Investment

Macroeconomics I
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Introduction: After studying consumption, we now turn to the second component of aggregate demand: investment. Through the neoclassical model, we analyze the intertemporal decisions behind capital accumulation and their impact on the economy. For this, we introduce the role of firms, production, and capital as an accumulable factor.

1. Production in the Neoclassical Model

We now introduce firms into our infinite-horizon neoclassical model. Firms use capital to produce goods and services, which they rent from households in exchange for a rental cost R. Capital is accumulated by households, who decide how much to consume today and how much to save in the form of capital. Thus, firms choose the amount of capital to use in production, and households decide how much to accumulate in order to maximize utility.

1.1. The firm's problem

We assume output is determined by a neoclassical production function of the form $F(K)$, where:

- $F' > 0$: the marginal product of capital is positive.
- $F'' < 0$: diminishing marginal returns to capital.

In this basic version, there is no labor (it will be introduced later). Each period, output is sold at a normalized price of 1 and firms pay a rental rate R_t for the use of capital. The firm chooses K_t to maximize its profits in that period:

$$\max_{K_t} \; F_t(K_t) - R_t K_t$$

The first-order condition (optimal capital) is:

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$$F_t'(K_t) = R_t$$

That is, capital is accumulated up to the point where its marginal product equals its rental cost.

1.2. The household's problem

In a given period t, households choose how much to consume C_t and how much capital K_{t+1} to accumulate. In other words, the agent begins the period with a capital stock K_t chosen in the previous period and decides how much to accumulate for the next, so K_{t+1} is the choice variable. Capital depreciates at a rate $\delta \in [0, 1]$. The capital accumulation law is defined as:

$$I_t = K_{t+1} - K_t + \delta K_t$$

That is, total investment in period t is used to replenish depreciated capital δK_t and to increase the capital stock $K_{t+1} - K_t$. This equation is known as the **law of motion of capital**.

The household receives income Y_t each period, in addition to rental income from capital. The intratemporal budget constraint is:

$$C_t + I_t = Y_t + R_t K_t$$

Replacing the capital accumulation law:

$$C_t + K_{t+1} - K_t + \delta K_t = Y_t + R_t K_t$$

The household maximizes its intertemporal utility:

$$\max_{\{C_t\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t u(C_t)$$

subject to the infinite sequence of intratemporal budget constraints. The Lagrangian of the problem is:

$$\mathcal{L} = \sum_{t=0}^{\infty} \beta^t u(C_t) + \sum_{t=0}^{\infty} \lambda_t \left[Y_t + R_t K_t - C_t - K_{t+1} + K_t - \delta K_t\right]$$

The first-order conditions (FOCs) are:

$$\frac{\partial \mathcal{L}}{\partial C_t} = \beta^t u'(C_t) - \lambda_t = 0$$

$$\frac{\partial \mathcal{L}}{\partial C_{t+1}} = \beta^{t+1} u'(C_{t+1}) - \lambda_{t+1} \cdot \frac{1}{1+r} = 0$$
$$\frac{\partial \mathcal{L}}{\partial K_1} = -\lambda_t + \lambda_{t+1} \left(R_{t+1} + 1 - \delta\right) = 0$$

From the first two we again obtain the Euler equation:

$$u'(C_t) = \beta(1+r)u'(C_{t+1})$$

From the first and third:

$$u'(C_t) = \beta(R_{t+1} + 1 - \delta)u'(C_{t+1})$$

Moreover, observing that both optimality conditions share terms except for those within the parentheses on the right-hand side, we obtain another optimality condition:

$$1 + r = R_{t+1} + 1 - \delta$$
$$r = R_{t+1} - \delta$$

This condition equates the opportunity cost of capital (LHS) with its net return. It is called net because the household receives R_t per unit of capital rented but must spend δ on its replacement. Therefore, the household accumulates capital until the return is high enough to cover both the depreciation cost and the opportunity cost. Combining this optimality condition with the firm's FOC yields the capital market equilibrium:

$$r = F'(K_{t+1}) - \delta$$

2. Investment Theory and Tobin's q

Tobin's q theory offers a powerful alternative way to think about investment. Instead of starting from the perspective of the producer or the consumer, it focuses on the market value of capital relative to its replacement cost. The central idea is that firms will invest if the market value of capital exceeds the cost of acquiring it.

2.1. Definition of Tobin's q

It is defined as:

$$q \equiv \frac{\text{Market value of capital}}{\text{Replacement cost of capital}}$$

If $q > 1$, the market value of capital exceeds its physical cost, making investment profitable. If $q < 1$, firms prefer not to invest or even to sell capital.

