Teleported CNOT Gate between GKP Qubit States

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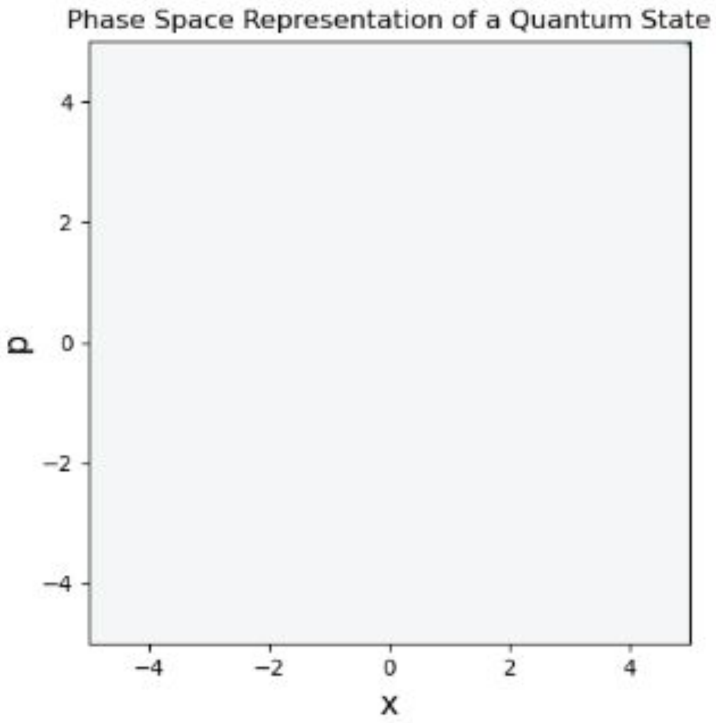
Quantum Harmonic Oscillator

$$\hat{H} = \frac{\hat{p}^2}{2m} + \frac{1}{2}m\omega^2\hat{x}^2$$
$$\hat{H} = \hbar\omega(\hat{a}^\dagger\hat{a} + \frac{1}{2}),$$

$$\hat{a}^\dagger |n\rangle = \sqrt{n+1}\,|n+1\rangle$$
$$\hat{a}\,|n\rangle = \sqrt{n}\,|n-1\rangle\,.$$

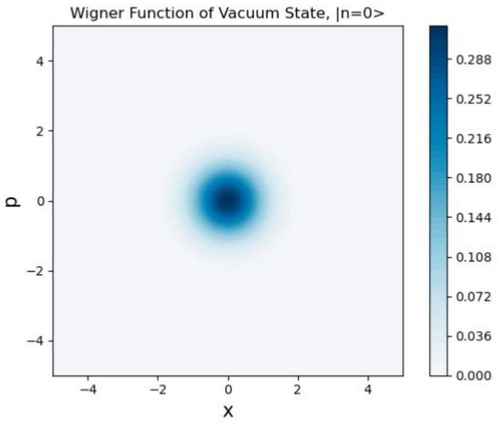
$$\hat{x} = \sqrt{\frac{\hbar}{2m\omega}}(\hat{a}^\dagger + \hat{a})$$
$$\hat{p} = i\sqrt{\frac{\hbar m\omega}{2}}(\hat{a}^\dagger - \hat{a}).$$

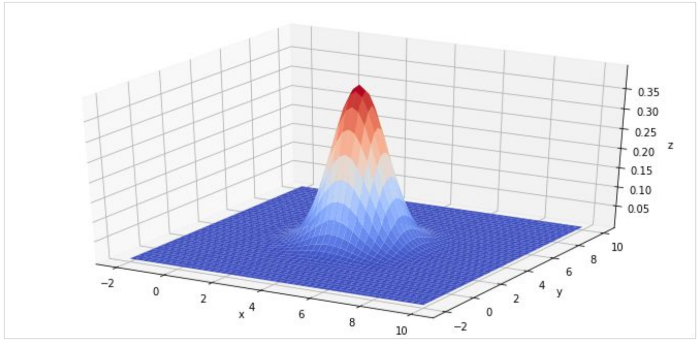


Continuous Variable Quantum States - Gaussian States

- There is an alternative representation of a quantum state (from a wave function or a density operator) as a quasi-probability distribution in phase space; known as a Wigner function.
- It is possible to obtain real distributions of the quadrature operators from the Wigner function.

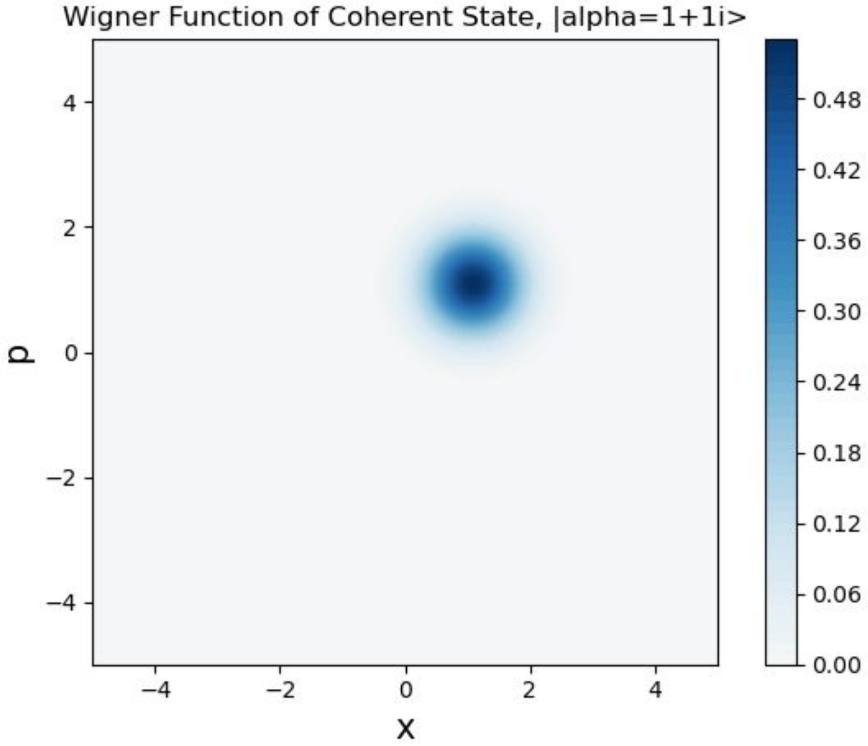
|n=0>





More Gaussian States - Coherent States

$$\hat{a}\,|\alpha\rangle = \alpha\,|\alpha\rangle$$



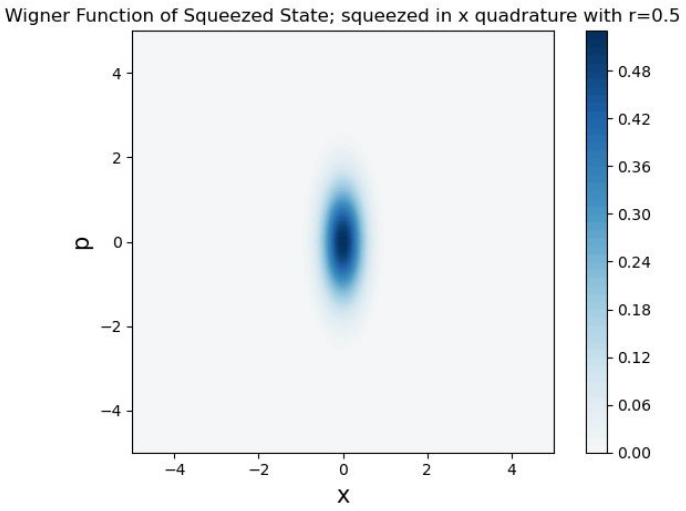
Quantum information processing with continuous variables

- Displacement Gate is specified by a complex parameter alpha.
  - This gate modifies the vector of means but not the covariance matrix.

$$D^\dagger(\alpha)\hat{x}D(\alpha) = \hat{x} + \sqrt{2\hbar}\mathrm{Re}(\alpha)\hat{\mathbf{1}},$$
$$D^\dagger(\alpha)\hat{p}D(\alpha) = \hat{p} + \sqrt{2\hbar}\mathrm{Im}(\alpha)\hat{\mathbf{1}}.$$

- Squeezing Gate
  - Does not change the vector of means but rather the covariance matrix.

$$S^\dagger(z)\hat{x}_\phi S(z) = e^{-r}\hat{x}_\phi, \quad S^\dagger(z)\hat{p}_\phi S(z) = e^{r}\hat{p}_\phi$$



Two-mode squeezing gate

- Does not squeeze both modes, rather it entangles them.

$$\hat{X}_a(t) \pm \hat{X}_b(t) = [\hat{X}_a(0) \pm \hat{X}_b(0)]e^{\pm r};$$
$$\hat{P}_a(t) \pm \hat{P}_b(t) = [\hat{P}_a(0) \pm \hat{P}_b(0)]e^{\mp r}.$$

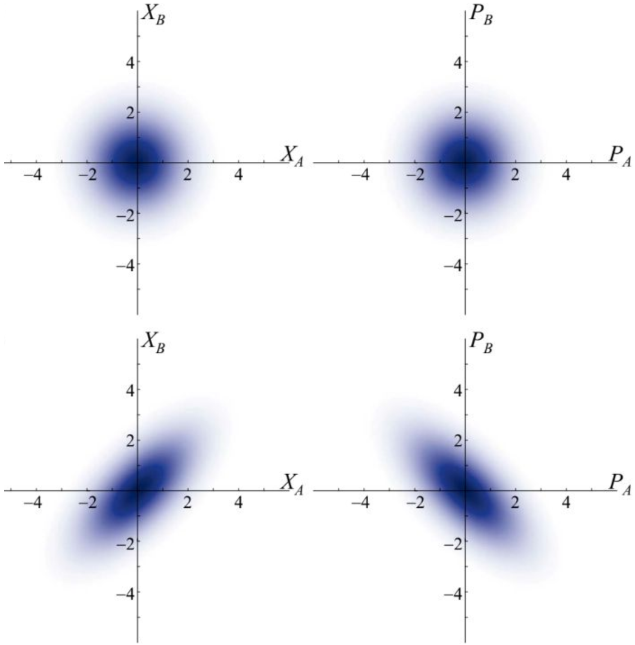


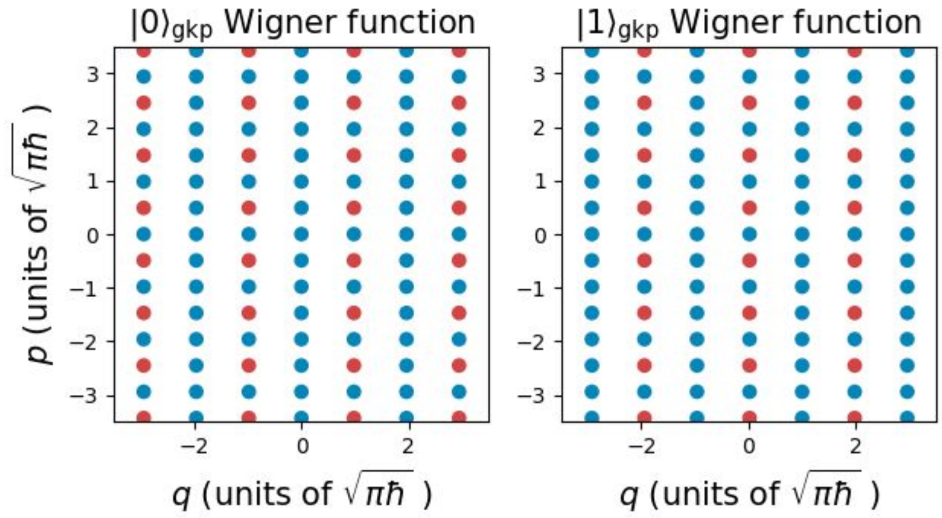
Image taken from “Squeezed Light”

Homodyne Measurements

- Used to measure the quadrature of a mode (position or momentum).

Non-Gaussian States - GKP States

- Non-Gaussian States are those whose Wigner function is not a Gaussian (but could be a linear combination of Gaussians).
- Ideal GKP (Gottesman-Kitaev-Preskill) States can be thought of a infinite lattices of delta functions in phase space.



$$|\psi\rangle_{gkp} = \cos\frac{\theta}{2}|0\rangle_{gkp} + e^{-i\phi}\sin\frac{\theta}{2}|1\rangle_{gkp}$$

Image taken from: Strawberry Fields Tutorial “Studying realistic bosonic qubits using the bosonic backend”

Finite Energy GKP States

- Finite Energy GKP States are obtained by applying a damping operator with parameter ε to ideal GKP states.

$$|\psi_\epsilon\rangle \equiv E(\epsilon)\,|\psi_I\rangle \qquad E(\epsilon) \equiv e^{-\epsilon\hat{n}}$$

- They acquire two widths: one of them specifies the variance in the individual peaks (σ), while the other specifies the variance of the distribution containing the full lattice (Δ).

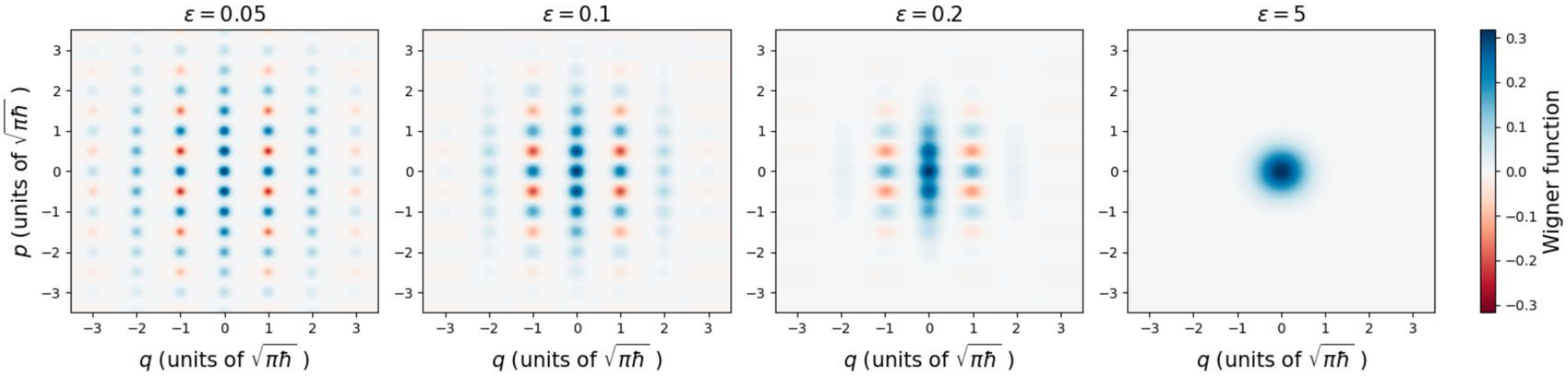


Image taken from: Strawberry Fields Tutorial “Studying realistic bosonic qubits using the bosonic backend”

Quantum Error Correction with GKP States

- GKP States can be implemented with quantum error correction.
  - A prerequisite for fault-tolerant quantum computing.
  - Demonstrated in “Quantum error correction of a qubit encoded in grid states of an oscillator”.

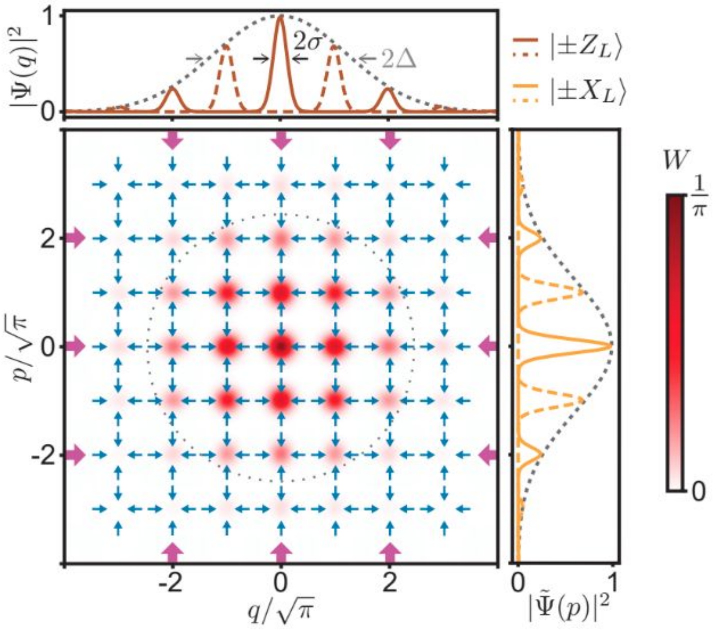


Image taken from:
“Quantum error correction of a qubit encoded in grid states of an oscillator”

Universal Quantum Computing with Continuous Variables?

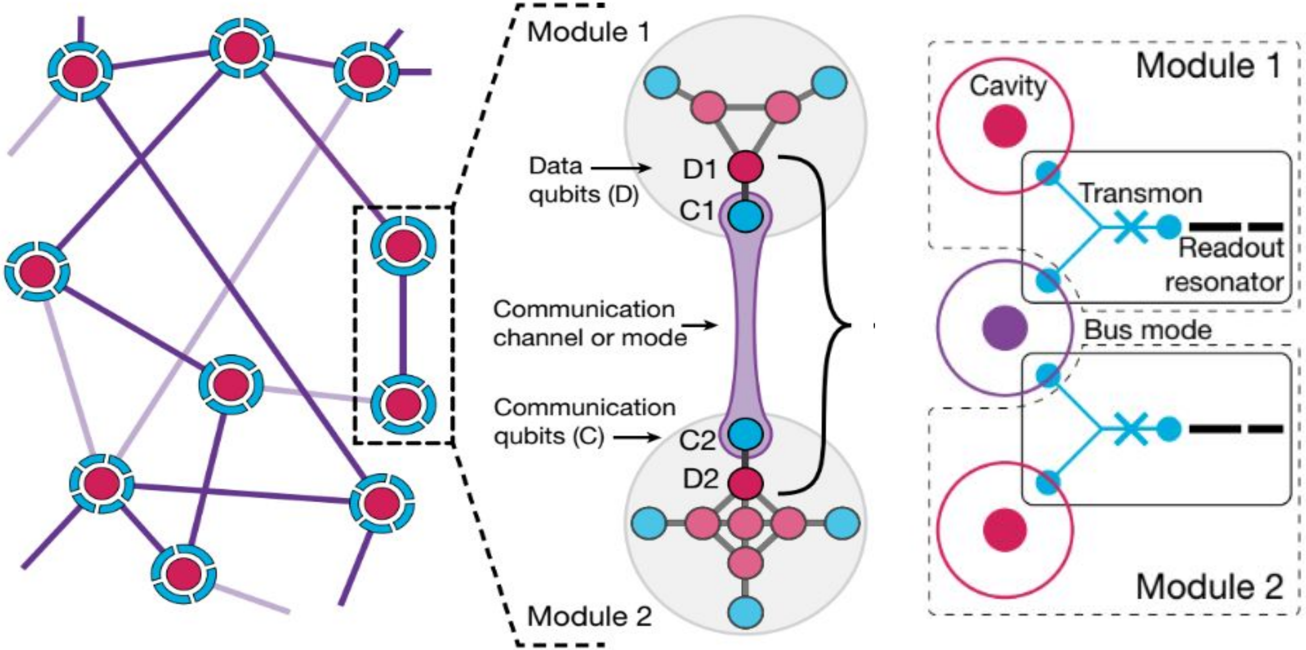
- It has been demonstrated that universal quantum computation is available with continuous variables.
    - Shown in
- Experimentally, it is complicated two apply two-qubit gates in non-neighboring components.
- Solution?
    - Quantum Gate Teleportation

