ASYMMETRIC INFORMATION, SIGNALING AND SCREENING

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1. INTRODUCTION

Information is often asymmetrically held by market participants. In the used car market, the seller of a used car has more information about the car's quality than a prospective buyer. When a firm hires a worker, the firm's knowledge about the worker's innate ability is less than that of the worker. When an automobile insurance company insures an individual, the individual knows more about his/her own driving skills than the insurance company. We will see that in the presence of asymmetric information, market often fails to be Pareto optimal. The tendency for inefficiency can become very severe due to a phenomenon called adverse selection. Adverse selection arises when an informed individual's trading decision depend on his/her privately held information in a manner that adversely affects market participants. This phenomenon was first pointed out by Akerlof [1].

2. THE MARKET FOR LEMONS

There is a big price difference between new and used cars. We try to provide an explanation for this phenomenon rather than claiming that this phenomenon is mainly due to the pure joy of owning a new car.[1] Information about specific used cars create information asymmetries, that is, a seller knows more while a buyer knows less. It is this asymmetry that leads to something like the Gresham's Law. Gresham's Law states that bad money drives good money out of circulation. In the used car market this means that bad used cars drive out good used cars from the market.

2.1. **A simple example.** Suppose that there are two types of used cars: Good used cars ('peaches') and bad used cars ('lemons') whose valuations are measured in some monetary units (dollars, rupees etc). Assume that sellers and buyers are risk neutral. Also assume the following: (a) Supply of cars is fixed, that is, there are n_l lemons and n_p peaches. Moreover, assume that $n_l = 2n_p$. (b) Supply of potential buyers is infinite with each buyer willing to buy one car. (c) Valuation of a peach to a buyer is $v_{bp} = 3000$. (d) Valuation of a peach to a seller is $v_{sp} = 2500$, $v_{bp} > v_{sp}$. (e) Valuation of a lemon to a buyer is $v_{bl} = 2000$. (f) Valuation of a lemon to a seller is $v_{sl} = 1000$, $v_{bl} > v_{sl}$.

Suppose that both buyers and sellers know about the quality of the car, that is, we have a situation of *complete information*. Since the sellers are in the short side of the market, they have all the bargaining power which means that in equilibrium peaches will be sold at a price of $P_p = 3000$ and lemons will be sold at a price of $P_l = 2000$.

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[1]The two examples to follow are from Kreps [2].

If we have the other extreme, that is, neither the seller nor the buyer knows about the quality of the car, then we have *incomplete information*. In this scenario both buyers and sellers form expectations about the valuation of the cars using their available information. Each seller assumes that the car that the seller owns is a peach with probability $\frac{1}{3}$ and a lemon with probability $\frac{2}{3}$. From our risk neutrality assumption it follows that the expected worth of a car to any seller is $E(V_s) = \frac{1}{3}v_{sp} + \frac{2}{3}v_{sl} = \frac{1}{3} \times 2500 + \frac{2}{3} \times 1000 = 1500$. Similarly, each buyer assumes that the car that the buyer is buying is a peach with probability $\frac{1}{3}$ and a lemon with probability $\frac{2}{3}$. From our risk neutrality assumption it also follows that the expected worth of a car to any buyer is $E(V_b) = \frac{1}{3}v_{bp} + \frac{2}{3}v_{bl} = \frac{1}{3} \times 3000 + \frac{2}{3} \times 2000 = 2333.33$. Again since sellers are in the short side of the market, the market clearing price is $P^* = 2333.33$.

In reality sellers have complete knowledge about the quality of the car and buyers have no knowledge. What will be the market clearing price? *Peach market breaks down and only lemons are traded in the market.* The reason is the following:

(i) At any price $P \in [0, 1000)$ no car is supplied by the sellers.
(ii) At $P \in [1000, 2500)$ only lemons will be offered for sale. A rational buyer will assume that the car for sale is a lemon and hence is worth 2000 to the buyer.
(iii) If $P \geq 2500$, then both lemons and peaches will arrive in the market. Buyer concludes that the car has an expected worth of $E(V_b) = 2333.33 < 2500$.
(iv) There is no demand at price above 2000. The reason being the following:
  (a) At a price above 2333.33 there is no demand whatsoever as no buyer is willing to pay that much.
  (b) At price below 2500 the only cars available for trade are lemons. Hence $P^{**} = 2000$.

In equilibrium only lemons are put for sale at a price $P^{**} = 2000$. Further gains from trade are theoretically possible between the owners of peaches and buyers, but these gains cannot be realized because buyers cannot be sure that they are not getting lemons. Thus, we have a Pareto sub-optimal outcome. The story would not be that bad if we assume that $n_p = 2n_l$ (instead of $n_l = 2n_p$). In this case, if we assume that a car is a peach with probability $\frac{2}{3}$ and a lemon with probability $\frac{1}{3}$, then the expected valuation of the car to any potential buyer is $E(V_b) = \frac{2}{3}v_{bp} + \frac{1}{3}v_{bl} = \frac{2}{3} \times 3000 + \frac{1}{3} \times 2000 = 2666.67 > 2500 = v_{sp} > v_{sl}$. In this case, the equilibrium price is $\hat{P} = 2666.67$ and all the $n_p + n_l$ cars are traded. However, owners of peaches are unhappy as the presence of lemons has stopped them from getting (3000-2666.67)=333.33 extra.

2.2. **Another example.** This example is to illustrate the possibility that adverse selection can become worse the greater the number of qualities and the smaller the "valuation gaps" (that is, the difference between any given car's worth to a buyer and a seller). Imagine that there are 10001 cars of different qualities. The quality level is uniform between the two extremes. In particular, each buyer's valuation across different qualities are the following:

$$V_b(1) = 3000, V_b(2) = 2999.9, \ldots, V_b(10000) = 2000.1, V_b(10001) = 2000.$$

and each seller's valuation across different qualities are the following:

$$V_s(1) = 2900, V_s(2) = 2899.9, \ldots, V_s(10000) = 1900.1, V_s(10001) = 1900.$$

Observe that $V_b(i) - V_s(i) = 100$ for all $i = 1, \ldots, 10001$. What will be the equilibrium? First note that at price 1900 exactly one car will be supplied, at price 1901, 11 cars will be supplied and so on and finally at price 2900 all the 10001 cars will be supplied.

How will the buyers argue? At a price P between 3000 and 2000, buyers rationally argues that sellers who value their own cars at P or less are willing to sell. Hence, a car being sold at price P has a quality level that makes it worth between $\{2000, P + 100\}$ to the buyers where each value in this range is equally likely. Therefore, the average car being sold is worth $\frac{2000+P+100}{2}$ implying that the average quality of the car to the buyer is $\mu(P) = \frac{2100+P}{2}$. Hence, if $P > 2100$, then $\mu(P) < P$ and there is *no demand*. If $P < 2100$, then $\mu(P) > P$ there is *infinite demand*. Thus, $P^* = 2100$ is the equilibrium price since $P^* = 2100 = \mu(P^*) = \mu(2100)$. The number of cars offered for sale is $(2100 - 1900) \times 10 + 1 = 2001$ out of 10001. Thus, in comparison to the earlier example where 67% of the used cars $\left(\frac{n_l}{n_l+n_p} = \frac{2}{3}\right)$ were traded in equilibrium, here (because of too many varieties of cars) only 20% of the cars (2001 out of 10001) are traded in equilibrium.

Finally, observe that if valuation of all qualities for the buyers is reduced by 50, that is, $\hat{V}_b(i) = V_b(i) - 50$ for all $i = 1, \ldots, 10001$ so that $\hat{V}_b(i) - V_s(i) = 50$ for all $i = 1, \ldots, 10001$. A car being sold at price P has a quality level that makes it worth between $\{1950, P + 50\}$ to the buyers where each value in this range is equally likely. The average quality at any price $P > 2000$ is $\hat{\mu}(P) = \frac{2000+P}{2} < P$. The average quality at any price $P < 2000$ is $\hat{\mu}(P) = \frac{2000+P}{2} > P$. Thus, the equilibrium price is $\hat{P}^* = 2000$. In this case the number of cars offered for sale is $(2000 - 1900) \times 10 + 1 = 1001$ out of 10001 which has gone down in comparison to the earlier case. This shows that as the valuation gap between buyers and sellers goes down the adverse selection problem becomes severe. Earlier, when the variety specific gap was 100, 20% of the cars (2001 out of 10001) were traded in equilibrium and when this variety specific gap is 50 only 10% of the cars (1001 out of 10001) are traded in equilibrium.

3. The labor market model of adverse selection

Consider a competitive industry with many identical risk-neutral firms that hires workers. Each firm produces the same output using a constant returns to scale technology where labor is the only input. Thus, the production function of each firm is $y = f(L)$ and it is homogeneous of degree one (since constant returns to scale technology is equivalent to production function being homogenous of degree one). Recall that production function is homogeneous of degree one means that $f'(L)L = f(L) \Rightarrow f'(L) = \frac{f(L)}{L} \Rightarrow MP_L = AP_L \equiv \theta$. Moreover, $y = \theta L$ where θ represents the marginal (average) productivity of labor. Each firm maximizes profit and acts as a price taker and we normalize the price of output to unity.

Suppose that there are N workers. Workers differ in the number of units of output they produce. We assume that a worker's productivity is θ and that θ lies in the compact interval $[\underline{\theta}, \overline{\theta}]$ where $0 \leq \underline{\theta} < \overline{\theta} < +\infty$. Let $F(\theta)$ be the proportion of workers with productivity θ or less. F is non-degenerate so that there are at least two types of workers. We further assume that $f(\theta)$ is the density function associated with F such that $f(\theta) > 0$ for all $\theta \in [\underline{\theta}, \overline{\theta}]$. Workers seek to maximize the amount that they earn from their labour (in units of money). Workers chooses to work either at a firm or at home. Let $r(\theta)$ be the amount of income of a worker of type θ if the worker works at home. Thus, $r(\theta)$ is the opportunity cost to a worker of type θ of accepting employment. Clearly, a rational worker will accept employment in a firm if and only if the wage w that the worker earns from the firm is at least as large as $r(\theta)$.[2]

3.1. **Competitive equilibrium with known worker types.** The labor of each type of worker is a distinct good (as worker types are known) and hence there is a distinct equilibrium wage $w^*(\theta)$ for each type θ. The two main features of the competitive equilibrium are the following:

(C1) $w^*(\theta) = \theta$ for all $\theta \in [\underline{\theta}, \overline{\theta}]$ since, given constant returns to scale, a firm selects $w^*(\theta)$ for a worker of type θ to maximize $[y(\theta) - w(\theta)L \Leftrightarrow (\theta - w(\theta))L]$.

(C2) The set of worker types accepting employment is $\Theta = \{\theta : r(\theta) \leq \theta\}$.

We can have other competitive equilibria such as $w^*(\theta) = \theta$ for all $\theta \in [\underline{\theta}, \overline{\theta}]$ such that $\theta \geq r(\theta)$ and $w^*(\theta) \in [\underline{\theta}, r(\theta))$ for all $\theta \in [\underline{\theta}, \overline{\theta}]$ such that $\theta < r(\theta)$. For expositional simplicity we rule out such equilibria and concentrate on the equilibrium wages that are equal to workers' marginal productivity.

What can we say about the labor allocation under the competitive equilibrium given by (C1) and (C2)? From the first fundamental theorem of welfare economics we know that every competitive equilibrium is Pareto optimal. We claim that the labor allocation under the competitive equilibrium given by (C1) and (C2) is Pareto optimal. Any allocation of labor that leads to Pareto optimality must maximize aggregate surplus (that is, consumer surplus and producer's surplus). Define $I(\theta) = 1$ if the type θ worker accepts the job in a firm and $I(\theta) = 0$ if the type θ worker works at home. Thus, $I(\theta)$ represents the worker of type θ's decision of accepting or not accepting a job. The total surplus generated by a worker of type θ under the competitive equilibrium is

$$I(\theta)\{[\theta - w^*(\theta)] + w^*(\theta)\} + (1 - I(\theta))r(\theta) = I(\theta)(\theta - r(\theta)) + r(\theta).$$

Therefore, we can write the aggregate surplus in the industry as

$$\int_{\underline{\theta}}^{\overline{\theta}} N[I(\theta)(\theta - r(\theta)) + r(\theta)]f(\theta)d\theta = N\int_{\underline{\theta}}^{\overline{\theta}} [I(\theta)(\theta - r(\theta)) + r(\theta)]dF(\theta).$$

Observe that the aggregate surplus is maximized by setting $I(\theta) = 1$ whenever $\theta \geq r(\theta)$ and $I(\theta) = 0$ for $\theta < r(\theta)$. Thus, if a labor allocation is Pareto optimal, then the set of worker types employed by the firm must be $\{\theta : \theta \geq r(\theta)\}$ which is indeed the case in (C2). Hence, the competitive equilibrium ((C1) and (C2)) leads to a Pareto optimal labor allocation.

3.2. **Asymmetric information and competitive equilibrium.** If the workers' type is unknown, then equilibrium wage must be type independent, that is, we cannot have wage contingent on the productivity level θ. The type set of workers that are willing to accept employment at some wage w is given by $\Theta(w) = \{\theta : r(\theta) \leq w\}$. Thus, $\Theta(w)$ for each w represents the supply side of labor. The

[2]The labor market model is from Mas-Colell, Whinston and Green [3], Chapter 13.

demand for labor depends on $\mu(w)$ which is the firm's belief about the average productivity of the workers who accept employment. Therefore, the demand for labor is defined as

$$z(w) = \begin{cases} 0 & \text{if } \mu(w) < w \\ [0, +\infty] & \text{if } \mu(w) = w \\ +\infty & \text{if } \mu(w) > w \end{cases}$$

A competitive equilibrium in this model is a wage rate w^* and a type set Θ^* of workers' type who accept employment such that:

(CA1) $\Theta^* = \{\theta : r(\theta) \leq w^*\}$ and
(CA2) $w^* = E(\theta \mid \theta \in \Theta^*)$.

The first observation is that *the market equilibrium need not be efficient, that is, the labor allocation need not be Pareto optimal*. For example, let $r(\theta) = \bar{r}$ where $\bar{r} \in (\underline{\theta}, \overline{\theta})$ (or $F(\bar{r}) \in (0,1)$). Recall that Pareto optimal labor allocation would require that all workers with $\theta \in [\bar{r}, \overline{\theta}]$ choose to work. If $w \geq \bar{r}$, then all workers are willing to work and if $w < \bar{r}$, then no worker is willing to work. However, since worker types θ is unknown, the wage offer cannot be type contingent. Therefore, $\Theta(w)$ is either $[\underline{\theta}, \overline{\theta}]$ or $\emptyset$. In the first case, $E(\theta \mid \theta \in \Theta(w)) = E(\theta)$ for all $w \geq \bar{r}$ and so by (CA2), $w^* = E(\theta)$. Therefore, in general, if $E(\theta) \geq \bar{r}$, then all workers accept employment and we have *too much* employment compared to the Pareto optimal labor allocation, and, if $E(\theta) < \bar{r}$, then no worker accepts employment and we have *too little* employment compared to the Pareto optimal labor allocation.

How inefficient can the market equilibrium be? We show that striking breakdown of efficiency can arise when $r(\theta)$ varies with θ. In this context, adverse selection occurs when only relatively less capable workers are willing to accept a firm's employment offer at any given wage. Because of adverse selection the market may fail completely despite the fact that every worker type should work at a firm. Let $r(\theta) \leq \theta$ for all $\theta \in [\underline{\theta}, \overline{\theta}]$ and assume that $r(\theta)$ is increasing in θ. Recall that Pareto optimal labor allocation requires that every worker type should be employed since $r(\theta) \leq \theta$ for all $\theta \in [\underline{\theta}, \overline{\theta}]$. From the equilibrium conditions (CA1) and (CA2) we know that $w^* = E(\theta \mid r(\theta) \leq w^*)$. Given $r(\theta)$ is increasing in θ it follows that $g(w) := E(\theta \mid r(\theta) \leq w) = \int_{\underline{\theta}}^{r^{-1}(w)} \theta f(\theta) d\theta / \int_{\underline{\theta}}^{r^{-1}(w)} f(\theta) d\theta = \int_{\underline{\theta}}^{r^{-1}(w)} \theta f(\theta) d\theta / F(r^{-1}(w))$. The function $g(w)$ varies continuously with w for all $w \in [r(\underline{\theta}), +\infty]$ and $g(w)$ is increasing in w for $w \in [r(\underline{\theta}), r(\overline{\theta})]$ since $r(\theta)$ is increasing in θ. The function $g(w)$ has minimum value of $\underline{\theta}$ for $w = r(\underline{\theta})$ and has a maximum value of $E(\theta)$ for $w \geq r(\overline{\theta})$. Competitive equilibrium is obtained by locating the fixed point, that is, by locating w^* such that $g(w^*) = w^*$. In equilibrium $\Theta^* = \{\theta : r(\theta) \leq w^*\}$ is the type set accepting employment. The average productivity of the type group Θ^* is exactly equal to w^*. As long as $g(w)$ is continuous in w over the interval $[\underline{\theta}, \overline{\theta}]$ we will always get a fixed point (due to Brower's Fixed Point Theorem). If $E(\theta) < r(\overline{\theta})$, that is, $E(\theta) = r(\tilde{\theta})$ with $\tilde{\theta} \in (\underline{\theta}, \overline{\theta})$, then the equilibrium is *not efficient*. To make the best worker participate $w^* \geq r(\overline{\theta})$, but because $E(\theta) < r(\overline{\theta})$ this cannot happen. Thus, the presence of low productivity worker drives wage down below $r(\overline{\theta})$ which in turn drives out the best workers from the market and hence the average productivity of the remaining workers is lower thereby reducing the wage that firms are willing to pay (see Figure 1). This process may continue. Potentially very far. We can have a situation where $w^* = \underline{\theta}$. Since $\underline{\theta}$ is a point which is a set of measure zero, no worker is hired even though Pareto optimal labor allocation requires that all workers are hired.

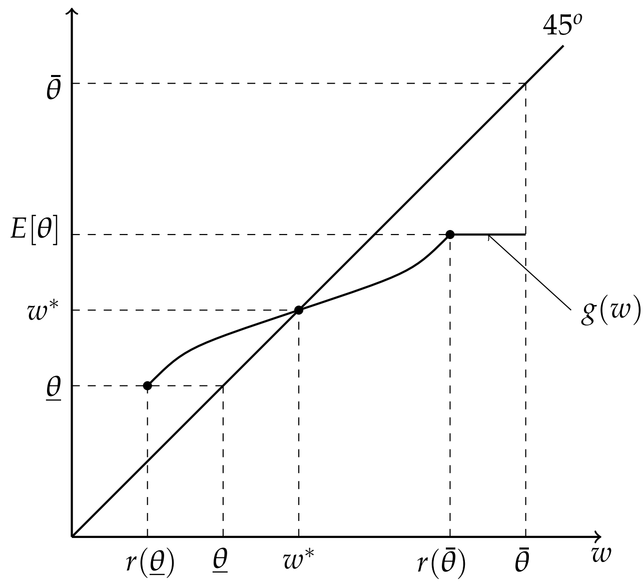


FIGURE 1. Example of an inefficient equilibrium

To see that $w^* = \underline{\theta}$ is a possibility consider the following example. Let $r(\theta) = \alpha\theta$ where $\alpha \in \left(\frac{1}{2}, 1\right)$ and let θ follow uniform distribution over the interval $[0, 2]$. Then $r(\underline{\theta}) = r(0) = 0$ and $r(\theta) < \theta$ for all $\theta \in (\underline{\theta}, \overline{\theta}]$. Given any $w \in [0, 2]$,

$$g(w) = E(\theta \mid \alpha\theta \leq w) = E\left(\theta \mid \theta \leq \frac{w}{\alpha}\right) = \frac{0 + \frac{w}{\alpha}}{2} = \frac{w}{2\alpha} < w, \ \forall\ w > 0.$$

Hence $g(0) = 0$ is the only equilibrium (see Figure 2).

