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HW1

3.5) X is a discrete random variable with probability distribution:

$\cdot P\{X = x_1\} = \frac{1}{4}$ and $P\{X = x_2\} = \frac{3}{4}$

a) find x_1, and x_2 so that $E[X] = 0$ and $var(X) = 10$.

$$E[X] = \sum_{i=1}^{2} x_i \cdot P(X = x_i) = x_1 \cdot \left(\frac{1}{4}\right) + x_2\left(\frac{3}{4}\right) = \frac{x_1}{4} + \frac{3x_2}{4} = 0$$

$$\Rightarrow \frac{x_1}{4} = \frac{-3x_2}{4} \Rightarrow \boxed{x_1 = -3x_2}$$

$$var(X) = cov(X,X) = E[X \cdot X] - \cancel{E[X] \cdot E[X]}^{0} = E[XX]$$

and $E[XX] = \sum_{i=1}^{2}\sum_{j=1}^{2} x_i \cdot x_j \, P(X = x_i, X = x_j)$

$$= x_1^2 P(X = x_1) + x_2^2 P(X = x_2) = x_1^2 \cdot (1/4) + x_2^2 (3/4) = 10$$

$$\Rightarrow \frac{3x_2^2}{4} = 10 - \frac{x_1^2}{4} \Leftrightarrow 3x_2^2 = 40 - x_1^2 = 40 - (-3x_2)^2$$

$$\Rightarrow 3x_2^2 = 40 - 9x_2^2 \Leftrightarrow 3x_2^2 + 9x_2^2 = 40 \Leftrightarrow 12x_2^2 = 40 \Leftrightarrow x_2 = \pm\sqrt{\frac{40}{12}}$$

$$\Rightarrow x_2 = \pm\sqrt{\frac{20}{6}} = \pm\sqrt{10/3} \quad \therefore \boxed{x_2 = \pm\sqrt{10/3}}$$

if $x_2 = \sqrt{10/3}$ then $x_1 = -3\left(\sqrt{10/3}\right) = -3\sqrt{10/3}$

$\Rightarrow (x_1, x_2) = \left(-3\sqrt{10/3}, \sqrt{10/3}\right)$. If $x_2 = -\sqrt{10/3}$

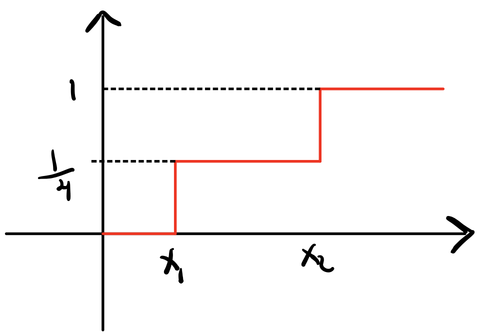
$\Rightarrow \quad x_1 = -3x_2 = -3\left(-\sqrt{10/3}\right) = 3\sqrt{10/3} \Rightarrow (x_1, x_2) = \left(3\sqrt{10/3}, -\sqrt{10/3}\right)$

$\therefore \left(x_1 = 3\sqrt{10/3}, x_2 = -\sqrt{10/3}\right)$ or $\left(x_1 = -3\sqrt{10/3}, x_2 = \sqrt{10/3}\right)$

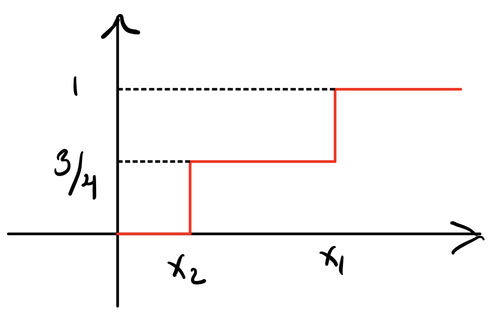
b)

Sketch the CDF of X

Case 1 $\quad x_1 < x_2$:



Case 2 : $\quad x_2 < x_1$:



6.

X: life in hours of a certain kind of radio tube

Density function:

$$f_X(y) = \begin{cases} 100/y^2, & y \geq 100 \\ 0, & y < 100 \end{cases}$$

a) $P(X \geq 250\text{hrs}) = ?$

$$\int_{250}^{\infty} \frac{100}{y^2}\, dy = -\frac{100}{y}\Big|_{250}^{\infty} = -\cancel{\frac{100}{\infty}}^{\,0} + \frac{100}{250} = 0.40$$

$\therefore P(X \geq 250) = 0.40$ or 40%

b) $$E[X] = \int_{100}^{\infty} y \cdot f_X(y)\, dy = \int_{100}^{\infty} y \cdot \frac{100}{y^2}\, dy = \int_{100}^{\infty} \frac{100}{y}\, dy$$

$$= 100\ln(y)\Big|_{100}^{\infty} = 100\ln(\infty) - 100\ln(100) = \infty$$

3.7)

X is a random variable. X can take any value of $\{-2,-1,0,1,2\}$. We have: $P\{-1 < X < 2\} = 0.4$

$P\{X=0\} = 0.3$

$P\{|X| \leq 1\} = 0.6$

$P\{X \geq 2\} = P\{X=1 \text{ or } X=-1\}$

a) PMF of X:

$P(-1 < X < 2) = P(X=0 \cup X=1) = P(X=0) + P(X=1) = 0.4$

$\Rightarrow P(X=1) = 0.4 - P(X=0) = 0.4 - 0.3 = 0.1 \quad \therefore P(X=1) = 0.1$

$P(|X| \leq 1) = P(X=-1 \cup X=0 \cup X=1) = P(X=-1) + P(X=0) + P(X=1) = 0.6$

$\Rightarrow P(X=-1) = 0.6 - P(X=0) - (X=1) = 0.6 - 0.3 - 0.1 = 0.6 - 0.4 = 0.2$

$\therefore P(X=-1) = 0.2$

$P(X \geq 2) = P(X=2) = P(X=1 \cup X=-1) = P(X=1) + P(X=-1) = 0.1 + 0.2 = 0.3$

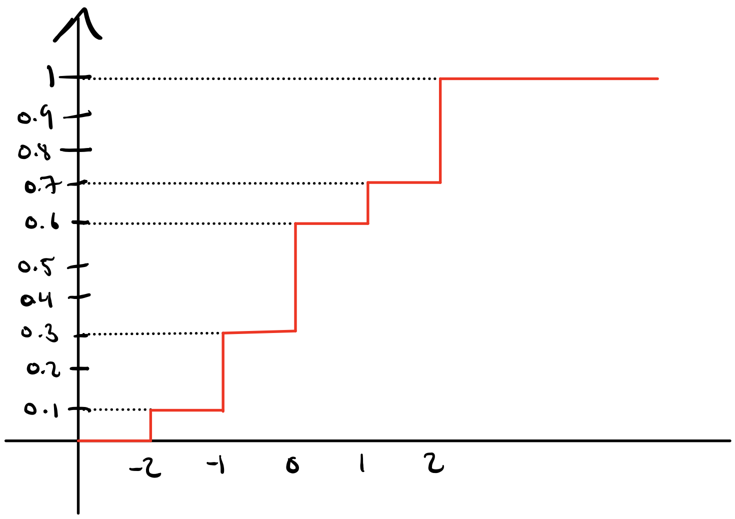
$\therefore P(X=2) = 0.3$

$P(X=-2) = 1 - P(X \neq -2) = 1 - (0.3 + 0.3 + 0.2 + 0.1) = 1 - 0.9 = 0.1$

$\therefore P(X=-2) = 0.1$

Then, the distribution function of X is $f_X(x) = \begin{cases} 0.1, & x = -2 \\ 0.2, & x = -1 \\ 0.3, & x = 0 \\ 0.1, & x = 1 \\ 0.3, & x = 2 \end{cases}$

b) Graph CDF of X:



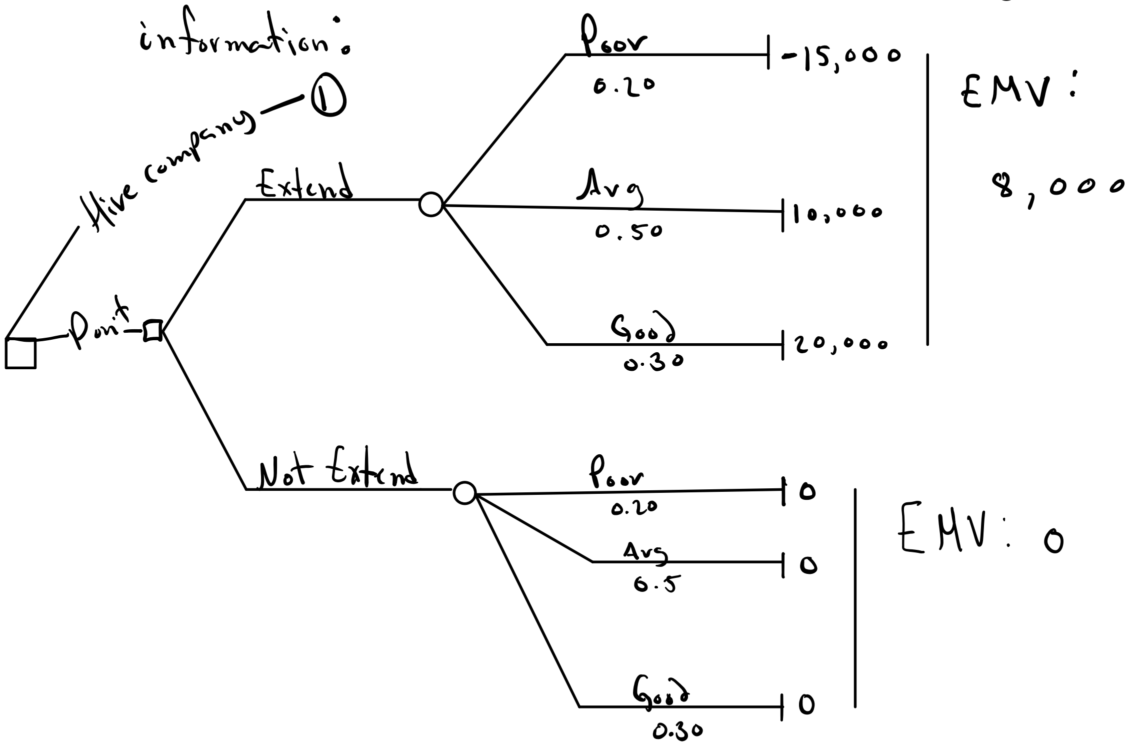
c) $E[X] = \sum_{i=-2}^{2} x_i \cdot P(X=x_i) = \sum_{i=-2}^{2} i \cdot P(X=i)$

$= -2\cdot(0.1) - 1(0.2) + 1(0.1) + 2(0.3) = 0.3$

$\therefore E[X] = 0.3$

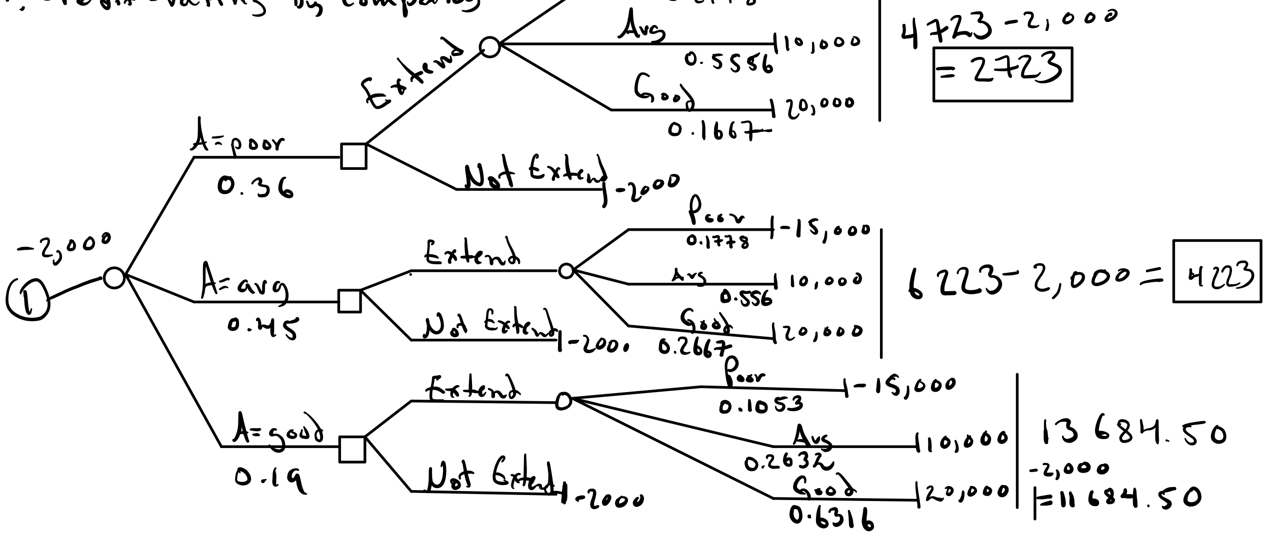
14.3 $P(\text{poor}) = 0.20$, $P(\text{avg}) = 0.50$, $P(g) = 0.30$

a) Decision tree without credit-rating organization information:



