Real Business Cycle

Macroeconomics 3
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Introduction: Following the seminal contributions of Kydland and Prescott (1982) and Prescott (1986), Real Business Cycle (RBC) theory emerged as the dominant framework for analyzing macroeconomic fluctuations. Over time, it became a core component of modern macroeconomic thinking, influencing both methodology and interpretation.

Methodologically, RBC theory established the use of dynamic stochastic general equilibrium (DSGE) models as the standard analytical tool. It replaced ad hoc behavioral equations with the optimality conditions of intertemporal decisions made by rational agents.

The most groundbreaking aspect of the RBC approach, however, was conceptual. It revolved around three central propositions:

- **Business cycles as efficient outcomes.** Fluctuations in output and employment could be seen as the economy's efficient response to real exogenous shocks—particularly technology shocks—in a setting with perfect competition and no market frictions. Under this view, economic cycles did not reflect misallocations and did not justify stabilization policies.
- **The central role of technology shocks.** The standard RBC model was able to generate realistic dynamics in macro variables solely from variations in total factor productivity, challenging the traditional view that linked technology only to long-run growth and not to business cycle fluctuations.
- **Minimal role for monetary factors.** In most RBC models, money plays no role at all. The explanation of macroeconomic fluctuations is entirely real, with no need for nominal variables or a monetary sector.

Now, a step-by-step development of the baseline RBC model is presented, as well as the interpretation of its results.

1. Model Assumptions

The central assumptions of the RBC model, as mentioned above, are the following:

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- Perfectly competitive markets.
- Fully flexible prices and wages.
- Economic fluctuations are explained by real shocks, especially to total factor productivity.
- There is no money, nominal variables, or monetary policy.

2. Households

Households choose consumption C_t, labor L_t, investment I_t, and capital K_t to maximize their intertemporal utility subject to budget and capital accumulation constraints. Households own the capital and the firms, so they receive income from labor, capital rental, and profits.

$$\max_{\{C_t, I_t, L_t, K_t\}} E_0 \sum_{t=0}^{\infty} \beta^t U(C_t, 1 - L_t)$$

subject to:

$$\begin{gathered} C_t + I_t \leq W_t L_t + R_t K_{t-1} + \Pi_t \\ K_t = (1-\delta) K_{t-1} + I_t \end{gathered}$$

Utility function (logarithmic case):

$$U(C_t, 1 - L_t) = \gamma \log(C_t) + \psi \log(1 - L_t)$$

Household's Lagrangian:

$$\begin{aligned} \mathcal{L} = E_0 \sum_{t=0}^{\infty} \beta^t \Big[& \gamma \log(C_t) + \psi \log(1 - L_t) \\ & + \lambda_t \left(W_t L_t + R_t K_{t-1} - C_t - I_t \right) \\ & + \mu_t \left((1-\delta) K_{t-1} + I_t - K_t \right) \Big] \end{aligned}$$

First-order conditions:

$$
\begin{aligned}
\frac{\partial \mathcal{L}}{\partial C_t}: &\quad \frac{\gamma}{C_t} = \lambda_t &(1)\\
\frac{\partial \mathcal{L}}{\partial L_t}: &\quad -\frac{\psi}{1-L_t} + \lambda_t W_t = 0 &(2)\\
\frac{\partial \mathcal{L}}{\partial I_t}: &\quad -\lambda_t + \mu_t = 0 &(3)\\
\frac{\partial \mathcal{L}}{\partial K_t}: &\quad -\mu_t + E_t\beta\left[\lambda_{t+1}R_{t+1} + \mu_{t+1}(1-\delta)\right] = 0 &(4)
\end{aligned}
$$

Combining (1), (3), and (4) we obtain the **Euler equation**:

$$\frac{\gamma}{C_t} = \beta E_t\left[\frac{\gamma}{C_{t+1}}(R_{t+1} + 1 - \delta)\right]$$

