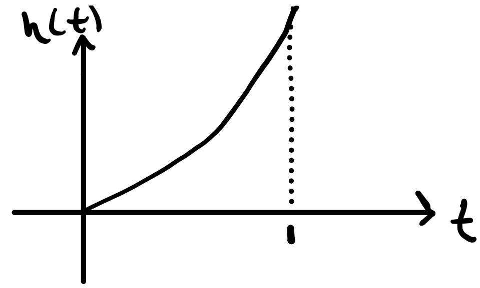
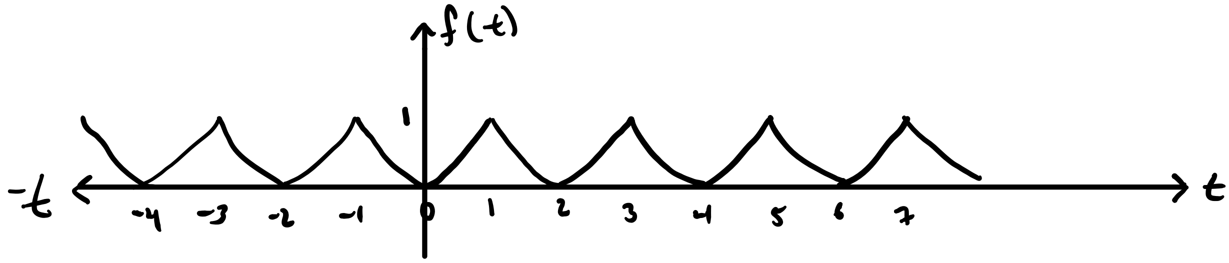
1.) t^2 , $t \in [0,1]$

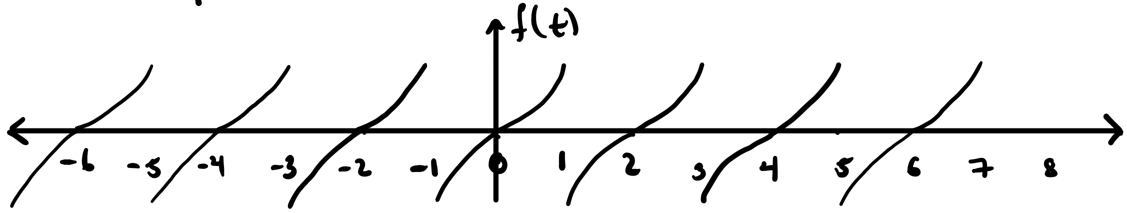


even periodic extension:



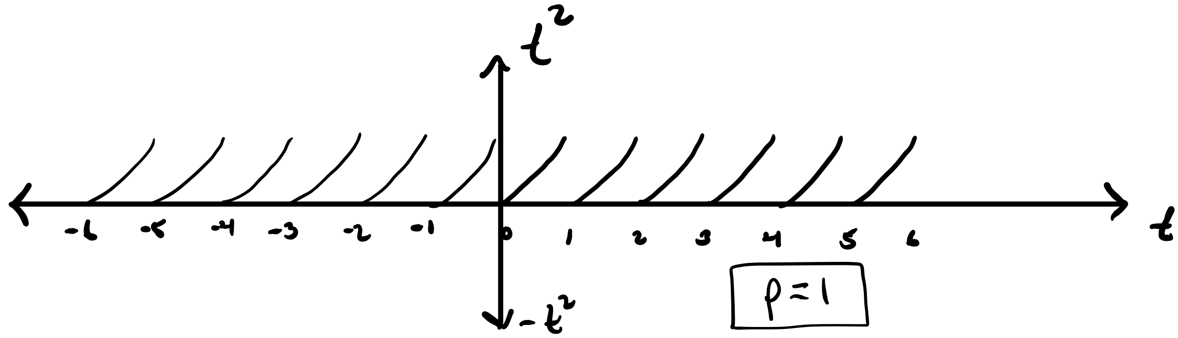
period = 2

odd periodic extension:



period $p = 2$

Periodic Extension:



