ISE/OR 560: Stochastic Models in Industrial Engineering

Homework 0 – Probability Review
Due Friday, Sept. 9th at 11:59pm

This homework is to be completed ***individually*** and the score will be **based on the problems attempted**, not the problems that are correctly answered. *Note: Problem 12 is required for credit.*

1. Brownstown Central High School has 500 seniors (300 boys and 200 girls). The number who pass and fail the final exam in a particular course is given below:

   Number of students

	Fail	Pass	Total
Boys	28	272	300
Girls	10	190	200

   If one of the 500 seniors is chosen at random (i.e., if each senior has a 1/500 chance of being chosen), D and A denote the events that the senior fails or passes, respectively; while B and G denote the events that the senior is a boy or girl. Calculate each of the following probabilities:

   a. P (D)

   b. P (A or B)

   c. P (A and B)

   d. P (B|A)

   e. P (D|G)

   f. P (A| not G)

   g. Is the event denoted by D independent of the event denoted by G? Why, or why not?

   h. Is the event denoted by A independent of the event denoted by G? Why, or why not?

2. A machine repair can be performed either on time or late, and either satisfactorily or unsatisfactorily. The probability of a repair being both on time and satisfactory is 0.26. The probability of a repair being on time is 0.76. The probability of a repair being satisfactory is 0.41.

   a. What is the probability of a repair being both late and unsatisfactory?

   b. What is the probability that a repair is on time, given that the repair is satisfactory?

3. Seventy percent of the light aircraft that disappear while in flight in a certain country are subsequently discovered. Of the aircraft that are discovered, 60% have an emergency locator, whereas 90% of the aircraft not discovered do not have such a locator. Suppose a light aircraft has disappeared.

   a. If it has an emergency locator, what is the probability that it will not be discovered?
   b. If it does not have an emergency locator, what is the probability that it will be discovered?

4. Determine whether the events in the following situations are independent:

   a. In a normal 52 card deck pick one card at random. Let E be the event that the selected card is an ace and let F be the event that it is a spade.

   b. Two fair coins are flipped. Let E be the event that the first coin lands heads. Let F be the event that the second lands tails.

   c. Suppose two fair dice are tossed. Let E denote the event that the sum of the dice is 7 and let F denote the event that the first die equals 4.

5. If $P(A|B) = 1$, must $A = B$? Use a Venn diagram to explain your answer.

6. A batch of 500 containers of orange juice contains 5 that are past the expiration date. Two are selected at random, without replacement, from the batch.

   a. What is the probability that the second one selected is expired given that the first one was expired?

   b. What is the probability that both are expired?

   c. What is the probability that neither are past expiration?

7. Suppose that a table of part counts is as follows:

		Conforms? Yes	No
Supplier	**1**	ka	kb
	2	a	b

Where a, b, and k are positive integers. Let A denote the event that a part is from supplier 1 and let B denote the event that a part conforms to specifications.

a. What is $P(A|B)$?
b. What is $P(\bar{B}|A)$?
c. Are A and B independent? Explain mathematically.

8. Customers are used to evaluate preliminary product designs. In the past, 95% of highly successful products received good reviews, 60% of moderately successful products received good reviews, and 10% of poor products received good reviews. In addition, 40% of products have been highly successful, 35% have been moderately successful, and 25% have been poor products.

a. What is the probability that a product attains a good review?

b. If a new design attains a good review, what is the probability that it will be a highly successful product?

c. If a product does not attain a good review, what is the probability that it will be a highly successful product?

9. A card is drawn from a standard pack of 52 well-shuffled cards (which contains 4 kings). What is the probability that it is a king? Without replacing this first king card, a second card is drawn. What is the probability that the second card pulled is a king? What is the probability that the first four cards drawn from a standard deck of 52 well-shuffled cards are all kings? Once drawn, a card is not replaced in the deck.

10. A card is drawn at random from a standard deck of 52 well-shuffled cards. Let A be the event that the card drawn is a queen and let B the event that the card pulled is red. Find the probabilities of the following events and state in words what they represent.

    a. $A \cap B$
    b. $A \cup B$

11. A factory has three machines that manufacture widgets. The percentages of a total day's production manufactured by the machines are 10%, 35%, and 55%, respectively. Furthermore, it is known that 5%, 3%, and 1% of the outputs of the respective three machines are defective. What is the probability that a randomly selected widget at the end of the day's production runs will be defective?

12. ***Required Problem***: Find one example of a stochastic process in practice and summarize the example on a single powerpoint slide. On the slide: (1) Briefly describe the example, (2) explain what is uncertain, (3) explain why you think it is a stochastic process. This example can be from industry, it can be from the newspaper, it can be something that you have personally experienced, etc.