MA 501 SP22 Lubkin HW1 due Wed 1/26/22

Objectives:

- Practice with Euler's formula
- Comfort with sinh and cosh
- Review of solution of linear constant coefficient ODE

1. Of these three boundary value problems, one has no solution, one has exactly one solution, and one has an infinite number of solutions. Which is which?

a. $\dfrac{d^2u}{dx^2} + u = 0, \quad u(0) = 0, \quad u(\pi) = 0;$

b. $\dfrac{d^2u}{dx^2} + u = 1, \quad u(0) = 0, \quad u(1) = 0;$

c. $\dfrac{d^2u}{dx^2} + u = 0, \quad u(0) = 0, \quad u(\pi) = 1.$

2. Use Euler's formula to write

(a) $\sin(x)$

(b) $\cos(x)$

in terms of complex exponentials. Show your reasoning.

3a. Use Euler's formula (and the results from Problem 1?) to prove the trig identity

$$\sin(a+b) = \sin(a)\cos(b) + \cos(a)\sin(b)$$

3b. Use the definitions of $\sinh$ and $\cosh$ to prove the identity

$$\sinh(a+b) = \sinh(a)\cosh(b) + \cosh(a)\sinh(b)$$

4. What is i^i ? Is it real, imaginary, or what? Simplify it until you have a good answer. Show your reasoning.

5. For each of the the following ODEs in $y(x)$, (i) Find the general solution by the methods we used in class, (ii) verify that your answer is a solution to its ODE, by differentiating and plugging in. (iii) verify that your solution is *not* a solution to the other ODEs (except in the case of the trivial solution), by differentiating and plugging in. (Note: Maple may be helpful in taking the derivatives.)

(a) $y'' + 5y' - 2y = 0$

(b) $y'' - 2y' + 5y = 0$

(c) $y'' - 2y' + 5y = 2x$