Consumption and Saving

Macroeconomics I
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Introduction: The first component of aggregate demand we will study is consumption. To do so, as in the other chapters, we rely on the neoclassical model. This model uses mathematics and microfoundations to understand how agents make optimal decisions by characterizing the problem they face. It is a way of supporting observed behaviors and facts in the real economy.

1. Introduction to the Neoclassical Model

We consider a basic two-period model to study the intertemporal decisions of a representative agent who wishes to allocate their consumption between today and the future. This structure is sufficient to introduce fundamental concepts such as the intertemporal budget constraint, expected utility, optimal consumption choice, and consumption smoothing.

1.1. Intratemporal Budget Constraints

Imagine an agent who lives for two periods: as young (period 1) and old (period 2), and receives exogenous income[1] Y_1 and Y_2 in each period, respectively. As in real life, the agent does not need to consume all their income in the same period—they can save or borrow at a real interest rate r, freely accessing financial markets (no constraints). Based on this, we can state our intratemporal budget constraints for each period. A *budget constraint* is an equation that balances sources of funds with their uses (like a budget), and *intratemporal* refers to within a single period. We have:

- In the first period, the constraint is:

$$\underbrace{C_1 + S}_{\text{Uses}} = \underbrace{Y_1}_{\text{Sources}}$$

  where C_1 is present consumption and S is saving (if $S < 0$, this is borrowing).

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[1] Exogenous means it is not determined within the model. That is, we do not delve into how this income is determined, it is simply received. In later models we will expand and make income an endogenous variable, meaning the agent's earnings will be determined by decisions within the model.

- In the second period, the agent must consume what they saved plus the interest earned on their savings. If S were negative, i.e., the agent borrowed in the first period, this second term would be negative, representing a negative source (a use):

$$C_2 = Y_2 + (1 + r)S$$

1.2. Intertemporal Budget Constraint

From the intratemporal budget constraints, we can form a single intertemporal budget constraint (IBC). Here, *intertemporal* refers to more than one period. The procedure is simple: the connector in both equations is savings S, so we solve for it in the first equation $S = Y_1 - C_1$ and substitute into the second-period equation. We get:

$$C_2 = Y_2 + (1 + r)(Y_1 - C_1) \Rightarrow C_1 + \frac{C_2}{1 + r} = Y_1 + \frac{Y_2}{1 + r}$$

This is the intertemporal budget constraint: the present value of consumption must equal the present value of income. *Present value* means that resources do not have the same value over time. That is, receiving a given amount of income today is not the same as receiving that amount tomorrow, because of the **opportunity cost**. If you received Y today, you could save it at rate r and have $(1+r)Y > Y$ tomorrow. That's why we divide the second-period terms by $(1 + r)$ to express everything in present value and have all terms in the same units.

1.3. Utility Function

The agent chooses C_1 and C_2 to maximize intertemporal utility:

$$\max_{C_1, C_2} \; u(C_1) + \beta u(C_2)$$

where $u(\cdot)$ is an increasing and concave utility function (i.e., $u' > 0$, $u'' < 0$). In other words, an extra unit of consumption always brings more utility (first derivative positive), but the utility increases at a decreasing rate (second derivative negative). This is known as diminishing marginal utility and is modeled to reflect the concept of *satiety*. Going from owning no cars to one increases happiness/utility a lot; going from one to two also increases it, but less. Having 30 versus 31 cars likely increases utility too, but much less than getting the first 30. That's satiety.

Meanwhile, $\beta \in (0, 1)$ is the subjective discount factor, which reflects the agent's impatience. Just as resources have a time value due to opportunity cost, the utility received from consumption also depends on the period in which it is enjoyed. Having a Porsche at 20 is not the same as at 50. Clearly, agents are impatient and value present consumption more. That's why second-period utility is multiplied by a $\beta < 1$, which reduces (or discounts) future utility. A lower value of β (closer to 0) implies the individual values future consumption less, i.e., is more impatient.