Combining (1) and (2) we obtain the intratemporal labor-consumption condition:

$$\frac{\psi}{1-L_t} = \frac{\gamma}{C_t}W_t$$

3. Firms

Firms operate in perfect competition and rent capital and labor from households to produce final goods. The production function is Cobb-Douglas:

$$Y_t = A_t K_{t-1}^{\alpha} L_t^{1-\alpha}$$

Profits:

$$\Pi_t = Y_t - W_t L_t - R_t K_{t-1}$$

Since firms rent capital and hire labor each period, and this decision is independent of other periods, the Lagrangian is posed as a static maximization problem (for any period t):

$$\mathcal{L} = A_t K_{t-1}^{\alpha} L_t^{1-\alpha} - W_t L_t - R_t K_{t-1}$$

First-order conditions:

$$
\begin{aligned}
\frac{\partial \mathcal{L}}{\partial L_t}: &\quad W_t = (1-\alpha)A_t\left(\frac{K_{t-1}}{L_t}\right)^{\alpha} = (1-\alpha)\frac{Y_t}{L_t}\\
\frac{\partial \mathcal{L}}{\partial K_{t-1}}: &\quad R_t = \alpha A_t\left(\frac{L_t}{K_{t-1}}\right)^{1-\alpha} = \alpha\frac{Y_t}{K_{t-1}}
\end{aligned}
$$

These are the labor and capital demand equations, respectively. The firm will hire workers until the marginal product equals the marginal cost of the last worker (same for capital).

4. Stochastic Process

Now, we introduce the stochastic process, which will introduce unexpected *shocks* to productivity. This process consists of an AR(1) as follows:

$$\log A_t = \rho \log A_{t-1} + \varepsilon_t, \quad \varepsilon_t \sim N(0, \sigma^2)$$

Where ϵ_t corresponds to white noise. White noise is characterized by following a normal distribution, being uncorrelated with its past values, having constant variance over time, and a zero expected value. In other words, it is expected to be zero on average, but sometimes it takes a different value, and it is unpredictable when it will happen (that's why it's a *shock*).

5. General Equilibrium

In equilibrium, we have the optimality conditions of each agent, households and firms, as well as other economy-wide conditions/constraints. The general equilibrium is characterized by the following equations:

1. Euler equation: $\frac{1}{C_t} = \beta E_t \left[\frac{1}{C_{t+1}} (R_{t+1} + 1 - \delta) \right]$
2. Intratemporal condition: $\frac{\psi}{1-L_t} = \frac{1}{C_t} W_t$
3. Capital accumulation: $K_t = (1-\delta)K_{t-1} + I_t$
4. Goods market equilibrium: $Y_t = C_t + I_t$
5. Production: $Y_t = A_t K_{t-1}^{\alpha} L_t^{1-\alpha}$
6. Labor demand: $W_t = (1-\alpha)\frac{Y_t}{L_t}$
7. Capital demand: $R_t = \alpha \frac{Y_t}{K_{t-1}}$
8. Stochastic process: $\log A_t = \rho \log A_{t-1} + \varepsilon_t$

Endogenous variables: $Y_t, C_t, I_t, K_t, L_t, W_t, R_t, A_t$.
Exogenous variable: $\varepsilon_t \sim \mathcal{N}(0, \sigma^2)$

6. Steady State

To find the steady-state solution, we solve all model variables as functions of parameters. Sometimes it's hard to know where to start. We begin with the variable that carries the shock: Total Factor Productivity A.

$$\log(A_t) = \rho_A \log(A_{t-1}) + \epsilon_t^A$$

In steady state, the economy is in equilibrium, so $\epsilon^A = 0$. Also, we can drop time subscripts. We get:

$$\log(A^{ss}) = \rho_A \log(A^{ss})$$
$$A^{ss} = 1$$

We continue with the Euler Equation, where time subscripts $t, t+1$ also drop, leading to:

$$U_C^{ss} = \beta U_C^{ss}\left(1 - \delta + R^{ss}\right) \quad \Rightarrow \quad R^{ss} = \frac{1}{\beta} - 1 + \delta$$

From Capital Demand:

$$R^{ss} = \alpha \cdot \frac{Y^{ss}}{K^{ss}} \quad \Rightarrow \quad \frac{K^{ss}}{L^{ss}} = \left(\frac{\alpha A^{ss}}{R^{ss}}\right)^{\frac{1}{1-\alpha}}$$

We already know A^{ss} and R^{ss}, so we have the ratio K^{ss}/L^{ss} as a function of parameters, but we have not exactly determine each separately. Still, this ratio will help us to solve the rest of the system. From Labor Demand:

$$W^{ss} = (1-\alpha) \cdot A^{ss} \cdot \left(\frac{K^{ss}}{L^{ss}}\right)^{\alpha}$$

Capital Accumulation:

$$K^{ss} = (1-\delta)K^{ss} + I^{ss} \quad \Rightarrow \quad \frac{I^{ss}}{L^{ss}} = \delta \frac{K^{ss}}{L^{ss}}$$

Production Function:

$$\frac{Y^{ss}}{L^{ss}} = A^{ss}\left(\frac{K^{ss}}{L^{ss}}\right)^{\alpha}$$

Market Clearing:

$$Y^{ss} = C^{ss} + I^{ss} \quad \Rightarrow \quad \frac{C^{ss}}{L^{ss}} = \frac{Y^{ss}}{L^{ss}} - \frac{I^{ss}}{L^{ss}}$$

Labor Supply:

$$\gamma W^{ss}\left(C^{ss}\right)^{-1} = \frac{\psi}{1 - L^{ss}} \quad \Rightarrow \quad \psi \frac{L^{ss}}{1 - L^{ss}} = \gamma W^{ss}\left(\frac{C^{ss}}{L^{ss}}\right)^{-1}$$

$$L^{ss} = \frac{\frac{\gamma}{\psi}\left(\frac{C^{ss}}{L^{ss}}\right)^{-1} W^{ss}}{1 + \frac{\gamma}{\psi}\left(\frac{C^{ss}}{L^{ss}}\right)^{-1} W^{ss}}$$

Where W^{ss} and the ratio C^{ss}/L^{ss} are functions of parameters, and replacing their forms, we have found the steady state of labor L^{ss}. With this value, we clear and solve for the rest of the variables of the system.

7. Shock on Productivity

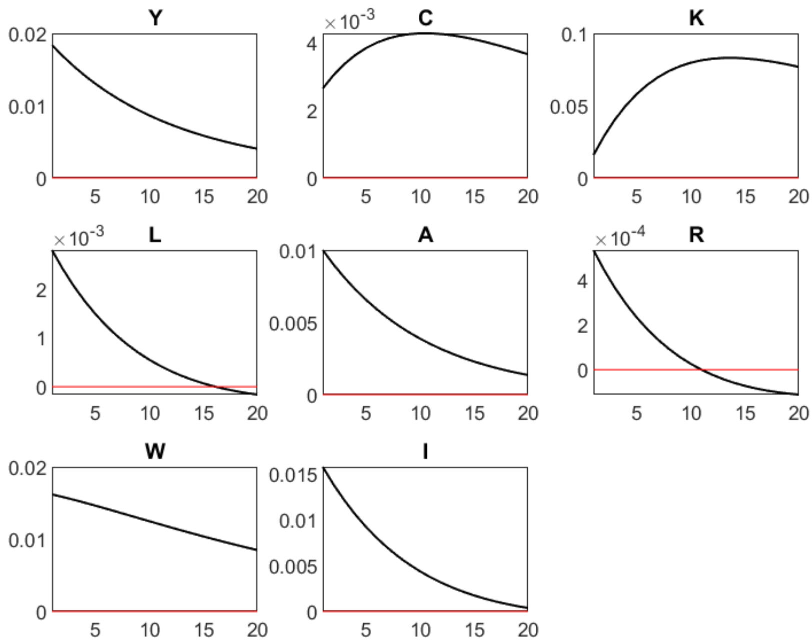

Figura 1: Impulse Response Functions to a positive shock in productivity

Now, we simulate a positive shock in productivity and look at the impulse response functions (IRFs). These allow us to observe the trajectories of our variables in response to a model perturbation. The positive productivity shock causes an immediate upward deviation from equilibrium in productivity itself. This higher productivity then drives output above its steady state level, which in turn raises both consumption and investment. Increased investment leads to capital accumulation, raising the marginal product of labor. Consequently, labor demand increases, bringing the labor market to a new equilibrium with higher wages and more hours worked. Since we have a ρ_A between 0 and 1, the productivity shock will dissipate over time, and the variables will return to their equilibrium trajectories (the red lines).