2(c) Proving we have an orthogonal set:

$$phin := \cos\left(\frac{(2\cdot n-1)\cdot x\cdot \mathrm{Pi}}{2\cdot L}\right):$$

$$phim := \cos\left(\frac{(2\cdot m-1)\cdot x\cdot \mathrm{Pi}}{2\cdot L}\right):$$

$$int(phin\cdot phim, x=0..L)\mathrm{assuming}(m \neq n, n, integer, m, integer)$$

$$0 \tag{1}$$

Therefore, we have an orthogonal set. If m = n, we have:

$$int\left(phin^2, x=0..L\right)\mathrm{assuming}(n, integer)$$

$$\frac{L}{2} \tag{2}$$

(d) Finding the coefficients:

$$cn := \frac{int(f(x)\cdot phin, x=0..L)}{int\left(phin^2, x=0..L\right)}\mathrm{assuming}(n, integer, n>0)$$

$$cn := \frac{2\left(\int_0^L f(x)\cos\left(\frac{(2\,n-1)\,x\,\pi}{2\,L}\right)\mathrm{d}x\right)}{L} \tag{3}$$

(e) For initial condition f(x) = M, the solution is:

$$cn := \frac{int(M\cdot phin, x=0..L)}{int\left(phin^2, x=0..L\right)}\mathrm{assuming}(n, integer, n>0)$$

$$cn := -\frac{4\,(-1)^n M}{(2\,n-1)\,\pi} \tag{4}$$

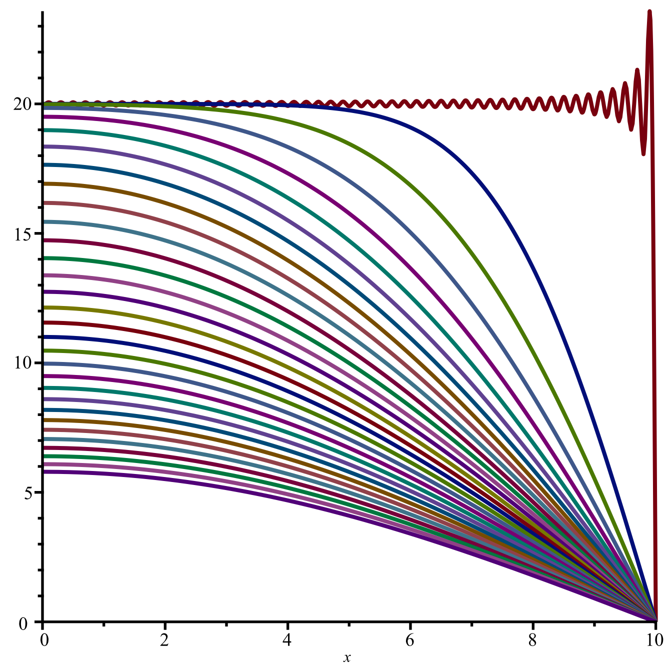
$$u := Sum\left(phin\cdot cn\cdot \exp\left(-\left(\frac{\mathrm{Pi}\cdot(2\cdot n-1)}{2\cdot L}\right)^2\cdot \mathrm{D}\cdot t\right), n=1..\mathrm{infinity}\right)$$

$$u := \sum_{n=1}^{\infty}\left(-\frac{4\cos\left(\frac{(2\,n-1)\,x\,\pi}{2\,L}\right)(-1)^n M\,\mathrm{e}^{-\frac{\pi^2(2\,n-1)^2\,\mathrm{D}\,t}{4\,L^2}}}{(2\,n-1)\,\pi}\right) \tag{5}$$

(f) Plotting:

$$uxt := subs\left(M=20, L=10, \mathrm{D}=1, phin\cdot cn\cdot \exp\left(-\left(\frac{\mathrm{Pi}\cdot(2\cdot n-1)}{2\cdot L}\right)^2\cdot \mathrm{D}\cdot t\right)\right):$$

$psum := sum(uxt, n=1..100):$
$with(plots):$
$animate(psum, x=0..10, t=0..20):$
$curves := [seq(subs(t=2\cdot m, psum), m=0..30)]:$
$plot(curves, x=0..10)$



20
15
10
5
0
0
2
4
6
8
10
x