(iv)

$u(x,0)=0$ and $\frac{\partial u}{\partial t}(x,0) = g(x)$ where

$$g(x) = \begin{cases} M, & \frac{L}{4} < x < \frac{L}{2} \\ 0, & \text{else} \end{cases}$$

we know $u(x,t) = \sum_{n=1}^{\infty} \sin\left(\frac{n \cdot \pi}{L} \cdot x\right) \cdot T_n(t)$ where

$$T_n(t) = a_n \sin\left(\frac{c n \pi}{L} \cdot t\right) + b_n \cos\left(\frac{c n \pi}{L} \cdot t\right)$$

$$u_n(x,0) \Rightarrow T_n(0) = a_n \cancel{\sin(0)}^{\,0} + b_n \cdot 1 = 0$$

$$\boxed{b_n = 0}$$

$$\Rightarrow T_n(t) = a_n \sin\left(\frac{c n \pi}{L} \cdot t\right)$$

then

$$\frac{\partial u}{\partial t}(x,0) = g(x) = \sum_{n=1}^{\infty} \sin\left(\frac{n \cdot \pi}{L} \cdot x\right) \cdot a_n \cdot \frac{c \cdot n \cdot \pi}{L}$$

where g is piecewise. Let $K_n = \frac{a_n \cdot c \cdot n \cdot \pi}{L}$

$$\Rightarrow g(x) = \sum_{n=1}^{\infty} \sin\left(\frac{n \cdot \pi}{L} \cdot x\right) \cdot K_n$$

by Maple, we end up with:

$$u(x,t) = \sum_{n=1}^{\infty} \frac{-M\left(\cos\left(\frac{n\pi}{2}\right) - \cos\left(\frac{n\pi}{4}\right)\right)L}{n^2 \pi^2 c} \cdot \sin\left(\frac{n\pi}{L} \cdot x\right) \cdot \sin\left(\frac{cn\pi}{L} t\right)$$

with period (in t) $\frac{2\cancel{\pi}}{cn\cancel{\pi}} \cdot L = \frac{2L}{cn}$ and common period $2L$.

Maple: