INDIAN STATISTICAL INSTITUTE
Mid-Semestral Examination: 2015-16

Course name: MSQE II
Subject name: Incentives and Organisations
Date: 7.9.15
Maximum marks: 30
Duration: 2 hours

Q1. There is a firm (principal) and a large group of workers (agents) of measure 1. A worker's productivity or ability level, θ, is private information, and $\theta = 1$ or $\theta = 2$. The proportion of workers that have high productivity ($\theta = 2$) is $\alpha \in (0,1)$. A worker can be hired at any output level $q \geq 0$. A worker's payoff is $u(q, w; \theta) = w - \frac{q^2}{2\theta}$, where the second term is the cost of producing output q, and w is wage. The firm's profit is $v(q, w; \theta) = q - w$ for a worker hired at w to produce q. If the firm does not hire any workers, it earns 0 profits. Any worker has payoff 0 if not hired.

Suppose the principal offers a two-item menu of contracts, one for each type, and each agent decides whether to reject or accept. The firm has the capacity to hire all the workers.

a) Derive the optimal full employment and shutdown contracts. [5]

b) When does the principal prefer full employment? [5]

c) Suppose there were two competing firms with no capacity constraints, each simultaneously offering contract menus. Any agent can choose to accept a contract from any one firm, or reject both. Consider a symmetric subgame perfect equilibrium with full employment and separating contracts. Suppose in this equilibrium both firms make zero profits overall, i.e., on both items in the menu together. Show that each firm then must make zero profit on each item on the menu. [5]

Q2. As in the Akerlof framework, suppose there are many identical risk-neutral price-taking firms producing a homogeneous good with price = 1 using a CRS production function. Labor is the only input. There are many workers, of measure 1. The number of units a worker can produce if employed by a firm, θ, is private information. $[\underline{\theta}, \bar{\theta}]$ is the set of possible worker types, where $0 \leq \underline{\theta} < \bar{\theta}$. The proportion of workers with productivity of θ or less is given by the increasing and continuously differentiable distribution function $F(\theta)$, with derivative $f(\theta)$. The outside option for wokers is given by $r(\theta)$.

Suppose $r(.)$ is a continuous and increasing function and that there exists $\hat{\theta} \in (\underline{\theta}, \bar{\theta})$ such that $r(\theta) < \theta$ for $\theta < \hat{\theta}$, while $r(\theta) > \theta$ for $\theta > \hat{\theta}$.

a) Derive the competitive equilibrium with observable worker types. [10]

b) Show that a competitive equilibrium with unobservable worker types will be Pareto inefficient. [5]

Indian Statistical Institute

Mid-Semester Examination: 2015-2016

MS(QE) II: 2015-2016

Industrial Organization

Date: 07/09/2015 Maximum Marks: 40 Duration: 3 Hours

Answer any FOUR questions.

1. (a) Let the market demand function be $Q = Ap^{-\varepsilon}$, $\varepsilon > 1, A > 0$. The cost function of the monopolist is linear. A consumer at a distance $x \geq 0$ from the monopolist's trading centre requires to pay tx $(t > 0)$ as transport cost per unit of output to be bought. Under price discrimination what f.o.b. price will the monopolist charge to the consumer? If x goes up, find its effect on the f.o.b. price. Interpret the result.

(b) Suppose value of an object, v, is distributed uniformly over [0, 1]. The buyers valuation for the object is $U_B = \theta_B v$ and that for the sellers, $U_S = \theta_S v$. Suppose the sellers know the quality of the good, but buyers do not; buyers form a belief about the quality of the product based on the price charged by the seller. Show that if $\theta_B < 2\theta_S$ the market for the product will cease to exist.

[5+5=10]

2. A monopolist produces a durable good which does not depreciate in the first period but lasts for only two periods, and resale market exists in the second period, that is, the goods purchased in the first period can be sold in the second period. Assume that the cost of production is zero and both the monopolist and the consumers have the same time-preference given by the discount factor δ where $0 < \delta \leq 1$. The consumption demand for the good in period t is given by the function $X_t = 1 - p_t$ where p_t is the user price (that is, the marginal willingness to pay for consumption or use of the good).

(a) If x_1 and x_2 be the monopoly sales in two periods under "selling" strategy, derive the optimal time consistent monopoly sales and ownership prices in two periods.

(b) Would the monopolist gain by leasing the products instead of selling?

[6+4=10]

3. A restaurant sells mixed fried rice (R) and chili chicken (C). The marginal costs of each plate of R and C are respectively Rs 40 and Rs 20. There are five types of customers: A, B, C, D and E; however, each type has the same number of customers. Each customer buys at most one plate of R and C provided that the price of the product of any item does not exceed its reservation price. The reservation prices of each type customers are given below:

Customer Types	Res. Price for R	Res. Price for C
A	Rs 40	Rs 10
B	Rs 50	Rs 30
C	Rs 70	Rs 30
D	Rs 20	Rs 60
E	Rs 50	Rs 50

(a) Find the profit maximizing prices of R and C when these are sold separately.

(b) If the restaurant sells R and C only in a 'Value Meal' bundle consisting one unit of each, find the profit maximizing price of a Value Meal.

(c) Suppose the restaurant sells R and C both separately as well as in a Value Meal bundle. Find the respective profit-maximizing prices of each of R and C and that of a Value Meal.

[3+3+4=10]

4. Suppose two firms, 1 and 2, produce one good each at marginal cost c_i $(i = 1, 2)$. Each firm has a monopoly power in the production of its goods. The demand curve is $q = D(p)$ where $p \equiv p_1 + p_2$ is the price of the composite good and p_i is the price of good i $(i = 1, 2)$. Let $c \equiv c_1 + c_2$ and $\varepsilon = -D'p/D$ be the price elasticity of demand. Answer the following.

(a) Derive the optimal p for the horizontally integrated structure in terms of c and ε.

(b) Consider the non-integrated structure and suppose that that firm 1 chooses its price first and takes into account the effect of its choice on firm 2's price. Show that the composite price in this case will be given by $p = c/(1 - \frac{1}{\varepsilon})^2$.

(c) Suppose now that the two firms choose their prices simultaneously and non-cooperatively. Show that $p = c/(1 - \frac{2}{\varepsilon})$.

[3+3+4=10]

5. Consider a monopoly manufacturer which sells its products to a retailer which in turn resells these products to the final users. The market demand for the product is linear and is given by $p = a - bq,\ a,b > 0$. The unit cost of producing the goods by the manufacturer is $c,\ 0 < c < a$; the retailer has no additional retailing cost.

(a) Suppose the manufacturer gives a linear contract $\{r\}$ under which the retailer can buy any amount of the good at price r per unit. Find the optimal values of r, p and profits of the manufacturer and the retailer.

(b) Suppose the manufacturer gives a two-part tariff contract $\{F,r\}$ where F is a fixed fee and r is the price charged per unit. Again solve for F, r, p and profits of the manufacturer and the retailer.

(c) If both the firms integrate to form a single firm, what will happen to industry profits and consumers' welfare compared to (a) and (b) above?

[3+4+3=10]

INDIAN STATISTICAL INSTITUTE

Mid-Semester Examination: (2015-2016)

MS(QE) II

Selected Topics I

Date: 14. 09. 2015 Maximum Marks: 40 Duration: 2 hrs 30 minutes

(1) Consider the labor market signaling model where the marginal productivity of a worker is $\theta \in \{2, 6\}$ and $Pr(\theta = 6) = \frac{1}{2}$. The cost of education is $c(e, \theta) = \frac{e^2}{2\theta}$ for all $e \geq 0$. Let $u(w, e; \theta) = w - c(e, \theta)$ be the utility of a worker of type θ who chooses education level e and receives wage w. Assume that both worker types earn zero by staying home, that is $r(2) = r(6) = 0$.

(a) Consider the belief function

$$\mu^a(e) = \begin{cases} 1 & \text{if } e \geq e^*, \\ 0 & \text{if } 0 \leq e < e^*. \end{cases}$$

Find all possible values of e^* for which we can have a separating equilibrium. Justify your answer. **(6)**

(b) Consider the belief function $\mu^b(e) = \frac{e - \max(0, e-4)}{4}$ for all $e \geq 0$. Can you find a separating equilibrium for the belief function $\mu^b(e)$? Justify you answer. **(7)**

(2) In the two-stage screening game with unknown worker types, show that in any equilibrium (separating or pooling), firms earn zero profits. Then show that in this game we cannot have any pooling equilibrium. **(7+5)**

(3) Consider the labour market model where the effort level of the tenant is neither observable nor verifiable. Derive the second best contract. **(15)**

INDIAN STATISTICAL INSTITUTE

Semestral Examination: (2015-2016)

MS(QE) II

Selected Topics I

Date: 23.11.2015 Maximum Marks: 60 Duration: 3 hrs.

