Freeman’s Centralities Summary

This is a summary of the exelent work of Linton C. Freeman in the article Centrality in Social Networks Conceptual Clarification, where he explains the 3 main centralities applied to social network analysis.

(a) Degree Centrality

The degree of a point p_i is simply the count of the number of other points, $p_j(i \neq j)$, that are adjacent to it and with which it is, therefore, in direct contact. [4]

(a.1) **Degree**

- According to [3].
- *“Count of the number of adjacencies for a point p_k”*
- Where $a(p_i, p_k) = 1$ if and only if p_i and p_k are connected by a line, 0 otherwise.

$$C_D(p_k) = \sum_{i=1}^{n} a(p_i, p_k)$$

(a.2) **Relative Degree Centrality**

- According to [4].
- For a non-reflexive undirected graph.
- Where $n - 1$ is the maximum of $C_D(p_k)$, in other words, the maximum amount of adjacent points a given point p_k can have.

$$C'_D(p_k) = \frac{\sum_{i=1}^{n} a(p_i, p_k)}{n-1}$$

(b) Betweenness Centrality

Betweeness is equal to the number of shortest paths from all vertices to all others that pass through that node.

(b.1) **Betweenness**

- According to [1] and [2].
- Where $\sigma(i, j)$ is the count of shortest chains between i and j.
- Where $\sigma(i, u, j)$ is the count of shortest chains between i and j passing by u.

$$\forall i \neq u \neq j, \sigma(i, u, j) < 0, Iu = \sum_{(i,j)} \frac{\sigma(i, u, j)}{\sigma(i, j)}$$

(b.2) **Potential Betweenness**

- According to [4].
- For $p_i \neq p_j$ and where there is at least 1 geodesic linking both points.
- Where g_{ij} is the number of geodesics linking p_i and p_j.
- The probability of using any one of those is:

$$\frac{1}{g_{ij}}$$

(b.3) **Potential of Control in Betweenness**

- According to [4].
- The potential of point p_k for control of information passing between p_i and p_j.
- Where $g_{ij}(p_k) =$ the number of geodesics linking p_i and p_j that contain p_k.

$$b_{ij}(p_k) = \frac{1}{g_{ij}} \times g_{ij}(p_k) = \frac{g_{ij}(p_k)}{g_{ij}}$$

Is the probability we seek; it is the probability that point p_k falls on a randomly selected geodesic linking p_i to p_j [4].

(b.4) **Betweenness Centrality**

- Same as (b.1) but explained with (b.2) and (b.3).
- *To determine the overall centrality of a point p_k, we sum its partial betweenness values for all unordered pairs of points* [4].
- Where $i \neq j \neq k$.
- Where $i < j$
- Where n is the number of points in the graph.

$$C_B(p_k) = \sum_{i}^{n} \sum_{j}^{n} b_{ij}(p_k)$$

(b.5) **Relative Betweenness Centrality**

- According to [4] the maximum value taken by $C_B(p_k)$ is achieved only by the central point in a star. It is:

$$\frac{n^2 - 3n + 2}{2}$$

- Therefore, the relative centrality of any point in the graph may be expressed as a ratio.

$$C'_B(p_k) = \frac{2C_B(p_k)}{n^2 - 3n + 2}$$

- Values of $C'_B(p_k)$ may be compared between graphs. A star or wheel, for example, of any size will have a center point with $C'_B(p_k) = 1$; all other points will yield $C'_B(p_k) = 0$.

(c) Closeness Centrality

Is the average size of geodesics of one node to the rest of the nodes in the graph.

(c.1) **Closeness**

- According to [4].
- Where $d(p_i, p_k) =$ the number of edges in the geodesic linking p_i and p_k.
- Where n is the number of points in the graph.

$$C_C(p_k) = \frac{\sum_{i=1}^{n} d(p_i, p_k)}{n - 1}$$

References

[1] L. Freeman, “A set of measures of centrality based on betweenness.” Sociometry, vol. 40, pp. 35–41, 1977.

[2] M. Newman, “A measure of betweenness centrality based on random walks,” Social Networks, vol. 27, no. 1, pp. 39 – 54, 2005.

[3] J. Nieminen, “On centrality in a graph” Scandinavian Journal of Psychology, 15:322-336, 1974.

[4] L. Freeman, “Centrality in Social Networks Conceptual Clarification” Social Networks, pp. 215-239, 1979.