Consumer choice in a market economy

Anuj Bhowmik

Economic Research Unit
Indian Statistical Institute
203 Barackpore Trunk Road
Kolkata 700108
India

Email: anuj.bhowmik@isical.ac.in,
anujbhowmik09@gmail.com

Homepage: http://www.isical.ac.in/∼anuj.bhowmik/

Microeconomic Theory I
Semester I, 2013

Outline

Outline

Outline

1. Basic elements of the consumer's decision problem
2. Comparative statics
3. Walrasian demand function and the weak axiom
4. Walrasian demand function and rational preference

Outline

1. Basic elements of the consumer's decision problem
2. Comparative statics
3. Walrasian demand function and the weak axiom
4. Walrasian demand function and rational preference

Mas-Colell, A., Whinston, M. D. and Green, J. (1995).
Microeconomic Theory

Introduction

We study the consumer demand in the context of a market economy. Market economy is an economy where the goods and services (that the consumer may acquire) are either available for purchase at known prices or, are available for trade for other goods at known rates of exchange.

Commodities

The decision problem faced by the consumer in a market economy is to choose consumption levels of the various *commodities* (*goods and services*) that are available for purchase in the market.

We assume that the number of commodities is finite and equal to L (indexed by $\ell = 1, \cdots, L$).

A *commodity vector* (or, *commodity bundle*) is a list of the different commodities,

$$x = \begin{bmatrix} x_1 \\ \vdots \\ x_L \end{bmatrix} \in \mathbb{R}^L.$$

The consumption set

The consumption set is $X \subseteq \mathbb{R}^L$ whose elements are the consumption bundles that the individual can conceivably consume given the physical constraints imposed by the consumer's environment (for example, supplying of several types of labor to an amount totaling more than 24 hours in a day is impossible).

To keep things straightforward, let

$$X = \mathbb{R}^L_+ = \left\{ x \in \mathbb{R}^L \mid x_\ell \geq 0 \text{ for all } 1 \leq \ell \leq L \right\}.$$

A special feature of $X = \mathbb{R}^L_+$ is that it is convex, that is, if $x \in \mathbb{R}^L_+$ and $x' \in \mathbb{R}^L_+$ then $\alpha x + (1 - \alpha)x' \in \mathbb{R}^L_+$ for any $\alpha \in [0, 1]$.

Competitive Budgets

An individual's consumption choice is limited to those commodity bundles that he can afford.

The L-commodities are traded in the market at rupee prices that are publicly quoted (principle of completeness or, universality of markets). In general, a price vector is

$$p = \begin{bmatrix} p_1 \\ \vdots \\ p_L \end{bmatrix} \in \mathbb{R}^L,$$

where $p_l < 0$ simply means that the buyer is paid to consume the commodity.

The two assumptions are the following:

(A1) $p >> 0$, that is $p_\ell > 0$ for all $\ell \in \{1, \cdots, L\}$.

(A2) Individuals are *price takers*, that is, the prices are beyond the influence of the consumers.

A consumption bundle $x \in X = \mathbb{R}^L_+$ is *affordable* if its total cost does not exceed the consumer's wealth $w > 0$, that is,

$$p.x = p^T x = \begin{bmatrix} p_1 & \ldots & p_L \end{bmatrix} \begin{bmatrix} x_1 \\ \vdots \\ x_L \end{bmatrix} = \sum_{\ell=1}^{L} p_\ell x_\ell \leq w.$$

The *Walrasian budget set* is

$$B(p, w) = \left\{x \in \mathbb{R}^L_+ : p.x \leq w\right\}.$$

It is the set of feasible consumption bundles for the consumer who faces market price p and has wealth w. The set $\left\{x \in \mathbb{R}^L_+ : p.x = w\right\}$ is called the *budget hyperplane* and for $L = 2$, it is called the *budget line*.

The slope of the budget line captures the rate of exchange between the two commodities.