(1) Consider the social aggregation problem, where $A = \{x, y\}$ is the set of alternatives, $|A| = 2$, N is the finite set of agents and $|N| \geq 2$.

(a) Define non-triviality, symmetry, neutrality and positive responsiveness of a social welfare function.

(b) Show that if a social welfare function satisfies symmetry, neutrality and positive responsiveness, then it must be the majority rule social welfare function.

(c) Can you find a social welfare function that satisfies non-triviality and positive responsiveness but fails to satisfy Pareto? Justify your answer. **(8+10+5=23)**

(2) Consider the social aggregation problem, where A is the set of alternatives, $|A| \geq 3$, N is the finite set of agents and $|N| \geq 2$.

(a) Define 'almost decisiveness' and 'decisiveness'.

(b) A social ordering satisfies *positive responsiveness* if the following holds: If $R_i = R_i' \; \forall \; i \in N \setminus \{j\}$ and either (yP_jx and $xR_j'y$) or (xI_jy and $xP_j'y$), then xRy implies $xP'y$. Show that if a social ordering satisfies unrestricted domain and positive responsiveness, then 'almost decisiveness' and 'decisiveness' are equivalent. **(5+15=20)**

(3) Define the pivotal mechanism for the pure public goods problem. Show that this mechanism is dominant strategy incentive compatible and feasible. **(2+10+5=17)**

INDIAN STATISTICAL INSTITUTE
Final Examination: 2015-16

Course name: MSQE II
Subject name: Incentives and Organisations
Date: 27.11.15.
Maximum marks: 50
Duration: 3 hours

Answer all questions

Q1. Suppose there is a risk-averse agent who has wealth w, and can face a damage d, with $w > d > 0$. Damage is observable. There are two possible types of agents, 1 and 2, depending on the probability of facing the damage. Let p_1 and p_2 be the respective probabilitites for the two types of agents, with $0 < p_1 < p_2 < 1$, and $a \in (0,1)$ be the probability the agent is of type 1. Suppose also there is a risk-neutral monopoly insurer who can offer insurance to the agent.

(a) Suppose the agent's type is publicly observable. Characterize the optimal contract. [5]

(b) Suppose the agent's type is not publicly observable, but is private information to the agent. Characterize the optimal contract. [15]

Q2. (i) Consider the binary-effort hidden-action model with a risk-averse agent and a risk-neutral principal. The cost to the agent of high effort ($e = 1$) is c, and the agent's outside option gives utility 0. There are two possible output levels q_h and q_l, with $q_h > q_l > 0$. Output is publicly observable, and the probability of getting q_h is π_e, with $1 > \pi_1 > \pi_0 > 0$.

(a) Characterize the optimal contract for implementing high effort (with hidden action). [10]

(ii) Suppose the principal can observe an extra signal y of the agent's effort, which is independent of output, conditional on effort. y can be either y_h or y_l, with $y_h > y_l > 0$. The probability of observing y_h, in the event the principal wishes to obtain this extra signal, is β_e, with $1 > \beta_1 > \beta_0 > 0$. Obtaining this signal costs k, and the decision to obtain the signal has to be made before any contract is offered.

