MA 501 SP22 Lubkin HW2 due Fri 2/4/22

Objectives:

- Consolidating the concepts of periodic functions, orthogonality of functions, odd and even functions
- Consolidating the concept of Fourier series through practice and observation
- Moving beyond plug-and-chug calculations, to critically assess a problem (especially for problems 7, 8, 9)
- More independence using Maple

1. Show that e^x is the sum of an even and an odd function. What are they?

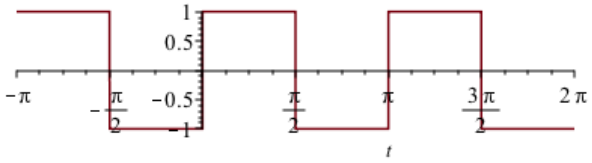
2. Working directly from the definition of a *periodic function of period T*, prove that the sum of two functions, which are both periodic and both of period *T*, is also periodic with period *T*.

3. Is the function $\sin(x)+\sin(\pi x)$ periodic? Why or why not? Explain (briefly).

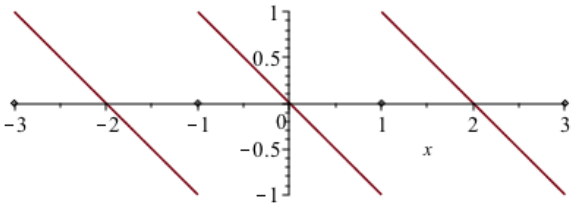
4. Show that the functions $\sin(\frac{2\pi nt}{T})$ and $\cos(\frac{2\pi mt}{T})$ are orthogonal on $t\in(0,T)$ if *m* and *n* are integers. Does it still hold true for *m*=0? (Warning: if you use Maple, I want to see intermediate steps, to be sure you're not secretly/accidentally dividing by 0.)

5. Consider the set of three functions $\{f_1,f_2,f_3\}$ with $f_1(x)=1$, $f_2(x)=x$, $f_3(x)=\frac{1}{2}(3x^2-1)$. Do they form an orthogonal set of functions on $x\in(0,1)$? Do they form an orthogonal set of functions on $x\in(-1,1)$?

6. For each of the following odd periodic functions, (a) State the period *T*, (b) Give a general form for the appropriate Fourier terms $\phi_n(t)=\sin(\cdots t)$ (fill in the ...), (c) Determine the Fourier coefficients, (d) Plot the partial sums P_N for $N=1,2,4,8,16$ nonzero terms, all on the same axes. Any comments on the quality of the partial sums? For all of your plots, be sure I can see at least 2 full periods. Note that the points at the jumps don't matter.



square wave



sawtooth

7. [no Maple] What is the Taylor series for $g(x)=x^3+1$ about the point *x*=0?

8. [no Maple] What is the Fourier series for $f(t)=1+\sin(t)$? (Hint: You don't need to find any inner products.)

9. [no Maple] What is the Fourier series for $f(t)=\sin(t)\cos(t)$? (Hint: Using a trig identity will show that you don't need to find any inner products.)