# INDIAN STATISTICAL INSTITUTE

## BACK PAPER EXAMINATION: (2013-2014)

## MSQE I and M.Stat II

## Microeconomic Theory II

Date: 02.08.14 Maximum marks: 100 Duration: 3 Hours

**Note:** Answer all questions.

**Note:** $\mathbb{R}^\ell$ denotes the $\ell$-dimensional Euclidean space. Assume that

$$\mathbb{R}^\ell_+ = \left\{x = (x^1, \cdots, x^\ell) \in \mathbb{R}^\ell : x^i \geq 0 \text{ for all } 1 \leq i \leq \ell\right\}$$

and

$$\mathbb{R}^\ell_{++} = \left\{x = (x^1, \cdots, x^\ell) \in \mathbb{R}^\ell : x^i > 0 \text{ for all } 1 \leq i \leq \ell\right\}.$$

Throughout, a preference relation is assumed to be rational.

Q1. Let $\mathbb{R}_+$ denote the set of wealth and $U$ be a twice-differentiable strictly increasing utility function on $\mathbb{R}_+$ for a decision maker. For any fixed amount of money $\omega$ and positive number $\varepsilon$, the probability premium is denoted by $\pi(\omega, \varepsilon, U)$ and the certainty equivalence and expected value for a lottery $L$ are denoted by $CE(L, U)$ and $\mathbb{E}(L)$ respectively. Show that the following properties are equivalent:

(i) The decision maker is risk averse.

(ii) $\pi(w, \varepsilon, U) \geq 0$ for all $w \geq 0$ and all $\varepsilon > 0$.

(iii) $CE(L, U) \leq \mathbb{E}(L)$ for all lottery $L$. [18]

Q2. Suppose that $\succeq$ and $\omega$ are a continuous preference relation and an initial endowment of an agent, respectively. If the consumption set of the agent is $\mathbb{R}^\ell_+$, then show that the demand set $D(p, \omega, \succeq) \neq \emptyset$ for all prices $p \in \mathbb{R}^\ell_{++}$. [10]

Q3. Consider an economy $\mathscr{E} = \left\{N; \mathbb{R}^\ell_+; (\succeq_i, \omega_i)_{i\in N}\right\}$, where $N$ is the set of agents containing $n$ many elements; $\mathbb{R}^\ell_+$ is the consumption set of each agent; and $\succeq_i$ and $\omega_i$ are the preference and initial endowment of agent $i$, respectively.

(i) Suppose that $\sum_{i\in N} p\cdot\omega_i \neq 0$ for some $p \in \mathbb{R}^\ell$. If an allocation is supported by the price $p$, then show that it is weakly Pareto optimal allocation. [10]

(ii) If $\succeq_i$ is continuous and monotone, then show that the set of Pareto optimal allocations is a compact subset of $\mathbb{R}^{n\ell}$. [12]

(iii) Let $N = \{1, 2\}$ and $\ell = 2$. Suppose that the preference relation $\succeq_i$ is represented by a utility function $U_i$ for $i = 1, 2$. Given that

$$\begin{cases} \omega_1 = (2, 8), & U_1(x, y) = \min\{2x, y\}; \\ \omega_2 = (6, 0), & U_2(x, y) = \min\{x, 3y\}. \end{cases}$$



P.T.O

Find the set of Walrasian equilibrium of $\mathscr{E}$. [10]

(iv) Assume that $\succeq_i$ is continuous, strictly monotone for all $i \in N$ and $\sum_{i \in N} \omega_i \in \mathbb{R}^\ell_{++}$. If $x = (x_1, \cdots, x_n)$ is an allocation of $\mathscr{E}$ and $y \succeq_i x_i$ implies $p \cdot x \geq p \cdot \omega_i$ for all $i \in N$ and for some non-zero price $p$, then show that $x_i$ is in agent $i$'s demand set $D_i(p, \succeq_i, \omega_i)$ for all $i \in N$. [15]

(v) Suppose that $0 < \varepsilon < 1$. Show that there is closed ball $B$ such that the excess demand set $\zeta(p) \subseteq B$ for all prices $p \in \mathbb{R}^\ell_{++}$ satisfying $\varepsilon \leq p^h \leq 1$ for any $1 \leq h \leq \ell$. [15]

(vi) Suppose that $\mathscr{E}_r$ is the $r$-fold replicated economy of $\mathscr{E}$ and $\succeq_i$ is convex for all $i \in N$. Let

$$x = (x_{11}, \cdots, x_{1r}, x_{21}, \cdots, x_{2r}, \cdots, x_{n1}, \cdots, x_{nr})$$

be a Walrasian allocation of $\mathscr{E}_r$. Show that the allocation $\hat{x} = (\hat{x}_1, \cdots, \hat{x}_n)$ of $\mathscr{E}$, defined by

$$\hat{x}_i = \frac{1}{r} \sum_{j=1}^{r} x_{ij},$$

is a Walrasian allocation of $\mathscr{E}$. [10]

