4.7. P antisimétrica, n impar $\quad P = -P^t \quad P \in \mathbb{R}^{n\times n}$

$|P| = |-P^t| = (-1)^n |P^t| = (-1)^n |P|$

$|P| = (-1)^n |P| \qquad n$ impar $\Rightarrow (-1)^n = -1$

$|P| = -|P| \Leftrightarrow 2|P| = 0 \Rightarrow \boxed{|P| = 0}$

T.P. N°3

b) $W = \{ p \in P_4 \,/\, p = 0$ ó $gr(p) = 2 \}$ $\qquad W$ subespacio: 0_v, $(p+q) \in W$, $\alpha . p \in W$

$p \in W \Rightarrow p(x) = a_2 x^2 + a_1 x + a_0$

1) se cumple.

2) $p(x) \in W \Rightarrow p(x) = a_2 x^2 + a_1 x + a_0 \ \wedge\ a_2 \neq 0$

$q(x) \in W \Rightarrow q(x) = b_2 x^2 + b_1 x + b_0 \ \wedge\ b_2 \neq 0$

$p(x) + q(x) = \underbrace{(a_2 + b_2)}_{\neq 0} x^2 + (a_1 + b_1) x + (a_0 + b_0)$

$p(x) = x^2 + 2$

$q(x) = -x^2 + 5x + 6$

$p(x) + q(x) = 5x + 8 \notin W$

3d) $W = \{ A \in \mathbb{R}^{3\times 3} / tr(A) = 0 \}$ $\qquad tr(A) = a_{11} + a_{22} + a_{33} = 0$

1) $0_{3\times 3} \in W$ ✓ pues $tr(0_{3\times 3}) = 0$

2) $A \in W \Rightarrow tr(A) = 0$

$B \in W \Rightarrow tr(B) = 0$

¿$A + B \in W$? $\quad$ Si $A + B \in W \Rightarrow tr(A+B) = 0$

$tr(A+B) = (a_{11} + b_{11}) + (a_{22} + b_{22}) + (a_{33} + b_{33}) =$

$= (a_{11} + a_{22} + a_{33}) + (b_{11} + b_{22} + b_{33}) =$

$= 0 + 0 = 0 \Rightarrow A + B \in W$

3) $\alpha . A \in W \qquad tr(\alpha . A) = \alpha a_{11} + \alpha a_{22} + \alpha a_{33} =$

$= \alpha \underbrace{(a_{11} + a_{22} + a_{33})}_{0} = \alpha\, 0 = 0 \Rightarrow$

$\alpha A \in W$

Dependencia Lineal:

$\{\bar{u}_1, \bar{u}_2, \bar{u}_3, \dots, \bar{u}_n\}$ n vectores

$\alpha_1 \bar{u}_1 + \alpha_2 \bar{u}_2 + \alpha_3 \bar{u}_3 + \dots + \alpha_n \bar{u}_n \Rightarrow$ combinación lineal.

$\bar{v} \,/\, \bar{v} = \alpha_1 \bar{u}_1 + \alpha_2 \bar{u}_2 + \dots + \alpha_n \bar{u}_n$. Si existen los $\alpha_i \in \mathbb{R} \Rightarrow \bar{v}$ es combinación lineal de los vectores u_i

Ej: Decidir si el vector $(2,5,3)$ es C.L de los vectores $\{(1,1,1), (1,0,1), (0,0,1)\}$

$(2,5,3) = \alpha_1 (1,1,1) + \alpha_2 (1,0,1) + \alpha_3 (0,0,1)$

$(2,5,3) = (\alpha_1, \alpha_1, \alpha_1) + (\alpha_2, 0, \alpha_2) + (0, 0, \alpha_3)$

$(2,5,3) = (\alpha_1 + \alpha_2, \alpha_1, \alpha_1 + \alpha_2 + \alpha_3)$

$\begin{cases} 2 = \alpha_1 + \alpha_2 & \Rightarrow \alpha_2 = -3 \\ 5 = \alpha_1 \\ 3 = \alpha_1 + \alpha_2 + \alpha_3 & \Rightarrow \alpha_3 = 1 \end{cases}$

