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HW1

3.5) X is a discrete random variable with probability distribution:

$\cdot P\{X = x_1\} = \frac{1}{4}$ and $P\{X = x_2\} = \frac{3}{4}$

a) find x_1, and x_2 so that $E[X] = 0$ and $var(X) = 10$.

$$E[X] = \sum_{i=1}^{2} x_i \cdot P(X = x_i) = x_1 \cdot \left(\tfrac{1}{4}\right) + x_2\left(\tfrac{3}{4}\right) = \frac{x_1}{4} + \frac{3x_2}{4} = 0$$

$$\Rightarrow \frac{x_1}{4} = \frac{-3x_2}{4} \Rightarrow \boxed{x_1 = -3x_2}$$

$$var(X) = cov(X, X) = E[X \cdot X] - \cancelto{0}{E[X] \cdot E[X]} = E[XX]$$

and $E[XX] = \sum_{i=1}^{2}\sum_{j=1}^{2} x_i \cdot x_j \, P(X = x_i, X = x_j)$

$$= x_1^2 P(X = x_1) + x_2^2 P(X = x_2) = x_1^2 \cdot (1/4) + x_2^2 (3/4) = 10$$

$$\Rightarrow \frac{3x_2^2}{4} = 10 - \frac{x_1^2}{4} \Leftrightarrow 3x_2^2 = 40 - x_1^2 = 40 - (-3x_2)^2$$

$$\Rightarrow 3x_2^2 = 40 - 9x_2^2 \Leftrightarrow 3x_2^2 + 9x_2^2 = 40 \Leftrightarrow 12x_2^2 = 40 \Leftrightarrow x_2 = \pm\sqrt{\frac{40}{12}}$$

$$\Rightarrow x_2 = \pm\sqrt{\frac{20}{6}} = \pm\sqrt{10/3} \quad \therefore \boxed{x_2 = \pm\sqrt{10/3}}$$

if $x_2 = \sqrt{10/3}$ then $x_1 = -3\left(\sqrt{10/3}\right) = -3\sqrt{10/3}$

$\Rightarrow (x_1, x_2) = \left(-3\sqrt{10/3}, \sqrt{10/3}\right)$. If $x_2 = -\sqrt{10/3}$

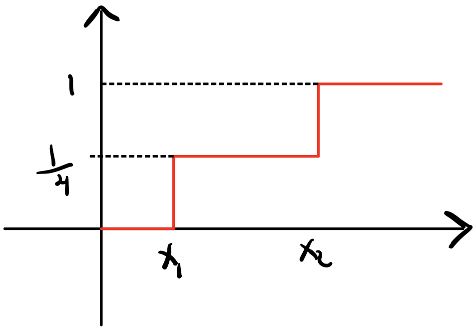
$\Rightarrow \ x_1 = -3x_2 = -3\left(-\sqrt{10/3}\right) = 3\sqrt{10/3} \Rightarrow (x_1, x_2) = \left(3\sqrt{10/3}, -\sqrt{10/3}\right)$

$\therefore \left(x_1 = 3\sqrt{10/3}, x_2 = -\sqrt{10/3}\right)$ or $\left(x_1 = -3\sqrt{10/3}, x_2 = \sqrt{10/3}\right)$

b)

Sketch the CDF of X

Case 1 $\quad x_1 < x_2$:



Case 2 : $\quad x_2 < x_1$: