period 1 is:

$$C_1 + B_1^h - B_0^h = \Pi_1(r_0, A_1) + r_0 B_0^h.$$

In period 2, the household's budget constraint is:

$$C_2 + B_2^h - B_1^h = \Pi_2(r_1, A_2) + r_1 B_1^h.$$

Assuming the transversality condition $B_2^h = 0$ holds, we can combine these constraints into a single intertemporal budget constraint:

$$C_1 + \frac{C_2}{1 + r_1} = (1 + r_0) B_0^h + \Pi_1(r_0, A_1) + \frac{\Pi_2(r_1, A_2)}{1 + r_1}.$$

The household chooses C_1 and C_2 to maximize utility:

$$\max_{C_1, C_2} \quad U(C_1) + \beta U(C_2),$$

subject to the intertemporal budget constraint. The first-order condition yields the standard Euler equation:

$$\frac{U'(C_1)}{\beta U'(C_2)} = 1 + r_1$$

Effect of a Temporary Increase in Productivity

Suppose the productivity of capital in period 1 rises temporarily from A_1 to $A_1' > A_1$, while A_2 and r_1 remain unchanged. Since capital is predetermined, the increase in A_1 leads to higher output and higher firm profits Π_1, which are transferred to households. This acts as a temporary income shock.

As in the endowment model, households respond by raising consumption in period 1, but by less than the increase in income—implying a rise in savings. This savings may be used to finance future consumption. Thus, a temporary productivity shock leads to an increase in both consumption and savings.

Effect of an Anticipated Future Productivity Increase

Suppose now that in period 1, households learn that productivity in period 2 will increase from A_2 to $A_2' > A_2$, while A_1 and r_1 remain constant. Then, expected future profits rise from $\Pi_2(r_1, A_2)$ to $\Pi_2(r_1, A_2')$, effectively increasing lifetime income.

Because consumption is a normal good, consumption in both periods increases. However, because present income remains the same, then the increase in consumption comes along with a reduction in savings.

Effect of an Increase in the Interest Rate

Consider an increase in the interest rate from r_1 to $r_1' > r_1$, holding A_1 and A_2 constant. The household was initially borrowing:

$$C_1 - \Pi_1(r_0, A_1) > 0.$$

At the higher interest rate, firms invest less, and thus their profits in period 2 decline: $\Pi_2(r_1', A_2) < \Pi_2(r_1, A_2)$. This reduces expected household income in the future. The household also faces a higher cost of borrowing. Therefore, there are three negative effects on C_1:

1. **Substitution effect:** higher r_1 incentivizes saving more today to consume more tomorrow.
2. **Wealth effect:** borrowing becomes more expensive, making the household effectively poorer.
3. **Income effect:** future firm profits decline, reducing expected income.

As a result, period-1 consumption falls, and household saving increases (i.e., borrowing decreases).

Collecting these insights, we can summarize the behavior of consumption in period 1 as a function of interest rates and productivity levels:

$$C_1 = C(r_1^-, A_1^+, A_2^+),$$

where C is decreasing in r_1, and increasing in both current (A_1) and future (A_2) productivity.

3. The Saving Schedule

In the present model, there is no government spending, so public saving is zero: $G_1 = 0$. As a result, national saving is simply defined as the difference between national income and private consumption:

$$S_1 = Y_1 - C_1.$$

National income in period 1 consists of two components: net investment income from the country's initial foreign asset position, and output produced domestically. Thus, we can write:

$$Y_1 = r_0 B_0 + Q_1,$$

where B_0 denotes the country's net foreign asset position at the beginning of period 1, and Q_1 is the output produced in that period.

The country's net foreign asset position can be decomposed as the sum of households' net assets and firms' net debt:

$$B_0 = B_0^h - D_0^f.$$

Both B_0^h and D_0^f are predetermined before period 1, and the interest rate r_0 is exogenous. Thus, net investment income $r_0 B_0$ is exogenously given. Similarly, output in period 1 is given by:

$$Q_1 = A_1 F(I_0),$$

where I_0 is the predetermined stock of capital and A_1 is an exogenous productivity factor realized in period 1. Hence, national income is a function of productivity:

$$Y_1 = Y(A_1^+).$$

Combining this with the household's consumption decision, we obtain the saving function:

$$S_1 = Y(A_1^+) - C(r_1^-, A_1^+, A_2^+).$$

This expression shows how national saving responds to different variables:

- It is **increasing in the interest rate** r_1: a higher interest rate induces households to consume less today and save more.
- It is **increasing in current productivity** A_1: although both income and consumption rise, the increase in income exceeds the increase in consumption, as households smooth consumption over time.
- It is **decreasing in expected future productivity** A_2: since higher expected future income leads households to consume more in the present, reducing saving.