1.4. Maximization Problem

Once we have defined the function to maximize and the constraints, we can formulate the maximization problem. For that, we use a Lagrangian with a single constraint (since we have already combined the two intratemporal constraints into a single intertemporal one that contains the information from both). An alternative is to work with both intratemporal constraints separately, in which case we would use two Lagrange multipliers[2]:

$$\mathcal{L} = u(C_1) + \beta u(C_2) + \lambda \left(Y_1 + \frac{Y_2}{1+r} - C_1 - \frac{C_2}{1+r} \right)$$

As in any maximization problem, we differentiate with respect to the variables the agent can choose, in this case, C_1 and C_2. Our first-order conditions (FOCs) are:

$$\frac{\partial \mathcal{L}}{\partial C_1} = u'(C_1) - \lambda = 0$$
$$\frac{\partial \mathcal{L}}{\partial C_2} = \beta u'(C_2) - \lambda \cdot \frac{1}{1+r} = 0$$

Solving for λ from the first and substituting into the second, we get the Euler equation:

$$u'(C_1) = \beta(1+r)u'(C_2)$$

This optimality condition is known as the **Euler equation**, and it balances the marginal utility of one more unit of consumption today (left-hand side) with the marginal utility of future consumption, adjusted by the discount factor β and financial return $1+r$ (right-hand side). If the agent does not consume that unit today, they can save it and receive $(1+r)$ units tomorrow, increasing utility by $(1+r)u'(C_2)$, which must be discounted by β to express it in today's utility. Remember that $u'(C)$ is decreasing in C, so if the right-hand side is greater, the individual should save more today (reduce C_1) to consume more tomorrow.

1.5. Optimal consumption and intertemporal wealth

Using the Euler equation and the intertemporal constraint, we can derive the expressions for optimal consumption. Let:

$$W \equiv Y_1 + \frac{Y_2}{1+r}$$

be intertemporal wealth. And if $u(C) = \ln(C)$, then $u'(C) = 1/C$, our Euler equation becomes:

[2]It's a matter of preference, but if multiple constraints can be combined into one, it's easier to work with a single multiplier. However, sometimes it is also useful to work with two multipliers. The key is that if we want to place a + before the multiplier, then inside the parentheses we must put sources minus uses.

$$C_2 = \beta(1+r)C_1$$

Replacing the Euler equation into the intertemporal budget constraint and solving for C_1:

$$C_1 + \frac{\beta(1+r)C_1}{(1+r)} = W$$

$$C_1^* = \frac{1}{1+\beta}W$$

$$C_2^* = \beta(1+r)C_1^*$$

These expressions correspond to optimal present and future consumption. These are optimal equations because they depend solely on exogenous variables (Y_1, Y_2, and r) and parameters (β). The optimal equations indicate that optimal consumption depends on the present value of total income W multiplied by a fraction that depends on the impatience factor. The agent **smooths consumption** over time: they do not necessarily consume what they earn in each period but instead allocate consumption based on their entire intertemporal wealth.

1.6. Consumption smoothing

The Euler equation reveals two opposing forces:

- The desire to consume today: reflected in a $\beta < 1$.
- The incentive to postpone consumption: if $r > 0$, saving yields a return.

The opposition of these two forces causes them to largely offset each other, leading to similar levels of consumption in both periods ($\beta(1+r) \Rightarrow 1$). This phenomenon is known as **consumption smoothing**. When $\beta(1+r) = 1$, these forces are balanced and consumption is perfectly flat: $C_1 = C_2$. This particular case is known as **perfect consumption smoothing**. If, for example, first-period income drops by one unit, optimal first-period consumption only drops by $1/(1+\beta) < 1$ units, reflecting that even if income fluctuates, the individual seeks to keep consumption stable over time, as long as they have access to financial markets.

2. Adjustments to Shocks

One of the central strengths of the intertemporal consumption model is its ability to analyze how agents respond to different types of shocks. In this section, we evaluate three relevant cases: (1) an increase in the interest rate, (2) an increase in present income, and (3) an increase in future income.

2.1. Increase in the interest rate r

Suppose that initially the individual was in equilibrium with an interest rate r, and now this rate increases to $r' > r$. There are two opposing effects on consumption:

- **Substitution effect:** A higher r makes present consumption more expensive relative to future consumption, as saving now yields a higher return, encouraging the individual to consume less today and more tomorrow. This induces a fall in C_1 and an increase in C_2.
- **Income effect:** Depends on whether the individual is a saver or a borrower.
  - If the agent is a **saver** ($S > 0$), a higher r increases their total future income due to the higher return on savings: this generates a positive income effect, allowing for increased consumption in both periods.
  - If the agent is a **borrower** ($S < 0$), the higher r makes debt more expensive. This reduces their future disposable income and generates a negative income effect, leading to a reduction in consumption in both periods.

Net effect: It depends on the relative magnitude of both effects and the type of agent. The substitution effect is independent of the agent type and reflects the substitution of present consumption for future consumption (or vice versa if the interest rate decreases). The income effect depends on the type of agent and will move both present and future consumption up or down depending on whether the agent feels richer or poorer. Since the substitution effect is the same for all agents, while the income effect differs, it is usually assumed that the substitution effect dominates. That is, when the interest rate rises, present consumption is substituted for future consumption, and the opposite happens when the interest rate falls.

2.2. Positive shock in present income Y_1

An unexpected increase in first-period income raises intertemporal wealth W. The individual can use this additional income to increase both C_1 and C_2. If the agent wants to smooth consumption, they will prefer to distribute the additional income across both periods. This is reflected in the optimal consumption equations:

$$\frac{dC_1}{dY_1} = \frac{1}{1+\beta}, \quad \frac{dC_2}{dY_1} = \frac{\beta(1+r)}{1+\beta}$$

Present consumption increases but by less than the income shock ($1/(1+\beta) < 1$). The remainder is saved to allow for more consumption in the future. This behavior is consistent with the **consumption smoothing theory**: in the face of transitory shocks, consumption reacts less than income.

2.3. Positive shock in future income Y_2

In this case, the individual anticipates higher income in the second period. Given that financial markets allow them to borrow today and repay tomorrow, the agent adjusts both future and current consumption:

$$\frac{dC_1}{dY_2} = \frac{1}{(1+\beta)(1+r)}, \quad \frac{dC_2}{dY_1} = \frac{\beta(1+r)}{(1+\beta)(1+r)}$$

Thus, once again the agent distributes the increase in future income across both consumption periods. The impact on present and future consumption will depend on how much the individual values present versus future consumption (i.e., on β and r).

In summary:

- An increase in r generates a negative substitution effect on C_1, but its income effect depends on the financial profile of the agent.
- An increase in Y_1 leads to an increase in consumption in both periods, but smoothed.
- An increase in Y_2 also raises present consumption, through higher borrowing or reduced saving.

3. Financial Constraints

So far we have assumed that the agent has unlimited access to financial markets: they can save or borrow freely at the interest rate r. In practice, however, many individuals face **liquidity constraints** or borrowing limits. We analyze how this affects consumption and intertemporal smoothing.

3.1. No-borrowing constraint

Suppose the agent cannot borrow: they are constrained to $S \geq 0$, that is:

$$C_1 \leq Y_1$$

This condition limits present consumption to today's available income. The intertemporal budget constraint remains:

$$C_1 + \frac{C_2}{1+r} \leq Y_1 + \frac{Y_2}{1+r}$$

But now it is additionally imposed that:

$$C_1 \leq Y_1$$

3.2. Consequences for optimal consumption

In this new context, the maximization problem is:

$$\max_{C_1, C_2} \; u(C_1) + \beta u(C_2)$$

subject to:

$$C_1 + \frac{C_2}{1+r} \leq Y_1 + \frac{Y_2}{1+r}, \quad C_1 \leq Y_1$$

If in the unconstrained optimum it turns out that $C_1^* \leq Y_1$, the constraint is non-binding, and the solution is the same as before (the agent already chose to consume less than their income in the first period, i.e., they did not borrow). But if $C_1^* > Y_1$, then the agent would like to borrow to smooth consumption but cannot. In this case, the no-borrowing constraint is binding and it is imposed that $C_1 = Y_1$, which is the closest the agent can get to the optimal level, so $S = 0$.

