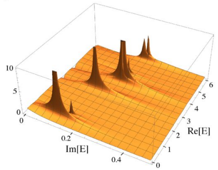
Entanglement in composite normal & super - conducting nanowires

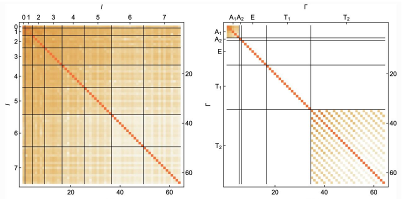
By Francisco Estrella

Some of my research life

- Estrella, F. Barr, A. Reichl, L.: “Quasibound States of Tetrahedral Quantum Structures.” Physical Review A 101.022706 (February 2020)

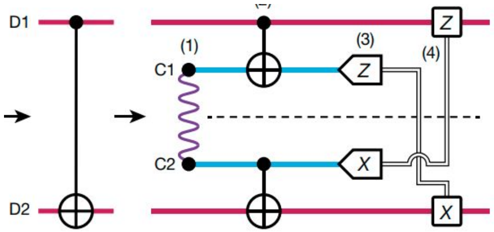


- Estrella, F. Barr, A. Reichl, L. Furman, A.: “The scattering symmetries of tetrahedral quantum structures.” European Physical Journal D 76: 83 (May 2022)



- Simulation of a teleported CNOT gate with

  GKP (Gottesman-Kitaev-Preskill) qubits



Outline

- System
  - Wave function
- Scattering theory
  - S matrix
  - Transfer matrix method
- Current and cross-correlated shot noise
  - Surprising results
  - Relation to entanglement
- Explanation via quasi-bound states
- Current work
  - Selecting relevant scattering amplitudes and regions of complex energy plane
- Summary

Composite normal & super - conducting nanowires

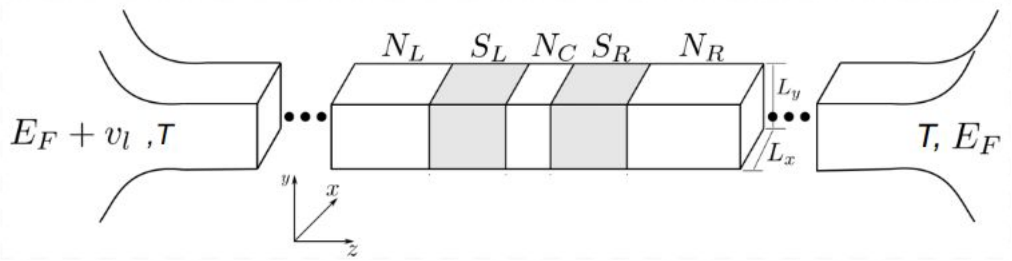


Figure from:
“Positive cross-correlated shot noise and quasibound states in an NSNSN geometry”.
L. Reichl, C. Ostrove. Physical Review B (2021)

- Assumptions
    - Nanowire is much longer than it is wide
    - Each superconducting segment is much longer than it is wide
    - Cross section is the same for each segment
    - Difference between Fermi energies is small
    - For simplicity, walls are taken as “infinite hard walls”
    - No impurities are present that can couple different transverse modes

Superconductors “LSCO” and “BSCCO”

Need:

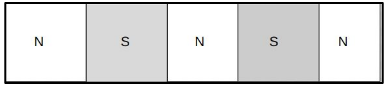
$L_x, L_y > \xi$

$T < T_c$

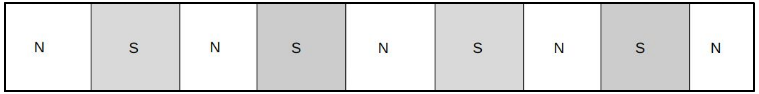
Compound:	$La_{2-x}Sr_xCu_4(LSCO)$	$Bi_2Sr_2CaCu_2O_{8+x}(BSCCO)$
$\Delta(E_H)$:	2.2×10^{-4}	5.3814×10^{-4}
$E_F(E_H)$:	9.35×10^{-4}	1.307×10^{-3}
$\xi(a_B)$:	65	30.24
$T_c(K)$:	38	96

Table from:
“Physical properties of high-temperature superconductors.”.
Wesche, R. John Wiley and Sons. (2021)

- Will mainly show results for BSCCO in NSNSN



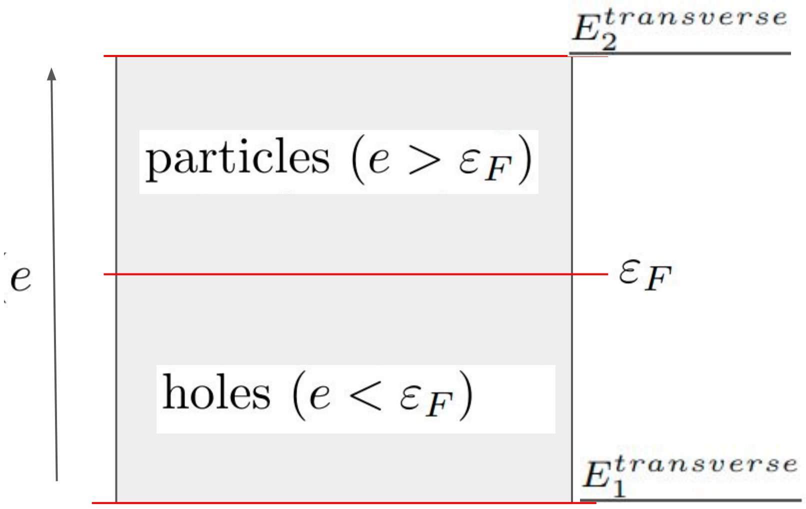
- Currently studying BSCCO in NSNSNSNSN



“Particles” and “holes”

- Propagating charge carriers have energies around the Fermi energy

$$\boxed{\varepsilon_F = \frac{E_2^{transverse} - E_1^{transverse}}{2}}$$



$$\delta E = E_2^{transverse} - \varepsilon_F = \varepsilon_F - E_1^{transverse}$$

Fermi-Dirac distribution in the thermal reservoirs

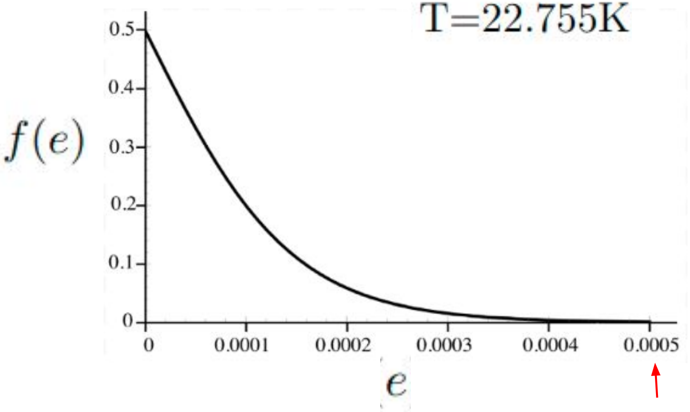
For $L_x = 144a_B$ and $L_y = 120a_B$ in BSCCO:

$\delta E = 0.0005E_H$

Atomic units

$1a_B = .052917\text{nm}$

$1E_H = 27.716eV = 4.3 \times 10^{-18}J$



$f(\delta E) \approx 0.001$

$$f(e) = \frac{1}{e^{(e-\varepsilon_F)/k_B T}}$$

Wave function in normal conducting segments

$$\psi_{normal}(z) = \frac{A_p}{\sqrt{q_p}}\begin{bmatrix}1\\0\end{bmatrix}e^{iq_p z} + \frac{B_h}{\sqrt{q_h}}\begin{bmatrix}0\\1\end{bmatrix}e^{-iq_h z} + \frac{C_p}{\sqrt{q_p}}\begin{bmatrix}1\\0\end{bmatrix}e^{-iq_p z} + \frac{D_h}{\sqrt{q_h}}\begin{bmatrix}0\\1\end{bmatrix}e^{iq_h z}$$

Particle states

A_p

C_p

Hole states

B_h

D_h

$$q_p = \sqrt{\frac{2m_N^*}{\hbar^2}(\varepsilon_F + e)} \qquad 0 \le e \le \varepsilon_F$$

$$q_h = \sqrt{\frac{2m_N^*}{\hbar^2}(\varepsilon_F - e)} \qquad 0 \le e \le \varepsilon_F$$

m_N^* = effective mass in normal segment
$\hbar$ = reduced Planck's constant
ε_F = Fermi energy

