Finding coefficients for initial condition (i):

$$\text{phi} := \sin\left(\frac{n\cdot \text{Pi}\cdot x}{L}\right):$$

$$an := \frac{int\left(\frac{M\cdot x}{L}\cdot \text{phi}, x=0..\frac{L}{2}\right) + int\left(\left(0-M+\frac{M\cdot x}{L}\right)\cdot \text{phi}, x=\frac{L}{2}..L\right)}{int\left(\phi^2, x=0..L\right)} \text{assuming}(M>0, n, integer)$$

$$an := \frac{2\left(-\frac{L\left(n\pi\cos\left(\frac{n\pi}{2}\right)-2\sin\left(\frac{n\pi}{2}\right)\right)M}{2\pi^2 n^2} - \frac{L\left(n\pi\cos\left(\frac{n\pi}{2}\right)+2\sin\left(\frac{n\pi}{2}\right)\right)M}{2\pi^2 n^2}\right)}{L} \quad \textbf{(1)}$$

Coefficients are:

$$an := simplify(an)$$

$$an := -\frac{2M\cos\left(\frac{n\pi}{2}\right)}{n\pi} \quad \textbf{(2)}$$

(a) Solution u(x,t):

$$u := M - \frac{M\cdot x}{L} + Sum\left(an\cdot \text{phi}\cdot \exp\left(-\left(\frac{n\cdot \text{Pi}}{L}\right)^2\cdot \text{D}\cdot t\right), n=1..\text{infinity}\right)$$

$$u := M - \frac{Mx}{L} + \sum_{n=1}^{\infty}\left(-\frac{2M\cos\left(\frac{n\pi}{2}\right)\sin\left(\frac{n\pi x}{L}\right)e^{-\frac{n^2\pi^2 \text{D}\, t}{L^2}}}{n\pi}\right) \quad \textbf{(3)}$$

$$plot\left(100 - \frac{100\cdot x}{10}, x=0..10\right):$$

(b) Partial sum, animation, plotting solution u(x,t) curves:

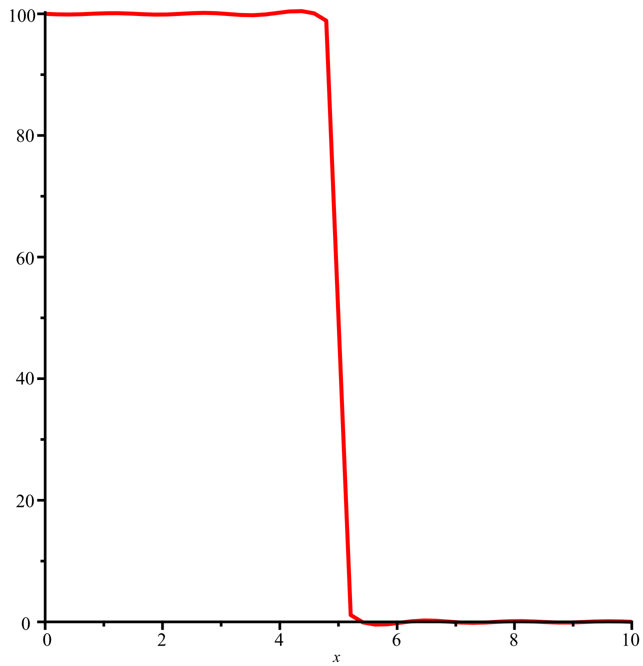
$$uss := subs\left(M=100, L=10, M-\frac{M\cdot x}{L}\right):$$

$$uh := subs\left(M=100, L=10, \text{D}=1, an\cdot \text{phi}\cdot \exp\left(-\left(\frac{n\cdot \text{Pi}}{L}\right)^2\cdot \text{D}\cdot t\right)\right):$$

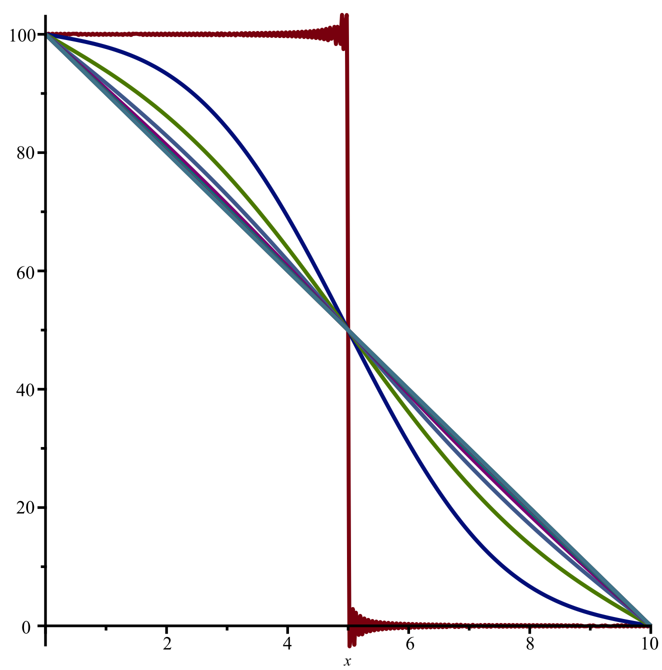
$$psum := uss + sum(uh, n=1..300):$$

$$with(plots):$$

$$animate(psum, x=0..10, t=0..40)$$



$curves := [seq(subs(t=2\cdot m, psum), m=0..10)]:$
$plot(curves, x=0..10)$



As t goes to infinity, the temperature u(x,t) approach the steady state.

(c) Steady state and asymptotic solution:

$$u(x, \text{infinity}) = M - \frac{M\cdot x}{L}$$

$u(x, t)$ is approximately equal to:

$$M - \frac{M\cdot x}{L} + subs\left(n=2, an\cdot \text{phi}\cdot \exp\left(-\left(\frac{n\cdot \text{Pi}}{L}\right)^2\cdot \text{D}\cdot t\right)\right)$$

$$M - \frac{M x}{L} - \frac{M \cos(\pi) \sin\left(\frac{2 \pi x}{L}\right) e^{-\frac{4 \pi^2 D t}{L^2}}}{\pi} \tag{4}$$