In his paper Akerlof [1] took $\alpha = \frac{2}{3}$ that resulted in $g(w) = \frac{3w}{4} < w$ for all $w > 0$. One can provide an iterative argument behind the $g(0) = 0$ equilibrium. Suppose in **Stage 0**, firms wants to attract all types of workers $\theta \in [0, 2]$. This requires that $E(\theta) = 1 = w^0$. But at wage $w^0 = 1$, only workers with $r(\theta) \leq w^0 = 1$ will be willing to work. This means that $\frac{2}{3}\theta \leq w^0 = 1$ which implies that $\theta \leq \frac{3}{2}$. Therefore, the types accepting jobs lie in the interval $\left[0, 2 \times \frac{3}{4}\right]$. As a result, in **Stage 1**, the firms will try to adjust their wage offer by calculating the average productivity of types lying in the interval $\left[0, 2 \times \frac{3}{4}\right]$. This gives $E\left(\theta \mid \theta \leq \frac{3}{2}\right) = \frac{3}{4} = w^1$. For a wage offer $w^1 = \frac{3}{4}$ all workers with $r(\theta) \leq \frac{3}{4}$ will join implying that types accepting jobs lie in the interval $\left[0, 2 \times \left(\frac{3}{4}\right)^2\right]$. Thus, in **Stage 2** the wage offer will be $w^2 = \left(\frac{3}{4}\right)^2$ and so on. In **Stage n** the firms will try to adjust their earlier (that is, **Stage n-1**) wage offer by calculating the average productivity of types lying in the interval $\left[0, 2 \times \left(\frac{3}{4}\right)^n\right]$ and this will lead to a wage offer of $w^n = \left(\frac{3}{4}\right)^n$. Thus as $n \to +\infty$, the relevant interval $\left[0, 2 \times \left(\frac{3}{4}\right)^n\right]$ as well as the wage offer $w^n = \left(\frac{3}{4}\right)^n$ shrinks to 0 giving us the required result in the limits.

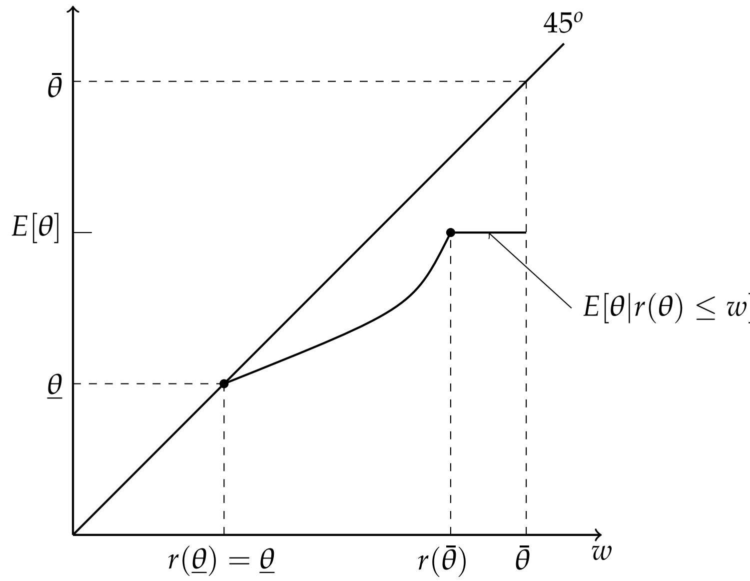


FIGURE 2. Akerlof's example of extreme inefficiency

Consider a situation where $r(\theta)$ is continuous and increasing and that there exists $\hat{\theta} \in (\underline{\theta}, \overline{\theta})$ such that $r(\theta) < \theta$ for all $\theta \in [\underline{\theta}, \hat{\theta})$ and $r(\theta) > \theta$ for all $\theta \in (\hat{\theta}, \overline{\theta}]$. By continuity of $r(\theta)$ it follows that $r(\hat{\theta}) = \hat{\theta}$. One can show that a *competitive equilibrium with unknown types will necessarily lead to Pareto inefficient outcome*. To see this note first that for Pareto optimal outcome we require that $\Theta^* = [\underline{\theta}, \hat{\theta}]$. Since types are unknown, equilibrium wage offer cannot be type contingent and therefore, to get Pareto efficient labor allocation in equilibrium, we must have $E(\theta \mid r(\theta) \leq w^*) = w^*$ with $w^* = \hat{\theta}$. But this is not possible since, using $r(\hat{\theta}) = \hat{\theta}$, we get $E(\theta \mid r(\theta) \leq \hat{\theta}) < \hat{\theta}$ from the following step:

$$E(\theta \mid r(\theta) \leq \hat{\theta}) = \frac{\int_{\underline{\theta}}^{r^{-1}(\hat{\theta})} \theta f(\theta) d\theta}{\int_{\underline{\theta}}^{r^{-1}(\hat{\theta})} f(\theta) d\theta} = \frac{\int_{\underline{\theta}}^{\hat{\theta}} \theta f(\theta) d\theta}{F(\hat{\theta})} < \frac{\int_{\underline{\theta}}^{\hat{\theta}} \hat{\theta} f(\theta) d\theta}{F(\hat{\theta})} = \frac{\hat{\theta} F(\hat{\theta})}{F(\hat{\theta})} = \hat{\theta}.$$

Hence, the equilibrium must necessarily lead to Pareto sub-optimal labor allocation.

Finally consider a *positive selection* version in which $r(\theta)$ is continuous and decreasing in θ. We can make the following observations.

(1) *The more capable workers are the ones choosing to work at any given wage*. Suppose that firms make a wage offer of w. All workers of type θ with $r(\theta) \leq w$ will work. Let θ^* be such that $r(\theta^*) = w$. Then since $r(\theta) \leq r(\theta^*) = w$ for all $\theta \geq \theta^*$, all workers with $\theta \in [\theta^*, \overline{\theta}]$ will accept to work. Hence, the more capable workers are the ones willing to work at any given wage rate.

(2) *If $r(\theta) > \theta$ for all θ, then the resulting competitive equilibrium is Pareto efficient*. Note that if $r(\theta) > \theta$ for all θ, then Pareto efficient labor allocation implies that $\Theta^* = \emptyset$. Firms can offer a wage of at most $\overline{\theta}$. At this wage $w = \overline{\theta}$, worker of type $\overline{\theta}$ would not work since $r(\overline{\theta}) > \overline{\theta} = w$.

Moreover any worker with type $\theta \in [\underline{\theta}, \overline{\theta})$ will also not work since $r(\theta) > r(\overline{\theta}) > \overline{\theta} = w$ for all $\theta \in [\underline{\theta}, \overline{\theta})$. Thus, no worker of any type will want to work even if they are offered the maximum wage $\overline{\theta}$. Thus, $\Theta^* = \emptyset$ and we have Pareto efficient labor allocation.

(3) *If there exists a $\hat{\theta}$ such that $r(\theta) < \theta$ for $\theta > \hat{\theta}$ and $r(\theta) > \theta$ for $\theta < \hat{\theta}$, then any equilibrium with strictly positive employment necessarily involve too much employment relative to the Pareto optimal labor allocation*. Pareto optimal labor allocation in this context means that $\Theta^* = [\hat{\theta}, \overline{\theta}]$. If $w = \hat{\theta}$, only workers of type $\theta \geq \hat{\theta}$ will accept the wage w and work. But at $w = \hat{\theta}$, $E(\theta \mid \theta \geq \hat{\theta}) > \hat{\theta}$ because of the following reason:

$$E(\theta \mid \theta \geq \hat{\theta}) = \frac{\int_{\hat{\theta}}^{\overline{\theta}} \theta f(\theta) d\theta}{\int_{\hat{\theta}}^{\overline{\theta}} f(\theta) d\theta} = \frac{\int_{\hat{\theta}}^{\overline{\theta}} \theta f(\theta) d\theta}{1 - F(\hat{\theta})} > \frac{\int_{\hat{\theta}}^{\overline{\theta}} \hat{\theta} f(\theta) d\theta}{1 - F(\hat{\theta})} = \hat{\theta}.$$

Therefore, at $w = \hat{\theta}$, firms demand more workers than there are in supply and the market will not clear. If $w < \hat{\theta}$, only workers of type $\theta > \theta^* > \hat{\theta}$ with $r(\theta^*) = w$, will accept the wage w. But $E(\theta \mid \theta \geq \theta^*) > \theta^* = w$ which implies that firms demand more workers than there are in supply and the market will not clear. Thus, market clearing can be obtained only if $w > \hat{\theta}$ which implies that some workers of type $\theta < \hat{\theta}$ will accept the job as well and hence, if we have any competitive equilibrium, then there will be over employment compared to the Pareto optimal labor allocation.