3.3. Economic interpretation

The main implication is that borrowing-constrained agents cannot smooth their consumption in response to shocks. For example, if they receive low income in period 1 and high income in period 2, they would like to borrow today to level their consumption across periods. But since they cannot borrow, they are forced to consume very little in the present, despite having high future income. Moreover, if the credit constraint is binding and present income falls for some reason, the agent will have to reduce consumption by the same amount, since they cannot access credit.

This result aligns with empirical observations, where individuals with low access to credit exhibit higher sensitivity of consumption to current income. In addition, financial constraints create a **role for public policy**: if individuals cannot smooth their consumption, temporary transfer programs (like unemployment insurance or social transfers) can significantly increase welfare by alleviating these frictions.

In the next section, we will extend the analysis to an infinite horizon and examine how consumption responds to transitory and permanent shocks, which will allow us to formally introduce the **Permanent Income Hypothesis** and the **Life-Cycle Model**.

4. Permanent Income Hypothesis

We now extend the analysis to an infinite horizon. The agent lives forever and seeks to maximize intertemporal utility by choosing their consumption path $\{C_t\}_{t=0}^{\infty}$, subject to their income flow $\{Y_t\}_{t=0}^{\infty}$ and the possibility of saving or borrowing.

4.1. The consumer's problem with infinite horizon

The consumer maximizes:

$$\max_{\{C_t\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t u(C_t)$$

subject to the sequence of intratemporal budget constraints for any period t:

$$S_{t+1} + C_t = Y_t + (1+r)S_t$$

where S_t represents the level of savings at the beginning of period t, and r is the constant interest rate. S_{t+1} is the amount saved for the next period. How do we construct the intertemporal budget constraint (IBC)? We could iterate and substitute forward, but it's easier to recall the definition: the present value of uses must equal the present value of sources[3]:

$$\underbrace{\sum_{t=0}^{\infty} \frac{C_t}{(1+r)^t} + \lim_{T\to\infty} \frac{S_{T+1}}{(1+r)^{T+1}}}_{\text{PV of Uses}} = \underbrace{S_0(1+r) + \sum_{t=0}^{\infty} \frac{Y_t}{(1+r)^t}}_{\text{PV of Sources}}$$

On the right-hand side, we have the present value of consumption plus the terminal savings. On the left, we have initial wealth (savings) and its return, plus the present value of the income stream.

4.2. No-Ponzi condition

To avoid solutions where the agent finances consumption only through increasing debt, we impose the **No-Ponzi condition**:

$$\lim_{T\to\infty} \frac{S_{T+1}}{(1+r)^T} \geq 0$$

This ensures that accumulated debt does not grow faster than the agent's ability to repay. Otherwise, the agent could keep borrowing and consuming beyond their means indefinitely. This constraint requires that the present value of savings at the end of life ($T \to \infty$) is zero or positive.

4.3. Transversality condition

The no-Ponzi condition is a constraint, but an optimality condition that always emerges in solution is the **transversality condition**. This is intuitive: since only consumption

[3] If in doubt, you can try iterating. Start at $t = 1$, connect it to the constraint in $t = 2$ via S_2, then to $t = 3$, and so on until you derive the general form.

provides utility and savings do not, the agent will choose to consume everything and leave no unspent savings. That is, the agent will consume as much as possible to maximize utility, leaving a zero stock of savings:

$$\lim_{T\to\infty} \frac{S_{T+1}}{(1+r)^T} = 0$$

4.4. Maximization

Again, there are two equivalent ways to set up the Lagrangian: working with the infinite sequence of intratemporal constraints (in which case we have a Lagrange multiplier λ_t for each period), or working with the single intertemporal budget constraint (in which case we have a single multiplier λ, without the subscript). Since the two-period model used the latter, we now use the first:

$$\mathcal{L} = \sum_{t=0}^{\infty} \beta^t u(C_t) + \sum_{t=0}^{\infty} \lambda_t \left[Y_t + (1+r)S_t - S_{t+1} - C_t\right]$$

What variables do we differentiate with respect to for the FOCs? We must think about the variables chosen in any period t. The agent chooses C_t and S_{t+1}:

$$\frac{\partial \mathcal{L}}{\partial C_t} = \beta^t u'(C_t) - \lambda_t = 0$$

$$\frac{\partial \mathcal{L}}{\partial S_{t+1}} = -\lambda_t + (1+r)\lambda_{t+1} = 0$$

Combining both, we obtain the Euler equation:

$$u'(C_t) = \beta(1+r)u'(C_{t+1}) \quad \forall t$$

4.5. Perfect consumption smoothing

If $\beta(1+r) = 1$, then the Euler equation implies:

$$u'(C_t) = u'(C_{t+1}) \Rightarrow C_t = C_{t+1} \quad \forall t$$

That is, consumption is constant over time: $C_t = C$. This is a case of **perfect consumption smoothing**, where the agent keeps their consumption level stable over time. We will work with this case to simplify the IBC[4]:

$$\sum_{t=0}^{\infty} \frac{C}{(1+r)^t} = S_0(1+r) + \sum_{t=0}^{\infty} \frac{Y_t}{(1+r)^t}$$

[4]In moving to the next step, recall the convergence of the geometric series: $1 + C + C^2 + \cdots = \frac{1}{1-C}$

$$C\sum_{t=0}^{\infty}\frac{1}{(1+r)^t} = S_0(1+r) + \sum_{t=0}^{\infty}\frac{Y_t}{(1+r)^t}$$

$$C\left[1+\frac{1}{1+r}+\frac{1}{(1+r)^2}+\dots\right] = S_0(1+r) + \sum_{t=0}^{\infty}\frac{Y_t}{(1+r)^t}$$

$$C\frac{(1+r)}{r} = S_0(1+r) + \sum_{t=0}^{\infty}\frac{Y_t}{(1+r)^t}$$

$$C = \frac{r}{1+r}\left[S_0(1+r) + \sum_{t=0}^{\infty}\frac{Y_t}{(1+r)^t}\right]$$

Consumption is then a function of the entire income path—intertemporal wealth. This is the **Permanent Income Hypothesis**: the agent does not change consumption significantly in response to changes in current income, but rather to changes in permanent income. This will become clearer in the next section where we analyze transitory and permanent shocks.

4.6. Transitory vs. permanent shocks

The model predicts different responses depending on the type of shock. Assume $r = 4\,\%$ to visualize magnitudes:

- **Transitory shock:** Suppose an increase of one unit in Y_0. Since the agent bases decisions on the entire income path, C increases only slightly. We differentiate C_0 with respect to Y_0:

$$\frac{dC_0}{dY_0} = \frac{r}{1+r} = \frac{0{,}04}{1{,}04} = 0{,}038$$

- **Permanent shock:** Now suppose that income in all periods increases by one unit. Consumption now increases by one unit as well. The total change is calculated as:

$$\Delta C_1 = \frac{r}{1+r}\left[\Delta\sum_{t=0}^{\infty}\frac{Y_t}{(1+r)^t}\right] = \frac{r}{1+r}\left[\Delta Y_0 + \frac{\Delta Y_1}{(1+r)} + \frac{\Delta Y_2}{(1+r)^2}\right]$$

$$\Delta C_1 = \frac{r}{1+r}\left[\Delta\sum_{t=0}^{\infty}\frac{Y_t}{(1+r)^t}\right] = \frac{r}{1+r}\left[1 + \frac{1}{(1+r)} + \frac{1}{(1+r)^2}\right]$$

$$\Delta C_1 = \frac{r}{1+r}\left[\Delta\sum_{t=0}^{\infty}\frac{Y_t}{(1+r)^t}\right] = \frac{r}{1+r}\left[\frac{1+r}{r}\right]$$

$$\Delta C_1 = 1$$

  We confirm that consumption changes by the same amount as income.

