Uncertainty in an Open Economy

International Macroeconomics
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Introduction: In the preceding chapters, we analyzed the behavior of the current account in deterministic environments. In this chapter, we extend our framework by introducing uncertainty into the analysis. Uncertainty is a key feature of real-world economies and can have significant implications for both savings and investment decisions. We will study how increased or decreased volatility in economic fundamentals—such as productivity or income—affects the current account. This framework will help us understand episodes like the U.S. Great Moderation and its possible connection to the emergence of sustained current account deficits.

1. The Great Moderation

A number of researchers have documented that the volatility of U.S. output declined significantly starting in the early 1980s. This phenomenon has become known as the *Great Moderation*.

The most commonly used measure of volatility in macroeconomic data is the standard deviation. According to this measure, postwar U.S. output growth became half as volatile after 1983. Specifically, the standard deviation of quarter-to-quarter real per capita output growth was 1.2 percent over the period 1947 to 1983, but only 0.6 percent over the period 1984 to 2017. Some economists believe that the Great Moderation ended in 2007, just before the onset of the Global Financial Crisis, while others argue that the Great Moderation is still ongoing, as the volatility of output has returned to pre-crisis levels.

Researchers have proposed three competing explanations for the Great Moderation: *good luck*, *good policy*, and *structural change*.

- The **good-luck hypothesis** claims that, by chance, the U.S. economy has been subject to smaller shocks since the early 1980s.
- The **good-policy hypothesis** attributes the decline in volatility to the implementation of sound macroeconomic policies, beginning with Paul Volcker's aggressive stance against inflation in the early 1980s and continuing with Alan Greenspan's focus on maintaining low inflation.

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- The **structural change hypothesis** suggests that advances in technology, particularly in inventory management and financial markets, allowed firms to better smooth production and sales, thus dampening the business cycle.

The onset of the Great Moderation coincided with a significant change in the U.S. current account, as the country began running large and persistent deficits. This raises an important question: is the timing of the Great Moderation and the emergence of protracted current account deficits a coincidence, or does a causal relationship exist between the two? To investigate this, we will explore how changes in output uncertainty affect current account behavior using our theoretical framework.

2. An Open Economy with Uncertainty

Let us now introduce uncertainty into our baseline open economy model. We examine how households adjust their consumption and saving decisions when future income is risky. Intuitively, even when the expected value of income remains unchanged, the concavity of the utility function implies that the household dislikes risk. Thus, the household may engage in *precautionary saving*, reducing present consumption to hedge against adverse future outcomes.

We assume that the endowment in period 1 is Q, and that the endowment in period 2 is stochastic. Specifically, the household expects with equal probability either a positive or negative shock of size $\sigma > 0$. The future endowment is therefore:

$$Q_2 = \begin{cases} Q + \sigma & \text{with probability } \frac{1}{2} \\ Q - \sigma & \text{with probability } \frac{1}{2} \end{cases}$$

We assume the household starts with no initial wealth:

$$B_0 = 0$$

and for simplicity the interest rate is zero ($r_1 = 0$), and the utility discount factor is one ($\beta = 1$). The intertemporal budget constraint becomes:

$$C_1 + C_2 = Q + Q_2$$

Since Q_2 is uncertain, consumption in period 2 is also uncertain and depends on the realization of the shock.

Let us define consumption in period 2 for the two states of the world:

$$\begin{aligned} C_2^g &= 2Q + \sigma - C_1 \quad \text{(good state)} \\ C_2^b &= 2Q - \sigma - C_1 \quad \text{(bad state)} \end{aligned}$$

The household maximizes expected utility, subject to these contingent intertemporal budget constraints:

$$\max_{C_1} \quad \ln(C_1) + \frac{1}{2}\ln(C_2^g) + \frac{1}{2}\ln(C_2^b)$$

Replacing the equations of the two states, and maximizing for C_1, the first-order condition is:

$$\frac{1}{C_1} = \frac{1}{2}\left(\frac{1}{2Q+\sigma-C_1} + \frac{1}{2Q-\sigma-C_1}\right)$$

This condition equates the marginal utility of present consumption to the expected marginal utility of future consumption.

