

7). A: part is from supplier 1, B: part conforms to specifications.

$$a) P(A|B) = ?$$

$$\Rightarrow P(A|B) = \frac{P(B|A) \cdot P(A)}{P(B)}, \quad \cancel{P(B|A) = k_a} \quad P(B \cap A) =$$
$$P(A) = k_a + k_b$$
$$P(B) = k_a + a$$
$$= \frac{P(A \cap B)}{P(B)}$$

$$\Rightarrow P(A|B) = \frac{P(B \cap A)}{P(B)} = \frac{k_a}{k_a + a},$$

$$\therefore P(A|B) = \frac{k_a}{k_a + a}$$

$$b) P(\bar{B}|A) = \frac{P(A|\bar{B}) \cdot P(\bar{B})}{P(A)} = \frac{P(A \cap \bar{B})}{\cancel{P(A)} P(A)}$$

$$\text{and } P(A \cap \bar{B}) = k_b, P(A) = k_a + k_b$$

$$\Rightarrow P(\bar{B}|A) = \frac{k_b}{k_a + k_b}$$

$$\therefore P(\bar{B}|A) = \frac{k_b}{k_a + k_b}$$

c) If A and B are independent, then $P(A|B) = P(A)$

note that
$$P(A|B) = \frac{\kappa a}{\kappa a + a} = \frac{\kappa a}{a(\kappa + 1)} = \frac{\kappa}{\kappa + 1}$$

and
$$P(A) = \kappa a + \kappa b = \kappa(a + b)$$

$$\Rightarrow P(A|B) = \frac{\kappa}{\kappa + 1} \neq \kappa(a + b) = P(A)$$

\therefore Events A and B are not independent.