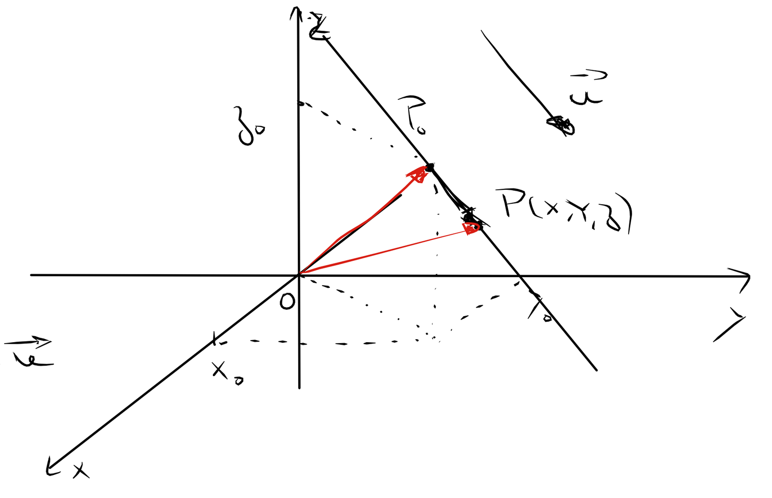
Ecuación de la recta



Datos $\begin{cases} P_0(x_0, y_0, z_0) \\ \bar{u} = (u_x, u_y, u_z) \end{cases}$

$\overline{P_0P} \parallel \bar{u} \iff \overline{P_0P} = \lambda \vec{u}$
$\lambda \in \mathbb{R}$

$\overline{OP} = \overline{OP_0} + \overline{P_0P}$

$\boxed{\overline{OP} = \overline{OP_0} + \lambda \vec{u}}$ → Ecuación vectorial de la recta

$(x, y, z) = (x_0, y_0, z_0) + \lambda (u_x, u_y, u_z)$

$(x, y, z) = (x_0, y_0, z_0) + (\lambda u_x, \lambda u_y, \lambda u_z)$

$(x, y, z) = (x_0 + \lambda u_x, y_0 + \lambda u_y, z_0 + \lambda u_z)$

$\begin{matrix} E_1 \\ E_2 \\ E_3 \end{matrix} \begin{cases} x = x_0 + \lambda u_x \\ y = y_0 + \lambda u_y \\ z = z_0 + \lambda u_z \end{cases}$ → Ecuaciones paramétricas

$\vec{u} = (u_x, u_y, u_z)$ → vector director.

$E_1 \Rightarrow \lambda = \dfrac{x - x_0}{u_x}$ $\quad u_x \neq 0$ $\qquad E_3 \Rightarrow \lambda = \dfrac{z - z_0}{u_z}$ $\quad u_z \neq 0$

$E_2 \Rightarrow \lambda = \dfrac{y - y_0}{u_y}$ $\quad u_y \neq 0$

$\dfrac{x - x_0}{u_x} = \dfrac{y - y_0}{u_y} = \dfrac{z - z_0}{u_z}$ → Ecuaciones simétricas.

21 $P_0(1, -1, 3)$ $\quad \vec{u} = (-2, 3, 4)$ $\qquad v = v_0 + at$

$\overline{OP} = \overline{OP_0} + \lambda \vec{u}$

→ $(x, y, z) = (1, -1, 3) + \lambda(-2, 3, 4)$

Puntos de r.
$\lambda = 2 \to P_1(-3, 5, 11)$
¿$P_2(-9, 14, 20) \in r$?

$(-9, 14, 20) = (1, -1, 3) + \lambda(-2, 3, 4)$

$\begin{cases} -9 = 1 - 2\lambda \to \lambda = 5 \\ 14 = -1 + 3\lambda \checkmark \\ 20 = 3 + 4\lambda \otimes \end{cases}$ No es de la r

$\begin{cases} x = 1 - 2\lambda \\ y = -1 + 3\lambda \\ z = \underbrace{3}_{P_0} + \underbrace{4}_{\bar{u}}\lambda \end{cases}$ → $\dfrac{x-1}{-2} = \lambda$, $\dfrac{y+1}{3} = \lambda$, $\dfrac{z-3}{4} = \lambda$

$\dfrac{x - 1}{-2} = \dfrac{y + 1}{3} = \dfrac{z - 3}{4}$ (1, -1, 3 above; $\bar{u}$ below)

b) $P_1(1, -3, 1)$ $\qquad P_2(1, 3, -4)$

$\overline{P_1P_2} = \bar{u} \to \bar{u} = (0, 6, -5)$

$(x, y, z) = (1, -3, 1) + \lambda(0, 6, -5)$ → $\begin{cases} x = 1 \\ y = -3 + 6\lambda \\ z = 1 - 5\lambda \end{cases}$

