

$$(ii) u(x, 0) = 0$$

$$\frac{\partial u}{\partial t}(x, 0) = f(x)$$

$$\text{where } f(x) = \begin{cases} \frac{2M}{L} \cdot x, & 0 < x < \frac{L}{2} \\ \frac{2M(L-x)}{L}, & \frac{L}{2} < x < L \end{cases}$$

We know :

$$X(x) = \sin\left(\frac{n \cdot \pi}{L} \cdot x\right)$$

$$T(t) = a \sin(c \nu t) + b \cos(c \nu t), \quad \nu = \frac{n \cdot \pi}{L}$$

$$\text{since } u(x, t) = X(x)T(t)$$

$$u(x, 0) = X(x) \cdot T(0) = 0$$

$$\Rightarrow a \sin(0) + b \cos(0) = 0 = b \Rightarrow b = 0$$

$$\Rightarrow T(t) \text{ becomes: } T(t) = a \sin(c \nu t)$$

$$\text{and } T'(t) = a \frac{c n \cdot \pi}{L} \cdot \cos\left(\frac{n \cdot \pi c}{L} \cdot x\right)$$

since $\frac{\partial u}{\partial t}(x, 0) = f(x)$

$$\Rightarrow \sum_{n=1}^{\infty} \frac{a_n \cdot c \cdot n \cdot \pi}{L} \cdot \sin\left(\frac{n \cdot \pi}{L} \cdot x\right) = g(x)$$

let $k_n = \frac{a_n \cdot c \cdot n \cdot \pi}{L} \Rightarrow a_n = \frac{k_n}{c \cdot n \cdot \pi} \cdot L$

then $\sum_{n=1}^{\infty} \frac{a_n \cdot c \cdot n \cdot \pi}{L} \cdot \sin\left(\frac{n \cdot \pi}{L} \cdot x\right) = \sum_{n=1}^{\infty} k_n \sin\left(\frac{n \cdot \pi}{L} \cdot x\right)$

and $u_n(x, t) = \sum_{n=1}^{\infty} \sin\left(\frac{n \cdot \pi}{L} \cdot x\right) \cdot a_n \cdot \sin\left(\frac{c \cdot n \cdot \pi}{L} \cdot t\right)$

with period $\frac{2\pi}{c \cdot n \cdot \pi} \cdot L = \frac{2L}{cn}$