(iv)

$u(x,0)=0$ and $\frac{\partial u}{\partial t}(x,0)=g(x)$ where

$$g(x) = \begin{cases} M, & \frac{L}{4} < x < \frac{L}{2} \\ 0, & \text{else} \end{cases}$$

we know $u(x,t) = \sum_{n=1}^{\infty} \sin\left(\frac{n\cdot\pi}{L}\cdot x\right)\cdot T_n(t)$ where

$T_n(t) = a_n \sin\left(\frac{cn\pi}{L}\cdot t\right) + b_n \cos\left(\frac{cn\pi}{L}\cdot t\right)$

$u_n(x,0) \Rightarrow T_n(0) = a_n \cancel{\sin(0)}^{0} + b_n\cdot 1 = 0$

$\boxed{b_n = 0}$

$\Rightarrow T_n(t) = a_n \sin\left(\frac{cn\pi}{L}\cdot t\right)$

then

$\frac{\partial u}{\partial t}(x,0) = g(x) = \sum_{n=1}^{\infty} \sin\left(\frac{n\cdot\pi}{L}\cdot x\right)\cdot a_n\cdot \frac{c\cdot n\cdot\pi}{L}$

where g is piecewise. Let $K_n = \frac{a_n\cdot c\cdot n\cdot\pi}{L}$

$\Rightarrow g(x) = \sum_{n=1}^{\infty} \sin\left(\frac{n\cdot\pi}{L}\cdot x\right)\cdot K_n$

by Maple, we end up with:

$$u(x,t) = \sum_{n=1}^{\infty} \frac{-M\left(\cos\left(\frac{n\pi}{2}\right) - \cos\left(\frac{n\pi}{4}\right)\right)L}{n^2 \pi^2 c} \cdot \sin\left(\frac{n\pi}{L}\cdot x\right) \cdot \sin\left(\frac{cn\pi}{L} t\right)$$

with period (in t) $\frac{2\cancel{\pi}}{cn\cancel{\pi}} \cdot L = \frac{2L}{cn}$ and common period $2L$.

Maple:

> $kn := \dfrac{int\left(M\cdot\sin\left(\dfrac{n\cdot\text{Pi}\cdot x}{L}\right), x=\dfrac{L}{4}..\dfrac{L}{2}\right)}{int\left(\sin^2\left(\dfrac{n\cdot\text{Pi}\cdot x}{L}\right), x=-L..L\right)}$ assuming$(L > 0, n > 0, n, integer)$

$$kn := -\frac{M\left(\cos\left(\frac{n\pi}{2}\right) - \cos\left(\frac{n\pi}{4}\right)\right)}{n\pi} \tag{1}$$

> $an := \dfrac{kn\cdot L}{c\cdot n\cdot\text{Pi}}$

$$an := -\frac{M\left(\cos\left(\frac{n\pi}{2}\right) - \cos\left(\frac{n\pi}{4}\right)\right)L}{n^2\pi^2 c} \tag{2}$$

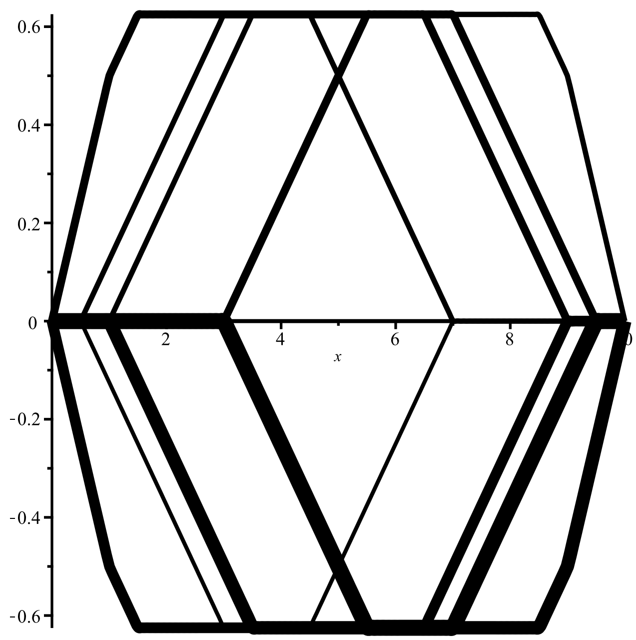
> $with(plots)$:

> $psum := subs\left(M=1, L=10, c=1, sum\left(an\cdot\sin\left(\dfrac{n\cdot\text{Pi}\cdot x}{L}\right)\cdot\sin\left(\dfrac{c\cdot n\cdot\text{Pi}\cdot t}{L}\right), n=1..200\right)\right)$:

> $animate(psum, x=0..50, t=0..20)$:

> $curve := \{seq(subs(t=2\cdot m, psum), m=0..10)\}$:

> $plot(curve, x=0..10, thickness=[1, 2, 3, 4, 5, 6], color=black)$



>