Quasi- Newton. 170920 

1) Muestre que la 2ª cond fuerte de Wolfe implica la cond de curvatura: $s_k^T y_k > 0$

$x_{k+1} = x_k + \alpha_k p_k$

Nota: $s_k = \alpha_k p_k$ , $y_k = \nabla f_{k+1} - \nabla f_k$

$\boxed{\text{Quiero: } \alpha_k p_k^T(\nabla f_{k+1} - \nabla f_k) > 0}$

2ª cond fuerte de Wolfe: $|\nabla f(x_k + \alpha_k p_k)^T \cdot p_k| \leq c_2 |\nabla f_k^T p_k|$

→ reesc. Wolfe y considerando $\cancel{\nabla f}$: $x_{k+1} \overset{def.}{=} x_k + \alpha_k p_k$

$$\Rightarrow \quad |\nabla f_{k+1}^T \cdot p_k| \leq c_2 |\nabla f_k^T p_k|$$

$$\Leftrightarrow \quad -|\nabla f_{k+1}^T \cdot p_k| \geq -c_2 |\nabla f_k^T p_k|$$

$$\Leftrightarrow \quad \nabla f_{k+1}^T \cdot p_k \geq -c_2 |\nabla f_k^T p_k|$$

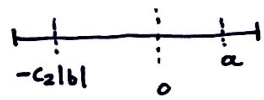
$$\Leftrightarrow \quad \nabla f_{k+1}^T p_k - \nabla f_k^T p_k \geq -c_2 |\nabla f_k^T p_k| - \nabla f_k^T p_k$$

$$\Leftrightarrow \quad (\nabla f_{k+1}^T - \nabla f_k^T) p_k \geq c_2 \nabla f_k^T p_k - \nabla f_k^T p_k$$

$$'' \quad \geq \underbrace{(c_2 - 1)}_{<0} \underbrace{\nabla f_k^T p_k}_{<0} > 0$$

$c_2 \in (0,1)$

$a \geq -c_2 |b|$




si $b < 0$

$\Rightarrow -c_2|b| = c_2 b$

rec: $\nabla f_k^T p_k < 0$ (éstos decendientes)

$$\Rightarrow \quad (\nabla f_{k+1}^T - \nabla f_k^T) p_k > 0$$

Transponiendo:

$$p_k^T (\nabla f_{k+1} - \nabla f_k) > 0$$

mult. por $\alpha_k > 0$

$$\alpha_k p_k^T (\nabla f_{k+1} - f_k) > 0 \quad \equiv \quad s_k^T y_k > 0 \quad ▲$$

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2) Inversas,

Sabemos:

$$* \; H_{k+1} = (I - \rho_k S_k Y_k^T) H_k (I - \rho_k Y_k S_k^T) + \rho_k S_k S_k^T$$

$$\rho_k = \frac{1}{Y_k^T S_k}$$

$$B_{k+1} = B_k - \frac{B_k S_k S_k^T B_k}{S_k^T B_k S_k} + \underbrace{\frac{Y_k Y_k^T}{Y_k^T S_k}}_{\rho_k Y_k Y_k^T}$$

$*$ sabemos $B_{k+1} \cdot S_k = Y_k$ → Marco con $*$ donde se usa.

$$\Rightarrow \; B_{k+1} \cdot H_{k+1} = B_{k+1}\left((I - \rho_k S_k Y_k^T) H_k (I - \rho_k Y_k S_k^T) + \rho_k S_k S_k^T\right)$$

$$= (B_{k+1} - \rho_k Y_k Y_k^T) H_k (I - \rho_k Y_k S_k^T) + \rho_k Y_k S_k^T$$

sust la formula de $B_{k+1}$

$$= \left(B_k - \frac{B_k S_k S_k^T B_k}{S_k^T B_k S_k}\right) H_k (I - \rho_k Y_k S_k^T) + \rho_k Y_k S_k^T$$

$$= \left(I - \frac{B_k S_k S_k^T}{S_k^T B_k S_k}\right)(I - \rho_k Y_k S_k^T) + \rho_k Y_k S_k^T$$

$$= I - \cancel{\rho_k Y_k S_k^T} - \frac{B_k S_k S_k^T}{S_k^T B_k S_k}(I - \rho_k Y_k S_k^T) + \cancel{\rho_k Y_k S_k^T}$$

$$= I - \frac{B_k S_k S_k^T}{S_k^T B_k S_k} + \underbrace{\frac{B_k S_k S_k^T}{S_k^T B_k S_k} \cdot \rho_k Y_k S_k^T}$$

$$= \frac{B_k S_k S_k^T \rho_k Y_k S_k^T}{S_k^T B_k S_k}$$

$$= \frac{B_k S_k \cancel{S_k^T} \left(\cancel{\frac{1}{Y_k^T S_k}}\right) \cancel{Y_k} S_k^T}{S_k^T B_k S_k}$$

simetría $I$

$$= \frac{B_k S_k S_k^T}{S_k^T B_k S_k}$$

$$= I \; \blacktriangle$$

∴ $H_{k+1}$ es inversa de $B_{k+1}$

# Gradiente Conjugado

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1) p. def: si $A$ es sim. pos. def $\Rightarrow$ ~~$z^T$~~ $z^T A z > 0 \quad \forall z$

Demuestre que si los vectores no nulos $p_i^T A p_j = 0 \ \forall i \neq j$

y $A$ es sim. pos. Def. $\Rightarrow$ los $p_i$ son LI

$\rightarrow$ Supongamos que no son LI y que $A \in M^{n \times n}$

Spg.

$$\Rightarrow \quad p_n = \alpha_1 p_1 + \alpha_2 p_2 + \dots + \alpha_{n-1} p_{n-1} \quad \text{con } \alpha_i \text{ escalares}$$

Como $A$ es sim. pos def. se cumple que $p_i^T A p_i > 0$, ~~sea~~

usando $p_n$ tenemos

$$p_n^T A p_n = p_n^T A (\alpha_1 p_1 + \dots + \alpha_{n-1} p_{n-1}) > 0$$

$$\Leftrightarrow \quad \underbrace{\alpha_1 p_n^T A p_1}_{0} + \underbrace{\alpha_2 p_n^T A p_2}_{0} + \dots + \underbrace{\alpha_{n-1} p_n^T A p_{n-1}}_{0} > 0 \quad \rightarrow \text{por Hip.}$$

$$\Rightarrow 0 > 0 \quad \nabla$$

$\therefore \ \{p_1, \dots, p_n\}$ forman un conj. LI ▲

2) Como el método usa la sucesión $x_{k+1} = x_k + \alpha_k p_k$

$\Rightarrow$ todos los $p_i$ (que son LI) ~~son las direcciones de descenso~~

$\Rightarrow$ al ir iterando vemos que solo $\exists$ $n$ posibles pasos pues hay $n$ $p_i$ y estos son LI.