$(2,5,3) = 5.(1,1,1) + (-3)(1,0,1) + 1.(0,0,1)$

5b $\quad p(t) = t^2 + 4t - 3 \quad$ C.L

$\{ t^2 - 2t + 5,\ 2t^2 - 3t,\ t + 3 \}$

$t^2 + 4t - 3 = \alpha_1 (t^2 - 2t + 5) + \alpha_2 (2t^2 - 3t) + \alpha_3 (t + 3)$

$t^2 + 4t - 3 = \alpha_1 t^2 - 2\alpha_1 t + 5\alpha_1 + 2\alpha_2 t^2 - 3\alpha_2 t + \alpha_3 t + 3\alpha_3$

$t^2 + 4t - 3 = t^2 (\alpha_1 + 2\alpha_2) + t(-2\alpha_1 - 3\alpha_2 + \alpha_3) + (5\alpha_1 + 3\alpha_3)$

$\begin{cases} 1 = \alpha_1 + 2\alpha_2 \\ 4 = -2\alpha_1 - 3\alpha_2 + \alpha_3 \\ -3 = 5\alpha_1 + 3\alpha_3 \end{cases} \Rightarrow \begin{cases} \alpha_1 = -3 \\ \alpha_2 = 2 \\ \alpha_3 = 4 \end{cases}$

$\alpha_1 \bar{u}_1 + \alpha_2 \bar{u}_2 + \alpha_3 \bar{u}_3 + \dots + \alpha_n \bar{u}_n = 0_v$

- sistema determinado $\Rightarrow \bar{u}_1, \bar{u}_2, \dots, \bar{u}_n \rightarrow$ son linealmente independientes.

$\{\bar{u}_1, \bar{u}_2, \bar{u}_3, \dots, \bar{u}_n\}$ conjunto linealmente independiente L.I

- sistema indeterminado $\Rightarrow \bar{u}_1, u_2, \dots, \bar{u}_n$ son linealmente dependientes L.D

$\{u_1, u_2, \dots, u_n\}$ conjunto linealmente dependiente L.D

Ej: Analizar la dependencia de los siguientes vectores.

1) $\{(1,2,1), (1,0,1), (1,-1,0)\}$

$\alpha_1 (1,2,1) + \alpha_2 (1,0,1) + \alpha_3 (1,-1,0) = (0,0,0)$

$(\alpha_1, 2\alpha_1, \alpha_1) + (\alpha_2, 0, \alpha_2) + (\alpha_3, -\alpha_3, 0) = (0,0,0)$

$(\alpha_1 + \alpha_2 + \alpha_3, 2\alpha_1 - \alpha_3, \alpha_1 + \alpha_2) = (0,0,0)$

$\begin{cases} \overbrace{\alpha_1 + \alpha_2}^{0} + \alpha_3 = 0 \\ 2\alpha_1 - \alpha_3 = 0 \\ \alpha_1 + \alpha_2 = 0 \end{cases}$ $\Rightarrow \alpha_3 = 0 \Rightarrow \alpha_1 = 0 \Rightarrow \alpha_2 = 0$

S.C.D $\Rightarrow$ Los vectores son L.I

$\left\{ \begin{pmatrix} -2 & 3 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ -2 & 0 \end{pmatrix}, \begin{pmatrix} -2 & 5 \\ -3 & 0 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \right\}$ L.D. $\rightarrow$ Existe al menos un vector que es C.L. del resto

$\begin{cases} -2\alpha_1 - 2\alpha_3 + \alpha_4 = 0 \\ 3\alpha_1 + \alpha_2 + 5\alpha_3 = 0 \\ \alpha_1 - 2\alpha_2 - 3\alpha_3 = 0 \end{cases}$ $\Rightarrow$ S.C.I $\Rightarrow$ tiene infinitas soluciones.