Decision tree with Credit-Rating Organization info:

A: credit-rating by company



Calculating probabilities: Let good = g , average = avg , poor = p

$$P(A=p) = P(A=p \cap p) + P(A=p \cap avg) + P(A=p \cap good)$$
$$P(A=p = P(A=p|p) \cdot P(p) + P(A=p|avg) \cdot P(avg) + P(A=g|g) \cdot P(g)$$
$$P(A=p) = (0.50) \cdot (0.20) + (0.40)(0.50) + (0.20)(0.30) = 0.36$$

For $P(A=avg)$:

$$P(A=avg) = P(A=avg|p) \cdot P(p) + P(A=avg|avg) \cdot P(avg) + P(A=avg|g) \cdot P(g)$$
$$P(A=avg) = (0.40)(0.20) + (0.50)(0.50) + (0.40)(0.30) = 0.45$$
$$\therefore P(A=avg) = 0.45$$

for $P(A=g)$:

$$P(A=g|p) \cdot P(p) + P(A=g|avg) P(avg) + P(A=g|g) \cdot P(g)$$
$$= (0.10)(0.20) + (0.10)(0.50) + (0.40)(0.3) = 0.19$$

- $P(poor|A=poor) = \dfrac{P(A=p|p) \cdot P(p)}{P(A=p)} = \dfrac{(0.5)(0.20)}{0.36} = 0.277\overline{8}$
- $P(avg|A=p) = \dfrac{P(A=p|avg) \cdot P(avg)}{P(A=p)} = \dfrac{(0.40)(0.50)}{0.36} = 0.555\overline{6}$
- $P(g|A=p) = \dfrac{P(A=p|g) \cdot P(g)}{P(A=p)} = \dfrac{(0.20)(0.30)}{0.36} = 0.166\overline{7}$

- $P(p|A=avg) = \dfrac{P(A=avg|p) \cdot P(p)}{P(A=avg)} = \dfrac{(0.40) \cdot (0.20)}{0.45} = 0.1778$
- $P(avg|A=avg) = \dfrac{P(A=avg|avg) \cdot P(avg)}{P(A=avg)} = \dfrac{(0.50) \cdot (0.5)}{0.45} = 0.5556$

- $P(g \mid A=avg) = \frac{P(A=avg \mid g) \cdot P(g)}{P(A=avg)} = \frac{(0.40)(0.30)}{0.45} = 0.2667$

- $P(p \mid A=g) = \frac{P(A=g \mid p) \cdot P(p)}{P(A=g)} = \frac{(0.10)(0.20)}{0.19} = 0.1053$
- $P(avg \mid A=g) = \frac{P(A=g \mid avg) \cdot P(avg)}{P(A=g)} = \frac{(0.10)(0.5)}{0.19} = 0.2632$
- $P(g \mid A=g) = \frac{P(A=g \mid g) \cdot P(g)}{P(A=g)} = \frac{(0.40)(0.30)}{0.19} = 0.6316$

(c) Optimal policy:

Pay the credit-rating company and extend credit to the dress manufacturer.

(d)

$EMV(\text{No info}) = 8000$

$EMVPI = (0.5) \cdot 10{,}000 + 0.30(20000)$

$= 11{,}000$

$11{,}000 - 8{,}000 = 3{,}000$

∴ The max we are willing to pay for perfect information is 3,000.

Prior Prob.

