MA 501 SP22 Lubkin Project 2 due Fri 3/25/22

Objectives:

- More independence deriving solutions
- Practice with the finer points of applied math (estimates, good vs. bad plots)
- Consolidating understanding of the significance of BC through observation
- Experience with the Laplace equation, breaking down problems into subproblems, and when to re-derive something
- Experience visualizing and displaying functions of 2 spatial variables
- Practice solving a PDE from scratch
- Consolidating understanding of behavior of the heat equation
- Deepen understanding of the role of orthogonality in representing a function as a series
- More independence using Maple

As always, give your solutions using the parameters given. Only use specific parameter values for the portion of the problem that must be numerical, such as plots.

1. Solve the Laplace equation $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=0$ for the specified BC on the rectangle $x \in (0,a)$, $y \in (0,b)$.

Plot both a 3D surface and a flat projection with filled contours (as we did in class). Make sure your solution looks right in terms of satisfying the BC. Make $a \neq b$ in your plots. You must use constrained scaling in *x* and *y*. Choose your *a* and *b* so the 3D plots don't look ridiculous with constrained scaling.

(a) u(0,y) = 0, u(a,y) = 0, u(x,0)=50, u(x,b) = 100

(b) u(0,y)=0, u(a,y)=50, u(x,0)=100, u(x,b)=0

(c) $\frac{\partial u}{\partial x}(0,y)=0$, $\frac{\partial u}{\partial x}(a,y)=0$, u(x,0)=50, u(x,b)=100 (insulated on 2 sides, fixed temp on the other 2). You will have to go back a few steps in our derivation since the type of BC is different! This problem will require you to think.

(d) u(0,y)=0, u(a,y)=0, u(x,0)=0, $u(x,b)=\sin(3\pi x/a)$. Hint: don't calculate any inner products.

2. Solve the heat equation in 2D on a rectangle (non-square!),

$$\frac{\partial u}{\partial t}=D\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right),\ x \in (0,a),\ y \in (0,b), \text{ with } u=0 \text{ on all boundaries, and } u(x,y,0)=M.$$

(a) Write the solution. Animate solutions (`animate3d`) to see the behavior and confirm that your result is correct. I do not need to see your animation.

(b) Turn in both 3D plots and flat contour plots at different times to show the behavior. Include $t = 0$. Required: *Make sure the z scaling and color scaling are consistent over time so you can see the decay. Maple will rescale them unless you read the help files and figure out how to pin them! I do not want to see plots that look the same at different times. And as always, make sure that the scaling in x is the same as the scaling in y (scaling=constrained).*

(c) Write an expression for the average temperature as a function of time.

(d) Taking a 1-term approximation of your solution (c), give an expression for the time for the average temperature to cool to half the initial average temperature. (Your result should depend on general *a* and *b*.)

(e) Give the specific time from (d) for a square.

(f) Sometimes an approximation or estimate gives valuable information without a complicated simulation. Consider the everyday problem of cooling a cup of tea. This problem is roughly equivalent. We can use it to get useful order-of-magnitude estimates. Use the result from (e) as an estimate of the characteristic cooling time of a cup of diameter a, and your own experience as to how long it takes for a cup of tea to cool significantly (that gives you an estimate for D). Using this estimate for D, make a small table of cooling times:

	diameter	cooling time
spoonful		
cup		
pot of soup		
home hot water tank		