Problem Set: MLE, GMM, and ARMA

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Guidelines
No hand-written answers. Perform estimation from scratch using your own code (i.e., do not use built-in regression, GMM, or MLE functions, though you may use a built-in numerical optimizer and built-in variance-covariance functions). Include your code with your solutions.

Questions

1. Download the following six US factors from my website: be_me, ret_12_1, market_equity, qmj, ope_be, rvol_21d. Use data from 1975-2020.

   - Plot and test the first 12 autocorrelations and partial autocorrelations. Interpret your findings.
   - Study the dynamics of each time series with univariate analysis using ARMA(1,0), ARMA(1,1), ARMA(0,1), and ARMA(2,0) models. Estimate models via:
     (a) regression
     (b) exact MLE assuming a Gaussian model for each series (write down the exact likelihood)

2. **PhD*** The data set `wsj-counts.csv` contains occurrence frequency of various phrases from *The Wall Street Journal*. Let $c_{i,t}$ denote the counts of phrase i at time t. Consider an autoregressive dynamic model for expected phrase counts:

$$E_{t-1}[c_{i,t}] = \delta_0 + \delta_1 c_{i,t-1}$$

   Tasks:

   - Estimate this models via:
     (a) OLS regression
     (b) Exact MLE assuming a Poisson model for each series (hint: you must represent the conditional expectation within the Poisson distribution). Write down the exact likelihood.

(c) Describe and explain differences in your results from regression versus MLE.

- Test estimated ARMA parameters using OLS, ML-based Wald test, likelihood ratio test, and LaGrange multiplier test. Explain how you set up each of these tests.

3. **PhD*** For each phrase i in `wsj-counts.csv`, estimate the Poisson parameter λ_i via GMM using the Poisson model mean and variance as the moment conditions.

   - Write down your formulation of the GMM problem.
   - Describe and explain differences in your results from GMM versus MLE.
   - Perform an asymptotic test of the overidentifying restriction. Interpret your results.