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Wave Height Variation Across Beaches of Arbitrary Profile

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An intuitive expression for the spatial change in energy flux associated with waves breaking in the surf zone is developed. Using shallow water linear wave theory, analytical solutions for wave height transformation due to shoaling and breaking on a flat shelf, a plane slope, and an "equilibrium" beach profile are derived and then compared to laboratory data with favorable results. The effect of beach slope on wave decay is included explicitly, while wave steepness effects are included implicitly by specification of the incipient conditions. Set-down/set-up in the mean water level, bottom friction losses, and bottom profiles of arbitrary shape are introduced, and solutions are obtained numerically. The model is calibrated and verified using laboratory data with very good results for the wave decay but not so favorable results for set-up. A test run on a prototype scale profile containing two bar and trough systems demonstrates the model's ability to describe the shoaling, breaking, and wave re-forming process commonly observed in nature. Bottom friction is found to play a negligible role in wave decay in the surf zone when compared to shoaling and breaking.

INTRODUCTION

A major problem encountered in modeling nearshore wave-induced phenomena is the description of wave parmeters subsequent to the initiation of wave breaking. Specifically, wave height and its spatial gradients generate or have direct impact on sediment mobilization and suspension, littoral currents in both the alongshore and onshore/offshore directions, wave-induced set-down/set-up in the mean water level, and forces on coastal structures. While the "0.78" criterion (ratio of breaker height to water depth equals 0.78) appears to provide a reasonable prediction of incipient breaking on mildly sloping beaches, data show that this criterion does not hold farther into the surf zone [*Horikawa and Kuo*, 1966; *Nakamura et al.*, 1966; *Street and Camfield*, 1966; *Divoky et al.*, 1970]. In fact, these data show that such a similarity model is especially inappropriate on mild slopes, just where many coastal scientists assume that it is most valid. Another shortcoming of this and most other representations developed to date is that they are not applicable on nonmonotonic beach profiles such as those containing bar/trough formations. Such a model, capable of describing wave transformation across beaches of irregular profile shape, is essential to an adequate understanding of nearshore hydrodynamics and sediment transport.

During the past two decades a number of laboratory, field, and analytical studies have been carried out to develop a realistic model of wave height transformation across the surf zone. The steady state equation governing energy balance for waves advancing directly toward shore is

$$\partial ECg/\partial x = -\delta(x) \tag{1}$$

in which E is the wave energy per unit surface area, Cg is the group velocity, and δ is the energy dissipation rate per unit surface area due to boundary shear, turbulence due to breaking, etc. The central problem in previous studies has been the development of a rational and universally valid formulation for δ. The most physically appealing approach, first advanced by *Le Méhauté* [1962], has been the approximation of a breaking wave as a propagating bore (or hydraulic jump), in which case, δ is given by

$$\delta = \frac{\rho g}{4}\frac{(BH)^3}{h^2}Q \tag{2}$$

where ρ is the mass density of water, g is gravity, H is wave height, h is the water depth, Q is the transport of water across the bore, and B is a parameter representing the fraction of the wave height that is due to breaking. Equation (2) in slightly different forms has been used to represent periodic water waves in the laboratory [*Divoky et al.*, 1970; *Hwang and Divoky*, 1970; *Svendsen et al.*, 1978; *Svendsen*, 1984], aperiodic water waves in the laboratory [*Battjes and Janssen*, 1978], and aperiodic waves in nature [*Thornton and Guza*, 1983].

Other approaches for wave energy dissipation have included that of *Horikawa and Kuo* [1966] in which the internal energy dissipation is represented in terms of turbulent velocity fluctuations which are assumed to decay exponentially with distance from the wave break point. A different recent approach by *Mizuguchi* [1981] applies the analytical solution for internal energy dissipation due to viscosity [*Lamb*, 1932] with the molecular kinematic viscosity replaced by the eddy viscosity, which must be estimated based on the wave and beach profile characteristics.

Understandably, the extension of approaches developed for periodic waves to aperiodic waves introduces complexities, primarily with respect to representation of the probability distribution function. *Battjes and Janssen* [1978], in a laboratory study of breaking random waves over plane and barred beaches, employed (2), interpreting H as H_{rms}, and introduced two assignable constants. A truncated Rayleigh wave height distribution was assumed with a finite probability of the maximum truncating wave height occurring, resulting in a delta function at this limit. Battjes and Janssen included the effect of wave set-up and demonstrated good agreement between the predicted and measured wave height distribution. Following *Collins* [1970], *Kuo and Kuo* [1974] modified the truncated wave height probability distribution by omitting the delta

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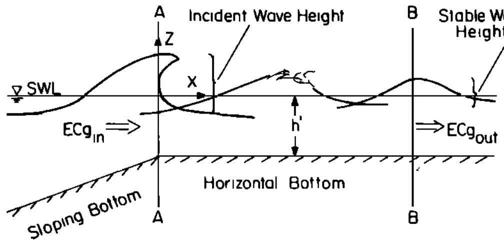

Fig. 1. Shelf beach idealization of the surf zone. Breaking is initiated at section A-A and continues until the stable wave condition is attained at section B-B.

function at the upper wave height. *Goda* [1975] adopted a modified probability distribution in which the distribution decreases linearly to zero over a specified upper range of the wave heights. On the basis of field measurements, *Thornton and Guza* [1983] present convincing data that the unmodified Rayleigh distribution is applicable seaward of and across the entire surf zone. At each location within the surf zone they define a subpopulation for the waves that are breaking.

None of the models developed and evaluated to date provide a demonstrated, completely general capability for representing wave transformation across the surf zone. Certainly, the waves in this zone are not linear, and some of the difficulty may be in the application of linear theory, although other theories (cnoidal and solitary) have been used by some investigators.

One of the features that is not represented in most models is that of the wave height stabilizing at some value in a uniform depth following the initiation of breaking. The laboratory data of *Horikawa and Kuo* [1966], general observations, and intuition support such a phenomenon, yet none of the energy dissipation models based on the moving hydraulic jump predict this effect. Although the model by *Mizuguchi* [1981] includes this stabilization, the difficulties of estimating the representative eddy viscosity and of rationally applying the model on a beach of nonuniform slope argue for a simpler model.

In the present paper we concentrate on the development and evaluation of a somewhat intuitive model originally proposed by *Dally* [1980] which includes the wave height "stabilization." Analytical solutions for wave decay due to breaking on a flat shelf, a plane slope, and an equilibrium beach profile are derived. Bottom friction, wave-induced set-up, and beach profiles of arbitrary shape are introduced, and the governing equations are transformed for numerical solution. The model is then calibrated using laboratory data, verified both qualitatively and quantitatively, and tested at prototype scale. Although one might question the value of a regular wave model and the significance of comparison against laboratory data when several random wave models and some field data are available [e.g., *Thornton and Guza*, 1983], the field data are much more limited. Also, because these data contain both breaking and nonbreaking waves, the dependencies of breaker decay on beach slope and wave steepness appear only as trends which, although acknowledged (see, for example, *Thornton et al.* [1985] and *Battjes and Janssen* [1978]), are not included in the formulation of the random wave models. These dependencies are clearly discernible and tractable in the monochromatic laboratory data, and the role of beach slope even appears explicitly in the analytical solutions of the model described herein. Thus it seems that there is still much to be learned from a regular wave study before moving on to the problem of greater interest in nature.

MODEL DEVELOPMENT

Consider a beach profile that rises from deep water in a gently sloping manner and at some point in shallow water becomes horizontal (see Figure 1). Consider further a wave propagating onto this profile with characteristics such that breaking starts at the point where the bottom becomes horizontal. The wave will not instantaneously stop breaking because the bottom becomes horizontal (as dictated by the "0.78" criterion), but breaking would continue until some stable wave height is attained. Breaking would be most intense just shoreward of line AA and would decrease until the approximate stable wave height is reached at line BB. The rate of energy dissipation per unit plan area $\delta(x)$ used in (1) is assumed to be proportional to the difference between the local energy flux and the stable energy flux, that is,

$$\frac{\partial ECg}{\partial x} = \frac{-K}{h'} [ECg - ECg_s] \tag{3}$$

ECg is now taken to be the depth-integrated, time-averaged energy flux as given by shallow water linear wave theory, K is a dimensionless decay coefficient, h' is the still water depth, and ECg_s is the energy flux associated with the stable wave that the breaking wave is striving to attain.

Horikawa and Kuo [1966] conducted laboratory tests with a bottom configuration identical to the one described above. As shown in Figure 2, their data indicate a stable wave criterion given by

$$H_s = \Gamma h' \tag{4}$$

where H_s is the stable wave height and Γ is a dimensionless coefficient whose value appears to lie somewhere between 0.35 and 0.40.

Examination of another figure in their paper (see Figure 3 of this paper), where wave height was plotted versus still water depth for a uniform beach slope of 1/65, revealed that the breaking waves tended to approach asymptotically the line $H = 0.5\ h'$. In any event, (4) appears to be a reasonable supposition, and (3) can then be written:

$$\frac{\partial[H^2(h')^{1/2}]}{\partial x} = \frac{-K}{h'} [H^2(h')^{1/2} - \Gamma^2(h')^{5/2}] \tag{5}$$

where Cg is taken as $(gh')^{1/2}$. It should be noted that (3), (4), and (5) can be applied to a bottom of varying depth and slope

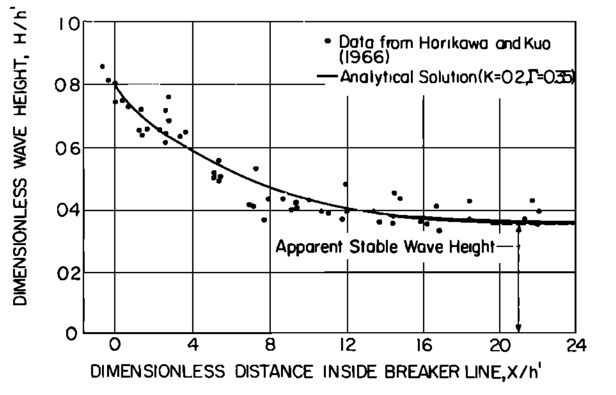

Fig. 2. Experimental results of *Horikawa and Kuo* [1966] for waves breaking on a shelf, as shown in Figure 1. The waves approach the stable wave height indicated. Analytical solution (18) with $K = 0.2$ and $\Gamma = 0.35$ is displayed as the solid line.

because shoaling is included implicitly. (If $K = 0$, the model satisfies conservation of energy, i.e., Green's law.)

ANALYTICAL SOLUTIONS

The problem to be addressed ultimately includes set-up, bottom friction, and beach profiles of irregular shape and subsequently must be solved numerically. However, closed form solutions which exist for the simpler case of breaking on beaches of more idealized shapes, without including set-up, are both enlightening and potentially valuable for future analytical work with wave-induced currents and sediment transport. Equation (5) can be rewritten as

$$\frac{\partial G}{\partial x} + \frac{K}{h'} G = K\Gamma^2 h'^{3/2} \tag{6}$$

where

$$G = H^2(h')^{1/2} \tag{7}$$

The general solution to the differential equation

$$\frac{\partial G}{\partial x} + P(x)G = Q(x) \tag{8}$$

is given by

$$G \exp\left(\left(\int P\,dx\right) = \int Q \exp\left(\int P\,dx\right)dx + C \tag{9}$$

and by comparison of (8) to (6) we see that in our case,

$$P(x) = K(h')^{-1} \qquad Q(x) = K\Gamma^2(h')^{3/2} \tag{10}$$

and

$$\int P\,dx = K \int (h')^{-1}\,dx \tag{11}$$

$$\int Q \exp\left(\int P\,dx\right)dx = K\Gamma^2 \int (h')^{3/2} \exp\left[K \int (h')^{-1}\,dx\right]dx \tag{12}$$

Uniform Depth

For the idealized beach with a horizontal bottom described in the previous section given by

$$h'(x) = \text{const} = h' \tag{13}$$

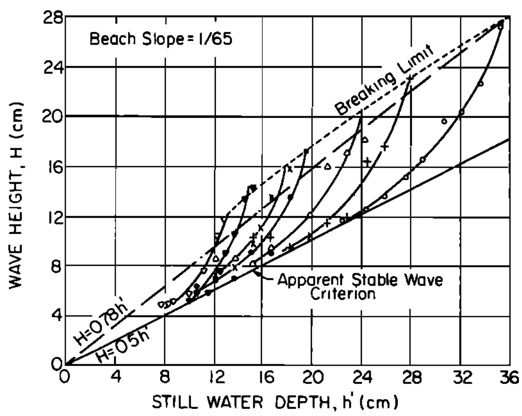

Fig. 3. Experimental results as presented by *Horikawa and Kuo* [1966] for waves breaking on a plane slope of 1/65. All waves tend to approach the stable wave criterion $H = 0.5h'$.

(11) is found to be equal to

$$\frac{K}{h'} x \tag{14}$$

and (12) is found to be equal to

$$\Gamma^2(h')^{5/2} \exp(Kx/h') \tag{15}$$

and from (9),

$$G = \left[\exp\left(-K\frac{x}{h'}\right)\right]\left[\Gamma^2(h')^{5/2}\exp\left(K\frac{x}{h'}\right) + C\right] \tag{16}$$

Applying the initial condition

$$G = G_b = H_b{}^2(h')^{1/2} \qquad x = 0 \tag{17}$$

to evaluate the constant C, the decay in dimensionless wave height in water of uniform depth is

$$\frac{H}{h'} = \left\{\left[\left(\left(\frac{H}{h'}\right)_b^2 - \Gamma^2\right)\exp\left(-K\frac{x}{h'}\right)\right] + \Gamma^2\right\}^{1/2} \tag{18}$$

where the subscript b denotes conditions at incipient breaking and x has its origin at the breaker line and is directed onshore. This expression dictates that the energy flux decays exponentially across the surf zone, never quite reaching the stable wave state known to exist. However, (18) may still be valid because internal and bottom friction losses could be accountable for the last bit of energy dissipation required to reach the stable condition. Note that if $K = 0$ (no breaking), the wave height remains constant as would be expected. Equation (18) is plotted in Figure 2 with $K = 0.2$, $\Gamma = 0.35$, and $(H/h')_b = 0.8$.

Uniform Slope

The same procedure is applied to determine the analytical solution for the breaker model on a plane beach given by

$$h'(x) = h_b' - mx \tag{19}$$

where m is the beach slope. Equation (11) is evaluated as equal to

$$-\frac{K}{m}\ln(h_b' - mx) \tag{20}$$

and (12) is equal to

$$-\frac{K}{m}\frac{\Gamma^2}{(5/2 - K/m)}(h_b' - mx)^{(5/2 - K/m)} \tag{21}$$

Inserting these expressions into (9), applying the initial condition

$$G = G_b = H_b'(h_b')^{1/2} \qquad x = 0 \tag{22}$$

and casting the result into dimensionless form yields

$$\frac{H}{H_b} = \left[\left(\frac{h'}{h_b'}\right)^{(K/m - 1/2)}(1 + \alpha) - \alpha\left(\frac{h'}{h_b'}\right)^2\right]^{1/2} \tag{23}$$

where

$$\alpha = \frac{K\Gamma^2}{m(5/2 - K/m)}\left(\frac{h'}{H}\right)_b^2 \tag{24}$$

Note that the solution is invalid if $K/m = 5/2$. For this special case, (11) becomes equal to

$$-5/2 \ln(h_b' - mx) \tag{25}$$

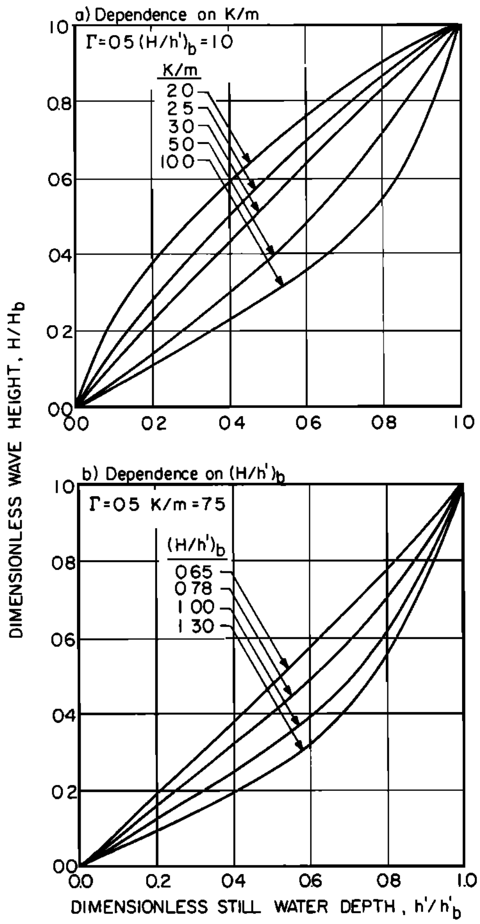

Fig. 4. Dependence of analytical solution (23) and (27) on K/m and $(H/h')_b$ for waves breaking on a plane beach. (H/h') tends to increase for increasing beach slope and increasing incipient condition (decreasing wave steepness).

and (12) becomes equal to

$$-5/2\ \Gamma^2 \ln\,(h_b' - mx) \tag{26}$$

and the solution is

$$\frac{H}{H_b} = \left(\frac{h'}{h_b'}\right)\left[1 - \beta \ln\left(\frac{h'}{h_b'}\right)\right]^{1/2} \tag{27}$$

where

$$\beta = \frac{5}{2}\Gamma^2\left(\frac{h'}{H}\right)_b^2 \tag{28}$$

Also, note that if K is set equal to zero, (23) becomes

$$\frac{H}{H_b} = \left(\frac{h_b'}{h'}\right)^{1/4} \tag{29}$$

which is simply Green's law. If $\alpha = -1.0$, (23) reverts to the common similarity model $H \sim h'$. Equations (23) and (27) are plotted in Figures 4*a* and 4*b* for several values of K/m and $(H/h')_b$. Figure 5 compares (23) to the data presented by *Horikawa and Kuo* [1966]. Here $K = 0.17$ and $\Gamma = 0.5$, which are the recommended values for use with the "still water model" on a plane beach of slope less than approximately 1/20. Note that the dependence of breaker decay on beach slope appears explicitly in (23) once the coefficient K is chosen and, as Figures 4*a* and 5 indicate, has the correct trend of increasing (H/h') as beach slope increases. Any further judgment as to the correctness of this dependence must wait until set-up is included in the model formulation. The role of wave steepness is contained implicitly in the solution through the specification of the incipient condition. Waves of low steepness have greater values of $(H/h')_b$ [*Weggel*, 1972] and maintain higher values of (H/h') farther into the surf zone. Converting Figure 4*b* to dimensional form demonstrates this, as will the results presented later.

"Equilibrium" Beach Profile

The final profile shape to be solved in closed form is the shape which seems to best represent "equilibrium" beach profiles as determined by *Dean* [1977] and is expressed by

$$h'(x) = A(L - x)^{2/3} \tag{30}$$

where A is a parameter dependent on fluid and sediment characteristics, L is the distance from the still water line to the breaker line, and the origin of x remains at the breaker line directed onshore.

The integration of P is straightforward and (11) is equal to

$$-\frac{3K}{A}(L - x)^{1/3} \tag{31}$$

while to integrate (12), the substitution is made

$$u = (L - x)^{1/3} \tag{32}$$

and the expression is integrated by parts which yields (12) equal to

$$-3K\Gamma^2 A^{3/2} \exp - \left(\frac{3K}{A}u\right) S(u) \tag{33}$$

where

$$S(u) = \sum_{n=0}^{5} (-1)^n \frac{5!u^{(5-n)}}{(5-n)!(-3K/A)^{n+1}} \tag{34}$$

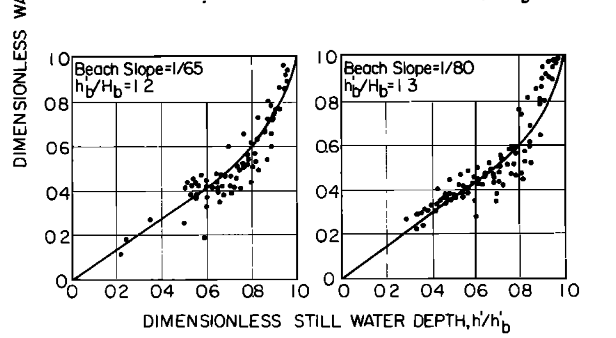

Fig. 5. Comparison of analytical solution (23) to wave decay data on a plane beach as presented by *Horikawa and Kuo* [1966] for various beach slopes ($K = 0.17$, $\Gamma = 0.5$). Set-up effects are distinguishable on the steeper beaches.