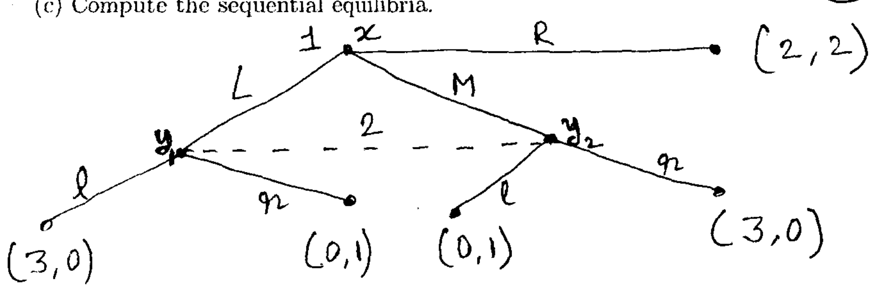
# INDIAN STATISTICAL INSTITUTE

Mid-Semestral Examination : 2014-15

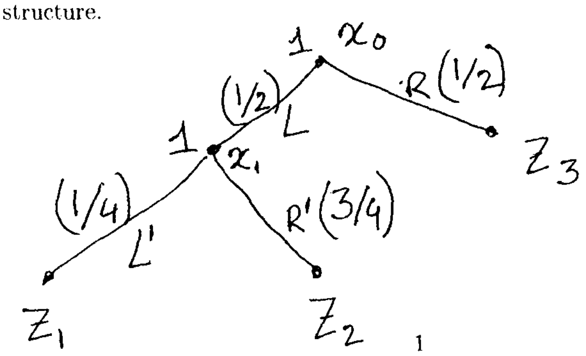
Course Name: M.S. (Q.E.) I YEAR / M.STAT. II YEAR
Subject Name: Game Theory I
Date: 28-08-2014 Maximum Marks: 50 Duration: 2 hours

**Problem 1.** Let $(b, \beta)$ be a consistent assessment in an extensive form game $\Gamma$. Show that $(b, \beta)$ is Bayesian consistent. (5)

**Problem 2.** Consider the following extensive form game below.
(a) Determine the strategic form of this game.
(b) Compute the Nash equilibria and subgame perfect Nash equilibria. (10)
(c) Compute the sequential equilibria.




**Problem 3.** Determine all mixed strategies that are outcome equivalent with the behavioral strategy represented in the following one-player extensive form structure. (5)



**Problem 4.** Justify your answer by a proof or a counterexample.
(a) Existence of Nash Equilibrium in pure strategies implies existence of subgame perfect Nash Equilibrium in pure strategies.
(b) Nash Equilibrium in behavioural strategies always exists. (Remember: The set of behavioural strategies is a strict subset of the set of mixed strategies.) 10

**Problem 5.** There are $n$ individuals who witness a crime. Everybody would like the police to be called. If this happens, each individual derives satisfaction $v > 0$ from it. Calling the police has a cost of $c$, where $0 < c < v$. The police will come if at least one person calls. Hence, this is an $n$-person game in which each player chooses from $\{C, N\}$ : $C$ means 'call the police' and $N$ means 'do not call the police'. The payoff to person $i$ is 0 if nobody calls the police, $v - c$ if $i$ (and perhaps others) call the police, and $v$ if the police is called but not by person $i$.
(a) What are the Nash equilibria of this game in pure strategies? In particular, show that the game does not have a symmetric Nash equilibrium in pure strategies (a Nash equilibrium is symmetric if every player plays the same strategy).
(b) Compute the symmetric Nash equilibrium or equilibria in mixed strategies.
(c) For the Nash equilibrium/equilibria in (b), compute the probability of the crime being reported. What happens to this probability if $n$ becomes large? 10

**Problem 6.** Consider a seller of a used car and a potential buyer of that car. Suppose that quality of the car, $\theta$, is a uniform draw from $[0, 1]$. This quality is known to the seller, but not to the buyer. Suppose that the buyer can make an offer $p \in [0, 1]$ to the seller, and the seller can then decide whether to accept or reject the buyer's offer.
Payoffs are as follows:

$$\begin{aligned} u_S &= p \quad \text{if offer accepted} \\ &= \theta \quad \text{if offer rejected} \end{aligned}$$

$$\begin{aligned} u_B &= a + b\theta - p \quad \text{if offer accepted} \\ &= 0 \quad \text{if offer rejected} \end{aligned}$$

Assume that $a \in [0, 1)$, that $b \in (0, 2)$ , and that $a+b > 1$. These assumptions imply that for all $\theta$, it is more efficient for the buyer to own the car.

