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HW3

Problem 1

Define $N(1) = N \sim Poisson(\lambda * 1) = Poisson(8)$, where N(1) is the maximum number of customers that will arrive to the store in an 1 hour interval.
Then, we have the following:

$$\Pr(N \geq 3) = 1 - \Pr(N = 2) - \Pr(N = 1) - \Pr(N = 0) = 1 - \Sigma_{i=0}^{2} e^{-8*1} * \frac{(8)^i}{i!} = 0.9866$$

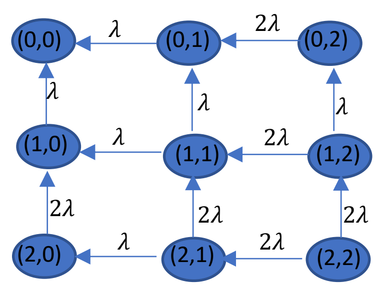
Problem 2

1. State space (given):
   S = {(0, 0), (0, 1), (0, 2), (1, 0), (1, 1), (1, 2), (2, 0), (2, 1), (2, 2)}.

2. States of the CTMC that implies a flight crash: (0,0) (0,1) (0,2) (1,0) (2,0).

3. Rate Matrix:

i|j	0,0	0,1	0,2	1,0	1,1	1,2	2,0	2,1	2,2
0,0	0	0	0	0	0	0	0	0	0
0,1	λ	0	0	0	0	0	0	0	0
0,2	0	2λ	0	0	0	0	0	0	0
1,0	λ	0	0	0	0	0	0	0	0
1,1	0	λ	0	λ	0	0	0	0	0
1,2	0	0	λ	0	2λ	0	0	0	0
2,0	0	0	0	2λ	0	0	0	0	0
2,1	0	0	0	0	2λ	0	λ	0	0
2,2	0	0	0	0	0	2λ	0	2λ	0



Problem 3

Since $X(0) = 4$ machines, we have that $\Pr(X(9) = 4|X(0) = 4) = 0.8971$ by the given transition probability matrix P(9) .

Both worker will be busy if the system is on state 0, 1 or 2. Calculating for the long-run fraction of the time that both workers will be busy is the same as calculating the proportion of time the system will be in state 0,1 or 2. Therefore, we have to calculate $p_j = \lim_{t\to\infty} P(X(t) = j)$ for $j = 0,1,2$, and then add them.

$$\lambda = \frac{1}{2}, \mu = \frac{1}{72}$$

$$R = \begin{array}{cccccc} i|j & 0 & 1 & 2 & 3 & 4 \\ 0 & 0 & 2\lambda & 0 & 0 & 0 \\ 1 & \mu & 0 & 2\lambda & 0 & 0 \\ 2 & 0 & 2\mu & 0 & 2\lambda & 0 \\ 3 & 0 & 0 & 3\mu & 0 & \lambda \\ 4 & 0 & 0 & 0 & 4\mu & 0 \end{array}$$ then we have the following: $$\begin{gathered} r_0 = 2\lambda \\ r_1 = \mu + 2\lambda \\ r_2 = 2\mu + 2\lambda \\ r_3 = 3\mu + \lambda \\ r_4 = 4\mu \end{gathered}$$

System of equations:

$(1) p_0 * r_0 = \Sigma_{i=0}^{4} p_i r_{i,0} = p_1 * \mu$

$(2) p_1 r_1 = p_0 2\lambda + p_2 2\mu$

$(3) p_2 r_2 = p_1 2\lambda + p_3 3\mu$

$(4) p_3 r_3 = p_2 2\lambda + p_4 4\mu$

$(5) p_4 r_4 = p_3 \lambda$

$(6) \Sigma p_i = 1$

Since we have N balanced equations, just N-1 will be independent. Therefore, to find each p_i we will use equations $(1) - (4)$ and equation $\Sigma p_i = 1$. Solving in MATLAB: