

3.4)

$X$  is a random variable.  $X$  can take any value of  $\{-2, -1, 0, 1, 2\}$ . We have:  $P\{-1 < X < 2\} = 0.4$

$$P\{X=0\} = 0.3$$

$$P\{|X| \leq 1\} = 0.6$$

$$P\{X > 2\} = P\{X=1 \text{ or } X=-1\}$$

a) PMF of  $X$ :

$$P(-1 < X < 2) = P(X=0 \cup X=1) = P(X=0) + P(X=1) = 0.4$$

$$\Rightarrow P(X=1) = 0.4 - P(X=0) = 0.4 - 0.3 = 0.1 \therefore P(X=1) = 0.1$$

$$P(|X| \leq 1) = P(X=-1 \cup X=0 \cup X=1) = P(X=-1) + P(X=0) + P(X=1) = 0.6$$

$$\Rightarrow P(X=-1) = 0.6 - P(X=0) - P(X=1) = 0.6 - 0.3 - 0.1 = 0.6 - 0.4 = 0.2$$

$$\therefore P(X=-1) = 0.2$$

$$P(X > 2) = P(X=2) = P(X=1 \cup X=-1) = P(X=1) + P(X=-1) = 0.1 + 0.2 = 0.3$$

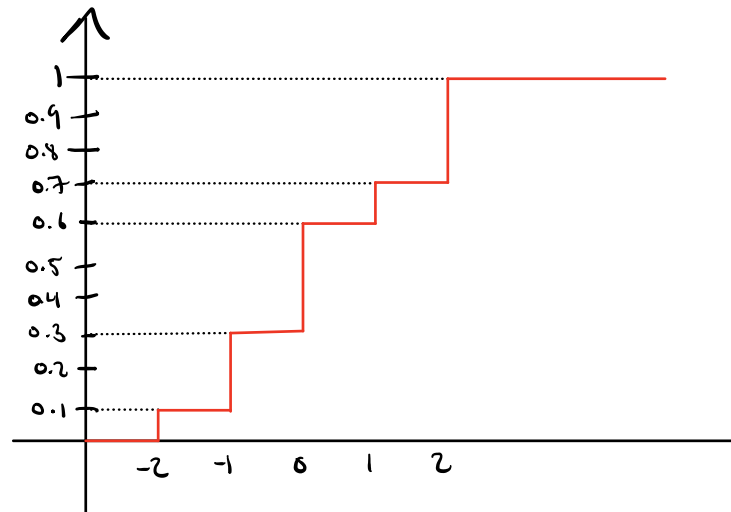
$$\therefore P(X=2) = 0.3$$

$$P(X=-2) = 1 - P(X \neq -2) = 1 - (0.3 + 0.3 + 0.2 + 0.1) = 1 - 0.9 = 0.1$$

$$\therefore P(X=-2) = 0.1$$

Then, the distribution function of  $X$  is  $f_X(x) = \begin{cases} 0.1, & x = -2 \\ 0.2, & x = -1 \\ 0.3, & x = 0 \\ 0.1, & x = 1 \\ 0.3, & x = 2 \end{cases}$

b) Graph CDF of  $X$ :



$$c) E[X] = \sum_{i=-2}^2 x_i \cdot P(X=x_i) = \sum_{i=-2}^2 i \cdot P(X=i)$$

$$= -2 \cdot (0.1) - 1 \cdot (0.2) + 1 \cdot (0.1) + 2 \cdot (0.3) = 0.3$$

$$\therefore E[X] = 0.3$$