$\boxed{p=1}$

For the even periodic extension:

$$t^2 \sim \sum_{n=0}^{\infty} A_n \phi_n \quad , \quad \phi_n(t) = \left\{ \cos\left(\frac{\cancel{\ell}\pi n t}{\cancel{\ell}}\right) \right\}$$

$$t^2 \sim \sum_{n=0}^{\infty} A_n \cos(\pi n x) \quad . \quad f(t) = t^2$$

$$A_0 = \frac{\int_{-1}^{1} t^2 \cdot 1 \, dt}{\int_{-1}^{1} 1 \, dt} = \frac{\left.\frac{t^3}{3}\right|_{-1}^{1}}{\left. t \right|_{-1}^{1}} = \frac{\frac{1}{3} + \frac{1}{3}}{1+1} = \frac{\frac{2}{3}}{2}$$

$$= \frac{1}{3} = A_0$$

Our even Fourier Series is:

$$f(t) \sim \frac{1}{3} + \sum_{n=1}^{\infty} A_n \cos(n\pi t)$$

Using Maple to find A_n:

$$A_n = \frac{\int_{-1}^{1} t^2 \cdot \overbrace{\cos(n\pi t)}^{\text{even}} \, dt}{\int_{-1}^{1} \underbrace{\cos^2(n\pi t)}_{\text{even}} \, dt} = \frac{\cancel{2} \int_{0}^{1} t^2 \cos(n\pi t) \, dt}{\cancel{2} \int_{0}^{1} \cos^2(n\pi t) \, dt}$$

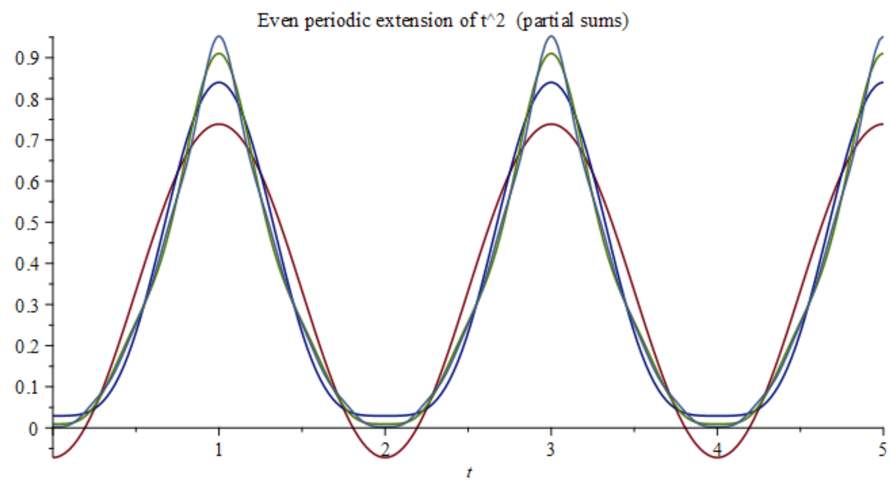
$$A_n = \frac{4(-1)^n}{(n\cdot\pi)^2} \qquad n > 1.$$

Answer for even extension:

$\therefore$ Fourier extension is $f(t) \sim \frac{1}{3} + \sum_{n=1}^{\infty} \frac{4(-1)^n}{(n\cdot\pi)^2}\cdot\cos(\pi n t)$

(a) period $T = \frac{2\cancel{\pi}}{\cancel{\pi} n} = \frac{2}{n}$, common period $= 2$

$$\phi_n(t) = \cos(\pi n t), \quad A_0 = \frac{1}{3}, \quad A_n = \frac{4(-1)^n}{(n\cdot\pi)^2}, \; n > 0$$



For the odd periodic extension:

$$f(t) = t^2 \sim \sum_{n=1}^{\infty} B_n \sin\left(\frac{2\pi n t}{p}\right) \qquad p = 2$$

Fourier terms:

$$\phi_n(t) = \sin\left(\frac{\cancel{2}\pi n t}{\cancel{2}}\right) = \sin(\pi n t)$$

