ISFA

Basket CDS pricing

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1 Introduction

The credit default swap is a simple derivative that revolutionized the transfer of credit risk. A credit default swap is a bilateral contract in which one party purchases protection from another party against losses from the default of a reference entity for a defined period of time. It can be seen as a tool to transfer credit risk from an entity to another. Like a CDS, a basket default swap is a derivative security tied to an underlying *basket* of corporate bonds or other assets subject to credit risk. Basket Default Swaps can be classified into nth-to-default, n-out-of-m-to-default and all-to-default. In this paper, we will focus on the Nth-to-default swap (NTD). In the first part we will derive the nth-to-default swap spread formula and deal with the problem of estimating default probabilities and hazard rates from CDS spreads using the model developed by JP Morgan. We will then analyse the impact of multiple factors on the valuation of the NTD swaps.

2 Bootstrapping hazard rates and survival probabilities

2.1 Credit Default Swap Spread

To obtain this protection, the CDS buyer makes regular payments, known as the **premium leg**, to the protection seller. These payments are made until a credit event occurs or until maturity, whichever occurs first[1]. The value of these premium is equal to the default swap spread. If a credit event occurs before maturity, a payment is made by the CDS seller, which is known as the **protection leg**.

The CDS spread is determined at the initial date so that the expected values of these two flows are equal by absence of arbitrage. We introduce some notations:

- $\tau \geq 0$ is the default time
- R is the recovery rate, which is either a random variable taking values in $[0, 1]$ or a stochastic process evaluated at the date τ.
- $s > 0$ is the CDS spread.
- T_0 is the initial date of the CDS and $T > 0$ the CDS maturity.
- $(T_1, \ldots, T_N = T)$ are the fixed payment dates, with $\Delta T = T_i - T_{i-1}$ constant for all $i = 1, \ldots, N$. We will suppose that $\Delta T = 1$ year.
- A function $\beta(t)$ is introduced, taking values in $\{1, \ldots, N\}$ such that $t \in [T_{\beta(t)-1}, T_{\beta(t)}]$.
- $r = (r_t, t \geq 0)$ is the short term interest rate and $D(t, T) = \exp\left(-\int_t^T r_s \, ds\right)$. the discount factor.

For the seller of the CDS, the future flow he will receive viewed at a date $t < \min(T, \tau)$ is given by:

[1] O'Kane, *Valuation of credit default swaps*, Lehman Brothers, 2003

$$s\left(\mathbf{1}_{\{\tau>T_{\beta(t)}\}}(T_{\beta(t)}-t)D(t,T_{\beta(t)})+\sum_{i=1}^{N}\mathbf{1}_{\{\tau>T_i\}}\Delta T D(t,T_i)+\mathbf{1}_{\{\tau\leq T\}}(\tau-T_{\beta(t)-1})D(t,\tau)\right)$$

If we approximate this by a continuous flow: $\int_t^T S\mathbf{1}_{\{\tau>u\}}D(t,u)\,du$. To calculate the CDS spread at the initial date: at $t=0$, we have

$$\mathbb{E}^*\left[\int_0^T s\mathbf{1}_{\{\tau>u\}}D(0,u)\,du\right]=\mathbb{E}^*\left[\mathbf{1}_{\{\tau\leq T\}}D(0,\tau)\right]$$

$$\Rightarrow S=\frac{\mathbb{E}^*[\mathbf{1}_{\{\tau\leq T\}}(1-R)D(0,\tau)]}{\mathbb{E}^*\left[\int_0^T \mathbf{1}_{\{\tau>u\}}D(0,u)\,du\right]}$$

However, in practice, we are going to use a discrete simplification used by JP Morgan that assumes that the interest rate process is independent of the default process and the default leg pays at the end of each accrual period. Concerning the premium leg, JP Morgan assumes that defaults occur midway during each payment period, but the accrual payment is made at the end of the periods.

For N periods (meaning N payments), the present value of the **premium leg payments** can be written as:

$$PV(Premium)=\sum_{i=1}^{N} Spread_N\times Notional\times D(0,T_i)\times S(T_i)\times\Delta t_i$$

Similarly, the present value of the **default leg payments** can be written as follows:

$$PV(Default)=\sum_{i=1}^{N}(1-R)\times D(0,T_i)\times[S(T_{i-1})-S(T_i)])$$

where R is the recovery rate, i.e. the amount that can be recovered.

Using the equality between these two prices, we can find the **spread value** as follows:

$$Spread_N=\frac{\sum_{i=1}^{N}(1-R)\times D(0,T_i)\times[S(T_{i-1})-S(T_i)])}{\sum_{i=1}^{N} Notional\times D(0,T_i)\times S(T_i)\times\Delta t_i}$$

It can be re-written in terms of the default probability, $F(T_i)$. We then have:

$$Spread_N=\frac{\sum_{i=1}^{N}(1-R)\times D(0,T_i)\times[F(T_i)-F(T_{i-1})])}{\sum_{i=1}^{N} Notional\times D(0,T_i)\times(1-F(T_i))\times\Delta t_i}$$

2.2 Overview of Survival probabilities and hazard rates

A key component in credit risk derivatives is the default time τ which is assumed to be a stopping time in credit risk models We will define the default probability up to time τ as the cumulative probability distribution function of τ, namely,

$$F(t)=\text{Prob}(\tau\leq t).$$

The corresponding survival probability, i.e. the probability that no default occurs until time τ, is

$$S(t) = 1 - F(t) = \text{Prob}(\tau > t).$$

Finally, the hazard rate corresponding to default time τ can be defined as the deterministic function λ such that

$$S(t) = \exp\left(-\int_0^t \lambda(u)\,du\right).$$

The hazard rate (or default intensity) $\lambda(t)$ is defined as a non-random function designed to model the rate of default over time. It is determined by the derivative of the logarithm of the survival probability, $S(t)$, as follows:

$$\lambda(t) = -\frac{d}{dt}\ln S(t)$$

which reflects the instantaneous risk of default at time t assuming no default has occurred prior.

Additionally, the hazard rate can be expressed through the relationship between the cumulative distribution function $F(t)$ and its derivative:

$$\lambda(t) = \frac{F'(t)}{1 - F(t)}$$

This expression underscores the dynamics of the default probability over time, emphasizing how changes in the perceived risk of default influence the shape of the hazard curve.

2.3 Deriving a Hazard Rate Curve

The valuation of single-credit derivatives largely hinges on the accurate construction of a survival probability curve, denoted as $S(T_1), S(T_2), ..., S(T_N)$. This process resembles the development of any financial term structure and employs comparable techniques:

- Initially, determine as many $S(T_i)$ values as possible from available market data.
- For any missing values, employ interpolation or extrapolation methods to ensure consistency within the curve.

A robust valuation model is essential for this initial phase, transforming theoretical $S(T_i)$ into practical securities valuations that align with observed market structures. By inversely applying this model, one can infer $S(T_i)$ from existing market prices or spreads. Given that the number of available market data points is typically less than the required $S(T_i)$, it is customary to assume that hazard rates remain constant between the maturities where data is available. This assumption facilitates the following structured approach:

- Begin by estimating $S(T_i)$ for the security with the earliest maturity.
- Extend this estimate incrementally to subsequent maturities using new market data points as they become available.

This methodical approach, known as term structure bootstrapping, is preferred for its precision and the natural curve it produces without imposing artificial constraints often found in other fitting methods, thereby providing a more accurate reflection of market conditions.

The CDS spread formula just derived can be used to bootstrap either the survival probabilities or default probabilities, seeing that CDS spreads and discount factors are directly observable quantities. One starts by finding $S(T_1)$ and go through the bootstrapping to arrive to $S(T_N)$ by increasing the CDS's maturity by using the iterative formula below:

$$S(T_N) = \frac{\sum_{i=1}^{N-1} D(0,T_i)[LS(T_{i-1}) - (L + \Delta t_i \times Spread_N) \times S(T_i)]}{\sum_{i=1}^{N} Notional \times D(0,T_N) \times (\Delta t_i \times Spread_N)} + \frac{S(T_{N-1}) \times L}{L + \Delta t_N \times Spread_N}$$

where $L = 1 - R$ the proportion of loss given the default.

Now that we have the bootstrapped generic formula, we are able to extract the term structure of hazard rates. Assuming that the survival probablity of the reference entity at each time T_N is given by:

$$S(T_N) = e^{-\int_0^{T_N} \lambda(u)du}$$

$$\implies lnS(T_N) = -\int_0^{T_N} \lambda(u)du = -\sum_{i=1}^{N} \lambda_i (T_i - T_{i-1})$$

$$\implies \lambda_N = \frac{1}{T_N - T_{N-1}}[-lnS(T_N) - \sum_{i=1}^{N-1} \lambda_i \times (T_i - T_{i-1})]$$

3 Nth to default CDS Pricing

A *basket credit default swap* is a portfolio of CDS containing multiple reference entities. A *k-th-to-default* CDS provides a payoff only when the k-th default occurs. Hence, copulas are important factors in modelling joint default time probability.

3.1 Copula

Let $U_1, U_2, ..., U_n$ be n uniform random variables with a correlation matrix Σ, the joint distribution or **copula function**, noted C, is classically defined as:

$$C(U_1, U_2, ..., U_n|\Sigma) = \mathbb{P}(u_1 \leq U_1, u_2 \leq U_2, ..., u_n \leq U_n)$$

Basically a copula function can link uni-variate marginal distributions and can describe the multivariate distribution, excluding their marginals. Sklars' theorem shows the existence of such a function such as if $F(x_1, ..., x_n)$ is a joint multivariate distribution function with marginal distribution functions $U_1 = F_1(x_1)$, ..., $U_n = F_n(x_n)$, we have:

$$F(x_1, ..., x_n) = C(F_1(x_1), ..., F_n(x_n)|\Sigma)$$

Note that if F_i is continuous then C is unique. Then if we know the marginal distributions and their correlations we can construct the joint distribution through use of the appropriate copula function. In this work the joint default probabilities are modelled using Gaussian and Student T copula functions.

3.2 Correlation

Copula methods require deriving linear correlation matrices from historical spread data. The aim isn't to capture correlations of price movements, but rather the correlation of intrinsic default risk.

This study considers two different elliptical copulas (the Gaussian and the Student t copulas) and correlation methods as the foundation for copula techniques:

- Pearson's correlation.
- Spearman's rho rank correlation.
- Kendall's tau correlation.

Each of these correlation methods require input data to be structured in a certain way.

Input Datasets

There are 3 distinct input datasets for the 3 correlation methods:

1. Kendall's tau:
   - Dataset: ΔCDS spreads (denoted as X^{Hist}).
   - Description: Weekly changes in CDS spreads.
2. Spearman's rho:
   - Dataset: X^{Hist} converted to uniform distribution, $U^{Hist} \sim U(0,1)$.
   - Description:
     - Generate pseudo-samples/scores by fitting an Empirical Cumulative Distribution Function (ECDF) to $X^H ist$.
     - Steps involved:
       (a) Estimate $X^H ist$'s Empirical Probability Density Function (EPDF) via kernel density estimation.
       (b) Smoothen the EPDF using an appropriate kernel function.
       (c) Adjust the bandwidth as needed to achieve uniformity.
3. Pearson's:
   - Dataset: U^{Hist} converted to a normal distribution, $Z^{Hist} \sim N(0,1)$
   - Description: Convert the pseudo-samples to conform to a standard normal distribution.

Correlation Methodologies Explained

Kendall's tau

Kendall's tau is a non-parametric rank correlation measure that measures the relationships between large values in two variables. It calculates the difference between the number of concordant and discordant pairs, then normalizes this difference by the total number of pairs.

A pair of observations is considered concordant if the rank order for both elements is the same. Conversely, a pair of observations is considered discordant if the rank order for both elements is not the same.

To illustrate, suppose there are a pair of observations (x_i,y_i) and (x_j,y_j):

- The pair is concordant if ($x_i > x_j$ Λ $y_i > y_j$) V ($x_i < x_j$ Λ $y_i < y_j$)
- The pair is disconcordant if ($x_i > x_j$ Λ $y_i < y_j$) V ($x_i < x_j$ Λ $y_i > y_j$)

Given N as the total number of pairs, N_c as the number of concordant pairs, and N_d as the number of discordant pairs, the formula for Kendall's tau estimation is:

$$\rho_\tau = \frac{N_c - N_d}{N(N-1)/2}$$

Spearman's rho
Spearman's rho is another non-parametric rank correlation measure that calculates the linear correlation of the associated empirical cumulative distribution functions. The ECDF provides the ranks of the underlying data points. Validating the uniformity of the ECDF is crucial for the accuracy of this correlation method.

Given B pairs of observations and variables X and Y, where $X$and Y are ascending-ordered ranked variables and x_i and y_i are the i^{th} observations in each variable, the formula is:

$$\rho_s = 1 - \frac{6\sum_{i=1}^{N}(x_i - y_i)^2}{N(N^2-1)}$$

Pearson
Lastly, the commonly used Pearson correlation coefficient is the linear relationship between two continuous variables, X and Y:

$$\rho = \frac{Cov(X,Y)}{\sqrt{Var(X)Var(Y)}}$$

3.3 An empirical formula for the kth-to-default swap spread

Payments for the Nth to default spreads are calculated in the same manner as for regular CDS. After the relevant default occurs, a settlement is made. The swap then terminates, and no further payments are made by either party.

