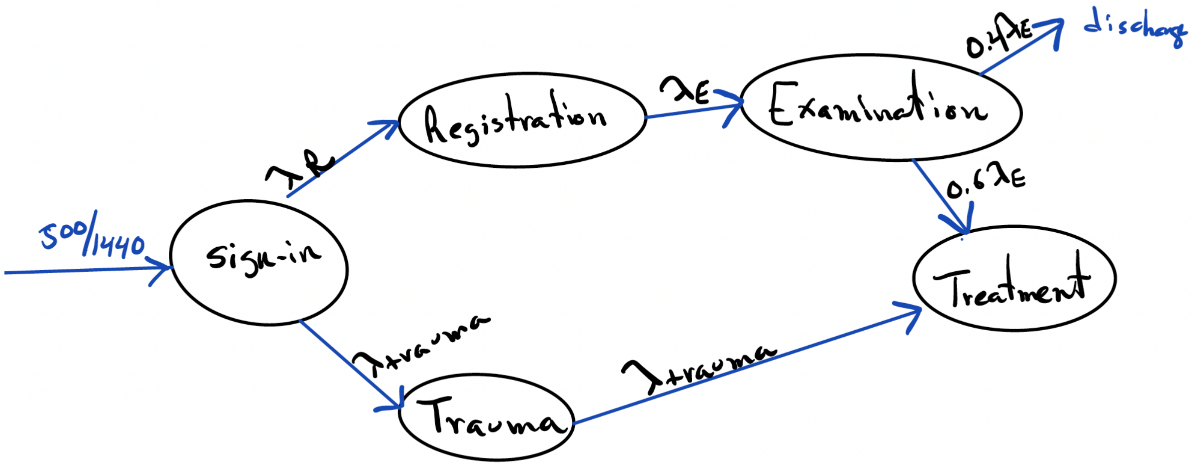
Problem 2

(a)
Queueing model:



Summary of arrival rates and service rates:

Station	Arrival rate For 8% trauma (per min)	Arrival Rate for 12% trauma (per min)	Service rate (per min)
Sign-in	0.3472	0.3472	0.3333
Registration	0.3194	0.3056	0.2
Examination	0.3194	0.3056	0.0625
Trauma	0.02778	0.04167	0.0111
Treatment	0.2194	0.225	For 8%: 0.0683 For 12%: 0.0653

Computation of arrival rates (table 1):

Station	**For 8% trauma patients**	**12% trauma patients**
Sign-in	$\frac{500}{1440} \approx 0.3472$	$\frac{500}{1440} \approx 0.3472$
Registration (λ_R)	$\frac{500}{1440} * 0.92$ $= \frac{460}{1440} \approx 0.3194$	$0.88 * \frac{500}{1440} = \frac{440}{1440} \approx 0.3056$
Examination (λ_E)	$\frac{460}{1440} \approx 0.3194$	$\frac{440}{1440} \approx 0.3056$
Trauma (λ_{trauma})	$0.08 * \frac{500}{1440}$ $= \frac{40}{1440} \approx 0.02778$	$0.12 * \frac{500}{1440} = \frac{60}{1440} \approx 0.04167$
Treatment($\lambda_{trauma} + 0.6\lambda_E$)	$\frac{40 + 0.6 * 460}{1440}$ $\frac{316}{1440} \approx 0.2194$	$\frac{60 + 0.60 * 440}{1440} = \frac{324}{1440} = 0.225$

Staff needed to just keep-up (table 2):

For 8% trauma patients	For 12% Trauma Pacients
$C_{\ sign-in} = 2\ staff$	$C_{\ sign-in} = 2\ staff$
$C_{trauma} = 3\ staff$	$C_{trauma} = 4\ staff$
$C_{Regisration} = 2\ staff$	$C_{Regisration} = 2\ staff$
$C_{examination} = 6\ staff$	$C_{examination} = 5\ staff$
$C_{treatment} = 4\ staff$	$C_{treatment} = 4\ staff$

Computations for the needed staff just to keep up:

Computations of staff for 8% trauma:	Computations for 12% trauma:
$c_{sign-in} > \left(\frac{\frac{500}{1440}}{\frac{1}{3}}\right) = \frac{1500}{1440} = 1.041$	$c_{sign-in} > \left(\frac{\frac{500}{1440}}{\frac{1}{3}}\right) = 1.041$
$c_{trauma} > \left(\frac{\frac{40}{1440}}{\frac{1}{90}}\right) = 2.5$	$c_{trauma} > \left(\frac{\frac{60}{1440}}{\frac{1}{90}}\right) = 3.75$
$C_{Registration} > \left(\frac{\frac{460}{1440}}{\frac{1}{5}}\right) = 1.597$	$C_{Registration} > \left(\frac{\frac{440}{1440}}{\frac{1}{5}}\right) = 1.52$
$C_{examination} > \left(\frac{\frac{460}{1440}}{\frac{1}{16}}\right) = 5.11,$	$C_{examination} > \left(\frac{\frac{440}{1440}}{\frac{1}{16}}\right) = 4.8889$
$C_{treatment} > ceil\left(\frac{\frac{316}{1440}}{\mu_{treatment}}\right) = 3.21$	$C_{treatment} > \frac{\frac{324}{1440}}{\frac{1}{0.12 * 30 + 0.88 * 13.3}} = 3.443$

Note: $\frac{1}{\mu_{treatment}} = 0.08 * 30min + 0.92 * 13.3 = 14.636\ min \rightarrow \mu_{treatment} = \frac{1}{14.636}$

(b)

For Rayleigh distribution: mean = $\sigma * \sqrt{\frac{\pi}{2}} \rightarrow \sigma = mean * \frac{\sqrt{2}}{\sqrt{\pi}}$

Registration station: $\sigma = 5 * \frac{\sqrt{2}}{\sqrt{\pi}}$ =3.9894

variance = $\frac{4-\pi}{2} * \sigma^2$

Treatment Station:

By the theory, we have:

$$\sigma_{trauma} = 0.12 * 30 * \frac{\sqrt{2}}{\sqrt{pi}} = 3.5107$$

$$\sigma_{non-trauma} = 0.88 * 13.3 * \frac{\sqrt{2}}{\sqrt{\pi}} = 9.3384$$

Note: for the next exercises
Let X_1 be the random variable that represent the contact time of the patient that comes from the trauma station. Similarly, X_2 for the patients that come from the examination station. By the exam problem, X_1 and X_2follow the Rayleigh distribution. Then, by the theory, X_1^2, and X_2^2 both follow exponential distribution with $\lambda_1 = \frac{1}{2*\sigma_1^2}$, and $\lambda_2 = \frac{1}{2*\sigma_2^2}$.

Let $Y = X_1^2 + X_2^2$, then by the theory we have that $Y \sim P(\lambda = \lambda_1 + \lambda_2 = 0.0663)$. The variance of $Y = 0.0663 = \lambda$.
We will use this variance number to model the trauma station as a M/G/c for the remaining exercises. Also note that our service rate will be λ in this case.

(c)

Station	**Waiting time (in minutes)**	**Root mean squared deviation**
Sign-in	1.117	0.8138
Registration	3.641	0.6359
Examination	69.296	3.6197
Trauma	321.695	159.8475
Treatment	8.538	0.4308

(d)

Station	**Minimum number of staff**	**Root mean squared deviation**
Sign-in	2	0.8138
Registration	2	0.5537
Examination	6	0.7335
Trauma	8	0.5475
Treatment	4	0.4308

(e)
Increasing staff:

Station	**Minimum number of staff**	**Root mean squared deviation (adding 1 to the optimal staff #)**	**Root mean squared deviation (adding 2 to the optimal staff #)**
Sign-in	3,4	0.9743	0.9960
Registration	3,4	0.9564	0.9917
Examination	7,8	0.9237	0.9737
Trauma	9,10	0.8595	0.9585
Treatment	5,6	0.8921	0.9694
Total Sum:		4.6060	4.8893

We can see that if we keep increasing the staff, the objective function will keep growing.

Computation using MATLAB:

Question (e):

```
station=[6,10,15,2,15];calc_a=[0.154,0.436,1.144,0.281,1.619]

calc_b=[0.024,0.083,0.394,0.083,0.459]
rm_1=zeros(1,5);rm_2=zeros(1,5);
for i=1:5
    rm_1(i)=rmse(calc_a(i),station(i));
    rm_2(i)=rmse(calc_b(i),station(i));
end
rm_1'
rm_2'
sum(rm_1)
sum(rm_2)
```

```
function y = rmse(obtained,expected)
x=obtained;y=expected;
y = sqrt((x-y)^2)/y;
end
```

```
calc_a = 1×5
      0.1540    0.4360 ...

calc_b = 1×5
      0.0240    0.0830 ...

ans = 5×1
      0.9743
      0.9564
      0.9237
      0.8595
      0.8921

ans = 5×1
      0.9960
      0.9917
      0.9737
      0.9585
      0.9694
```