Quantum Gate Teleportation	Quantum State Teleportation
Requires Shared Entanglement	Requires Shared Entanglement
Requires Classical Communication	Requires Classical Communication
Implements an effective gate between two distant locations	Send a quantum state from one location to another

Pause and Review

- So far we have considered the theoretical description of continuous variable quantum computing.
- Questioned about universal quantum computing and pointed out an important experimental issue.
- We will pause on continuous variables for a second and revisit a recent experiment where a teleported gate was implemented at a distance.
- Later, we will link this experiment with continuous variable quantum computing.

Teleported CNOT Gate Experiment - Architecture



All images taken from: “Deterministic teleportation of a quantum gate between two logical qubits”

Teleported CNOT Gate

- First time that this circuit was demonstrated deterministically.
- The encodings used in the data qubits consisted of the two lowest Fock levels and the binomial code defined by

$$|0_L\rangle = |2\rangle,\ |1_L\rangle = \frac{|0\rangle + |4\rangle}{\sqrt{2}}$$

- The teleported gate was achieved with 79% fidelity.

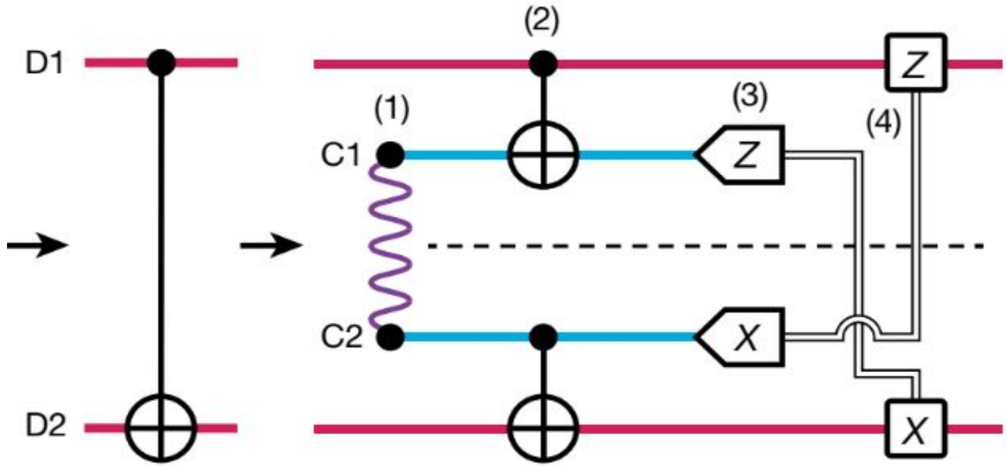


Image taken from:
“Deterministic teleportation of a quantum gate between two logical qubits”

Can a continuous variable encoding be used in the data qubits?

- In particular could we encode the qubit states using GKP states?
- First we need a continuous variable version of a CNOT gate.
  - The answer is the CX (Controlled-X) gate (also known addition gate, sum gate, or QND gate).

$$
\begin{aligned}
\mathrm{CX}(s)^{\dagger}\hat{x}_1\mathrm{CX}(s) &= \hat{x}_1 \\
\mathrm{CX}(s)^{\dagger}\hat{p}_1\mathrm{CX}(s) &= \hat{p}_1 - s\,\hat{p}_2 \\
\mathrm{CX}(s)^{\dagger}\hat{x}_2\mathrm{CX}(s) &= \hat{x}_2 + s\,\hat{x}_1 \\
\mathrm{CX}(s)^{\dagger}\hat{p}_2\mathrm{CX}(s) &= \hat{p}_2
\end{aligned}
$$

$$\mathrm{CX}(s)|x_1, x_2\rangle_x = |x_1, x_2 + sx_1\rangle_x.$$

- Is there a teleported version of this gate?

Teleported CX (or QND) Gate

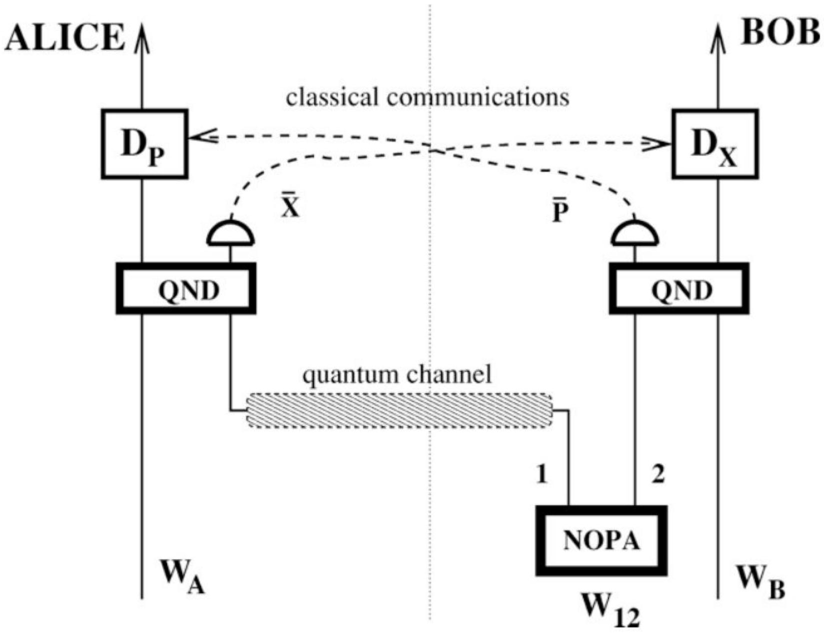


Image taken from: “Continuous-variable quantum nondemolishing interaction at a distance”

- G is the parameter of the two local QND gates.

$$\boxed{X'_A = X_A, \quad P'_A = P_A - G^2 P_B} - G(P_1 - P_2)$$

$$\boxed{X'_B = X_B + G^2 X_A} + G(X_1 + X_2), \quad P'_B = P_B$$

$$\mathrm{CX}(s)^\dagger \hat{x}_1 \mathrm{CX}(s) = \hat{x}_1$$
$$\mathrm{CX}(s)^\dagger \hat{p}_1 \mathrm{CX}(s) = \hat{p}_1 - s\,\hat{p}_2$$
$$\mathrm{CX}(s)^\dagger \hat{x}_2 \mathrm{CX}(s) = \hat{x}_2 + s\,\hat{x}_1$$
$$\mathrm{CX}(s)^\dagger \hat{p}_2 \mathrm{CX}(s) = \hat{p}_2$$

- This circuit approaches an ideal QND gate with parameter G^2.

