**Problem 1**

2. . We have that is a DTMC if the following properties are true:

(i)

(ii)

Claim 1: satisfies (i):

Proof:

It follows from the definition of on exercise 1 that for all n .

This proves 1.

Claim 2:

Proof:

Without loss of generality, assume .

Let Note that

Let and let .

Then:

, because is a series of independent and identically distributed random variables. Also note that for any , we can write in terms of .

Since are iid random variables, we also have

From

But

where

Therefore is a DTMC.

3. :

A picture containing calendar

Description automatically generated

Then we have the following for the transition matrix P:

Where

Note that is well defined since:

Case

If the inventory of is i, where then by exercise 1 we have that

in either case.

Since , and production happens after the demand is either met or not met, we have that for

Case :

Note that means going from to where . Note that this is impossible since if we have items in the inventory at time , then the demand is always met (since . Note that is a contradiction with the formula for , since if ,

then

which is a contradiction since .

Hence

Case

Then means going from to where is satisfied. Since

the demand at time n+1 is always met and .

By using the relation derived in problem 1, we have:

Then, by exercise 2, we have:

This proves claim 2.

**Problem 2**



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We have that is a DTMC if the following properties are true:

(i)

(ii)

Claim 1: satisfies (i):

Proof:

Since each is a nonnegative integer,

This proves claim 1.

Claim 2:

Proof:

If

Let .Then:

The above is equivalent to since

If , then .

Note is true, since the inequality is always true.

.(\*)

Similarly, (\*\*)

Hence by (\*) and by (\*\*) we get

This proves claim (ii)

1. Transition Probability Matrix:

Like problem 1, let

**Problem 3**

1. Show .

Claim 1: :

Proof:

The definition of = drug administered to patient X at time n,

Claim proved.

Claim 2:

Proof:

Let be any two drugs from the state space

Then (\*)

since the successive patients are independent of each other.

Similarly, .(\*\*)

This proves claim 2.

is a DTMC.

1. Derive the transition matrix.

In essence, we have the following:

**Problem 4**

For we have:

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Then for N=10 the transition probability P is:

Table

Description automatically generated

NOTE: since this matrix was generated with MATLAB, for then the location (i+1,j+1) in the matrix above will give you .

1. Since

For n=0:

since is given.

For n=5 transitions, have only five possible states i.e. .

Then

where the marginal probability is:

. But

By theory, we have is the marginal probability of .

Then, using MATLAB: