Stochastic System: What and Why

Hong Wan, Fall 2022

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Definition

A stochastic model represents a situation where **uncertainty** is present. In other words, it's a model for a process that has some kind of randomness.

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https://www.random.org/dice/

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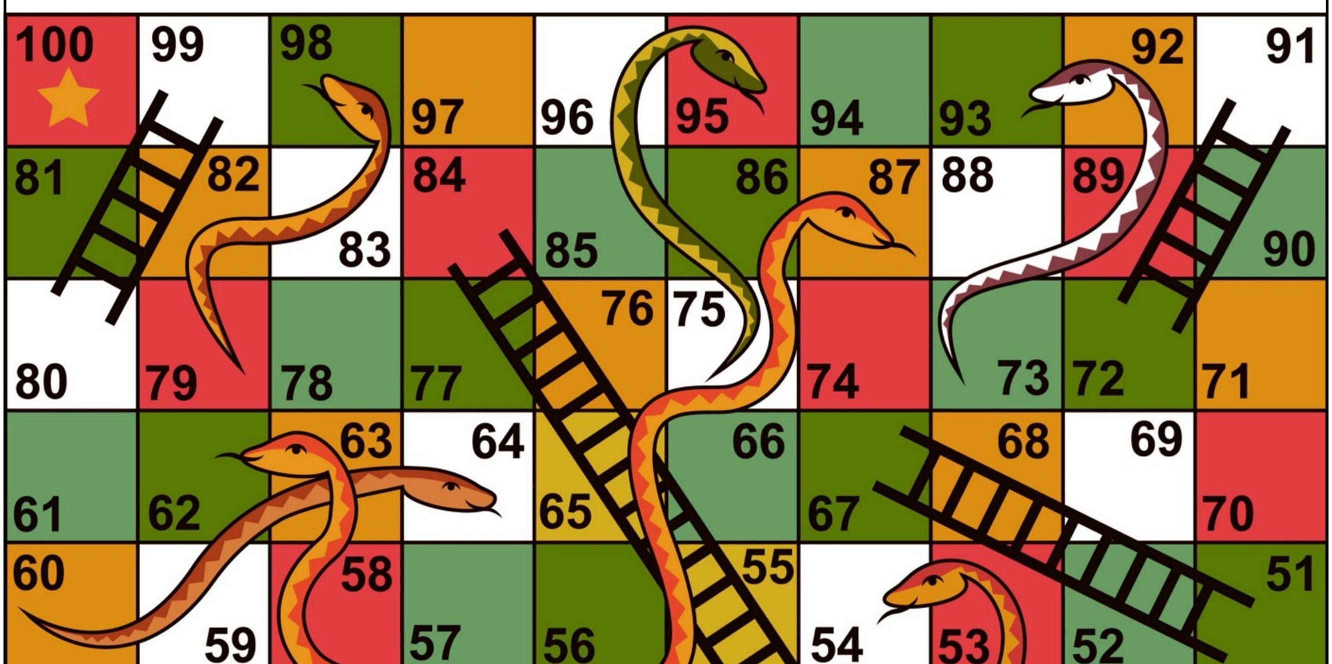

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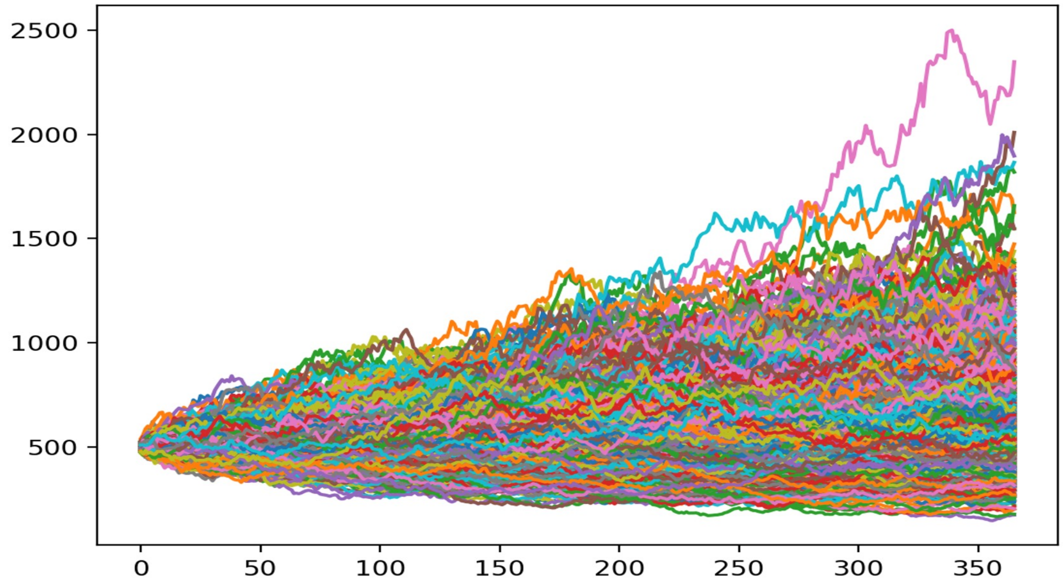