$\left(\begin{array}{cccc|c} -2 & 0 & -2 & 1 & 0 \\ 3 & 1 & 5 & 0 & 0 \\ 1 & -2 & -3 & 0 & 0 \end{array}\right) \begin{array}{l} \\ E_2 \\ E_3 \end{array}$

$\left(\begin{array}{cccc|c} -2 & 0 & -2 & 1 & 0 \\ 0 & 2 & 4 & 3 & 0 \\ 0 & -4 & -8 & 1 & 0 \end{array}\right) \begin{array}{l} \\ E_2' = 2E_2 + 3E_1 \\ E_3' = 2E_3 + E_1 \end{array}$

$\left(\begin{array}{cccc|c} -2 & 0 & -2 & 1 & 0 \\ 0 & 2 & 4 & 3 & 0 \\ 0 & 0 & 0 & 7 & 0 \end{array}\right) \quad E_3'' = 2E_2' + E_3'$

$-2\alpha_1 - 2\alpha_3 + \cancel{\alpha_4} = 0$

$2\alpha_2 + 4\alpha_3 + \cancel{3\alpha_4} = 0$

$\cancel{7\alpha_4 = 0}$

$\begin{cases} \alpha_1 + \alpha_3 = 0 \\ \alpha_2 + 2\alpha_3 = 0 \end{cases} \Rightarrow \begin{array}{l} \alpha_1 = -\alpha_3 \\ \alpha_2 = -2\alpha_3 \\ \alpha_4 = 0 \end{array}$

$\left\{ \begin{pmatrix} -2 & 3 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ -2 & 0 \end{pmatrix}, \begin{pmatrix} -2 & 5 \\ -3 & 0 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \right\}_{L.D.}$

$\underbrace{\begin{pmatrix} -2 & 3 \\ 1 & 0 \end{pmatrix}}_{\bar{u}} = \overset{-2}{\alpha_1} \underbrace{\begin{pmatrix} 0 & 1 \\ -2 & 0 \end{pmatrix}}_{\bar{v}_1} + \overset{1}{\alpha_2} \underbrace{\begin{pmatrix} -2 & 5 \\ -3 & 0 \end{pmatrix}}_{\bar{v}_2} + \overset{0}{\alpha_3} \underbrace{\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}}_{\bar{v}_3}$

$\begin{cases} -2 = -2\alpha_2 + \alpha_3 \\ 3 = \alpha_1 + 5\alpha_2 \\ 1 = -2\alpha_1 - 3\alpha_2 \\ 0 = 0 \end{cases}$

$\bar{u} = -2v_1 + 1.v_2 + \overset{\bar{0}}{\cancel{0 v_3}}$

$\bar{u} + 2v_1 = v_2$

$2v_1 = v_2 - \bar{u}$

$v_1 = \frac{1}{2}(v_2 - \bar{u}) = \frac{1}{2} v_2 - \frac{1}{2} \bar{u}$

6. d) L.D.

e) $\alpha . 0_v = 0_v \qquad$ L.I (única solución)

$\{0_v\}_{LD}$.

$\{(1,1,2), (0,0,0), (1,1,5)\}_{L.D}$

7) $\alpha_1 \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + \alpha_2 \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} + \alpha_3 \begin{pmatrix} 0 & k \\ 1 & -1 \end{pmatrix} + \alpha_4 \begin{pmatrix} 0 & 3 \\ -k & 1 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$

$\begin{vmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & k & 3 \\ 0 & 1 & 1 & -k \\ 1 & 0 & -1 & 1 \end{vmatrix} \neq 0$

8) a) $\bar{u} \neq \bar{0} \wedge \bar{v} \neq \bar{0} \Rightarrow \{\bar{u}, \bar{v}, \underbrace{\bar{u} \times \bar{v}}_{\bar{0}}\}_{LI}$ (F)

si $\bar{u} // \bar{v} \Rightarrow \bar{u} \times \bar{v} = \bar{0} \Rightarrow \{\bar{u}, \bar{v}, \bar{u} \times \bar{v}\}_{LD}$

b) $\{u, v\}_{LI} \quad \{u, w\}_{LI} \quad \{v, w\}_{LI} \Rightarrow$

$\Rightarrow \{u, v, w\}_{LI}$

$\{(1,0), (0,1)\} \quad \{(1,0), (1,1)\} \quad \{(0,1), (1,1)\} \Rightarrow$

$\Rightarrow \{(1,0), (0,1), (1,1)\}_{L.D}$ (F)

$(1,1) = 1.(1,0) + 1.(0,1)$

c) $\{\bar{u}, \bar{v}, \bar{w}\}_{L.I} \Rightarrow \{\bar{u} + \bar{v}, \bar{v} + \bar{w}, \bar{w}\}_{L.I}$ (V)

$\underset{0}{\alpha_1} (\bar{u} + \bar{v}) + \underset{0}{\alpha_2} (\bar{v} + \bar{w}) + \underset{0}{\alpha_3} \bar{w} = 0_v$

$\alpha_1 \bar{u} + \alpha_1 \bar{v} + \alpha_2 \bar{v} + \alpha_2 \bar{w} + \alpha_3 \bar{w} = 0_v$

$\alpha_1 \bar{u} + \bar{v}(\alpha_1 + \alpha_2) + \bar{w}(\alpha_2 + \alpha_3) = 0_v$

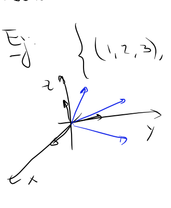
$\begin{cases} \alpha_1 = 0 \ ✓ \\ \alpha_1 + \alpha_2 = 0 \\ \alpha_2 + \alpha_3 = 0 \end{cases} \Rightarrow \alpha_2 = 0 \Rightarrow \alpha_3 = 0$

Generadores:

$H = \{v_1, v_2, \dots, v_n\} \subseteq V$

H es un sistema de generadores de V sii todo vector de V, se puede escribir como una C.L de los vectores de H.

Ej: $\{(1,2,3), (1,1,1), (0,1,1)\}_{LI}$ ¿Es un sistema de generadores de $\mathbb{R}^3$?



$\alpha_1 (1,2,3) + \alpha_2 (1,1,1) + \alpha_3 (0,1,1) = (0,0,0)$

$\begin{cases} \alpha_1 + \alpha_2 = 0 \\ 2\alpha_1 + \alpha_2 + \alpha_3 = 0 \\ 3\alpha_1 + \alpha_2 + \alpha_3 = 0 \end{cases} \qquad \begin{vmatrix} 1 & 1 & 0 \\ 2 & 1 & 1 \\ 3 & 1 & 1 \end{vmatrix} \neq 0 \quad$ LI

9) $A = \{(1,2,3), (0,1,1)\}_{LI} \rightarrow$ genera un subespacio de $\mathbb{R}^3$ $\rightarrow$ plano

$\bar{v} \in A \qquad \bar{v} = (-2,-1,-3)$

$(-2,-1,-3) = \alpha_1 (1,2,3) + \alpha_2 (0,1,1)$

$\begin{cases} -2 = \alpha_1 \xrightarrow{E_2} \alpha_2 = 3 \\ -1 = 2\alpha_1 + \alpha_2 \\ -3 = 3\alpha_1 + \alpha_2 \end{cases}$

