

By exercise 1 (a) we know the solution to u1:

$$u1(x, y) = \text{Sum} \left(An \cdot \sin \left(\frac{n \cdot \text{Pi} \cdot x}{a} \right) \cdot \sinh \left(\frac{n \cdot \text{Pi} \cdot (b - y)}{a} \right), n = 1.. \infty \right)$$

Calculating Bn:

$$\text{phi} := \sin \left(\frac{n \cdot \text{Pi} \cdot x}{a} \right) :$$

$$An0 := \frac{\text{int}(100 \cdot \text{phi}, x = 0..a)}{\text{int}(\phi^2, x = 0..a)} \text{assuming}(n > 0, n, \text{integer})$$

$$An0 := - \frac{200 \left((-1)^n - 1 \right)}{\pi n} \quad (1)$$

$$An := \frac{An0}{\sinh \left(\frac{n \cdot \text{Pi} \cdot b}{a} \right)}$$

$$An := - \frac{200 \left((-1)^n - 1 \right)}{\pi n \sinh \left(\frac{n \pi b}{a} \right)} \quad (2)$$

For u2, we know that we have to change x for y, y for x, a for b, and b for a in the solution u1:

$$u2(x, y) := \text{Sum} \left(Dn \cdot \sin \left(\frac{n \cdot \text{Pi} \cdot y}{b} \right) \cdot \sinh \left(\frac{n \cdot \text{Pi} \cdot x}{b} \right), n = 1.. \text{infinity} \right)$$

Calculating the coefficients:

$$\text{psi} := \sin \left(\frac{n \cdot \text{Pi} \cdot y}{b} \right) :$$

$$Dn0 := \frac{\text{int}(50 \cdot \text{psi}, y = 0..b)}{\text{int}(\psi^2, y = 0..b)} \text{assuming}(n, \text{integer}, n > 0) :$$

$$Dn := \frac{Dn0}{\sinh \left(\frac{n \cdot \text{Pi} \cdot a}{b} \right)} \text{assuming}(n, \text{integer})$$

$$Dn := - \frac{100 \left((-1)^n - 1 \right)}{\pi n \sinh \left(\frac{n \pi a}{b} \right)} \quad (3)$$

Then, our final solution is:

$$u(x, y) = u1 + u2 =$$

$$\text{Sum} \left(An \cdot \sin \left(\frac{n \cdot \text{Pi} \cdot x}{a} \right) \cdot \sinh \left(\frac{n \cdot \text{Pi} \cdot (b - y)}{a} \right), n = 1.. \infty \right) + \text{Sum} \left(Dn \cdot \sin \left(\frac{n \cdot \text{Pi} \cdot y}{b} \right) \cdot \sinh \left(\frac{n \cdot \text{Pi} \cdot x}{b} \right), n = 1.. \text{infinity} \right)$$

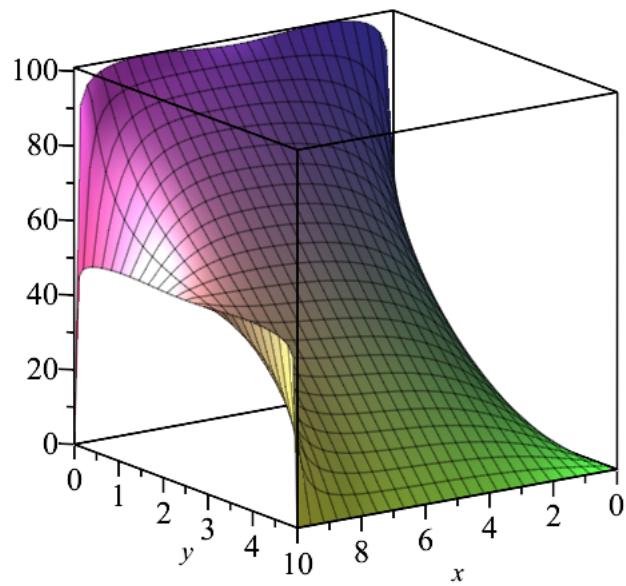
Plotting the solution:

with(plots) :

$$\text{psum} := \text{sum} \left(An \cdot \sin \left(\frac{n \cdot \text{Pi} \cdot x}{a} \right) \cdot \sinh \left(\frac{n \cdot \text{Pi} \cdot (b - y)}{a} \right), n = 1..100 \right) + \text{sum} \left(Dn \cdot \sin \left(\frac{n \cdot \text{Pi} \cdot y}{b} \right) \cdot \sinh \left(\frac{n \cdot \text{Pi} \cdot x}{b} \right), n = 1..100 \right) :$$

$$\text{psum} := \text{subs}(a = 10, b = 5, \text{psum}) :$$

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plot3d(psum, x=0..10, y=0..5)
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contourplot(psum, x=0..10, y=0..5, scaling=constrained, coloring=[blue, red], filledregions=true)
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