Decision Analysis 2: Value of Information

ISE/OR 560, Fall 2022

In the previous lecture

We know how to make decisions with perfect information.

- Determine the best portfolio selection for the investor.

Event	Portfolio Selection A	B	C	Prob.
Economy Declines	$500	-$2,000	-$7,000	0.30
No change	$1,000	$2,000	-$1,000	0.50
Economy Expands	$2,000	$5,000	$20,000	0.20

Perfect Information

- Perfect information is complete foresight of "which state of nature is really going to occur"
- **Sequence of events:**
  - Obtain perfect information
  - Now that we know what state of nature is going to happen, pick the best alternative for that state of nature
- Usually not realistic, but the value of perfect information provides a bound on the value of any information

Example: Real Estate Investment

- Investor choosing between 3 real estate investments, with 2 states of nature.
- Based on forecasts, the investor is able to estimate a 0.60 probability that good economic conditions will prevail and a 0.40 probability that poor economic conditions will prevail

Decision (alternatives)	States of nature		
	Good econ. (j=1)	Bad econ. (j=2)	
Apt. bldg. (a_1)	50	30	
Office bldg. (a_2)	100	-40	
Warehouse (a_3)	30	10	
Probability			

Perfect Information Example 1

- Now suppose we knew which type of economy was going to occur, and we choose best investment for each
- Calculate EMV with perfect information

  EMV (with PI)=
- EVPI (Expected Value of Perfect Information): **EVPI =**

Decision (alternatives)	States of nature		EMV
	Good econ.	Bad econ.	
Apt. bldg.	50	30	
Office bldg.	100	-40	
Warehouse	30	10	
Probability	.60	.40	
Best alt., given state of nature			

Perfect Information and EVPI

- Expected Value of Perfect Information

  EVPI = EMV with perfect information

  - EMV with no information

- Maximum we'd be willing to pay for any kind of information (Consultant, Market research, Trade insider, etc)

- Related: EVSI: Expected Value of Sample (or Imperfect) Information

Sample

Often, there will be imperfect information available:

Expert forecasts

Market research

Consultants

Simulation.....

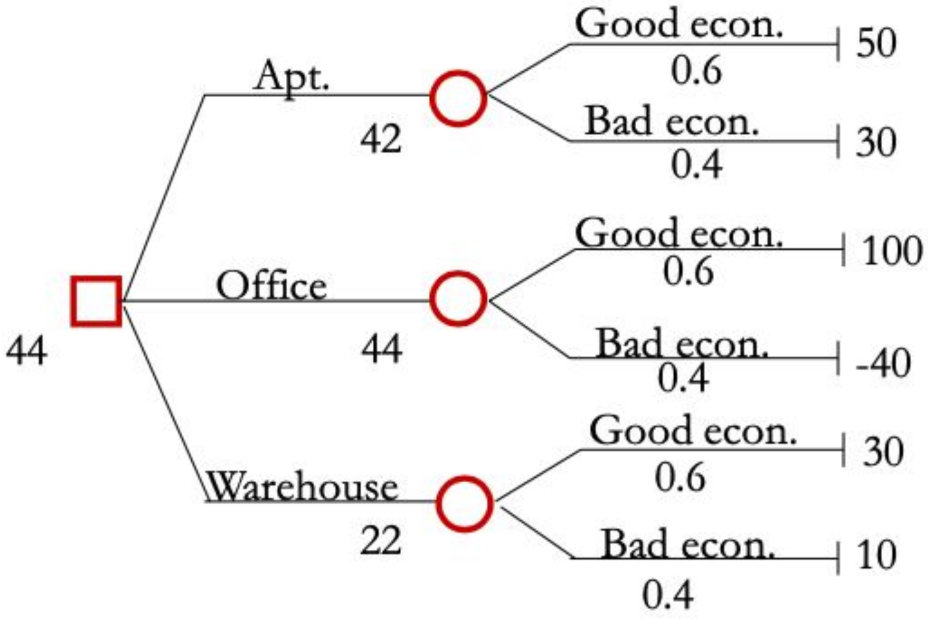
Question: How best to use sample info, and how much to pay for it?

Sample/Imperfect Information: Example 1

- **Economic analyst provides ratings on economy:**
  P (positive) or N (negative)
- **Sequence of events:**
  1. Investor decides whether to use analyst info.
  2. Analyst provides rating (P or N)
  3. Investor decides where to invest
  4. Economy turns out to be good or bad

Decision Tree without Analyst info.

- Without analyst info. (Tree A)



How do we decide the information value

- What probabilities do we need?
  - Marginal: probability of sample information outcomes
  - Posterior probability: updated probabilities of events with new information

Prior Belief

Given: **Prior**

- Probabilities assigned to each state of nature: $P(S_1)$, $P(S_2)$, …
- Probabilities determined prior to acquiring additional information

How Good is the Sample Information?

Given: Likelihood (conditional probability)

Conditional probability indicates accuracy of sample information

Probability of observing a sample outcome, given a state of nature

Known and Unknown Probabilities

- What probabilities do we already have?
  - **Prior:** prob. of states of nature before new information

    P(Good**), P(B**ad**)**
  - **Likelihood**: Accuracy of new information

    P(P|G), P(N|G), P(P|B), P(N|B)
- What probabilities do we need?
  - **Marginal:** Prob. of sample information type

    P(Positive), **P(N**egative)
  - **Posterior:** prob. of states of nature after new information

    P(Good|**P**ositive**)**, **P(B|P)**, **P(G|N)**, **P(B|N)**

Marginal Probabilities: Total Probability Rules

Marginal

- Probability of sample outcomes
- Probability of positive report, probability of negative report

Posterior: Bayes' rule

Conditional probability of state of nature given sample information

Calculating Posterior

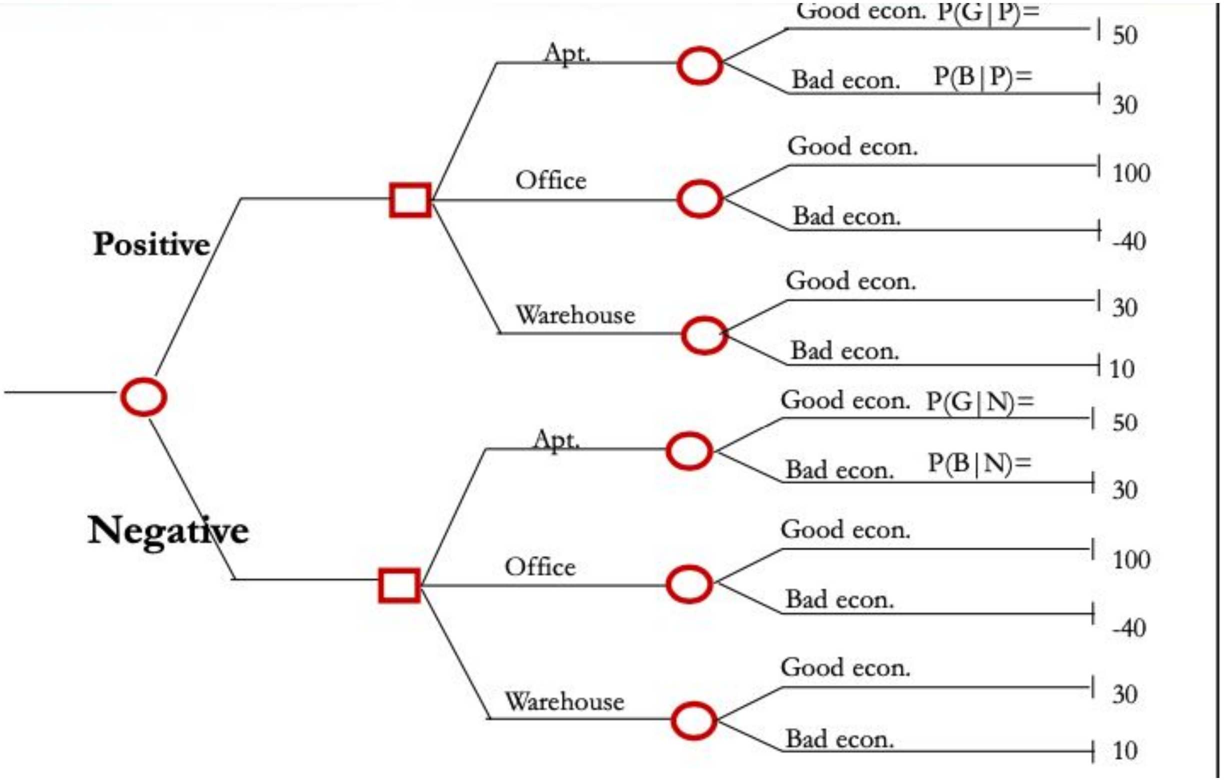
- If analyst report is **Positive**

S.O.N	Priors	Likelihoods	Joint	Posterior
Good	P(G)=	P(P|G)=	$P(P\cap G)$=	P(G|P)=
Bad	P(B)=	P(P|B)=	$P(P\cap B)$=	P(B|P)=
			P(P)=	

- If analyst report is **Negative**

S.O.N	Priors	Likelihoods	Joint	Posterior
Good	P(G)=	P(N|G)=	$P(N\cap G)$=	P(G|N)=
Bad	P(B)=	P(N|B)=	$P(N\cap B)$=	P(B|N)=
			P(N)=	

Tree B Analyst Information



EVSI: Expected Values of Sample Information

EMV(w.o. info) = $44,000 (the optimal action: office)

In real estate example,
If analyst is positive, invest in ( ) with EMV =
If analyst is negative, invest in ( ) with EMV =

EMV(with samp. info) =

EVSI: Maximum amount to pay for the sample information
EVSI =

Value of Sample Information

Original states of nature $S_1, S_2, \ldots S_n$

Sample information $I_1, I_2, \ldots I_k$

Key idea: Sample information updates probabilities of states of nature.

Priors — **Bayes' rule** → **Posterior**

Original $P(S_j)$ → After info. I_k: $P(S_i|I_k)$

Solve a decision tree with updated probabilities.

If sample information was perfect (no error), then

Generally, sample information is imperfect.

Therefore,