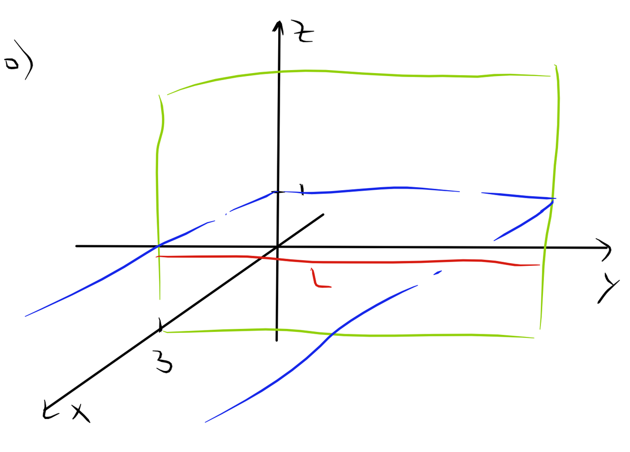
$\dfrac{y + 3}{6} = \dfrac{z - 1}{-5}, \quad x = 1$

c) $L \parallel$ eje de ordenadas (y) $\quad (3, 2, 1)$

$\bar{u} = (0, 1, 0)$ $\qquad P_0(3, 2, 1)$

$(x, y, z) = (3, 2, 1) + \lambda(0, 1, 0)$

$\begin{cases} x = 3 \\ y = 2 + \lambda \\ z = 1 \end{cases}$

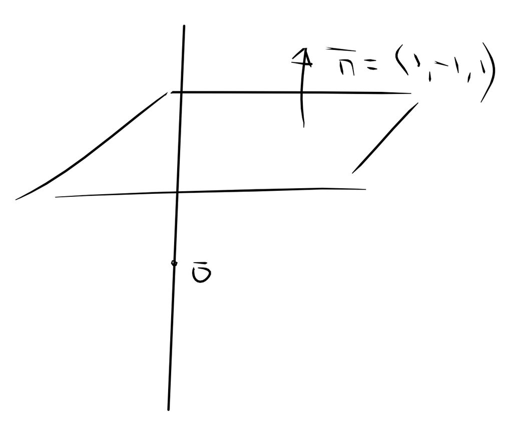


d) $P_0(0, 0, 0)$ $\qquad \vec{u} = (1, 1, 1)$ $\qquad \|\bar{u}\| = \sqrt{1^2 + 1^2 + 1^2} = \sqrt{3}$

$(x, y, z) = \lambda(1, 1, 1)$

$\begin{cases} x = \lambda \\ y = \lambda \\ z = \lambda \end{cases}$ $\qquad x = y = z$

e) $P_0(0, 0, 0)$ $\qquad L \perp \alpha : x - y + z - 1 = 0$



$\vec{u} = (1, -1, 1)$

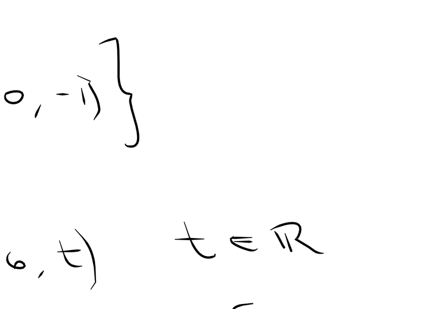
$(x, y, z) = \lambda(1, -1, 1)$

$\begin{cases} x = \lambda \\ y = -\lambda \\ z = \lambda \end{cases}$ $\qquad x = \dfrac{y}{-1} = z$

22 $\pi_1.\ x - y - z + 1 = 0$

$\pi_2.\ x - 2y - 3z - 2 = 0$

Datos → $\begin{cases} P_0 \\ \bar{u} \end{cases}$



$\bar{u} = \bar{n}_1 \times \bar{n}_2 = (1, 2, -1)$ ✓ $\qquad \begin{vmatrix} i & j & k \\ 1 & -1 & -1 \\ 1 & -2 & -3 \end{vmatrix} = (1, 2, -1)$

$z = 0 \to \begin{cases} x - y + 1 = 0 \\ x - 2y - 2 = 0 \end{cases}$ $\to P_0(-4, -3, 0)$ ✓

$(x, y, z) = (-4, -3, 0) + \lambda(1, 2, -1)$

25 $L: (x, y, z) = (1, 0, -1) + \lambda(2, 3, 4)$ $\Rightarrow \begin{cases} x = 1 + 2\lambda \\ y = 3\lambda \\ z = -1 + 4\lambda \end{cases}$

$\pi.\ 2x - 2y + 3z + 1 = 0$

$2(1 + 2\lambda) - 2(3\lambda) + 3(-1 + 4\lambda) + 1 = 0$

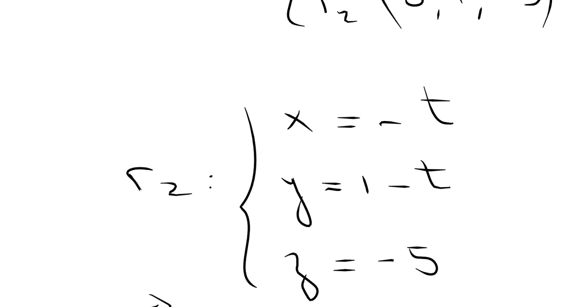
$2 + 4\lambda - 6\lambda - 3 + 12\lambda + 1 = 0$