In the absence of uncertainty, we had:

$$C_1 = Q, \quad C_2 = Q$$

Now consider whether $C_1 = Q$ still solves the FOC under uncertainty. Substituting into the uncertain FOC:

$$\frac{1}{Q} \stackrel{?}{=} \frac{1}{2}\left(\frac{1}{Q+\sigma} + \frac{1}{Q-\sigma}\right)$$

Multiply both sides by Q:

$$1 \stackrel{?}{=} \frac{Q}{2}\left(\frac{1}{Q+\sigma} + \frac{1}{Q-\sigma}\right) = \frac{1}{2}\left(\frac{Q}{Q+\sigma} + \frac{Q}{Q-\sigma}\right)$$

Using a common denominator:

$$1 \stackrel{?}{=} \frac{1}{2}\left(\frac{Q(Q-\sigma)+Q(Q+\sigma)}{Q^2-\sigma^2}\right) = \frac{1}{2}\cdot\frac{2Q^2}{Q^2-\sigma^2} = \frac{Q^2}{Q^2-\sigma^2}$$

For any positive value of σ, it holds that:

$$1 < \frac{Q^2}{Q^2-\sigma^2}$$

Therefore:

$$\frac{1}{Q} < \frac{1}{2}\left(\frac{1}{Q+\sigma} + \frac{1}{Q-\sigma}\right)$$

So $C_1 = Q$ does not satisfy the FOC. To restore the equality, the household must **increase** the left-hand side of the FOC. Since $\frac{1}{C_1}$ is decreasing in C_1, this implies:

$$\boxed{C_1 < Q}$$

Hence, the optimal consumption in period 1 is *lower* than under certainty. This is the hallmark of **precautionary saving**: despite no change in expected lifetime resources, uncertainty induces the household to consume less in the present in order to buffer against adverse outcomes in the future. This result is intuitive, since the household is risk averse due to the concavity of its utility function.

The trade balance in period 1 is:

$$TB_1 = Q - C_1 \quad \Rightarrow \quad \boxed{TB_1^{\text{uncertainty}} > TB_1^{\text{certainty}}}$$

The current account in period 1 is:

$$CA_1 = TB_1 + r_0 B_0 = TB_1 \quad (\text{since } B_0 = 0) \quad \Rightarrow \quad \boxed{CA_1^{\text{uncertainty}} > CA_1^{\text{certainty}}}$$

The model thus predicts that an increase in future income uncertainty causes:

- A decline in optimal current consumption ($C_1^* \downarrow$)
- An increase in saving and trade balance ($S_1 \uparrow,\ TB_1 \uparrow$)
- An improvement in the current account ($CA_1 \uparrow$)

This analysis helps us understand how changes in volatility, even without changing expected fundamentals, can influence the external balance of an open economy. Once again, the requirement for precautionary savings depends on the convexity of the marginal utility of consumption. In other words, it is important that the third derivative of the period utility function to be positive.

3. Complete Asset Markets and the Current Account

So far, our analysis featured an open economy with *incomplete* markets: households could only trade a single risk-free bond with identical payoff across states. As a result, they were exposed to uninsurable income risk and responded with precautionary saving. We now consider a richer environment — one with **complete asset markets**, where households can freely insure against future income shocks by trading *state-contingent claims*. This setup allows the household to perfectly smooth consumption across states without reducing current consumption for precautionary reasons.

3.1. State-Contingent Claims

Suppose the household lives in a two-period, two-state economy. In period 2, either a “good” or “bad” state realizes with equal probability. Let:

- P_g: price (in period-1 units) of an asset that pays 1 unit of good in period 2 if the **good** state occurs, 0 otherwise.
- P_b: price of an asset that pays 1 unit if the **bad** state occurs, 0 otherwise.

- B_g, B_b: quantity of each asset purchased in period 1.

The household is allowed to choose positive or negative positions in each claim, i.e., it may both buy or sell contingent claims.