Let N be the notional amount, s the spread, n the number of securities making up the basket, and R the recovery rate. The par spread of the k-th-to-default swap is derived by equating $DL = PL$:

$$s = \frac{PV(DL)}{PV(PL \cdot s)} = \frac{(1-R)\sum_{i=1}^{N} Z(0,t_i)(S(t_{i-1}) - S(t_i))}{\sum_{i=1}^{N} Z(0,t_i)S(t_i)\Delta t_i} \tag{1}$$

where:

- $PV(DL)$: the present value of the default legs.
- $PV(PL)$: the present value of the premium legs.
- $S(t_i)$: the arbitrage-free survival probability of the reference entity from $t = 0$ to t_i (premium payment time).
- $Z(0, t_i)$: Zero Coupon Bond / Discount factor from 0 to t_i (risk-free).
- Δt_i: Day count fraction between premium payment dates.

Total Expected Loss: By introducing the Total Expected Loss as an expectation over the joint distribution:

$$E[F_k(t)] = L_k, \quad L_i - L_{i-1} = \frac{1}{n} \times \text{Notional} \tag{2}$$

with n the number of reference entities avec $F_k(t)$ the CDF of default.

- **No default:** If no default occurs, the protection seller receives $s \times N \times \frac{1}{n}$.
- **First to default (FTD):** If one of the reference entities defaults before the maturity of the contract, the protection seller pays $N \times \frac{1}{n} \times (1 - R)$.
- **Kth to default (KTD):**

$$s = \frac{(1-R)(Z(0,\tau_k) \times \frac{1}{n}) \sum_{i=1}^{k} Z(0,\tau_i)(\tau_i - \tau_{i-1}) \times \frac{n-(i-1)}{n}}{\sum_{i=1}^{N} Z(0,\tau_i)(S(\tau_{i-1}) - S(\tau_i))} \tag{3}$$

3.4 Monte Carlo Model Implementation

The implementation of the Monte Carlo model for simulating basket credit default swaps involves the following steps:

1. Generate uniform random variables $U = (U_1, U_2, U_3, U_4, U_5)$.
2. Transform the uniform variables to standard normal variables $Z = (Z_1, Z_2, Z_3, Z_4, Z_5)$.
3. Produce correlated standard normal variables through the Cholesky decomposition of the correlation matrix obtained from market data, indicated by $Z_c = A^T A$.
4. Convert the correlated standard normal variables to correlated uniform variables employing Gaussian or Student's T distribution (with degrees of freedom ν), given by $U_c = \Phi(Z_c)$ or $U_c = t_\nu(Z_c)$.
5. Determine the default times by inverting the correlated uniform variables according to the survival probability function $\tau_i = F_i^{-1}(U_{ci})$.
6. Set default times exceeding the maturity of the basket swap ($T = 5$) to 999, signifying no default within the product's maturity.

7. Organize the default times into a sequential vector $\tau = (\tau_1, \tau_2, \tau_3, \tau_4, \tau_5)$.

8. Compute the k-th to default spread vector using the previously discussed methods$S = (S_1, S_2, S_3, S_4, S_5)$.

9. Repeat the above steps a significant number of times to average the spreads and estimate the expected k-th to default spread.

This procedure encapsulates the systematic approach to modeling and simulating the behavior of basket CDS using Monte Carlo techniques, emphasizing the generation and transformation of random variables to accurately assess default risks.

4 Numerical application and sensitivity analysis

4.1 Data exploration

For our numerical application, we will use the daily CDS spreads of 5 companies from January 2015 to September 2021 and up to 10 different tenors (from 1Y to 10Y spreads):

- Wells Fargo (US bank)
- Morgan Stanley (US bank)
- Barclays (UK bank)
- Bank of America (US bank)
- UBS (Switzerland bank)

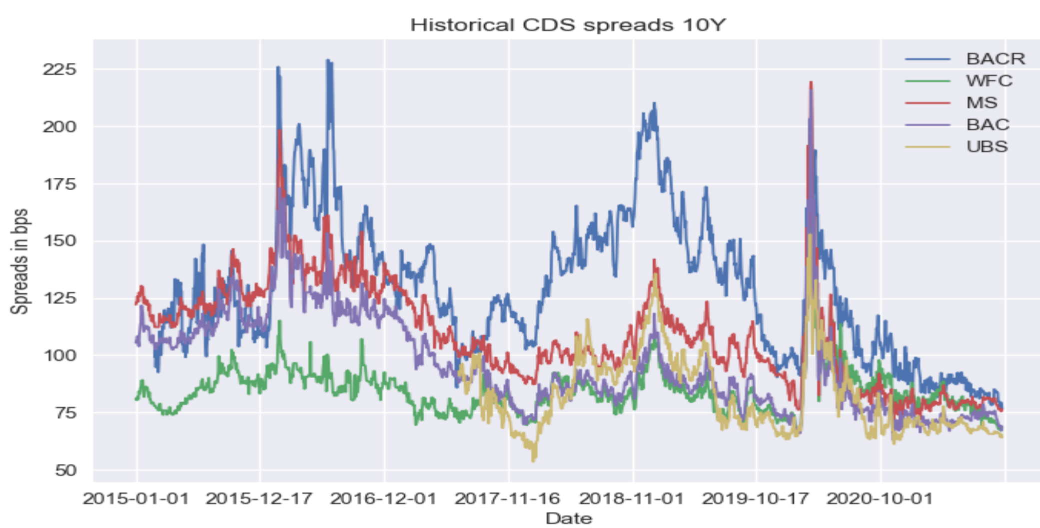

Figure 1: Historical 10Y CDS spreads

As we can see with the Credit curve in the figure below, credit spreads increase with maturity which makes sense as the longer the contract is, the more the contract buyer is exposed to the company's credit risk. Investors demands higher risk premiums due to a long credit risk exposition.

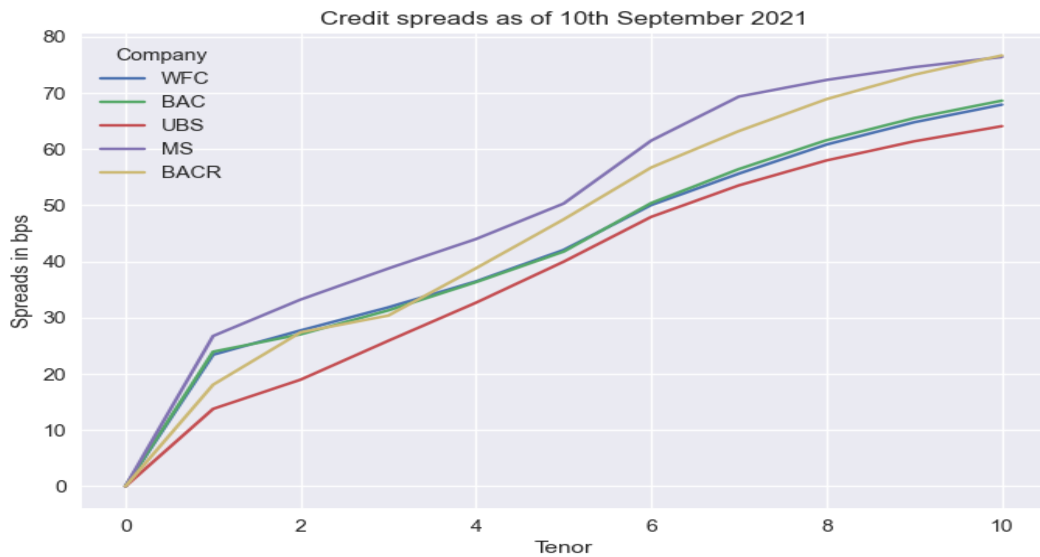

Figure 2: Credit curve

4.2 Bootstrapping survival probabilities and hazard rates

In order to derive the survival probabilities curve, the discount factor term structure for each maturity and the credit curve obtained above are needed. For practicality, the assumption made is that all companies operate in the US as we will use the US treasury yield curve [2] from the 10th of September, 2021 (the last data point known in our cds dataset). With the bootstrap method, the Survival probabilities term structure over the next ten years starting from 2021 is extracted.

On the one hand, as expected, the survival probabilities are decreasing with time. However, there seems to a a small dispersion in survival probabilities over time, which can be explained by the fact that they are all banks operating in the same areas. We can infer that our portfolio is not that diversified.

On the other hand, the hazard rates are increasing over time. The longer the timeframe, the greeater the uncertainty about a company's future economic health, making it hard to predict financial stability over time.

[2] Daily Treasury Yield Curve Rates for 2023

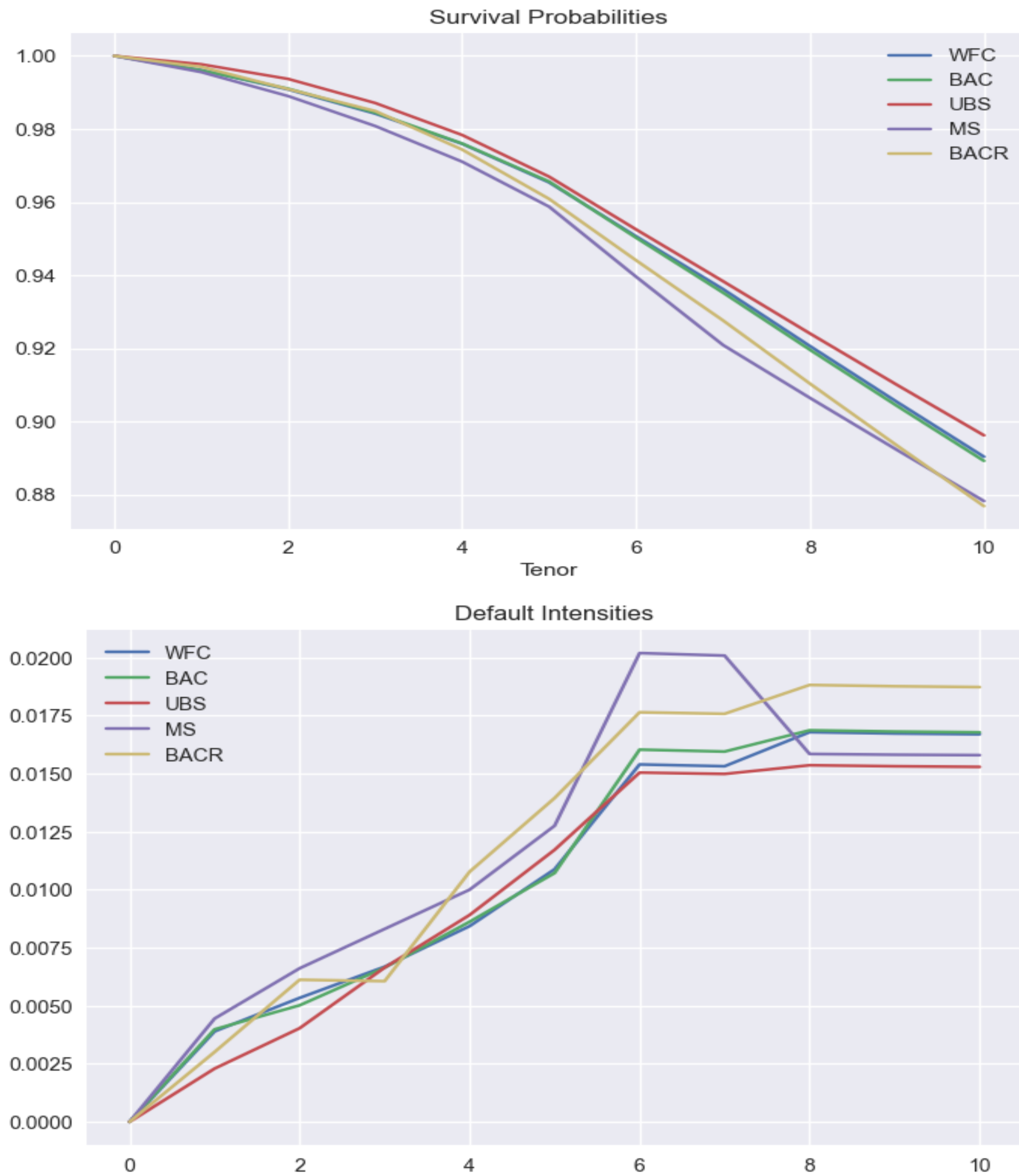

Figure 3: Survival probabilities and Hazard rates term structures

4.3 Correlation Matrix

One of the key component in computing the NTD price is the correlation matrix used, as it will be used to generate samples from copula.

Kendall Tau Correlation

The pair plot shows no strong dependence on the extremes. However, the individual distributions do not conform to a normal distribution.

