(ii) $u(x,0) = 0$

$$\frac{\partial u}{\partial t}(x,0) = f(x)$$

where $f(x) = \begin{cases} \frac{2M}{L} \cdot x, & 0 < x < \frac{L}{2} \\ \frac{2M(L-x)}{L}, & \frac{L}{2} < x < L \end{cases}$

We know:

$$X(x) = \sin\left(\frac{n \cdot \pi}{L} \cdot x\right)$$

$$T(t) = a \sin(c \nu t) + b \cos(c \nu t), \quad \nu = \frac{n \cdot \pi}{L}$$

since $u(x,t) = X(x) T(t)$

$$u(x,0) = X(x) \cdot T(0) = 0$$

$$\Rightarrow a \sin(0) + b \cos(0) = 0 = b \Rightarrow b = 0$$

$\Rightarrow T(t)$ becomes: $T(t) = a \sin(c \nu t)$

and $T'(t) = a \frac{c\, n \cdot \pi}{L} \cdot \cos\left(\frac{n \cdot \pi c}{L} \cdot x\right)$

since $\frac{\partial u}{\partial t}(x,0) = f(x)$

$$\Rightarrow \sum_{n=1}^{\infty} \frac{a_n \cdot c_n \cdot \pi}{L} \cdot \sin\left(\frac{n \cdot \pi}{L} \cdot x\right) = g(x)$$

let $K_n = \frac{a_n c_n \cdot \pi}{L} \Rightarrow a_n = \frac{K_n}{c \cdot n \cdot \pi} \cdot L$

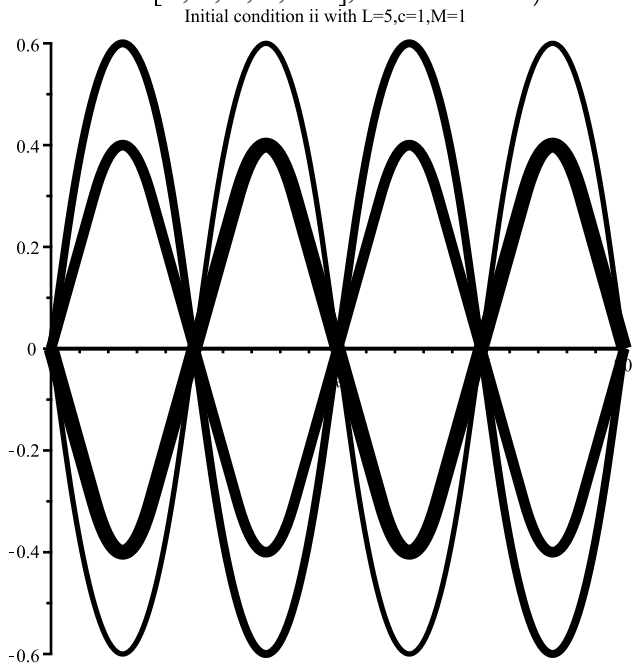
then $\sum_{n=1}^{\infty} \frac{a_n \, c \cdot n \cdot \pi}{L} \cdot \sin\left(\frac{n \cdot \pi}{L} \cdot x\right) = \sum_{n=1}^{\infty} K_n \sin\left(\frac{n \cdot \pi}{L} \cdot x\right)$

and $u_n(x,t) = \sum_{n=1}^{\infty} \sin\left(\frac{n \cdot \pi}{L} \cdot x\right) \cdot a_n \cdot \sin\left(\frac{c \cdot n \cdot \pi}{L} \cdot t\right)$

with period $\frac{2\pi}{c_n \pi} \cdot L = \frac{2L}{c_n}$

in t

```
> kn :=
      int( 2·M·x/L · sin( n·Pi·x/L ), x = 0 .. L/2 ) + int( 2·M·(L − x)/L · sin( n·Pi·x/L ), x = L/2 .. L )
      ------------------------------------------------------------------------------------------------------ :
                                        int( sin^2( n·Pi·x/L ), x = −L .. L )

> simplify(kn) :

> kn := simplify(kn) :

> an := kn·L/(c·n·Pi) :

> with(plots) :

> psum := subs( M = 1, L = 5, c = 1, sum( an·sin( n·Pi·x/L ) · sin( c·n·Pi·t/L ), n = 1 .. 100 ) ) :

> curves := {seq(subs(t = 2·m, psum), m = 0 .. 10)} :

> plot(curves, x = 0 .. 20, thickness = [1, 2, 3, 4, 5.6], color = black)
```




```
> animate(psum, x = 0 .. 20, t = 0 .. 10) :

>
```