$-3 = 3(-2) + 3$ ✓ $\Rightarrow \bar{v} \in A$

Hallar el subespacio gen$\{(1,2,3), (0,1,1)\}$

$(x, y, z) = \alpha_1 (1,2,3) + \alpha_2 (0,1,1)$

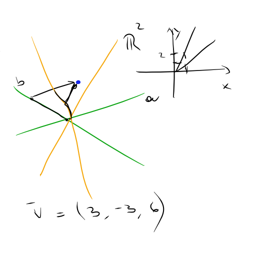
$\begin{cases} x = \alpha_1 \\ y = 2\alpha_1 + \alpha_2 \\ z = 3\alpha_1 + \alpha_2 \end{cases} \Rightarrow - \begin{cases} y = 2x + \alpha_2 \\ z = 3x + \alpha_2 \end{cases}$

$y - z = -x \Rightarrow x + y - z = 0$

$A = \{ \bar{x} \in \mathbb{R}^3 / x + y - z = 0 \}$

b) $A = \{(1,2), (1,1)\}_{L.I} \quad \mathbb{R}^2 \quad \bar{v} = (3,4)$

gen$\{(1,2), (1,1)\} = \mathbb{R}^2$



c) $A = \{(1,-1,-2), (-2,2,4)\}_{LD} \qquad \bar{v} = (3,-3,6)$

$-2(1,-1,-2) = (-2,2,4)$

$\{(1,-1,-2)\} \rightarrow$ genera una recta $(3,-3,6)$

$(3,-3,6) = \alpha (1,-1,-2)$

$3 = \alpha \qquad 6 = -2\alpha$

$-3 = -\alpha$

Base y dimensión

Una base es un conjunto de generadores L.I

$B = \{v_1, v_2, v_3, \dots v_n\} \qquad [V] \rightarrow$ base del E.V V.

Dimensión = cantidad de vectores que hay en la base.

$\dim V = n$.

E.V	Base Canónica (E)	$\dim V$
$\{0_v\}$	—	0
$\mathbb{R}^2$	$\{(1,0), (0,1)\}$	2
$\mathbb{R}^3$	$\{(1,0,0), (0,1,0), (0,0,1)\}$	3
$\mathbb{R}^4$	$\{(1,0,0,0), (0,1,0,0), (0,0,1,0), (0,0,0,1)\}$	4
$\vdots$	$\vdots$	
$\mathbb{R}^n$	$\{(1,0,0,\dots,0), (0,1,0,\dots,0), \dots (0,0,\dots,1,0), (0,0,\dots,0,1)\}$	n
P_0	$\{1\}$	1
P_1	$\{1, x\}$	2
P_2	$\{1, x, x^2\}$	3
$\vdots$	$\vdots$	$\vdots$
P_n	$\{1, x, x^2, x^3, \dots, x^n\}$	$n+1$
$\mathbb{R}^{2\times 2}$	$\left\{\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}\right\}$	4
$\mathbb{R}^{3\times 2}$	$\left\{\begin{pmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 0 & 1 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 1 \end{pmatrix}\right\}$	6
$\vdots$		
$\mathbb{R}^{n\times m}$	$\left\{\begin{pmatrix} 1 & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 \\ \vdots & & & \vdots \\ 0 & 0 & \cdots & 0 \end{pmatrix}, \begin{pmatrix} 0 & 1 & \cdots & 0 \\ 0 & 0 & \cdots & 0 \\ \vdots & & & \vdots \\ 0 & 0 & \cdots & 0 \end{pmatrix}, \dots \begin{pmatrix} 0 & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 \\ \vdots & & & \vdots \\ 0 & 0 & \cdots & 1 \end{pmatrix}\right\}$	$n.m$

Coordenadas:

$B \rightarrow$ base $\quad B = \{v_1, v_2, v_3, \dots, v_n\}$

Las coordenadas de un vector $\bar{u}$, en la base B,

$[\bar{u}]_B = \begin{pmatrix} \alpha_1 \\ \alpha_2 \\ \vdots \\ \alpha_n \end{pmatrix} \Big/ \ \bar{u} = \alpha_1 v_1 + \alpha_2 v_2 + \dots + \alpha_n v_n$.

