3. a $\sin(a+b) = \sin(a)\cos(b) + \cos(a)\sin(b)$

by exercise 2: $\sin(x) = \left(\frac{e^{ix} - e^{-ix}}{2}\right) \cdot -i$

$$\Rightarrow \sin(a+b) = \left(\frac{e^{i(a+b)} - e^{-i(a+b)}}{2}\right) \cdot -i$$

Aside:

For $e^{i(a+b)} = e^{ia} \cdot e^{ib}$

$= (\cos(a) + i\sin(a)) \cdot (\cos(b) + i\sin(b)) = \cancel{\cos(a) \cdot \cos(b)} + \cancel{\cos(a) \cdot i\sin(b)} + \cancel{i\sin(a) \cdot \cos(b)} - \cancel{\sin(a)\sin(b)}$

For $e^{-i(a+b)} = e^{i(-a-b)} = e^{-ia} \cdot e^{-ib}$

$= (\cos(a) - i\sin(a)) \cdot (\cos(b) - i\sin(b))$

$= \cancel{\cos(a) \cdot \cos(b)} - \cancel{\cos(a) \cdot i\sin(b)} - \cancel{i\sin(a)\cos(b)} - \cancel{\sin(a) \cdot \sin(b)}$

then $e^{i(a+b)} - e^{-i(a+b)}$

$= i\cos(a)\sin(b) + i\cos(a)\sin(b)$

$+ i\sin(a)\cos(b) + i\sin(a)\cos(b)$

$= 2 \cdot i \cos(a)\sin(b) + 2 \cdot i \sin(a) \cdot \cos(b)$

$\therefore e^{i(a+b)} - e^{-i(a+b)} = 2 \cdot i (\cos(a)\sin(b) + \sin(a)\cos(b))$

then

$$\left(e^{i(a+b)} - e^{-i(a+b)}\right) \cdot \frac{-i}{2} = \frac{-i}{\cancel{4}} \cdot \cancel{2} i \cdot \big( \cos(a) \sin(b) + \sin(a) \cos(b) \big)$$

$$= \cos(a) \cdot \sin(b) + \sin(a) \cos(b) = \sin(a+b)_{\,//}$$

3.b show

$$\sinh(a+b) = \sinh(a) \cosh(b) + \cosh(a) \sinh(b)$$

$$\sinh(a+b) = \frac{e^{a+b} - e^{-a-b}}{2} = \frac{2(e^{a+b} - e^{-a-b})}{4}$$

$$= \frac{(e^{a+b} - e^{-a-b}) + (e^{a+b} - e^{-a-b}) + 0 + 0}{4}$$

$$= \frac{(e^{a+b} - e^{-a-b}) + (e^{a+b} - e^{-a-b}) + e^{a-b} - e^{a-b} + e^{b-a} - e^{b-a}}{4}$$

$$= \frac{\overbrace{\left(e^{a+b} + e^{a-b} - e^{b-a} - e^{-a-b}\right)}^{= (e^{a} - e^{-a})(e^{b} + e^{-b})} + \underbrace{\left(e^{a+b} - e^{a-b} + e^{b-a} - e^{-a-b}\right)}_{(e^{a} + e^{-a})(e^{b} - e^{-b})}}{4}$$

$$= \underbrace{\frac{(e^{a} - e^{-a})}{2}}_{=\sinh(a)} \cdot \underbrace{\frac{(e^{b} + e^{-b})}{2}}_{=\cosh(b)} + \underbrace{\frac{(e^{a} + e^{-a})}{2}}_{=\cosh(a)} \cdot \underbrace{\frac{(e^{b} - e^{-b})}{2}}_{=\sinh(b)}$$

$$\Rightarrow \sinh(a+b) = \sinh(a)\cosh(b) + \cosh(a)\sinh(b)$$