(a) Show that the unique BNE is for the buyer to offer $p = a/(2 - b)$ and the seller to accept if and only if $p \geq \theta$.
(b) Note that if $a = 0$, then trade never occurs despite it the fact that there are always gains from trading. Explain this from game theoretic point of view. 10

# INDIAN STATISTICAL INSTITUTE

## MID-SEMESTRAL EXAMINATION: (2014-2015)

## MSQE I and M.Stat II

## Microeconomic Theory I

Date: 02.09.2014 Maximum marks: 40 Duration: 2 Hours

**Note:** Answer **all** questions.

**Note:** Throughout, $\mathbb{R}^L$ is the $L$-dimensional Euclidean space. Let

$$\mathbb{R}^L_+ = \{x = (x_1, \cdots, x_L) \in \mathbb{R}^L : x_i \geq 0 \text{ for all } 1 \leq i \leq L\}$$

and

$$\mathbb{R}^L_{++} = \{x = (x_1, \cdots, x_L) \in \mathbb{R}^L : x_i > 0 \text{ for all } 1 \leq i \leq L\}.$$

Q1. Let $\succsim$ be a *preference relation* over a non-empty set of alternatives $X$. Suppose also that $\succ$ and $\sim$ are the *strict preference relation* and the *indifference relation*, respectively, associated with $\succsim$. Prove or disprove the following statements. In case of disprove, give an example to show that the statement is incorrect.

(i) If $U$ and $V$ are utility functions representing $\succsim$, then the function $f := \min\{U, U+V\}$, defined by $f(x) = \min\{U(x), U(x)+V(x)\}$ for all $x \in X$, is also a utility function representing $\succsim$.

(ii) If $U : X \to \mathbb{R}$ is a function, then "$U$ is a utility function representing $\succsim$" is equivalent to "$x \sim y \Leftrightarrow U(x) = U(y)$ for all $x, y \in X$".

(iii) If $\succsim$ is rational, then the choice structure generated by $\succsim$ satisfies *WARP*.

(iv) Suppose that $X$ is a countable set. Then $\succsim$ can be represented by a utility function.

(v) Suppose that $X$ is a finite set and $U$ is a utility function representing $\succsim$. For any non-empty subset $B$ of $X$, let $|B|$ denote the number of elements in $B$. Assume that $\mathscr{B}$ denotes the set of non-empty subsets of $X$. Define a choice rule $\mathbb{C}(\cdot)$ on $\mathscr{B}$ by

$$\mathbb{C}(B) := \left\{x \in B : U(x) > U(y) \text{ for all } y \in D \text{ for some } D \subseteq B \text{ with } |D| > \frac{1}{2}|B|\right\}.$$

Then $(\mathscr{B}, \mathbb{C}(\cdot))$ satisfies *WARP*. [20]

Q2. Answer **all** questions.

(i) Show that if a demand function $x : \mathbb{R}^L_{++} \times \mathbb{R}_{++} \to \mathbb{R}^L_+$ satisfies *WARP* then it is homogeneous of degree zero.

(ii) Show that for a demand function $x : \mathbb{R}^L_{++} \times \mathbb{R}_{++} \to \mathbb{R}^L_+$ satisfying Walras's law, *WARP* holds if and only if *WARP* holds for all compensated price changes.

(iii) Show that the demand point $x(p, \omega)$ for a price-wealth combination $(p, \omega)$ is not necessarily a demand point for a price-wealth combination $(p', \omega')$, where $\omega' = p' \cdot x(p, \omega)$. [12]

Q3. Answer **all** questions.

(i) Suppose that $X$ is a non-empty set of alternatives and $(\mathscr{B}, \mathbb{C}(\cdot))$ is a choice structure of $X$ that satisfies *WARP*. Show that $\succ^*$ and $\succ^{**}$ are identical, that is, for any $x, y \in X$, $x \succ^* y \Leftrightarrow x \succ^{**} y$, where $\succ^*$ is the *strict revealed preference relation* and $x \succ^{**} y \Leftrightarrow [x \succeq^* y \text{ and } y \not\succeq^* x]$.

(ii) Show that *WARP* is not a sufficient condition to ensure the existence of a rationalizing preference relation. [8]

# INDIAN STATISTICAL INSTITUTE

First Semestral Examination : 2014-15

Course Name: M.S. (Q.E.) I YEAR / M.STAT. II YEAR
Subject Name: Game Theory I
Date: 6-11-2014 Maximum Marks: 50 Duration: 2 hours

**Problem 1.** Consider the following version of prisoner dilemma game.

| 1/2 | C | D |
|---|---|---|
| C | 4,4 | 0,6 |
| D | 6,0 | 2,2 |

(a) Is it true that the outcome (C,C) forever can be a subgame perfect equilibrium of the infinitely repeated prisoners dilemma game? If yes, provide the equilibrium strategies. Find the restriction on the discount factor $\delta$.
(b) What do you think about the other Subgame Perfect Equilibria that are possible in this repeated prisoners dilemma game (use folk theorem)? Plot graphically: (i) The set of subgame perfect Nash Equilibria payoffs, (ii) The set of feasible payoffs.
(c) Consider the following tit-for-tat (TFT) strategy (row player version)
First round: Play C.
Second and later rounds: If the history from the last round is (C,C) or (D,C) play C. If the history from the last round is (C,D) or (D,D) play D.
(i) Is it true that TFT supports (C,C) forever as an NE in the Infinitely Repeated PD? - Justify your answer by a proof.
(ii) Is it true that TFT as an equilibrium strategy is Subgame Perfect?
- Justify your answer by a proof.

(20)

**Problem 2.** Consider the Independent Private Values auction model with $n$ bidders and value distribution function $F$.
(a) Find the optimum bidding strategies in first price auction.
(b) Do these strategies form an equilibrium? If yes, is it unique? Justify your answer.
(c) Calculate the expected revenue using the optimum bidding strategy (do not use revenue equivalence).

(15)

**Problem 3.** Justify by a proof or counter example:
(a) Evolutionary stable strategy always exits for a $2 \times 2$ symmetric game.
(b) Evolutionary stable strategy always exits for a symmetric game that has a pure strategy Nash equilibrium.
(c) Evolutionary stable strategies always constitute a Nash Equilibrium.

(10)

**Problem 4.** We know that the Nash bargaining solution is the *unique* bargaining solution that satisfies the axioms: (i) Pareto Efficiency, (ii) Symmetry, (iii) Invariance to Equivalent, (iv) Payoff Representations Independence of Irrelevant Alternatives. Provide examples to show that all these four axioms are necessary for the uniqueness of the Nash bargaining solution.

(15)

# INDIAN STATISTICAL INSTITUTE

## SEMESTRAL EXAMINATION: (2014-2015)

## MSQE I and M.Stat II

## Microeconomic Theory I

Date: 10.11.2014 Maximum Marks: 60 Duration: 3 Hours

**Note:** Answer any **five** questions.

**Note:** Throughout, $\mathbb{R}^\ell$ is the $\ell$-dimensional Euclidean space. Let

$$\mathbb{R}^\ell_+ = \{x = (x_1, \cdots, x_\ell) \in \mathbb{R}^\ell : x_i \geq 0 \text{ for all } 1 \leq i \leq \ell\}$$

and

$$\mathbb{R}^\ell_{++} = \{x = (x_1, \cdots, x_\ell) \in \mathbb{R}^\ell : x_i > 0 \text{ for all } 1 \leq i \leq \ell\}.$$

Q1. Let $\succeq$ be a rational preference relation over $\mathbb{R}^\ell_+$.

(i) Assume that $L(x) = \{y \in \mathbb{R}^\ell_+ : x \succeq y\}$ and $R(x) = \{y \in \mathbb{R}^\ell_+ : y \succeq x\}$ for all $x \in \mathbb{R}^\ell_+$. If $L(x)$ and $R(x)$ are closed subsets of $\mathbb{R}^\ell_+$ then show that $\succeq$ is continuous.

(ii) Give an example of a discontinuous rational preference relation.

(iii) Show that $\succ$ is transitive. [6+3+3]

Q2. Answer **all** questions.

(i) Show that the demand function $x : \mathbb{R}^\ell_{++} \times \mathbb{R}_{++} \to \mathbb{R}^\ell_+$ satisfies *WARP* if and only if for any wealth level $\omega$ and prices $p, p'$ we have $p' \cdot x(p, \omega) > \omega$ if $p \cdot x(p', \omega) \leq \omega$ and $x(p, \omega) \neq x(p', \omega)$.

(ii) Prove or disprove: If $U : \mathbb{R}^\ell_+ \to \mathbb{R}$ is a utility function then $V : \mathbb{R}^\ell_+ \to \mathbb{R}$, defined by $V(x) = \frac{U(x)}{1+U(x)}$, is a utility function.

(iii) Suppose that $e : \mathbb{R}^\ell_{++} \times \mathbb{R} \to \mathbb{R}^\ell_+$ is the *expenditure function* associated with a continuous utility function $U$ on $\mathbb{R}^\ell_+$. Show that $e(p, \cdot)$ is strictly increasing for all $p \in \mathbb{R}^\ell_{++}$ and $e(\cdot, u)$ is non-decreasing in $p_j$ for all $1 \leq j \leq \ell$ and $u > U(0)$. [3+3+6]

Q3. Answer **all** questions.