We say the market is complete because there exists one asset per state of nature. In such environments, the household can construct any payoff profile across states. For instance, to ensure x units of goods in the good state and y units in the bad state, it simply chooses $B_g = x$ and $B_b = y$. The cost in period 1 is:

$$P_g B_g + P_b B_b$$

A useful implication is that any other asset — like a risk-free bond that pays the same in all states — is *redundant*, as its payoff can be replicated by a portfolio of state-contingent claims. To see this, consider a bond that pays $1 + r_1$ units in every state. A portfolio of $1 + r_1$ units of each contingent claim replicates this. The cost is:

$$(P_g + P_b)(1 + r_1)$$

In equilibrium, no arbitrage implies this cost must equal 1 (the bond's price), so:

$$1 + r_1 = \frac{1}{P_g + P_b}$$

3.2. The Household's Problem

Let initial income be Q, and assume again no initial wealth:

$$B_0 = 0$$

Period-1 budget constraint:

$$C_1 + P_g B_g + P_b B_b = Q$$

Period-2 endowment depends on the state:

$$Q_2 = \begin{cases} Q + \sigma & \text{(good state)} \\ Q - \sigma & \text{(bad state)} \end{cases}$$

Therefore, consumption in period 2 in each state is:

$$C_2^g = Q + \sigma + B_g \tag{1}$$
$$C_2^b = Q - \sigma + B_b \tag{2}$$

Lifetime utility is expected utility:

$$U = \ln(C_1) + \frac{1}{2}\ln(C_2^g) + \frac{1}{2}\ln(C_2^b)$$

Substituting the period 1 budget constraint and the two state equations into the utility function, the household chooses B_g and B_b to solve:

$$\max_{B_g, B_b} \quad \ln(Q - P_g B_g - P_b B_b) + \frac{1}{2}\ln(Q + \sigma + B_g) + \frac{1}{2}\ln(Q - \sigma + B_b)$$

The first-order conditions are:

$$\frac{\partial U}{\partial B_g} = -\frac{P_g}{C_1} + \frac{1}{2}\cdot\frac{1}{C_2^g} = 0 \quad \Rightarrow \quad \frac{P_g}{C_1} = \frac{1}{2}\cdot\frac{1}{C_2^g} \tag{3}$$

$$\frac{\partial U}{\partial B_b} = -\frac{P_b}{C_1} + \frac{1}{2}\cdot\frac{1}{C_2^b} = 0 \quad \Rightarrow \quad \frac{P_b}{C_1} = \frac{1}{2}\cdot\frac{1}{C_2^b} \tag{4}$$

These optimality conditions have an intuitive interpretation. For each claim:

- The left-hand side is the opportunity cost of purchasing in period 1 a state contingent claim that pays one unit of consumption in the respective state in period 2.
- The right-hand side is the marginal expected benefit: the marginal utility of receiving an extra unit of consumption in the corresponding state, discounted by the probability of that state.

In the incomplete market model with only one bond:

$$\frac{1}{C_1} = \mathbb{E}\left[\frac{1}{C_2}\right]$$

This single equation summarizes intertemporal substitution between C_1 and an average C_2. Under complete markets, we obtain one condition *per state*, allowing the household to fully tailor its portfolio across states. This eliminates the need for precautionary saving.

In particular, even if future income is uncertain, the household can achieve perfect insurance — say, by choosing B_g and B_b so that:

$$C_2^g = C_2^b$$

which equalizes consumption across states and maximizes expected utility due to the concavity of the utility function.

Therefore, in contrast to the incomplete markets case, under complete markets:

$$\boxed{C_1^{\text{complete}} > C_1^{\text{incomplete}}} \quad \text{and} \quad \boxed{CA_1^{\text{complete}} < CA_1^{\text{incomplete}}}$$

That is, the household consumes more in period 1 (no precautionary motive), saves less, and the current account improves less or may even be in deficit depending on insurance terms.

3.3. Free Capital Mobility

As in our earlier model with incomplete markets, we now assume **free international capital mobility**. This means that domestic households can trade state-contingent claims at the same prices available in world markets. Formally, letting P_g^* and P_b^* be the world prices of the assets that pay in the good and bad states respectively, we must have:

$$P_g = P_g^* \qquad \text{and} \qquad P_b = P_b^*$$

Just like before, the risk-free interest rate in the world market is determined by the cost of constructing a portfolio that pays one unit in every state. That is:

$$1 + r^* = \frac{1}{P_g^* + P_b^*} \tag{5}$$

Now, to ensure that foreign investors do not make arbitrage profits (i.e., zero expected profits), we must impose the following logic.

Suppose a foreign investor sells B_g and B_b units of the contingent claims in period 1. Their revenue is:

$$\text{Revenue} = P_g^* B_g + P_b^* B_b$$

They invest this revenue in a world risk-free bond, earning $(1 + r^*)(P_g^* B_g + P_b^* B_b)$ in period 2. But they owe the households 1 unit in the good state and 1 unit in the bad state, depending on which occurs. So, their net profit in each state is:

$$\begin{aligned} \text{If good state:} \quad & (1 + r^*)(P_g^* B_g + P_b^* B_b) - B_g \\ \text{If bad state:} \quad & (1 + r^*)(P_g^* B_g + P_b^* B_b) - B_b \end{aligned}$$

The expected profit is the average across states:

$$\text{Expected profit} = \left[(1 + r^*) P_g^* - \frac{1}{2} \right] B_g + \left[(1 + r^*) P_b^* - \frac{1}{2} \right] B_b$$

The no-arbitrage condition requires that expected profits be zero *for all possible portfolios* (B_g, B_b). This is only possible if the coefficients on both B_g and B_b are zero, implying:

$$(1 + r^*) P_g^* = \frac{1}{2}, \qquad (1 + r^*) P_b^* = \frac{1}{2}$$

Thus, the prices of the state-contingent claims are equal:

$$P_g^* = P_b^* = \frac{1}{2} \quad \text{if } r^* = 0$$

This means each contingent claim costs one-half of a unit of good in period 1, and the world interest rate is zero.

3.4. Equilibrium in the Complete Asset Market Economy

We now return to the household's optimization problem and substitute $P_g = P_b = \frac{1}{2}$ into the first-order conditions derived earlier:

$$\begin{aligned} P_g &= \frac{1}{2} = \frac{1}{2} \cdot \frac{C_1}{C_2^g} \quad \Rightarrow \quad C_1 = C_2^g \\ P_b &= \frac{1}{2} = \frac{1}{2} \cdot \frac{C_1}{C_2^b} \quad \Rightarrow \quad C_1 = C_2^b \end{aligned}$$

Therefore, under complete markets and free capital mobility:

$$C_1 = C_2^g = C_2^b = C^* \tag{6}$$

This means that the household is able to perfectly smooth consumption across time and across states. This is a key insight: state-contingent claims eliminate the need for precautionary saving, because households can insure directly against bad states.

To determine the optimal portfolio (B_g, B_b), we plug this result into the period-2 consumption equations:

$$\begin{aligned} C_2^g &= Q + \sigma + B_g = C^* \\ C_2^b &= Q - \sigma + B_b = C^* \end{aligned}$$

Solving these:

$$\begin{aligned} B_g &= C^* - Q - \sigma = Q - Q - \sigma = -\sigma \\ B_b &= C^* - Q + \sigma = Q - Q + \sigma = \sigma \end{aligned}$$

The household takes a *short position* in the claim that pays in the good state, and a *long position* in the claim that pays in the bad state. Intuitively, they borrow from the good state (when income is high) and save into the bad state (when income is low), transferring resources to equalize consumption across states.

Since:

$$C_1 = Q \quad \Rightarrow \quad TB_1 = Q - C_1 = 0$$

and the household starts with zero foreign assets:

$$CA_1 = TB_1 = 0$$

Thus, under complete markets (and the perfect consumption smoothness assumptions), the current account is always zero — regardless of the level of uncertainty σ. This contrasts sharply with the incomplete markets case, where an increase in uncertainty leads to more saving and a current account surplus.

However, even though the *net* international asset position is zero:

$$B_g + B_b = -\sigma + \sigma = 0$$

the household is still engaging in active **gross positions**. It saves in bad-state assets and borrows in good-state assets.

Crucially, these gross positions grow with the level of uncertainty. The more volatile the environment (i.e., the larger σ), the more the household wants to hedge, increasing both $|B_g|$ and B_b. But in the real world, why might households (or countries) not achieve this perfect smoothing?

- **Incomplete markets:** Not all possible risks can be insured — especially when the number of potential shocks (e.g., political, environmental, technological) is effectively infinite.
- **Policy restrictions:** Governments may limit access to international derivatives, or prohibit certain kinds of borrowing.
- **Transaction costs, enforcement issues, or information asymmetries** may also prevent full insurance.

Therefore, the benchmark result that uncertainty does not affect the current account under complete markets is elegant but likely unrealistic. Most real economies operate under incomplete markets, making precautionary saving and a link between uncertainty and the current account more empirically relevant.