

6. For the sawtooth function:

(a) period $T = 2$

Let x represent time for this one...

(b) General form for $\phi_n(x) = \sin\left(\frac{2\pi nx}{2}\right) = \sin(\pi nx)$

(c) Fourier coefficients:

$$\frac{\int_{-1}^1 -x \sin(\pi nx) dx}{\int_{-1}^1 \sin^2(\pi nx) dx} = A_n \quad \text{for } n \neq 0$$

$$= \frac{\left(\frac{x \cos(\pi nx)}{\pi n} - \frac{\sin(\pi nx)}{\pi^2 n^2} \right) \Big|_{-1}^1}{\frac{\pi n - \sin(\pi n)}{\pi n}}$$

$$\Rightarrow A_n = \frac{(2\pi n \cos(\pi n) - 2 \sin(\pi n))}{\pi^2 n^2} \cdot \frac{\pi n}{\pi n - \sin(\pi n)}$$

$$A_n = \frac{2\pi n \cos(\pi n) - 2 \sin(\pi n)}{\pi n (\pi n - \sin(\pi n))} \quad \text{for } n \neq 0$$

$$f \sim \sum_{n=1}^{\infty} \frac{2\pi n \cos(\pi n) - 2 \sin(\pi n)}{\pi n (\pi n - \sin(\pi n))} \cdot \sin(\pi nx)$$