Bogoliubov-de-Gennes (BdG) Theory

$$\boxed{\begin{pmatrix} \frac{\hbar^2 k^2}{2m_s} - \varepsilon_F & \Delta \\ \Delta & -\frac{\hbar^2 k^2}{2m_s} + \varepsilon_F \end{pmatrix} \begin{pmatrix} u(e) \\ v(e) \end{pmatrix} = e \begin{pmatrix} u(e) \\ v(e) \end{pmatrix}}$$

particles $(e > \varepsilon_F)$

$$e = \sqrt{\left(\frac{\hbar^2 k^2}{2m_s} - \varepsilon_F\right)^2 + \Delta^2}$$

$$k_p = \sqrt{\frac{2m_s}{\hbar^2}} \sqrt{\varepsilon_F + \sqrt{e^2 - \Delta^2}} \quad \text{and} \quad \begin{pmatrix} u_p(e) \\ v_p(e) \end{pmatrix} = \begin{pmatrix} u_o \\ v_o \end{pmatrix}$$

holes $(e < \varepsilon_F)$

$$e = -\sqrt{\left(\frac{\hbar^2 k^2}{2m_s} - \varepsilon_F\right)^2 + \Delta^2}$$

$$k_h = \sqrt{\frac{2m_s}{\hbar^2}} \sqrt{\varepsilon_F - \sqrt{e^2 - \Delta^2}} \quad \text{and} \quad \begin{pmatrix} u_h(e) \\ v_h(e) \end{pmatrix} = \begin{pmatrix} v_o \\ u_o \end{pmatrix}$$

$$u_o^2 = \frac{1}{2}\left(1 + \frac{\sqrt{e^2 - \Delta^2}}{e}\right) \quad \text{and} \quad v_o^2 = \frac{1}{2}\left(1 - \frac{\sqrt{e^2 - \Delta^2}}{e}\right).$$

Wave function in superconducting segments

$$\psi_{super}(z) = \frac{A_p^S}{\sqrt{k_p}} \begin{bmatrix} u_o \\ v_o \end{bmatrix} e^{ik_p z} + \frac{B_h^S}{\sqrt{k_h}} \begin{bmatrix} v_o \\ u_o \end{bmatrix} e^{-ik_h z} + \frac{C_p^S}{\sqrt{k_p}} \begin{bmatrix} u_o \\ v_o \end{bmatrix} e^{-ik_p z} + \frac{D_h^S}{\sqrt{k_h}} \begin{bmatrix} v_o \\ u_o \end{bmatrix} e^{ik_h z}$$

Particle states

A_p^S

C_p^S

Hole states

B_h^S

D_h^S

$$k_h = \sqrt{\frac{2m_s}{\hbar^2}} \sqrt{\varepsilon_F - \sqrt{e^2 - \Delta^2}} \qquad k_p = \sqrt{\frac{2m_s}{\hbar^2}} \sqrt{\varepsilon_F + \sqrt{e^2 - \Delta^2}}$$

m_S = effective mass in superconducting segment
$\hbar$ = reduced Planck's constant
ε_F = Fermi energy
Δ = superconducting gap

Scattering theory

- Scattering matrix (S matrix) relates "outgoing" states to "incoming" states

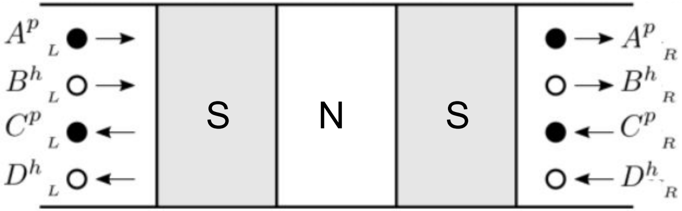
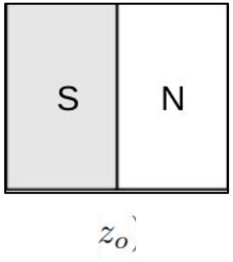


Figure from:
"Positive cross-correlated shot noise and quasibound states in an NSNSN geometry".
L. Reichl, C. Ostrove. Physical Review B (2021)

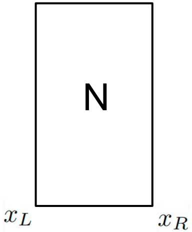
$$\begin{bmatrix} C_p^L \\ D_h^L \\ A_p^R \\ B_h^R \end{bmatrix} = \mathbf{S} \cdot \begin{bmatrix} A_p^L \\ B_h^L \\ C_p^R \\ D_h^R \end{bmatrix}$$

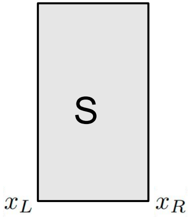
- No impurities => Wave function and its derivative are continuous at interfaces



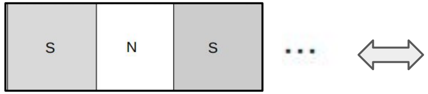
$$\psi_{normal}(z_o) = \psi_{super}(z_o)$$
$$\psi'_{normal}(z_o) = \psi'_{super}(z_o)$$

Transfer matrix method



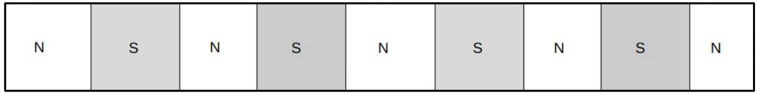


$$\psi_{normal}(x_R) = \mathbf{T}_N \cdot \psi_{normal}(x_L) \qquad \psi_{super}(x_R) = \mathbf{T}_S \cdot \psi_{super}(x_L)$$



$$\mathbf{S} \approx \mathbf{T}_S \cdot \mathbf{T}_N \cdot \mathbf{T}_S \ldots$$

- This allows us to study NSNSNSNSN!



The scattering matrix (S matrix)

$$\bar{\bar{S}} = \begin{pmatrix} r_{pp}^{LL} & r_{ph}^{LL} & t_{pp}^{LR} & t_{ph}^{LR} \\ r_{hp}^{LL} & r_{hh}^{LL} & t_{hp}^{LR} & t_{hh}^{LR} \\ t_{pp}^{RL} & t_{ph}^{RL} & r_{pp}^{RR} & r_{ph}^{RR} \\ t_{hp}^{RL} & t_{hh}^{RL} & r_{hp}^{RR} & r_{hh}^{RR} \end{pmatrix}$$

- Example: transmission of a particle in NSN

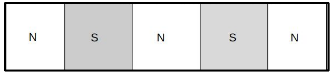
$$\begin{aligned} t_{pp}^{RL} &= \frac{4}{\mathrm{DEN}} (u_o^2 - v_o^2) \sqrt{q_p^L} \sqrt{q_p^R} \\ &\times \left(e^{iL_S k_p - iL_S q_p^R} k_h\, v_o^2 P_{ph}^L P_{ph}^R - e^{iL_S k_h - iL_S q_p^R} k_p\, u_o^2 P_{hh}^L P_{hh}^R \right. \\ &\left. + e^{-iL_S k_h - iL_S q_p^R} k_p\, u_o^2 M_{hh}^L M_{hh}^R - e^{-iL_S k_p - iL_S q_p^R} k_h\, v_o^2 M_{ph}^L M_{ph}^R \right) \end{aligned}$$

$$P_{y,z}^X = k_y + q_z^X \text{ and } M_{y,z}^X = k_y - q_z^X$$

$$X = L, R,\ y = p, h \text{ and } z = p, h$$

$$\begin{aligned} DEN = &\ 8 k_h k_p u_o^2 v_o^2 \left(q_h^L + q_p^L\right)\left(q_h^R + q_p^R\right) \\ + &\ e^{iL_S k_p - iL_S k_h} (P_{hp}^L P_{ph}^L v_o^2 - M_{hh}^L M_{pp}^L u_o^2)(M_{hh}^R M_{pp}^R u_o^2 - P_{hp}^R P_{ph}^R v_o^2) \\ + &\ e^{iL_S k_h + iL_S k_p} (M_{pp}^L P_{hh}^L u_o^2 - M_{hp}^L P_{ph}^L v_o^2)(M_{pp}^R P_{hh}^R u_o^2 - M_{hp}^R P_{ph}^R v_o^2) \\ + &\ e^{iL_S k_h - iL_S k_p} (M_{hp}^L M_{ph}^L v_o^2 - P_{hh}^L P_{pp}^L u_o^2)(P_{hh}^R P_{pp}^R u_o^2 - M_{hp}^R M_{ph}^R v_o^2) \\ + &\ e^{-iL_S k_h - iL_S k_p} (M_{hh}^L P_{pp}^L u_o^2 - M_{ph}^L P_{hp}^L v_o^2)(M_{hh}^R P_{pp}^R u_o^2 - M_{ph}^R P_{hp}^R v_o^2) \end{aligned}$$

Transmission of a particle in BSCCO NSNSN



Atomic units

$$1a_B = .052917\text{nm}$$
$$1E_H = 27.716eV = 4.3 \times 10^{-18}J$$
$$m_N^* = m_S = m_e = 1$$
$$\hbar = 1$$

BSCCO parameters

$$\varepsilon_F = 1.307 \times 10^{-3} E_H$$
$$\Delta = 5.3814 \times 10^{-4} E_H$$

System parameters

$$S_L = S_R = 200a_B$$
$$N_C = 980a_B$$
$$T = 22.76K$$
$$\nu_L = \nu_R = 0$$

$e < \Delta$ (tunneling!)