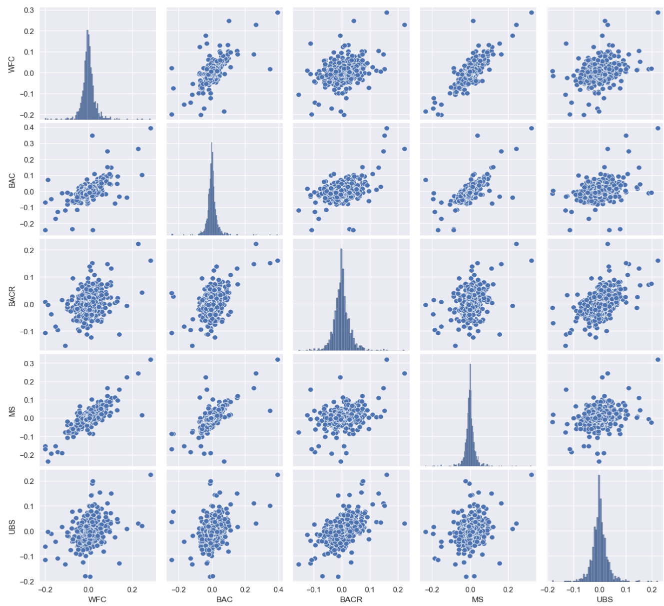

Figure 4: $\Delta Spreads$ pair-plot and Kendall Tau Correlation

Spearman Rho Correlation The data is transformed into pseudo samples using the method explained in section 3.2. The pseudo samples distributions seem to resemble to a uniform distribution. The scatter plots show an overall higher correlated structure between the variables than the Kendall tau correlation. This would result in the NTD spread being higher with Spearman correlation than the Kendall correlation as higher correlation means the portfolio will behave more like a single credit asset and there will be a higher co-concentration of earlier default times.

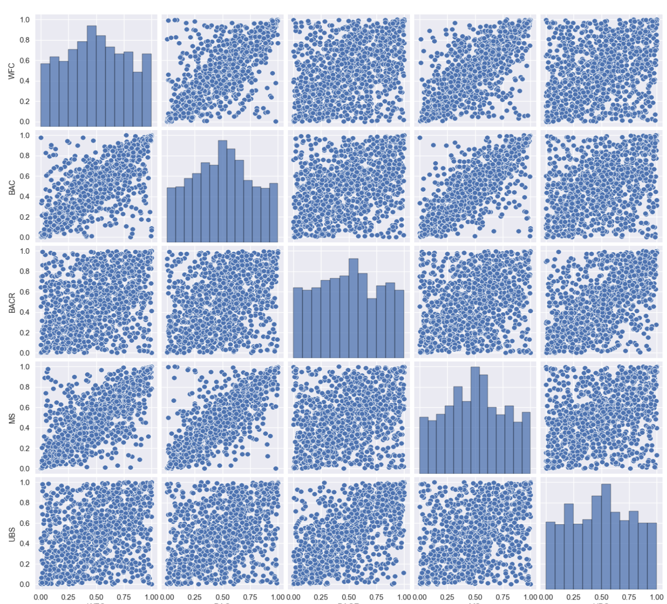

Figure 5: Pseudo samples pair-plot and Spearman Rho Correlation

Pearson Rho Correlation

The data look a lot more like normal distributions than with the two precedent methods but with variables being more correlated. Hence, we infer that the spreads will be higher than the with the other 2 methods.

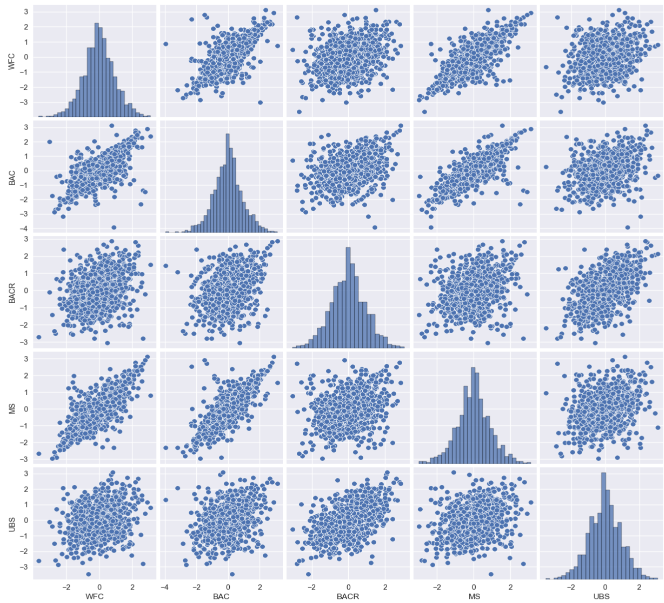

Figure 6: Z^{Hist} pair-plots and Pearson Rho Correlation

Overall, scatter plots show positive correlation as data points seem to follow the first diagonal, which can be explained by the fact that they are all banks, hence their credit spreads will likely move in the same direction.

4.4 Nth to default spreads and sensitivity analyses

4.4.1 Monte Carlo Convergence

Using the Monte-Carlo method exposed in section 4.2, the NTD spreads are be calculated. The spreads shown are the average spreads the each NTD swap (as there are 5 entities in our basket, the NTD contracts go up to the 5th to default swap).

As shown in the figure above, the spreads seem to converge under 20 000 simulations for the 2nd to default to the 5th to default. However, the spread for the 1st to default doesn't seem stable under 100 000 simulations. For this study, as simulating 100 000 spreads takes nearly 30 minutes, the rest of the analysis will be done with 100 000 simulations.

Furthermore, spreads diminish when moving over from the 1TD to the 5TD swap. There is a higher chance that the 1TD contract will activate, as it only requires one default, than the other swaps which are less likely to be activated, as more default are less likely to happen.

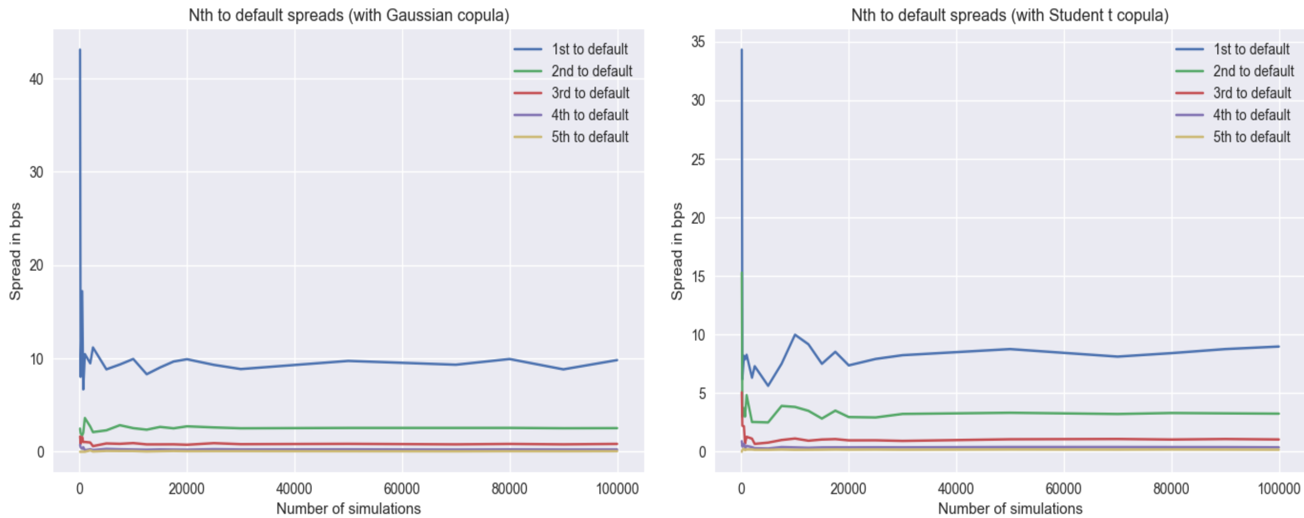

Figure 7: Monte Carlo NTD Spread convergence

4.4.2 Impact of correlation

The impact of the correlation method used is shown in the graph below. The historical daily 10Y CDS spreads were used as data to determine the correlation matrices. As expected earlier, the spreads are higher with the Pearson correlation method than with the Kendall and Spearman methods. This is due to the higher tail dependencies as shown in the Pearson pair plot above. This higher tail dependency implies that there is a high likelihood that the extreme events such as credit defaults come together, which means that there is a higher chance that successive credit events chain up. Also, the spreads converge to 0 for all three methods.

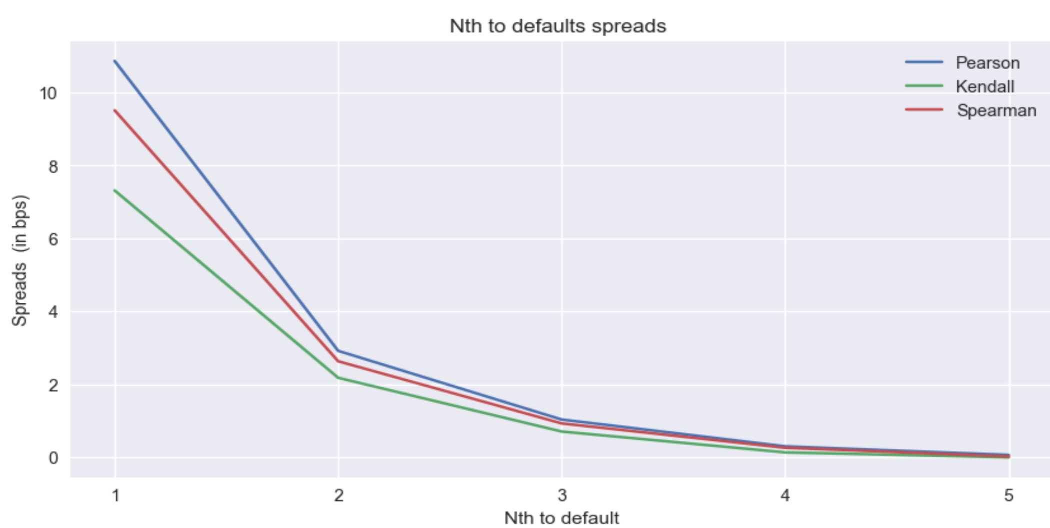

Figure 8: NTD spreads depending on the correlation method

This is also illustrated when the correlation increases. The figure below shows how spreads behave when correlation matrix factors increase or decreasing all at once. The higher the correlation between variables, the higher the spreads are.

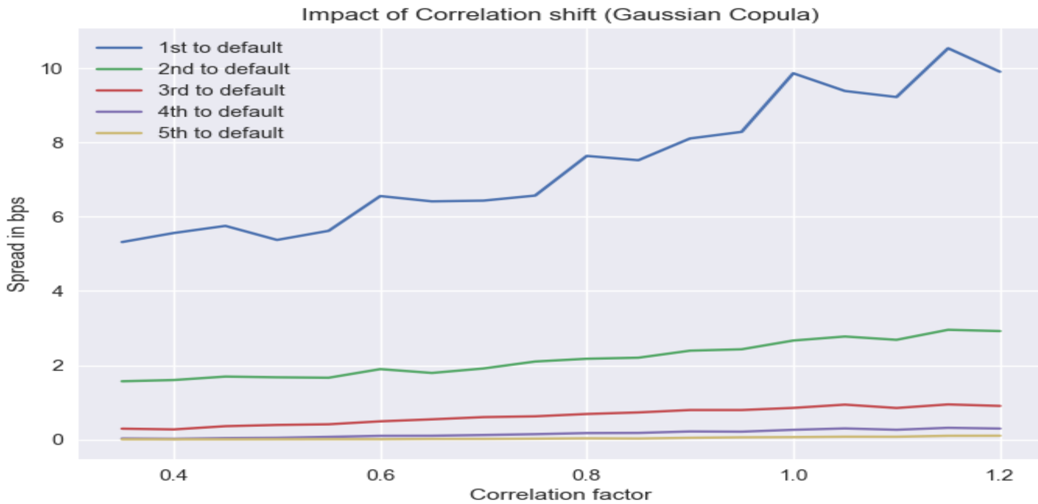

Figure 9: Impact of correlation factors

4.4.3 Impact of copula method

In this section, the impact of copula generator is shown through Monte Carlo simulations. First, the Student t copula degree of freedom has to be estimated. By maximum likelihood, the degree of freedom is 2.

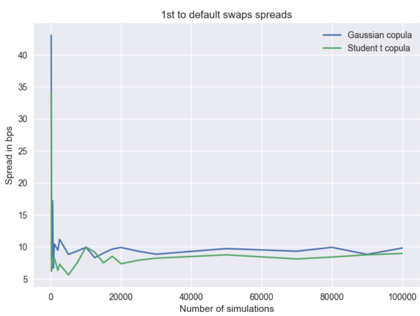

Figure 10: 1st-to-default

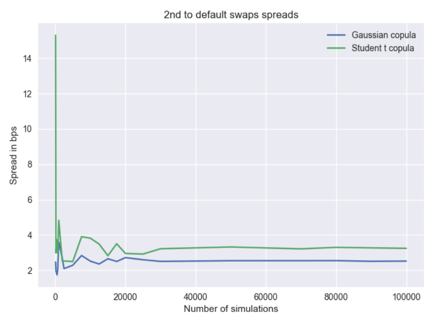

Figure 11: 2nd-to-default

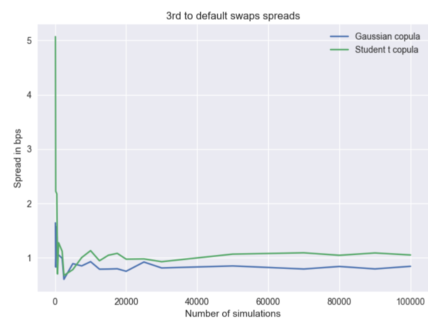

Figure 12: 3rd-to-default

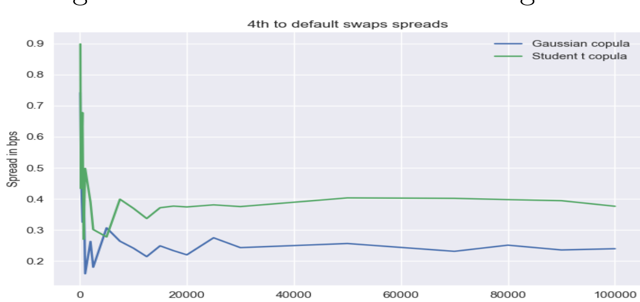

Figure 13: 4th-to-default

Figure 14: 5th-to-default

Overall, the Student copula method prices the NTDs higher than Gaussian copula method, except for the 1st to default swap. These differences are more likely due to Student T's copula fatter tails, providing a higher number of multiple defaults.
However, the spread differences between the two copulas seem to narrow down as the number of defaults is needed as shown in the table below.

