Rational Expectations

Macroeconomics 3
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Introduction: Rational expectations emerged as a fundamental concept in the evolution of macroeconomic thought starting in the 1970s, challenging previous models based on adaptive expectations. The concept was formally introduced by John F. Muth in his paper *"Rational Expectations and the Theory of Price Movements"* published in 1961. In this work, Muth proposed that economic agents do not form their expectations simply by looking at the past, but rather use all available information efficiently and consistently with the underlying economic model. This idea proved disruptive for the economic context of the time, as it implied that systematic government policies could not consistently manipulate agent decisions if they correctly anticipated their effects. According to rational expectations theory, agents make decisions considering three pillars: their rationality, all available information, and accumulated experience. The defining quote of this theory is:

> *"You can fool some people all the time, and all people some of the time, but you cannot fool all the people all the time."*

1. A First Model: Muth 1961

Muth proposes a simple model to illustrate how economic agents form rational expectations in a market context. The model is based on an isolated market for a non-storable good. The system consists of demand, supply, and market equilibrium equations, in that order:

$$C_t = -\beta p_t \tag{1}$$

$$P_t = \gamma p_t^e + u_t \tag{2}$$

$$P_t = C_t \tag{3}$$

where P_t represents the quantity produced in period t; C_t is the quantity consumed; p_t is the market price in period t; p_t^e is the price expected to prevail in period t, based on information available through period $t-1$; and u_t is an error term capturing random disturbances, such as variations in agricultural yields due to weather. All variables are

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expressed as deviations from their equilibrium values. From these equations, it is key to understand that the producer determines the quantity to produce with some lag, so they will base their production on what they expect the price to be. Since production is fixed ex-ante, the producer's final outcome will vary once supply and demand interact. Solving the equilibrium and isolating p_t yields:

$$p_t = -\frac{\gamma}{\beta}p_t^e - \frac{1}{\beta}u_t \tag{4}$$

The mathematical way to incorporate expectations is through the expected value, with the subscript t added to denote that it is the expected value with information available through that period. Taking expectations on both sides:

$$\mathbb{E}_t[p_t] = -\frac{\gamma}{\beta}p_t^e \tag{5}$$

The rational expectations hypothesis posits that agents use all available information efficiently, so their expectations coincide with the predictions of the relevant theoretical model. Formally, this implies that:

$$p_t^e = \mathbb{E}_t[p_t] \tag{6}$$

Substituting (5) into (6), we obtain the rationality condition:

$$p_t^e = -\frac{\gamma}{\beta}p_t^e \tag{7}$$

If $-\frac{\gamma}{\beta} \neq 1$, this equality only holds if $p_t^e = 0$, meaning the expected price equals the equilibrium price (recall that variables are expressed as deviations from their equilibrium values). This result reveals a fundamental property: when disturbances (u_t) are unpredictable (white noise), rational agents anticipate that the price will match its equilibrium value.

However, this problem is of little interest, as we have assumed the shock u_t in the supply equation is completely unpredictable. The case becomes more interesting when shocks have predictable components. Suppose u_t follows an autoregressive process:

$$u_t = \sum_{i=0}^{\infty} w_i \epsilon_{t-i}, \quad \epsilon_t \sim iid(0, \sigma^2) \tag{8}$$

where the weights w_i determine the temporal structure of the disturbances. The key to solving the system lies in recognizing that both the observed price (p_t) and the expectation (p_t^e) are linear functions of past shocks $\{\epsilon_{t-i}\}_{i=0}^{\infty}$. This allows us to express:

$$p_t = \sum_{i=0}^{\infty} W_i \epsilon_{t-i} \tag{9}$$

where the coefficients $\{W_i\}$ capture how historical shocks affect the current price. The rational expectation p_t^e, being formed with information through $t-1$, excludes the contemporaneous shock ϵ_t:

$$p_t^e = \mathbb{E}[p_t|\Omega_{t-1}] = \sum_{i=1}^{\infty} W_i \epsilon_{t-i} \tag{10}$$

Substituting these expressions into the original equilibrium equation (4):

$$\underbrace{\sum_{i=0}^{\infty} W_i \epsilon_{t-i}}_{p_t} = -\frac{\gamma}{\beta} \underbrace{\sum_{i=1}^{\infty} W_i \epsilon_{t-i}}_{p_t^e} - \frac{1}{\beta} \underbrace{\sum_{i=0}^{\infty} w_i \epsilon_{t-i}}_{u_t} \tag{11}$$

For this equality to hold for any realization of $\{\epsilon_t\}$, the coefficients of each ϵ_{t-i} must match on both sides. This generates an infinite system of equations:

$$\text{For } \epsilon_t : \quad W_0 = -\frac{1}{\beta} w_0 \tag{12}$$

$$\text{For } \epsilon_{t-k} (k \geq 1) : \quad W_k = -\frac{\gamma}{\beta} W_k - \frac{1}{\beta} w_k \tag{13}$$

Solving (12) directly gives us the first coefficient:

$$W_0 = -\frac{1}{\beta} w_0 \tag{14}$$

For $k \geq 1$, we rearrange (13):

$$W_k + \frac{\gamma}{\beta} W_k = -\frac{1}{\beta} w_k \implies W_k \left(1 + \frac{\gamma}{\beta}\right) = -\frac{1}{\beta} w_k \tag{15}$$

Multiplying both sides by β and solving for W_k:

$$W_k(\beta + \gamma) = -w_k \implies W_k = -\frac{w_k}{\beta + \gamma} \tag{16}$$

This result shows how the structural parameters (β, γ) determine the optimal weight that rational agents assign to each historical shock.