Entanglement and Low Variances

- Entanglement can be imposed using a two-mode squeezing gate.

$$\hat{X}_a(t) \pm \hat{X}_b(t) = [\hat{X}_a(0) \pm \hat{X}_b(0)]e^{\pm r};$$
$$\hat{P}_a(t) \pm \hat{P}_b(t) = [\hat{P}_a(0) \pm \hat{P}_b(0)]e^{\mp r}.$$

- It will then be a matter of finding the two-mode squeezing gate's parameter that yields a low variance in these variables.

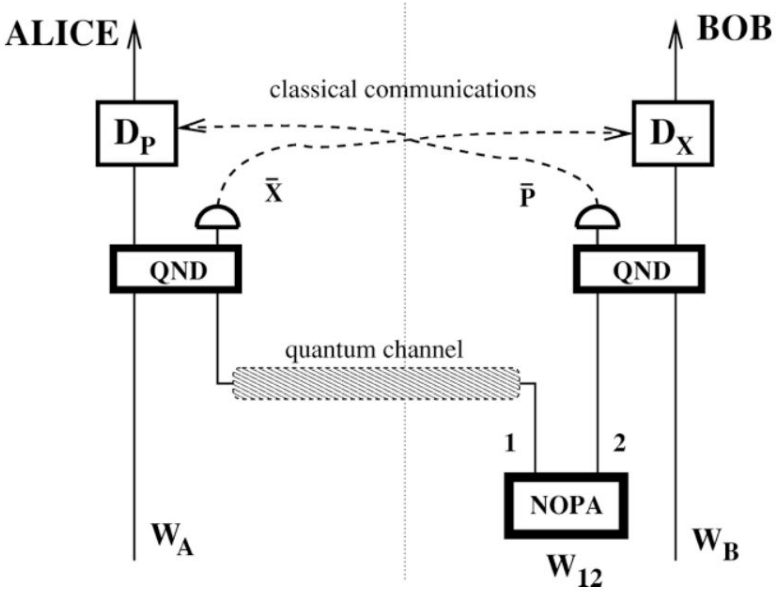
Pause and Review

- We are comparing the ideal CX (or QND) circuit with its teleported form.
- We are only considering the case in which the teleported circuit approaches the ideal QND gate with a parameter of s=1.
- What tool can be used to compare the circuits? Fidelity.

$$\mathrm{CX}(s)|x_1, x_2\rangle_x = |x_1, x_2 + sx_1\rangle_x.$$

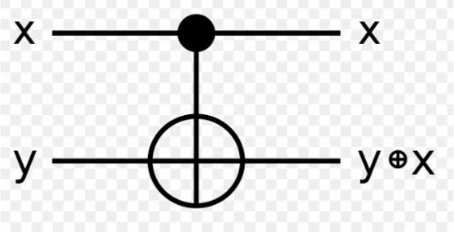
$$\mathrm{CX}(s)^\dagger \hat{x}_1 \mathrm{CX}(s) = \hat{x}_1$$
$$\mathrm{CX}(s)^\dagger \hat{p}_1 \mathrm{CX}(s) = \hat{p}_1 - s\,\hat{p}_2$$
$$\mathrm{CX}(s)^\dagger \hat{x}_2 \mathrm{CX}(s) = \hat{x}_2 + s\,\hat{x}_1$$
$$\mathrm{CX}(s)^\dagger \hat{p}_2 \mathrm{CX}(s) = \hat{p}_2$$

= ?



Fidelity Between Both Circuits

- The objective is to plot fidelity as a function of the two-mode squeezing parameter for all input states of a CNOT truth table using GKP qubit states.
  - This will yield the maximum value of the two-mode squeezing parameter for which the teleported circuit becomes an effective CX gate.



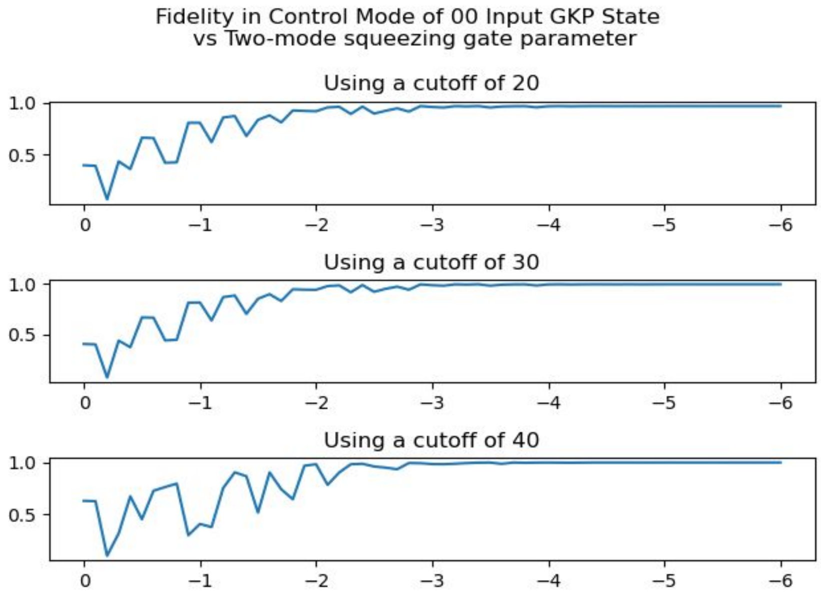
input		output	
x	y	x	y+x
$\|0\rangle$	$\|0\rangle$	$\|0\rangle$	$\|0\rangle$
$\|0\rangle$	$\|1\rangle$	$\|0\rangle$	$\|1\rangle$
$\|1\rangle$	$\|0\rangle$	$\|1\rangle$	$\|1\rangle$
$\|1\rangle$	$\|1\rangle$	$\|1\rangle$	$\|0\rangle$

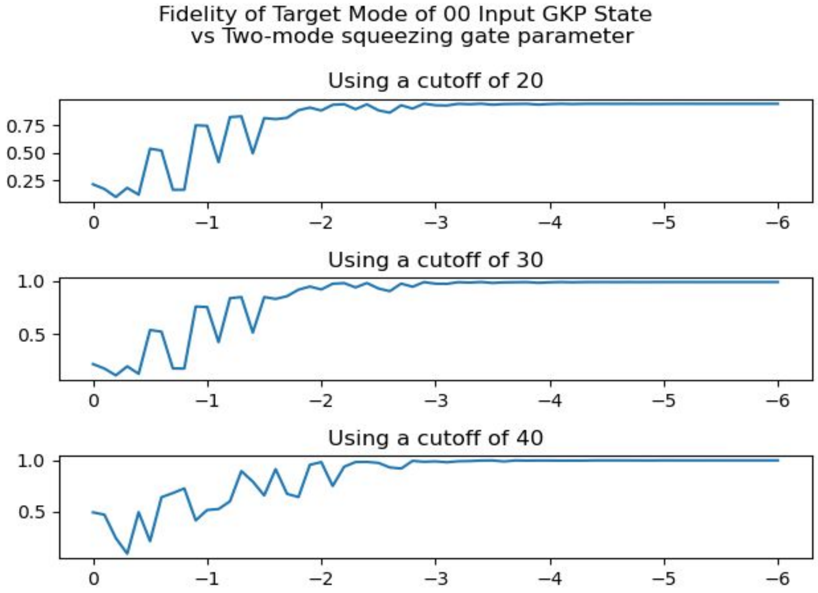
Image taken from: Wikipedia CNOT Gate Article

- Fidelity is a quantity that tells how close a quantum state is to another.

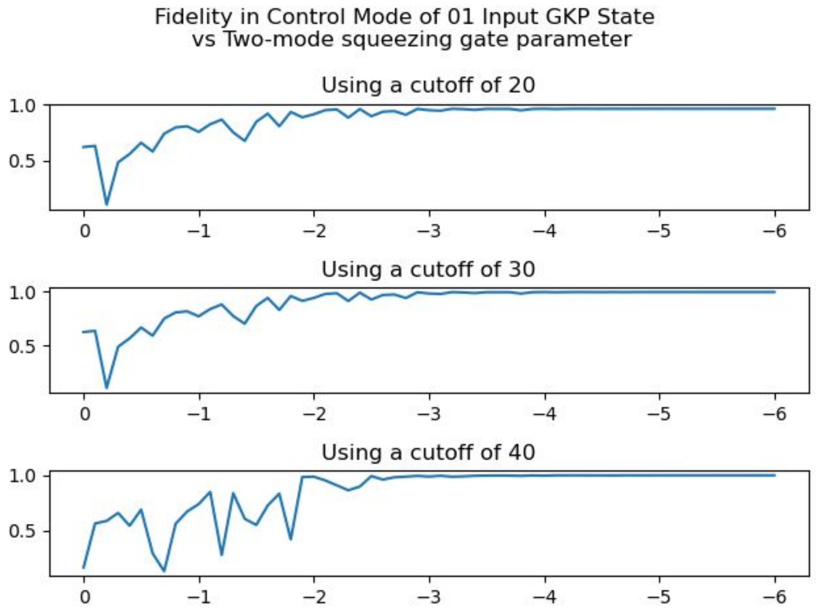
$$F(\rho,\sigma) = \left(\operatorname{tr}\sqrt{\sqrt{\rho}\,\sigma\sqrt{\rho}}\right)^2.$$

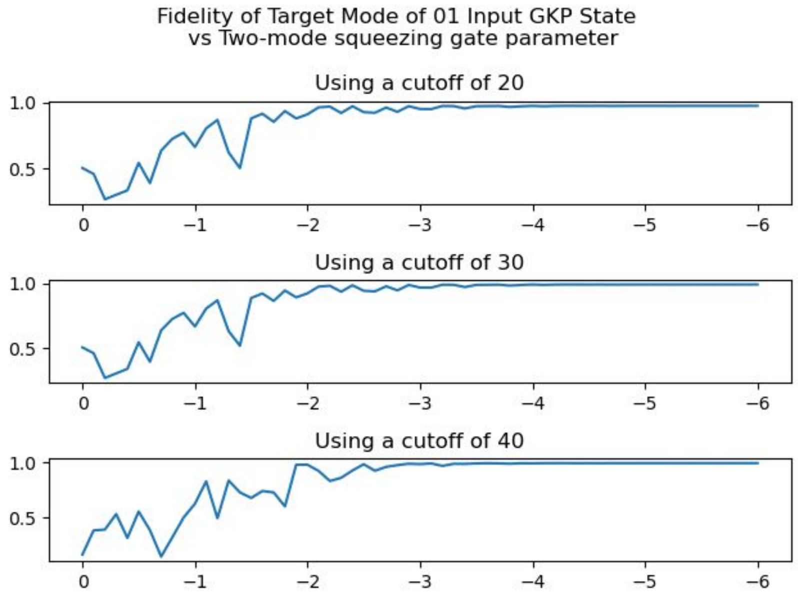
Fidelity 00 Input State





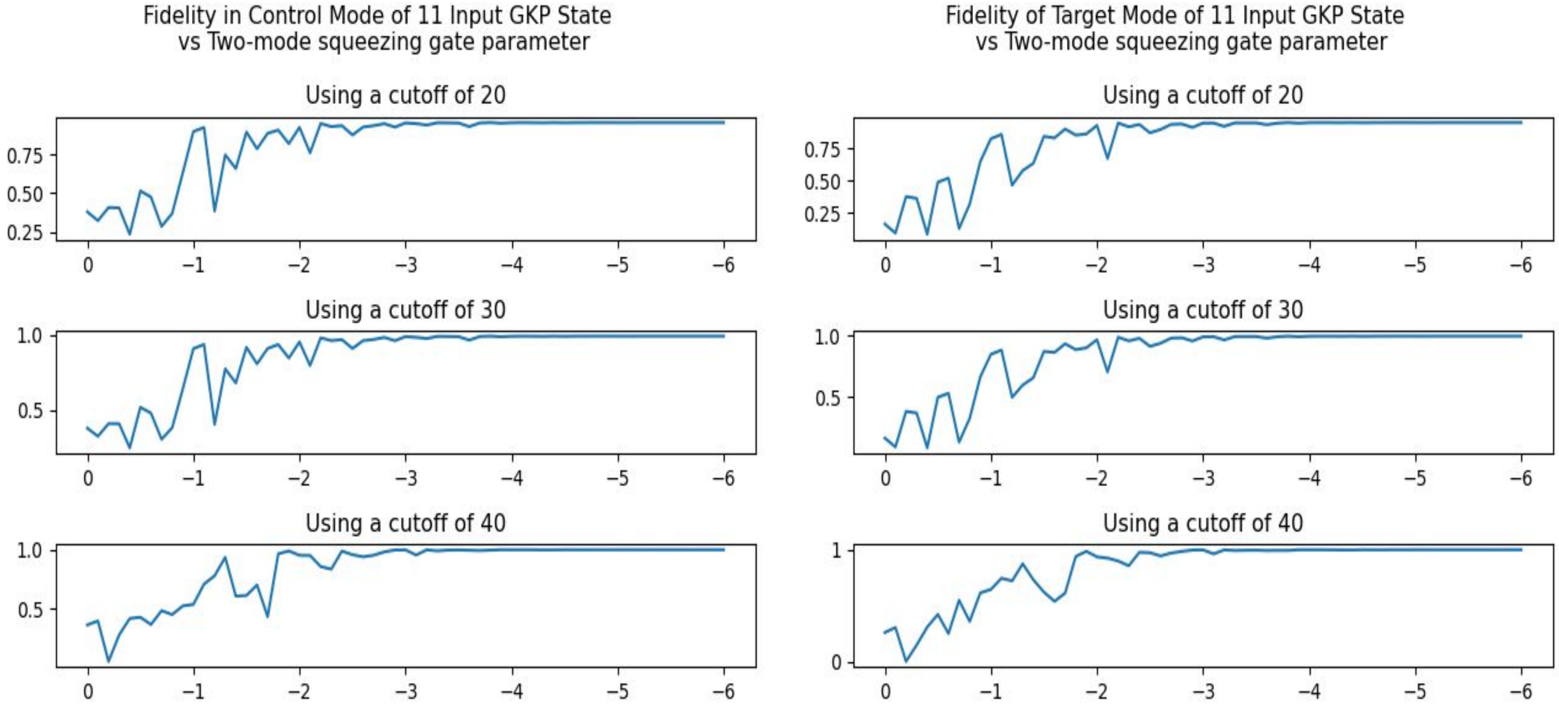
Fidelity 01 Input State





Fidelity 10 Input State

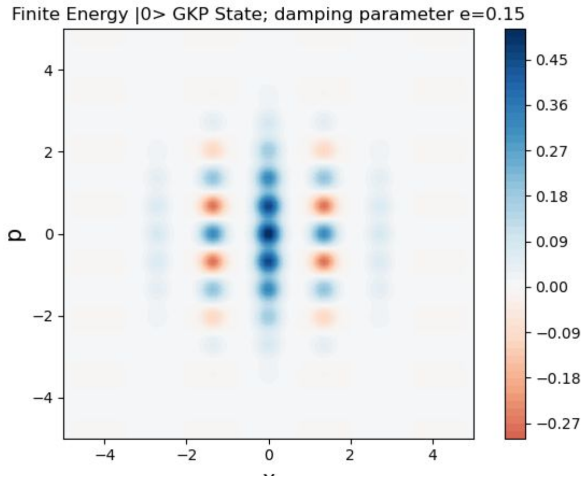
Fidelity 11 Input State

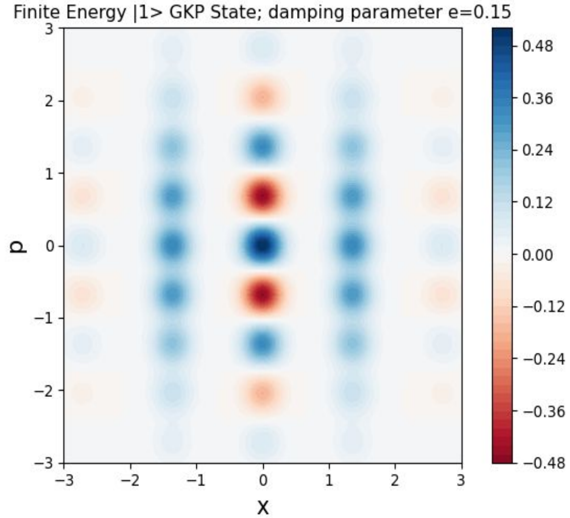


Verifying the CNOT Truth Table

- Even if the circuits became 99% close in terms of fidelity, how can we guarantee that we have created a CNOT gate?
    - By inspecting the output Wigner functions.

input		output	
x	y	x	y+x
$\|0\rangle$	$\|0\rangle$	$\|0\rangle$	$\|0\rangle$
$\|0\rangle$	$\|1\rangle$	$\|0\rangle$	$\|1\rangle$
$\|1\rangle$	$\|0\rangle$	$\|1\rangle$	$\|1\rangle$
$\|1\rangle$	$\|1\rangle$	$\|1\rangle$	$\|0\rangle$





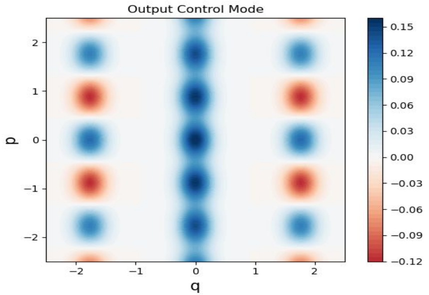
Verifying the CNOT Truth Table

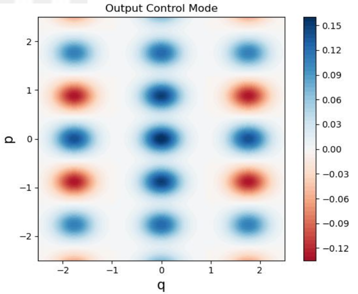
- In the following, input GKP states with ε=0.15 were used.
- The Wigner functions were extracted from the CX circuit.

