For initial condition (ii) u(x,0) = M*x/L :

(a) Calculating our solution u(x,t)

$$an := simplify\left(\frac{int\left(\left(\frac{M\cdot x}{L} - M + \frac{M\cdot x}{L}\right)\cdot \sin\left(\frac{n\cdot \mathrm{Pi}\cdot x}{L}\right), x = 0 ..L\right)}{int\left(\sin^2\left(\frac{n\cdot \mathrm{Pi}\cdot x}{L}\right), x = 0 ..L\right)} \text{assuming}(M > 0, n, integer)\right)$$

$$an := -\frac{2\,M\left((-1)^n + 1\right)}{n\,\pi} \tag{1}$$

$$u := M - \frac{M\cdot x}{L} + Sum\left(an\cdot \sin\left(\frac{n\cdot \mathrm{Pi}\cdot x}{L}\right)\cdot \exp\left(-\left(\frac{n\cdot \mathrm{Pi}}{L}\right)^2\cdot \mathrm{D}\cdot t\right), n = 1 ..\text{infinity}\right)$$

$$u := M - \frac{M\,x}{L} + \sum_{n=1}^{\infty}\left(-\frac{2\,M\left((-1)^n + 1\right)\sin\left(\frac{n\,\pi\,x}{L}\right)\mathrm{e}^{-\frac{n^2\,\pi^2\,\mathrm{D}\,t}{L^2}}}{n\,\pi}\right) \tag{2}$$

(b) Partial sum, animation, and plots:

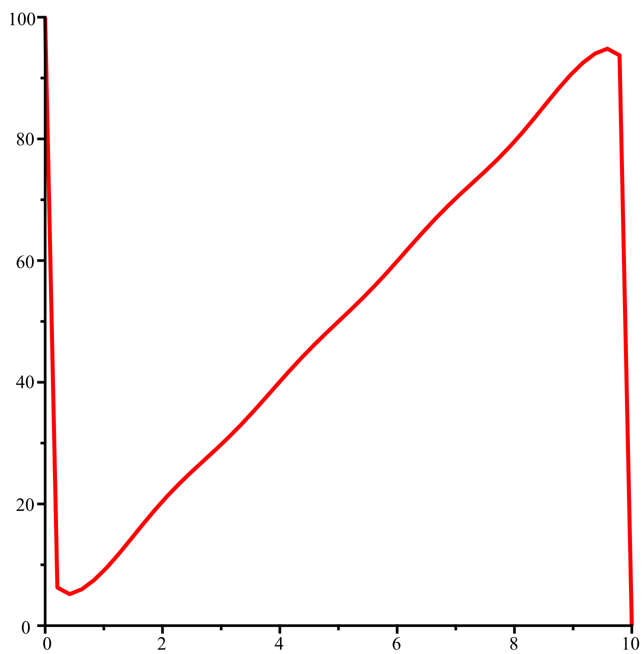
$$uss := subs\left(M = 100, L = 10, M - \frac{M\cdot x}{L}\right) :$$

$$uh := subs\left(M = 100, L = 10, \mathrm{D} = 1, an\cdot \sin\left(\frac{n\cdot \mathrm{Pi}\cdot x}{L}\right)\cdot \exp\left(-\left(\frac{n\cdot \mathrm{Pi}}{L}\right)^2\cdot \mathrm{D}\cdot t\right)\right) :$$

$psum := uss + sum(uh, n = 1 ..200) :$

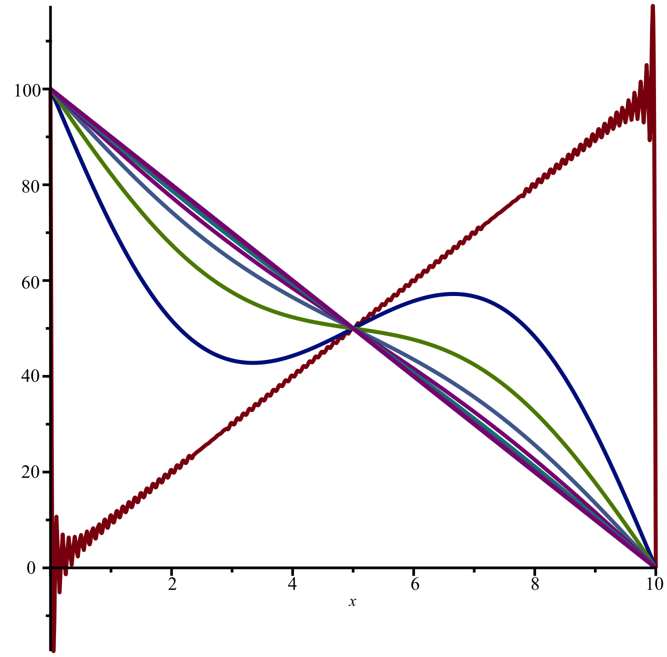
$with(plots) :$

$animate(psum, x = 0 ..10, t = 0 ..20)$



$curves := [seq(subs(t = 2\cdot m, psum), m = 0 ..20)] :$

$plot(curves, x = 0..10)$



As t gets bigger, the temperature apporaches the linear steady state condition uss(x) = M-M*x/L.

(c)
Steady state is:

$uss := M - \frac{M \cdot x}{L}$

$$uss := M - \frac{M x}{L} \tag{3}$$

Then the solution u(x,t) is approximately equal to:

$an2 := subs(n = 2, an)$

$$an2 := -\frac{2 M}{\pi} \tag{4}$$

$u := uss + subs\left(n = 2, \sin\left(\frac{n \cdot \mathrm{Pi} \cdot x}{L}\right) \cdot \exp\left(-\left(\frac{n \cdot \mathrm{Pi}}{L}\right)^2 \cdot \mathrm{D} \cdot t\right)\right) \cdot an2$

$$u := M - \frac{M x}{L} - \frac{2 \sin\left(\frac{2 \pi x}{L}\right) \mathrm{e}^{-\frac{4 \pi^2 \mathrm{D}\, t}{L^2}} M}{\pi} \tag{5}$$