# TL32: Entornos matemáticos

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# 1. Ecuación No. 1

$$\begin{aligned}
h_{\omega H}(\delta) &= \min_{z\in(0,1]} \log_q \frac{f_{\omega_H}(z)}{z^\delta} \\
&= \min_{z\in(0,1]} (\log_q f_{\omega_H}(z) - \log_q z^\delta) \\
&= \min_{z\in(0,1]} (\log_q(1+(q-1)z) - \delta \log_q z) \\
&= \log_q \left(1+(q-1)\frac{\delta}{(q-1)(1-\delta)}\right) - \delta \log_q \left(\frac{\delta}{(q-1)(1-\delta)}\right) \\
&= \log_q \left(\frac{1}{1-\delta}\right) - \delta \log_q \delta + \delta \log_q (q-1) + \delta \log_q (q-1) \\
&= \delta \log_q \frac{1}{\delta} + (1-\delta) \log_q \frac{1}{1-\delta} + \delta \log_q (q-1).
\end{aligned}$$

# 2. Ecuación No. 2

$$\begin{aligned}
ab &= [x_1,x_2]qx_2[x_1,x_2][x_1,x_2]x_1 + q^{-1}qx_2[x_1,x_2][[x_1,x_2]+q^{-1}x_2x_1][x_1,x_2]x_1 \\
&= [x_1,x_2]qx_2[x_1,x_2][x_1,x_2]x_1 + x_2[x_1,x_2][x_1,x_2][x_1,x_2]x_1 \\
&\quad + x_2[x_1,x_2]q^{-1}x_2x_1[x_1,x_2]x_1.
\end{aligned}$$

# 3. Ecuación No. 3

$$[x_i,x_j] = 0, \quad si\ |i-j|>1; \tag{1}$$

$$[[x_i,x_{i+1}],x_{i+1}] = 0, \quad si\ 1\le i<n; \tag{2}$$

$$[x_i,[x_i,x_{i+1}]] = 0, \quad si\ 1\le i<n. \tag{3}$$

# 4. Ecuación No. 4

$$\begin{aligned} [x_i, x_j] &= 0, \quad si\ |i-j| > 1; \\ [[x_i, x_{i+1}], x_{i+1}] &= 0, \quad si\ 1 \leq i < n; \\ [x_i, [x_i, x_{i+1}]] &= 0, \quad si\ 1 \leq i < n. \end{aligned} \tag{1}$$

# 5. Ecuación No. 5

$$\begin{aligned} e^{i\theta_1} e^{i\theta_2} &= (\cos\theta_1 + i\,\mathrm{sen}\,\theta_1)(\cos\theta_2 + i\,\mathrm{sen}\,\theta_2) \\ &= (\cos\theta_1 \cos\theta_2 - \mathrm{sen}\,\theta_1\,\mathrm{sen}\,\theta_2) + i(\cos\theta_1\,\mathrm{sen}\,\theta_2 + \mathrm{sen}\,\theta_1 \cos\theta_2) \\ &= \cos(\theta_1 + \theta_2) + i\,\mathrm{sen}\,(\theta_1 + \theta_2) \\ &= e^{i(\theta_1+\theta_2)}. \end{aligned}$$