$10\lambda = 0 \Rightarrow \lambda = 0 \longrightarrow \begin{cases} x = 1 \\ y = 0 \\ z = -1 \end{cases}$

$L \cap \pi = \{(1, 0, -1)\}$

26 $r_1 : (x, y, z) = (-t, 6, t)$ $\quad t \in \mathbb{R}$

$r_2 \begin{cases} (1, 2, -5) \\ (0, 1, -5) \end{cases}$



$r_1 : \begin{cases} \bar{u}_1 = (-1, 0, 1) \\ P_1 \quad (0, 6, 0) \end{cases}$ $\qquad r_2 : \begin{cases} \bar{u}_2 = \overline{P_1P_2} = (-1, -1, 0) \\ P_2 (0, 1, -5) \end{cases}$

$r_1 \cap r_2 \to$ $r_1 : \begin{cases} x = 0 - \lambda \\ y = 6 \\ z = 0 + \lambda \end{cases}$ $\qquad r_2 : \begin{cases} x = -t \\ y = 1 - t \\ z = -5 \end{cases}$

No puedo usar λ en r_2 porque ya lo usé en r_1

$r_1 \cap r_2 \quad \begin{cases} -\lambda = -t \\ 6 = 1 - t \\ \lambda = -5 \to t = -5 \end{cases}$ $\qquad 6 = 1 - (-5)$ $\quad \underline{6 = 6}$

$\lambda = -5 \Rightarrow r_1 \Rightarrow (5, 6, -5)$ $\qquad t = -5 \Rightarrow r_2 \Rightarrow (5, 6, -5)$

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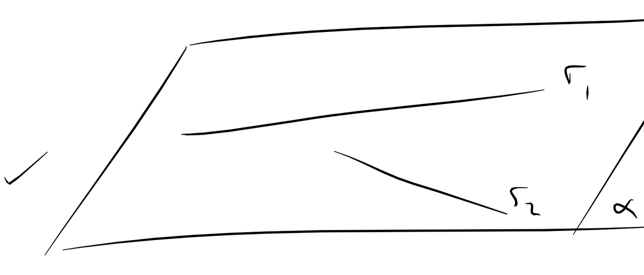
$r_1 \begin{cases} x = 3 - 2\lambda \\ y = 4 - 2\lambda \\ z = 3\lambda \end{cases}$ $\qquad r_2 \begin{cases} x = t \\ y = 2t \\ z = 3t \end{cases}$

$\alpha : \begin{cases} P_0 \\ \vec{n} \end{cases}$ ✓



$\bar{n} = \bar{u}_1 \times \bar{u}_2$

$\bar{n} = (-2, -2, 3) \times (1, 2, 3) =$

$= (-12, 9, -2)$

$-12x + 9y - 2z + D = 0$

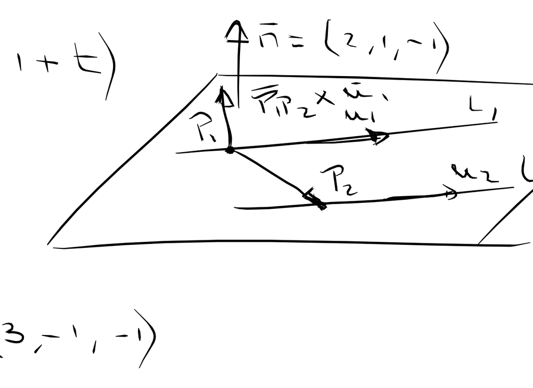
$(0, 0, 0) \in \alpha \Rightarrow -12.0 + 9.0 - 2.0 + D = 0 \Rightarrow D = 0$

$\boxed{-12x + 9y - 2z = 0}$

Hallar el plano que contiene a las rectas.

$L_1 \quad (x, y, z) = (2 + t, 3 - t, 1 + t)$

$L_2: \quad \dfrac{x - 3}{2} = \dfrac{y + 1}{-2} = \dfrac{z + 1}{2}$



$P_1(2, 3, 1)$ $\qquad P_2(3, -1, -1)$

$\overline{P_1P_2} = (1, -4, -2)$

$\overline{P_1P_2} \times \bar{u}_1 = \begin{vmatrix} i & j & k \\ 1 & -4 & -2 \\ 1 & -1 & 1 \end{vmatrix} = (-6, -3, 3) \parallel (2, 1, -1)$

$P(2, 3, 1)$ $\qquad 2x + y - z + D = 0$

$2.2 + 3 - 1 + D = 0 \Rightarrow D = -6$

$2x + y - z - 6 = 0$