(i) Prove or disprove: If $U$ is a continuous utility function representing a preference relation $\succeq$ over $\mathbb{R}^\ell_+$ then $\succeq$ is continuous.

(ii) Let $U : \mathbb{R}^{\ell}_{+} \to \mathbb{R}$ be a continuous utility representation of a locally non-satiated preference $\succeq$ over $\mathbb{R}^{\ell}_{+}$. Given a $(p, w) \in \mathbb{R}^{\ell}_{++} \times \mathbb{R}_{++}$, the *utility maximization problem* (*UMP*) of the consumer is the following:

$$\sup\{U(x) : x \in B(p, \omega)\}$$

where $B(p, w) = \{x \in \mathbb{R}^{\ell}_{+} : p \cdot x \leq w\}$ is the budget set for $(p, w)$. The expenditure minimization problem (*EMP*) for any given $p \in \mathbb{R}^{\ell}_{++}$ and any $u > U(0)$ is the following:

$$\inf\left\{p \cdot x : x \in \mathbb{R}^{\ell}_{+}, U(x) \geq u\right\}.$$

Show that if $x$ is the optimal solution of the *UMP* when $\omega > 0$, then $x$ is the optimal solution of the *EMP* when the required level of utility is $U(x)$. Show further that the minimum expenditure level in this *EMP* is $\omega$.

(iii) Give an example of an economy to show that a weakly Pareto optimal allocation is not necessarily a Pareto optimal allocation if all consumers have increasing utility functions and exactly one consumer has a discontinuous utility function.

[2+5+5]

Q4. Let $Y \subseteq \mathbb{R}^{\ell}$ be a production set.

(i) Show that if $y$ is profit maximizing for some $p \in \mathbb{R}^{\ell}_{++}$ then $y$ is efficient. Does the same conclusion hold if the condition $p \in \mathbb{R}^{\ell}_{++}$ is replaced with $p \in \mathbb{R}^{\ell}_{+} \setminus \{0\}$?

(ii) Find the additive closure of $Y = \{(-x, y_1, y_2) : x, y_1, y_2 > 0, y_1 + y_2 \leq x^3\}$.

(iii) Show that for a single-output technology, $Y$ is convex if and only if the production function $f$ is concave. [4+3+5]

Q5. Answer **all** questions.

(i) Find the cost function of a single output technology whose production function is given by $f(z) = z_1 + z_2$, where $z \in \mathbb{R}^{\ell-1}_{+}$ for $\ell \geq 3$.

(ii) Suppose that $Y$ denotes the aggregate production set of production sets $Y_1, \cdots, Y_m$. If $\pi$ and $\pi_i$ are profit functions for $i = 1, \cdots, m$ then show that $\pi(p) = \sum_{i=1}^{m} \pi_i(p)$ for $p \in \mathbb{R}^{\ell}_{++}$. [6+6]

Q6. Answer **all** questions.

(i) Show that if $U$ is a linear utility function, then

$$U(\hat{L}) = \sum_{i=1}^{m} U(L_i)$$

is satisfied for every compound lottery $\hat{L} = [q_1(L_1), \cdots, q_m(L_m)]$.

(ii) Let $\mathscr{L}$ denote the set of simple lotteries over $\mathcal{O} = \{a, b, c\}$. Define a preference relation $\succeq$ over $\mathscr{L}$ by

$$L_1 \succeq L_2 \text{ if either } [p_3 < p_3'] \text{ or } [p_3 = p_3' \text{ and } p_2 < p_2'],$$

where

$$L_1 = [p_1(a), p_2(b), p_3(c)] \text{ and } L_2 = [p_1'(a), p_2'(b), p_3'(c)].$$

Show that preference relation violates the continuity axiom of the Von Neumann-Morgenstern expected utility representation theorem.

(iii) Consider an economy with $I$ consumers and $J$ firms. Each consumer's preference is represented by a quasi-linear utility function $U_i(m_i, x_i) = m_i + \phi_i(x_i)$, where $m_i$ denotes the numeraire and $\phi_i(x_i) = 2x_i^{\frac{1}{2}}$. Firm $j$'s cost function is defined by $c_j(q_j) = \frac{q_j^2}{2}$. Assume that agent $i$'s initial endowment is $(a_i, 0)$, where $a_i > 0$. Find the sufficient condition for the existence of a partial equilibrium and then find an equilibrium price. [4+4+4]

# INDIAN STATISTICAL INSTITUTE
## Second Semestral Examination: (2014-2015)
## MS (Q.E.) I Year
## Macroeconomics I

Date: 27.04.15 Maximum Marks 60 Duration 3 hours

**Group A**
**Answer any two**

1. Rate of economic growth in a two sector planned economy varies positively with the investment share allocated to the investment good producing sector.— Examine the validity of the statement in the light of the Feldman-Mahalanobis model. [15]

2. Derive the optimum income tax rate in the endogenous model developed by Barro and interpret the derived result. [12+3]

3. Consider a two sector economy with the following two equations of motion.

$$\dot{K}_1 = K_1K_2 - 9$$

$$\dot{K}_2 = K_1 + K_2 - 10$$

Here $K_i$ is the capital stock of the $i$ th sector for i=1, 2.

a) Construct the phase diagram.

b) Examine the existence, uniqueness and stability of the steady-state equilibrium in this model. [7+2+2+4]

**Group B**
**Answer all questions**

1. Show how equilibrium unemployment is sustained in the Shapiro- Stiglitz model. What do you think would happen to equilibrium unemployment if the firms were to experience a technological progress (by this I simply mean that firms could produce more, say twice, the output they produced earlier, for any given combination of inputs). [12+3]

P.T.O

2. a) In a flex price, monopolistically competitive equilibrium of the Blanchard-Kiyotaki kind; show that money is neutral.

Also show that in such a model the monopolistically competitive output is smaller than the competitive output.

b) Consider an economy with the representative agent having the utility function:

$U = [C^{\alpha}(1-L)^{1-\alpha}]^{\gamma}\left[\frac{M}{P}\right]^{1-\gamma}$, $0<\alpha, \gamma<1$

Where $C = n\left[\frac{1}{n}\sum_{1=1}^{n} c_i{}^{\rho}\right]^{1/\rho}$, $0<\rho<1$ and $c_i$ is the consumption of the $i^{th}$ variety. $L$ is the labour supply, $P$ is the price index of the varieties. Each agent is endowed with one unit of labour, thereby $(1-L)$ is the leisure enjoyed. $M$ is the money balances (and suppose $M_0$ is the initial endowment of money). The household budget constraint is given by:

$PC + w(1-L) + M = M_0 + w + \pi - T$ where $w$ is the money wage rate and $\pi$ is the economy wide profits and $T$ is the taxes. Production of varieties is given by:

$$Y_i = 0 \; if \; L_i \leq F$$
$$= \frac{L_i - F}{k} \; if \; L_i > F \quad \text{where } k>0$$

$Y_i$ is the output of $i^{th}$ variety and $L_i$ is the labour employed in the production of the $i^{th}$ variety.

Assume that there are no costs in adjusting prices (i.e. prices are fully flexible) and that there is no entry/exit of firms (fixed $n$).

(i) Derive the multiplier of a balanced budget ($PG=T$) increase in government expenditure where $G$ takes the form:

$G = n\left[\frac{1}{n}\sum_{1=1}^{n} g_i{}^{\rho}\right]^{1/\rho}$ and $g_i$ is the government consumption of the $i^{th}$ variety.

(ii) What would be the effect of such increase in government expenditure on $P$ ?

[Hint: Try to write down the goods market equilibrium ($Y=C+G$) in a form which does not involve money balances. That would require a look into the money market equilibrium ( $M = M_0$ ).]

[10+5]

## INDIAN STATISTICAL INSTITUTE

**SEMESTRAL EXAMINATION: (2014-2015)**

**MSQE I and M.Stat II**

**Microeconomic Theory II**

Date: 05/05/2015 Maximum marks: 60+10 Duration: 3 Hours

**Note:** Answer **all** questions.

**Note:** Let $\mathbb{R}^\ell$ denote the $\ell$-dimensional Euclidean space. Assume that

$$\mathbb{R}^\ell_+ = \{x = (x^1, \cdots, x^\ell) \in \mathbb{R}^\ell : x^i \geq 0 \text{ for all } 1 \leq i \leq \ell\}$$

and

$$\mathbb{R}^\ell_{++} = \{x = (x^1, \cdots, x^\ell) \in \mathbb{R}^\ell : x^i > 0 \text{ for all } 1 \leq i \leq \ell\}.$$

Q1. Answer **all** questions.

(i) Let $\mathbb{R}_+$ denote the domain of wealth and $N = \{1, 2\}$ denote the set of agents. Suppose that $U_i : \mathbb{R}_+ \to \mathbb{R}$ be a strictly monotonically increasing and twice differentiable function for $i \in N$. If agent 1 is more risk averse than agent 2, then show that there is a strictly increasing and concave function $V : Im(U_2) \to \mathbb{R}$ such that $U_1 = V \circ U_2$, where $Im(U_2) = \{U_2(x) : x \in \mathbb{R}_+\}$ is the image of $U_2$.

