(iii)

We have $u(x,0) = 0$

$\frac{\partial u}{\partial t}(x,0) = g(x)$ where

$$g(x) = \begin{cases} M, & \frac{L}{4} < x < \frac{L}{2} \\ 0, & \text{else} \end{cases}$$

$$\Rightarrow T(0) = \cancel{a \sin(0)}^{0} + b \cos(0) = 0 = b$$

$$\therefore b = 0$$

$$\Rightarrow T(t) = a \sin\left(\frac{c \cdot n \cdot \pi}{L} \cdot t\right)$$

$\frac{du}{dt}(x,0) = g(x)$ and $T'(0) = \frac{ac n \cdot \pi}{L} \cdot \cos(0)$

$$\Rightarrow g(x) = \sum_{n=1}^{\infty} \sin\left(\frac{n \cdot \pi}{L} \cdot x\right) \cdot \underbrace{\frac{a_n c \cdot n \cdot \pi}{L}}_{= K_n}$$

$$\Rightarrow g(x) = \sum_{n=1}^{\infty} \sin\left(\frac{n \cdot \pi}{L} \cdot x\right) \cdot K_n$$

$$u_n(x,t) = \sum_{n=1}^{\infty} \sin\left(\frac{n\cdot\pi}{L}\cdot x\right)\cdot b_n \cdot \cos\left(\frac{n\pi c}{L}\cdot t\right) \quad \text{for IC}$$

$$u_n(x,0) = g(x) = \sum_{n=1}^{\infty} \sin\left(\frac{n\pi}{L}\cdot x\right)\cdot b_n \quad \text{where}$$

$$g(x) = \begin{cases} M, & \frac{L}{4} < x < \frac{L}{2} \\ 0, & \text{else} \end{cases}$$

by Maple, we end up with

$$u(x,t) = \sum_{n=1}^{\infty} -\frac{M\left(\cos\left(\frac{n\pi}{2}\right) - \cos\left(\frac{n\pi}{4}\right)\right)}{n\pi} \cdot \sin\left(\frac{n\pi}{L}\cdot x\right)\cdot \cos\left(\frac{c\cdot n\pi}{L}\cdot t\right)$$

period in t : $\frac{2\pi}{cn\pi} L = \frac{2L}{cn}$

Maple :

> $bn := \dfrac{int\left(M\cdot\sin\left(\dfrac{n\cdot\mathrm{Pi}\cdot x}{L}\right), x=\dfrac{L}{4}..\dfrac{L}{2}\right)}{int\left(\sin^2\left(\dfrac{n\cdot\mathrm{Pi}\cdot x}{L}\right), x=-L..L\right)}\ \text{assuming}\ (L>0, n, integer, n>0)$

$$bn := -\frac{M\left(\cos\left(\frac{n\,\pi}{2}\right)-\cos\left(\frac{n\,\pi}{4}\right)\right)}{n\,\pi} \tag{1}$$

> bn

$$-\frac{M\left(\cos\left(\frac{n\,\pi}{2}\right)-\cos\left(\frac{n\,\pi}{4}\right)\right)}{n\,\pi} \tag{2}$$

> $phin := \sin\left(\dfrac{n\cdot\mathrm{Pi}\cdot x}{L}\right)$

$$phin := \sin\left(\frac{n\,\pi\,x}{L}\right) \tag{3}$$

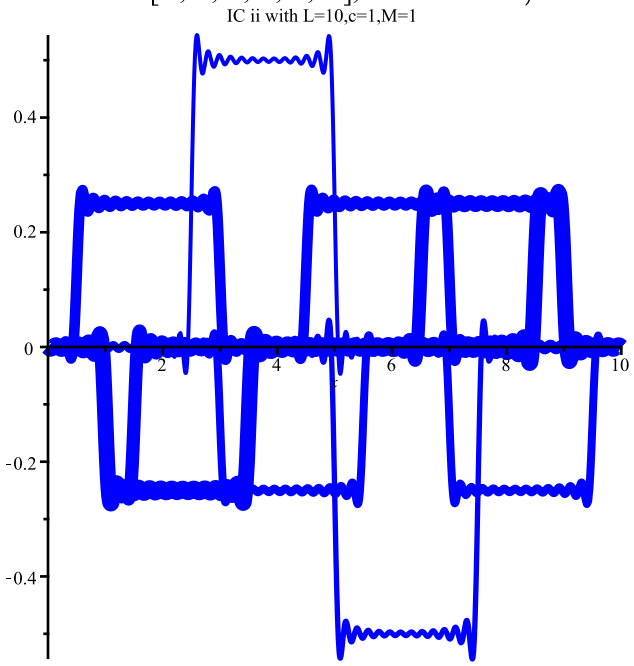
> $fs := subs\left(M=1, L=10, c=1, sum\left(bn\cdot phin\cdot\cos\left(\dfrac{n\cdot\mathrm{Pi}\cdot c\cdot t}{L}\right), n=1..100\right)\right):$

> $with(plots):$

> $animate(fs, x=0..10, t=0..20):$

> $curves := \{seq(subs(t=2\cdot m, fs), m=0..10)\}:$

> $plot(curves, x=0..10, thickness=[1,2,3,4,5,6], color=blue)$



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