Exercise 1, boundary conditions (a):

Boundary conditions for u1(x,y) are: $u(x,0) = u(0,y) = u(a,y) = 0$ **and** $u(x,b) = 100$ (upper edge of rectangle)

We know the solution is:

$$u1(x,y) = Sum\left(Bn \cdot \sin\left(\frac{n \cdot \text{Pi} \cdot x}{a}\right) \cdot \sinh\left(\frac{n \cdot \text{Pi} \cdot y}{a}\right), n = 1 \,..\text{infinity}\right)$$

Calculating Bn:

$$\text{phi} := \sin\left(\frac{n \cdot \text{Pi} \cdot x}{a}\right) :$$

$$Bn0 := \frac{int(100 \cdot \text{phi}, x = 0 \,..a)}{int(\phi^2, x = 0 \,..a)} \text{assuming}(n, integer, n > 0) :$$

$$Bn := \frac{Bn0}{\sinh\left(\frac{n \cdot \text{Pi} \cdot b}{a}\right)}$$

$$Bn := -\frac{200\left((-1)^n - 1\right)}{\pi\, n \sinh\left(\frac{n\,\pi\, b}{a}\right)} \qquad \textbf{(1)}$$

Now, For u2(x,y) we have the following BC: $u(x,b) = u(0,y) = u(a,y) = 0$ **and** $u(x,0) = 50$ (bottom edge of the rectangle).

To solve this, we define a new coordinate $w = b - y$, as done in class. Then, let $u2(x,y) = U2(x,w)$ (same temperature for the same point). By work done in class, we know $U2(x, y)$ satisfies the Laplace equation $\frac{d^2U}{dx^2} + \frac{d^2U}{dw^2} = 0$. Then, our solution is:

$$Sum\left(An \cdot \sin\left(\frac{n \cdot \text{Pi} \cdot x}{a}\right) \cdot \sinh\left(\frac{n \cdot \text{Pi} \cdot (b - y)}{a}\right), n = 1 \,..\text{infinity}\right)$$

Calculating An:

$$An := \left(\frac{1}{\sinh\left(\frac{n \cdot \text{Pi} \cdot b}{a}\right)}\right) \cdot \left(\frac{int(50 \cdot \text{phi}, x = 0 \,..a)}{int(\phi^2, x = 0 \,..a)}\right) \text{assuming}(n > 0, n, integer)$$

$$An := -\frac{100\left((-1)^n - 1\right)}{\pi\, n \sinh\left(\frac{n\,\pi\, b}{a}\right)} \qquad \textbf{(2)}$$

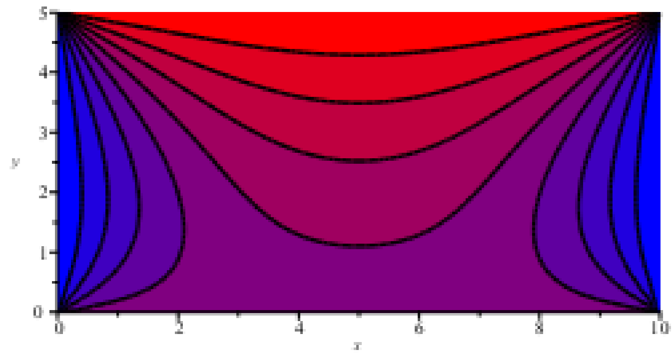
Plotting the solution:

$$psum := sum\left(Bn \cdot \sin\left(\frac{n \cdot \text{Pi} \cdot x}{a}\right) \cdot \sinh\left(\frac{n \cdot \text{Pi} \cdot y}{a}\right) + An \cdot \sin\left(\frac{n \cdot \text{Pi} \cdot x}{a}\right) \cdot \sinh\left(\frac{n \cdot \text{Pi} \cdot (b - y)}{a}\right), n = 1 \,..200\right) :$$

$psum := subs(a = 10, b = 5, psum) :$

$with(plots) :$

$contourplot(psum, x = 0 \,..10, y = 0 \,..5, scaling = constrained, coloring = [blue, red], filledregions = true)$



$plot3d(psum, x = 0\,..10, y = 0\,..5)$