

11.)

Let  $M_1$ : output is from machine 1 ;  $M_2$ : Output is from machine 2

$M_3$ : output is from machine 3.

$$P(M_1) = 0.10$$

$$P(M_2) = 0.35$$

$$P(M_3) = 0.55$$

Let  $D$ : output is defective. Then:  $P(D|M_1) = 0.05$ ,

$$P(D|M_2) = 0.03, P(D|M_3) = 0.01$$

$E$ : randomly selected widget is defective.

$$P(E) = P((D \cap M_1) \cup (D \cap M_2) \cup (D \cap M_3)) = P(D \cap M_1) + P(D \cap M_2) + P(D \cap M_3) \\ - P(D \cap M_1 \cap M_2 \cap M_3)$$

$$\text{since } D \cap M_1 \cap M_2 \cap M_3 = \emptyset \Rightarrow P(D \cap M_1 \cap M_2 \cap M_3) = 0$$

$$\Rightarrow P(E) = P(D \cap M_1) + P(D \cap M_2) + P(D \cap M_3) = \sum_{i=1}^3 P(D|M_i) \cdot P(M_i)$$

$$\Rightarrow P(E) = (0.05)(0.10) + (0.03)(0.35) + (0.01)(0.55) = 0.021$$

$\therefore 2.1\%$  is the probability that a randomly selected widget is defective.