Exercise 2(a):
Solution:

$$phix := \sin\left(\frac{n\cdot \mathrm{Pi}\cdot x}{a}\right):$$

$$phiy := \sin\left(\frac{m\cdot \mathrm{Pi}\cdot y}{b}\right):$$

$$Knm := \left(\frac{n\cdot \mathrm{Pi}}{a}\right)^2 + \left(\frac{m\cdot \mathrm{Pi}}{b}\right)^2:$$

$$u := Sum(Sum(Bnm\cdot phix\cdot phiy\cdot \exp(-\mathrm{D}\cdot Knm\cdot t), n = 1..\mathrm{infinity}), m = 1..\mathrm{infinity})$$

$$u := \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} Bnm \sin\left(\frac{n\,\pi\,x}{a}\right)\sin\left(\frac{m\,\pi\,y}{b}\right) e^{-\mathrm{D}\left(\frac{n^2\pi^2}{a^2}+\frac{m^2\pi^2}{b^2}\right)t} \quad \textbf{(1)}$$

Finding coefficients:

$$Bnm := \frac{int(int(phix\cdot phiy\cdot M, y = 0..b), x = 0..a)}{int\left(int\left(phix^2\cdot phiy^2, y = 0..b\right), x = 0..a\right)} \mathrm{assuming}(n > 0, m > 0, n, integer, m, integer, a > 0, b > 0)$$

$$Bnm := -\frac{4\,M\left((-1)^m - 1 - (-1)^{n+m} + (-1)^n\right)}{m\,\pi^2\,n} \quad \textbf{(2)}$$

$$test := subs(m = 1, n = 1, Bnm)$$

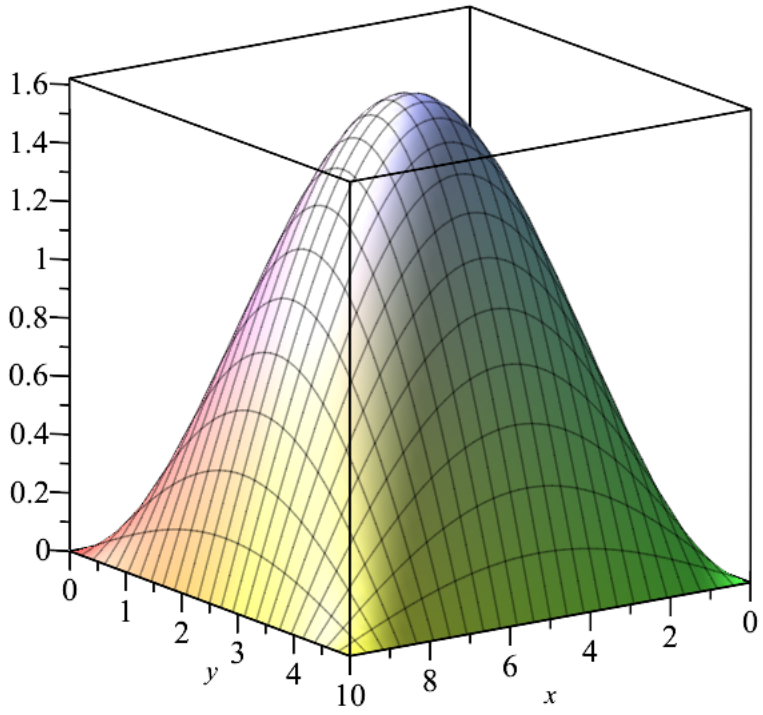
$$test := \frac{16\,M}{\pi^2} \quad \textbf{(3)}$$

$$with(plots):$$

$$psuma := Bnm\cdot phix\cdot phiy\cdot M\cdot \exp(-\mathrm{D}\cdot Knm\cdot t):$$