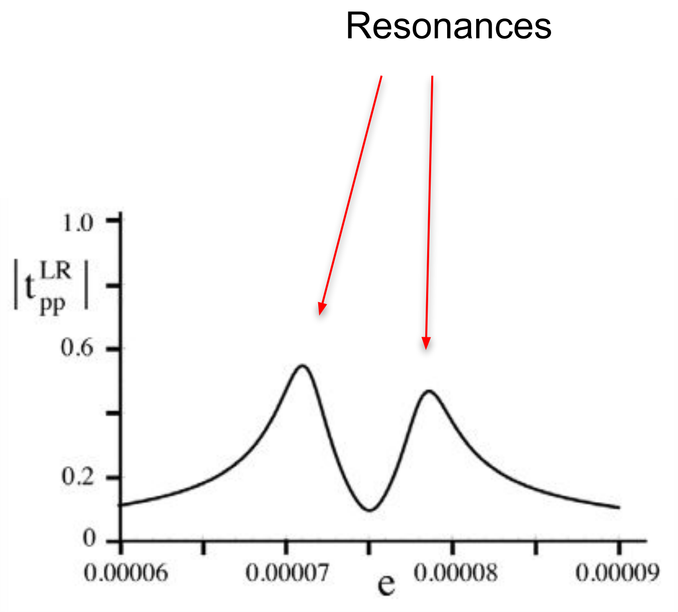
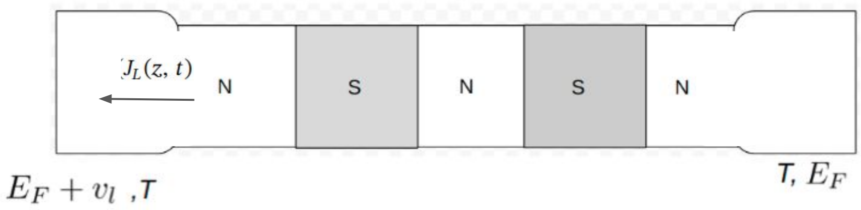


Figure from:
“Signatures of electron entanglement in a NSNSN BSCCO nanowire”.
L. Reichl. Physica B (2022)

Average current



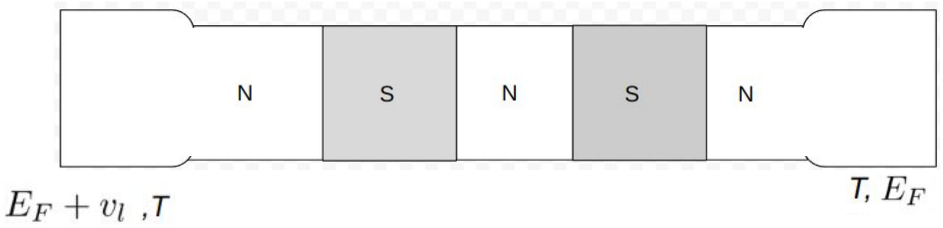
- Consider field operator in left normal lead

$$
\begin{aligned}
\hat{\psi}_L(z, t) \quad = & \sqrt{\frac{1}{2\pi}\frac{m_L}{\hbar^2}}\, e^{-i\varepsilon_F t/\hbar} \int_0^{\varepsilon_F} de \Bigg[\frac{1}{\sqrt{q_p^L}}\, e^{-iet/\hbar} \Big(\hat{a}^L_{e,p} e^{iq_p^L z} \\
& + r_{pp}^{LL}(e)\hat{a}^L_{e,p} e^{-iq_p^L z} + t_{pp}^{LR}(e)\hat{a}^R_{e,p} e^{-iq_p^L z} \\
& + r_{ph}^{LL}(e)\hat{a}^L_{e,h} e^{-iq_p^L z} + t_{ph}^{LR}(e)\hat{a}^R_{e,h} e^{-iq_p^L z} \Big) \\
& + \frac{1}{\sqrt{q_h^L}}\, e^{+iet/\hbar} \Big(\hat{a}^{L\dagger}_{e,h} e^{-iq_h^L z} + r_{hp}^{LL}(e)\hat{a}^{L\dagger}_{e,p} e^{iq_h^L z} \\
& + t_{hp}^{LR}(e)\hat{a}^{R\dagger}_{e,p} e^{iq_h^L z} + r_{hh}^{LL}(e)\hat{a}^{L\dagger}_{e,h} e^{iq_h^L z} + t_{hh}^{LR}(e)\hat{a}^{R\dagger}_{e,h} e^{iq_h^L z} \Big) \Bigg]
\end{aligned}
$$

$$
\langle J_L(z, t)\rangle = \frac{\hbar e}{2im_L}\left\langle \psi_L^\dagger(z, t)\frac{d\psi_L(z, t)}{dz} - \frac{d\psi_L^\dagger(z, t)}{dz}\psi_L(z, t) \right\rangle
$$

m_L = effective mass in left normal lead
$\hbar$ = reduced Planck's constant
e = charge of electron
ε_F = Fermi energy

Propagating current



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$$\hbar = 1$$

BSCCO parameters

$$\varepsilon_F = 1.307 \times 10^{-3} E_H$$
$$\Delta = 5.3814 \times 10^{-4} E_H$$

System parameters

$$S_L = S_R = 200a_B$$
$$N_C = 980a_B$$
$$T = 22.76K$$
$$\boxed{\nu_L = 0.00002E_H}$$
$$\nu_R = 0$$

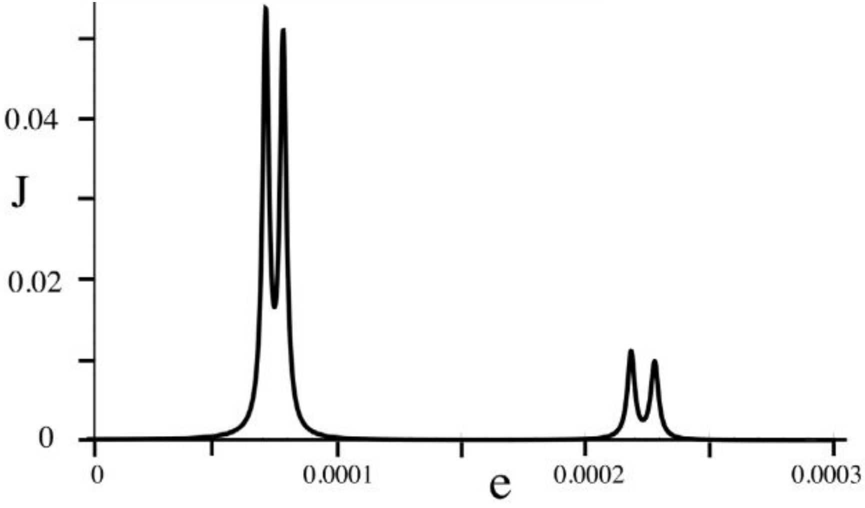
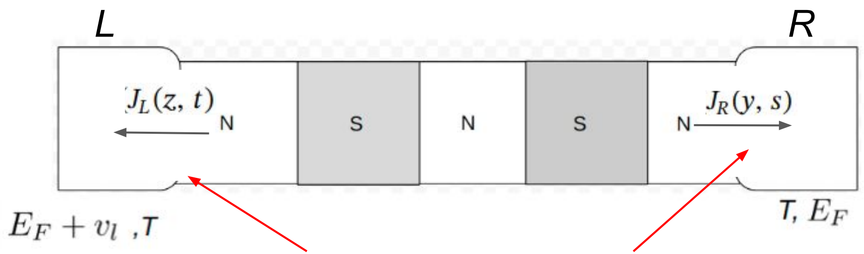


Figure from:
“Signatures of electron entanglement in a NSNSN BSCCO nanowire”.
L. Reichl. Physica B (2022)

Cross-correlated shot noise



Correlations in fluctuations of these currents

$$S^{LR}(z, y; t, s) = \frac{1}{2}\langle (J_L(z,t) - \langle J_L\rangle)(J_R(y,s) - \langle J_R\rangle) + (J_R(y,s) - \langle J_R\rangle)(J_L(z,t) - \langle J_L\rangle)\rangle .$$

F.T.

$$S^{LR}(\omega = 0) \equiv S^{LR}_{pp} + S^{LR}_{hh} + S^{LR}_{ph}$$

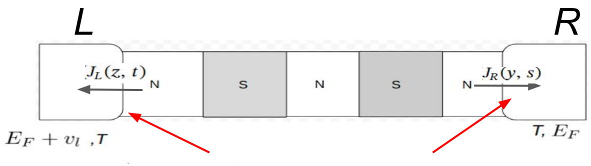
$$\begin{aligned} S^{LR}_{pp} = \frac{m_L m_R}{\pi^2\hbar^4} \int de \Big[& F^L_p N^L_p \left((-|r^{LL}_{hp}|^2 + |r^{LL}_{pp}|^2 - 1)(-|t^{RL}_{hp}|^2 + |t^{RL}_{pp}|^2) \right) \\ & + F^R_p N^R_p \left((-|r^{RR}_{hp}|^2 + |r^{RR}_{pp}|^2 - 1)(-|t^{LR}_{hp}|^2 + |t^{LR}_{pp}|^2) \right) \\ & + (F^L_p N^R_p + F^R_p N^L_p) \Big(\mathrm{Re}\left[r^{LL}_{hp} r^{RR}_{hp} t^{LR*}_{hp} t^{RL*}_{hp} \right] - \mathrm{Re}\left[r^{LL*}_{hp} r^{RR*}_{pp} t^{RL}_{pp} t^{LR}_{hp} \right] \\ & + \mathrm{Re}\left[r^{LL}_{pp} r^{RR}_{pp} t^{LR*}_{pp} t^{RL*}_{pp} \right] - \mathrm{Re}\left[r^{LL*}_{pp} r^{RR*}_{hp} t^{RL}_{hp} t^{LR}_{pp} \right] \Big) \Big] \end{aligned}$$

$$N_p(e) = N_h(e) = \left(1 + e^{e/k_B T}\right)^{-1}$$

$$F^L_p = 1 - N^L_p \text{ and } F^L_h = 1 - N^L_h$$

m_L = effective mass of electron in left normal lead
m_R = effective mass of electron in in right normal lead

