

Note: It make sense to let the temperature at $r = \text{infinite}$ to be equal to 25 (assuming that we have a unitary circle $L=1$, otherwise we would have $25L$), since we have the BC at $r=L$ to be 100 on $1/4$ of the ring. It is reasonable to assume that the heat will dissipate evenly inside the other 3 parts of the ring, arriving to 25 degrees on each quadrant of the ring.

with(plots) :

phisin := $\sin(n \cdot x)$:

phicos := $\cos(n \cdot x)$:

$$An := \frac{\int \left(100 \cdot phisin, x = 0 .. \frac{\text{Pi}}{2} \right)}{\int (phisin^2, x = 0 .. 2 \cdot \text{Pi})} \text{assuming}(n > 0, n, \text{integer})$$

$$An := - \frac{100 \left(-1 + \cos\left(\frac{n \pi}{2}\right) \right)}{n \pi} \quad (1)$$

$$Bn := \frac{\int \left(100 \cdot phicos, x = 0 .. \frac{\text{Pi}}{2} \right)}{\int (phicos^2, x = 0 .. 2 \cdot \text{Pi})} \text{assuming}(n, \text{integer}, n > 0)$$

$$Bn := \frac{100 \sin\left(\frac{n \pi}{2}\right)}{n \pi} \quad (2)$$

addcoords(zcylindrical, [z, r, θ], [r cos(θ), r sin(θ), z])

psum1 := $\text{sum}\left(\left(\frac{1}{r^n}\right) \cdot An \cdot phisin, n = 1 .. 100\right)$:

psum2 := $\text{sum}\left(\left(\frac{1}{r^n}\right) \cdot Bn \cdot phicos, n = 1 .. 100\right)$:

psum := $25 + psum1 + psum2$:

contourplot(psum, r = 1 .. 3, x = 0 .. 2 · Pi, coords = zcylindrical, scaling = constrained, filledregion = true)