The Walrasian budget set is convex, that is, if $x \in B(p, x)$ and $x' \in B(p, w)$ then for any $\alpha \in [0, 1]$, $\alpha x + (1 - \alpha)x' \in B(p, w)$.

What is a sufficient restriction on an arbitrary X that leads to convexity of the Walrasian budget set $B(p, w)$?

Exercise

Consider an extension of the Walrasian budget set to an arbitrary consumption set X: $B(p, w) = \{x \in X : p.x \leq w\}$. Show that if X is a convex set, then $B(p, w)$ is as well.

Solution: Suppose that $x, x' \in B(p, w)$. Consider any $\alpha \in [0, 1]$ and the commodity bundle $x'' = \alpha x + (1 - \alpha)x'$. Since X is convex, $x'' \in X$. Moreover, since $x, x' \in B(p, w)$, $p.x \leq w$ and $p.x' \leq w$. Therefore,

$$p.x'' = \alpha p.x + (1 - \alpha)p.x' \leq \alpha w + (1 - \alpha)w = w$$

implying that $x'' \in B(p, w)$. Thus, $B(p, w)$ is convex.

Demand functions

The consumer's *Walrasian (or, market or, ordinary) demand correspondence* is denoted by $D : \mathbb{R}^\ell \times \mathbb{R}_+ \rightrightarrows X$.

Here, $D(p, w)$ assigns a set of chosen consumption bundles for each price-wealth pair (p, w).

Implicit here is the fact that if $x \in D(p, w)$, then x is necessarily affordable. If D is single-valued, that is, $D(p, w)$ is singleton, then it is called the *demand function* and is denoted by $x : \mathbb{R}^\ell \times \mathbb{R}_+ \to X$. Usually, the demand function is represented as

$$x(p, w) = \begin{bmatrix} x_1(p, w) \\ \vdots \\ x_L(p, w) \end{bmatrix}.$$

Definition

The Walrasian demand correspondence D is *homogeneous of degree zero* if

$$D(\alpha p, \alpha w) = D(p, w)$$

for any $(p, w) >> 0$ and any $\alpha > 0$.

Homogeneity of degree zero says that if prices and wealth change in the same proportion, then individual's consumption choice does not change. To understand this, first note that

$$B(p, w) = B(\alpha p, \alpha w).$$

This means that a change in prices and wealth from (p, w) to $(\alpha p, \alpha w)$ leads to no change in the consumer's set of feasible consumption bundles. Homogeneity of degree zero says that individual choice depends only on the set of feasible points.

Definition

The Walrasian demand correspondence D satisfies *Walras' law* if for every $(p, w) >> 0$ and $x \in D(p, w)$, we have $p.x = w$.

Walras's law says that the consumer fully expends his wealth. Intuitively, this is a reasonable assumption to make as long as there is some good that is clearly desirable.

The examination of a change in outcome ($x(p, w)$) in response to a change in underlying economic parameters (p, w) is known as the comparative statics analysis.

Wealth effects

Fix $\bar{p} >> 0$. The function $x(\bar{p}, \cdot) : \mathbb{R} \to \mathbb{R}^L_+$ is a function of just w and is called the consumer's *Engel function*.

Its image is $E_{\bar{p}} = \{x(\bar{p}, w) : w > 0\}$ and is called the *wealth expansion path*.

Choose a commodity $\ell \in \{1, \cdots, L\}$. It is

1. *normal* at (p, w) if $\frac{\partial}{\partial w} x_\ell(p, w) \geq 0$.
2. *inferior* if $\frac{\partial}{\partial w} x_\ell(p, w) < 0$.

If $\frac{\partial}{\partial w} x_\ell(p, w) \geq 0$ for all $\ell \in \{1, \cdots, L\}$ and $(p, w) >> 0$, then we say that *demand is normal*.

Wealth effects

Fix $\bar{p} >> 0$. The function $x(\bar{p}, \cdot) : \mathbb{R} \to \mathbb{R}^L_+$ is a function of just w and is called the consumer's *Engel function*.

Its image is $E_{\bar{p}} = \{x(\bar{p}, w) : w > 0\}$ and is called the *wealth expansion path*.

Choose a commodity $\ell \in \{1, \cdots, L\}$. It is

1. *normal* at (p, w) if $\frac{\partial}{\partial w} x_\ell(p, w) \geq 0$.
2. *inferior* if $\frac{\partial}{\partial w} x_\ell(p, w) < 0$.

If $\frac{\partial}{\partial w} x_\ell(p, w) \geq 0$ for all $\ell \in \{1, \cdots, L\}$ and $(p, w) >> 0$, then we say that *demand is normal*.

Wealth effects

Fix $\bar{p} >> 0$. The function $x(\bar{p}, \cdot) : \mathbb{R} \to \mathbb{R}^L_+$ is a function of just w and is called the consumer's *Engel function*.

Its image is $E_{\bar{p}} = \{x(\bar{p}, w) : w > 0\}$ and is called the *wealth expansion path*.

Choose a commodity $\ell \in \{1, \cdots, L\}$. It is

1. *normal* at (p, w) if $\frac{\partial}{\partial w} x_\ell(p, w) \geq 0$.
2. *inferior* if $\frac{\partial}{\partial w} x_\ell(p, w) < 0$.

If $\frac{\partial}{\partial w} x_\ell(p, w) \geq 0$ for all $\ell \in \{1, \cdots, L\}$ and $(p, w) >> 0$, then we say that *demand is normal*.

Wealth effects

Fix $\bar{p} >> 0$. The function $x(\bar{p}, \cdot) : \mathbb{R} \to \mathbb{R}^L_+$ is a function of just w and is called the consumer's *Engel function*.

Its image is $E_{\bar{p}} = \{x(\bar{p}, w) : w > 0\}$ and is called the *wealth expansion path*.

Choose a commodity $\ell \in \{1, \cdots, L\}$. It is

1. *normal* at (p, w) if $\frac{\partial}{\partial w} x_\ell(p, w) \geq 0$.
2. *inferior* if $\frac{\partial}{\partial w} x_\ell(p, w) < 0$.

If $\frac{\partial}{\partial w} x_\ell(p, w) \geq 0$ for all $\ell \in \{1, \cdots, L\}$ and $(p, w) >> 0$, then we say that *demand is normal*.

Wealth effects

Fix $\bar{p} >> 0$. The function $x(\bar{p}, \cdot) : \mathbb{R} \to \mathbb{R}^L_+$ is a function of just w and is called the consumer's *Engel function*.

Its image is $E_{\bar{p}} = \{x(\bar{p}, w) : w > 0\}$ and is called the *wealth expansion path*.

Choose a commodity $\ell \in \{1, \cdots, L\}$. It is

1. *normal* at (p, w) if $\frac{\partial}{\partial w} x_\ell(p, w) \geq 0$.
2. *inferior* if $\frac{\partial}{\partial w} x_\ell(p, w) < 0$.

If $\frac{\partial}{\partial w} x_\ell(p, w) \geq 0$ for all $\ell \in \{1, \cdots, L\}$ and $(p, w) >> 0$, then we say that *demand is normal*.

Wealth effects

Fix $\bar{p} >> 0$. The function $x(\bar{p}, \cdot) : \mathbb{R} \to \mathbb{R}^L_+$ is a function of just w and is called the consumer's *Engel function*.

Its image is $E_{\bar{p}} = \{x(\bar{p}, w) : w > 0\}$ and is called the *wealth expansion path*.

Choose a commodity $\ell \in \{1, \cdots, L\}$. It is

1. *normal* at (p, w) if $\frac{\partial}{\partial w} x_\ell(p, w) \geq 0$.
2. *inferior* if $\frac{\partial}{\partial w} x_\ell(p, w) < 0$.