Consider a simple two-period model with the following characteristics:

- Capital is purchased in the first period at price P_K.
- It yields a return in the second period: $F(K)$.
- It can be sold in the second period at future price P_K^{future}.
- There are no taxes or inflation adjustments.

The market value of capital today is the present value of the benefits it generates (net of depreciation), discounted at the rate r:

$$\mathrm{MV}_K = \frac{F'(K) + (1-\delta)P_K^{future}}{1+r}$$

Assuming the future price of capital equals today's price $P_K^{future} = P_K$, then:

$$\mathrm{MV}_K = \frac{F'(K) + (1-\delta)P_K}{1+r}$$

Tobin's q can be expressed as:

$$q = \frac{\mathrm{MV}_K}{P_K} = \frac{F'(K)}{(1+r)P_K} + \frac{1-\delta}{1+r}$$

This expression connects investment incentives to the marginal product of capital. A higher $F'(K)$ leads to a higher q, incentivizing greater investment. In his 1981 paper *Taxation and Corporate Investment: A q-Theory Approach*[1], Summers presents a more complete version of this model, incorporating taxes, financial market conditions, and other features that enrich the basic framework.

3. Neoclassical Model with Adjustment Costs

In the basic neoclassical model, firms can adjust their capital stock instantly without frictions. This leads to unrealistic predictions, such as large investment jumps in response to small incentive changes. To address this issue, we introduce **adjustment costs**, which make capital changes costly and gradual.

3.1. Motivation

Changing the capital stock is not simple. It involves: building new infrastructure, reorganizing internal processes, training or hiring workers, among other things. Adjustment costs capture the inefficiencies of investing heavily over a short period.

[1] You can access the paper here

3.2. Model structure

Instead of assuming households own the capital, we now take the firm's perspective and assume firms own capital. Their problem becomes intertemporal: maximize the present value of profits subject to capital accumulation and an adjustment cost I. Profits in a generic period t are given by:

$$\Pi_t = F(K_t) - I_t - \phi(I_t, K_t)$$

The intertemporal problem is:

$$\max_{\{I_t\}} \sum_{t=0}^{\infty} \frac{1}{(1+r)^t} \left[F(K_t) - I_t - \phi(I_t, K_t)\right]$$

subject to:

$$K_{t+1} = (1-\delta)K_t + I_t$$

Where $\phi(I_t, K_t)$ is the capital adjustment cost. A common functional form is:

$$\phi(I_t, K_t) = \frac{\gamma}{2}\left(\frac{I_t}{K_t}\right)^2 K_t = \frac{\gamma}{2} \cdot \frac{I_t^2}{K_t}$$

where $\gamma > 0$ measures the intensity of adjustment costs. This term penalizes rapid changes in the capital stock.

3.3. Maximization

The Lagrangian is:

$$\mathcal{L} = \sum_{t=0}^{\infty} \frac{1}{(1+r)^t} \left[F(K_t) - I_t - \frac{\gamma}{2} \cdot \frac{I_t^2}{K_t} + \lambda_t \left((1-\delta)K_t + I_t - K_{t+1}\right)\right]$$

Taking derivatives with respect to I_t and K_{t+1}:

$$\frac{\partial \mathcal{L}}{\partial I_t} = -1 - \gamma \cdot \frac{I_t}{K_t} + \lambda_t = 0 \Rightarrow \lambda_t = 1 + \gamma \cdot \frac{I_t}{K_t}$$

$$\frac{\partial \mathcal{L}}{\partial K_{t+1}} = -\lambda_t + \frac{1}{1+r}\left[F'(K_{t+1}) + \lambda_{t+1}(1-\delta) + \frac{\gamma}{2}\left(\frac{I_{t+1}}{K_{t+1}}\right)^2\right] = 0$$

Rearranging:

$$\frac{F'(K_{t+1})}{1+r} = 1 + \gamma \cdot \frac{I_t}{K_t} - \frac{1}{1+r}\left[\left(1 + \gamma \cdot \frac{I_{t+1}}{K_{t+1}}\right)(1-\delta) + \frac{\gamma}{2}\left(\frac{I_{t+1}}{K_{t+1}}\right)^2\right]$$

A bit lengthy, but the intuition is not much more complex: capital will accumulate up to the point where the present value of the marginal return on capital (LHS) equals the present value of the total marginal cost of capital (RHS). The total cost now includes not only investment and depreciation, but also the adjustment costs from the ϕ term.

As a result, investment becomes gradual rather than instantaneous: sharp changes (i.e., large I_{t+1}/K_{t+1} ratios) greatly increase adjustment costs.

Incorporating adjustment costs resolves the issue in the basic model where investment was overly sensitive to shocks. However, we're still far from perfectly modeling real-world investment dynamics. In the next section, we will explore additional investment theories that help explain its fluctuations and deviations from the optimal level.

4. Other Investment Theories

In addition to the neoclassical approach, there are several theories that seek to explain why investment does not always reach its optimal level and why it responds differently than expected. These theories complement the previous framework by incorporating frictions, expectations, or specific constraints. The main ones are summarized below.

4.1. Inventory Model

This model starts from the idea that firms face costs when adjusting their production levels and want to avoid interruptions or fluctuations in supply. Therefore, they hold inventories that act as a buffer between production and investment. Investment becomes a tool to maintain a desired level of inventories:

- If firms anticipate an increase in future demand, they may invest today to build up inventories.
- Investment responds to the gap between desired and actual inventory levels.
- This model helps explain the **pro-cyclicality** of investment: when demand expectations are optimistic, firms increase both capital and inventories.

4.2. Accelerator Approach

The **accelerator model** is based on a direct relationship between investment and the growth rate of output. The intuition is simple: if demand for goods is growing, more installed capacity is needed — and thus more capital.

$$I_t = I_0 + \alpha(Y_t - Y_{t-1})$$

where $\alpha > 0$ measures the sensitivity of investment to changes in output, and I_0 is a base level of investment.

Investment therefore depends on the variation in output (or demand), not on its absolute level. For investment to exceed the base level, output must not only be high, but also growing.

4.3. Financial Imperfections: Asymmetric Information

In practice, many firms cannot finance all their investment with internal resources and must rely on credit. In the presence of **asymmetric information**, lenders cannot perfectly assess a firm's quality, which leads to:

- **Adverse selection:** riskier projects are more likely to seek external funding.
- **Moral hazard:** once financed, the firm may change its behavior (e.g., take on more risk or use funds for purposes different from those declared).

In response to these risks, credit providers (the less-informed party) increase the **cost of financing**. Higher financing costs reduce the level of investment, keeping it below what it would be in the absence of financial imperfections.

4.4. Animal Spirits

Finally, a fundamental yet difficult-to-model component in macroeconomics is the psychological one: **animal spirits**, a term introduced by Keynes. It refers to emotional impulses or sentiments that drive investment (e.g., optimism, confidence, fear, panic). Though not always grounded in rational fundamentals, these forces can have real effects on the business cycle. Expectations may form through collective mechanisms, social networks, business leadership, or public narratives.

These elements help explain episodes of investment bubbles, sudden collapses due to panic, or disconnects between investment and interest rates, among other phenomena.

5. Uncertainty and Irreversibility

Beyond the structural models of investment discussed earlier, there are additional mechanisms that help explain real-world investment decisions. These include environmental or strategic factors that influence firms' actual behavior.

5.1. Uncertainty and the Value of Waiting

Under uncertainty, the decision to invest becomes an option: it can be made today or delayed until more information is available. This introduces the concept of the **value of waiting**. If investment today is irreversible (i.e., cannot be undone without losses), the investor may prefer to wait for greater clarity about the economic environment. In volatile or uncertain contexts, investment may be postponed even if the net present value (NPV) is positive[2], because the risks are greater and investors are typically risk-averse.

5.2. Irreversibility

Irreversibility amplifies the previous effect. Investing often involves **sunk costs** that cannot be recovered once incurred. These could be due to specific installation costs or equipment that loses value outside the firm or sector.

The presence of irreversibility justifies a **precautionary** investment behavior: firms avoid investing in contexts where the risk of regret is high.

5.3. General Conclusion

Investment is one of the most volatile and strategic components of aggregate demand. Its behavior cannot be fully understood through neoclassical marginal incentives alone.

- **Real frictions** (adjustment costs, uncertainty, irreversibility) and **financial constraints** (asymmetric information, credit limits) distort optimal decisions.
- **Psychological and social factors** (animal spirits, confidence, self-fulfilling expectations) can amplify or suppress investment even without real or rational fundamentals.

To capture this complexity, modern macroeconomics combines micro-founded models with behavioral elements and institutional rigidities, allowing for a better understanding of investment cycles and their role in aggregate fluctuations.

[2]The NPV is a measure of investment calculated as the difference between the present value of the project's future cash flows and the present value of the investment cost. The cash flows are discounted at the opportunity cost of capital. If the NPV is positive, the project is worth pursuing, as it represents the best available investment opportunity.