Games of Decision
Open Research Draft

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***Abstract*—Collective decisions in intelligent multi-agent systems are done every day on different domains, choosing new laws, accepting github pull requests, doing business contracts or choosing family vacations location; normally a decision system is set before taking such decisions: democracy, autocracy or something in between. They all are important for the multi-agent system in order to progress, and also if abstracted they all have common parameters, which when changed the whole decision system changes. This open research draft proposes such abstract model, enlists some of its applications, shows how it can model some popular political systems and proposes several new decision systems. Also research opportunities are exposed for future work.**

Keywords*—*multi-agent, decision, system, model, influence.

I. Introduction

Decision systems refer to the broad concept of a community of agents that face the necessity of collectively decide over an issue. How can they self organize in order to achieve such task? How can a notion of "justice" be preserved? What is the most efficient method? How can it be automated? All of these are questions that this research line tries to answer.

The most obvious domain for decision systems is government and political systems. New political systems have been proposed like [2]. Also [3] tries to adapt system theory to political science. But both efforts lack mathematical rigor and a true abstract model for decision systems.

Crowdsourced public project planning is another domain example, in [1] it is argued the medium of the Web as an appropriate technology for harnessing widely distributed talent and contributions, but still the lack of a mathematical model impedes further decision system design and development.

The field that fits our research is game theory; the audience and applications for game theory has grown dramatically in recent years, and now spans disciplines as diverse as political sciences, biology, psychology economics, linguistics, sociology and computer science, among others [6].

A similar subfield to our research is decision theory; decision theory is the analysis of the behavior of an individual (agent) facing nonstrategic uncertainty [4], but the difference with this research line is that we are modeling collective decision making (not individual) in a multi-agent environments, even some times independently of agent utility.

The rest of the draft will continue with the proposal of a formal mathematical model for multi-agent collective decision making, then validating it by modeling well known political systems, also new decision systems are proposed based on the model, and finally future research opportunities are enlisted.

II. A basic abstract model

One could define a decision instance as a set of mutually exclusive events, which can be voted by the members of an intelligent multi-agent community, and after the decision process involving agent votes, the elected event is considered the resolved event. Then a game emerges by letting the agents be the players, and where each agent receives utility depending on the event which resolves in the decision. Since utility does not change depending on what other agents vote on, we can create a modified version of a Normal Form Game where each row represents an agent and each column an event from the decision instance, the intersections at cell (x, y) represent the utility obtained by agent x when event y passes. More formally:

- *N is a finite set of n agent, indexed by i;*
- *$A = N \times E$, where E is a finite set of events available to each agent. Each tuple $a = (i, e) \in A$ is called a vote;*
- *$u = (u_1, \ldots, u_n)$, where $u_i : A \mapsto \mathbb{R}$ is a real-valued utility (or payoff) function for agent i.*

This formalization abstracts the concept of a decision game, but we need to expand it in order to model decision resolution. In a decision system a decision resolves depending on two factors, 1) the influence of each member, which represents the power or counting voice measure of the agent, and 2) a function that determines which final event resolves after influence has been accumulated on the different possible events when agents vote. Adding these two abstractions we obtain a complete model for decision systems:

Definition 1.0 (Normal form decision game). *A (finite, n-person) normal-form decision game is a tuple $(N, A, u, \alpha, \delta)$, where:*

- *N is a finite set of n agents, indexed by i;*
- *$A = N \times E$, where E is a finite set of events available to each agent, indexed by m. Each tuple $a = (i, e) \in A$ is called a vote, and when a game has been resolved $a_i = e$ is known as the vote of agent i;*
- *$u = (u_1, \ldots, u_n)$, where $u_i : A \mapsto \mathbb{R}$ is a real-valued utility (or payoff) function for agent i.*
- *$\alpha : N \mapsto \mathbb{R}$ is a real-valued function that assigns an initial influence value to every agent.*
- *$\delta : E_{inf} \mapsto E$, where each $e_{inf} = (s_1, \ldots, s_m) \in E_{inf}$ is a tuple with possible summation of influence s_j for each event e_j after agents vote; α is the mapping from the accumulated influence on each event to the final resolved event of the decision instance.*

In the future, we will refer to N, A and u as the *context of the decision* and to α and δ as the *decision system*. Also we will call α the influence allocation function and δ the resolution function.

