IC (i):
Calculating coefficients bn:

$$bn := \frac{int\left(M\cdot\sin\left(\frac{n\cdot\text{Pi}\cdot x}{L}\right), x=0..L\right)}{int\left(\sin^2\left(\frac{n\cdot\text{Pi}\cdot x}{L}\right), x=0..L\right)} \text{assuming}(M>0, n, integer)$$

$$bn := -\frac{2\,M\left((-1)^n - 1\right)}{\pi\,n} \quad \textbf{(1)}$$

(a) u(x,t) is equalt to:

$$u := bn\cdot\sin\left(\frac{n\cdot\text{Pi}\cdot x}{L}\right)\cdot\exp\left(-\left(\frac{n\cdot\text{Pi}}{L}\right)^2\cdot\text{D}\cdot t\right)$$

$$u := -\frac{2\,M\left((-1)^n - 1\right)\sin\left(\frac{n\,\pi\,x}{L}\right)\mathrm{e}^{-\frac{n^2\pi^2\,\text{D}\,t}{L^2}}}{\pi\,n} \quad \textbf{(2)}$$

(b) Then, calculating the average temperature at t=0:

$$avg := \left(\frac{1}{L}\right)\cdot int\left(bn\cdot\sin\left(\frac{n\cdot\text{Pi}}{L}\cdot x\right)\cdot\exp\left(-\left(\frac{n\cdot\text{Pi}}{L}\right)^2\cdot\text{D}\cdot t\right), x=0..L\right)$$

$$avg := \frac{2\,M\left((-1)^n - 1\right)\mathrm{e}^{-\frac{n^2\pi^2\,\text{D}\,t}{L^2}}\left(\cos(n\,\pi) - 1\right)}{n^2\,\pi^2} \quad \textbf{(3)}$$

$$avg := subs(t=0, M=100, L=10, \text{D}=1, avg)$$

$$avg := \frac{200\left((-1)^n - 1\right)\mathrm{e}^0\left(\cos(n\,\pi) - 1\right)}{n^2\,\pi^2} \quad \textbf{(4)}$$

$psum := sum(avg, n=1..1000):$
$evalf(psum)$

$$99.95947149 \quad \textbf{(5)}$$

(c) Expression for calculating the average temperature at time t:

$$avg\_expr := \frac{\exp\left(-\left(\frac{n\cdot\text{Pi}}{L}\right)^2\cdot\text{D}\cdot t\right)}{L}\cdot int\left(-\frac{2\cdot M\cdot\left((-1)^n - 1\right)}{\text{Pi}\cdot n}\cdot\sin\left(\frac{n\cdot\text{Pi}}{L}x\right), x=0..L\right)$$

$$avg\_expr := \frac{2\,M\left((-1)^n - 1\right)\mathrm{e}^{-\frac{n^2\pi^2\,\text{D}\,t}{L^2}}\left(\cos(n\,\pi) - 1\right)}{n^2\,\pi^2} \quad \textbf{(6)}$$

(d) The slowest decaying term occurs when n = 1. Then
$initial\_value := 100:$
$final\_value := 0.1\cdot initial\_value$

$$final\_value := 10.0 \quad \textbf{(7)}$$

$equ := subs(M=100, n=1, L=10, \mathrm{D}=1, avg\_expr) - final\_value$

$$equ := -\frac{400\,\mathrm{e}^{-\frac{\pi^2 t}{100}}\,(\cos(\pi)-1)}{\pi^2} - 10.0 \quad \textbf{(8)}$$

$solve(equ, t)$

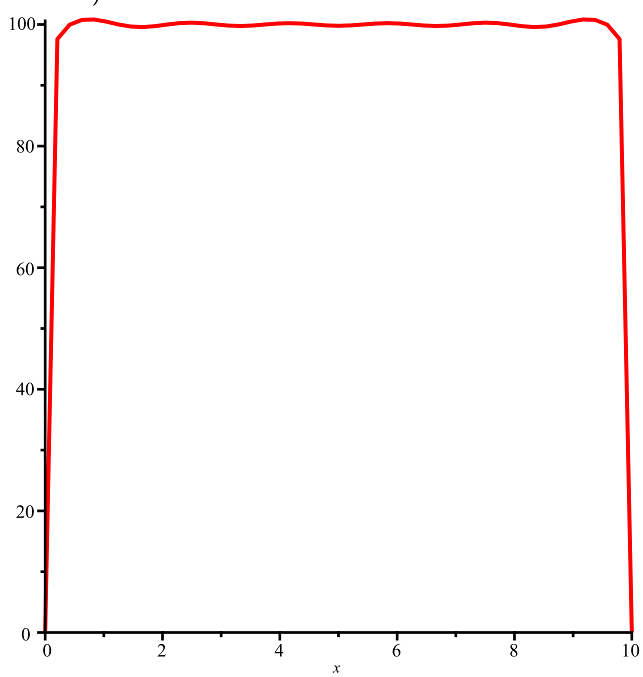
$$21.20213514 \quad \textbf{(9)}$$

(e) Creating Partial Sums:

$with(plots):$

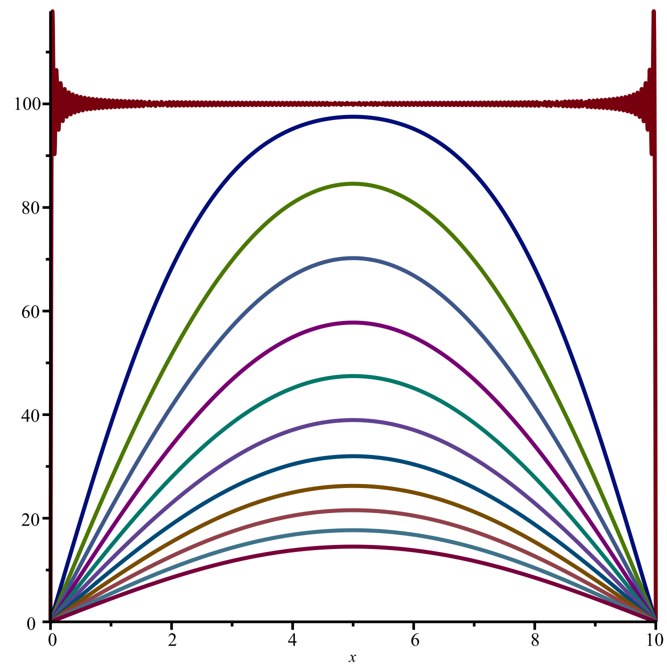
$psum := subs(M=100, \mathrm{D}=1, L=10, sum(u, n=1..300)):$

$animate(psum, x=0..10, t=0..23)$



$curves := [seq(subs(t=2\cdot m, psum), m=0..11)]:$

$plot(curves, x=0..10)$



We can see that approaches 21.20213514 time units, the amplitud of the curves (temperature) get smaller. We can also see the Gibbs phenomenon for the t=0 curve.