(b) Define a contract in this environment. [5]

(c) When will the principal choose to obtain this extra signal, given that he wishes to implement the high effort? [15]

Indian Statistical Institute

Semestral Examination: 2015-2016

MS(QE) II: 2015-2016

Industrial Organization

Date: 30/11/2015 Maximum Marks: (20+40) Duration: $3/(2\frac{1}{2})$ Hours

Group A: [Only for those candidates who have not submitted assignments in time]

1. Consider a duopoly market with market demand for the product given by $p = 2 - x_1 - x_2$, where is x_i is output of firm i. Firm 1's MC is 1 and this is common knowledge. However firm 2's MC is determined by Nature. It is 5/4 with probability ½ and ¾ with probability ½. Firms choose quantities simultaneously and non-cooperatively. Find the expected payoff of each firm prior to nature's move in the following cases;
   (i) At the stage of production neither firm knows firm 2's cost. [7]
   (ii) At the stage of production firm 2's cost is private information.[7]
   (iii) At the stage of production both firms know their costs and this is common knowledge. [6]

Group B: Answer ALL questions

[Maximum $2\frac{1}{2}$ Hours for answering this group]

2. Suppose that firms build only an integer number of plants, 0, 1, 2, ... Building $k > 0$ number of plants costs $3.5k$. Each plant can produce exactly one unit of output. There is no variable cost of production. The market demand for the product is $p = 6 - X$, where X is the industry total output.
   (i) If there is a single firm, how many plants will the monopolist install? [3]
   (ii) If there are two firms to decide the number of plants simultaneously, how many plants will each firm build in equilibrium? [3]
   (iii) Out of these two firms if firm 1 builds plant first before firm 2, how many plants will they install? [4]

3. Consider the following leader-follower structure when firms choose quantities. The market demand function for the homogenous good is given by $p = \frac{1}{X}$, where X is industry output. Each firm has MC=1. The firms first simultaneously decide whether they will choose Stackelberg leader output or Stackelberg follower output, and in the second stage they play accordingly. Solve the game. (Consider only the first order conditions.) [10]

4. Consider the Hotelling linear city model. Consumers are uniformly distributed over the unit length of a city and each consumer buys at most one unit of a product supplied by two private firms. Price of the product is fixed at p higher than the unit cost of production. But the consumers are to pay a transport cost. If a consumer located at a distance x from the left, buys a product from firm I located at a distance l_i from the left, the transport cost is $t|x - l_i|$. So firms compete in market shares.

   (a) Find the optimal locations of the firms. What is the total transport cost to be incurred by the consumers all together? [5]

   (b) If the social planner decides the location of the firms to minimize the total transport cost incurred by all consumers together, what will be the optimal locations of the firms? [5]

5. Consider an n-firm Cournot model with homogeneous goods; $n \geq 2$. The market demand function is linear and there is no cost of production. Now consider formation of a k- firm cartel; $1 < k \leq n$. Thus the game is played among $(n - k)$ independent firms and the cartel of k firms.

   (i) Derive the payoffs of each independent firm and each cartel member. [3]

   (ii) Now examine the following proposition. If there are at least three firms in the industry, a stable cartel cannot be formed, hence all firms will remain independent; if there are just two firms in the industry, the two firms form a cartel. [7]

Indian Statistical Institute

Mid-semester Examination 2016

Course name: **Political Economy**

Subject name: **Economics**

Date: **23 February 2016**

Maximum marks: **40**

Duration: **2 hours 30 minutes**

1. This question pertains to direct aggregation of individual preferences $\{\succsim_i\}_i$ over alternatives in some set X to obtain a social preference by applying the majority rule, $\succsim_{MR}$. For simplicity, you may assume $X \subsetneqq R$, discrete and finite. Either prove or give a counterexample for each of the following three statements, clearly indicating whether you are proving or giving a counterexample: **(5 x 3 = 15 points)**

(i) If $\succsim_{MR}$ is intransitive over any $x, y, z \subsetneqq X$, then exactly one of the following must hold:
(a) $x \succsim_{MR} y \succsim_{MR} z \succsim_{MR} x$
(b) $y \succsim_{MR} x \succsim_{MR} z \succsim_{MR} y$.

(ii) If $\succsim_i$ is complete $\forall i$, then $\succsim_{MR}$ is complete.

(iii) If individual preferences are *single-peaked* (SP), then $\succsim_{MR}$ is transitive. That is, SP $\Longrightarrow \succsim_{MR}$ transitive.