(ii) Consider the utility function $U : \mathbb{R}_+ \to \mathbb{R}$ defined by $U(x) = x^2$. Find the certainty equivalence and the probability premium for lotteries

$$L_1 = \left[\frac{1}{2}(25), \frac{1}{2}(5)\right] \text{ and } L_2 = \left[\frac{1}{2}(36), \frac{1}{2}(16)\right].$$

Compare the probability premium of $L_1$ and $L_2$. [6+4]

Q2. Consider an economy $\mathscr{E} = \{I; \mathbb{R}^\ell_+; (\succsim_i, \omega_i)_{i \in I}\}$, where $I$ is the set of agents containing $m$ many elements; $\mathbb{R}^\ell_+$ is the consumption set of each agent; and $\succsim_i$ and $\omega_i$ are the preference and initial endowment of agent $i$, respectively. Suppose further that $\succ_i$ and $\sim_i$ are the strict preference and indifference relations associated with a rational preference relation $\succsim_i$ for all $i \in I$. A price is an element of $\mathbb{R}^\ell \setminus \{0\}$. Assume

$\mathscr{W}(\mathscr{E})$: the set of Walrasian equilibrium allocations of $\mathscr{E}$;

$\mathscr{C}(\mathscr{E})$: the core of $\mathscr{E}$;

$\mathscr{I}(\mathscr{E})$: the set of individually rational allocations of $\mathscr{E}$;

$\mathscr{P}(\mathscr{E})$: the set of Pareto optimal allocations of $\mathscr{E}$.

(i) Suppose that $\succeq_i$ is continuous and strictly monotone for all $i \in N$, and $\sum_{i \in I} \omega_i \in \mathbb{R}^\ell_{++}$. Show that every quasiequilibrium allocation is a Walrasian equilibrium allocation. [10]

(ii) Suppose that $\omega_i = \omega_j$ and $\succeq_i = \succeq_j$ for all $i, j \in N$. Let $\succeq_i$ be continuous, strictly convex and strictly monotone for $i \in N$. Show that $\mathscr{W}(\mathscr{E}) = \mathscr{I}(\mathscr{E}) = \mathscr{C}(\mathscr{E})$. Further, show that if $(x_1, x_2, \cdots, x_m) \in \mathscr{C}(\mathscr{E})$, then $x_i \sim \omega_i$ for all $i \in I$. [10]

(iii) Let $I = I_1 \cup I_2$ and $I_1 \cap I_2 = \emptyset$, where each $I_i$ has at least two agents. Put

$$\mathscr{E}_1 = \left\{I_1; \mathbb{R}^\ell_+; (\succeq_i, \omega_i)_{i \in I_1}\right\} \text{ and } \mathscr{E}_2 = \left\{I_2; \mathbb{R}^\ell_+ : (\succeq_i, \omega_i)_{i \in I_2}\right\}.$$

Suppose that $\bar{y} = (y_i : i \in I_1)$ and $\bar{z} = (z_i : i \in I_2)$ are Walrasian equilibrium allocations of $\mathscr{E}_1$ and $\mathscr{E}_2$, respectively. Show that $(\bar{y}, \bar{z})$ is an allocation of $\mathscr{E}$. Prove or disprove $(\bar{y}, \bar{z}) \in \mathscr{C}(\mathscr{E})$. [8]

(iv) Assume that $N = \{1, 2\}$ and $\ell = 2$. Suppose that $\succeq_i$ is represented by a utility function $U_i$ for $i = 1, 2$. Let

$$\begin{cases} \omega_1 = (2, 8), & U_1(x, y) = \min\{2x, y\}; \\ \omega_2 = (6, 0), & U_2(x, y) = \min\{x, 3y\}. \end{cases}$$

Find the set of Walrasian equilibrium allocations of $\mathscr{E}$. [6]

(v) If $\succeq_i$ is strictly convex for all $i \in I$, then show that $\mathscr{W}(\mathscr{E}) \subseteq \mathscr{P}(\mathscr{E})$. [6]

(vi) If $\succeq_i$ is continuous and monotone for all $i \in I$, then show that $\mathscr{P}(\mathscr{E})$ is a compact subset of $\mathbb{R}^{m\ell}$. [10]

Q3. Recall that a *production set* is a non-empty closed convex subset $Y$ of $\mathbb{R}^\ell$ such that $Y \cap \mathbb{R}^\ell_+ = \{0\}$ and there is some $a \in \mathbb{R}^\ell_+$ such that $y \leq a$ for all $y \in Y$. Show that for any $p \in \mathbb{R}^\ell_{++}$, there is some $y_0 \in Y$ such that $p \cdot y \leq p \cdot y_0$ for all $y \in Y$. Does the same conclusion hold if $p \in \mathbb{R}^\ell_+ \setminus \{0\}$? [10]