The function is : $f(t) = \begin{cases} t^2, & 0 < t < 1 \\ -t^2, & -1 < t < 0 \end{cases}$

$$B_n = \frac{\int_{-1}^{1} f \cdot \sin(\pi n t)\, dt}{\int_{-1}^{1} \underbrace{\sin^2(\pi n t)}_{\text{even}}\, dt} \qquad = 2\int_0^1 \sin^2(\pi n t)\, dt$$

$$\Rightarrow B_n = \frac{\int_{-1}^{0} -t^2 \sin(n\pi t)\, dt + \int_0^1 t^2 \sin(n\pi t)\, dt}{2 \cdot \int_0^1 \sin^2(n\pi t)\, dt}$$

Using Maple, we arrive to :

$$B_n = \frac{-2\pi^2 n^2 (-1)^n + 4(-1)^n - 4}{n^3 \cdot \pi^3} \qquad n \geq 1, \ n \text{ integer}$$

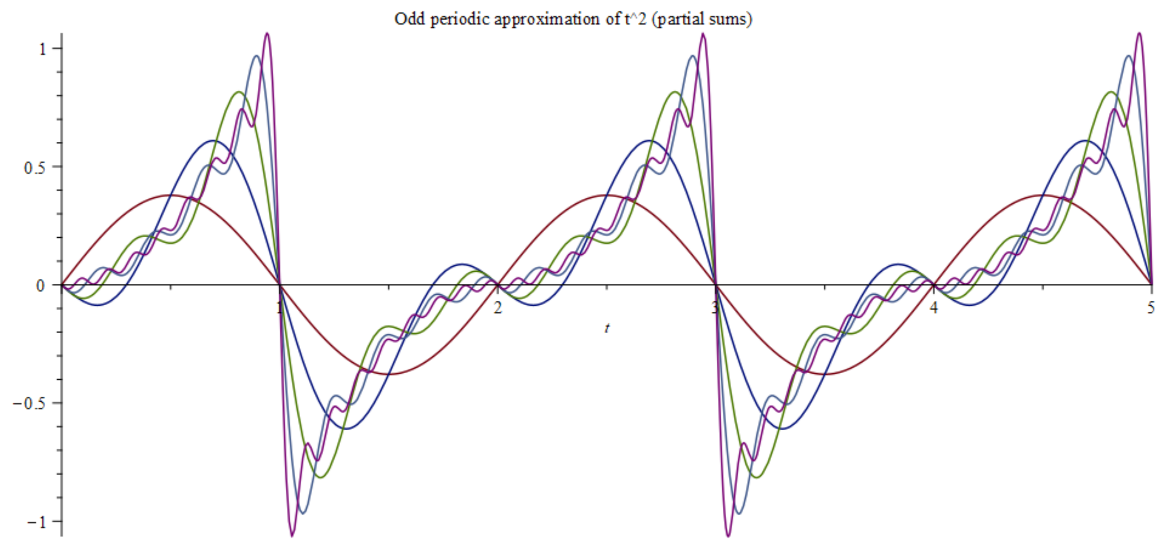
Odd :

(b) Extension period $T = \frac{2\cancel{\pi}}{\cancel{\pi} n} = \frac{2}{n}$

Fourier terms $\phi_n(x) = \sin(n \cdot \pi t)$

(c) Fourier Coefficients : (using Maple)

$$B_n = \frac{-2\pi^2 n^2 (-1)^n + 4(-1)^n - 4}{n^3 \pi^3} \qquad n > 0$$



Observation: the quality (i.e. looks more like the sketch) is better if n is big.

Periodic Extension:

period $p=1$

$$\phi_n(x) = \{ \cos(2\pi n x), \sin(2\pi n x) \}$$

F.S. is given by

$$A_0 + \sum_{n=1}^{\infty} A_n \cos(2\pi n x) + B_n \sin(2\pi n x)$$

where $A_0 = \frac{1}{3}$, $A_n = \frac{1}{(\pi \cdot n)^2}$

$B_n = \frac{-1}{\pi n}$.