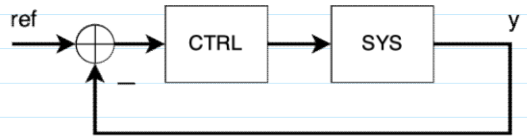
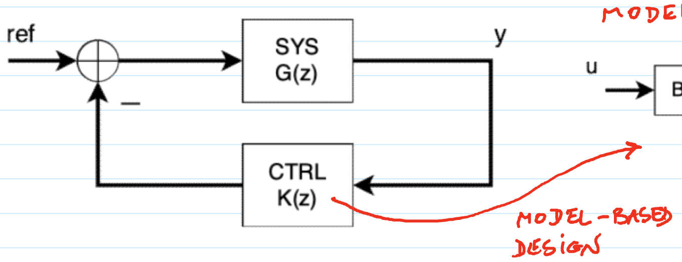
Computer Control Lab Project 2020/21 - Session 3

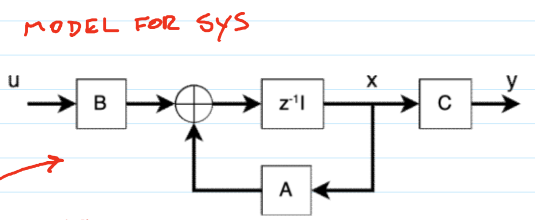
Monday, 9 November 2020 13:18

Part 2: Designing the feedback control system

Overview







$$x(k+1) = Ax(k) + Bu(k)$$
$$y(k) = Cx(k) + \underbrace{D}_{=0}u(k)$$

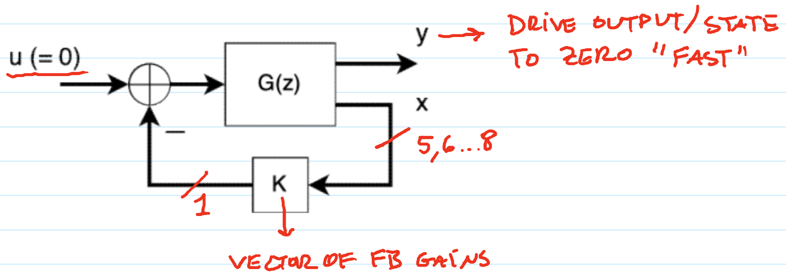
Design methodologies for the controller

- Heuristic (PID...)
- Optimization-based (LQR, LQE, LQG...)

Designing the controller in 3 steps:

1. Design the regulator (LQR)
2. Design the observer (LQE)
3. Merge regulator and observer (LQG)

1. Design the regulator (Linear Quadratic Regulator)



[K, ..., p] = dlqr(A, B, Q, R)

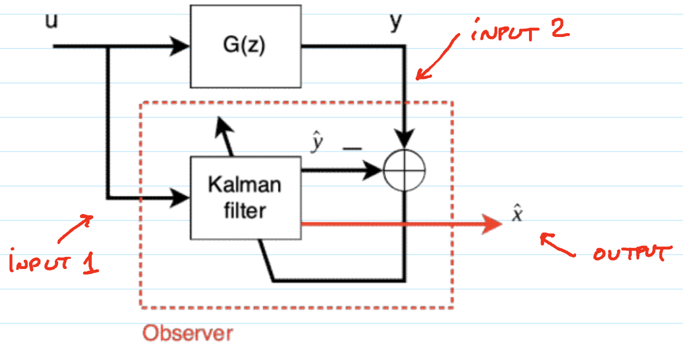
VECTOR OF FB GAINS

CLOSED-LOOP POLES → EIG(A - BK)

SS MODEL

"TUNING KNOBS"

2. Design the observer (Linear Quadratic Estimator)



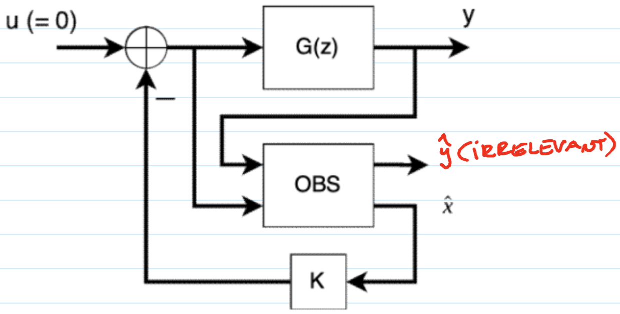
[M, ..., q] = dlqe(A, B1, C, QE, RE)

$\underbrace{\text{A, B1, C}}_{\text{RELEVANT PART OF SS MODEL}}$ $\underbrace{\text{QE, RE}}_{\text{"TUNING KNOBS"}}$

M: VECTOR OF (INTERNAL) FEEDBACK GAINS

q: CLOSED-LOOP POLES $\longrightarrow$ EIG$(A - MCA)$

3. Merge regulator and observer (LQG)



CLOSED-LOOP DYNAMICS: $\underbrace{\text{EIG}(A - BK)}_{\text{REGULATOR POLES}} \cup \underbrace{\text{EIG}(A - MCA)}_{\text{OBSERVER POLES}}$

Bottom line: If regulator and estimator are good, merged system should be good as well

Setting the tuning knobs for steps 1 and 2

1. Controller/regulator (dlqr(A, B, Q, R))

$$x(k+1) = A\,x(k) + B\,u(k)$$
$$y(k) = C\,x(k) + \underbrace{D}_{=0}\,u(k)$$

LQR COST FUNCTION

$$\underset{\text{FB GAINS}}{J(K)} = \sum_{\substack{k=0 \\ \text{TIME}}}^{+\infty} \Big(\underbrace{x^T(k)\,Q\,x(k)}_{\text{"NORM" OF } x(k)} + \underset{=\frac{1}{\rho}}{R}\,u^2(k) \Big) = \sum_{k=0}^{+\infty} \Big(y^2(k) + \frac{1}{\rho}\,u^2(k) \Big)$$

$$Q = C^T C$$

$$\underbrace{x^T(k)\,C^T}_{y(k)}\,\underbrace{C\,x(k)}_{y(k)} = y^2(k)$$

Two limit cases for the tuning parameter R (or rho):

① $R \approx 0,\ \rho \to \infty$

Inexpensive control
"Nervous" controller
Closed-loop poles near asymptotic positions

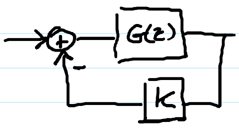
② $R \to \infty,\ \rho \to 0$

Expensive control
"Slow" controller
(Stable) poles near initial positions

Symmetric Root Locus: A tool for analyzing closed-loop pole positions

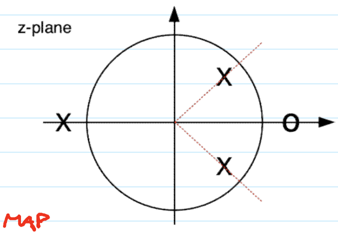
- Closed-loop poles of the regulator are the **stable roots** of

$$1 + \rho G(z) G(z^{-1}) = 0$$

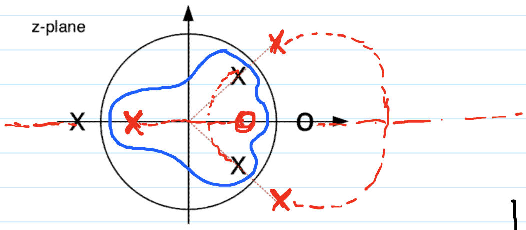


CL POLES : ROOTS OF

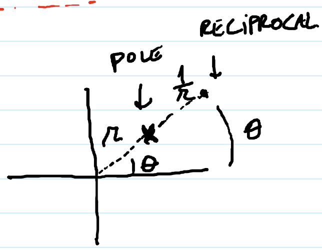
$$1 + K G(z) = 0$$



PZ MAP OF $G(z)$



PZ MAP OF $G(z)G(z^{-1})$



Important! For any choice of R, the closed-loop system is **stable**

2. Observer (dlqe(A, B1, C, QE, RE))

$$x(k+1) = A x(k) + B u(k) + B_1 w(k)$$
$$y(k) = C x(k) + v(k)$$

→ $w(k)$ PROCESS NOISE $\quad cov(w(k)) = Q_E$

→ $v(k)$ MEASUREMENT NOISE $\quad cov(v(k)) = R_E$

COST FUNCTION $\quad H(M) = E\{ \| x(k) - \hat{x}(k) \|^2 \}$

M → FB GAINS OF OBSERVER

Choosing QE and RE

Ⓐ $Q_E = q_E I, \quad B_1 = I, \quad R_E = 1 \qquad$ (DEMO CODE)

→ Ⓑ $B_1 = B \quad (x(k+1) = A x(k) + B(\underbrace{u(k) + w(k)}_{w(k) \text{ is "INPUT NOISE"}})), \quad Q_E = 1, \quad R_E = \dfrac{1}{\rho_E}$

With this choice of tuning knobs the closed-loop poles of the observer are given by the **same** Symmetric Root Locus of LQR

$$1 + \rho_E G(z) G(z^{-1}) = 0$$

Limiting behavior:

$\rho_E \to \infty$

Reliable output y (v is small)
"Nervous" observer

$\rho_E \approx 0$

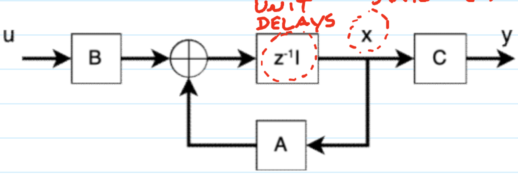
Noisy output
"Slow" observer

Observer architecture(s)

State-space model for G(z):

UNIT DELAYS

STATE $x(k)$



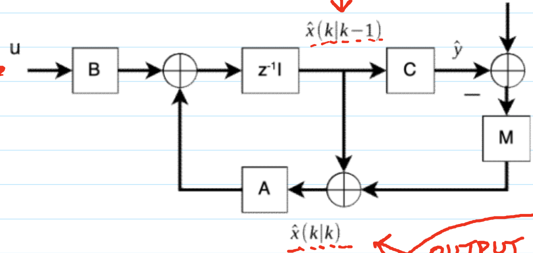
$$x(k+1) = A\,x(k) + B\,u(k)$$
$$y(k) = C\,x(k)$$

OUTPUT FOR A PREDICTIVE OBS. → $\hat{x}(k|k-1)$ IS UNAFFECTED BY $y(k)$

INPUT 2

Observer:

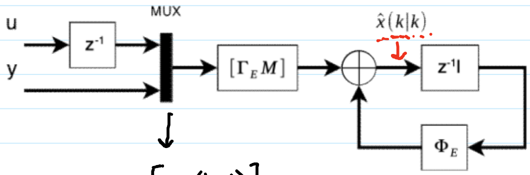
INPUT 1



OUTPUT FOR A CURRENT OBSERVER

PREFERRED

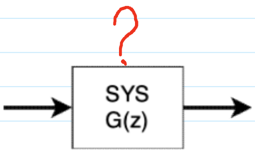
$\Leftrightarrow$



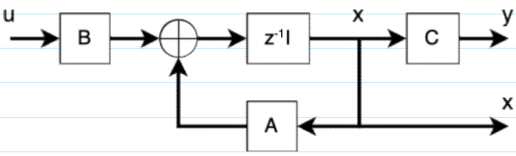
$\begin{bmatrix} u(k-1) \\ y(k) \end{bmatrix}$

(DEMO CODE)

Designing and testing the control system in incremental steps:

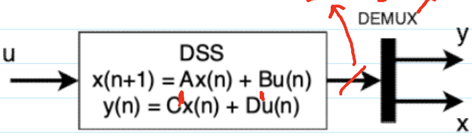


1^{ST} PHASE (DESIGN)



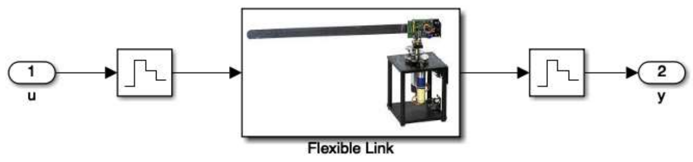
$\Leftrightarrow$

$\begin{bmatrix} y(k) \\ x(k) \end{bmatrix}$ $[1 \;\; \text{LENGTH}(A)]$

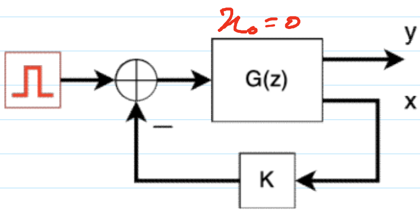


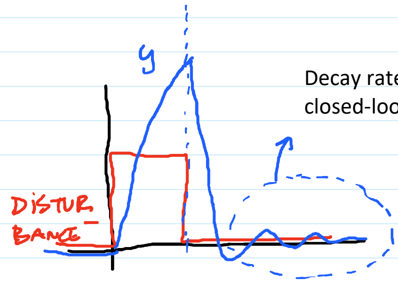
2^{ND} PHASE ("REAL" TESTING)

$A,\ B,\ C' = \begin{bmatrix} C \\ I \end{bmatrix},\ D' = \begin{bmatrix} D(=0) \\ 0 \end{bmatrix}$



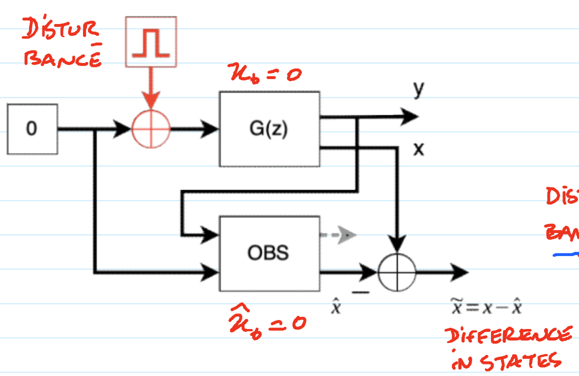
a. Design and validate the regulator

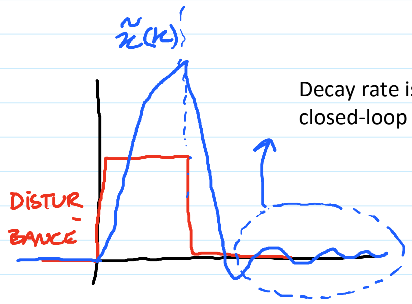




Decay rate is consistent with closed-loop poles eig(A - BK)

b. Design and validate the observer





Decay rate is consistent with closed-loop poles eig(A - MCA)

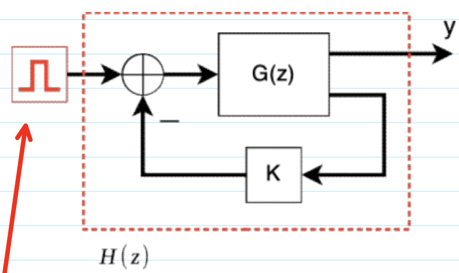
c. Merge and validate the full control system

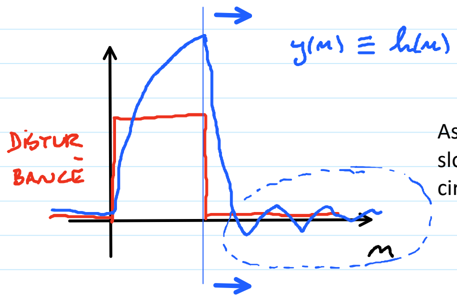
LQE is 2...10x FASTER THAN LQR

i. Check loop gain (ganho de malha), compare with the ideal one for LQR
ii. Check stability margins
iii. Check transient response

d. Add external input
e. Test on "real" system
f. Iterate

Measuring (exponential) decay rates

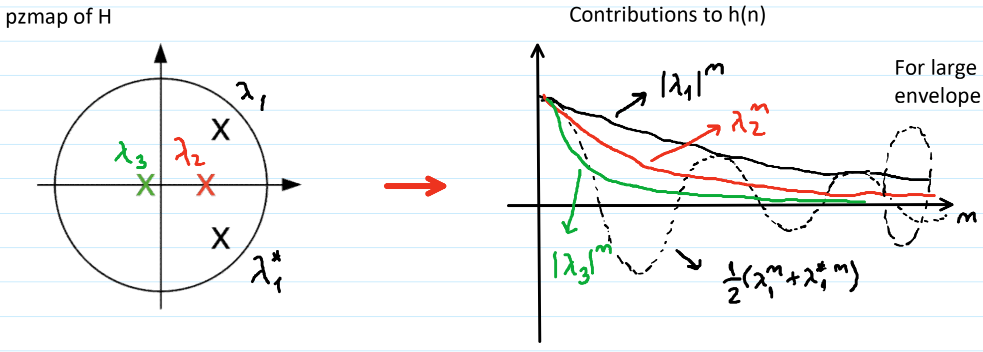




Asymptotic decay dominated by slow pole(s) closest to the unit circle $\max_i |\lambda_i|$

When the disturbance ends, we observe in y the **impulse response** h of the system

$H(z)$, POLES $\{\lambda_i\} \longleftrightarrow h(m) = \sum_i h_i \lambda_i^m$



For large n the dominant envelope is $|\lambda_1|^n$

Use log scale