Utilizing (31) and (33) in (9)

$$G = -3K\Gamma^2 A^{3/2} S(u) + C \exp\left(\frac{3K}{A} u\right) \tag{35}$$

and applying the initial condition

$$G = G_b = H_b^2 (h_b')^{1/2} \qquad u = L^{1/3} \qquad x = 0 \tag{36}$$

to evaluate the constant, and the decay in wave height due to breaking on an equilibrium beach profile in dimensionless form is

$$\frac{H}{H_b} = \left\{ \left[-\Gamma^2 \left(\frac{H}{h'}\right)_b^{-2} 120 \sum_{n=0}^{5} \left(\chi - \left(\frac{h'}{h_b'}\right)^{[(4-n)/2]} \right) \cdot \frac{\phi^n}{(5-n)!} \right] + \chi \right\}^{1/2} \tag{37}$$

where

$$\chi = \left(\frac{h'}{h_b'}\right)^{-1/2} \exp\left\{ \frac{1}{\phi} \left[\left(\frac{h'}{h_b'}\right)^{1/2} - 1 \right] \right\} \tag{38}$$

and ϕ is a similarity parameter given by

$$\phi = A/3KL^{1/3} \tag{39}$$

The effects of the incipient conditions and the parameter ϕ on breaker decay on an equilibrium beach profile given by (37) are shown in Figure 6.

SET-UP AND BOTTOM FRICTION

During initial examination of the complete raw data set collected by Horikawa and Kuo it was noticed that in all cases where measurements were taken in the inner portion of the surf zone, as the still water depth approached zero, the wave height did not. This may also be apparent from Figure 5. To model this phenomenon better, including wave-induced set-up and set-down of the mean water level is necessary: the same conclusion reached originally by *Hwang and Divoky* [1970]. From *Longuet-Higgins and Stewart* [1963] the slope of the mean water level $\bar{\eta}$ is given by

$$\frac{\partial \bar{\eta}}{\partial x} = \frac{-1}{\rho g (h' + \bar{\eta})} \frac{\partial S_{xx}}{\partial x} \tag{40}$$

where the onshore momentum flux S_{xx} in shallow water is

$$S_{xx} = \frac{3}{16} \rho g H^2 \tag{41}$$

Equation (40) becomes

$$\frac{\partial \bar{\eta}}{\partial x} = \frac{-3}{16} \frac{1}{(h' + \bar{\eta})} \frac{\partial H^2}{\partial x} \tag{42}$$

and can be used in conjunction with a slightly different form of (3):

$$\frac{\partial ECg}{\partial x} = \frac{-K}{h} [ECg - ECg_s] \tag{43}$$

and (5)

$$\frac{\partial [H^2 (h)^{1/2}]}{\partial x} = \frac{-K}{h} [H^2 (h)^{1/2} - \Gamma^2 (h)^{5/2}] \tag{44}$$

in which h, the mean water depth given by

$$h = h' + \bar{\eta} \tag{45}$$

has replaced the still water depth h'.

Energy dissipation due to bottom friction can be incorporated in an elementary form for completeness. The average rate of energy dissipation per plan area due to bottom friction for shallow water [see *Putnam and Johnson*, 1949] is expressed by

$$\delta_{BF} = \rho \frac{f H^3}{12\pi} \left(\frac{g}{h}\right)^{3/2} \tag{46}$$

where f is a drag coefficient dependent on flow and bottom/sediment characteristics and shallow water linear wave theory has been applied. (The bottom shear stress is defined as $\tau = \rho (f/2) u |u|$).

NUMERICAL SOLUTION

Introducing set-up, beach profiles of more realistic shape, or bottom friction to the model renders the equations unsolvable analytically. Therefore a numerical scheme was developed which is capable of describing the one-dimensional transformation of wave height over bottoms of arbitrary shape due to shoaling, breaking, re-forming, and bottom friction, including the effects of set-up in mean water level. Equation (44) was finite differenced using a central average for each of the quantities on the right-hand side, and the set-up/set-down equation

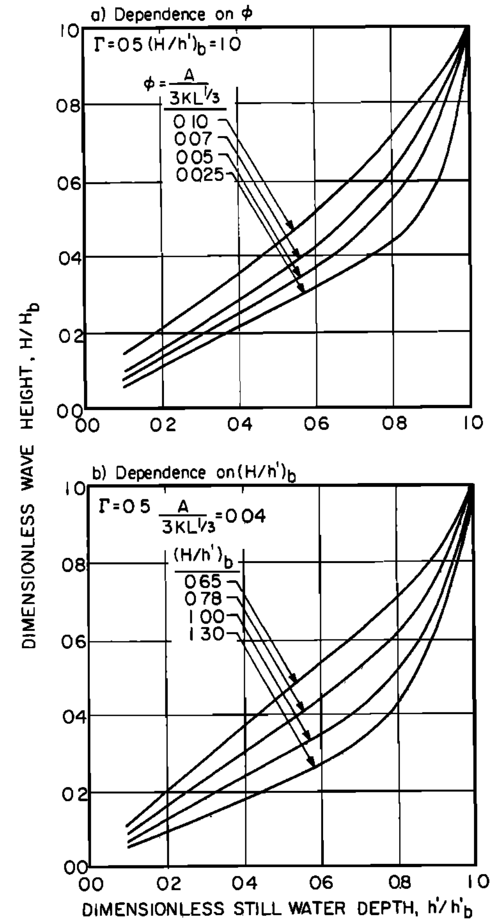

Fig. 6. Dependence of analytical solution (37) on similarity parameter ϕ and $(H/h')_b$ for waves breaking on an "equilibrium" beach profile.

TABLE 1. Breakdown of Data Used in Model Calibration

Slope	Number of Waves	Number of Data Points
1/80	56	500
1/65	12	96
1/30	17	173

(42) was treated in a similar manner. Because the mean water level at each spatial step is required but not known a priori, the program iterates until the updated value for the mean water depth is very close to the previous value. In the calibration runs it usually required only one or two iterations but never more than three for the difference in estimates to become less than a millimeter.

Early in this investigation the decay in wave energy due to bottom friction was included in the model in an uncoupled fashion. That is, after utilizing the scheme described, additional energy was then extracted using a finite-differenced form of (46) and energy flux considerations. As will be shown subsequently, for realistic values of the drag coefficient the energy dissipation due to bottom friction was found to be negligible for all cases examined, and this mechanism was therefore dropped from the model. However, it should be noted that bottom friction can be included in a directly coupled fashion by subtracting the centrally averaged form of (46) from the right-hand side of the finite-differenced (44).

