5. a) $y'' + 5y' - 2y = 0$, $y(x) = Ce^{\lambda x}$

$$y' = C\lambda e^{\lambda x}$$
$$y'' = C\lambda^2 e^{\lambda x}$$

$$y'' + 5y' - 2y = C\lambda^2 e^{\lambda x} + 5C\lambda e^{\lambda x} - 2Ce^{\lambda x}$$

$$= Ce^{\lambda x}(\lambda^2 + 5\lambda - 2) = 0 \Rightarrow \lambda = \frac{-5 \pm \sqrt{25 - 4(-2)}}{2}$$

$$\Rightarrow \lambda = \frac{-5 \pm \sqrt{33}}{2} \Rightarrow \lambda = \frac{-5 + \sqrt{33}}{2}, \frac{-5 - \sqrt{33}}{2}$$

Eigenfunctions: $u_1(x) = c_1 e^{\left(\frac{-5+\sqrt{33}}{2}\right)x}$

$$u_2(x) = c_2 e^{\left(\frac{-5-\sqrt{33}}{2}\right)x}$$

General function: $u_1 + u_2 = u(x) = c_1 e^{\left(\frac{-5+\sqrt{33}}{2}\right)x} + c_2 e^{\left(\frac{-5-\sqrt{33}}{2}\right)x}$

Let $a = \frac{-5+\sqrt{33}}{2}$, $b = \frac{-5-\sqrt{33}}{2}$ · $y'' - 2y' + 5y$:

$$\cancel{5c_1e^{ax}} + \cancel{5c_2e^{bx}} + a^2c_1e^{ax} + b^2c_2e^{bx} - \cancel{2ac_1e^{ax}} - \cancel{2bc_2e^{bx}} = 0$$

$$= c_1e^{ax}\left(5 + a^2 - 2a\right) + c_2e^{bx}\left(5 + b^2 - 2b\right) = 0 \quad (*)$$

$$e^{ax} = e^{\frac{-5x+\sqrt{33}x}{2}} = e^{-\frac{5x}{2}} \cdot e^{\frac{\sqrt{33}x}{2}}$$

$$e^{bx} = e^{\frac{-5x-\sqrt{33}x}{2}} = e^{-\frac{5x}{2}} \cdot e^{-\frac{\sqrt{33}x}{2}}$$

since $e^{-\frac{5x}{2}} \neq 0$ divide both sides of $(*)$ by $e^{-\frac{5x}{2}}$

$$c_1e^{\frac{\sqrt{33}x}{2}}\left(5 + a^2 - 2a\right) + c_2e^{\frac{-\sqrt{33}x}{2}}\left(5 + b^2 - 2b\right) = 0$$

$$5c_1 + a^2 - 2a = 5 + \frac{25 - 10\sqrt{33} + 33}{4} + \frac{10 - 2\sqrt{33}}{2}$$

$$= 5 + \frac{\cancel{25} + \cancel{33} - 10\sqrt{33} + \cancel{20} - 4\sqrt{33}}{4} = 5c_1 + \frac{45 + 33 - 14\sqrt{33}}{4}$$

$$= 5c_1 + \frac{78 - 14\sqrt{33}}{4} \approx 4.3940$$

$$\Rightarrow c_1e^{\frac{\sqrt{33}}{2}x}\left(4.3940\right) + c_2e^{-\frac{\sqrt{33}}{2}x}\left(5 + b^2 - 2b\right) = 0$$

$$5 + b^2 - 2b = 5 + \frac{\cancel{25} + 10\sqrt{33} + \cancel{33}}{4} + \frac{10 + 2\sqrt{33}}{2}$$

$$= 5 + \frac{58 + 10\sqrt{33}}{4} + \frac{10 + 2\sqrt{33}}{2} = 5 + \frac{58 + 10\sqrt{33} + \cancel{20} + 4\sqrt{33}}{4}$$

$$5 + \frac{78 + 14\sqrt{33}}{4} \approx 44.6060$$

$$\Rightarrow c_1 e^{\frac{\sqrt{33}}{2}x}(4.3940) + c_2 e^{-\frac{\sqrt{33}}{2}x}(44.6060) = 0$$

$$\Rightarrow c_1 e^{\frac{\sqrt{33}}{2}x} \cdot e^{\frac{\sqrt{33}}{2}x}(4.3940) + c_2 \frac{e^{\frac{\sqrt{33}}{2}x}}{e^{\frac{\sqrt{33}}{2}x}}(44.6060) = 0$$

$$\Rightarrow c_1 e^{\frac{33}{4}x^2}(4.3940) + c_2(44.6060) = 0$$

$$\Rightarrow c_1 e^{33/4 x^2} = -c_2 \frac{(44.6060)}{(4.3940)}$$

$$\Rightarrow e^{33/4 \cdot x^2} = \left(\frac{-c_2(44.6060)}{c_1\ 4.3940}\right)$$

$$\Rightarrow \frac{33}{4} \cdot x^2 = \ln\left(-\frac{c_2(44.6060)}{c_1(4.3940)}\right)$$

then $y'' - 2y' + 5y = 0 \iff \frac{33}{4} \cdot x^2 = \ln\left(\frac{-c_2(44.6060)}{c_1(4.3940)}\right)$

for some constant c_1 and c_2 i.e. $\frac{33}{4} \cdot x^2$ is constant

which is <u>not true</u>.

check (c) $y'' - 2y' + 5y = 2x$, by our previous analysis:

$$\Rightarrow u'' - 2u' + 5u = \frac{33}{4}x^2 - \ln\left(\frac{-c_2(44.6060)}{c_1(4.3940)}\right) = 2x$$

$$\text{if } \frac{33}{4}x^2 = 2x + \ln\left(\frac{-c_2}{c_1}\cdot\left(\frac{44.6060}{4.3940}\right)\right)$$

$\Rightarrow \frac{33}{4}x^2$ is equal to a linear function $\Rightarrow\Leftarrow$

$\therefore u(x) = c_1 e^{\left(\frac{-5+\sqrt{33}}{2}\right)x} + c_2 e^{\left(\frac{-5-\sqrt{33}}{2}\right)x}$ is not solution

for $y'' - 2y' + 5y = 2x$ //