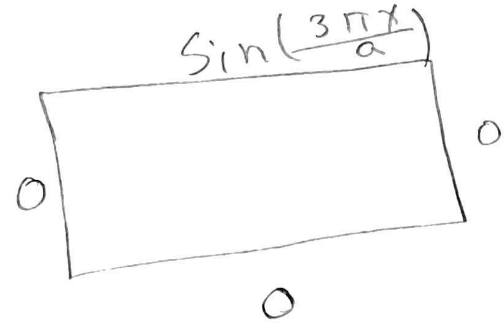
1.d)



$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$$ Let $u(x,y) = X(x)\cdot Y(y)$, we know

we arrive to $X'' + kX = 0$ $\quad k = \nu^2 > 0$

$Y'' - kY = 0$

$X(x) = A\cos(\nu x) + B\sin(\nu x)$ is a solution of $X'' + kX = 0$

then $X(0) = 0 = A\cos(0) + B\sin(0) \Rightarrow A = 0$

$\Rightarrow X(x) = B\sin(\nu x)$

$X(a) = 0 = B\sin(\nu a) \Rightarrow \boxed{\nu = \frac{n\pi}{a}}$

$\therefore X(x) = B_n \sin\left(\frac{n\pi}{a} x\right)$

$Y'' - kY = 0$ we know $Y(y) = \text{const}\cdot\sinh(\nu y) + \text{const}\cdot\cosh(\nu y)$

is a solution.

$Y(0) = 0 = \text{const}\cdot 0 + \text{const}\cdot 1 \Rightarrow Y(y) = \sinh(\nu y)$

$= \sinh\left(\frac{n\pi y}{a}\right)$

$\therefore u_n(x,y) = X(x)\cdot Y(y) = B_n \sin\left(\frac{n\pi x}{a}\right)\cdot \sinh\left(\frac{n\pi y}{a}\right)$

PDE ✓ BC ✓✓✓ $\therefore u(x,y) = \sum_{n=1}^{\infty} B_n \sin\left(\frac{n\pi x}{a}\right)\cdot \sinh\left(\frac{n\pi y}{a}\right)$

for BC $u(x,b) = \sin\left(\frac{3\pi x}{a}\right)$:

$$\sin\left(\frac{3\pi x}{a}\right) = \sum_{n=1}^{\infty} B_n \sin\left(\frac{n\pi x}{a}\right)\cdot \sinh\left(\frac{n\pi b}{a}\right)$$

$$\Rightarrow B_n = \frac{1}{\sinh\left(\frac{n\pi b}{a}\right)}\cdot \frac{\left(\sin\left(\frac{3\pi x}{a}\right), \sin\left(\frac{n\pi x}{a}\right)\right)}{\left(\sin\left(\frac{n\pi x}{a}\right), \sin\left(\frac{n\pi x}{a}\right)\right)}$$

Since $\left\{\sin\left(\frac{n\pi x}{a}\right)\right\}$ is an $\perp$ set, we have

$$\boxed{B_3 = \frac{1}{\sinh\left(\frac{3\pi b}{a}\right)}}$$

Maple: