(iii)

We have $u(x,0) = 0$

$$\frac{\partial u}{\partial t}(x,0) = g(x) \text{ where}$$

$$g(x) = \begin{cases} M, & \frac{L}{4} < x < \frac{L}{2} \\ 0, & \text{else} \end{cases}$$

$$\Rightarrow T(0) = \cancel{a \sin(0)}^{0} + b\cos(0) = 0 = b$$

$$\therefore b = 0$$

$$\Rightarrow T(t) = a \sin\left(\frac{c \cdot n \cdot \pi}{L} \cdot t\right)$$

$$\frac{\partial u}{\partial t}(x,0) = g(x) \quad \text{and} \quad T'(0) = \frac{a c n \cdot \pi}{L} \cdot \cos(0)$$

$$\Rightarrow g(x) = \sum_{n=1}^{\infty} \sin\left(\frac{n \cdot \pi}{L} \cdot x\right) \cdot \underbrace{\frac{a_n c \cdot n \cdot \pi}{L}}_{= K_n}$$

$$\Rightarrow g(x) = \sum_{n=1}^{\infty} \sin\left(\frac{n \cdot \pi}{L} \cdot x\right) \cdot K_n$$

$$u_n(x,t) = \sum_{n=1}^{\infty} \sin\left(\frac{n \cdot \pi}{L} \cdot x\right) \cdot b_n \cdot \cos\left(\frac{n \pi c}{L} \cdot t\right) . \quad \text{fa IC}$$

$$u_n(x,0) = g(x) = \sum_{n=1}^{\infty} \sin\left(\frac{n\pi}{L} \cdot x\right) \cdot b_n \quad \text{where}$$

$$g(x) = \begin{cases} M, & \frac{L}{4} < x < \frac{L}{2} \\ 0, & \text{else} \end{cases}$$

by Maple, we end up with

$$u(x,t) = \sum_{n=1}^{\infty} -\frac{M\left(\cos\left(\frac{n\pi}{2}\right) - \cos\left(\frac{n\pi}{4}\right)\right)}{n\pi} \cdot \sin\left(\frac{n\pi}{L} \cdot x\right) \cdot \cos\left(\frac{c \cdot n\pi}{L} \cdot t\right)$$

period in t : $\frac{2\pi}{cn\pi} L = \frac{2L}{cn}$

Maple :