$$psuma := subs(n = 1, m = 1, \mathrm{D} = 4, a = 10, b = 5, M = 1, psuma):$$

$$animate3d(psuma, x = 0..10, y = 0..5, t = 0..1, frames = 100)$$



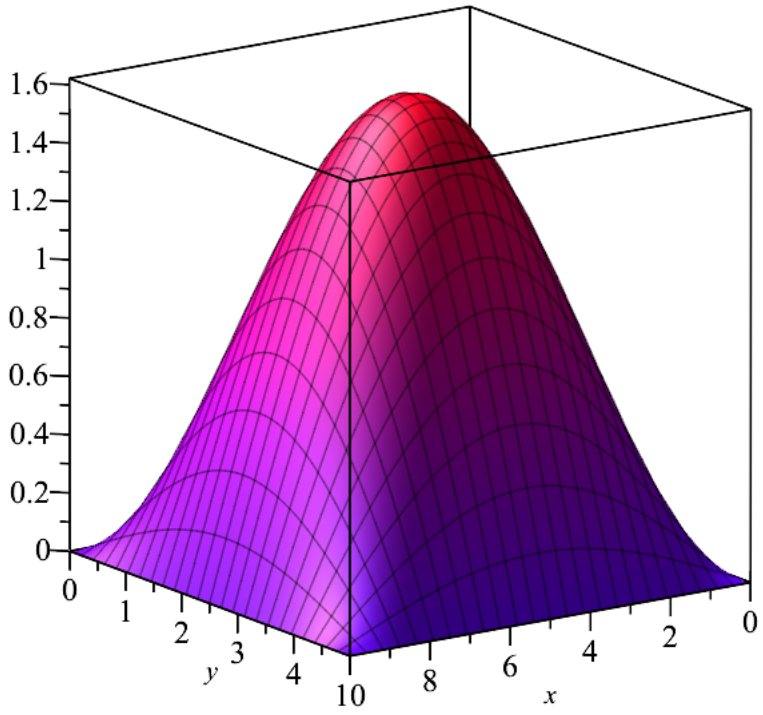
Exercercise 2(b):

For t=0 and n=1, m=1:

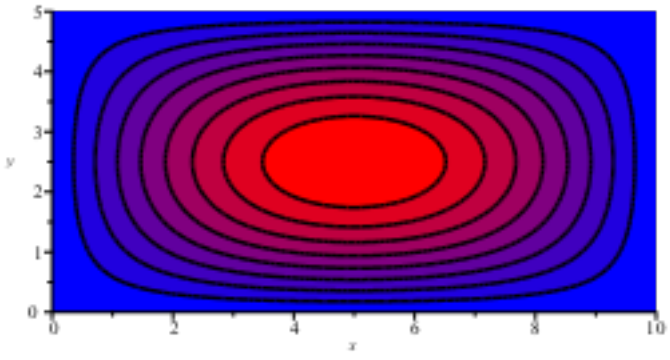
$psumb := Bnm \cdot phix \cdot phiy :$

$psumb := subs(a = 10, b = 5, n = 1, m = 1, t = 0, M = 1, \mathrm{D} = 4, psumb) :$

$plot3d(psumb, x = 0..10, y = 0..5)$



contourplot(*psumb*, $x = 0..10$, $y = 0..5$, *scaling* = *constrained*, *coloring* = [*blue*, *red*], *filledregions* = *true*)

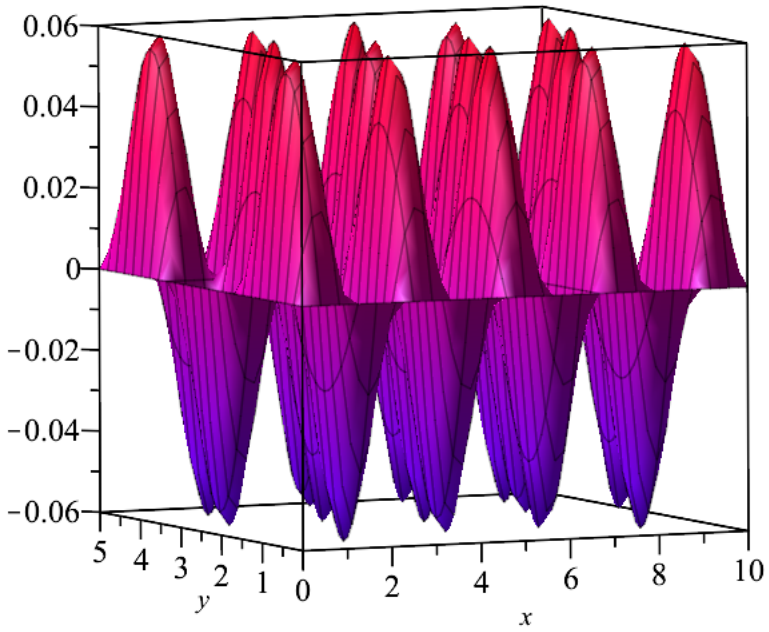


For t=10 and n=9, m=3:

$psumbii := Bnm \cdot phix \cdot phiy \cdot \exp(-D \cdot Knm \cdot t)$:

$psumbii := subs(n = 9, m = 3, a = 10, b = 5, t = 0, M = 1, D = 4, psumbii)$:

$plot3d(psumbii, x = 0..10, y = 0..5)$



$contourplot(psumbii, x = 0..10, y = 0..5, scaling = constrained, coloring = [blue, red], filledregions = true)$