A. A decision game: the battle of the sexes

A well-known simple game in Game Theory is the battle of the sexes, the metaphor behind the game goes like this: a husband and wife wish to go to the movies, and they can select among two movies: "Lethal Weapon (LW)" and "Wondrous Love (WL)". They much prefer to go together rather than to separate movies, but while the wife (p_1) prefers LW, the husband (p_2) being the romantic that he is, he prefers WL. The normal-form game matrix looks like the following:

	LW	WL
LW	2, 1	0, 0
WL	0, 0	1, 2

But this game alone does not reflect the collective decision that the wife and husband are making together, neither does the system that they will use to decide. But by applying our proposed model we can achieve such task.

Let the woman be the dominant in the relationship, so $\alpha(p_1) = 2$ and $\alpha(p_2) = 1$, and the decision system a simple one: the event which accumulates more influence will be the one that resolves, so

$$\delta((LW_{inf}, WL_{inf})) = max_e(LW_{inf}, WL_{inf})$$

where max_e returns the event with most influence collected. Then the normal-form decision game matrix is as follows:

	LW	WL
p_1: 2	2	1
p_2: 1	1	2

We added in the players column the influence of each player, the first row reads "p_1: 2" because the woman has 2 of influence value. Being δ as it is and assuming that both players are rational (they will vote for the event that yields the most utility), then it is evident that Lethal Weapon will be the event that resolves, because then:

$$A = \{(1, LW), (2, WL)\}$$

and

$$\delta((LW_{inf}, WL_{inf})) = \delta((2, 1)) = max_e(2, 1) = LW$$

III. Applications: modeling political systems

One of the most important applications of decision systems in human communities is government, in order to model a decision system using our model we just need to define α and δ; in this section we will use our decision systems abstraction to model three well known political systems: autocracy, democracy and representative democracy.

A. Autocracy

An autocracy is a system of government in which supreme power is concentrated in the hands of one person [5]. This becomes fairly straightforward to model by letting the influence allocation be $\alpha(i) = 0$ for any i different to the autocracy dictator, and $\alpha(dictator) = 1$. For the resolution function we just return the event that accumulated 1 of influence, which will be the one that the dictator chose, because it is the only agent with influence more than 0.

B. Democracy

The term originates from the Greek (dmokrata) "rule of the people" [7]. This means that every agent will be allocated the same positive non-zero influence value, so for any decision instance using a democratic system:

$$\alpha(i) = a$$

where $a > 0$. For decision resolution we use the max_e function we used in the last section, which effectively yields the event with most influence accumulated, so

$$\delta(E_{inf}) = max_e(E_{inf})$$

C. Representative Democracy

Representative democracy is a variety of democracy founded on the principle of elected officials representing the whole community, officials are elected through a democratic collective decision of the community, and then officials decide further decision instances using similar democratic systems between them. This encourages the modeling of decision instance composition and instance dependency.

Lets divide the representative democracy system as a 2-phased system, phase 1) elects officials, and phase 2) resolves further decision instances using elected officials. As one may observe all instances on phase 2 depend on phase 1, because the decision context of phase 2 instances are limited by the resolution of phase 1. In this specific case phase 1 subsets the community, and further instances in phase 2 use such subset instead of the original community.

More formally, let phase 1 be the decision game $P_1 = (N, A, u, \alpha, \delta)$, let C be the set of candidates, A some subset of the powerset of candidates $A \subset \mathcal{P}(C)$, and α and δ the democratic system already defined. Then the democratically elected officials will be $\delta(A_{inf})$, and any decision game $(M, B, v, \alpha', \delta')$ will be a democratic representative decision game of P_1 iff $M = \delta(A_{inf})$.

IV. Applications: modeling new systems

In the last section we saw how we could use decision games to mathematically understand better existent political systems, but another obtained advantage is to further design and develop new decision systems for various domains and applications, which should exceed "vanilla" systems in complexity, correctness and effectiveness. The leverage and opportunity that this brings is:

- **Framework**: The decision games model gives the parts that can be interchanged to obtain different decision systems, better understand the domain, and ultimately solve the issue that begs for the decision instance. The *decision context* is well defined by a normal-form game, and the *decision system* is well modeled by α and δ. A decision system can be designed by implementation of its influence allocation and its resolution system.
- **Specialization**: The framework parts allows for the design of specialized systems that fit well the domain and decision instance type.
- **Modularity and composition**: As seen in the last section, decision instances can be composed to create a dependency graph, creating complex but powerful decision flows. One important product of these is the concept of meta-decision systems, which are decision systems to decide decision systems; when a new issue arrives a community can first decide on what system they should use to resolve the instance, or decide on systems that will be used when types of instances appear.
- **Automation**: Once we have the framework, issue specialization, and composition of decision systems, the integration of information technologies can be used to automate decision flows, and enable agile and distributed decision making.

Some examples of decision system design reflecting these 4 properties will be exposed in the next subsections.

A. Geolocation based influence allocation

When issues are geolocated, agents near the issue may need more influence when voting for a decision instance. A strategy for this types of instances would be to allocate the influence over the community as function of the distance between the issue center and the agents location. Linear, logarithmic or exponential functions can be used.

B. Reputation based influence allocation

Domains like open software development may want to automate repository contributions without the need of a centralized control of the repository, then a distributed decision system for pull requests acceptance and issue control systems can be implemented by allocating influence based on a reputation function, reputation features can be extracted from systems like social networks, for example GitHub or Stack Overflow.

C. Low inertia resolution

As suggested by Elon Musk in [8], a good idea when creating laws may be: letting to remove laws be easier than creating them. To model this we will define two types of decision instances, law creation and law removal, and every decision instance to be binary, which means the set of possible events contains just 2 events, decision resolves positive or negative; for law creation this means law is created or is dropped, and for law removal it means the law is removed or prevails. Now for any law creation instance, δ will resolve positive if more than 60% of the sum of all influence votes positive, and for any law removal instance, δ resolves positive if more than 40% of the sum of all influence votes positive.

D. Agent attribute based influence allocation

In 2016 one of the biggest controversial political decisions was the election of the Brexit (Great Britain decision to exit the European Union), the used decision system was a pure/direct democratic system and the final result was 48.1% stay, 51.9% leave; a lot of the controversy comes from questioning the democratic system, specially when most of the voters in favor of staying were young people and most of the voters in favor of leaving were old, and then comes the argument that young people are the most affected by the decision, and that 17 or less years old did not have any influence and voting rights, making them the most affected and suffering from a type of social injustice.

Assume the last paragraph is right, a possible solution to social justice for similar instances would be to allocate influence based on how much the decision affects the agent, in this case features like age group could be factors for such allocation.

E. Competence based influence allocation

For sensible decisions like urban development (building bridges and designing the city layout), decision making can be automated by dynamically allocating influence based on a competence based system. Agents can achieve better ranking on the competence system and automatically gain influence to decide over the mentioned decision types.

V. Future Research

As of this draft, a list of possible research topics are:

- Study cases for geolocation based influence allocation.
- Study cases for reputation based influence allocation.
- Study cases for low inertia resolution.
- Study cases for agent attribute based influence allocation.
- Study cases for competence based influence allocation.
- Design and proposal of new decision systems.
- Study of decision instances and decision systems composition.
- Agent believes, study of the game an agent may be thinking he is playing.
- Ideologies, culture and corruption, and how they affect resolution.
- Ideologies, culture and corruption, and how they emerge from different decision systems and game mechanism.

VI. Conclusion

Decision models and systems are of core importance for humanity, we have been developing them since we first started living in community. In order to further understand this phenomena and develop the systems that will dictate our future, we need to mathematically model and understand them, so that automated and powerful systems are conceived. This research draft has proposed such model, and then proved its utility by modeling some political systems and also by proposing decision systems that address some domain specific issues.

Also a list of further research topics is proposed and exposed to the scientific and engineering community.

Humanity as the writing of this text is confronting several alarming challenges, and given the critical mass that population has reached, modern systems must be proposed and applied. This research hopes only for the best, for the future of the human as species, and for the survival of life in the universe.

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