To apply the model in a given situation, the following information is required: (1) the wave height and still water depth at a known nearshore location, (2) the wave height to water depth ratio at incipient breaking, (3) the bottom friction coefficient, and (4) the bottom profile. (Because the data used for calibration were collected on laboratory beaches of plane slope, only the incipient conditions and beach slope were required in this instance; however, items 1–4 are necessary for application of the model over more realistic bottom topography.) The breaking height to depth ratio is not easily predicted and was not treated in an extensive manner in this study. *Mallard* [1978] provides a more complete investigation. Assuming that the starting point is in shallow water and outside the surf zone, the set-down in mean water level, as given by *Longuet-Higgins and Stewart* [1963], is

$$\bar{\eta}_1 = -H_1^2/16h_1 \tag{47}$$

From these initial conditions the wave height will increase (with some losses due to bottom friction) as the wave moves shoreward until the incipient breaking criterion is reached. The wave then breaks to a location where local stability, as defined by (4), is achieved (if at all). On barred profiles the combination of the wave decay and the increasing water depth as the wave passes over the trough enable the wave to reach stability. The "re-formed" wave then shoals again until the breaking criterion is reached, and the process repeats until the mean water depth reaches an arbitrarily chosen small value (0.25 m is a reasonable choice at prototype scale).

CALIBRATION

The model is calibrated by determining the best values for the stable wave factor Γ and the wave decay factor K using a least squares procedure. The original raw laboratory data of waves breaking on plane slopes obtained and used by *Horikawa and Kuo* [1966] were examined. Starting at incipient breaking, they measured wave heights at known distances across the surf zone under monochromatic wave conditions for plane smooth rubber and concrete slopes of 1/20, 1/30, 1/65, and 1/80. The wave period varied from 1.2 to 2.3 s and the incipient breaker height from 7 to 27 cm. Although not specifically stated, the breakers must have spanned both the plunging and spilling types because the ratio of wave height to water depth at incipient breaking ranged from 0.63 to 1.67. The number of waves and the number of data points for each slope analyzed are presented in Table 1.

Data from the 1/20 slope were not included in the calibration because the measurements were taken too far apart for the model to remain numerically stable. The error function to be minimized is defined by

$$\varepsilon(\Gamma, K) = \left\{\left[\sum_{j=1}^{N} (H_{p_j}(\Gamma, K) - H_{m_j})^2\right]\left(\sum_{j=1}^{N} H_{m_j}^2\right)^{-1}\right\}^{1/2} \tag{48}$$

where H_{m_j} is the measured wave height, H_{p_j} is the wave height at that location as predicted by the numerical scheme for given incipient conditions and values of Γ and K, and N is the number of data points analyzed. An attempt to best fit Γ and K was made using a nonlinear, least squares error analysis. In effect, this method involves choosing initial values for Γ and K, fitting a parabolic surface to the error surface at that point, locating the minimum of the fitted surface, and using these new values of Γ and K for the next attempt. In this manner the procedure theoretically converges on the minimum of the error surface. However, this method was unsuccessful apparently because the error surfaces are too highly nonlinear; that is, a paraboloid is a bad approximation of the error surface. By calculating the error at regular intervals of K and Γ a discretized error surface can be generated whose low point occurs near the best fit values for the factors. Figure 7 displays a contour plot of the error surface for the data taken on the 1/65 slope. The surfaces for all three slopes were found to have recurved shapes with the 1/65 and 1/80 surfaces also containing saddle points. A one-wave, 16-point artificial "data" set was generated using the model on a 1/80 slope with Γ set equal to 0.35 and K equal to 0.10 (the best fit values from the real 1/80 data set). The corresponding error surface and contour plot were generated and found to have the same general shape and saddle point as the real data set. One can therefore conclude that the shape and saddle point are characteristics of

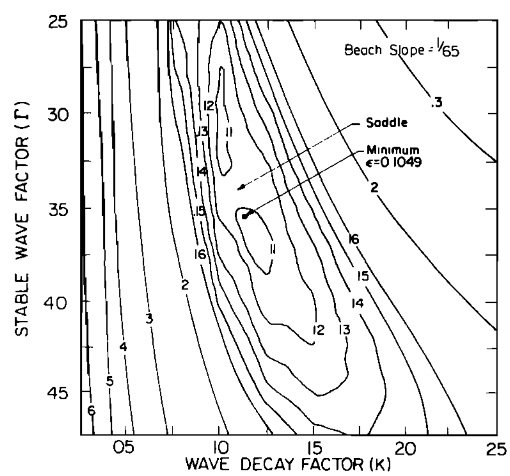

Fig. 7. Contour plot of error (48) between numerical solution and laboratory data as a function of wave decay factor K and stable wave factor Γ for 1/65 beach slope. Note that the surface is broad and flat around the minimum. Saddle feature is a characteristic of the model.

the model and are not due to problems with the data set. The best fit values for Γ and K for the three slopes analyzed are presented in Table 2.

The best fit values for the two factors do vary with beach slope, especially as the beach gets very steep. However, it would be preferable to choose single values for Γ and K which give satisfactory results for all beach slopes, allowing the model to be used on beach profiles of more realistic shape. Fortunately, the error surfaces for the three slopes tested are relatively broad and flat in the vicinity of their minimums, so the factors can be changed somewhat without excessively increasing the combined error. The procedure followed was to superimpose the contour plots for each of the three slopes and determine the location where the sum of the three error values is minimized. This point occurs where $\Gamma = 0.40$ and $K = 0.15$ (mean error is 0.1423), and it is recommended that these values be used in situations where the bottom slope varies over a wide range. If the beach is nearly planar, the values from Table 2 may be used accordingly.

The breaker model was not calibrated to prototype conditions due to a lack of suitable data. It would require the measurement of the breaker height distribution across the surf zone under truly monochromatic conditions. The waves should be normally incident to the bottom contours, and the beach profile must be known and preferably monotonic. A large wave tank with a regular wave generator would be ideal for this type of experiment. Calibration of the model in its present form using irregular waves would require following each individual breaker as it travels across the surf zone (hoping wave-wave interaction was not significant). Because the bubbles which dissipate the energy of a breaking wave are not scaled down properly in the laboratory, it is hypothesized that K will assume a lesser value under prototype conditions. It is unclear what change, if any, would occur in Γ.

RESULTS AND DISCUSSION

Laboratory Conditions

Wave height. Figure 8 displays a representative sample of model-predicted breaker decay as compared to the aforementioned laboratory data for plane beaches of 1/20, 1/30, 1/65, and 1/80 slopes. They are dimensional plots of wave height versus still water depth. In all cases the wave decay factor K was set equal to 0.15, and the stable wave factor Γ was taken to be 0.40. Bottom friction was considered negligible. Each curve was generated by specifying the wave height and still water depth at incipient breaking as given in the data and calculating stepwise ($\Delta x = 1/m$ cm) the set-up and decay profiles. It is not practical to display the data and model results in dimensionless form owing to the dependency of the shape of the decay profile on the incipient conditions as well as the set-up. Also, set-up measurements were not included in the Horikawa and Kuo experiments. Additional comparisons to this data set are available in the work by *Dally et al.* [1985].

Examination of these results shows that the model developed in this study appears to provide a good representation of breaking wave decay on plane beaches of laboratory scale. It is important to note that the model is in good quantitative agreement over the wide range of slopes tested (including the 1/20 slope not involved in the calibration) even with the two factors held constant at values that are not necessarily the best fit values for each particular slope. This is a result of the broad nature of the error surface in the vicinity of their minima, so that varying the empirical factors does not significantly affect the accuracy.

TABLE 2. Best Fit Values for Γ and K

Slope	Γ	K	Minimum Error
1/80	0.350	0.100	0.1298
1/65	0.355	0.115	0.1054
1/30	0.475	0.275	0.1165

The line $H = 0.78h'$ is plotted in each of the figures and appears to be a reasonable description of breaker decay only for the 1/30 slope. In fact, the similarity model ($H \sim h'$) so prevalent in the coastal literature seems to be approximately valid only for beaches of much greater slope than those commonly found in nature. As noted previously, the trend of the entire decay profile as well as the slope of the line approached asymptotically by the data increase with increasing beach slope. The model appears to describe this phenomena quite satisfactorily, and it can now be concluded with greater assurance that the general model given by (3) and the analytical solution (23) contain the correct dependence of breaker decay on beach slope. The wave height is determined by the sum of two opposing factors: the loss of wave height due to energy dissipation and the increase in wave height due to shoaling. If the beach slope is zero (no shoaling), the wave approaches the $H = 0.4h$ asymptote. Increasing the beach slope increases the effect of shoaling, resulting in an increase in the wave heights and the slope of the asymptote.

One might question the intersection of several of the measured decay profiles for waves with similar incipient breaking height to depth ratios but different incipient breaker heights. This phenomenon is also apparent in the profiles produced by the model. The greater total set-up associated with an initially larger wave [see *Bowen*, 1969] provides a greater mean water depth and so a larger stable wave height in the inner surf zone than a smaller wave. At this point the larger wave decays less rapidly, causing the decay profiles to intersect.

Characteristically, waves of the plunging type dissipate a large portion of their energy in a concentrated manner in the region just shoreward of the breaker line, while spilling breakers dissipate their energy at a slower rate. At incipient breaking, the height to depth ratio is usually greater than 1.0 for plunging waves and 0.78 or less for spilling. Figure 9 presents predicted wave height decay curves for two waves with the same initial breaking depth but different wave heights. The larger wave initially decays at a much greater rate than the smaller, and it appears that the model is at least qualitatively correct in dealing with the descriptive terms "plunging" and "spilling."

The effects of wave period and steepness are also apparent in Figures 8 and 9. Although the range in wave period in the data is limited (1.2–2.26 s), it appears that wave period and steepness are not primary factors in wave decay after breaking is initiated. Intuitively, this makes sense because breaking is a phenomenon restricted only to a portion of the wave crest. However, as mentioned before, steepness (which contains wave period) combined with beach slope determines the incipient breaking height to depth ratio [*Weggel*, 1972] and therefore affects decay through the initial condition. The upper curve in Figure 9 corresponds to a wave of low steepness and maintains a greater value of (H/h') aross the surf zone than the wave of higher steepness.

Figure 10 demonstrates the negligible effect bottom friction losses have on the wave decay profile when compared to

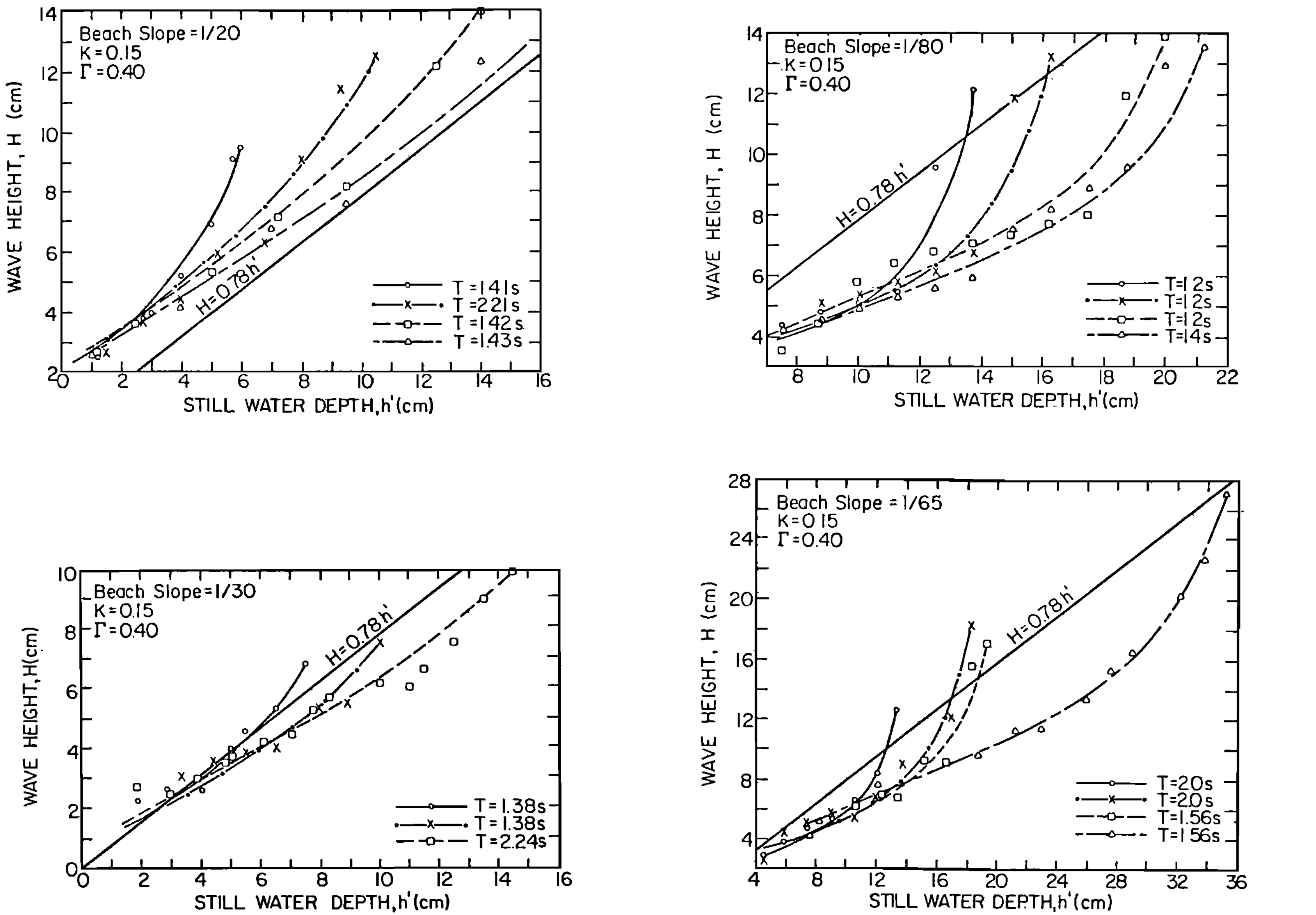

Fig. 8. Comparison of breaker model and "0.78" criterion to Horikawa and Kuo's laboratory data for 1/20, 1/30, 1/65, and 1/80 beach slope. Trend of (H/h') increases with increasing beach slope and increasing incipient condition (decreasing wave steepness).

