

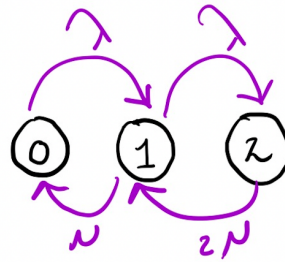
### Problem 1

(a)

The state space  $S=\{0,1,2\}$ .

(b)

Let  $\{X(t), t \geq 0\}$ .  $X$ : # of calls in the system



(c)

$$R = \begin{matrix} i \backslash j & 0 & 1 & 2 \\ \begin{matrix} 0 \\ 1 \\ 2 \end{matrix} & \begin{matrix} 0 \\ \mu \\ 0 \end{matrix} & \begin{matrix} \lambda \\ 0 \\ 2\mu \end{matrix} & \begin{matrix} 0 \\ \lambda \\ 0 \end{matrix} \end{matrix}$$

(d)

Arrival rate $\lambda$ and Service rate $\mu$	Values of $r_i = \sum_{j=0}^2 r_{i,j}$	System of Equations
$\lambda = \frac{9}{hr}; \mu = \frac{5}{hr}$	$r_0 = \lambda = 9$ $r_1 = \mu + \lambda = 14$ $r_2 = 2\mu = 10$	$p_0 + p_1 + p_2 = 1$ $p_0 r_0 = p_1 \mu$ $p_1 r_1 = p_0 \lambda + p_2 2\mu$ $p_2 r_2 = p_1 \lambda$

Using MATLAB to compute the steady state probabilities for state 0,1,and 2:

$$p_0 = 0.2262$$

$$p_1 = 0.4072$$

$$p_2 = 0.3665$$

Code:

```

lambda=9;
mu=5;
B=[1;0;0]
A = [1,1,1;lambda,-mu,0;-lambda,(mu+lambda),-2*mu]
p=A\B

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**(e)**

Utilization of the 2 technicians is 0.57. That means, we are using 57% of the system capacity.

**(f)**

The probability that both technicians are idle is  $P_0 = 0.226$ .

**(g)**

The probability that a caller receives a busy signal is the same as the probability of both technicians busy = 0.367.

**(h)**

$P(4) =$

**B = 3x3**

0.2262	0.4072	0.3665
0.2262	0.4072	0.3665
0.2262	0.4072	0.3665

Then the probability that both technicians are busy at 12:00 pm the same day is 0.3665

Code attached after exercise (i).

**(i)**

The long run expected number of calls in the system is  $L = 1.14$  calls.