MA 501 FA21 Lubkin
Project 1 due Tues 3/1

As explained in class, you may work with a partner on the project. If you do so, please submit one paper with both names. One of you should be the Moodle submitter. The other should submit to Moodle a statement of who you worked with, and I will mark one paper but enter grades in both places.

Objectives:

- Practice solving the heat equation
- Consolidating understanding of behavior of the heat equation and the significance of BC through observation
- Experience with inhomogeneous BC
- Practice thinking about asymptotic behavior and averages

1. Solve the heat equation $\frac{\partial u}{\partial t} = D\frac{\partial^2 u}{\partial x^2}$, $x \in (0,L)$, $u(0,t) = u(L,t) = 0$ for the initial conditions given.

   (a) Write the solution $u(x,t)$.

   (b) Write the average temperature at time 0.

   (c) Using your solution from (a), give an expression for the average temperature at time t.

   (d) Using the slowest-decaying term from (c), estimate how long it takes for the average temperature to decrease to 10% of its initial value.

   (e) Create a partial sum of satisfactory precision (in your opinion), look at an animation to see how it behaves, and plot solution curves at several times to illustrate the motion. If you can't indicate time by curve color or thickness, then hand-annotate the plot to indicate times. Make sure you run it until the time you estimated in (d).

   (i) $u(x,0) = M$ constant (as we did in class for M=100)

   (ii) $u(x,0) = \begin{cases} M, x < L/2 \\ 0, x > L/2 \end{cases}$ (similar to what we did in class)

2. Solve the heat equation $\frac{\partial u}{\partial t} = D\frac{\partial^2 u}{\partial x^2}$, $x \in (0,L)$, $\frac{\partial u}{\partial x}(0,t) = \frac{\partial u}{\partial x}(L,t) = 0$ for the initial conditions:

   (i) $u(x,0) = M$ constant

   (ii) $u(x,0) = \begin{cases} M, x < L/2 \\ 0, x > L/2 \end{cases}$

For each IC,

(a) Write the solution $u(x,t)$.

(b) Write the average temperature at time 0.

(c) Using your solution from (a), give an expression for the average temperature at time t.

(d) Using the slowest-decaying term from (c), estimate how long it takes for the average temperature to decrease to 10% of its initial value, or explain why this is a silly question.

(e) Create a partial sum of satisfactory precision (in your opinion), look at an animation to see how it behaves, and plot solution curves at several times to illustrate the motion. If you can't indicate time by curve color or thickness, then hand-annotate the plot to indicate times. Make sure you run it until at least the time you estimated in (d) (if applicable). Show multiple curves on the same axes but at different times, choosing parameters so as to clearly show what is happening.