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HWO

i. $N = 500$ seniors. 300 boys + 200 girls

Events: D: senior fails
A: senior passes

B: senior is a boy
G: senior is a girl

$$a) P(D) = P(\text{senior fails}) = \frac{28+10}{500} = \frac{38}{500} = 0.076$$

$$\begin{aligned} b) P(A \text{ or } B) &= P(A) + P(B) - P(A \cap B) \\ &= \frac{272+190}{500} + \frac{300}{500} - \frac{272}{300} \\ &= \frac{490}{500} = 0.98 \end{aligned}$$

$$c) P(A \cap B) = P(\text{senior passes} \cap \text{senior is a boy}) = \frac{272}{500} = 0.544$$

$$d) P(B|A) = P(\text{senior is a boy} \mid \text{senior passes})$$

$$= \frac{P(B \cap A)}{P(A)} = \frac{272/500}{462/500}$$

$$= \frac{272}{462} \approx 0.59$$

$$e) P(D|G) = \frac{P(D \cap G)}{P(G)} = \frac{10/500}{200/500} = \frac{10}{200} = \frac{1}{20} = 0.05$$

$$f) P(A \mid \text{not } G)$$

$G = \text{senior is a girl} \Rightarrow \sim G = B = \text{senior is a boy}$

$$P(A \mid \sim G) = P(A \mid B)$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{272/500}{300/500} = \frac{272}{300} \approx 0.91$$

g) D and G

cl_r D and G are independent events, then $P(D \cap G) = P(D) \cdot P(G)$

Note that $P(D \cap G) = 10/500 = 1/50 = 0.02$

$$\text{and } P(D) \cdot P(G) = \left(\frac{38}{500}\right) \cdot \left(\frac{200}{500}\right) = \frac{7600}{250,000} = \frac{76}{2,500} = 0.0304$$

$\Rightarrow P(D \cap G) \neq P(D) \cdot P(G) \therefore D \text{ and } G \text{ are not independent events.}$

• This means that if the event D or G occurs, it will affect the probability of the other event happening.
i.e. if D happens, then $P(G)$ changes, and vice versa.

h) are A and G independent events?

A and B are independent $\Leftrightarrow P(A|B) = P(A)$.

$$\begin{aligned} \text{Note that } P(A) &= \frac{462}{500} = 0.924 \quad \text{and } P(A|B) = \frac{P(A \cap B)}{P(B)} \\ &= \frac{272/500}{300/500} = \frac{272}{300} = 0.907. \therefore A \text{ and } B \text{ are not independent.} \end{aligned}$$

this means that $P(A|B) \neq P(A)$ and $P(B|A) \neq P(B)$

i.e. if event A occurs, $P(B)$ will change, and vice versa.