Optimization & Decision

Masters Degree in Mechanical Engineering

Project - Part 1 [EN]

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Group 32

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1 Introduction

The integration of renewable energy sources into the electrical grid presents challenges related to variability in power generation and demand fluctuations over time. Efficient energy management strategies are required to balance supply and demand while minimizing operational costs.

One approach involves optimizing battery dispatch to store excess renewable energy and discharge it when needed, reducing reliance on the power grid.

This project addresses the problem as a linear programming model to determine the optimal charging and discharging strategy for multiple batteries in a smart grid.

The problem is simplified to be solved using the Simplex method. The results from this approach provide insights into how an optimal energy management strategy can reduce costs and improve grid stability.

2 Mathematical Formulation and Simplex Method

2.1 Problem Formulation

The objective of this project is to optimize the dispatch of multiple batteries in a smart grid by minimizing energy costs while ensuring demand is met and battery constraints are satisfied. To simplify the model, all consumers and producers are aggregated into representative features:

- P_t^{prod}: Total renewable production at time t, summing all individual producer contributions.
- P_t^{load}: Total energy demand at time t, summing all individual consumer demands.

This combination, reduces the number of constraints, maintaining a linear formulation and making the problem solvable using the Simplex Method.

2.2 Decision Variables

The following decision variables are defined:

- $P_{i,t}^c$: Power charged into battery i at time t (kW), constrained by $0 \leq P_{i,t}^c \leq P_t^{prod}$.
- $P_{i,t}^d$: Power discharged from battery i at time t (kW), constrained by $0 \leq P_{i,t}^d \leq E_{i,t-1}$.
- $E_{i,t}$: Energy stored in battery i at time t (kWh), subject to $0 \leq E_{i,t} \leq E_{\max,i}$.
- P_t^{grid}: Power drawn from the grid at time t (kW), where $P_t^{grid} \geq 0$.

2.3 Objective Function

The goal is to minimize the total grid energy cost over the time horizon T:

$$\min \sum_{t \in T} C P_t^{grid} \tag{1}$$

where C is the constant electricity cost.

2.4 Constraints

2.4.1 Energy Balance:

$$E_{i,t} = E_{i,t-1} + \eta_c P^c_{i,t} - \frac{P^d_{i,t}}{\eta_d}, \quad \forall i, t > 0 \tag{2}$$

with the initial condition:

$$E_{i,0} = 150 \text{ kWh} \tag{3}$$

where η_c and η_d are the charging and discharging efficiencies.

2.4.2 Battery Limits:

$$0 \leq E_{i,t} \leq E_{\max,i}, \quad 0 \leq P^c_{i,t} \leq P^{prod}_t, \quad 0 \leq P^d_{i,t} \leq E_{i,t-1}, \quad \forall i, t \tag{4}$$

2.4.3 Grid Sufficiency Constraint:

$$\sum_{i=1}^{N} P^d_{i,t} + P^{grid}_t \geq P^{load}_t - P^{prod}_t, \quad \forall t \tag{5}$$

2.4.4 Power Balance Constraint:

$$P^{grid}_t + \sum_{i=1}^{N} P^d_{i,t} + P^{prod}_t \geq P^{load}_t, \quad \forall t \tag{6}$$

This ensures that total supply meets total demand while keeping the problem linear.

2.5 Solution Approach

To ensure computational efficiency, the system is modeled with a single representative battery. This simplification reduces problem size while preserving optimization structure. The problem remains solvable by Simplex because:

- The formulation consists only of linear constraints and a linear objective function.
- Decision variables (battery energy, charge/discharge power, and grid power) remain continuous and non-negative.
- The LP structure allows Simplex to efficiently find the optimal cost-minimizing solution.

3 Results and Analysis

The optimization results include:

- **Battery State of Charge (SoC)**: The evolution of $E_{i,t}$ over time, showing battery utilization.
- **Grid Power Import**: The total grid energy drawn, indicating external energy usage.
- **Charging and Discharging Patterns**: The behavior of $P^c_{i,t}$ and $P^d_{i,t}$ over time.
- **Total Cost**: The overall grid energy cost minimized by the model.

The figures below illustrate these results:

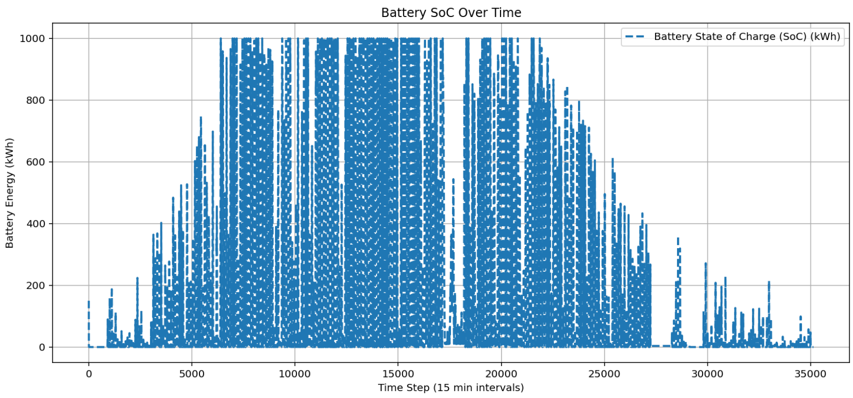

Figure 1: Battery State of Charge (SoC) Over Time

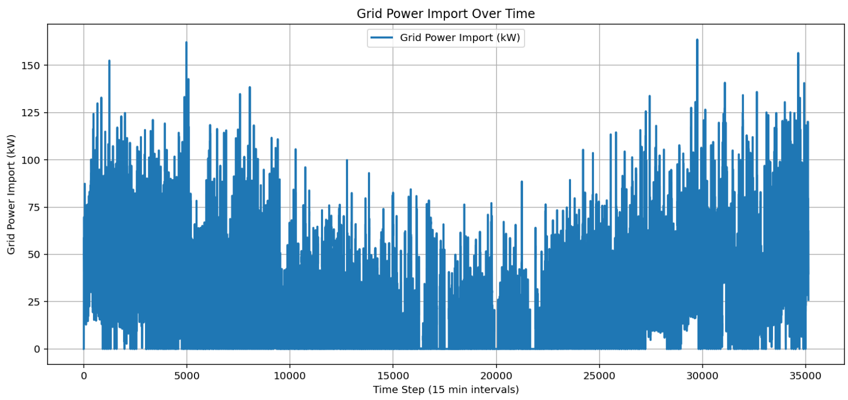

Figure 2: Grid Power Import Over Time

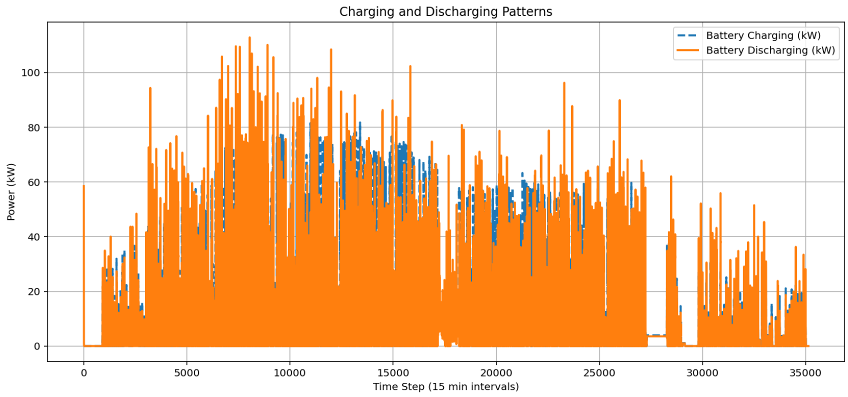

Figure 3: Charging and Discharging Patterns

4 Cost Analysis

Since the energy cost is constant, the main goal consists of using the battery as must as possible. By analyzing the 3, we can clearly see that the majority of the cycles correspond to discharging cycles, suggesting that the energy is being well managed.

GitHub Link

You can find the code on GitHub at:
https://github.com/FranciscoVPinto/OD_Proj