2. Suppose preferences of individuals for a service consists of two dimensions, p_1 and p_2. Suppose there are three individuals, a 'low' consumer, a 'middle' consumer and a 'high' consumer with bliss points $(1,1)$, $(2,3)$ and $(4,4)$ respectively. Suppose individuals can directly vote on alternative pairs. Prove or argue otherwise whether a 'median-voter' analogue may still hold good. That is, is it true that the $(2,3)$ alternative will beat all other alternatives in pairwise majority voting? Argue briefly (you may use diagrams). **(5 points)**

3. This question pertains to indirect aggregation of individual preferences in the context of a model of representative democracy as discussed in class. The standard assumptions of such a model remain the same, namely, there are n individuals in the economy each having well-behaved preferences over policy denoted by $\{\succsim_1, \succsim_2, \cdots, \succsim_n\}$. Moreover these preferences satisfy *extremal restriction* (ER). For simplicity, you may assume $X \subsetneqq R$, discrete and finite.

There are two candidates/parties/representatives, A and B who try to maximize objective functions u_A and u_B. However now, let the objective function of the representative be *net plurality* (NPL). For instance, candidate A's objective is to maximize

$$u_A = \#\{i : x_A \succ_i x_B\} - \#\{i : x_B \succ_i x_A\}.$$

We can define u_B likewise. Let $G_{NPL} := (\{A, B\}, X, \{u_A, u_B\})$. Consider the mixed extension of G_{NPL} and denote it by G'_{NPL}. Then show that

(i) G'_{NPL} has a Nash equilibrium. **(5 points)**

(ii) Any Nash equilibrium (p_A, p_B) of G'_{NPL} has support$(p_k) \subsetneq X^*$, $k = A, B$, where X^* is the set of Condorcet winners in X. **(10 points)**

4. This question pertains to part of the proof of the Sen and Pattanaik theorem (1969) as discussed in class.

Assume that individual preferences satisfy *extremal restriction* (ER). Suppose $\exists$ an individual i with strict preference between three alternatives x, y, z, such that $x \succ_i y \succ_i z$. Moreover assume that social ordering according to the majority rule satisfies the "forward cycle", that is, $x \succsim_{MR} y \succsim_{MR} z \succsim_{MR} x$. Under these circumstances, which kinds of individual preferences are feasible? **(5 points)**

(Note: The assumptions mentioned above are inconsistent and as we know, will lead to a contradiction. However, this question *does not* ask you to complete the proof and reach the contradiction.)

INDIAN STATISTICAL INSTITUTE
SEMESTRAL EXAMINATION, 2014-2015
M.S. (Q.E.) I, II Years and M. Math. II Year
Game Theory II

Date: 29.04.2016 Maximum Marks: 100 Time: 3 hours

Note: Answer Parts (A) and (B) in separate answer scripts. Clearly explain the symbols you use and state all the assumptions you need for any derivation. The paper carries 110 marks. You may attempt any part of any question. The maximum you can score is 100.

A

1. Define a voting game. Establish a necessary and sufficient condition for non-emptiness of the core of such a game in terms of a blocker. (10)

2. Consider the problem of allocating costs for providing some service to a set of customers. Assume that the following conditions hold: (a) all non-users of the service do not pay for it but all users should be charged equally; (b) the total cost of using the service is the sum of capital and operating costs, and (c) the service provider will recover the entire cost from the customers. Clearly demonstrate that there is a unique solution to this cost recovery game. (12)

3. Identify the relation between the nucleolus and the kernel of a coalition form game for the grand coalition structure by giving necessary preliminaries. (10)

4. State and prove the Bondareva-Shapley theorem by defining all necessary concepts. (16)

5. Formulate a bankruptcy situation, where there are $n > 1$ claims against an estate and the sum of the claims exceeds the estate's worth, as a coalition form game. Show that such a game is convex. (10)

P.T.O

6. Let $N=\{A_1,A_2,A_3\}$ be a set of 3 firms producing a homogenous output whose price function is $10-x_{A_1}-x_{A_2}-x_{A_3}$, x_{A_i} being the output of firm A_i. The maximum output a firm can produce is 3. The cost function of firm A_i is $(1+x_{A_i})$. The worth of any non-empty coalition $S\in 2^N$ is defined as $v(S)=\max_{x_{A_i},A_i\in S}\min_{x_{A_j},A_j\notin S}\sum_{A_i\in S}x_{A_i}\left(10-x_{A_1}-x_{A_2}-x_{A_3}\right)-\left(\sum_{A_i\in S}(1+x_{A_i})\right)$. Determine the numerical value of worth of each non-empty coalition. Also identify the set of core elements. (20)

7. (a) Show that the solution to the two-person bargaining problem satisfying the Nash axioms exists and is unique. (7)

(b) When do you say that a marriage matching is stable? (5)

B

1.(a) Show that for a weighted majority game, the problem of finding the number of swings for a particular player can be reduced to SUBSET SUM problem. Hence, determine the time complexity of finding the number of swings for a particular player.

(b) Suppose that the Gale-Shapley "men propose" algorithm is modified whereby in a round each unmatched man proposes to the second most preferred woman in his current list. Will this modified algorithm also result in a stable matching? Explain your answer.

© Assuming that the "men propose" strategy results in the best partners for men show that it results in the worst partners for women. (8+6+6)

Indian Statistical Institute

Second-semester Examination 2016

Course name: **MSQE II year**

Subject name: **Political Economy**

Date: **2 May 2016**

Maximum marks: **60**

Duration: **3 hours**

Instructions: **Answer all questions.**

1. Within the Palfrey-Rosenthal framework of strategic voting as discussed in class, and using simple numerical figures for team sizes, demonstrate the forces of "competition" and "free-riding" that individuals face when deciding whether or not to vote. (You may assume the coin-toss rule for breaking ties and all individuals facing an identical voting cost given by c.) **(10 points)**

(Hint: Consider using simple 2x2 games and their Nash equilibria to make your point.)

2. This question pertains to the characterization of "mixed-pure" equilibria as discussed by Palfrey and Rosenthal in the context of strategic voting. Suppose $M, N > 1$. (Recall M and N are the number of members in teams 1 and 2 respectively.) Let $k \in \mathbb{N}$ be such that $k \leq \min\{M-1, N\}$. Recall that c is the exogenously given cost of voting for each player. We know that if $c \leq \binom{M-1}{k}\left(\frac{k}{M}\right)^k\left(1-\frac{k}{M}\right)^{M-1-k}$, then there is an equilibrium of G_2 (status-quo rule) in which exactly k members of team 2 vote and all team 1 members vote with probability q. Show that q satisfies $c = \binom{M-1}{k}q^k(1-q)^{M-1-k}$. **(5 points)**

(Hint: This question only asks you to demonstrate the best response of a team 1 member, given the strategy of the other team - it *does not* ask you for the proof of the proposition.)

3. What is the expected turnout in a "$q-k$" or "mixed-pure" equilibrium as discussed by Palfrey and Rosenthal? What are its drawbacks when positive voting costs of individuals and large electorates are considered? **(5 points)**

4. This question pertains to a parameterized version of Feddersen and Sandroni's ethical voting model. Let the fraction of ethical agents in groups 1 and 2, $\tilde{q}_1$ and $\tilde{q}_2$ respectively, be

independently and identically distributed as $U[0,1]$. Let the fraction of the population in group 1 be deterministic and be given by $k \in (0, 1/2]$. Let cost of voting for each individual be random and be drawn from $U[0, \bar{c}]$. Let the payoff from 'doing one's part', D, be $> \bar{c}$. Let the social cost function be linear, that is $v(x) = x$. Recall w to be the parameter capturing the 'importance of election'. Also let parameters $\bar{c}, w$ and k satisfy $\frac{\bar{c}}{w} > \frac{1}{\sqrt{k(1-k)}}$.

(i) Show that the equilibrium fraction of ethical agents who vote in each group is given by:

$$\sigma_1^* = \left(\sqrt{\frac{w}{\bar{c}}}\right) \cdot \frac{1}{k^{1/4}(1-k)^{1/4}},$$

$$\sigma_2^* = \left(\sqrt{\frac{w}{\bar{c}}}\right) \cdot \frac{k^{1/4}}{(1-k)^{3/4}}.$$

(12 points)

(ii) From (i), what can you conclude about the participation rates of the minority and the majority? **(4 points)**

(iii) From (i), what can you conclude about the chances of winning of the minority versus that of the majority? **(4 points)**

5. This question deals with lobbying as discussed by Grossman and Helpman. They conclude that "both the policymaker and the interest group may benefit from having lobbying not be free." Consider a single interest group, a single policy variable and two possible states of the world, to elucidate the above statement. **(20 points)**