

4.

a)  $E$ : selected card is an ace  
 $F$ : selected card is a spade

$$P(E) = 4/52 = \frac{1}{13} \quad \text{and} \quad P(F) = 4/52 = \frac{1}{13}$$

$$\text{Since } E \cap F = \emptyset \Rightarrow P(E \cap F) = 0 \neq P(E) \cdot P(F) = \frac{1}{169}$$

$\therefore E$  and  $F$  are not independent

b)  $E$ : first coin lands head  
 $F$ : second coin lands tails

$$\Omega = \{HH, HT, TH, TT\}$$

$$E = \{HH, HT\} \Rightarrow P(E) = \frac{1}{4} + \frac{1}{4} = \frac{2}{4} = \frac{1}{2} = P(E)$$

$$F = \{HT, TT\} \Rightarrow P(F) = \frac{1}{2}$$

$$\text{Note that } F \cap E = \{HT\} \Rightarrow P(F \cap E) = \frac{1}{4}$$

$$\text{and } P(F) \cdot P(E) = \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4} = P(F \cap E)$$

$\therefore E$  and  $F$  are independent