For the initial condition (i) u(x,0) = M:
(a) Write the solution u(x,t):
For condition (i) we have the family of solutions:

$$u := \cos\left(\frac{n \cdot \mathrm{Pi}}{L} \cdot x\right) \cdot an \cdot \exp\left(-\left(\frac{n \cdot \mathrm{Pi}}{L}\right)^2 \cdot \mathrm{D} \cdot t\right) :$$

Finding coefficients an:

$$a0 := \frac{int\left(M \cdot \cos\left(\frac{n \cdot \mathrm{Pi} \cdot x}{L}\right), x = 0 ..L\right)}{int\left(\cos^2\left(\frac{n \cdot \mathrm{Pi} \cdot x}{L}\right), x = 0 ..L\right)} \text{assuming}(n = 0, M > 0, L > 0)$$

$$a0 := M \tag{1}$$

$$an := \frac{int\left(M \cdot \cos\left(\frac{n \cdot \mathrm{Pi} \cdot x}{L}\right), x = 0 ..L\right)}{int\left(\cos^2\left(\frac{n \cdot \mathrm{Pi} \cdot x}{L}\right), x = 0 ..L\right)} \text{assuming}(L > 0, n = 1)$$

$$an := 0 \tag{2}$$

Then, u(x,t) becomes:
$u := a0$

$$u := M \tag{3}$$

(the temperature is constant for all times)

(b) Average temperature at t = 0:

$$avg\_t0 := \left(\frac{1}{L}\right) \cdot int(u, x = 0 ..L)$$

$$avg\_t0 := M \tag{4}$$

Average temperature is constant. If we let M=100, then
$avg\_t0 := 100$

$$avg\_t0 := 100 \tag{5}$$

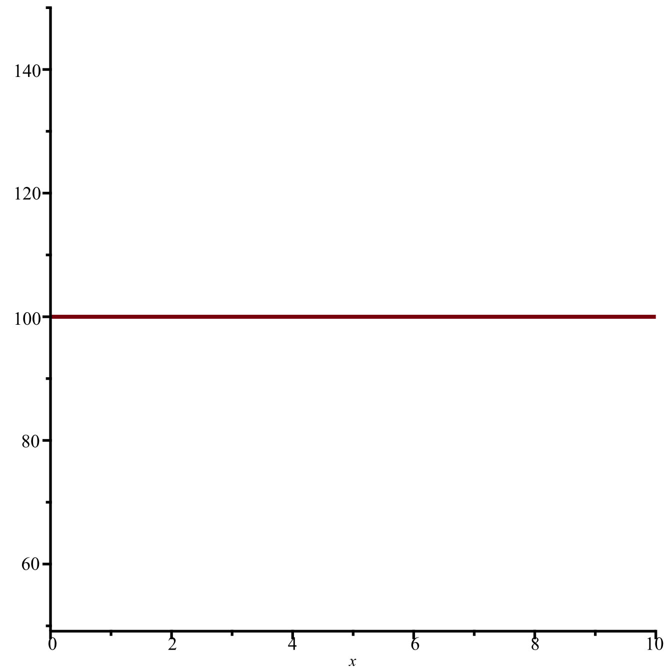
(c) Average temperature expression for any t (and M=100):
$avg\_t := 100$

$$avg\_t := 100 \tag{6}$$

(d) Since the temperature u(x,t) is always constant, it will never decrease

(e)Plotting temperature function u(x,t):

```
psum := 100 :
with(plots) :
plot(psum, x = 0 ..10)
```



140
120
100
80
60
0
2
4
6
8
10
x