3.3. **Multiple equilibria: A game theoretic approach.** The competitive equilibrium under unknown types, given by (CA1) and (CA2), need not be unique. Multiple equilibria can arise because there is virtually no restriction on the slope of the function $g(w) := E(\theta \mid r(\theta) \leq w)$. At any wage this slope depends on the density of the workers who are just indifferent about accepting employment and so it can vary greatly if this density varies. However, if there are multiple equilibria, then one can Pareto rank them. Firms earn zero expected profit in all equilibria and workers are better off if the wage is higher (those workers who do not work are indifferent and those workers who work are strictly better off). Thus, *the equilibrium with highest wage Pareto dominates all other equilibria*. The low wage Pareto dominated equilibria exists because of coordination failure: the wage is too low because firms expect that the productivity of workers accepting employment is poor, and, at the same time, only bad workers accept employment precisely because the wage is low. It seems unlikely that a model in which firms could change their offered wage would ever lead to a Pareto dominated equilibrium. To be more formal about this idea, consider the following game-theoretic model: The underlying market structure is common knowledge (that is, $F(\theta)$ and $r(\theta)$ are common knowledge). Market behavior is captured in the following two-stage game:

Stage 1: Two firms simultaneously announce their wage offers (the restriction to two firms is without loss of generality).

Stage 2: Workers decide whether to work for a firm and, if so, which firm (if they are indifferent between firms, they randomize, with equal probability and if they are indifferent between working in a firm and working at home they work in a firm).

The next proposition characterizes the sub-game perfect Nash equilibria (SPNEs') of this game for the adverse selection labor market model in which $r(\theta)$ is increasing with $r(\theta) \leq \theta$ for all $\theta \in [\underline{\theta}, \overline{\theta}]$ and $F(\theta)$ has an associated density $f(\theta)$ with $f(\theta) > 0$ for all $\theta \in [\underline{\theta}, \overline{\theta}]$.

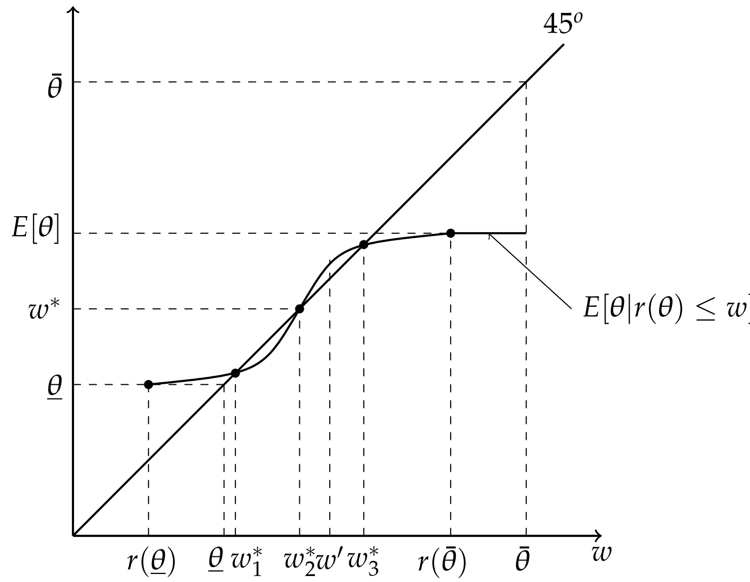


FIGURE 3. The case where the highest equilibrium wage is an SPNE outcome

Proposition 1. Let W^* denote the set of competitive equilibrium wages for the adverse selection labor market model and let $w^* = \max\{w : w \in W^*\}$.

(i) If $w^* > r(\underline{\theta})$ and there is an $\epsilon > 0$ such that $E(\theta \mid r(\theta) \leq w') > w'$ for all $w' \in (w^* - \epsilon, w^*)$, then there is a unique pure strategy SPNE of the two-stage game. In this SPNE, employed workers receive a wage of w^* and the workers in the type set $\Theta^* = \{\theta : r(\theta) \leq w^*\}$ accept employment.

(ii) If $w^* = r(\underline{\theta})$, then there are multiple pure strategy SPNEs. However, in every pure strategy SPNE each agent's payoff exactly equals his/her payoff in the highest wage competitive equilibrium.

Proof: The second stage strategy a worker of type θ accepts employment only at one of the highest wage firms if and only if the wage is at least $r(\theta)$.

(i) $w^* > r(\underline{\theta})$: *In any SPNE both firms must earn zero profits.* To see this suppose to the contrary that there is an SPNE in which a total of M workers are hired at a wage $\bar{w}$ and in which the aggregate profits of the two firms is $\pi = M[E(\theta \mid r(\theta) \leq \bar{w}) - \bar{w}] > 0$ implying $M > 0$ and $\bar{w} > r(\underline{\theta})$. In this case the weakly less profitable firm j (say) must be earning no more than $\frac{\pi}{2}$. But firm j can earn profits of at least $\pi(\alpha) = M[E(\theta \mid r(\theta) \leq \bar{w} + \epsilon) - (\bar{w} + \epsilon)]$ by, instead, offering a wage of $\bar{w} + \epsilon$ for $\epsilon > 0$. Since the function $E(\theta \mid r(\theta) \leq w)$ is continuous in w, the deviation profit $\pi(\epsilon)$ can be made arbitrarily close to π by choosing ϵ small enough. Thus, firm j would be better off by deviating, which yields a contradiction. Therefore, we must have $\pi \leq 0$. Because neither firm can have strictly negative profits in an SPNE (a firm can always offer a wage of zero), both firms must earn zero profits.

We now show that *in any SPNE, wage offered must be* w^*. From the zero profit condition it follows that if $\bar{w}$ is the highest wage rate offered by either firm then either $\bar{w} \in W^*$ or $\bar{w} < r(\underline{\theta})$. Suppose $\bar{w} < w^* = \max\{w : w \in W^*\}$. Then either firm can earn positive expected profit by deviating and offering any wage $w' \in (w^* - \epsilon, w^*)$. We conclude that the highest wage offered must equal w^* in any SPNE. With this wage offer no firm can benefit by unilaterally lowering its wage because then it gets no worker. To complete the argument, we must show that $E(\theta \mid r(\theta) \leq w) < w$ at every $w > w^*$, so that no unilateral deviation to a higher wage can yield a firm positive profits. By hypothesis w^* is the highest competitive wage. So there is no $w > w^*$ such that $E(\theta \mid r(\theta) \leq w) = w$. Therefore, because $E(\theta \mid r(\theta) \leq w)$ is continuous in w, $E(\theta \mid r(\theta) \leq w) - w$ must have the same sign for all $w > w^*$. Moreover, we cannot have $E(\theta \mid r(\theta) \leq w) > w$ for all $w > w^*$ because as $w \to \infty$, $E(\theta \mid r(\theta) \leq w) \to E(\theta)$ which under our assumption is finite. We must therefore have $E(\theta \mid r(\theta) \leq w) < w$ for all $w > w^*$. Finally, given equilibrium wage w^*, the workers in the type set $\Theta^* = \{\theta : r(\theta) \leq w^*\}$ accept employment. This completes the argument for (i).
(ii) $w^* = r(\underline{\theta})$: In this case, $E(\theta \mid r(\theta) \leq w) < w$ for all $w > w^*$, so that any firm attracting workers at a wage in excess of w^* incurs losses. Moreover, a firm must earn zero profits by announcing any $w \leq w^*$. Hence, the set of wage offers (w_1, w_2) in an SPNE is given by $\{(w_1, w_2) : w_j \leq w^* \text{ for } j = 1, 2\}$. In every one of these equilibria all agents earn exactly what they earn at the competitive equilibrium involving w^*. Both firms earn zero and a worker of type θ earns $r(\theta)$ for all $\theta \in [\underline{\theta}, \overline{\theta}]$. □

4. Signaling

Given the adverse selection problem of Sections 3, one expects that mechanisms will develop in the marketplace to help firms distinguish among workers. This seems plausible because both the firms and the high ability workers have incentives to try and accomplish this objective. The mechanisms that we examine in this section is that of *signaling*, which was first investigated by Spence [5]. The basic idea is that high ability workers may have actions they can take to distinguish themselves from their low ability counterparts.

4.1. **The Model.** Consider the labor market model where the marginal productivity of a worker is $\theta \in \{\theta_L, \theta_H\}$ where $0 < \theta_L < \theta_H$ and it is common knowledge that $\gamma = Pr(\theta = \theta_H) \in (0, 1)$. Before entering the job market a worker can get some education and the amount of education that a worker gets is observable. We assume that education does nothing for a worker's productivity. The cost of education is $c(e, \theta)$ where $c(e, \theta)$ is twice continuously differentiable function with $c(0, \theta) = 0$, $c_e(e, \theta) > 0$, $c_{ee}(e, \theta) \geq 0$, $c_\theta(e, \theta) < 0$ and $c_{e\theta}(e, \theta) < 0$. Let $u(w, e; \theta)$ denote the utility of worker of type θ who chooses education level e and receives wage w. We assume that $u(w, e; \theta) = w - c(e, \theta)$. A worker of type θ can earn $r(\theta)$ by working at home. We assume that $r(\theta_L) = r(\theta_H) = 0$. The unique equilibrium in the absence of any signal is $w^* = E(\theta) = \gamma\theta_H + (1-\gamma)\theta_L$ and this leads to Pareto optimal labor allocation.

Equilibrium Concept. The *beliefs*, which is common knowledge in the market, has the property that for each possible choice of e there exists $\mu(e) \in [0,1]$ such that

(i) firm i's belief that the worker is of type θ_H after seeing the choice of e is $\mu(e)$ and
(ii) after the worker has chosen e, firm j's belief ($j \neq i$) that the worker is of type θ_H and that firm i has chosen wage offer w is $\mu(e)\sigma_i^*(w \mid e)$ where $\sigma_i^*(w \mid e)$ is firm i's equilibrium probability of choosing wage offer w after observing education level e.