If $\frac{\partial}{\partial w} x_\ell(p, w) \geq 0$ for all $\ell \in \{1, \cdots, L\}$ and $(p, w) >> 0$, then we say that *demand is normal*.

Wealth effects(continued)

Normality assumption of normal demand makes sense if commodities are large aggregates (for example, food and shelter).

If they are very disaggregated (example, particular kinds of shoes) then because of substitution to higher quality goods as wealth increases, goods that become inferior at some level of wealth may be a rule rather than an exception.

The change in demand function with a change in wealth is summarized by

$$D_w x(p,w) = \begin{bmatrix} \frac{\partial}{\partial w} x_1(p,w) \\ \vdots \\ \frac{\partial}{\partial w} x_L(p,w) \end{bmatrix}.$$

Price effects

The partial derivative $\frac{\partial}{\partial p_k} x_\ell(p, w)$ is the price effect of p_k on the demand for good ℓ.

If $k = \ell$ then we have own price effect and if $k \neq \ell$ we have cross-price effect.

Although it may be natural to think that a fall in a good's price will lead the consumer to purchase more of it, the reverse situation is not economically impossible.

This kind of situation may arise for consumers with low income levels. For example, if price of potatoes fall then an individual eats other foods that also keep him from being hungry and we have $\frac{\partial}{\partial p_\ell} x_\ell(p, w) > 0$.

The change in demand function with respect to a change in prices is summarized by

$$D_p x(p,w) = \begin{bmatrix} \frac{\partial}{\partial p_1} x_1(p,w) & \dots & \frac{\partial}{\partial p_L} x_1(p,w) \\ \vdots & \dots & \vdots \\ \frac{\partial}{\partial p_1} x_L(p,w) & \cdots & \frac{\partial}{\partial p_L} x_L(p,w) \end{bmatrix}.$$

Implications of homogeneity and Walras' law

Proposition

If the Walrasian demand function $x(p, w)$ is homogeneous of degree zero, then for all $(p, w) >> 0$,

$$D_p x(p, w)p + D_w x(p, w)w = \underline{0}$$

or, equivalently

$$\sum_{k=1}^{L} \frac{\partial}{\partial p_k} x_\ell(p, w) \cdot p_k + \frac{\partial}{\partial w} x_\ell(p, w) \cdot w = 0 \text{ for all } \ell \in \{1, \cdots, L\}.$$

Sketch of the proof

Let $Y(\alpha) = x(\alpha p, \alpha w) - x(p, w)$. So, $Y(\alpha)$ is a $L \times 1$ vector. Since $x(p, w)$ is homogeneous of degree zero, $Y(\alpha) = \underline{0}$. Thus, $D_\alpha Y(\alpha) = \underline{0}$, which yields

$$\begin{bmatrix} \sum_{k=1}^{L} \frac{\partial}{\partial \alpha p_k} x_1(\alpha p, \alpha w) \cdot p_k + \frac{\partial}{\partial \alpha w} x_1(\alpha p, \alpha w) \cdot w \\ \vdots \\ \sum_{k=1}^{L} \frac{\partial}{\partial \alpha p_k} x_L(\alpha p, \alpha w) \cdot p_k + \frac{\partial}{\partial \alpha w} x_L(\alpha p, \alpha w) \cdot w \end{bmatrix} = \underline{0}.$$

By taking $\alpha = 1$, we get the result.

The *elasticities* of demand with respect to prices and wealth are given by

$$\varepsilon_{\ell k}(p, w) = \frac{\partial x_\ell(p, w)}{\partial p_k} \cdot \frac{p_k}{x_\ell(p, w)}$$

and

$$\varepsilon_{\ell w}(p, w) = \frac{\partial x_\ell(p, w)}{\partial w} \cdot \frac{w}{x_\ell(p, w)}.$$

These elasticities give the *percentage change* in demand for good ℓ per percentage change in the price of good k or wealth.

The expression for $\varepsilon_{\ell w}(p, w)$ can be read as $(\Delta x/x)/(\Delta w/w)$.

Elasticities are independent of the units chosen for measuring commodities and therefore provide a unit-free way of capturing changes in demand function.

The the equation of the previous proposition can be written as

$$\sum_{k=1}^{L} \frac{\partial}{\partial p_k} x_\ell(p, w) \frac{p_k}{x_\ell(p, w)} + \frac{\partial}{\partial w} x_\ell(p, w) \frac{w}{x_\ell(p, w)} = 0$$

for all $\ell \in \{1, \cdots, L\}$, which means

$$\sum_{k=1}^{L} \varepsilon_{\ell k}(p, w) + \varepsilon_{\ell w}(p, w) = 0$$

for all $\ell \in \{1, \cdots, L\}$.

Assume that $x(\cdot,\cdot)$ is homogeneous of degree zero, and satisfies Walras' law.

The family of the Walrasian budget sets is

$$\mathscr{B}^{\mathscr{W}} = \{B(p,w) : p >> 0, w > 0\}.$$

Note that $(\mathscr{B}^{\mathscr{W}}, x(\cdot,\cdot))$ is a choice function.

The *weak axiom of revealed preference (or, WARP)* holds for $(\mathscr{B}^{\mathscr{W}}, x(.,.))$ if the following condition is satisfied.

For any two price-wealth situations (p,w) and (p',w'), if $p \cdot x(p',w') \leq w$ and $x(p',w') \neq x(p,w)$ then $p' \cdot x(p,w) > w'$.

Proposition

The following statements are equivalent.

(i) $(\mathscr{B}^{\mathscr{W}}, x(\cdot,\cdot))$ or, simply $x(\cdot,\cdot)$ satisfies *WARP*.

(ii) For any $w > 0$ and all p, p' we have $p'.x(p,w) > w$ if $p.x(p',w) \leq w$ and $x(p',w) \neq x(p,w)$.

Proof. (i)$\Rightarrow$ (ii): Take $w' = w$ in the definition of *WARP*.

(ii)$\Rightarrow$ (i): Suppose (i) is not true. Then there exists (p,w) and (p',w') such that

$$p \cdot x(p',w') \leq w \text{ and } p' \cdot x(p,w) \leq w'.$$

By the fact that $x(\cdot,\cdot)$ is homogeneous of degree zero, we have

$$p \cdot x(\alpha p', \alpha w') \leq w \text{ for any } \alpha > 0.$$

By setting $\alpha = \frac{w}{w'}$ and defining $\bar{p} = \alpha p'$, we get $p.x(\bar{p}, w) \leq w$.

Now, $p' \cdot x(p, w) \leq w' \Rightarrow \bar{p}.x(p, w) \leq w$, which is a contradiction.

Implications of the Weak Axiom

The weak axiom has significant implications for the effects of price changes on demand.

The price changes effect the consumer in two ways. First, they alter the relative cost of different commodities. But, second, they also change the consumer's real wealth.

We now consider the second case. Let the consumer be originally facing prices p and wealth w and chooses consumption bundle $x(p, w)$. If prices change to p, we imagine that consumer wealth adjusted to $w' = p' \cdot x(p, w)$.

Thus, the wealth adjustment is $w' - w = (p' - p) \cdot x(p, w)$, that is, $\Delta w = \Delta p.x(p, w)$. This kind of wealth compensation is called *Slutsky wealth compensation*.

The budget hyperplane corresponding to (p', w') goes through the vector $x(p, w)$.

For a Walrasian demand function $x(\cdot, \cdot)$, we say that *WARP holds for all compensated price change* if for all pairs (p, w) and (p', w') such that $p'.x(p, w) = w'$ and $x(p, w) \neq x(p', w')$ we have $p.x(p', w') > w$.

Recall that when we just say that *WARP holds* then we mean that for all pairs (p, w) and (p', w') such that $p'.x(p, w) \leq w'$ and $x(p, w) \neq x(p', w')$ we have $p.x(p', w') > w$.

Lemma

For a demand function $x(\cdot,\cdot)$, *WARP* holds if and only if *WARP* holds for all compensated price changes.

Proof. Only if part: It follows from the definition.

If part: Suppose that *WARP* is violated, that is, there exist (p', w') and (p'', w'') such that $x(p', w') \neq x(p'', w'')$,

$$p' \cdot x(p'', w'') \leq w' \text{ and } p'' \cdot x(p', w') \leq w''.$$

If one of the inequalities hold with equality then we have a compensated price change which means that we are done.

So assume $p' \cdot x(p'', w'') < w'$ and $p'' \cdot x(p', w') < w''$. Choose some $\alpha \in (0, 1)$ such that

$$[\alpha p' + (1 - \alpha)p''] \cdot x(p', w') = [\alpha p' + (1 - \alpha)p''] \cdot x(p'', w'').$$

Define

$$p = \alpha p' + (1 - \alpha)p'' \text{ and } w = (\alpha p' + (1 - \alpha)p'') \cdot x(p', w').$$

Observe that

$$\alpha w' + (1 - \alpha)w'' > \alpha p' \cdot x(p', w') + (1 - \alpha)p'' \cdot x(p', w')$$

$$= p \cdot x(p', w') = w = p \cdot x(p, w) = \alpha p' \cdot x(p, w) + (1 - \alpha)p'' \cdot x(p, w).$$

Either $w' > p' \cdot x(p, w)$ or, $w'' > p'' \cdot x(p, w)$.

If $w' > p' \cdot x(p, w)$, we have $x(p', w') \neq x(p, w)$,

$$p \cdot x(p', w') = w \text{ and } p' \cdot x(p, w) < w'$$

which is a violation of *WARP* for the compensated price change from (p', w') to (p, w).

If $w'' > p'' \cdot x(p, w)$, we have $x(p'', w'') \neq x(p, w)$,

$$p \cdot x(p'', w'') = w \text{ and } p'' \cdot x(p, w) < w''$$

which is a violation of *WARP* for the compensated price change from (p'', w'') to (p, w).

Proposition

The demand function $x(p, w)$ satisfies *WARP* *if and only if* the following property holds: For any compensated price change from (p, w) to a new $(p', w') = (p', p' \cdot x(p, w))$, we have

$$(p' - p) \cdot [x(p', w') - x(p, w)] \leq 0$$

with strict inequality whenever $x(p, w) \neq x(p', w')$.

Proof: Only if part:

Case 1: $x(p, w) = x(p', w')$.

In this case, the result is immediate.

Case 2: $x(p, w) \neq x(p', w')$.

First, consider $p' \cdot [x(p', w') - x(p, w)]$. By Walras' law, $p' \cdot x(p', w') = w'$. Since w' is the compensated price change, $p' \cdot x(p, w) = w'$.

Hence,

$$p' \cdot [x(p', w') - x(p, w)] = p' \cdot x(p', w') - p' \cdot x(p, w) = 0.$$

Next, consider $p \cdot [x(p', w') - x(p, w)]$. Since compensated

price change from (p, w) to (p', w'), it follows from *WARP* that $p \cdot x(p', w') > w$.