We can summarize this relationship as:

$$S_1 = S(r_1^+; A_1^+, A_2^-).$$

4. The Current Account Schedule

In this economy with production, the current account in period 1 is defined as the difference between national saving and domestic investment:

$$CA_1 = S_1 - I_1.$$

Substituting the previously derived saving and investment schedules, we obtain:

$$CA_1 = S(r_1^+; A_1^+, A_2^-) - I(r_1^-; A_2^+).$$

Now, let's analyze how the current account responds to movements in any of these components.

Adjustment of the Current Account to Changes in the World Interest Rate

Suppose the world interest rate rises from r^* to $r^{*'} > r^*$. Since the domestic economy takes the world interest rate as given, this change affects both saving and investment. Specifically:

- Households increase saving because the return on bonds has risen.
- Firms reduce investment because the cost of borrowing is now higher.

As a result, the gap $S_1 - I_1$ widens, and the current account improves.

Adjustment of the Current Account to a Temporary Increase in Productivity

Consider next a temporary increase in productivity, that is, an increase in A_1 while A_2 remains unchanged. This shock raises period-1 profits and income:

- Households increase consumption, but not by the full amount of the income increase, which leads to higher saving.
- Investment remains unchanged, because the marginal return on capital in period 2 is unaffected.

Therefore, at a fixed interest rate r^*, we observe:

$$S_1' > S_1, \quad I_1' = I_1, \quad \Rightarrow \quad CA_1' > CA_1.$$

The current account improves due to an increase in saving with no change in investment.

Adjustment of the Current Account to an Anticipated Future Productivity Increase

Finally, suppose that in period 1 it becomes known that the productivity of capital will increase in period 2, i.e., A_2 increases while A_1 remains unchanged. This anticipated productivity improvement has two effects:

- **Investment rises**: firms respond to expected higher marginal productivity by increasing capital accumulation.
- **Saving falls**: households feel wealthier due to expected future income and increase present consumption.

At the unchanged world interest rate r^*, we observe:

$$S_1' < S_1, \quad I_1' > I_1, \quad \Rightarrow \quad CA_1' < CA_1.$$

The current account deteriorates because saving decreases while investment increases.

To keep it short, we can summarize the current account schedule as:

$$CA_1 = CA(r_1^+; A_1^+, A_2^-).$$

5. The Terms of Trade in the Production Economy

In Chapter 3, we established that terms-of-trade shocks are equivalent to endowment shocks in their impact on consumption and external accounts. In this section, we show that the same logic applies in a production economy: *terms-of-trade shocks behave like productivity shocks.*

Suppose, as before, that the good households prefer to consume is different from the good produced by firms. Let TT_1 and TT_2 denote the terms of trade in periods 1 and 2, defined as the relative price of the export good in terms of the import good.

Firm profits in periods 1 and 2 are then given by:

$$\Pi_1 = TT_1 A_1 F(I_0) - (1 + r_0) I_0,$$

$$\Pi_2 = TT_2 A_2 F(I_1) - (1 + r_1) I_1.$$

As before, the firm chooses I_1 in period 1 to maximize period-2 profits. The corresponding first-order condition is:

$$TT_2 A_2 F'(I_1) = 1 + r_1.$$

This is analogous to the condition in the one-good economy, with A_2 replaced by $TT_2 A_2$. Hence, the effect of a change in the period-2 terms of trade is equivalent to a change in the productivity factor. We can then write the investment schedule as:

$$I_1 = I(r_1^-; TT_2 A_2^+).$$

Similarly, the household's intertemporal budget constraint now depends on profits Π_1 and Π_2, which in turn depend on TT_1A_1 and TT_2A_2, respectively. Thus, we can express optimal consumption as:

$$C_1 = C(r_1^-, TT_1A_1^+, TT_2A_2^+).$$

National income in period 1 is given by:

$$Y_1 = r_0B_0 + TT_1A_1F(I_0),$$

which implies that saving, defined as $S_1 = Y_1 - C_1$, behaves analogously to the one-good economy:

$$S_1 = S(r_1^+; TT_1A_1^+, TT_2A_2^-).$$

Finally, since the current account is the difference between saving and investment, we obtain:

$$CA_1 = CA(r_1^+; TT_1A_1^+, TT_2A_2^-).$$

We conclude that in the production economy, as in the endowment economy, a deterioration in the terms of trade today or expected in the future has the same effect as a drop in current or expected productivity. As such, terms-of-trade shocks impact investment, consumption, saving, and the current account through their effect on the value of output and profits.

In this chapter, we have introduced production to our economy through the firm's maximization problem. The next step would be to introduce the government and public expenditures, but first, we will review uncertainty. As we have seen throughout this chapters, what agents expect in the future heavily impacts what they decide to do today, consequently moving the macro variables in the present. Thus, understanding uncertainty is key to enrich our model, and to see how variables respond to expected future shocks.