Towards Data ScienceGeometric Brownian Motion. A stochastic, non-linear process to… | by Roman Paolucci | Towards Data Science

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A Simulation Example

- Two components work as an active and spare, so the system fails if both are failed simultaneously.
- The lifetime of a component is equally likely 1, 2, 3, 4, 5 or 6 days.
- Repair takes exactly 2.5 days (only one repair at a time); the repaired component becomes the spare.
- What can we say about the time to failure (TTF) for this system?
- Discrete-event simulation updates the **state** of the system when **events** happen.
  - The *state* of the system is the number of functional components.
  - The *events* are the failure of a component and the completion of a repair.

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Clock	System State	Next Failure	Next Repair
0	2	3	∞
3	1	3+5=8	3+2.5=5.5
5.5	2	8	∞
8	1	8+3=11	8+2.5=10.5
10.5	2	11	∞
11	1	11+1=12	11+2.5=13.5
12	0	∞	13.5

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Clock	System State	Next Failure	Next Repair
0	2		∞

Your turn!
But let's do it with a spreadsheet & psuedorandom dice

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Outputs

- *Replications* are statistically independent repetitions of the model.
- We distinguish between *within-replication* and *across-replication* output data.
  - Your time of system failure *Y* and the number of functional components $\{S(t);\ t \geq 0\}$ are *within-replication* outputs.
  - Our times of system failure $Y_1, Y_2, \ldots, Y_n$, and the average number of functional components, $\bar{S}_1, \bar{S}_2, \ldots, \bar{S}_n$ from *n* replications are *across-replication* outputs.
- Notice that $\bar{S}$ is a *time-average* because $S(t)$ is a continuous-time output variable.

$$\bar{S} = \frac{1}{Y}\int_0^Y S(t)dt$$

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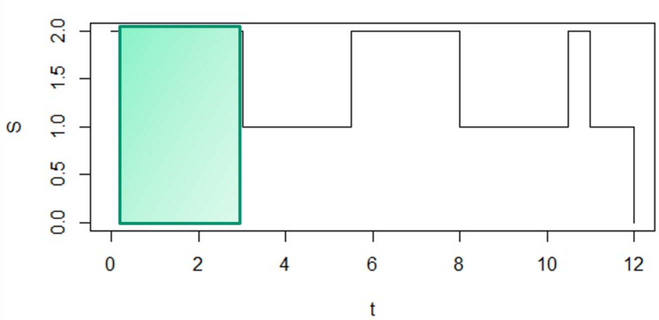
Time Averages

No calculus required!



$$\bar{S} = \frac{1}{Y}\int_0^Y S(t)dt$$

Average number of functional components

$$= \frac{1}{12}\left[\begin{array}{l}\boxed{2(3-0)} + 1(5.5-3) + 2(8-5.5) + \\ 1(10.5-8) + 2(11-10.5) + 1(12-11)\end{array}\right] = \frac{17}{12}$$

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Steady-State Simulation

- The "average number of functional components up to the first system failure" is not very meaningful.
- But if we let the simulation continue, then the system comes back up and starts functioning again.
- The "long-run average number of functional components" ***is*** a meaningful summary measure.
- But how do we let $T \rightarrow \infty$ in a simulation?

$$\lim_{T\to\infty} \frac{1}{T}\int_0^T S(t)dt$$

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