Condition (ii) adds an element of commonality to the firms' beliefs about the type of worker who has chosen e and requires that the firms' belief about each others' wage offers following e are consistent with the equilibrium strategies.

Definition 1. A set of strategies $(e^*(\theta_L), e^*(\theta_H), w^*(e))$ and a belief function $\mu : \Re_+ \to [0,1]$ giving the firms' common probability assessment that the worker is of high ability after observing education level e is a *Perfect Bayesian Equilibrium* (PBE) if

(a) Each worker's strategy is optimal given the firms' strategies.
(b) The belief function $\mu(e)$ is derived from the worker's strategy using Bayes rule where possible.
(c) The firms' wage offers following each choice e constitute a Nash equilibrium of the simultaneous-move wage offer game in which the probability that the worker is of high ability is $\mu(e)$.

We begin our analysis at the end game. Suppose after seeing some education level e, all firms attach a probability of $\mu(e)$ that the worker is of type θ_H. If so, the expected productivity of the worker is $E(\theta \mid e) = \mu(e)\theta_H + [1-\mu(e)]\theta_L$. For example, two natural beliefs that can occur are the following:

Belief 1:

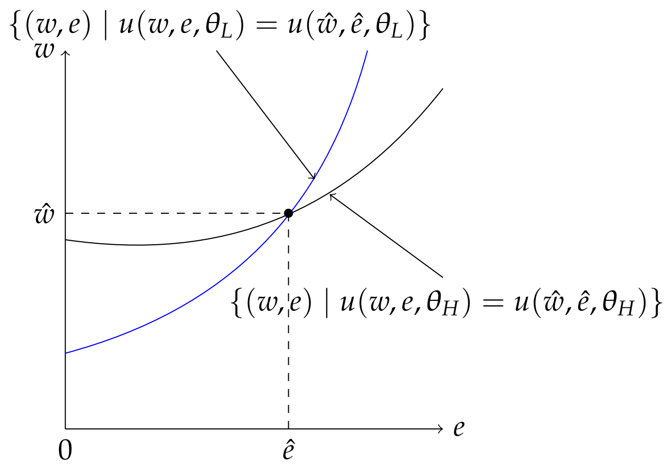
$$\mu^{(1)}(e) = \begin{cases} 1 & \text{if } e \geq e^*(>0) \\ 0 & \text{if } 0 \leq e < e^* \end{cases}$$

Belief 1 implies that $E(\theta \mid e) = \mu^{(1)}(e)\theta_H + [1-\mu^{(1)}(e)]\theta_L = \theta_L$ for all $e \in [0, e^*)$ and $E(\theta \mid e) = \mu^{(1)}(e)\theta_H + [1-\mu^{(1)}(e)]\theta_L = \theta_H$ for all $e \geq e^*$.

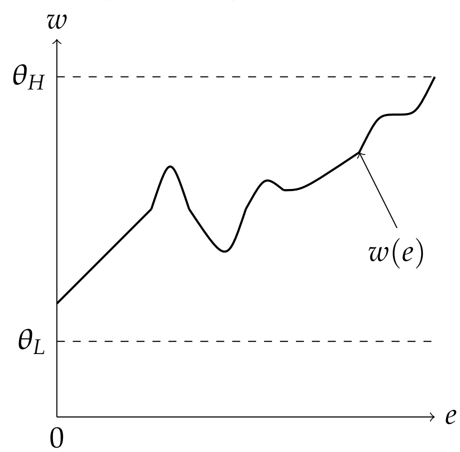
Belief 2: $\mu^{(2)}(e) = \gamma \in (0,1)$ for all $e \geq 0$. Belief 2 implies that $E(\theta \mid e) = \mu^{(2)}(e)\theta_H + [1-\mu^{(2)}(e)]\theta_L = \gamma\theta_H + (1-\gamma)\theta_L = E(\theta)$ for all $e \geq 0$.

In a simultaneous move wage offer game, the firms' (pure strategy) Nash equilibrium wage offers equal the worker's expected productivity (Bertrand pricing). Thus, in any (pure strategy) PBE, we must have both firms offering a wage $w^*(e) = \mu(e)\theta_H + [1-\mu(e)]\theta_L = E(\theta \mid e)$ for each $e \geq 0$.

Workers' Preferences. The first thing to note about the utility function $u(w, e; \theta) = w - c(e, \theta)$ for $\theta \in \{\theta_L, \theta_H\}$ is that the resulting indifference curves of the high and low ability workers intersect only once and that, where they do, the indifference curve of the high ability worker is flatter than the indifference curve of the low ability worker. This property of preferences, known as the *single crossing property*, plays an important role in separating high ability workers from the low ability workers.



(A) Indifference curves of the high and low-ability workers: the single crossing property.



(B) A wage schedule.

FIGURE 4

Equilibrium Education Choice. The optimal education choice of the workers is contingent upon whether we have a separating or pooling equilibrium.

(1) *Separating Equilibrium* in which the two types of workers choose different education levels.
(2) *Pooling Equilibrium* in which the two types of workers choose same level of education.

4.2. **Separating Equilibria.** Let $e^*(\theta) = (e^*(\theta_L), e^*(\theta_H))$ be the worker's equilibrium education choice as a function of type and $w^*(e)$ be the firms' equilibrium wage offer as a function of the education level.

O(1) *In any separating equilibrium, $w(e^*(\theta_H)) = \theta_H$ and $w(e^*(\theta_L)) = \theta_L$; that is each worker type receives a wage equal to his/her productivity level.*

In any PBE, beliefs on the equilibrium path must be correctly derived from the equilibrium strategies using Bayes rule. This implies that upon seeing the education level $e^*(\theta_L)$, firms' must correctly infer that the worker is of low ability with probability one. Likewise, upon seeing the education level $e^*(\theta_H)$, firms' must infer that the worker is of high ability with probability one. The resulting wages are then $w(e^*(\theta_L)) = \mu(e^*(\theta_L))\theta_H + (1-\mu(e^*(\theta_L)))\theta_L = \theta_L$ and $w(e^*(\theta_H)) = \mu(e^*(\theta_H))\theta_H + (1-\mu(e^*(\theta_H)))\theta_L = \theta_H$.

O(2) *In any separating equilibrium $e^*(\theta_L) = 0$; that is a low ability worker chooses to get no education.*

Suppose that, in equilibrium, a low ability worker chooses an education level $e' > 0$. According to observation **O(1)**, in doing so the worker gets a wage θ_L. However, the worker would receive a wage of at least θ_L by choosing $e = 0$ instead of e'. Since with $e = 0$ the low ability worker's cost is zero, the low ability worker is strictly better off by doing so. This contradicts $e' > 0$.

We first construct two forms of separating equilibrium and then from these two forms generate all other possible separating equilibria.

T(1): Let $e^{(1)} > 0$ be such that $U(\theta_L, 0; \theta_L) = U(\theta_H, e^{(1)}; \theta_L)$. Using the education level $e^{(1)}$ we construct the following separating equilibrium: $e^*(\theta_H) = e^{(1)}$, $e^*(\theta_L) = 0$ and $w^*(e)$ is a function such that

(1) $w^*(0) = \theta_L$,
(2) $w^*(e^{(1)}) = \theta_H$ and
(3) $w^*(e)$ is such that for all $e > 0$ ($e \neq e^{(1)}$),
 (a) $U(w^*(e), e; \theta_H) < min\{U(\theta_L, 0; \theta_L), U(\theta_H, e^{(1)}; \theta_H)\}$, and
 (b) $w^*(e) \in [\theta_L, \theta_H]$.

Observe that given the above specifications $\mu(e) = \frac{w^*(e)-\theta_L}{\theta_H-\theta_L} \in [0,1]$ for all $e \geq 0$. Moreover, $\mu(0) = 0$ and $\mu(e^{(1)}) = 1$ and $\mu(e) \neq 1$ for all $e \in (0, e^{(1)})$. The worker's maximize utility by selecting $e = 0$ if $\theta = \theta_L$ and by selecting $e = e^{(1)}$ if $\theta = \theta_H$. See Figure 5 (A) and (B).

There can be other beliefs (different from the one used in Figure 5 (B)) to generate the same equilibrium outcome as **T(1)**. For example, consider the following belief:

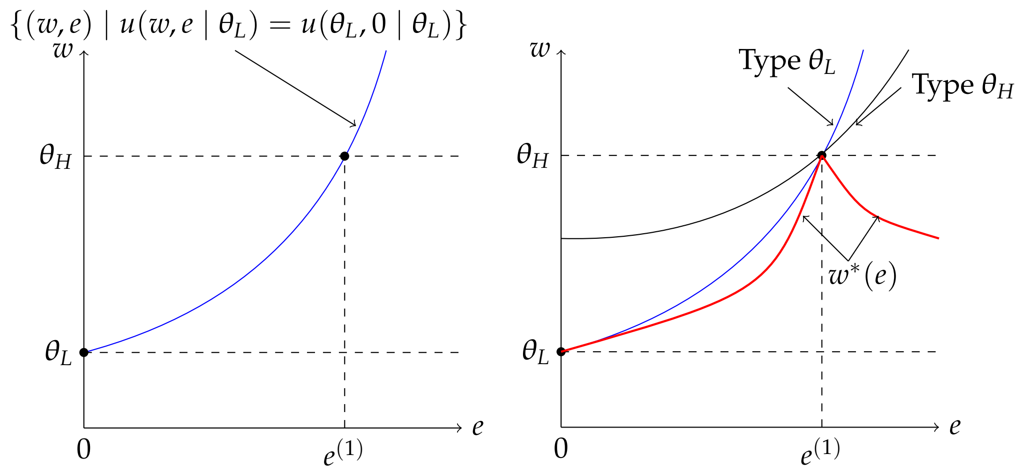
Belief I:

$$\mu^{(1)}(e) = \begin{cases} 1 & \text{if } e \geq e^{(1)} (> 0) \\ 0 & \text{if } 0 \leq e < e^{(1)} \end{cases}$$

Belief I implies that $E(\theta \mid e) = \mu^{(1)}(e)\theta_H + [1-\mu^{(1)}(e)]\theta_L = \theta_L$ for all $e \in [0, e^{(1)})$ and $E(\theta \mid e) = \mu^{(1)}(e)\theta_H + [1-\mu^{(1)}(e)]\theta_L = \theta_H$ for all $e \geq e^{(1)}$. With **Belief I**, we get the same equilibrium outcome as in **T(1)**. See Figure 6 (A).