This prediction—that consumption responds more to permanent than to transitory shocks—is one of the main empirical implications of the **Permanent Income Hypothesis**, proposed by Milton Friedman, and the **Life-Cycle Model** by Modigliani and Brumberg.

5. Consumption and Leisure Decisions

We now extend the two-period model by incorporating decisions about how much to work. The agent no longer receives exogenous income but instead chooses how much to work in each period, generating endogenous labor income. This allows us to analyze the interaction between consumption, saving, and leisure decisions.

As before, we assume the agent lives for two periods. They choose consumption C_1, C_2 and hours worked L_1, L_2 in each period. Additionally, the agent derives utility not only from consumption but also from leisure. The utility function is now:

$$U = u(C_1, O_1) + \beta u(C_2, O_2)$$

Labor income in each period is $w_t L_t$. Thus, the budget constraints are:

$$C_1 + S = w_1 L_1$$
$$C_2 = w_2 L_2 + (1 + r)S$$

Combining both, we obtain the intertemporal budget constraint:

$$C_1 + \frac{C_2}{1+r} = w_1 L_1 + \frac{w_2 L_2}{1+r}$$

Additionally, we must impose a time constraint. The agent has H hours in each period, split between leisure and work: $L_t + O_t = H$.

5.1. Optimization problem

The agent chooses C_1, C_2, O_1, O_2 to maximize:

$$\text{máx } \ u(C_1, O_1) + \beta u(C_2, O_2)$$

subject to:

$$C_1 + \frac{C_2}{1+r} = w_1 L_1 + \frac{w_2 L_2}{1+r}$$
$$L_t + O_t = H, \quad t = 1, 2$$

We can substitute the time constraints into the IBC via L_1 and L_2. The Lagrangian becomes:

$$\mathcal{L} = u(C_1, O_1) + \beta u(C_2, O_2) + \lambda \left[w_1(H - O_1) + \frac{w_2(H - O_2)}{1+r} - C_1 - \frac{C_2}{1+r} \right]$$

Taking derivatives with respect to C_1, C_2, O_1, O_2:

$$\frac{\partial \mathcal{L}}{\partial C_1} = u_C(C_1, O_1) - \lambda = 0$$
$$\frac{\partial \mathcal{L}}{\partial C_2} = \beta u_C(C_2, O_2) - \lambda \cdot \frac{1}{1+r} = 0$$
$$\frac{\partial \mathcal{L}}{\partial O_1} = u_O(C_1, O_1) + \lambda w_1 = 0$$
$$\frac{\partial \mathcal{L}}{\partial O_2} = \beta u_O(C_2, O_2) + \lambda \cdot \frac{w_2}{1+r} = 0$$

From the derivatives with respect to C_t and O_t, we obtain the **intratemporal optimality condition** for each period:

- For $t = 1$:
$$\frac{u_O(C_1, O_1)}{u_C(C_1, O_1)} = -w_1$$
- For $t = 2$:
$$\frac{u_O(C_2, O_2)}{u_C(C_2, O_2)} = -w_2$$

This condition shows that the agent chooses leisure O_t such that the marginal rate of substitution between leisure and consumption equals the real wage: each additional hour of leisure implies a loss of w_t consumption units, and the agent equates this cost with their marginal valuation of leisure.

From the conditions on C_1 and C_2, we obtain:

$$u_C(C_1, O_1) = \beta(1+r)u_C(C_2, O_2)$$

This follows the same logic as the Euler equation in the model without leisure.

5.2. Economic discussion

Now, let's explore how consumption and labor supply respond to changes in wages, preferences, or the interest rate using the first-order conditions.

- If the current wage increases ($w_1 \uparrow$), this generates:
  - **Substitution effect:** Replacing leisure with work (leisure becomes more expensive).
  - **Income effect:** The agent feels wealthier and consumes more of their "goods," meaning more leisure time.

- The net effect on L_1 (labor) depends on which effect dominates.

This concludes the analysis of the neoclassical consumption model. In the directed sessions, we will expand upon this basic model and introduce financial constraints, uncertainty, and multi-agent frameworks, gradually moving toward more realistic and dynamic environments.