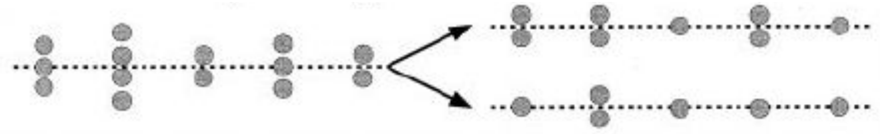
Sign of cross-correlated shot noise



Correlations in fluctuations of these currents

- For boson currents (such as photons):

$$S^{LR} > 0$$



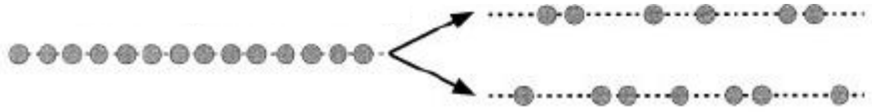
Current in L

Current in R

(fluctuations correlated)

- For fermion currents (such as electrons):

$$S^{LR} < 0$$



Current in L

Current in R

(fluctuations not correlated)

Figures from:
"Noise in mesoscopic physics".
T. Martinl, Physical Review B (2021)

Surprising results

Atomic units

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$$1E_H = 27.716eV = 4.3\times10^{-18}J$$
$$m_N^* = m_S = m_e = 1$$
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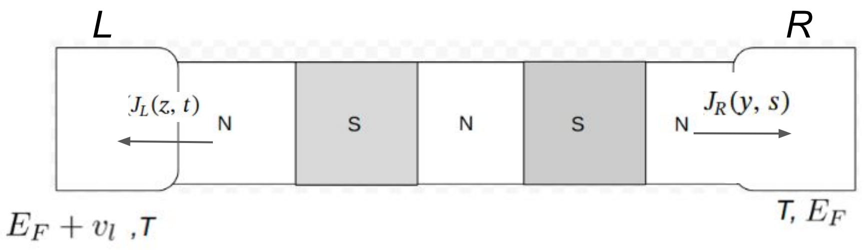
$$T = 22.76K$$
$$\nu_L = \nu_R = 0$$



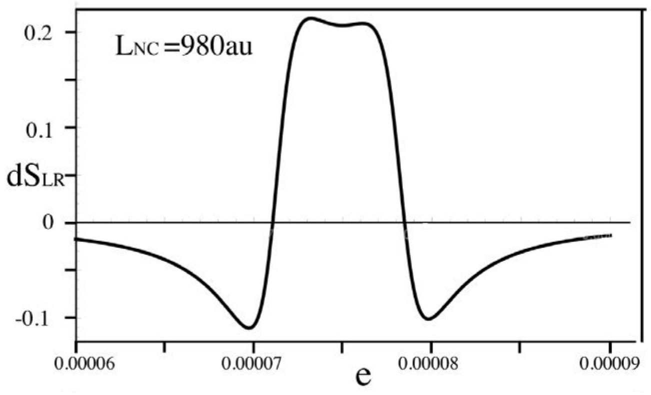


Figure from:
"Signatures of electron entanglement in a NSNSN BSCCO nanowire".
L. Reichl. Physica B (2022)

- Electrons are behaving as bosons!
- In pure normal conductors, dS_{LR} is negative. But it can become positive in hybrid normal & super - conducting structures ["Reversing the sign of current-current correlations". M, Büttiker. Quantum Noise in Mesoscopic Physics; Springer]

Cooper pair splitting

- Superconductors are characterized by "Cooper pairs" (pairings of electrons)
- In the BdG picture, these pairs form because of the coupling between particles and holes in the superconductor

$$\begin{pmatrix} u_o \\ v_o \end{pmatrix} \quad \begin{pmatrix} v_o \\ u_o \end{pmatrix} \Rightarrow$$

(k ,spin up & -k, spin down)

J = 0 (total angular momentum)

- "Cooper pair splitting" consists of the breaking of a Cooper pair such that the electrons travel in opposite directions

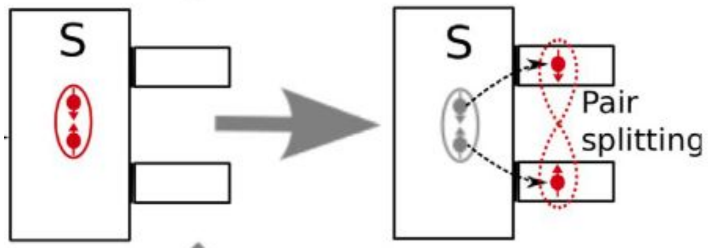
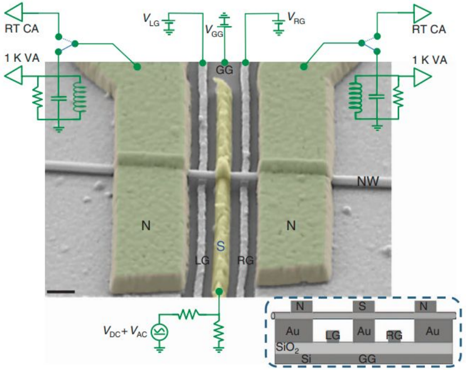


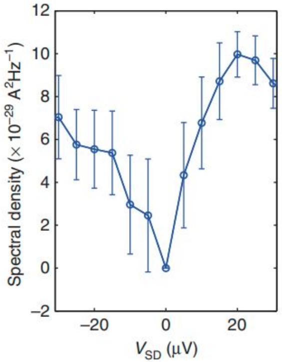
Figure from:
"Noise cross-correlation and Cooper pair splitting efficiency in multi-teminal superconductor junctions". Celis, J.A.; Gomez, S.; Herrera. Solid State Communications (2017)

Efficiency of Cooper pair splitting

- The efficiency of Cooper pair splitting has been observed in the positivity of the cross-correlated shot noise



Cross-correlated shot noise



Figures from:
"High-efficiency Cooper pair splitting demonstrated by two-particle conductance resonance and positive noise cross-correlation". Das, A.; Ronen, Y.; Heiblum, M.; Mahalu, D.; Kretinin, A. V.; Shtrikman, H. Solid State Communications (2012)

- Before we can make conclusions about this process for our system, we need to understand the causes for the positivity in our case

Normal behavior

Atomic units

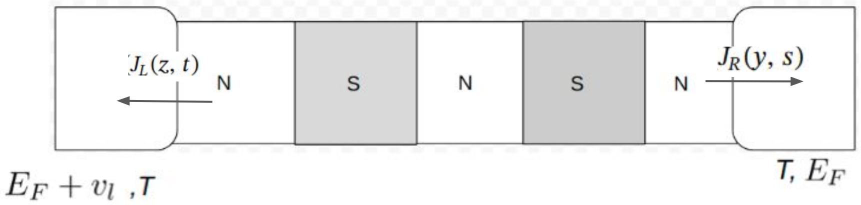
$$1a_B = .052917\text{nm}$$
$$1E_H = 27.716eV = 4.3 \times 10^{-18}J$$
$$m_N^* = m_S = m_e = 1$$
$$\hbar = 1$$

BSCCO parameters

$$\varepsilon_F = 1.307 \times 10^{-3} E_H$$
$$\Delta = 5.3814 \times 10^{-4} E_H$$

System parameters

$$S_L = S_R = 200a_B$$
$$N_C = 980a_B$$
$$T = 22.76K$$
$$\nu_L = \nu_R = 0$$



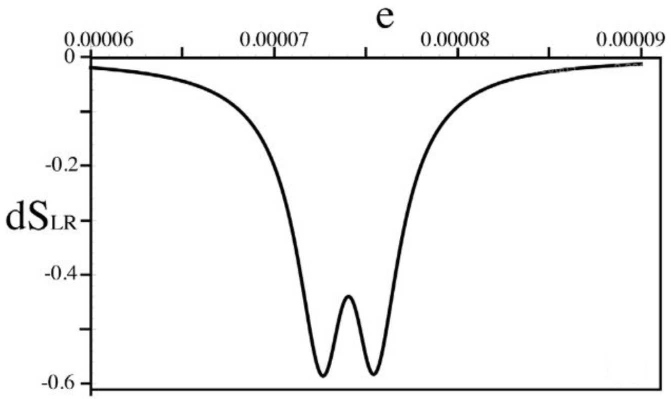


Figure from:
“Signatures of electron entanglement in a NSNSN BSCCO nanowire”.
L. Reichl. Physica B (2022)