T(2): Consider $e^{(2)} > 0$ such that $U(\theta_L, 0; \theta_H) = U(\theta_H, e^{(2)}; \theta_H)$. Using the education level $e^{(2)}$ we construct the following separating equilibrium: $e^*(\theta_H) = e^{(2)}$, $e^*(\theta_L) = 0$ and $w^*(e)$ is a function such that

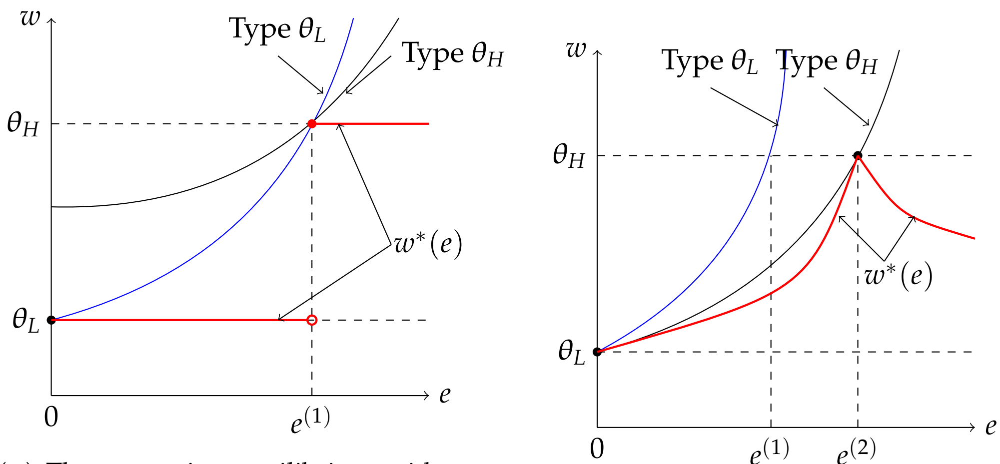
(1) $w^*(0) = \theta_L$,
(2) $w^*(e^{(2)}) = \theta_H$ and
(3) $w^*(e)$ is such that for all $e > 0$ ($e \neq e^{(2)}$),
 (a) $U(w^*(e), e; \theta_H) < U(\theta_H, e^{(2)}; \theta_H)$, and
 (b) $w^*(e) \in [\theta_L, \theta_H]$.



(A) Low-ability worker's outcome in the separating equilibrium **T(1)**.

(B) The separating equilibrium **T(1)**.

FIGURE 5



(A) The separating equilibrium with same choices as **T(1)** but generated by using **Belief I**.

(B) The separating equilibrium **T(2)**.

FIGURE 6

Given the above specifications $\mu(e) = \frac{w^*(e)-\theta_L}{\theta_H-\theta_L} \in [0,1]$ for all $e \geq 0$. Moreover, $\mu(0) = 0$ and $\mu(e^{(2)}) = 1$ and $\mu(e) \neq 1$ for all $e \in (0, e^{(2)})$. The worker's maximize utility by selecting $e = 0$ if $\theta = \theta_L$ and $e = e^{(2)}$ if $\theta = \theta_H$. See Figure 6 (B).

Consider the education levels $e^{(1)}$ and $e^{(2)}$ defined in **T(1)** and **T(2)** respectively. It follows that $0 < e^{(1)} < e^{(2)}$ for the following reason. From **T(1)** we get $\theta_L = U(\theta_L, 0; \theta_L) = U(\theta_H, e^{(1)}; \theta_L) = \theta_H - c(e^{(1)}, \theta_L)$ which implies **(a)** $e^{(1)} > 0$ and **(b)** $\theta_H - \theta_L = c(e^{(1)}, \theta_L) > c(e^{(1)}, \theta_H)$. From **T(2)** we get $\theta_L = U(\theta_L, 0; \theta_H) = U(\theta_H, e^{(2)}; \theta_H) = \theta_H - c(e^{(2)}, \theta_H)$ which implies **(c)** $c(e^{(2)}, \theta_H) = \theta_H - \theta_L$.

From **(b)** and **(c)** we get $c(e^{(2)}, \theta_H) > c(e^{(1)}, \theta_H)$ which implies **(d)** $e^{(1)} < e^{(2)}$. From **(a)** and **(d)** we get $0 < e^{(1)} < e^{(2)}$.

Exercise 1: Select any $\alpha \in (0, 1)$. By appropriately selecting the function $w^*(e)$, derive the separating equilibrium where $e^*(\theta_L) = 0$ and $e^*(\theta_H) = \alpha e^{(1)} + (1 - \alpha)e^{(2)}$.

O(3) *The only range of education level for which we get separating equilibrium is* $[e^{(1)}, e^{(2)}]$.

On the one hand, if we have $e^*(\theta_H) = \tilde{e} < e^{(1)}$. Then $w^*(\tilde{e}) = \theta_H$ and $e^*(\theta_H) = \tilde{e}$ (meant only for the high ability worker in a separating equilibrium) are such that $U(\theta_H, \tilde{e}; \theta_L) > U(\theta_L, 0; \theta_L)$ which contradicts our requirement (see **O(1)-O(2)**) that in any separating equilibrium the low ability worker selects $e^*(\theta_L) = 0$ and gets $w^*(e^*(\theta_L)) = \theta_L$. On the other hand, if we assume that $e^*(\theta_H) = \hat{e} > e^{(2)}$ and $w^*(e^*(\theta_H)) = \theta_H$, then we have $U(\theta_H, \hat{e}; \theta_H) < U(\theta_L, 0; \theta_H)$ which contradicts our assumption that in any separating equilibrium it is optimal for the high ability worker to select an education level of $\hat{e}$ and receive θ_H.

In any separating equilibrium, high ability workers are willing to get otherwise useless education simply because it allows them to distinguish themselves from the low ability workers and receive higher wages. Given that the marginal cost of education is higher for low ability workers, a high ability worker may find it worthwhile to get some positive education to raise the wage by some amount Δw, whereas a low ability worker may be unwilling to get the same level of education in return for the same incremental wage Δw. As a result firms can reasonably regard education level as a signal of worker quality. The set of separating equilibria obtained for different education levels $e^*(\theta_H) \in [e^{(1)}, e^{(2)}]$ can be Pareto ranked. In all these equilibria firms earn zero profit and the utility of low ability worker is θ_L. However, the utility of the high ability worker is contingent on the equilibrium education level $e^*(\theta_H) \in [e^{(1)}, e^{(2)}]$. In particular, lower level of education generates more utility for the high ability workers. Thus, the separating equilibrium with $e^*(\theta_H) = e^{(1)}$ Pareto dominates all other separating equilibria.

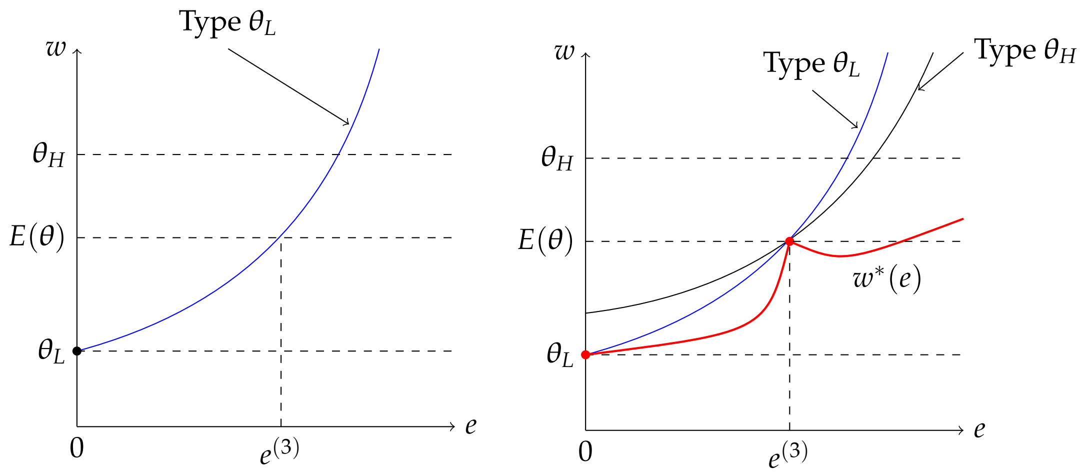
4.3. **Pooling Equilibria.**

In a pooling equilibrium, both types of workers choose the same level of education $e^*(\theta_L) = e^*(\theta_H) = e^*$. Since the firms' belief must be correctly derived from the equilibrium strategies using Bayes rule when possible, their beliefs when they see an education level e^* must assign a probability of γ to the worker being of type θ_H. Thus, in any pooling equilibrium, $w^*(e^*) = \gamma\theta_H + (1 - \gamma)\theta_L = E(\theta)$.

O(4) *The level of educations e^* that can arise in a pooling equilibrium lies in the interval $[0, e^{(3)})$ where $e^{(3)}$ is such that $U(\theta_L, 0; \theta_L) = U(E(\theta), e^{(3)}; \theta_L)$.*

To see **O(4)** consider any $e^* = \beta e^{(3)}$ where $\beta \in [0, 1)$ and define $w^*(e)$ in the following way:

(1) $w^*(e^*) = \gamma\theta_H + (1 - \gamma)\theta_L$,
(2) $w^*(e)$ is such that for all $e > 0$ ($e \neq e^*$),
 (a) $U(w^*(e), e; \theta_H) < \min\{U(w^*(e^*), e^*; \theta_H), U(w^*(e^*), e^*; \theta_L)\}$, and
 (b) $w^*(e) \in [\theta_L, \theta_H]$.



(A) The highest-possible education level $e^{(3)}$ in any pooling equilibrium.

(B) The pooling equilibrium with $e^{(3)}$.

FIGURE 7

One can easily verify that $\mu(e) \in [0,1]$ for all $e \geq 0$ and $\mu(e^*) = \mu(\beta e^{(3)}) = \gamma$. See Figure 7 (A) and (B) for a diagrammatic representation of the pooling equilibrium with the maximum possible equilibrium education choice $e^{(3)}$.

The set of pooling equilibria obtained for different education levels $e^* \in [0, e^{(3)})$ can be Pareto ranked. In all these equilibria firms earn zero profit. However, the utility of all workers is contingent on the equilibrium education level $e^* \in [0, e^{(3)}]$. In particular, lower level of education generates more utility. Thus, the pooling equilibrium with $e^* = 0$ Pareto dominates all other pooling equilibria. At $e^* = 0$ we have the no signaling equilibrium. Thus, pooling equilibria are weakly dominated by the no signaling equilibrium.

5. SCREENING

Firms can offer contracts to distinguish (or screen) between different types of workers. In the previous section informed parties (laborers) used education levels as a signaling devise and in this section uninformed parties (firms) use task levels as a screening devise. The screening model was first proposed by Rothschild and Stiglitz [4].

5.1. **The Model.** Consider the labor market model where the marginal productivity of a worker is $\theta \in \{\theta_L, \theta_H\}$ with $0 < \theta_L < \theta_H$ and it is common knowledge that $Pr(\theta = \theta_H) = \gamma \in (0,1)$. We assume that $r(\theta_L) = r(\theta_H) = 0$. The unique equilibrium in the absence of any signaling or screening is $w^* = E(\theta) = \gamma\theta_H + (1-\gamma)\theta_L$ and this leads to Pareto optimal labor allocation.

We assume that jobs of the workers may differ in the 'task level' required of the worker. Task level does not affect productivity of a worker. The utility of a worker of type θ is $U(w, t; \theta) = w - c(t, \theta)$ where w is the wage earned by the worker, $t \geq 0$ represents the task level and $c(t, \theta)$ is the cost associated with task level t for a worker of type $\theta \in \{\theta_L, \theta_H\}$. If we replace education level by task level in the cost function of the signaling model we get $c(t,\theta)$. We retain all the assumptions on the cost function that we had for the signaling model, that is $c(t,\theta)$ is twice continuously differentiable function with $c(0,\theta) = 0$, $c_t(t,\theta) > 0$, $c_{tt}(t,\theta) \geq 0$, $c_\theta(t,\theta) < 0$ and $c_{t\theta}(t,\theta) < 0$. The qualitative difference between the signaling model and the screening model is that the education level in the signaling model was a choice of the workers (informed parties) and the task level (to be taken up by a worker) in the screening model is a choice made by the firms (uninformed parties).

Two-Stage Screening Game: We consider the following two-stage screening game and study the pure strategy sub-game perfect Nash Equilibrium (SPNE) of this game.

Stage 1 Two firms simultaneously announce sets of offered contracts. A contract is a pair (w,t). Each firm may announce any finite number of contracts.

Stage 2 Workers of each type decides whether or not to accept a contract and, if so, which one.

We assume the following:

(a) If a worker is indifferent between telling the truth and misreporting, then the worker will tell the truth.
(b) Workers choose employment if they are indifferent between working and not working.
(c) If the workers most preferred contract is offered by both firms then the worker selects each firm with equal probability.

5.2. **Known Worker Types.** If the types of the workers are known then firms can offer type contingent offers like (w_i, t_i) for $i \in \{L, H\}$.

Proposition 2. In any SPNE of the screening game with known worker types, a type θ_i worker accepts contract $(w_i^*, t_i^*) = (\theta_i, 0)$ for $i \in \{L, H\}$ and firms earn zero profits.

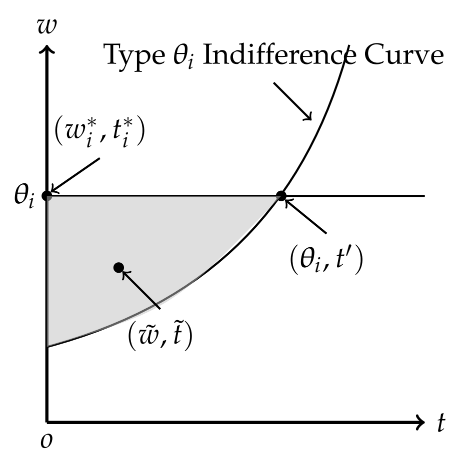


FIGURE 8. The equilibrium contract (w_i^*, t_i^*) for type θ_i with known worker types.

Proof of Proposition 2: Any contract (w_i^*, t_i^*) that workers of type θ_i accept in equilibrium must give zero profits, that is, $w_i^* = \theta_i$ for $i \in \{L, H\}$. To see this suppose that $w_i^* > \theta_i$. Then firms make losses and hence such an offer is not possible. Consider $w_i^* < \theta_i$ and let $\pi > 0$ be the aggregate profit earned by the two firms by employing θ_i type workers. Then there is at least one firm earning a profit which is less than or equal to $\frac{\pi}{2}$. This firm can deviate and offer $(w_i^* + \epsilon, t_i^*)$ where ϵ is positive and sufficiently small so that all θ_i type workers accept this new offer and the resulting profit of the deviating firm $\pi(\epsilon)$ is sufficiently close to π. Hence we cannot have $w_i^* < \theta_i$. Thus, $w_i^* = \theta_i$ for $i \in \{L, H\}$. Finally, if in equilibrium $(w_i^*, t_i^*) = (\theta_i, t')$ where $t' > 0$, then $\bar{u} \equiv U(\theta_i, t'; \theta_i) < U(\theta_i, 0; \theta_i) = \theta_i$ implying that any firm can deviate and offer $(\tilde{w}, \tilde{t})$ in the shaded region of Figure 8 such that $U(\tilde{w}, \tilde{t}; \theta_i) > U(\theta_i, t'; \theta_i) \equiv \bar{u}$. Therefore, all workers with productivity θ_i will join the deviating firm. The deviating firm earns strictly positive profits which implies that $(w_i^*, t_i^*) = (\theta_i, t')$ with $t' > 0$ cannot be an equilibrium choice. Hence $t_i^* = 0$ in equilibrium. □

5.3. **Unknown Worker Types.** If worker types are unknown then contracts like $(\theta_L, 0)$ and $(\theta_H, 0)$ are no more feasible. Moreover, we can have the possibility of separating and pooling equilibrium like in the signalling case.

Lemma 1. In any equilibrium (pooling or separating) both firms earn zero profits.

Proof: Let (w_L, t_L) and (w_H, t_H) be the contracts accepted by the low ability and high ability workers respectively. The two contracts can be the same one. Let aggregate profit of the two firms be $\pi > 0$. The weaker firm earns a profit which is less than or equal to $\frac{\pi}{2}$. This firm deviates and offers $(w_L + \epsilon, t_L)$ and $(w_H + \epsilon, t_H)$ where $\epsilon > 0$ and small enough. With these new offers interchange of workers' preference over contracts is not possible, that is if (w_i, t_i) is weakly preferred by type i to (w_j, t_j) for $i, j \in \{L, H\}$ and $i \neq j$, then $(w_i + \epsilon, t_i)$ is weakly preferred by type i to $(w_j + \epsilon, t_j)$. Thus the deviation by the weaker firm yields a profit $\pi(\epsilon)$ sufficiently close to π for small ϵ and hence $\pi(\epsilon) > \frac{\pi}{2}$. Therefore, we must have aggregate profit $\pi = 0$ and given each firm must earn non-negative profits it follows that profits of each firm must be zero in equilibrium. □

Lemma 2. No pooling equilibrium exists.

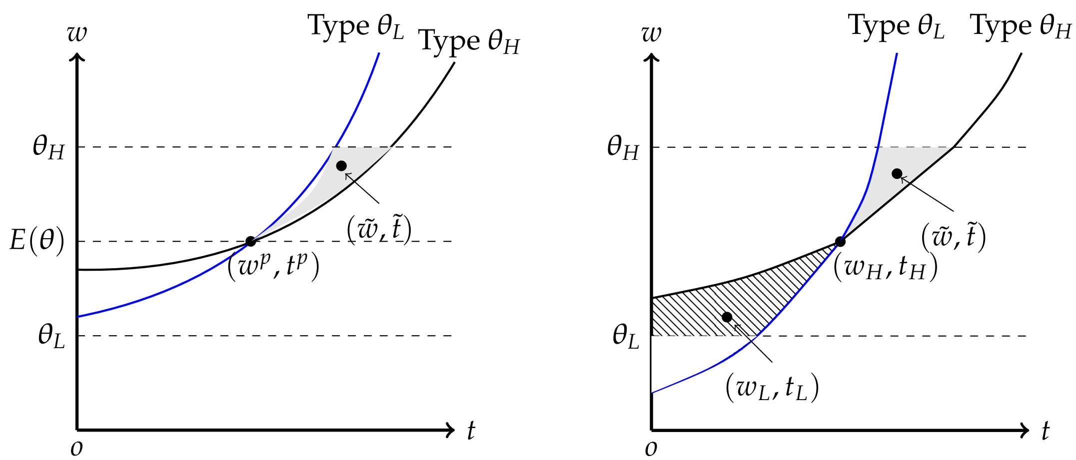
Proof: Given Lemma 1, in any pooling equilibrium $w^p = E(\theta) = \gamma\theta_H + (1-\gamma)\theta_L$. Let (w^p, t^p) be the pooling contract. Then any firm can deviate and offer a single contract $(\tilde{w}, \tilde{t})$ where $E(\theta) < \tilde{w} < \theta_H$, $\tilde{t} > t^p$ and, more importantly (a) $U(w^p, t^p; \theta_L) > U(\tilde{w}, \tilde{t}; \theta_L)$ and (b) $U(w^p, t^p; \theta_H) < U(\tilde{w}, \tilde{t}; \theta_H)$ (see Figure 9a). Due to single crossing property it is always possible to give a contract that simultaneously meets the two inequalities (a) and (b). Inequalities (a) and (b) imply that only the high ability workers prefer this new contract $(\tilde{w}, \tilde{t})$. Thus the deviating firm gets all the high ability workers and earns positive profits (since $\tilde{w} < \theta_H$). This is a contradiction to Lemma 1. □

Lemma 3. If (w_L, t_L) and (w_H, t_H) are the contracts signed by the low ability and high ability workers respectively in a separating equilibrium, then both contracts yield zero profits, that is, $w_L = \theta_L$ and $w_H = \theta_H$.

Proof: If $w_L < \theta_L$, then either firm can deviate to a single contract (w'_L, t_L) where $w_L < w'_L < \theta_L$ and earn positive profits.[3] By Lemma 1 this is not possible. Hence $w_L \geq \theta_L$ in any separating equilibrium.

[3]In particular, it must be noted that with (w'_L, t_L) the low ability workers will definitely join the firm and, depending on the nature of the utility function, the high-ability types may or may not join. In either case the deviating firm earns positive profits. The profit amount is higher when the high-ability types join and is lower when the high ability types do not join.

Suppose $w_H < \theta_H$. For separating equilibrium (Lemma 2) and zero profit condition (Lemma 1) it is necessary (w_L, t_L) satisfies (1) $U(w_L, t_L; \theta_L) \geq U(w_H, t_H; \theta_L)$, (2) $U(w_L, t_L; \theta_H) \leq U(w_H, t_H; \theta_H)$ and (3) $w_L > \theta_L$. Suppose firm l is offering (w_L, t_L). Then firm $k \neq l$ can deviate and offer a single contract $(\bar{w}, \bar{t})$ that satisfies (a) $U(\bar{w}, \bar{t}; \theta_L) < U(w_H, t_H; \theta_L)$, (2) $U(\bar{w}, \bar{t}; \theta_H) > U(w_H, t_H; \theta_H)$ and (3) $w_H < \bar{w} < \theta_H$ and $t_H < \bar{t}$. Finding such a $(\bar{w}, \bar{t})$ is possible due to single crossing. See Figure 9b. Given the offer $(\bar{w}, \bar{t})$ of firm k, none of the low ability workers are willing to accept employment in firm k and all the high ability workers will join firm k. Since $\bar{w} < \theta_H$, firm k makes positive profit which contradicts Lemma 1. Hence $w_H \geq \theta_H$. Since by Lemma 1 firms break even, we must have $w_L = \theta_L$ and $w_H = \theta_H$. □



(A) Non-existence of pooling equilibrium. (B) $w_H < \theta_H$ in a separating equilibrium is impossible.

FIGURE 9

Lemma 4. In any separating equilibrium, the low ability workers accept the contract $(\theta_L, 0)$.

Proof: By Lemma 3, $w_L = \theta_L$ in any separating equilibrium. If $(w_L, t_L) = (\theta_L, t')$ with $t' > 0$, then $u \equiv U(\theta_L, t'; \theta_L) < U(\theta_L, 0; \theta_L) = \theta_L$ and any firm can deviate and offer $(w'_L = u + \epsilon, 0)$ such that $u < w'_L < \theta_L$ and $U(w'_L, 0; \theta_L) = u + \epsilon > U(\theta_L, t'; \theta_L) \equiv u$. Therefore, all low ability workers will join the deviating firm and hence the deviating firm earns strictly positive profits from the contract $(w'_L = u + \epsilon, 0)$. Thus, $(w_L, t_L) = (\theta_L, t')$ with $t' > 0$ is not sustainable in equilibrium. □

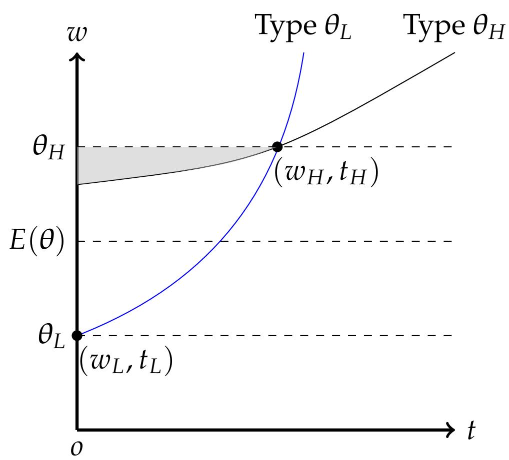
Lemma 5. In any separating equilibrium the high ability worker accepts the contract $(\theta_H, t^{(1)})$ where $t^{(1)}$ satisfies $u(\theta_H, t^{(1)}; \theta_L) = \theta_H - c(t^{(1)}, \theta_L) = \theta_L - c(0, \theta_L) = u(\theta_L, 0; \theta_L)$.

Proof: By Lemma 3 and 4 we know that $(w_L, t_L) = (\theta_L, 0)$ and $w_H = \theta_H$. Since the low ability worker is accepting the contract $(\theta_L, 0)$, the task level of the high ability worker must be at least $t^{(1)}$ where $t^{(1)}$ is that task level for which $u(\theta_H, t^{(1)}; \theta_L) = u(\theta_L, 0; \theta_L)$. If the contract of the high ability worker is (θ_H, t') where $t' > t^{(1)}$ then either firm can, in addition to the existing offers, propose another contract (w'', t'') where $E(\theta) < w'' < \theta_H$, $t'' < t'$ and, more importantly (i) $U(\theta_L, 0; \theta_L) > U(w'', t''; \theta_L)$ and (ii) $U(\theta_H, t'; \theta_H) < U(w'', t''; \theta_H)$. Due to single crossing property it is always possible to give a contract that simultaneously meets (i) and (ii). Inequalities (i) and (ii) imply that only the high ability workers prefer this new contract (w'', t''). The firm offering this new contract (w'', t'') attracts all the high ability workers and earn positive profits. Hence $t' > t^{(1)}$ is not possible in equilibrium. Therefore, the task level of the high ability worker is $t^{(1)}$. □

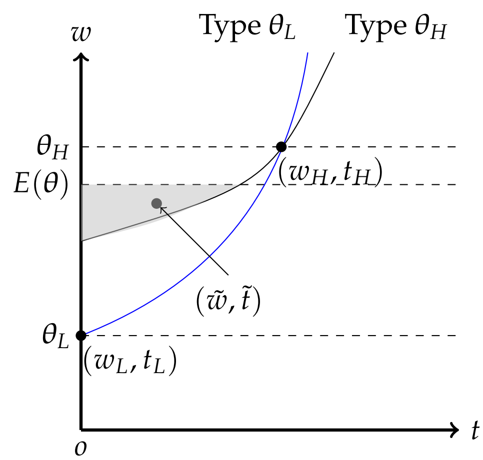
The conclusions of Lemma 1-5 is summarized in the next proposition.

Proposition 3. In any SPNE of the screening game with unknown worker types, the low ability worker accepts $(\theta_L, 0)$ and the high ability worker accepts $(\theta_H, t^{(1)})$ where $t^{(1)}$ satisfies $\theta_H - c(t^{(1)}, \theta_L) = \theta_L$.

Our analysis shows that the pure strategy SPNE, *if it exists*, is summarized by the conditions in Proposition 3. However, its existence depends on the utility difference between the contracts $(w_H = \theta_H, t_H = t^{(1)})$ and $(w = E(\theta), t = 0)$ for the high type. Specifically, if $U(E(\theta), 0; \theta_H) = E(\theta) \leq U(\theta_H, t^{(1)}, \theta_H)$ (as shown in Figure 10a), then the SPNE equilibirum exists and if $U(E(\theta), 0; \theta_H) = E(\theta) > U(\theta_H, t^{(1)}, \theta_H)$, then any firm can deviate and offer a single pooling contract like $(\tilde{w}, \tilde{t})$ (as shown in Figure 10b) and make positive profit implying non-existence of the separating equilibrium.



(A) Existence of the separating equilibrium



(B) Non-existence of the separating equilibrium.

FIGURE 10

REFERENCES

[1] Akerlof, George 1970. Market for 'Lemons': quality uncertainty and the market mechanism. The Quarterly Journal of Economics, **89**, 488-500.

[2] Kreps, David M. 1990. A Course in Microeconomic Theory. Princeton University Press.

[3] Mas-Colell, A., Whinston, M. and Green, J. 1996. Microeconomic Theory. Oxford University Press.

[4] Rothschild, M. and Stiglitz, J. 1976. Equilibrium in Competitive Insurance Markets: An Essay on the Economics of Imperfect Information. The Quarterly Journal of Economics, **90**, 629-649.

[5] Spence, A. M. 1973. Job market signaling. The Quarterly Journal of Economics, **87**, 355-374.

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