HW3-ISE560 Fall 2022, Due on Dec 9th, 2022

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Please note that this is a group homework, so each group needs to submit only one copy. Make sure that you have your group member's names on the submitted homework

Problem 1

Customers arrive at a service facility one at a time. Suppose that the total number of arrivals during a 1-hour period is a Poisson random variable with parameter 8. Compute the probability that at least three customers will arrive during 1 hour.

Problem 2

Airplane Reliability: A commercial jet airplane has four engines, two on each wing. Each engine lasts for a random amount of time that is an exponential random variable with parameter λ and then fails. If the failure takes place in flight, there can be no repair. The airplane needs at least one engine on each wing to function properly in order to fly safely. Model this system so that we can predict the probability of a trouble-free flight.

(Hint: Let $X_L(t)$ be the number of functioning engines on the left wing at time t, and let $X_R(t)$ be the number of functioning engines on the right wing at time t. The state of the system at time t is given by $X(t) = (X_L(t), X_R(t))$. Assuming the engine failures are independent of each other, $\{X(t), t \geq 0\}$ is a CTMC:

(1) What is the state space?

$$S = \{(0,0),(0,1),(0,2),(1,0),(1,1),(1,2),(2,0),(2,1),(2,2)\}.$$

(2) What states imply flight crash?

(3) Assume that the CTMC continues to evolve even after the airplane crashes, draw the CTMC diagram and derive the rate matrix.

Problem 3

Let us revisit the machine shop example with four machines and two repair persons we discussed in class. Suppose the lifetimes of the machines are exponential random variables with mean 3 days, while the repair times are exponential random variables with mean 2 hours. The shop operates 24 hours a day. Suppose all the machines are operating at 8:00 a.m. Monday. Compute the probability that all the machines are up at 5:00 p.m. on the same day. Also, compute the long-run fraction of the time that both repair persons are busy.

$$P(9) = \begin{bmatrix} .0002 & .0020 & .0136 & .1547 & .8295 \\ .0000 & .0006 & .0080 & .1306 & .8607 \\ .0000 & .0002 & .0057 & .1154 & .8787 \\ .0000 & .0002 & .0048 & .1070 & .8880 \\ .0000 & .0001 & .0041 & .0987 & .8971 \end{bmatrix}.$$