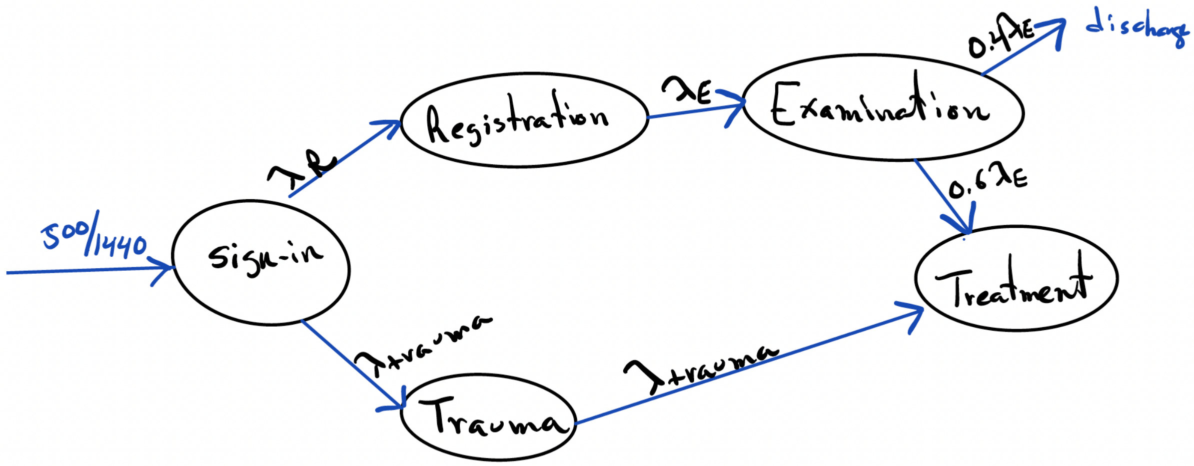
Problem 2

(a)
Queueing model:



Summary of arrival rates and service rates:

Station	Arrival rate For 8% trauma (per min)	Arrival Rate for 12% trauma (per min)	Service rate (per min)
Sign-in	0.3472	0.3472	0.3333
Registration	0.3194	0.3056	0.2
Examination	0.3194	0.3056	0.0625
Trauma	0.02778	0.04167	0.0111
Treatment	0.2194	0.225	For 8%: 0.0683 For 12%: 0.0653

Computation of arrival rates (table 1):

Station	**For 8% trauma patients**	**12% trauma patients**
Sign-in	$\frac{500}{1440} \approx 0.3472$	$\frac{500}{1440} \approx 0.3472$
Registration (λ_R)	$\frac{500}{1440} * 0.92$ $= \frac{460}{1440} \approx 0.3194$	$0.88 * \frac{500}{1440} = \frac{440}{1440}$ ≈ 0.3056
Examination (λ_E)	$\frac{460}{1440} \approx 0.3194$	$\frac{440}{1440} \approx 0.3056$
Trauma (λ_{trauma})	$0.08 * \frac{500}{1440}$ $= \frac{40}{1440} \approx 0.02778$	$0.12 * \frac{500}{1440} = \frac{60}{1440}$ ≈ 0.04167
Treatment($\lambda_{trauma} + 0.6\lambda_E$)	$\frac{40 + 0.6 * 460}{1440}$ $\frac{316}{1440} \approx 0.2194$	$\frac{60 + 0.60 * 440}{1440} = \frac{324}{1440}$ $= 0.225$

Staff needed to just keep-up (table 2):

For 8% trauma patients	For 12% Trauma Pacients
$C_{\ sign-in} = 2\ staff$	$C_{\ sign-in} = 2\ staff$
$C_{trauma} = 3\ staff$	$C_{trauma} = 4\ staff$
$C_{Regisration} = 2\ staff$	$C_{Regisration} = 2\ staff$
$C_{examination} = 6\ staff$	$C_{examination} = 5\ staff$
$C_{treatment} = 4\ staff$	$C_{treatment} = 4\ staff$

Computations for the needed staff just to keep up:

Computations of staff for 8% trauma:	Computations for 12% trauma:
$c_{sign-in} > \left(\frac{\frac{500}{1440}}{\frac{1}{3}}\right) = \frac{1500}{1440} = 1.041$	$c_{sign-in} > \left(\frac{\frac{500}{1440}}{\frac{1}{3}}\right) = 1.041$
$c_{trauma} > \left(\frac{\frac{40}{1440}}{\frac{1}{90}}\right) = 2.5$	$c_{trauma} > \left(\frac{\frac{60}{1440}}{\frac{1}{90}}\right) = 3.75$
$C_{Registration} > \left(\frac{\frac{460}{1440}}{\frac{1}{5}}\right) = 1.597$	$C_{Registration} > \left(\frac{\frac{440}{1440}}{\frac{1}{5}}\right) = 1.52$
$C_{examination} > \left(\frac{\frac{460}{1440}}{\frac{1}{16}}\right) = 5.11,$	$C_{examination} > \left(\frac{\frac{440}{1440}}{\frac{1}{16}}\right) = 4.8889$
$C_{treatment} > ceil\left(\frac{\frac{316}{1440}}{\mu_{treatment}}\right) = 3.21$	$C_{treatment} > \frac{\frac{324}{1440}}{\frac{1}{0.12 * 30 + 0.88 * 13.3}} = 3.443$

