MA 501 SP22 Lubkin HW4 due Fri 2/18/22

Objectives

- Consolidating understanding of behavior of the wave equation through observation
- Consolidating understanding of periodic extensions
- More independence using Maple

1. For the function t^2, $t \in [0,1]$, (a) sketch by hand its periodic extension, its odd periodic extension, and its even periodic extension. You will find the Fourier series for each of these 3 extensions. For each extension, (b) State the period *T*, and give general forms for the appropriate Fourier terms, (c) Determine the Fourier coefficients, (d) Plot the partial sums P_N for $N = 1,2,4,8,16$ terms, all on the same axes. Any comments on the quality of the partial sums? For all of your plots, be sure I can see at least 2 full periods.

2. Solve the wave equation $\dfrac{\partial^2 u}{\partial t^2} = c^2 \dfrac{\partial^2 u}{\partial x^2}$, $x \in (0,L)$, $u(0,t) = u(L,t) = 0$ for the initial conditions given.

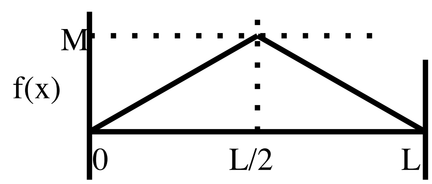
(a) Write the solution $u(x,t)$.

(b) State the period (in *t*, not in *x*).

Note that in (a)-(b), your answers should be in terms of the parameters given. You will only plug in representative numbers in (c), so you can make plots.

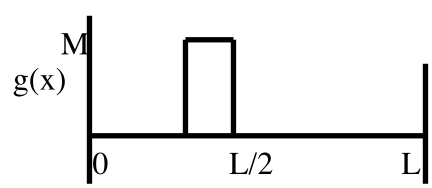
(c) Create a partial sum of satisfactory precision (in your opinion), *look* at an animation to see how it behaves, and *plot* solution curves at several times to illustrate the motion (no hand sketches for this problem). For each of (i) - (iv), show multiple curves on the same axes, each curve representing a specific time, including $t = 0$. Ideally, we should be able to see what the string does over a full period (in *t*). Do not show more domain than the domain given, since there's no string there.

(i) $u(x,0) = f(x)$ as illustrated at right, $\dfrac{\partial u}{\partial t}(x,0) = 0$



(ii) $u(x,0) = 0$, $\dfrac{\partial u}{\partial t}(x,0) = f(x)$ as illustrated at right

(iii) $u(x,0) = g(x)$ as illustrated at right, $\dfrac{\partial u}{\partial t}(x,0) = 0$



(iv) $u(x,0) = 0$, $\dfrac{\partial u}{\partial t}(x,0) = g(x)$ as illustrated at right.

For (iii) and (iv), exact position and width of pulse don't matter, but make sure it's off-center.