Ej: 12 a) $B = \{(1,1,0), (0,1,1), (1,0,1)\}$

$\begin{vmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{vmatrix} \neq 0$

$[(1,2,-1)]_B \qquad (1,2,-1) = \alpha_1 (1,1,0) + \alpha_2 (0,1,1) + \alpha_3 (1,0,1)$

$- \begin{cases} 1 = \alpha_1 + \alpha_3 \\ 2 = \alpha_1 + \alpha_2 \\ -1 = \alpha_2 + \alpha_3 \end{cases} \qquad \begin{array}{l} -1 = \alpha_3 - \alpha_2 \\ -1 = \alpha_2 + \alpha_3 \ + \\ -2 = 2\alpha_3 \end{array}$

$[(1,2,-1)]_B = \begin{pmatrix} 2 \\ 0 \\ -1 \end{pmatrix} \qquad \alpha_1 = 2 \Leftarrow \alpha_2 = 0 \Leftarrow \boxed{\alpha_3 = -1}$

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a) $B = \{ \underbrace{x^3 - 3x^2 + 3x - 1}_{P_1}, \underbrace{x^2 - 1}_{P_2}, \underbrace{x - 1}_{P_3}, \underbrace{1}_{P_4} \}$ $\quad$ base: L.I $\rightarrow$ base.

$\alpha_1 P_1 + \alpha_2 P_2 + \alpha_3 P_3 + \alpha_4 P_4 = 0_{(x)}$

$\begin{array}{l} x^3 \rightarrow \\ x^2 \rightarrow \\ x \rightarrow \\ T.I \rightarrow \end{array} \begin{cases} \alpha_1 = 0 \\ -3\alpha_1 + \alpha_2 = 0 \rightarrow \alpha_2 = 0 \\ 3\alpha_1 + \alpha_3 = 0 \rightarrow \alpha_3 = 0 \\ -\alpha_1 - \alpha_2 - \alpha_3 + \alpha_4 = 0 \rightarrow \alpha_4 = 0 \end{cases}$

$[2x^3 - 3x^2 + 4]_B = \begin{pmatrix} \gamma_1 \\ \gamma_2 \\ \gamma_3 \\ \gamma_4 \end{pmatrix} \qquad \begin{cases} \alpha_1 = 2 \\ -3\alpha_1 + \alpha_2 = -3 \Rightarrow \alpha_2 = 3 \\ 3\alpha_1 + \alpha_3 = 0 \Rightarrow \alpha_3 = -6 \\ -\alpha_1 - \alpha_2 - \alpha_3 + \alpha_4 = 4 \Rightarrow \alpha_4 = 3 \end{cases}$

$-2 - 3 + 6$

$[2x^3 - 3x^2 + 4]_B = \begin{pmatrix} 2 \\ 3 \\ -6 \\ 3 \end{pmatrix}$

$B = \{(1,1,1), (1,1,0), (1,1,-1)\}$

Hallar $\bar{u} \,/\, [\bar{u}]_B = \begin{pmatrix} 0 \\ -2 \\ 1 \end{pmatrix}$

$\bar{u} = 0.\cancel{(1,1,1)} + (-2)(1,1,0) + 1.(1,1,-1)$

$\bar{u} = (-2,-2,0) + (1,1,-1)$

$\bar{u} = (-1,-1,-1)$ ✓

Operaciones con subespacios

1) Intersección: $\cap$

S_1 subespacio; S_2 subespacio

$S_1 \cap S_2 = \{ \bar{x} \,/\, \bar{x} \in S_1 \wedge \bar{x} \in S_2 \} \rightarrow$ es un subespacio.

