# The Real Exchange Rate & Purchasing Power Parity

International Macroeconomics
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**Introduction:** Sometimes Europe seems cheaper than the U.S., and other times the reverse is true. When Europe is cheaper, Americans tend to travel there and import more; when the U.S. is cheaper, the opposite happens. This idea is captured by the **real exchange rate**, which compares the price of foreign goods to domestic goods. It tells us how many domestic baskets are needed to buy one foreign basket. When this rate equals one, purchasing power is equal across countries — a condition known as *purchasing power parity* (PPP). This chapter explores how the real exchange rate changes and why PPP often doesn't hold in practice.

## 1. The Real Exchange Rate and the Law of One Price

We say the *law of one price* holds when the same good sells for the same price in two countries once we account for the exchange rate. Formally, let:

- $P$: the price of a good in domestic currency,
- $P^*$: the price of the same good in foreign currency,
- $E$: the nominal exchange rate, defined as the domestic currency price of one unit of foreign currency.

Then, the law of one price holds when:

$$P = E \cdot P^*$$

That is, the price of the good at home equals the price of the foreign good converted to domestic currency. If $EP^* > P$, the good is more expensive abroad; if $EP^* < P$, the good is cheaper abroad.

Why Should LOOP Hold? The intuition behind LOOP is based on arbitrage. Suppose a good is cheaper in one country than in another. Traders would purchase it in the

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cheaper market and sell it in the more expensive one, making a profit in the process. This cross-border arbitrage would drive prices toward equality — enforcing LOOP.

However, in practice, this arbitrage process is rarely frictionless. Several factors cause persistent deviations from LOOP: transportation costs, distribution margins, tariffs and trade barriers or the non-tradability of some components. As a result, LOOP might not hold exactly, and deviations can persist for extended periods, especially for goods that use high non-tradable components for their production.

The *real exchange rate* (RER) is the concept that captures the relative price of goods abroad versus at home and is defined as:

$$e = \frac{E \cdot P^*}{P}$$

Where:

- $E$: nominal exchange rate (domestic currency per unit of foreign currency),
- $P^*$: foreign price level (in foreign currency per foreign good),
- $P$: domestic price level (in domestic currency per domestic good),
- $e$: real exchange rate (in units of domestic goods per foreign good).

To better understand the RER and this definition, we can do a unit analysis of its equation:

$$\underbrace{\left(\frac{\text{domestic currency}}{\text{foreign currency}}\right)}_{E} \cdot \underbrace{\left(\frac{\text{foreign currency}}{\text{foreign good}}\right)}_{P^*} \div \underbrace{\left(\frac{\text{domestic currency}}{\text{domestic good}}\right)}_{P} = \frac{\text{domestic good}}{\text{foreign good}}$$

Thus, $e$ tells us how many domestic goods are needed to buy one foreign good. If $e > 1$, foreign goods are relatively expensive. Whereas, if $e < 1$, foreign goods are relatively cheap. Lastly, if $e = 1$, the LOOP is satisfied at the aggregate level.

Intuitively, if $e > 1$, the domestic economy finds it costly to import, but may find it profitable to export — the domestic good is cheaper. Conversely, when $e < 1$, domestic consumers can buy more foreign goods, and imports are incentivized.

In practice, the real exchange rate differs from unity for prolonged periods, especially when there are structural differences in price levels across countries. While the law of one price might hold for highly tradable, standardized goods like oil or gold, it is less likely to hold for goods that involve local services, branding, or government intervention. Therefore, the real exchange rate remains a central object of interest for studying relative competitiveness and international price disparities.

# 2. Purchasing Power Parity (PPP)

Purchasing Power Parity (PPP) is a central concept in international macroeconomics and extends the idea of the Law of One Price from individual tradable goods to broader consumption baskets representative of household spending. It provides a benchmark for comparing the relative price levels of different countries, by indicating whether a currency is under- or overvalued. The real exchange rate plays a key role in this comparison, as it measures the relative cost of living across countries in a common unit.

Let:

- $P$: domestic price level (price of a representative consumption basket in domestic currency),
- $P^*$: foreign price level (same basket in foreign currency),
- $E$: nominal exchange rate (domestic currency per unit of foreign currency),
- $e$: real exchange rate.

Then, the real exchange rate is given by:

$$e = \frac{E \cdot P^*}{P}$$

This expression tells us how many baskets of domestic goods are required to purchase one foreign basket. If:

- $e > 1$: the foreign country is more expensive than the home country,
- $e < 1$: the foreign country is cheaper,
- $e = 1$: purchasing power parity holds.

## 2.1. Absolute Purchasing Power Parity

*Absolute PPP* holds when the same consumption basket costs the same amount in both countries once prices are expressed in a common currency. That is,

$$P = E \cdot P^* \quad \Rightarrow \quad e = 1$$

In this case, the purchasing power of the domestic currency is the same at home and abroad. The exchange rate that ensures absolute PPP holds is called the **PPP exchange rate**, denoted by $E_{PPP}$, and it satisfies:

$$E_{PPP} \cdot P^* = P \quad \Rightarrow \quad E_{PPP} = \frac{P}{P^*}$$

Comparing this benchmark with the actual market exchange rate $E$ allows us to assess currency valuation:

- If $E_{PPP} > E$, then $P > E \cdot P^*$: the domestic currency is *overvalued*, and the foreign currency is undervalued.
- If $E_{PPP} < E$, then $P < E \cdot P^*$: the domestic currency is *undervalued*, and the foreign currency is overvalued.

## 2.2. Relative Purchasing Power Parity

*Relative PPP* generalizes the idea of PPP by focusing on *changes* in the real exchange rate over time rather than on its level. We say that relative PPP holds if the real exchange rate remains constant over time:

$$\Delta e_t \equiv \Delta \left( \frac{E_t \cdot P_t^*}{P_t} \right) = 0$$

Where $\Delta e_t \equiv e_t - e_{t-1}$ denotes the change in RER over time. When relative PPP holds, the domestic and foreign price levels may differ, but they evolve at the same rate. In terms of the real exchange rate:

- If $\Delta e_t < 0$, the real exchange rate appreciates — the domestic country becomes relatively more expensive.
- If $\Delta e_t > 0$, the real exchange rate depreciates — the domestic country becomes relatively cheaper.

A useful interpretation is in terms of inflation and currency depreciation. Let:

- $\pi_t = \frac{P_t - P_{t-1}}{P_{t-1}}$: domestic inflation,
- $\pi_t^* = \frac{P_t^* - P_{t-1}^*}{P_{t-1}^*}$: foreign inflation,
- $\varepsilon_t = \frac{E_t - E_{t-1}}{E_{t-1}}$: nominal depreciation of the domestic currency.

Then, relative PPP implies:

$$\varepsilon_t = \pi_t - \pi_t^*$$

That is, the nominal depreciation rate of the domestic currency equals the inflation differential. If inflation is higher at home than abroad, the currency should depreciate proportionally to preserve relative purchasing power.

In practice, relative PPP is often a better approximation than absolute PPP, particularly over the medium to long run. Over short horizons, nominal rigidities, price stickiness, and financial market volatility can lead to deviations from both absolute and relative PPP.

## 2.3. Deviations from PPP

One important reason why purchasing power parity (PPP) does not hold in practice is that not all goods and services are internationally tradable. For many goods, particularly services, transportation costs and localization requirements make international trade unprofitable. These goods are referred to as *nontradables*. Examples include haircuts, restaurant meals, housing, certain health and educational services. However, not all services are nontradable—some, such as higher education or software development, are actively exported across borders.

The existence of nontradable goods generates systematic and persistent deviations from PPP. To understand this, consider that the consumption-based price index, $P$, reflects an average of prices for both tradables and nontradables. That is,

$$P = \varphi(P_T, P_N),$$

where $P_T$ and $P_N$ denote the prices of tradable and nontradable goods, respectively, and $\varphi(\cdot, \cdot)$ is increasing in both arguments and homogeneous of degree one. The homogeneity assumption implies that a uniform percentage increase in both tradable and nontradable prices leads to the same percentage increase in the overall price level.

Assume a symmetric price index structure in the foreign country:

$$P^* = \varphi(P_T^*, P_N^*),$$

where $P_T^*$ and $P_N^*$ are the foreign prices of tradables and nontradables.

Suppose the Law of One Price holds for tradables:

$$P_T = EP_T^*.$$

But the same does not generally apply for nontradables:

$$P_N \neq EP_N^*,$$

since nontradables are not subject to international arbitrage and are determined by domestic factors.

The real exchange rate is given by:

$$e = \frac{EP^*}{P} = \frac{E\varphi(P_T^*, P_N^*)}{\varphi(P_T, P_N)}.$$

Using the homogeneity of $\varphi(\cdot, \cdot)$, we normalize both numerator and denominator by tradable prices:

$$e = \frac{EP_T^* \cdot \varphi(1, P_N^*/P_T^*)}{P_T \cdot \varphi(1, P_N/P_T)}.$$

Substituting $P_T = EP_T^*$ from the Law of One Price for tradables:

$$e = \frac{\varphi(1, P_N^*/P_T^*)}{\varphi(1, P_N/P_T)}.$$

This expression highlights that deviations from PPP are driven by differences in the relative price of nontradables in terms of tradables across countries. In particular:

$$e < 1 \quad \text{if} \quad \frac{P_N^*}{P_T^*} < \frac{P_N}{P_T},$$

meaning the foreign consumption basket is cheaper when the relative price of nontradables is lower abroad.

Beyond the role of nontradables, several other factors contribute to violations of PPP:

- **Trade barriers:** Import tariffs, export subsidies, and quantitative restrictions (quotas) distort relative prices across countries and prevent price equalization.
- **Home bias in consumption:** Differences in preferences or habits across countries lead to consumption baskets that differ systematically, limiting the comparability of price indices.
- **Price stickiness and nominal rigidities:** In the short run, prices may not fully adjust to exchange rate movements, generating temporary deviations from PPP.

In conclusion, The RER and the PPP are key variables in international macroeconomics as they reflect directly or indirectly currency over- or sub-valuation, relative prince indices, costs and standards of living, among other factors. They are crucial to correctly compare the real cost of goods between economies, and to better understand international dynamics in trade and the exchange rate.