$$\frac{du}{dt} = D\frac{\partial^2 u}{\partial x^2} \quad , \quad x \in (0,L)$$

$$u(0,t) = 0 = u(L,t)$$

we know $u_n(x,t) = b_n e^{-\left(\frac{n\pi}{L}\right)^2 \cdot D \cdot t} \cdot \sin\left(\frac{n\pi}{L}\cdot x\right)$ is a family of solutions. and

$$u_n(0,t) = b_n \cdot e^{-\left(\frac{n\pi}{L}\right)^2 D t} \cdot 0 = 0$$

$$u_n(L,t) = b_n \cdot \sin(n\cdot\pi) \cdot e^{-\left(\frac{n\pi}{L}\right)^2 D t} = 0$$

$\therefore u_n(x,t) = \sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi}{L}\cdot x\right) \cdot e^{-\left(\frac{n\pi}{L}\right)^2 \cdot D \cdot t}$ satisfies B.C.

For the Initial Conditions (i) $u(x,0) = M$ constant

$$\text{(ii) } u(x,0) = \begin{cases} M, & x < L/2 \\ 0, & x > L/2 \end{cases}$$

For I.C. (i) $u(x,0) = M$:

$$u_n(x,0) = \sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi}{L}\cdot x\right)\cdot e^0 \stackrel{?}{=} M$$

$$\Rightarrow u_n(x,0) \stackrel{?}{=} \sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi}{L}\cdot x\right) = M$$

Using Maple :