Cross-correlated shot noise in NSNSNSNSN

Atomic units

$1a_B = .052917\text{nm}$

$1E_H = 27.716eV = 4.3 \times 10^{-18}J$

$m_N^* = m_S = m_e = 1$

$\hbar = 1$

BSCCO parameters

$\varepsilon_F = 1.307 \times 10^{-3} E_H$

$\Delta = 5.3814 \times 10^{-4} E_H$

System parameters

$S_1 = S_1 = S_3 = S_4 = 200a_B$

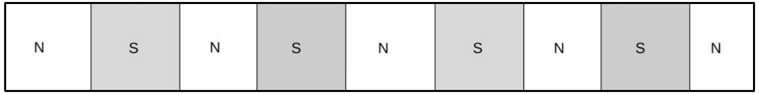
$N_L = N_R = 800a_B \; N_C = 1800a_B$

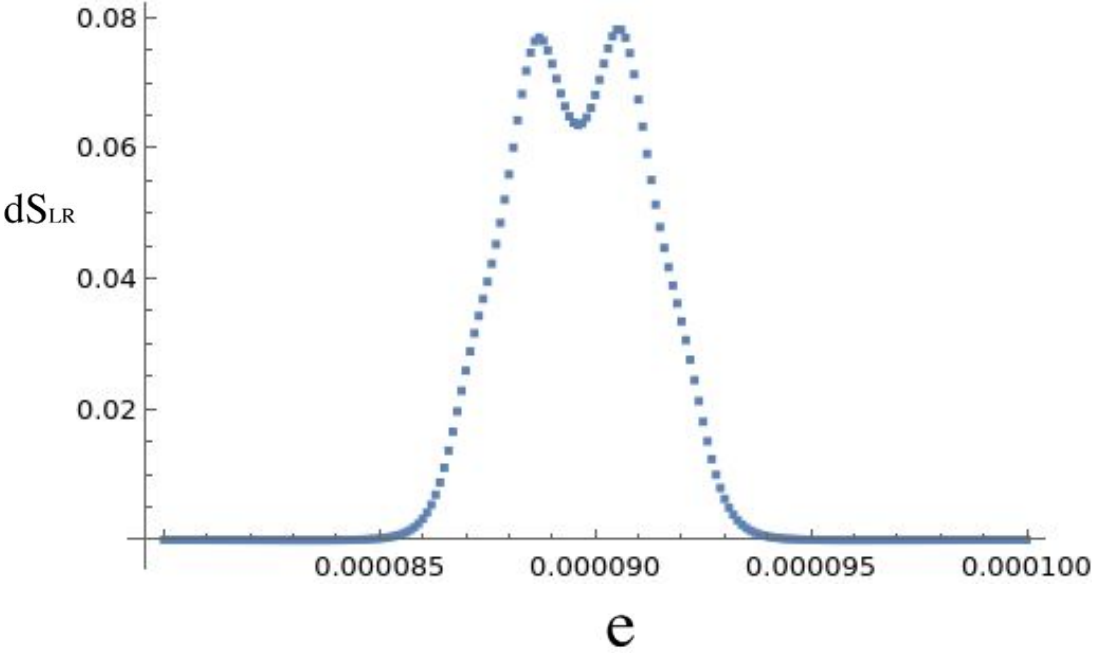
$T = 22.76K$

$\nu_L = 0.00002E_H$

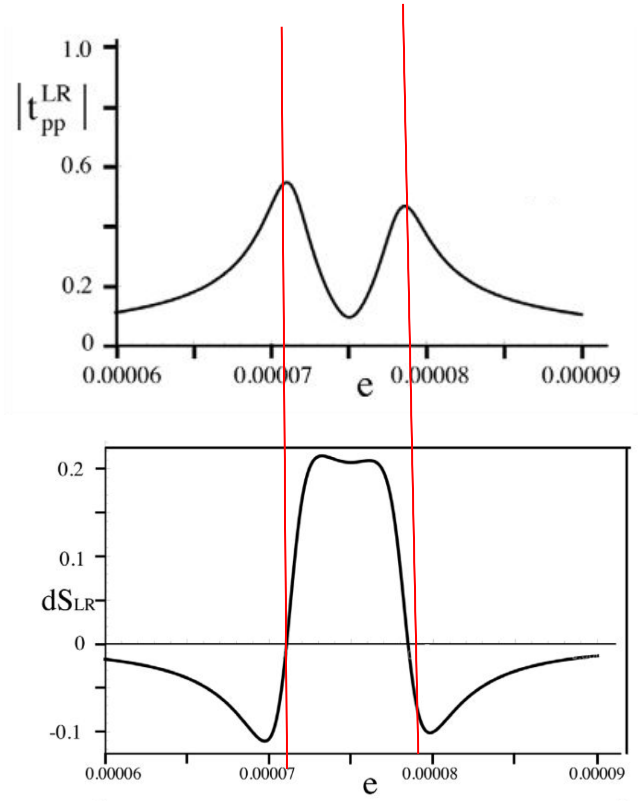
$\nu_R = 0$

- These are new results!





Common factor: resonances



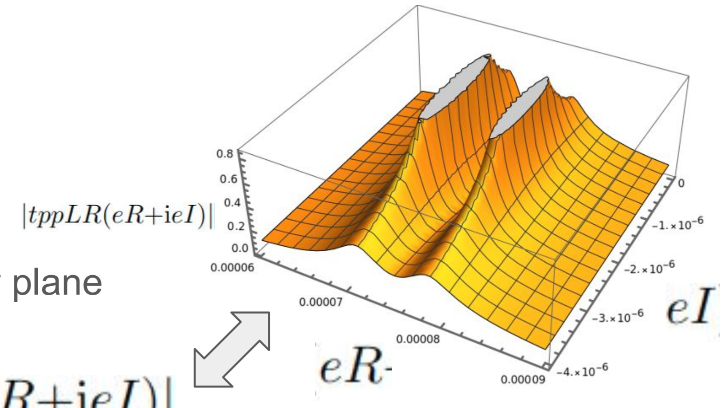
- Location of resonances can give us information about the positivity in cross-correlated shot noise

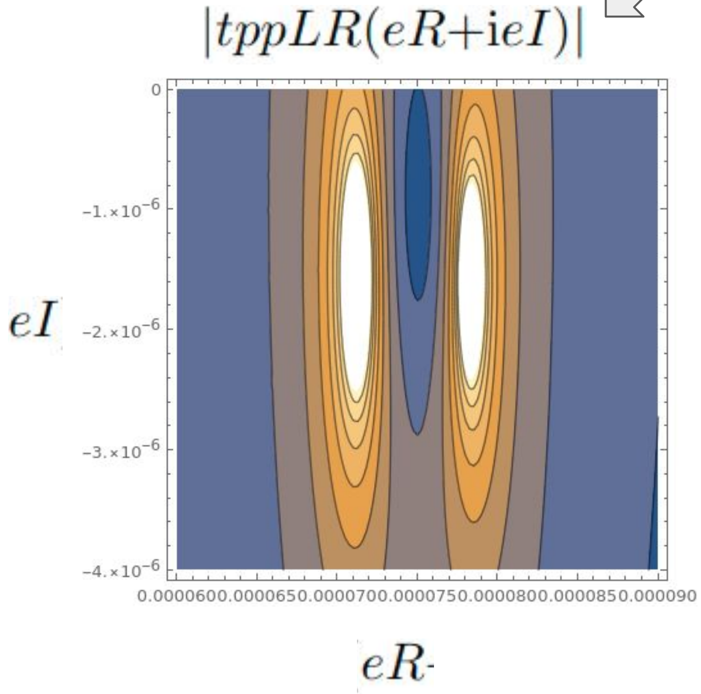
$e < \Delta$ (tunneling!) ⟺ Quasi-bound states

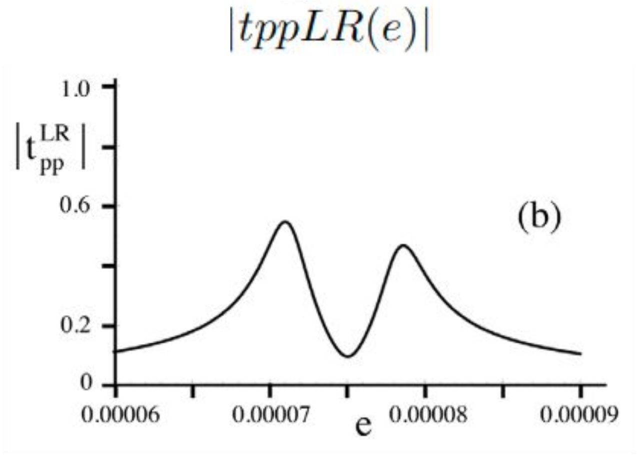
Figures from:
"Signatures of electron entanglement in a NSNSN BSCCO nanowire".
L. Reichl. Physica B (2022)

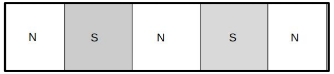
Quasi-bound states

- Quasi-bound states appear as poles in the complex energy plane





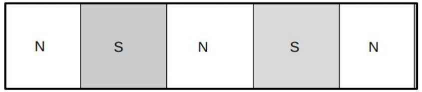




eR=real energy (positive)

eI=yields lifetime (lifetime=hbar/Ei)

For example… (LSCO in NSNSN)



Atomic units

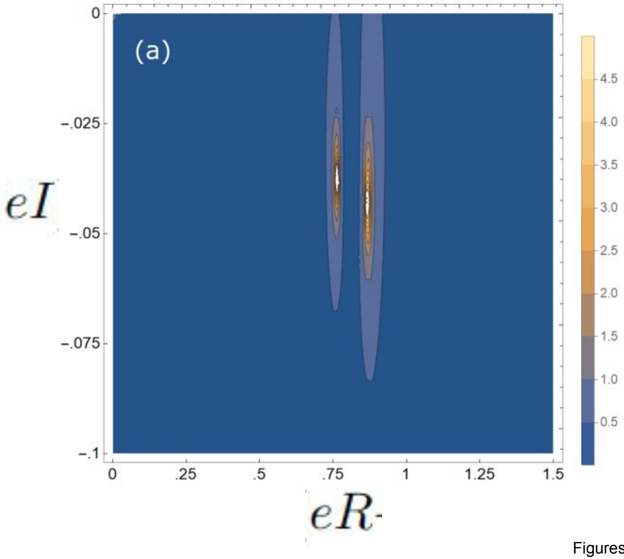
$$1a_B = .052917\text{nm}$$
$$1E_H = 27.716eV = 4.3 \times 10^{-18}J$$
$$m_N^* = m_S = m_e = 1$$
$$\hbar = 1$$

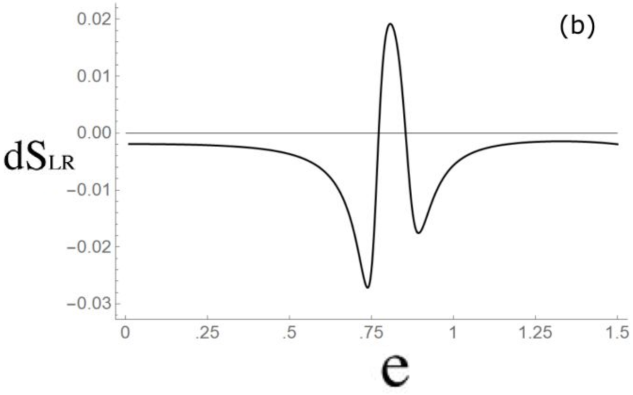
LSCO parameters

$$\varepsilon_F = 9.35 \times 10^{-4}E_H$$
$$\Delta = 2.2 \times 10^{-4}E_H$$

System parameters

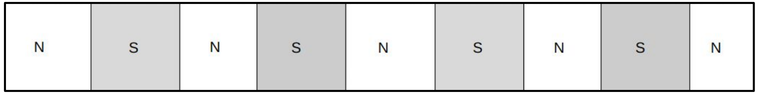
$$S_L = S_R = 390a_B$$
$$N_C = 325a_B$$
$$T = 16.3K$$
$$\nu_L = 0.00002E_H$$
$$\nu_R = 0$$



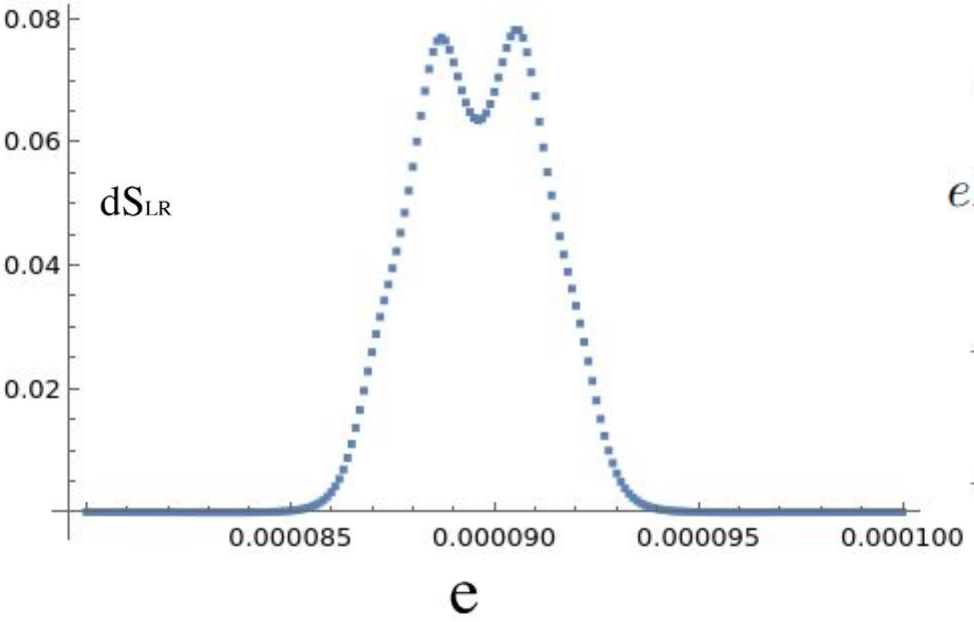


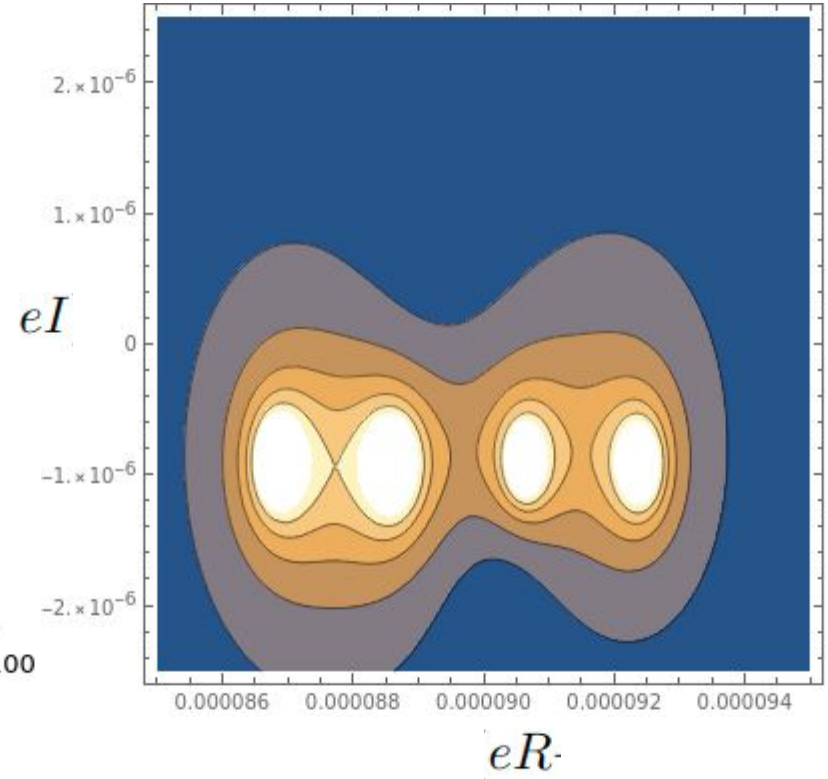
Figures from:
"Positive cross-correlated shot noise and quasibound states in an NSNSN geometry".
L. Reichl, C. Ostrove. Physical Review B (2021)

But?



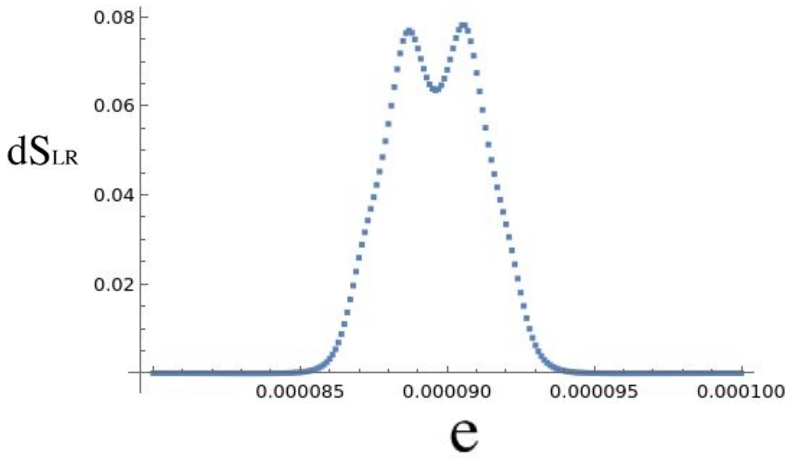
- BSCCO in NSNSNSNSN

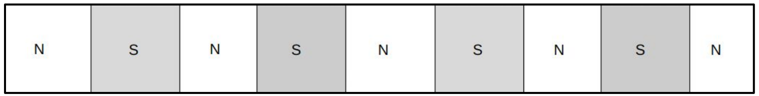


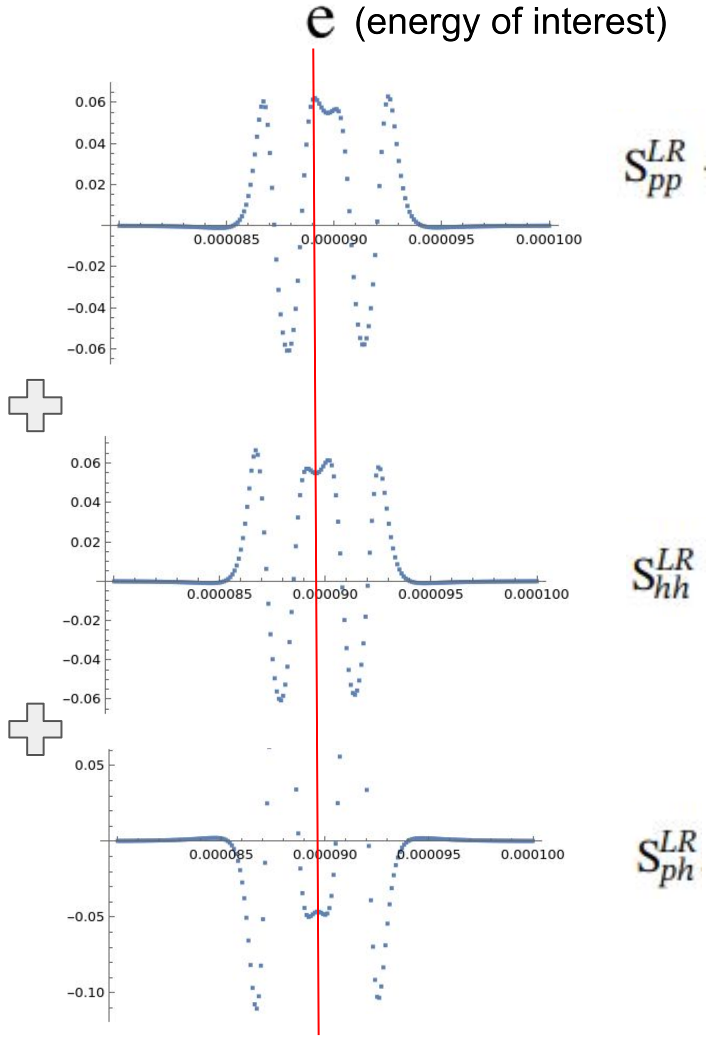


Current work: selecting energies

$$S^{LR}(\omega = 0) \equiv S^{LR}_{pp} + S^{LR}_{hh} + S^{LR}_{ph}$$







Current work: selecting scattering amplitudes

$$
\begin{aligned}
S_{pp}^{LR} &= \frac{m_L m_R}{\pi^2 \hbar^4} \int de \Big[F_p^L N_p^L \Big(\big(|r_{hp}^{LL}|^2 + |r_{pp}^{LL}|^2 - 1 \big) \big(|t_{hp}^{RL}|^2 + |t_{pp}^{RL}|^2 \big) \Big) \\
&+ F_p^R N_p^R \Big(\big(|r_{hp}^{RR}|^2 + |r_{pp}^{RR}|^2 - 1 \big) \big(|t_{hp}^{LR}|^2 + |t_{pp}^{LR}|^2 \big) \Big) \\
&+ (F_p^L N_p^R + F_p^R N_p^L) \Big(\mathrm{Re}\big[r_{hp}^{LL} r_{hp}^{RR} t_{hp}^{LR*} t_{hp}^{RL*} \big] - \mathrm{Re}\big[r_{hp}^{LL} r_{pp}^{RR*} t_{pp}^{RL} t_{hp}^{LR*} \big] \\
&+ \mathrm{Re}\big[r_{pp}^{LL} r_{pp}^{RR} t_{pp}^{LR*} t_{pp}^{RL*} \big] - \mathrm{Re}\big[r_{pp}^{LL} r_{hp}^{RR*} t_{hp}^{RL} t_{pp}^{LR*} \big] \Big) \Big]
\end{aligned}
$$

$$N_p(e) = N_h(e) = \left(1 + e^{e/k_B T}\right)^{-1}$$

$$F_p^L = 1 - N_p^L \text{ and } F_h^L = 1 - N_h^L$$

(term 1)

$$F_p^L N_p^L \Big(\big(|r_{hp}^{LL}|^2 + |r_{pp}^{LL}|^2 - 1 \big) \big(|t_{hp}^{RL}|^2 + |t_{pp}^{RL}|^2 \big) \Big)$$

(term 2)

$$+ F_p^R N_p^R \Big(\big(|r_{hp}^{RR}|^2 + |r_{pp}^{RR}|^2 - 1 \big) \big(|t_{hp}^{LR}|^2 + |t_{pp}^{LR}|^2 \big) \Big)$$

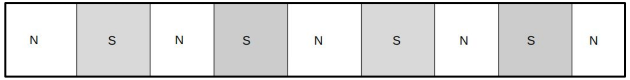
(term 3)

$$+ (F_p^L N_p^R + F_p^R N_p^L) \Big(\mathrm{Re}\big[r_{hp}^{LL} r_{hp}^{RR} t_{hp}^{LR*} t_{hp}^{RL*} \big] - \mathrm{Re}\big[r_{hp}^{LL} r_{pp}^{RR*} t_{pp}^{RL} t_{hp}^{LR*} + \mathrm{Re}\big[r_{pp}^{LL} r_{pp}^{RR} t_{pp}^{LR*} t_{pp}^{RL*} \big] - \mathrm{Re}\big[r_{pp}^{LL} r_{hp}^{RR*} t_{hp}^{RL} t_{pp}^{LR*} \big] \Big)$$

For the energy *e* of interest, evaluate each term

Term 1 (*e*)
Term 2 (*e*)
Term 3 (*e*)

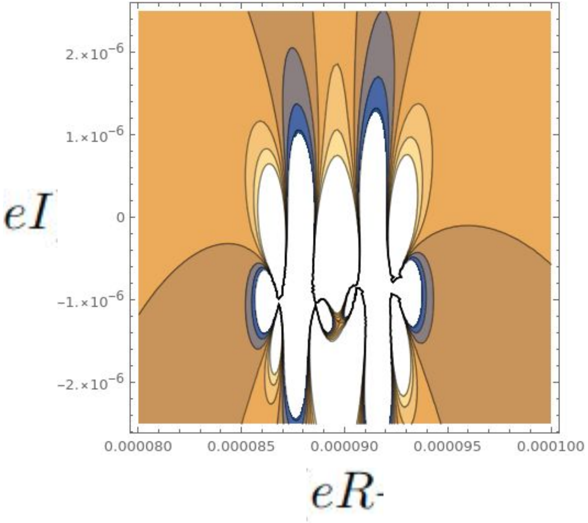
Select term with highest numerical contribution to use in the search of quasi-bound states



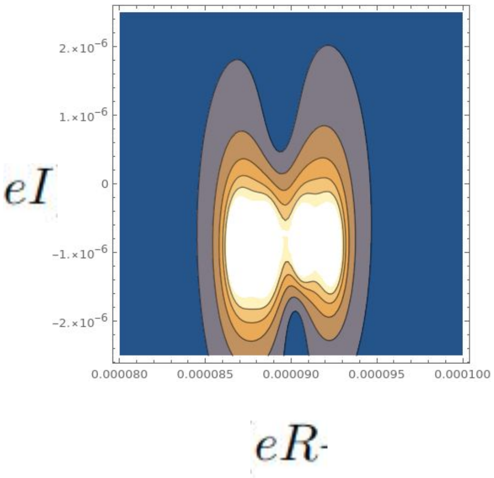
Current work: selecting scales in complex energy plane

- Can zoom in/out in complex energy plane to isolate poles

pole structure of term 3
In NSNSNSNSN

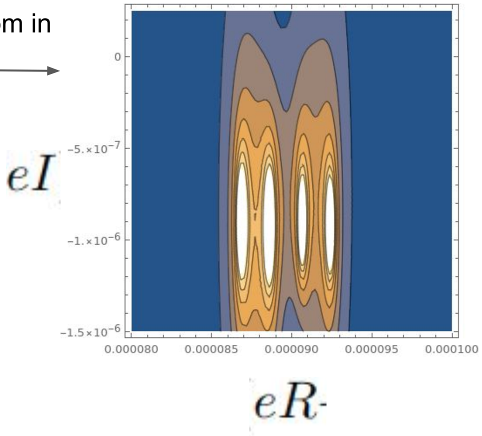


tppLR in
NSNSNSNSN



Zoom in

tppLR in
NSNSNSNSN



- The objective is to associate poles in these terms to regions of positivity in the contributions of the shot noise

Summary

- System and wave function
- Scattering theory
  - S matrix
  - Transfer matrix method
- Current and cross-correlated shot noise
  - Surprising results
  - Relation to entanglement
- Explanation via quasi-bound states
  - Search for poles of the S matrix elements
- Current work
  - Selecting relevant scattering amplitudes and regions of complex energy plane

References

1. Ostrove, C.; Reichl, L. (2021). Positive cross-correlated shot noise and quasibound states in an NSNSN geometry. Physical Review B, 103(22).

2. Wesche, R. (2015). Physical properties of high-temperature superconductors. John Wiley and Sons.

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Conservation of angular momentum => Entanglement

$s_1 = \frac{1}{2}$ and $s_2 = \frac{1}{2}$

$\alpha_1\,\alpha_2, \quad \alpha_1\,\beta_2, \quad \beta_1\,\alpha_2, \quad \beta_1\,\beta_2$

$s_1 + s_2 \equiv 1$ and $|s_1 - s_2| \equiv 0$

$$\alpha_1\,\alpha_2 \equiv |s = 1, m_s = 1, s_1 = \tfrac{1}{2}, s_2 = \tfrac{1}{2}\rangle$$

$$\beta_1\,\beta_2 \equiv |s = 1, m_s = -1, s_1 = \tfrac{1}{2}, s_2 = \tfrac{1}{2}\rangle$$

$$\boxed{\begin{aligned} |s = 1, m_s = 0, s_1 = \tfrac{1}{2}, s_2 = \tfrac{1}{2}\rangle &= \frac{1}{\sqrt{2}}[\alpha_1\,\beta_2 + \beta_1\,\alpha_2] \\ |s = 0, m_s = 0, s_1 = \tfrac{1}{2}, s_2 = \tfrac{1}{2}\rangle &= \frac{1}{\sqrt{2}}[\alpha_1\,\beta_2 - \beta_1\,\alpha_2]. \end{aligned}}$$

