1. $\frac{\partial u}{\partial t} = D \frac{\partial^2 u}{\partial x^2}$, $x \in (0, L)$ with BC $u(0,t) = M > 0$
$u(L,t) = 0$

We know:
$u(x,t) = u_{ss}(x) + u_h(x,t)$

we know $v_{ss}(x) = T_1 + \frac{1}{L}(T_2 - T_1)x$,

where $u(0,t) = u_{ss}(0) = T_1 = M$
$u(L,t) = u_{ss}(L) = T_2 = 0$

then $u(x,t) = M - \frac{Mx}{L} + u_h(x,t)$

Initial Condition (i) $u(x,0) = f(x) = \begin{cases} M, & x < L/2 \\ 0, & x > L/2 \end{cases}$

then $u(x,0) = f(x) = u_{ss}(x) + u_h(x,0)$

$\Rightarrow u_h(x,0) = f(x) - u_{ss}(x)$ ( Un homogeneous sol. of heat eq. )

then $f(x) - u_{ss}(x) = \sum_{n=1}^{\infty} a_n \sin\left(\frac{n\pi x}{L}\right)$

Using maple :