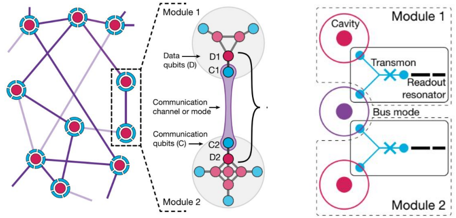
Teleported Controlled-NOT Gate Between GKP Qubits

Francisco J. Estrella The University of Texas at Austin, Department of Physics
Prof. Shyam Shankar The University of Texas at Austin, Department of Electrical and Computer Engineering

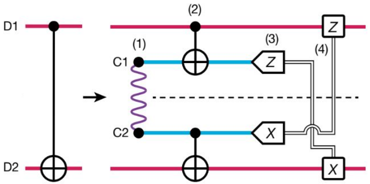
Abstract: In quantum computing, the controlled-NOT (CNOT) gate is essential to create entanglement. An architecture that implements the CNOT via a quantum gate teleportation protocol has been demonstrated experimentally with 79% fidelity[1]. In this experiment, qubits were encoded with discrete variables by using the number states of quantum harmonic oscillators. Would the same experiment work with a continuous variable qubit encoding? In particular, how about using Gottesman-Kitaev-Preskill (GKP) qubit states? These states have acquired great interest due to their capability for identifying and correcting errors[2]. The quest is then to simulate a teleported CNOT gate between GKP qubits to determine the feasibility of doing such an experiment.

Background:

1. The following architecture was used to demonstrate an effective CNOT gate experimentally[1]



2. The following figure shows the CNOT gate and its teleported form implemented in the architecture[1]



(1) Entanglement between C1 and C2
(2) Local CNOT gates
(3) Pauli Z and Pauli X measurements
(4) Feedforward operations

3. The following fidelity results were obtained when using the corresponding qubit encodings

79% +- 2% $|0_L\rangle = |2\rangle$ $|1_L\rangle = \frac{|0\rangle + |4\rangle}{\sqrt{2}}$

86% +- 2% $|0_L\rangle = |0\rangle$ $|1_L\rangle = |1\rangle$

4. Here, qubits were encoded using the number states of a quantum harmonic oscillator (a discrete variable):

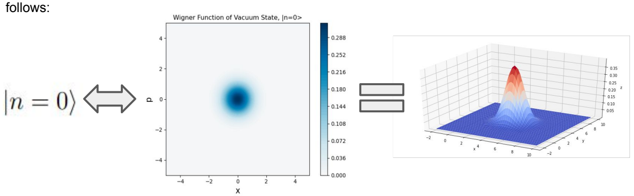
$$\{|0\rangle, |1\rangle, |2\rangle, \ldots, |n\rangle, \ldots\}$$

Research Question:

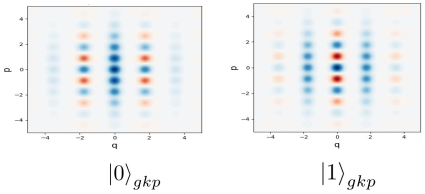
Would the same experiment work if qubits are encoded in continuous variables?

Methods:

1. Physical states of a quantum harmonic oscillator can also be represented with continuous variables using a *phase space* representation. For example, the *vacuum state* is described as follows:

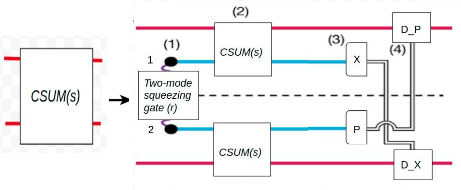


2. In this formalism, qubits can be encoded using *Gottesman-Kitaev-Preskill (GKP) states*. These look as patterns of Gaussian functions with positive and negative weights. The shape of the pattern is determined by the parameter ε[3]



For ε = 0.125

3. The version of a CNOT gate in continuous variables is known as the controlled-SUM (CSUM) gate and depends on a parameter s. The operations in the teleported form of the CSUM gate also depend on parameters as shown in the following figure[4]



(1) Entanglement between modes 1 and 2
(2) Local CSUM gates
(3) *X* and *P* measurements on modes 1 and 2
(4) Conditional displacements in *X* and *P*

4. The closeness between the two circuits can be quantified using a fidelity operation

$$F(\rho,\sigma) = \left(tr\sqrt{\sqrt{\rho}\sigma\sqrt{\rho}}\right)^2, 0 \leq F(\rho,\sigma) \leq 1$$

-ρ and σ represent the output states after the CSUM and teleported CSUM gates
-A value in the fidelity closer to 1 means that the gates approach each other

How does TACC help?

1. The fidelity operation is parametrized by *fixed* and *varying* parameters

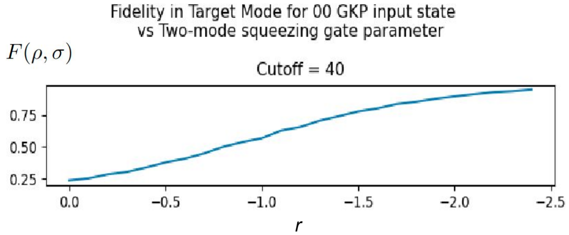
Fixed parameters	Cutoff to represent output states	CSUM gate parameter s	Input state	Number of circuit runs in each choice of parameters
Varying parameters	GKP states parameter ε	Two-mode squeezing gate parameter *r*	and more (not introduced here)	

2. A simulation with cutoff=40, s=1, a choice of input state, and a particular choice of the varying parameters takes ~ 3.5 minutes
-And this is only for 1 circuit run!

3. Testing the circuit sequentially with a range of varying parameter *r* of -2.4 ≤ *r* ≤ 0 (in steps of 0.1, so 25 parameters in total) while keeping all other parameters fixed and doing 500 runs in each choice of parameters takes an estimated time of
3.5 minutes * 25 parameters * 500 runs ~ 30 days.
-And we still need to test with more varying parameters like ε!!

4. Parallel computation allows us to distribute the circuit runs among several processors. For example, using 48 cores; the simulation time reduces to
500 runs/48 cores * 3.5minutes * 25 parameters ~ 15 hrs

5. With this method, it becomes possible to look at the fidelity as a function of the varying parameters as in the following figure:



References:

1. Chou, K.S., Blumoff, J.Z., Wang, C.S. et al. Deterministic teleportation of a quantum gate between two logical qubits. Nature 561, 368–373 (2018). https://doi.org/10.1038/s41586-018-0470-y
2. Campagne-Ibarcq, P., Eickbusch, A., Touzard, S. *et al*. Quantum error correction of a qubit encoded in grid states of an oscillator. *Nature* 584, 368–372 (2020). https://doi.org/10.1038/s41586-020-2603-3
3. Ilan Tzitrin, J. Eli Bourassa, Nicolas C. Menicucci, Krishna Kumar Sabapathy. Progress towards practical qubit computation using approximate Gottesman-Kitaev-Preskill codes. *Physical Review A* 101 (2020): 032315.
4. Filip, Radim. "Continuous-variable quantum nondemolishing interaction at a distance." *Physical Review A* 69 (2004): 052313.