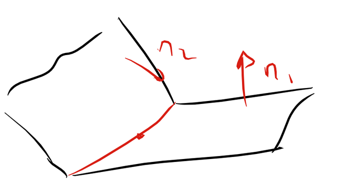
Recta $\left\{ \begin{array}{l} P_0 \\ \bar{u} \end{array} \right.$

$\overline{OP} = \overline{OP_0} + \lambda\, \bar{u}$

$\left\{ \begin{array}{l} x = x_0 + \lambda u_x \\ y = y_0 + \lambda u_y \\ z = z_0 + \lambda u_y \end{array} \right.$

$\dfrac{x - x_0}{u_x} = \dfrac{y - y_0}{u_y} = \dfrac{z - z_0}{u_z}$



$L \left\{ \begin{array}{l} \bar{u} = \bar{n}_1 \times \bar{n}_2 \\ P_0 \end{array} \right.$

$L_1 \parallel L_2 \iff \bar{u}_1 \parallel \bar{u}_2$

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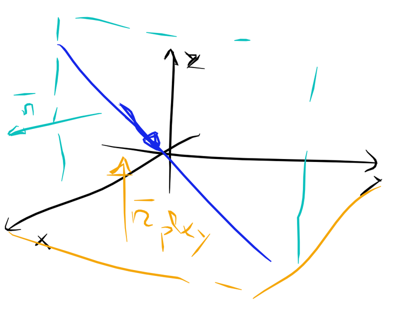
$r: (x, y, z) = (2, -2, 3) + \lambda (1, 2, -1)$



pl. $\perp$ pl xy $\wedge$ pl. contiene a r

$\vec{n} \perp n_{plxy} \wedge \vec{n} \perp \bar{u}$

$\vec{n} = \bar{n}_{plxy} \times \bar{u}$

pl xy $\Rightarrow z = 0$

$\bar{n}_{plxy} = (0, 0, 1)$

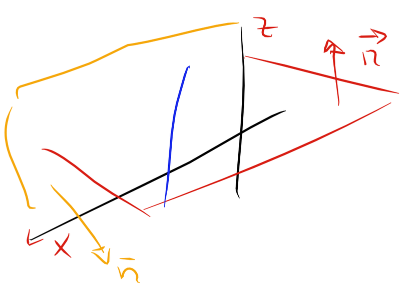
$\vec{n} = \begin{vmatrix} \check{i} & \check{j} & \check{k} \\ 0 & 0 & 1 \\ 1 & 2 & -1 \end{vmatrix} = -2\check{i} + \check{j} \Rightarrow \vec{n} = (-2, 1, 0)$

$-2x + y + D = 0$

$-2 \cdot 2 + (-2) + D = 0 \Rightarrow D = 6$

$\boxed{-2x + y + 6 = 0}$

$r: (x, y, z) = (2, -2, 3) + \lambda (1, 2, -1)$



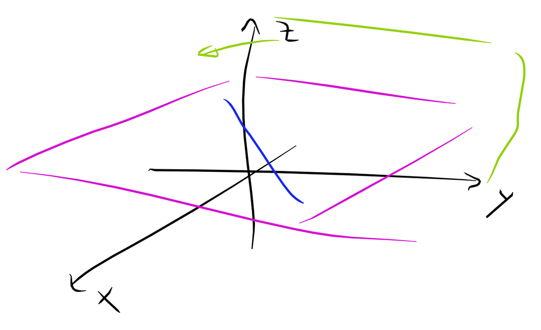
$\vec{n} \perp (1, 2, -1) \wedge \vec{n} \perp (0, 1, 0)$

$\vec{n} = \begin{vmatrix} \check{i} & \check{j} & \check{k} \\ 1 & 2 & -1 \\ 0 & 1 & 0 \end{vmatrix} = \check{i} + \check{k} = (1, 0, 1)$

$1 \cdot x + 1 \cdot z + D = 0$ $\qquad 2 + 3 + D = 0$

$x + z - 5 = 0$ $\qquad D = -5$

$r: (x, y, z) = (2, -2, 3) + \lambda (1, 2, -1)$



$\bar{n} \perp (1, 2, -1) \wedge \bar{n} \perp (1, 0, 0)$

$\bar{n} = \begin{vmatrix} \check{i} & \check{j} & \check{k} \\ 1 & 2 & -1 \\ 1 & 0 & 0 \end{vmatrix} = -\check{j} - 2\check{k} \parallel (0, 1, 2)$

$y + 2z + D = 0$ $\qquad -2 + 2 \cdot 3 + D = 0$

$D = -4$

$y + 2z - 4 = 0$

L_1 y L_2 son alabeadas sii no existe plano que las contenga. $\left\{ \begin{array}{l} L_1 \not\parallel L_2 \\ L_1 \cap L_2 = \{\} \end{array} \right.$

27|

$r: \left\{ \begin{array}{l} x - y + z = 0 \\ 2x - y + z = 2 \end{array} \right.$ $\qquad s: \left\{ \begin{array}{l} (3, 2, 4) \\ (k, 0, k) \end{array} \right.$

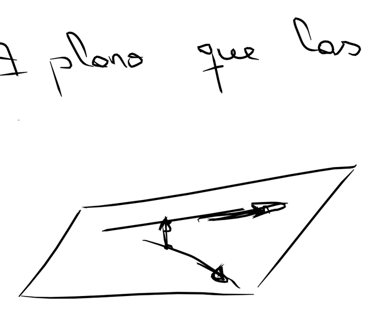
$z = 0 \Rightarrow \left\{ \begin{array}{l} x - y = 0 \\ 2x - y = 2 \end{array} \right.$

$r: (x, y, z) = (2, 2, 0) + \lambda (0, 1, 1)$

$s: (x, y, z) = (3, 2, 4) + \beta (3-k, 2, 4-k)$

ej $k = 1 \Rightarrow s: (3, 2, 4) + \beta (2, 2, 3)$

$k / \exists$ plano que las contenga y luego desecho ese valor



$\bar{u}_1 = (0, 1, 1)$

$\bar{u}_2 = (3-k, 2, 4-k)$

$\overline{P_r P_s} = (1, 0, 4)$

$(\bar{u}_1 \times \overline{P_r P_s}) \cdot \bar{u}_2 = 0$

$(4, 1, -1) \cdot (3-k, 2, 4-k) = 0$

$4(3-k) + 1(2) + (-1)(4-k) = 0$

$12 - 4k + 2 - 4 + k = 0$

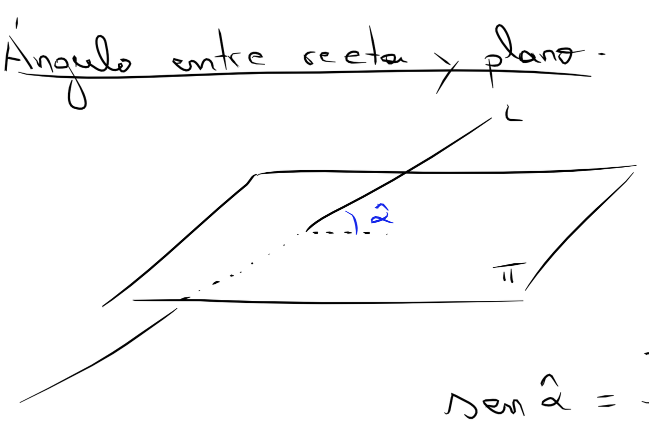
$10 - 3k = 0 \Rightarrow \boxed{k = \frac{10}{3}}$

Con $k = \frac{10}{3}$ existe pl. que las contiene $\therefore$ para que las rectas alabeadas $k \neq \frac{10}{3}$

$k \in \mathbb{R} - \left\{ \frac{10}{3} \right\}$

Ángulo entre recta y plano.

Datos $\left\{ \begin{array}{l} L \\ \pi \end{array} \right.$



$\hat{\alpha} = (L \hat{;} \pi)$

$\operatorname{sen} \hat{\alpha} = \dfrac{\bar{n} \cdot \bar{u}}{\|\bar{n}\| \, \|u\|}$ $\qquad 0 \leq \hat{\alpha} \leq \pi$

$(L \hat{;} \pi) = \operatorname{arc\,sen} \dfrac{\bar{n} \cdot \bar{u}}{\|\bar{n}\| \, \|\bar{u}\|}$

30) $L: \left\{ \begin{array}{l} x - 2y + z + 4 = 0 \\ x + 2y + 3z - 4 = 0 \end{array} \right.$ $\qquad \pi: 3x - 7y + 8z - 9 = 0$

$\bar{n} = (3, -7, 8)$

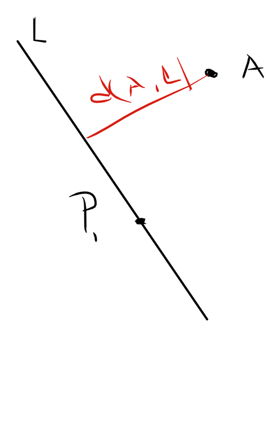
$\bar{u} = (1, -2, 1) \times (1, 2, 3) = (-8, -2, 4) \Rightarrow \|\bar{u}\| = \sqrt{84}$

$\|(3, -7, 8)\| = \sqrt{122}$

$\operatorname{sen} (L \hat{;} \pi) = \dfrac{(-8, -2, 4)(3, -7, 8)}{\sqrt{84} \cdot \sqrt{122}} = \dfrac{-24 + 14 + 32}{\sqrt{84} \cdot \sqrt{122}}$

Distancia de punto a recta.

