

Note: It make sense to let the temperature at $r = \text{infinite}$ to be equal to 25 (assuming that we have a unitary circle $L=1$, otherwise we would have $25L$), since we have the BC at $r=L$ to be 100 on $1/4$ of the ring. It is reasonable to assume that the heat will dissipate evenly inside the other 3 parts of the ring, arriving to 25 degrees on each quadrant of the ring.

with(plots) :

phisin := sin(n·x) :

phicos := cos(n·x) :

$$A_n := \frac{\int \left(100 \cdot \text{phisin}, x = 0 \dots \frac{\text{Pi}}{2} \right)}{\int (\text{phisin}^2, x = 0 \dots 2 \cdot \text{Pi})} \text{assuming}(n > 0, n, \text{integer})$$

$$A_n := - \frac{100 \left(-1 + \cos\left(\frac{n \pi}{2}\right) \right)}{n \pi} \quad (1)$$

$$B_n := \frac{\int \left(100 \cdot \text{phicos}, x = 0 \dots \frac{\text{Pi}}{2} \right)}{\int (\text{phicos}^2, x = 0 \dots 2 \cdot \text{Pi})} \text{assuming}(n, \text{integer}, n > 0)$$

$$B_n := \frac{100 \sin\left(\frac{n \pi}{2}\right)}{n \pi} \quad (2)$$

addcoords(zcylindrical, [z, r, θ], [r cos(θ), r sin(θ), z])

psum1 := sum $\left(\left(\frac{1}{r^n} \right) \cdot A_n \cdot \text{phisin}, n = 1 \dots 100 \right) :$

psum2 := sum $\left(\left(\frac{1}{r^n} \right) \cdot B_n \cdot \text{phicos}, n = 1 \dots 100 \right) :$

psum := 25 + psum1 + psum2 :

contourplot(psum, r = 1 .. 3, x = 0 .. 2·Pi, coords = zcylindrical, scaling = constrained, filledregion = true)