The wave function and energies

$$\hat{\psi}(x, y, z, t) = \frac{1}{\sqrt{\mathrm{L}_L}} \sum_{n_z=1}^{\infty} \sum_{\nu} \mathrm{e}^{-iE_{n_z,\nu}t/\hbar}\, \phi_\nu(x, y)(\hat{a}^L_{q_n,\nu}\mathrm{e}^{iq_n x} + \hat{b}^L_{q_n,\nu}\mathrm{e}^{-iq_n x}),$$

$$\phi_\nu(x, y) = \frac{2}{\sqrt{\mathrm{L_x L_y}}} \sin\left(\frac{n_x \pi x}{\mathrm{L_x}}\right) \sin\left(\frac{n_y \pi y}{\mathrm{L_y}}\right)$$

$$E^\alpha_{n,n_x,n_y} = e^\alpha_n + E^\alpha_{n_x,n_y}$$

$$e^\alpha_n = \frac{\hbar^2}{2m_\alpha}(q^\alpha_n)^2, \;\; q^\alpha_n = \frac{2\pi n}{\mathrm{L}\alpha}$$

$$E^{s,tr}_{n_x,n_y} = \frac{\hbar^2}{2m_s}\left(\frac{n_x^2\pi^2}{L_x^2} + \frac{n_y^2\pi^2}{L_y^2}\right)$$

Condition on widths

- Want e (total energy) to be less than the energy at which 2nd transverse mode opens
- Can set Fermi energy (“average energy of propagating particles”) to midpoint

$$E_F^\alpha \approx (E_{1,1}^\alpha + E_{2,1}^\alpha)/2$$

- Say Lx=1.2Ly=>

$$E_F = \frac{1}{2}(E_{1,1} + E_{2,1}) = \frac{1}{4m^*}\left(5\left(\frac{\pi}{1.2L_y}\right)^2 + 2\left(\frac{\pi}{L_y}\right)^2\right)$$

- Superconducting segments require Ly > coherence length

Condition on temperature

- Distribution for particles and holes

$$N_h(e) = (1 + e^{e/k_B T})^{-1}$$

- Want low probabilities of particles with energies away from the Fermi energy
  - E2,1-Ef = Ef-E1,1 = deltaE

$$(1 + e^{\delta E/(k_B T_0)})^{-1} = 0.001$$

- Superconducting segments require To < Tc

Components in cross-correlated shot noise

$$
\begin{aligned}
S_{pp}^{LR} = \frac{m_L m_R}{\pi^2 \hbar^4} \int de \Big[& F_p^L N_p^L \left((-|r_{hp}^{LL}|^2 + |r_{pp}^{LL}|^2 - 1)\,(-|t_{hp}^{RL}|^2 + |t_{pp}^{RL}|^2) \right) \\
& + F_p^R N_p^R \left((-|r_{hp}^{RR}|^2 + |r_{pp}^{RR}|^2 - 1)(-|t_{hp}^{LR}|^2 + |t_{pp}^{LR}|^2) \right) \\
& + (F_p^L N_p^R + F_p^R N_p^L) \Big(\mathrm{Re}\left[r_{hp}^{LL} r_{hp}^{RR} t_{hp}^{LR*} t_{hp}^{RL*}\right] - \mathrm{Re}\left[r_{hp}^{LL*} r_{pp}^{RR*} t_{pp}^{RL} t_{hp}^{LR}\right] \\
& + \mathrm{Re}\left[r_{pp}^{LL} r_{pp}^{RR} t_{pp}^{LR*} t_{pp}^{RL*}\right] - \mathrm{Re}\left[r_{pp}^{LL*} r_{hp}^{RR*} t_{hp}^{RL} t_{pp}^{LR}\right] \Big) \Big]
\end{aligned}
$$

$$N_p(e) = N_h(e) = \left(1 + \mathrm{e}^{e/k_B T}\right)^{-1}$$

$$F_p^L = 1 - N_p^L \text{ and } F_h^L = 1 - N_h^L$$

$$m_L = \text{effective mass of electron in left normal lead}$$
$$m_R = \text{effective mass of electron in in right normal lead}$$

$$
\begin{aligned}
S_{hh}^{LR} = \frac{m_L m_R}{\pi^2 \hbar^4} \int de \Big[& F_h^L N_h^L \left((|r_{hh}^{LL}|^2 - |r_{ph}^{LL}|^2 - 1)(|t_{hh}^{RL}|^2 - |t_{ph}^{RL}|^2) \right) \\
& + F_h^R N_h^R \left((|r_{hh}^{RR}|^2 - |r_{ph}^{RR}|^2 - 1)(|t_{hh}^{LR}|^2 - |t_{ph}^{LR}|^2) \right) \\
& + (F_h^L N_h^R + F_h^R N_h^L) \Big(\mathrm{Re}\left[r_{hh}^{LL} r_{hh}^{RR} t_{hh}^{LR*} t_{hh}^{RL*}\right] \\
& - \mathrm{Re}\left[r_{hh}^{LL*} r_{ph}^{RR*} t_{ph}^{RL} t_{hh}^{LR}\right] \\
& + \mathrm{Re}\left[r_{ph}^{LL} r_{ph}^{RR} t_{ph}^{LR*} t_{ph}^{RL*}\right] - \mathrm{Re}\left[r_{ph}^{LL*} r_{hh}^{RR*} t_{hh}^{RL} t_{ph}^{LR}\right] \Big) \Big]
\end{aligned}
$$

$$
\begin{aligned}
S_{ph}^{LR} = \frac{m_L m_R}{\pi^2 \hbar^4} \int de \Big[& (F_h^L N_p^L + F_p^L N_h^L) \Big(\mathrm{Re}\left[r_{hh}^{LL} r_{hp}^{LL*} t_{hp}^{RL} t_{hh}^{RL*}\right] \\
& - \mathrm{Re}\left[r_{hh}^{LL} r_{hp}^{LL*} t_{ph}^{RL*} t_{pp}^{RL}\right] \\
& - \mathrm{Re}\left[r_{ph}^{LL} r_{pp}^{LL*} t_{hh}^{RL*} t_{hp}^{RL}\right] + \mathrm{Re}\left[r_{ph}^{LL} r_{pp}^{LL*} t_{pp}^{RL} t_{ph}^{RL*}\right] \Big) \\
& + \mathrm{Re}\left[r_{ph}^{LL} r_{pp}^{RR} t_{pp}^{LR*} t_{ph}^{RL*}\right] - \mathrm{Re}\left[r_{ph}^{LL*} r_{hp}^{RR*} t_{hh}^{RL} t_{pp}^{LR}\right] \Big) \\
& + (F_p^L N_h^R + F_h^R N_p^L) \Big(\mathrm{Re}\left[r_{hp}^{LL} r_{hh}^{RR} t_{hh}^{LR*} t_{hp}^{RL*}\right] \\
& - \mathrm{Re}\left[r_{hp}^{LL*} r_{ph}^{RR*} t_{pp}^{RL} t_{hh}^{LR}\right] \\
& + \mathrm{Re}\left[r_{pp}^{LL} r_{ph}^{RR} t_{ph}^{LR*} t_{pp}^{RL*}\right] - \mathrm{Re}\left[r_{pp}^{LL*} r_{hh}^{RR*} t_{hp}^{RL} t_{ph}^{LR}\right] \Big) \Big]
\end{aligned}
$$

Particle-hole coupling => Cooper pair

$$H = \sum_{\alpha\beta k} \in_{\alpha\beta} (k) C^{+}_{k\alpha} C_{k\alpha} + \frac{1}{2} \sum_{\alpha\beta k} [\Delta_{\alpha\beta}(k) C^{+}_{k\alpha} C^{+}_{-k\beta} + hc]$$

General expression for Hamiltonian in superconductors

$$H(k) = \frac{1}{2} \sum_{k} \psi^{+}(k) H(k) \psi(k) \qquad H(k) = \begin{bmatrix} \in(k) & \Delta(k) \\ \Delta^{+}(k) & -\in(-k) \end{bmatrix}, \ \psi(k) = \begin{bmatrix} C_{k\alpha} \\ C^{+}_{-k\alpha} \end{bmatrix}$$

Hamiltonian in matrix form

$$H(k) = \sum_{x} e^{ikx} H(x); \ \psi(k) = \frac{1}{\sqrt{v}} \sum_{x} e^{ikx} \psi(x) \qquad \psi(x) = \begin{bmatrix} C_{\uparrow}(x) \\ C_{\downarrow}(x) \\ C^{+}_{\uparrow}(x) \\ C^{+}_{\downarrow}(x) \end{bmatrix}$$

(Fourier transformed)

$$H = \frac{1}{2} \sum_{k} (C^{+}_{k\uparrow}, C^{+}_{k\downarrow}, C_{-k\uparrow}, C_{-k\downarrow}) H(k) \begin{bmatrix} C_{k\uparrow} \\ C_{k\downarrow} \\ C_{-k\uparrow} \\ C_{-k\downarrow} \end{bmatrix}$$

Define “Bogoliubons” in terms of the creation and annihilation operators

$$c_{i\sigma} = \sum_{n}{}' (u^{n}_{i\sigma} \gamma_n - \sigma v^{n*}_{i\sigma} \gamma^{\dagger}_{n}), \quad c^{\dagger}_{i\sigma} = \sum_{n}{}' (u^{n*}_{i\sigma} \gamma^{\dagger}_{n} - \sigma v^{n}_{i\sigma} \gamma_n)$$

Excitations in the superconductor are represented as superpositions of electron and hole states

$$H_{eff} = \sum_{n} E_n \gamma^{\dagger}_{n} \gamma_n + E'_{\text{const}}$$