Datos



$d(A, L) = \dfrac{\|\bar{u} \times \overline{P_1 A}\|}{\|\bar{u}\|}$

31 a) $A (2, 1, -1)$ $\qquad \overline{P_1 A} = (1, -2, 1)$

$(x, y, z) = \underbrace{(1, 3, -2)}_{P_1} + \lambda (3, 0, -4)$

$d(A, L) = \dfrac{\|(3, 0, -4) \times (1, -2, 1)\|}{\|(3, 0, -4)\|} = \dfrac{\|(-8, -7, -6)\|}{5} = \dfrac{\sqrt{149}}{5}$

Distancia entre rectas alabeadas:

Datos $\left\{ \begin{array}{l} L_1 \\ L_2 \end{array} \right.$ alabeadas $\qquad d(L_1, L_2) = \dfrac{|\overline{P_1 P_2} \cdot (\bar{u}_1 \times \bar{u}_2)|}{\|\bar{u}_1 \times \bar{u}_2\|}$

$P_1 \in L_1 \wedge u_1$ directa de L_1

$P_2 \in L_2 \wedge u_2$ " " L_2

$L_1: \dfrac{x - 1}{2} = y + 2 = z - 3$ $\qquad L_2: \dfrac{x + 2}{-3} = y - 2 = \dfrac{z + 1}{2}$

$P_1 (1, -2, 3)$ $\qquad P_2 (-2, 2, -1)$

$\bar{u}_1 = (2, 1, 1)$ $\qquad \bar{u}_2 = (-3, 1, 2)$

$\overline{P_1 P_2} = (-3, 4, -4)$ $\qquad \bar{u}_1 \times \bar{u}_2 = (1, -7, 5)$

$|\overline{P_1 P_2} \cdot \bar{u}_1 \times \bar{u}_2| = |(-3, 4, -4)(1, -7, 5)| =$

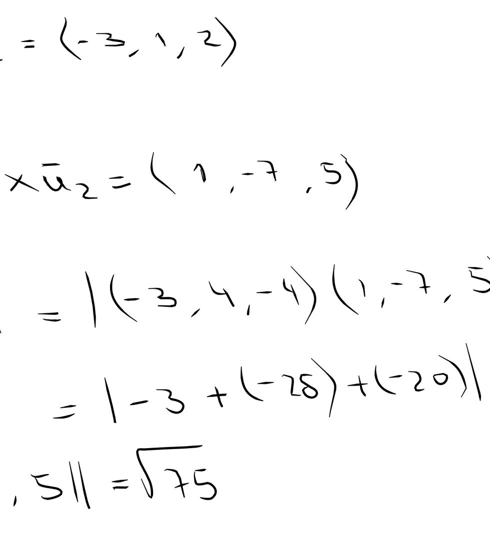
$= |-3 + (-28) + (-20)| = 51$

$\|\bar{u}_1 \times \bar{u}_2\| = \|(1, -7, 5)\| = \sqrt{75}$

$d(L_1, L_2) = \dfrac{|\overline{P_1 P_2} \cdot (u_1 \times u_2)|}{\|\bar{u}_1 \times \bar{u}_2\|} = \dfrac{51}{\sqrt{75}}$

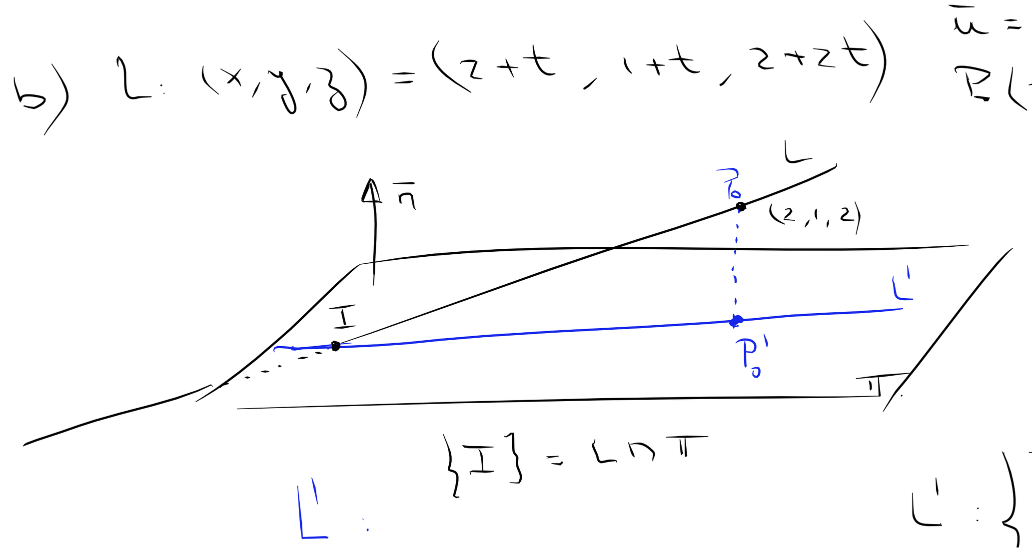
33| $\pi: 3x - 2y + 4z - 3 = 0$

a) $A (3, -1, 2)$



$L: \left\{ \begin{array}{l} \bar{n} \\ A \end{array} \right.$ $\qquad \{A'\} = L \cap \pi$

b) $L: (x, y, z) = (2 + t, 1 + t, 2 + 2t)$ $\qquad \bar{u} = (1, 1, 2)$ $\quad P_0 (2, 1, 2)$



$\{I\} = L \cap \pi$

$L':$ $\qquad L': \left\{ \begin{array}{l} I \\ P_0' \end{array} \right.$