input

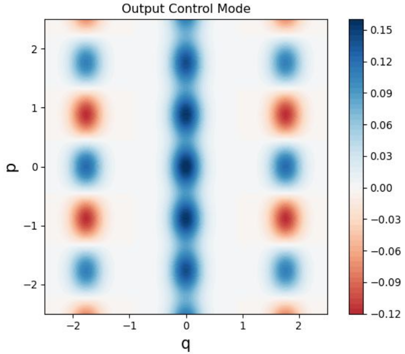
output

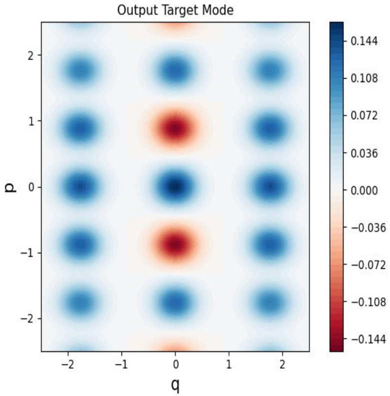
$|0\rangle \ |0\rangle$





$|0\rangle \ |1\rangle$



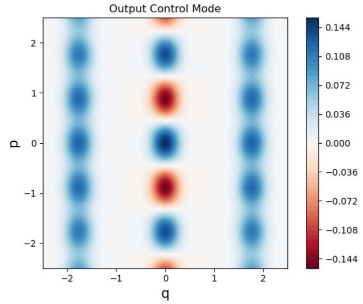


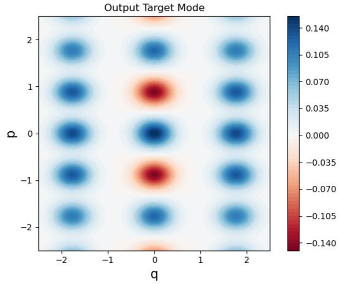
Verifying the CNOT Truth Table

input

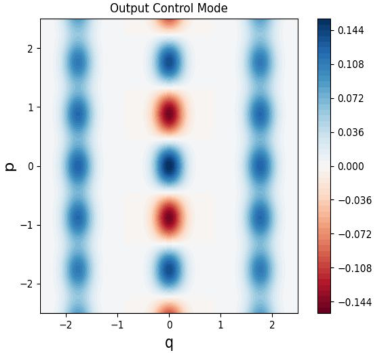
output

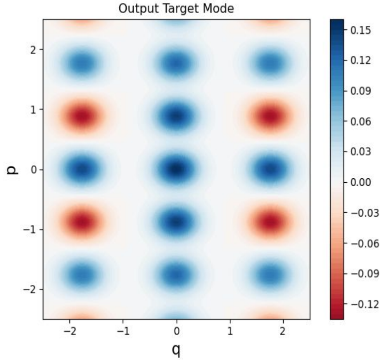
$|1\rangle \ |0\rangle$





$|1\rangle \ |1\rangle$

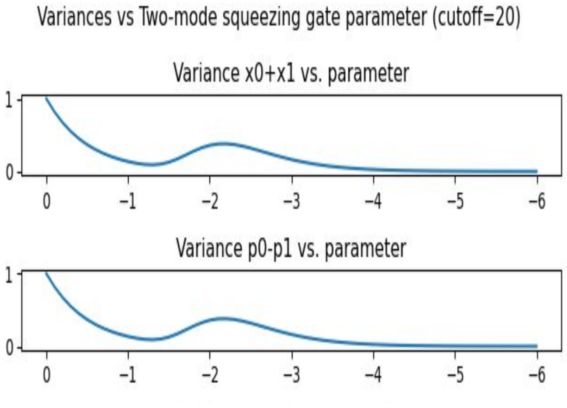




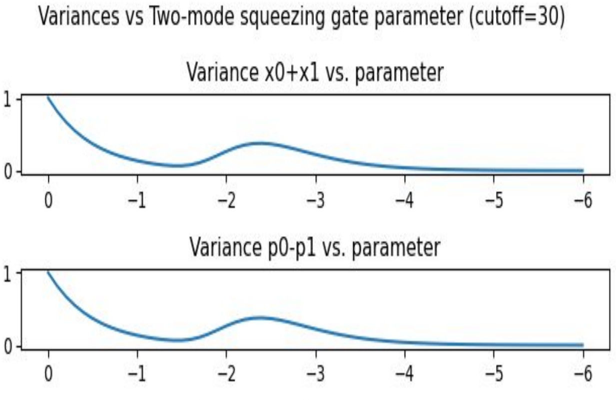
Variances as function of two-mode squeezing parameter

- This simulation had to be carried out in the Fock backend of the Strawberry Fields library from Xanadu.
- Plotted the variances of X1+X2 and P1-P2 as functions of the two-mode squeezing parameter for different choices of the cutoff (largest number state in Fock representation).
- Even with large cutoffs to represent the quantum states, the programs would not yield valuable information. They would hit an inflection point eventually, which should not occur.
- All that could be guessed is that the parameter has to be at most ~ -2.0.

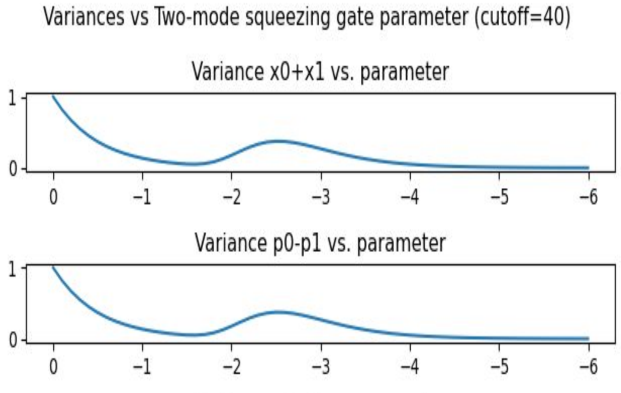
Variance vs Two-Mode Squeezing Gate Parameter



r (two-mode squeezing parameter)



r (two-mode squeezing parameter)



r (two-mode squeezing parameter)

Further Directions

- Understand issue in variance plots.
- Can incorporate common errors in the circuit.
  - Amplitude Damping
  - Phase Damping
  - Measurement Errors
- Only considered the case where the teleported circuit approaches an effective CX gate with parameter $s=1$.
- Only a particular case of GKP states (defined by a damping parameter of $\varepsilon=0.15$) were considered.

Summary

- Continuous Variable Quantum Computation was introduced.
  - This led to the introduction of non-gaussian GKP states, which are convenient to use for quantum error correction.
- An experiment that implemented a CNOT gate at a distance was introduced.
  - The question of implementing the same experiment but with an encoding based on GKP states was raised.
  - The continuous variable teleported version of the circuit was described.
- Simulations where made to test the experimental viability of this circuit in which the control and target modes are GKP states.
  - The input states where chosen such that the CNOT truth table can also be verified.
- Further directions were discussed.

References

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