Ex 2(c) Expression for average temperature as a function of time:

$$phix := \sin\left(\frac{n \cdot \mathrm{Pi} \cdot x}{a}\right) :$$

$$phiy := \sin\left(\frac{m \cdot \mathrm{Pi} \cdot y}{b}\right) :$$

$$Knm := \left(\frac{n \cdot \mathrm{Pi}}{a}\right)^2 + \left(\frac{m \cdot \mathrm{Pi}}{b}\right)^2 :$$

$$Bnm := -\frac{4\,M\left((-1)^m - 1 - (-1)^{n+m} + (-1)^n\right)}{m\,\pi^2\,n} :$$

$$avg := \frac{1}{a \cdot b} \cdot (int(int(Bnm \cdot phix \cdot phiy \cdot \exp(-\mathrm{D} \cdot Knm \cdot t), x = 0..a), y = 0..b)) \text{assuming}(n, integer, m, integer, n > 0, m > 0, a > 0, b > 0)$$

$$avg := -\frac{1}{m^2\,\pi^4\,n^2}\left(4\,M\,\mathrm{e}^{-\frac{\mathrm{D}\,\pi^2\,(a^2 m^2 + b^2 n^2)\,t}{a^2 b^2}}\left(3\,(-1)^m - 4 - 4\,(-1)^{n+m} + 2\,(-1)^n + (-1)^{2n+m} + 2\,(-1)^{2m+n}\right)\right) \quad \textbf{(1)}$$

Exercise 2(d):Taking a 1-term approximation of your solution (c), give an expression for the time for the average temperature to cool to half the initial average temperature:

One term approximation of average temperature:

$$avgd := subs(n = 1, m = 1, avg)$$

$$avgd := \frac{64\,M\,\mathrm{e}^{-\frac{\mathrm{D}\,\pi^2\,(a^2 + b^2)\,t}{a^2 b^2}}}{\pi^4} \quad \textbf{(2)}$$

Expression for the time for the average temperature to cool to half the initial average temperature:

$$\text{alpha} := subs(t = 0, avgd)$$

$$\alpha := \frac{64\,M\,\mathrm{e}^0}{\pi^4} \quad \textbf{(3)}$$

$$avg2 := \frac{\text{alpha}}{2} :$$

$$func := avgd - avg2$$

$$func := \frac{64\,M\,\mathrm{e}^{-\frac{\mathrm{D}\,\pi^2\,(a^2 + b^2)\,t}{a^2 b^2}}}{\pi^4} - \frac{32\,M}{\pi^4} \quad \textbf{(4)}$$

Then, the solution is:

$$func := solve(func, t)$$

$$func := \frac{\ln(2)\, a^2\, b^2}{D\, \pi^2\, (a^2 + b^2)} \tag{5}$$

Exercise 2 (e) Give the specific time from (d) for a square.

$func2e := subs(\ b = a, func)$

$$func2e := \frac{\ln(2)\, a^2}{2\, D\, \pi^2} \tag{6}$$

From exercise 2(e)

$spoonful := subs(a = 1, D = 30, func2e)$

$$spoonful := \frac{\ln(2)}{60\, \pi^2} \tag{7}$$

$cup := subs(a = 2, D = 30, func2e)$

$$cup := \frac{\ln(2)}{15\, \pi^2} \tag{8}$$

$potofsoup := subs(a = 4, D = 30, func2e)$

$$potofsoup := \frac{4\, \ln(2)}{15\, \pi^2} \tag{9}$$

$watertank := subs(a = 24, D = 30, func2e)$

$$watertank := \frac{48\, \ln(2)}{5\, \pi^2} \tag{10}$$