# INDIAN STATISTICAL INSTITUTE

## BACK PAPER EXAMINATION: (2014-2015)

## MSQE I and M.Stat II

## Microeconomic Theory II

Date: 13.07.15 Maximum marks: 100 Duration: 3 Hours

**Note:** Answer **all** questions.

**Note:** Let $\mathbb{R}^\ell$ denote the $\ell$-dimensional Euclidean space. Assume that

$$\mathbb{R}^\ell_+ = \{x = (x^1, \cdots, x^\ell) \in \mathbb{R}^\ell : x^i \geq 0 \text{ for all } 1 \leq i \leq \ell\}$$

and

$$\mathbb{R}^\ell_{++} = \{x = (x^1, \cdots, x^\ell) \in \mathbb{R}^\ell : x^i > 0 \text{ for all } 1 \leq i \leq \ell\}.$$

Q1. Let $\mathbb{R}_+$ denote the domain of wealth and $U$ be a twice-differentiable strictly increasing utility function for a decision maker. For any fixed amount of money $\omega$ and positive number $\varepsilon$, the probability premium is denoted by $\pi(\omega, \varepsilon, U)$ and the certainty equivalence for a lottery $L$ is denoted by $CE(L, U)$. Show that the following properties are equivalent:

(i) The decision maker is risk lover.
(ii) $\pi(w, \varepsilon, U) \leq 0$ for all $w \geq 0$ and all $\varepsilon > 0$ with $\omega - \varepsilon \geq 0$.
(iii) $CE(L, U) \geq \mathbb{E}(L)$ for all lottery $L$. [15]

Q2. Consider an economy $\mathscr{E} = \{N; \mathbb{R}^\ell_+; (\succeq_i, \omega_i)_{i \in N}\}$, where $N$ is the set of agents containing $n$ many elements; $\mathbb{R}^\ell_+$ is the consumption set of each agent; and $\succeq_i$ and $\omega_i$ are a rational preference and an initial endowment of agent $i$, respectively.

(i) If $\succeq_i$ is continuous, convex and strictly monotone for all $i \in N$, then show that an allocation is a Walsian equilibrium allocation if and only if it is an Edgeworth equilibrium. [20]

(ii) Let $N = \{1, 2\}$ and $\ell = 2$. Suppose that the preference relation $\succ_i$ is represented by a utility function $U_i$ for $i = 1, 2$. Given that

$$\begin{cases} \omega_1 = (2, 1), & U_1(x, y) = (y + 1)e^x; \\ \omega_2 = (2, 3), & U_2(x, y) = xy. \end{cases}$$

Find the set of Walrasian equilibrium of $\mathscr{E}$. [10]

(iii) Suppose that $\mathscr{E}_r$ is the $r$-fold replicated economy of $\mathscr{E}$ and $\succ_i$ is continuous, convex and strictly monotone for all $i \in N$. Let

$$x = (x_{11}, \cdots, x_{1r}, x_{21}, \cdots, x_{2r}, \cdots, x_{n1}, \cdots, x_{nr})$$

be a core allocation of $\mathscr{E}_r$ for some $r \geq 1$. Show that the allocation $\hat{x} = (\hat{x}_1, \cdots, \hat{x}_n)$ of $\mathscr{E}$, defined by

$$\hat{x}_i = \frac{1}{r}\sum_{j=1}^{r} x_{ij},$$

is a core allocation of $\mathscr{E}$. [15]

(iv) If $\{p_k : k \geq 1\} \subseteq \mathbb{R}^\ell_{++}$ satisfies $p_k \to p \in \mathbb{R}^\ell_{++}$, then show that there exists a bounded subset $B_i$ of $\mathbb{R}^\ell_+$ such that the demand set $D_i(p_k, \omega_i, \succeq_i) \subseteq B_i$ holds for each $i \in N$ and $k \geq 1$. [15]

(v) If an allocation is both a Walrasian equilibrium allocation and a quasiequilibrium allocation, then show that it is a Pareto optimal allocation. [10]

(vi) Let $\succeq_i$ be continuous and strictly monotone for all $i \in N$. Show that an allocation $(x_1, \cdots, x_n)$ is a core allocation if and only if there are no coalition $S$ and a set of vectors $\{y_i \in \mathbb{R}^\ell_+ : i \in S\}$ such that

$$\sum_{i \in S} y_i \leq \sum_{i \in S} \omega_i$$

and $y_i \succeq_i x_i$ for all $i \in S$ and $y_i \succ_i x_i$ for some $i \in S$. [15]