This result shows how rational expectations optimally incorporate the dynamic structure of the economic system. When shocks are permanent ($w_i = 1$ for all i), expectations take the form of a geometric moving average:

$$p_t^e = \frac{\beta}{\gamma} \sum_{j=1}^{\infty} \left(\frac{\gamma}{\beta + \gamma}\right)^j p_{t-j} \tag{17}$$

which coincides with Nerlove's adaptive expectations mechanism, but with an endogenous adjustment coefficient determined by the underlying model's structural parameters, the supply and demand sensitivities: β and γ. The elegance of Muth's approach lies in the fact that expectations are not imposed ad hoc, but emerge as an optimal solution given the economic environment.

2. From Cobweb to Rational Expectations

Since the early 20th century, economists faced the problem of modeling how agents formed price expectations. An iconic example is the Cobweb model, where farmers based their future price expectations on past prices. Let's revisit the basic model where supply responds to producer expectations:

$$p_t = \frac{\gamma}{\beta} p_t^e + u_t \tag{18}$$

If we assume static expectations $p_t^e = p_{t-1}$, the model's predictions have the opposite sign of farmers' expectations. Farmers expect high prices to continue and increase production. However, this causes prices to fall, generating fluctuations that contradict their expectations. This paradox reflected a fundamental flaw in traditional expectation models that led to systematic errors.

Muth's model emerged as a response to the inconsistencies of traditional expectation formation mechanisms. Historically, three main approaches had been proposed:

1. **Static expectations (Schultz-Tinbergen)**:

   $$p_t^e = p_{t-1} \tag{19}$$

   They generate cycles when $|\gamma/\beta| > 1$, contradicting evidence of stable markets. Muth showed these cycles are a consequence of specification: if agents consistently used the model to predict, they would deduce that $p_t^e = 0$, eliminating artificial fluctuations.

2. **Extrapolative expectations (Goodwin)**:

   $$p_t^e = p_{t-1} + \varrho(p_{t-1} - p_{t-2}) \tag{20}$$

   For $\varrho \in (-1, 1)$. While more flexible, Muth notes that the price expected by farmers and the price predicted by the model have opposite signs.

3. **Adaptive expectations (Nerlove)**:

   In this case, agents incorporate past prediction errors into future predictions:

   $$p_t^e = p_{t-1}^e + \eta(p_{t-1} - p_{t-1}^e) \tag{21}$$

   Which converge to a geometric average:

   $$p_t^e = \eta \sum_{j=0}^{\infty} (1 - \eta)^j p_{t-j} \tag{22}$$

Muth identified two key flaws in these approaches:

- *Lack of microfoundations*: The parameters ϱ and η are calibrated ad-hoc without connection to market structure (β, γ).
- *Informational inefficiency*: Reported expectations systematically underestimate actual changes (regression bias).

The rational expectations solution emerges naturally by requiring:

$$\mathbb{E}[(p_t - p_t^e)|\Omega_{t-1}] = 0 \quad \text{(Unbiasedness)} \tag{23}$$

$$\mathbb{E}[(p_t^e - \hat{p}_t)^2] \leq \mathbb{E}[(p_t^e - \tilde{p}_t)^2] \quad \forall \tilde{p}_t \quad \text{(Efficiency)} \tag{24}$$

Where $\hat{p}_t$ is any other predictor. Table 1 summarizes how rational expectations address empirical limitations:

Cuadro 1: Comparison of expectation mechanisms

Property	Adaptive	Extrapolative	Rational
Consistency with data	Partial	No	Yes
Efficient information use	No	No	Yes
Recognizes policy changes	No	No	Yes
Consistent with market structure	No	No	Yes

3. The Lucas Critique and Its Implications

Robert Lucas (1976) extended this logic to economic policy. If agents form rational expectations, any systematic policy rule will be incorporated into their decisions, neutralizing its effectiveness. Consider an economy that had historically maintained a constant money supply, but at t=0 the Central Bank decides to implement a new policy rule:

$$m_t = (1 + \theta)m_{t-1} + \epsilon_t \tag{25}$$

Imagine the money supply level at the time of the change was 100 and the new growth rate implemented is $\theta = 10\,\%$. At t=1, agents with adaptive expectations ($\eta = 1$) predict $m_1^e = m_0 = 100$, but the authority implements $m_1 = 1{,}1 \times 100 + 0 = 110$. Agents underestimate the policy by $10\,\%$. For t=2, they update:

$$m_2^e = m_1^e + (m_1 - m_1^e) = 100 + 10 = 110 \tag{26}$$

However, the policymaker implements $m_2 = 1{,}1 \times 110 = 121$. The error persists:

$$\mathbb{E}[m_t - m_t^e] = \theta m_{t-1} = 0{,}1 \times 110 = 11 \tag{27}$$

This bias is *consistent*: even though agents fully correct ($\eta = 1$), they always remain one step behind. With rational expectations, however, agents deduce the true rule and perfectly predict the systematic component:

$$m_t^e = (1+\theta)m_{t-1} \implies m_t - m_t^e = \epsilon_t \tag{28}$$

The difference is pure noise ($\mathbb{E}[\epsilon_t] = 0$), eliminating any predictable real effect of policy. Any attempt at systematic stimulation ($\theta > 0$) is immediately incorporated into expectations, neutralizing its real effect.

This result transformed economic policy design, emphasizing credibility and predictable rules over discretion.