MA501 SP22 Lubkin HW3 due Fri 2/11/22

Objectives:

- Solidifying classification of PDE
- Practice separating variables
- Practice with guess-and-test solutions

1. For each PDE:

(i) Is the PDE linear? If linear, is the PDE homogeneous? How many boundary conditions does it need? How many initial conditions?

(ii) Is it separable? If separation of variables can be done, separate the PDE into two ODEs. If not, show where separation fails. Do not solve.

(a) $\dfrac{\partial^2 w}{\partial x^2}+\dfrac{\partial^2 w}{\partial y^2}=0$ (Laplace equation)

(b) $\dfrac{\partial^2 w}{\partial x^2}+\dfrac{\partial^2 w}{\partial y^2}=A$ (Poisson equation; A is a parameter)

(c) $\frac{\partial u}{\partial t}+c(1+au)\frac{\partial u}{\partial x}=D\frac{\partial^2 u}{\partial x^2}$ (*a, c, D* are parameters)

(d) $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial x \partial y}+\frac{\partial^2 u}{\partial y^2}=0$

(e) $(\sin x)\frac{\partial^2 u}{\partial x^2}+(\cos y)\frac{\partial^2 u}{\partial y^2}=0$

2. For the wave equation,

$$\frac{\partial^2 u}{\partial t^2}=c^2\frac{\partial^2 u}{\partial x^2}$$

determine, by plugging in ("guess-and-test"), whether the following guesses are solutions. There are no BC or IC specified.

(a) $u_1(x,t)=f(x-ct)$ for some function f

(b) $u_2(x,t)=f(x+ct)$ for some function f

(c) $u_3(x,t)=\ f(x-ct)+f(x+ct)$ for some function f

Note that you should not pick a specific f, but consider any arbitrary f.