	MSE
1st to default	10.699923
2nd to default	8.572907
3rd to default	0.707834
4th to default	0.028418
5th to default	0.017507

Figure 15: MSE between the two Monte Carlo simulations: Gaussian VS Student t copula

4.4.4 Impact of Recovery rates

Recovery rate is an important factor in credit derivatives. Historically, the Recovery rate in credit derivatives for corporates is assumed to be 40%. For this factor analysis, 100 000 simulations have been made using the Gaussian copula and the Pearson correlation.

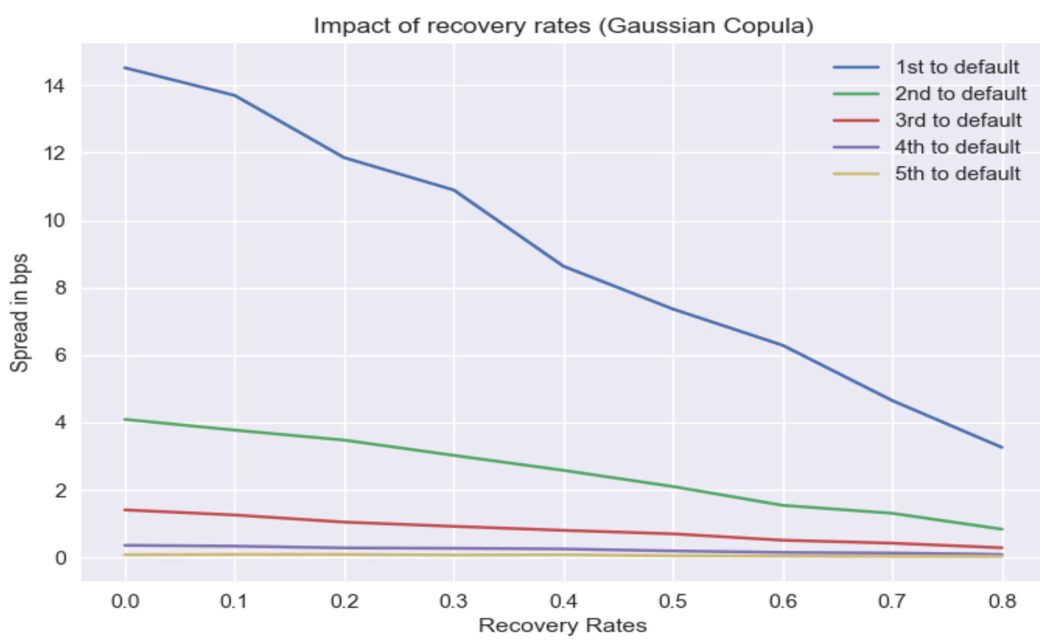

Figure 16: Impact of Recovery rates

All contracts spreads are decreasing as recovery rates increase. This is due to the fact that as as recovery rates increase, the total loss if default events occur will be less important. The need for credit risk protection will be reduced, hence the decrease of spreads. In addition, This makes sense as the NTD default spread is proportional to $(1 - R)$ in the formula derived in section 4.1.

The downward slope of the 1st to default swap is higher. It has the highest sensitivity to changes in recovery rates as it is the most likely impacted contract out the 5.

4.4.5 Impact of Credit spread shift

As discussed before, the CDS spreads are the most important data in modelling the NTD spreads, from calculating survival probabilities to calculating the spreads. Hence, the NTD spreads should be highly sensitive to credit spreads shift.

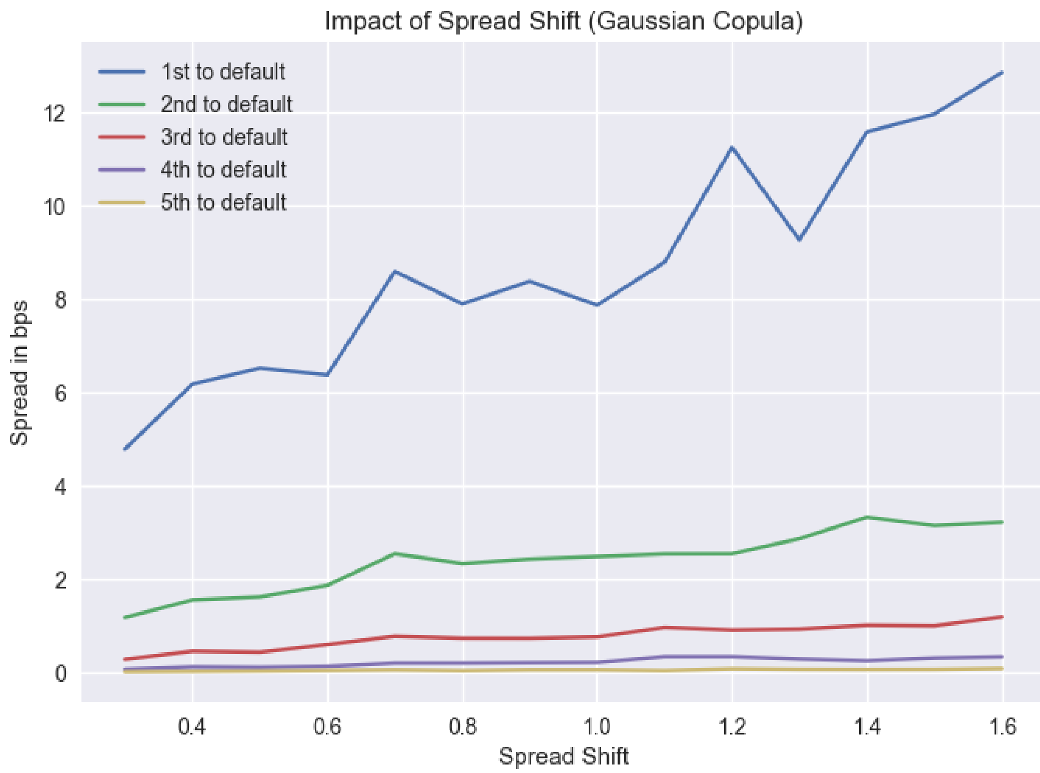

Figure 17: Impact of CDS Spreads

As we can see, the NTD spreads are increasing for all contracts as the CDS spreads are shifting. This is due to the fact that as entities' CDS spreads increase, their individual credit risk rises. The credit spreads curves also increase which has an impact on survival probability. There will be a greater credit curve steepness The higher the CDS spread is, the more likely the company is going to default. This curve steepness indicates a greater market-implied default risk.

5 Conclusion

The NTD swap price model inspired by the JP Morgan model of Hazard Rates bootstrapping is subject to multiple risk factors such as economical environment, entities own default risk, asset correlation, copula models and recovery rates. However, there are some drawbacks that should be noted:

- In order to have more precise prices, interpolating the hazard rates and the discount factors are to be considered.
- Our data is concentrated in one sector only, which significantly rises the credit default risk by contagion. Furthermore, only CDS spreads and yield curve are considered, whereas counterparty risk and additional exposition factors should be taken into account.
- This model relies on the bootstrapping method to derive hazard rates and survival probabilities. To obtain a more accurate pricing model, default times should be following a structural model as developed by Merton in 1973 or a reduced form model.

A Appendix

A.1 Yield curve and Discount factors

The Yield curve data was extracted from the US treasury site. From this curve, the discount factor curve needed for the calculation of survival probabilities is drawn.

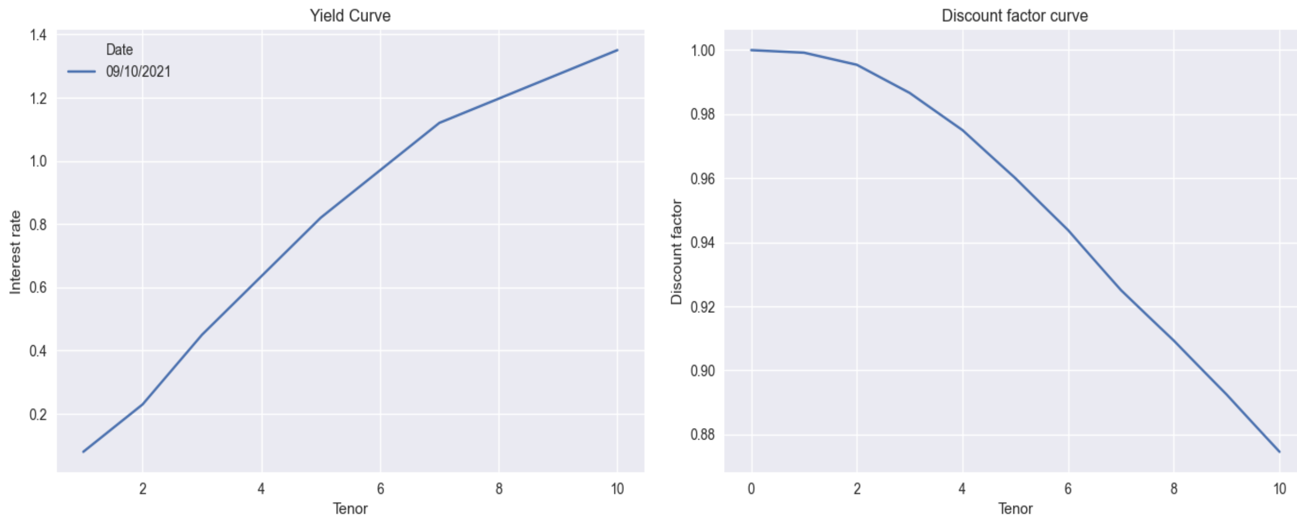

Figure 18: Monte Carlo NTD Spread convergence

As expected, the longer the maturity, the higher the interest rate will be for borrowers as the investors will ask a higher premium for their exposure to credit events.

A.2 Impact of Interest rates shift

An impact analysis of interest rates has also been conducted. This is motivated by the fact that credit derivatives, and financial products in general, are subject to macro economic cycles, which may lead in a change of interest rates.

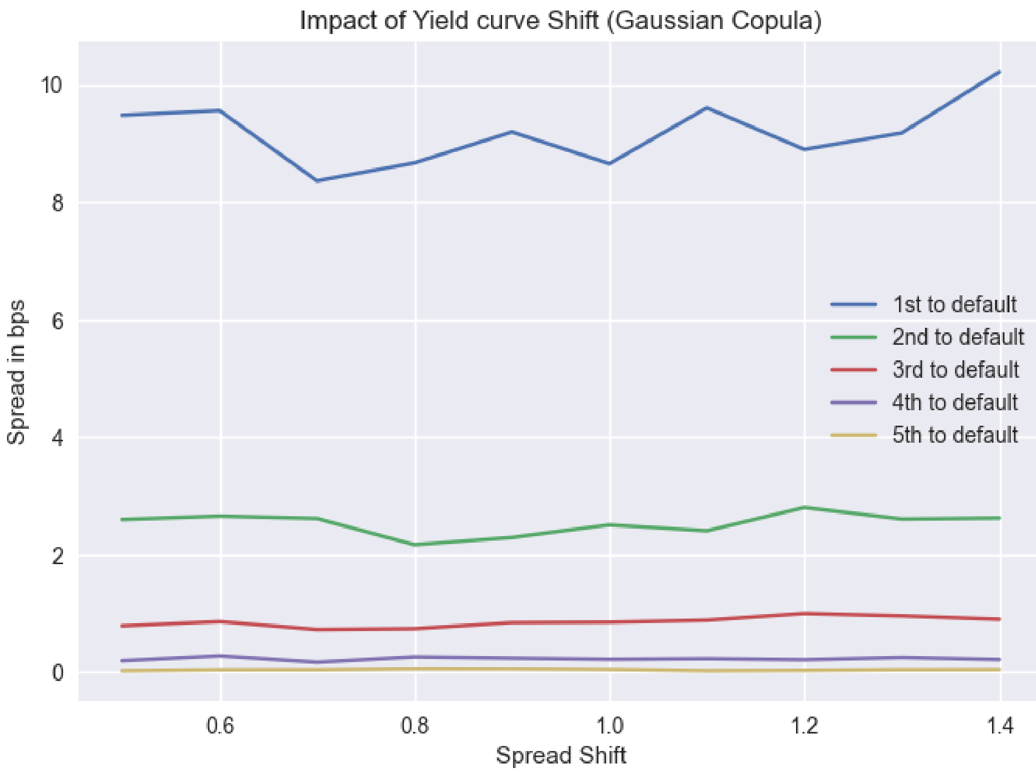

Figure 19: Impact of interest rates shift

After running 100 000 Monte Carlo simulations and shifting the yield curve with a factor varying from 0.5 to 1.4, we notice that the impact of these shifts on the NTD spreads are small on every NTD contract, if we exclude Monte Carlo variance.

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