$P(A) = 0.5$

$P(B) = 0.5$

Posterior Dist $\left[P(A|P) = \frac{P(P|A)P(A)}{P(P)}\right]$

$P(A|P) = \frac{(0.8)(0.5)}{(0.5)} = 0.8$

$P(B|P) = \frac{P(P|B)P(B)}{P(P)} = \frac{(0.2)(0.5)}{(0.5)} = 0.2$

$P(A|N) = \frac{P(N|A)P(A)}{P(N)} = \frac{(0.2)(0.5)}{(0.5)} = 0.2$

$P(B|N) = 0.8$

$P(P|A) = 0.8$

$P(P|B) = 0.2$

$P(N) = P(N|A)P(A) + P(N|B)P(B)$

$P(P) = P(P|A)P(A) + P(P|B)P(B)$

$= (0.8)(0.5) + (0.2)(0.5)$

$= 0.4 + 0.1 = 0.5$

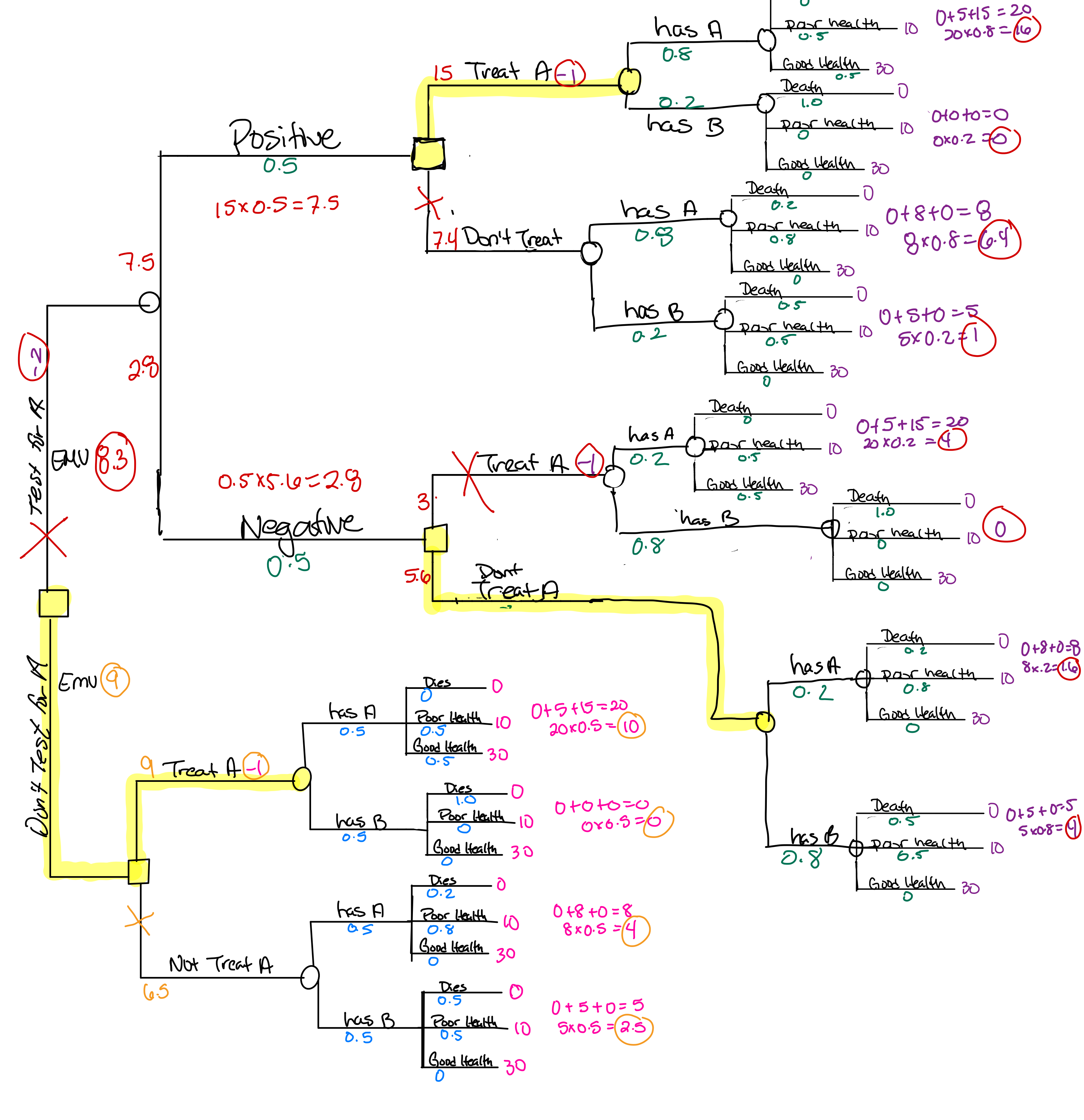
$1 = P(P) + P(N) \rightsquigarrow P(N) = 0.5$

$P(P|A) + P(N|A) = 1$

$0.8 + P(N|A) = 1$

$\rightsquigarrow P(N|A) = 0.2$

$P(A|P)P(P) = P(A \cap P)$



Test for A EUV : 8.3

Don't Test EUV : 9

Treat : 9

Don't treat : 6.5

The patient should not test for disease A. Then, the patient should receive treatment for disease A.