Note: $\frac{1}{\mu_{treatment}} = 0.08 * 30min + 0.92 * 13.3 = 14.636\ min \rightarrow \mu_{treatment} = \frac{1}{14.636}$

(b)

For Rayleigh distribution: mean = $\sigma * \sqrt{\frac{\pi}{2}} \rightarrow \sigma = mean * \frac{\sqrt{2}}{\sqrt{\pi}},$ and variance = $\frac{4-\pi}{2} * \sigma^2$

Registration station: $\sigma = 5 * \frac{\sqrt{2}}{\sqrt{\pi}}$ =3.9894

variance = $\frac{4-\pi}{2} * \sigma^2$

Treatment Station:

By the theory, we have that:

$$\sigma_{trauma} = 0.12 * 30 * \frac{\sqrt{2}}{\sqrt{pi}} = 3.5107$$

$$\sigma_{non-trauma} = 0.88 * 13.3 * \frac{\sqrt{2}}{\sqrt{\pi}} = 9.3384$$

Note for the next exercises:
Let X_1 be the random variable that represent the contact time of the patient that comes from the trauma station. Similarly, X_2 for the patients that come from the examination station. Since these two random variables are independent of each other,
Variance$(X_1 + X_2) = Var(X_1) + Var(X_2)$=$\frac{4-pi}{2} * (\sigma_1^2 + \sigma_2^2)$= 42.7190

We will use this variance number to model the trauma station as a M/G/c for the remaining exercises

(c)

Station	**Waiting time (in minutes)**	**FirstTreatment Standards**	**Root mean squared deviation**
Sign-in	1.117	6	0.8138
Registration	3.641	10	0.6359
Examination	69.296	15	3.6197
Trauma	321.695	2	159.8475
Treatment	11.614	15	0.2257
		Total:	165.1426

(d)

Station	**Minimum number of staff**	**Mean Waiting time (wQ) in minutes**	**FirstTreatment Waiting time Standards (minutes)**	**Root mean squared deviation**
Sign-in	2	1.117	6	0.8138
Registration	2	4.463	10	0.5537
Examination	6	3.998	15	0.7335
Trauma	8	0.905	2	0.5475
Treatment	4	11.614	15	0.2257
			Total:	2.8742

Note that if we add 1 minute (average time to move 1 patient between stations) to the waiting time of each station we still are below the FirstTreatment waiting standards for any station. Therefore, the time of moving patients from one station to another can be ignored.

(e)

Increasing staff:

Station	**Minimum number of staff**	**Root mean squared deviation (adding 1 to the optimal staff #)**	**Root mean squared deviation (adding 2 to the optimal staff #)**
Sign-in	3,4	0.9743	0.9960
Registration	3,4	0.9564	0.9917
Examination	7,8	0.9237	0.9737
Trauma	9,10	0.8595	0.9585
Treatment	5,6	0.8599	0.9604
Total Sum:		4.5739	4.8803

We can see that if we keep increasing the staff, the objective function will keep growing. That means that we have achieved the optimal level of staff required that best fulfil FirstTreatment waiting standards.

Computation using MATLAB:

Question (e):

```
station=[6,10,15,2,15];
calc_a=[0.154,0.436,1.144,0.281,2.101]

calc_b=[0.024,0.083,0.394,0.083,0.594]
rm_1=zeros(1,5);rm_2=zeros(1,5);
for i=1:5
    rm_1(i)=rmse(calc_a(i),station(i));
    rm_2(i)=rmse(calc_b(i),station(i));
end
rm_1'
rm_2'
sum(rm_1)
sum(rm_2)
```

```
function y = rmse(obtained,expected)
x=obtained;y=expected;
y = sqrt((x-y)^2)/y;
end
```

```
calc_a = 1×5
    0.1540    0.4360    1.1440    0.2810 ...

calc_b = 1×5
    0.0240    0.0830    0.3940    0.0830 ...

ans = 5×1
    0.9743
    0.9564
    0.9237
    0.8595
    0.8599

ans = 5×1
    0.9960
    0.9917
    0.9737
    0.9585
    0.9604

ans = 4.5739
ans = 4.8803
```