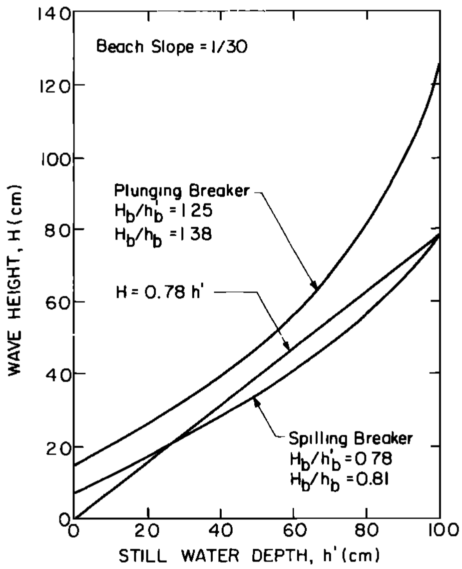

Fig. 9. Comparison of model-predicted wave decay for spilling and plunging breakers. Increasing $(H/h')_b$ (decreasing wave steepness) increases trend in (H/h').

breaking. The upper curve is a test case of the model without bottom friction. The lower curve was generated with the same conditions, except that greatly exaggerated bottom friction losses were included using the uncoupled scheme. The bottom drag coefficient f was set equal to a value (0.1) about 2 orders of magnitude greater than is realistic for the smooth rubber and concrete slopes used by Horikawa and Kuo in order for the two curves to be distinguishable from each other.

Set-down/set-up. The Horikawa and Kuo data set does not include measurements of set-down/set-up in mean water level and so data presented by *Bowen et al.* [1968] and *Bowen* [1969] were examined. Wave height and mean water level measurements were made on a relatively steep plane beach of 1/12 slope, and the results of one test are presented in Figure 11 along with decay and set-down/set-up as predicted by the model. As can be seen in Figure 11, the overall best fit values for K and Γ, as determined for the 1/80, 1/65, and 1/30 slopes and verified for the 1/20 slope ($K = 0.15$, $\Gamma = 0.40$), do not perform well on the unrealistically steep beach slope of 1/12. To evaluate better the model's capability for describing wave-

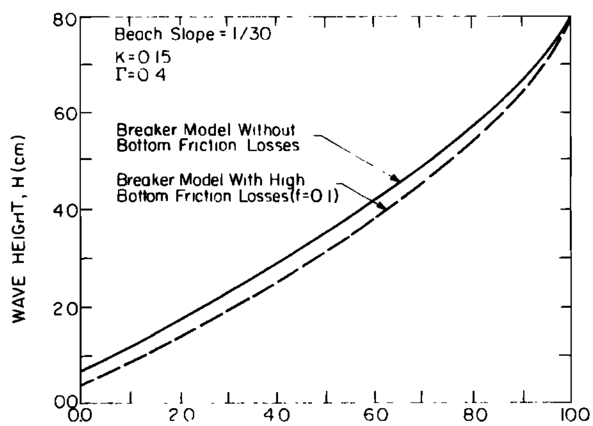

Fig. 10. Comparison of model-predicted wave decay with and without bottom friction losses. Bottom friction is negligible in the surf zone in most situations.

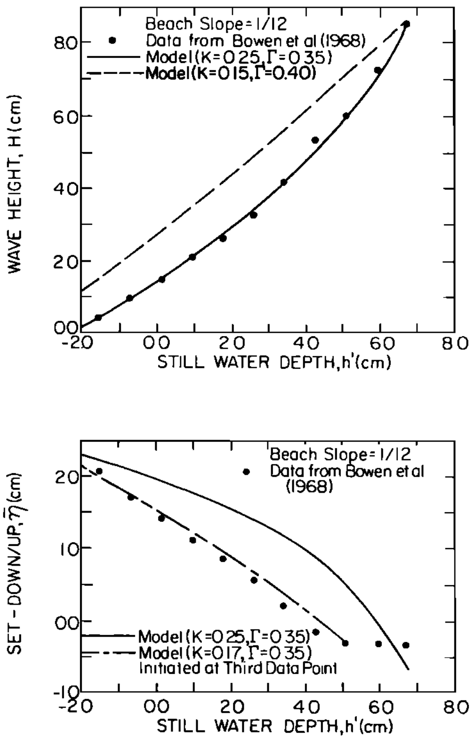

Fig. 11. Model-predicted wave decay and set-down/set-up as compared to laboratory data from *Bowen et al.* [1968]. Set-up comparison is substantially improved if decay and set-up are initiated at values of third data point as per *Svendsen* [1984].

induced set-up, K and Γ were changed to 0.25 and 0.35, respectively. The breaker decay now compares well, and the maximum set-up values are reasonable if the swash zone is neglected; however, the predicted set-down/set-up curves do not follow the data for two apparent reasons. First, (41) may not be a good representation of the onshore excess momentum flux for near-breaking and breaking conditions. Higher-order wave theories yield significantly less momentum flux for a given wave height than linear theory [see *Dean*, 1974; *Stive and Wind*, 1982], although *Svendsen* [1984] maintains that including the effects of a roller in the breaking wave representation again increases the momentum flux, and therefore the two effects may compensate each other. Second, as noted by *Bowen et al.* [1968], the measured set-down is nearly uniform for a distance after breaking is initiated, in this case up to the point where the curl of the plunging breaker touches down (the first three data points). Similar behavior was reported for random wave laboratory data by *Battjes* [1972] and *Battjes and Janssen* [1978]. It is interesting that even though the wave height is decreasing in this region, the momentum flux apparently is not. Perhaps this is because (strictly speaking) no energy is dissipated until the curl touches down and "white water" appears.

Since original submittal of this paper, additional observations and a similar explanation of this phenomenon were noted by *Battjes and Stive* [1985] for random waves and *Svendsen* [1984] for regular wave data. Both suggest the explanation that during initial breaking, the energy formerly represented by wave height is converted to organized and/or turbulent kinetic energy in the water column and not dissipated as rapidly as the decrease in wave height would dictate. This might cause the substantial lag in the initiation of set-up as compared to the initiation of breaking. *Svendsen* [1984]

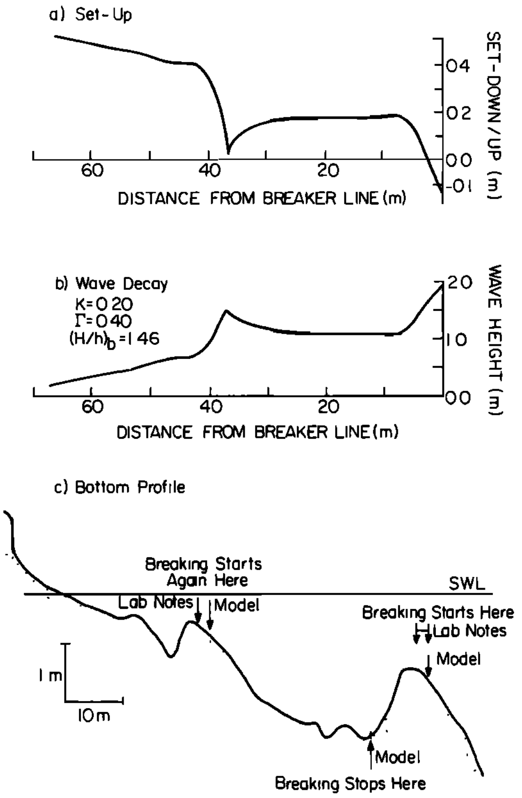

Fig. 12. Test of wave transformation model at prototype scale on large wave tank profile from *Saville* [1957]. Wave breaks, re-forms, shoals, and breaks again.

avoids this problem by somewhat artificially starting decay and set-up calculations landward of this region as dictated by the data for that particular experiment [*Svendsen*, 1984, Figures 14 and 15, pp. 323 and 324], and therefore is restricted to the inner portion of the surf zone. In fact, the case presented where set-up was permitted to start at incipient breaking [*Svendsen*, 1984, Figure 17, p. 325] has many of the same features as our comparison in Figure 11. Following Svendsen, if decay and set-up calculations are initiated at the values of the third data point with $K = 0.17$ and $\Gamma = 0.35$, decay almost identical to the curve shown and a more favorable comparison for set-up are produced (also displayed in Figure 11).

Large-Scale Conditions

Applying the results of the laboratory calibration to near-prototype conditions may be somewhat questionable because of expected scale effects in the turbulence associated with wave breaking, as previously discussed. However, in order to demonstrate use of the model for waves breaking on beach profiles containing bars and to lend some validity to the model for prototype situations, runs under large-scale conditions were carried out. Prototype scale beach profiles generated by *Saville* [1957] in the Beach Erosion Board large wave tank were utilized, and one is displayed in Figure 12*c*. The profile is characterized by two offshore bar/trough systems, along with a monotonic section in the nearshore region. Test conditions, although not completely documented, were taken from the lab notes to be wave period, $T = 3.75$ s; wave height at incipient breaking, $H_b = 1.83$ to 1.98 m; location of primary breaker line equal to 75.6 to 78.0 m from datum; location of secondary breaker line equal to 39.6 m from datum; and mean sediment diameter, $D = 0.2$ mm. These conditions placed the primary breaker line outside the crest of the first bar at a still water depth of 1.46 m. With an incipient breaker height of 1.91 m the set-down in mean water level given by (47) is -0.16 m, and the ratio of wave height to mean water depth at incipient breaking is 1.46. Γ was set equal to 0.4, but with $K = 0.15$, conditions would not permit the broken wave to re-form and break again as stated in the lab notes. It was necessary to increase K to a value of 0.2, which one may consider unfortunate in regards to our hopes of choosing single values for K and Γ. However, this increase is due to the extremely steep seaward faces of the bars (much steeper than those found in nature), and single best fit values as determined in the calibration may still be reasonably valid on realistic beach slopes. Following the procedure described by *Kamphuis* [1975] and assuming that the bottom was not rippled, the bottom friction factor f was found to be approximately 0.005. Initial runs showed that bottom friction caused decay only of the order of millimeters. Apparently, bottom friction plays an important role in wave decay only under nonbreaking conditions well outside the surf zone, and friction was again left out of the model for this test. Using these incipient conditions, the transformation of wave height was generated in a stepwise fashion across the entire surf zone ($\Delta x = 1.22$ m) until the mean water depth became less than 0.25 m. The distribution of model predicted set-down/set-up and wave height are shown in Figure 12*a* and 12*b*, respectively. Note that the wave reaches the stable criterion in the deepest portion of the outermost trough as might be expected, shoals on the inner bar until the incipient condition is again attained (at a location close to that quoted in the lab notes), and then breaks continuously until the shoreline is reached.

The results of the application of the model under large-scale conditions seem reasonably valid, at least in a qualitative sense. The example has demonstrated the ability of the model to describe wave breaking and re-forming, a commonly observed process on natural beaches. The predicted wave decay and set-down/set-up profiles are continuous and well behaved until the mean water depth becomes quite small ($h < 0.25$ m), where a swash zone model would be more appropriate.

SUMMARY AND CONCLUSIONS

On the basis of laboratory data collected by *Horikawa and Kuo* [1966] the parameters found to affect most the decay in wave height due to breaking in the surf zone are the beach slope and the ratio of wave height to water depth at incipient breaking. Wave-induced set-down/set-up in mean water level plays a smaller but by no means trivial role in governing the shape of the wave decay profile, especially near the still water line. The similarity model ($H \sim h'$) commonly used by the coastal profession appears to be reasonable only on steep beaches (1/20 to 1/30 at laboratory scale), and the "0.78" criterion predicts with marginal accuracy only for a 1/30 slope.

The model developed herein appears to describe qualitatively and quantitatively the wave transformation in the surf zone due to shoaling, breaking, and re-forming over a wide range of beach slopes (1/80 to 1/12) and incipient conditions ($0.63 \leq (H/h)_b \leq 1.67$). The analytical solutions which neglect set-up and bottom friction for the idealized profile shapes of uniform depth, uniform slope, and "equilibrium" beach profile provide valuable insight because the apparently correct dependence of breaker decay on beach slope appears explicitly, while the effect of wave steepness is contained implicitly through specification of the incipient conditions. The best fit values of the two assignable parameters in the model, Γ and K, were found to be relatively constant for beaches encompassing natural slope ranges (1/80 to 1/20). The greatest assets of the model are its simplicity and ease of application. Although it is most successful on profiles of monotonic shape, it is also employable when multiple bar/trough systems are present.

The model predicts maximum set-up values with reasonable accuracy for test cases presented by *Bowen* [1969]; however, it does not describe the distribution of set-down/set-up across the surf zone satisfactorily. This is attributed to the questionable assumption that onshore radiation stress can be described by using linear wave theory and the observed lag between incipient breaking and the initiation of set-up. More work is required in this area.

From calculations based on *Kamphuis* [1975] it can be concluded that bottom friction plays a negligible role in wave decay in the surf zone for most naturally occurring conditions when compared to the effects of breaking and shoaling. Bottom friction could be significant in nearshore regions that have very mild slopes or rough bottoms. This same conclusion was reached by *Thornton and Guza* [1983].

Of course, the model has certain limitations which might restrict its use. Reflection is not included, and although it appears valid for even 1/12 slopes, one must be careful when interpreting results for steep beaches. An elementary scheme utilizing reflection coefficients might be included. A breaking phenomenon not handled well by the model is wave "tripping" which commonly occurs when incipient breaking conditions are reached at or near the peak of a bar or submerged structure and the wave travels into the trough before breaking becomes fully developed. With the knowledge gained from this regular wave study, especially that pertaining to the effects of beach slope and wave steepness, the random wave problem can now be addressed with greater insight and confidence.

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