Ej: $S_1 = \{ \bar{x} \in \mathbb{R}^4 / x_1 + x_2 - x_3 = 0 = x_2 + x_4 \}$

$S_2 = \{ \bar{x} \in \mathbb{R}^4 / x_1 - x_2 - x_4 = 0 = x_3 + x_2 \}$

$S_1 \cap S_2 = \{ \bar{x} \in \mathbb{R}^4 / x_1 + x_2 - x_3 = 0 = x_2 + x_4 \wedge x_1 - x_2 - x_4 = 0 = x_3 + x_2 \}$

base y dimensión:

$\begin{cases} x_1 + x_2 - x_3 = 0 \\ x_2 + x_4 = 0 \\ x_1 - x_2 - x_4 = 0 \\ x_3 + x_2 = 0 \end{cases} \qquad S_1 \cap S_2 = \{(0,0,0,0)\}$

No tiene base.

$\dim(S_1 \cap S_2) = 0$

$S_1 = \{ \bar{x} \in \mathbb{R}^4 / x_1 + x_2 - x_3 = 0 \}$

$S_2 = \{ \bar{x} \in \mathbb{R}^3 / x_1 + x_2 - x_4 = 0 \}$

$S_1 \cap S_2$; $[S_1 \cap S_2]$; $\dim(S_1 \cap S_2)$

$S_1 \cap S_2 = \{ \bar{x} \in \mathbb{R}^4 / x_1 + x_2 - x_3 = 0 = x_1 + x_2 - x_4 \}$

$S_1 \cap S_2 \rightarrow \begin{cases} x_1 + x_2 - x_3 = 0 \Rightarrow x_1 + x_2 = x_3 \\ x_1 + x_2 - x_4 = 0 \Rightarrow x_1 + x_2 = x_4 \end{cases}$

$S_1 \cap S_2 = \{ \bar{x} \in \mathbb{R}^4 / x_3 = x_4 = x_1 + x_2 \}$

$\bar{x} \in (S_1 \cap S_2) \Rightarrow \bar{x} = (x_1, x_2, x_1 + x_2, x_1 + x_2)$

$\bar{x} = (x_1, 0, x_1, x_1) + (0, x_2, x_2, x_2)$

$\bar{x} = x_1 (1,0,1,1) + x_2 (0,1,1,1)$

$S_1 \cap S_2 = $ gen$\{(1,0,1,1), (0,1,1,1)\}$

$\{(1,0,1,1), (0,1,1,1)\}_{LI}$

$[S_1 \cap S_2] = \{(1,0,1,1), (0,1,1,1)\}$

$\dim(S_1 \cap S_2) = 2$

$S_1 = \{ \bar{x} \in \mathbb{R}^3 / x_1 - 2x_2 + x_3 = 0 \}$

$S_2 = \{ \bar{x} \in \mathbb{R}^3 / x_1 + x_2 - x_3 = 0 \}$

$S_1 \cap S_2$; $[S_1 \cap S_2]$; $\dim(S_1 \cap S_2)$

$S_1 \cap S_2 = \{ \bar{x} \in \mathbb{R}^3 / x_1 - 2x_2 + x_3 = 0 = x_1 + x_2 - x_3 = 0 \}$

$\begin{cases} x_1 - 2x_2 + x_3 = 0 \\ x_1 + x_2 - x_3 = 0 \end{cases} \ + \rightarrow 2x_1 - x_2 = 0 \qquad x_2 = 2x_1$

$- \rightarrow -3x_2 + 2x_3 = 0 \qquad 2x_3 = 3x_2 \qquad x_3 = \frac{3}{2} x_2$

$S_1 \cap S_2 = \{ \bar{x} \in \mathbb{R}^3 / x_2 = 2x_1 \wedge x_3 = \frac{3}{2} x_2 \}$

$\bar{x} \in (S_1 \cap S_2) \Rightarrow \bar{x} = (x_1, 2x_1, \frac{3}{2} . 2x_1) =$

$= (x_1, 2x_1, 3x_1) = x_1 (1,2,3)$

$S_1 \cap S_2 = $ gen$\{(1,2,3)\} \qquad 3.(\frac{1}{3}, \frac{2}{3}, 1)$

$[S_1 \cap S_2] = \{(1,2,3)\} \qquad \dim(S_1 \cap S_2) = 1$