By Walras' law, $p \cdot x(p, w) = w$. So, $p \cdot [x(p', w') - x(p, w)] > 0$. Thus,

$$(p' - p) \cdot [x(p', w') - x(p, w)] = -p \cdot [x(p', w') - x(p, w)] < 0.$$

If part: Suppose that *WARP* is violated. Then by the lemma, there exists compensated price change such that *WARP* is violated. Thus, there exists (p', w') and (p, w) such that $x(p, w) \neq x(p', w')$,

$$p' \cdot x(p, w) = w' \text{ and } p \cdot x(p', w') \leq w.$$

Since $x(\cdot, \cdot)$ satisfies Walras' law,

$$p' \cdot [x(p', w') - x(p, w)] = 0 \text{ and } p \cdot [x(p', w') - x(p, w)] \leq 0.$$

Hence, we have

$$(p' - p) \cdot [x(p', w') - x(p, w)] \geq 0$$

which is a violation of inequality since $x(p, w) \neq x(p', w')$.

How would a theory of demand that is based solely on the assumption of homogeneity of degree zero, Walras' law and the consistency requirement embodied in the *WARP* compare with the one based on rational preference maximization?

The two approaches (preference based approach and choice based approach) are not equivalent. The theory based on *WARP* is weaker than the theory based on preference maximization.

If the demand function $x(\cdot, \cdot)$ is consistent with a preference relation then $x(p, w) \succ y$ for all $y \in B(p, w) \setminus \{x(p, w)\}$.

The following example shows that given a Walrasian demand function having *WARP* there is no consistent rational preference relation.

Example

Let

1. $p^1 = (2, 1, 2)$, $p^2 = (2, 2, 1)$, $p^3 = (1, 2, 2)$;
2. $w^1 = w^2 = w^3 = 8$;
3. $x(p^1, w^1) = x^1 = (1, 2, 2)$, $x(p^2, w^2) = x^2 = (2, 1, 2)$, and $x(p^3, w^3) = x^3 = (2, 2, 1)$.

Example

Note that

- $p^3 \cdot x(p^2, w^2) = w^3 = 8$, $x(p^2, w^2) \neq x(p^3, w^3)$ and $9 = p^2 \cdot x(p^3, w^3) > w^2 = 8$. Thus, $x^3 \succ^* x^2$ which implies that $x^3 \succ x^2$.
- $p^2 \cdot x(p^1, w^1) = w^2 = 8$, $x(p^1, w^1) \neq x(p^2, w^2)$ and $9 = p^1 \cdot x(p^2, w^2) > w^1 = 8$. Thus, $x^2 \succ^* x^1$ which implies that $x^2 \succ x^1$.
- $p^1 \cdot x(p^3, w^3) = w^1 = 8$, $x(p^1, w^1) \neq x(p^3, w^3)$ and $9 = p^3 \cdot x(p^1, w^1) > w^3 = 8$. Thus, $x^1 \succ^* x^3$ which implies that $x^1 \succ x^3$.

Example

Hence we have $x^3 \succ x^2 \succ x^1 \succ x^3$ which is incompatible with the fact that $\succeq$ rational preference.

Proposition

If the Walrasian demand function $x : \mathbb{R}^\ell \times \mathbb{R}_+ \to \mathbb{R}^\ell_+$ is generated by a rational preference relation, then it must satisfy *WARP*.

Proof: Assume not. Then there exists $(p, w) >> 0$ and $(p', w') >> 0$ such that $x(p, w) \neq x(p', w')$,

$$p \cdot x(p', w') \leq w \text{ and } p' \cdot x(p, w) \leq w'.$$

For the budget set $B(p, w)$, $x(p, w), x(p', w') \in B(p, w)$ and

$$x(p', w') \neq x(p, w) \Rightarrow x(p, w) \succ x(p', w')$$

and for the budget set $B(p', w')$, $x(p, w), x(p', w') \in B(p', w')$ and

$$x(p, w) \neq x(p', w') \Rightarrow x(p', w') \succ x(p, w).$$

Thus, we have a violation of rationality of $\succeq$ over $\mathbb{R}^{\ell}_{+}$.

Hence, the Walrasian demand function $x(p, w)$ generated by the rational preference relation $\succeq$ must satisfy *WARP*.